

Explaining What Leads Up to Stock Market Crashes: A Phase Transition Model and Scalability Dynamics

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ABSTRACT

Markets are described as complex dynamical systems that may switch between two regimes. Efficient market prevails when rational traders make decisions independently based on all available public and private information. When information complexity increases traders resort to rule based trading and imitation rate increases. As a result of this herding behavior price bubbles build up. If not reverted such behavior leads to loss of traders' heterogeneity and at a critical point, crash occurs. Predictability of such critical points is described previously as log-periodic oscillation of prices. Multifractal analysis should detect the volatility cascade leading to crashes.

I. INTRODUCTION

Although Gene Fama championed the publication of Benoit Mandelbrot's article in the *Journal of Business* in 1963, he also wrote a critique and thence stayed on the path of the three dominant Finance paradigms, the efficient market hypothesis (EMH; Fama, 1970), the capital-asset pricing model (CAPM; Sharpe, 1964; Lintner, 1965; & Black, 1972), and the Black-Scholes (1973) options-pricing model. In parallel, Mandelbrot (1970, 1982, 1997, 1999, Mandelbrot & Hudson, 2004) kept applying his fractal geometry ideas to stock markets. His views have more recently been picked up by others (Peters, 1991, 1994; Rosser, 2000; Sornette, 2003; Malevergne & Sornette, 2005; Jondeau et al., 2007; Calvert and Fisher, 2008). Over the years these two competing Schools have remained contentious, seemingly arguing about the same turf, with little basis of reconciliation apparent. For example, in his 2004 book with Hudson, Mandelbrot says: "*Modern' financial theory is founded on a few, shaky myths that lead us to underestimate the real risk of financial markets*" and: "*Orthodox financial theory is riddled with false assumptions and wrong results*" (pp. ix, x). According to Fama (1998), however, until new and better paradigm is put forth, one cannot criticize EMH/CAPM. Fama reduces behavioral finance—and trading dynamics—to anomalies and over-/under reaction episodes that are normally distributed.

The mathematical descriptions of financial market behavior by each School are now equally robust. Still, EMH and other paradigms have successfully remained at the center of market analysis for most researchers in the Finance community, even though the Fractal School has continued to grow in numbers of participants and depth of mathematical analysis (Adler, Feldman & Taqqu, 1998; Rachev & Mittnik, 2000; Malevergne & Sornette, 2005; Jondeau, Poon, & Rockinger, 2006). Since we have just passed through another of what Martin Greenspan recently termed a once-in-a-century market crash (the first being the 1929 crash) our concern about what sets off unusual volatility sequences and occasional extreme crashes is surely timely and calls for further analysis of when and why the EMH view of market trading shifts into behaviors better fitting fractal mathematics.

Following EMH, we first distinguish between the *rational* and *noise* traders who, acting *independently* of each other, create efficient markets. Then we recognize *interdependent* trader behaviors, recently accounted for in the mathematics of what Malevergne and Sornette (2005, p. 100) call the “*copula*” of two random variables—which offers a “complete and unique description of the dependence structure existing between them.” **First**, we note that there are three kinds of equally relevant trading behaviors that our models and theories have to account for: (a) There are two kinds of trader behavior in which they act *independently* of each other, which are excellently represented by the formalizations of EMH—which includes both *rational* and *noise* traders; and (b) A third kind of trader behavior also exists—where trader behaviors are in fact *interdependent*. We will call this the *high-risk* trading category; it occurs when traders resort to rule-driven behavior—e.g., chartists, herding, information cascades, etc.—in short: behavioral traders. **Second**, we note, however, that Sornette and his colleagues are content to draw explanatory closure when they fit an equation to a few parameters representing log periodic oscillations of prices (Sornette, 2004). A number of quantitative researchers now fit extreme events into power law distributions of price changes to formalize non-linear patterns during periods of extreme volatility (e.g., Jondeau et al., 2007; Calvet and Fisher, 2008). But our question remains: *What are the various kinds of interdependent trader-induced causes that scale up into the fractal volatility sequences and occasional extreme events (crashes) we see in stock market behaviors?*

We do not offer an alternative to EMH/CAPM but extend the existing efficient market framework to accommodate situations with higher information complexity, interactions with positive feedback, and extreme events that cannot be simply explained by presuming independent-additive data points, and normal distributions. For example, the development of the “*herding behavior*” literature in finance (Banerjee, 1992; Bikhchandani, et al. 1992; Prechter, 1999; Brunnermeier, 2001; Rook, 2006;) marks a significant recognition that interdependent trader behavior may result in skew distributions of stock market prices and, therefore, offers the first underlying explanation of behavior that may begin as

Holland's (2002) "*tiny initiating events*," but later scales up into extremes. Building on this, we introduce several so-called "scale-free theories" that explain why some tiny events scale up into extremes while many others do not. In all of these, interdependent trader behavior is the critical element. Our scale-free theories come from a range of disciplines (Andriani & McKelvey, forthcoming).

We can explain why some herding lunges scale up and then die off whereas others scale up into significant crashes. Given the empirical base of each theory, we believe we can offer rather solid rationale as to why high-risk trading behavior is ever present as mild to fairly strong volatility cycles. Thus, interactions of many traders with different investment horizons may simply die out, while some others may scale up into significant crashes that may or may not result from macroeconomic events (Rosser, 2000). Which way it goes depends on the nature of exogenous and endogenous shocks (Sornette and Helmstetter, 2003). Any shock that is related to small events with cumulative effects (endogenous) has different price diffusion than an exogenous shock such as 9/11, a natural disaster, a political coup, or some shocking economic news such as the 2007 liquidity crisis. Standard economic theory postulates that continuous streams of news get incrementally incorporated into prices. In principle, large shocks should result from anomalous, very bad news, but the puzzle around 1987 is that there was no such news. Large market moves and strong bursts of volatility are not, then, always associated with external economic, political or natural events (White, 1996). Endogenous shocks can result from the combination and/or accumulation of many small shocks as what seems like random noise re-structures into different kinds of nonlinearities (Schroeder, 1991).

In this paper, we attempt to further legitimate the separate-but-equal status of EMH and Fractal Finance (FF). We do this by attempting to go beyond classic financial asset pricing theory to account for findings of behavioral aberrations arising where unrealistic assumptions are made of unbounded rationality and independent judgments among investors about future payoffs and choices made solely on those anticipated payoffs (Fama & French, 2007).

The objective of this paper is to encourage research on the nonlinear models giving some theoretical underpinning to the equations that mirror markets as complex dynamical systems. Instead of just an “either-or” view of EMH or FF, we especially focus on dynamics causing traders to shift from one regime to the other. Research of extreme events, and underlying scale-free dynamics, is of particular interest for the overall understanding of markets functioning as complex systems reveal their characteristics under stress better than in normal conditions. Baum and McKelvey (2006) also reveal the potential advantage of extreme value theory in modeling management phenomena and the increasing popularity in financial applications. Andriani and McKelvey (2007) reveal how misleading are the assumptions behind the econometric methods involving linear multiple regression.

We begin by reviewing both the EMH and FF views of market dynamics. Then we use a physical analogy to help define the axes of our “Financial Markets Phase Diagram.” In this diagram we depict and define the *Triple Point* where we see EMH-driven behaviors and the *Critical Point* at which major market crashes occur: i.e., 1929, 1987, and 2007+. We then turn our attention toward explaining the various kinds of nonlinearities in trader behavior occurring as “market mania” develops. We define four kinds of nonlinearities. We then apply concepts from complexity science, econophysics, and scale-free theory to zero in on the underlying causes of these market nonlinearities. A Conclusion follows.

II. BACKGROUND

EFFICIENT MARKET HYPOTHESIS

According to the efficient markets hypothesis (EMH) that dominates finance theory and empirical work, price changes are completely random and driven by unexpected news about fundamentals, i.e. EMH models belong to the class of theories of asset price dynamics which consider fluctuations to be a result of exogenous stochastic influences. Efficient markets imply the absence of detectable (and predictable) structure in the market. If markets are not efficient, then we should be able to find

structure in financial data and exploit it. While some evidence against efficiency can be found, empirical studies generally don't find structure that is exploitable.

A basic principle in modeling the stock market is rationality. Traders in general exhibit a rational behavior, trying to optimize their strategies based on the available information. However this behavior is only "boundedly rational" since the available information is incomplete and traders have limited abilities to process the available information. The process of decision-making is essentially "noisy". In fact, a noise free market with rational traders of infinite analyzing abilities would have very little trading, if any, since for a market to be active there should be individuals willing to take opposite sides of the trade for the same price based on differing assumptions about where the price is going next. Perfect information and analytical ability would inevitably lead all traders to the same assumptions.

Fama and French (2007) assert how disagreement and tastes can affect asset prices thus admitting that some assumptions in standard asset pricing models are unrealistic. Recent models emphasize the role of heterogeneous beliefs and expectations about future prices. Brock and Holmes (1998) recap two basic investor types in these heterogeneous agent models: 1) rational traders including buy and hold fundamentalists (who base their trades on the perceived intrinsic value of firms) and 2) noise traders who are basically chartists or technical market analysts, and others who trade on misperceived information, rumors, personal opinions, or who simply copy other traders with positive reputations. We may also include liquidity traders here, i.e. traders whose decision to "cash out" is based on a need for liquidity that is independent of market information.

Equilibrium theory is based on four assumptions a) price taking, where agents can not do anything to change the price; b) independent agents make utility maximizing decisions; c) market clearing, e.g. equality of aggregate demand and supply and d) rational decisions based on perfect information.

When efficient market rules, it sustains dynamic equilibrium and temporary deviations are viewed as anomalies that are randomly distributed, therefore predictions are impossible and nobody can consistently beat the market.

FRACTAL FINANCE—CHAOS

Complexity has always been part of our environment and many scientific fields have dealt with complex systems, which display variation without being purely random. Complex systems tend to be high dimensional and non-linear but may exhibit low-dimensional behavior. Financial markets have been shown to be similar to complex dynamical systems (Johansen et al., 2000). The different parts of complex systems are linked and thus affect one another. A complex system may exhibit deterministic and random characteristics with the level of complexity depending on the system's dynamics and its interactions with the environment.

One of the objectives in quantifying complex systems is to explain emergent structures, i.e. self-organization. Phase transition is a property of self-organizing systems that move from static or chaotic states to a semi-stable balance between these two states that can be more effective (Brown and Eisenhardt, 1998). Self-organized criticality is characterized by power-law distribution of events around the phase boundary. Sornette et al. (1996) argue that scale invariance and self-similarity are dominant concepts useful in describing the processes surrounding the October 1987 crash since this event could be seen as the result of a worldwide cooperative phenomenon, analogous to a critical phase transition in physics. Johansen et al. (2000) identify patterns of near-critical behavior years before market crashes. Similarly, the hierarchical or cascade model of traders with "crowd" or "herd" behavior illustrates the concept of criticality (Bak, 1996), where a large proportion of the actors simultaneously decide to sell their stocks (cf. Breyman et al., 2000).

Another piece of the puzzle is the observation of aperiodic long-term behavior that exhibits sensitive dependence on initial conditions and has limited predictability of the dynamics—characteristics of

chaos. This sensitivity to initial conditions means that two points in a chaotic system may move in vastly different trajectories in their phase space, even if the difference in their initial configurations is very small. Since some nonlinear dynamical systems under certain conditions exhibit chaos, *detection* of the emergence of chaos in the system (as opposed to prediction) might allow active control at a low cost leading not only to highly positive outcomes but also to the prevention of costly crisis situations.

In finance, Brock and Hommes (1998), Chen et al. (2001), Gaunersdorfer (2000), Lux (1995, 1998) and others suggest that heterogeneous beliefs of market players lead to market instability and complex endogenous price dynamics such as chaotic fluctuations. When a complex dynamical chaotic system becomes unstable, the system may split (bifurcation) in the presence of an attractor. Chaotic attractors are fractal and fractals have complex geometry with similarity at various scales. Such attractors may be seen as a subset of the domain wherein attraction defines a set of initial conditions.

At the time of its discovery, the phenomenon of chaotic motion was considered a mathematical oddity, but physicists have proven that chaotic behavior is much more widespread. Chaos is randomness operating through deterministic laws. But the question remains whether chaos may actually produce ordered structures and patterns that can be used in finance to make improvements over asset pricing models that are based on the assumption of randomness.

Theories of asset pricing dynamics that challenge the EMH orthodox view see fluctuations as arising from underlying systematic causes that can be related to nonlinear dynamic mechanisms. Such a dynamical system approach to asset pricing was introduced by Chiarella (1992) and the need for empirical techniques for the full range of nonlinear dynamic possibilities was suggested.

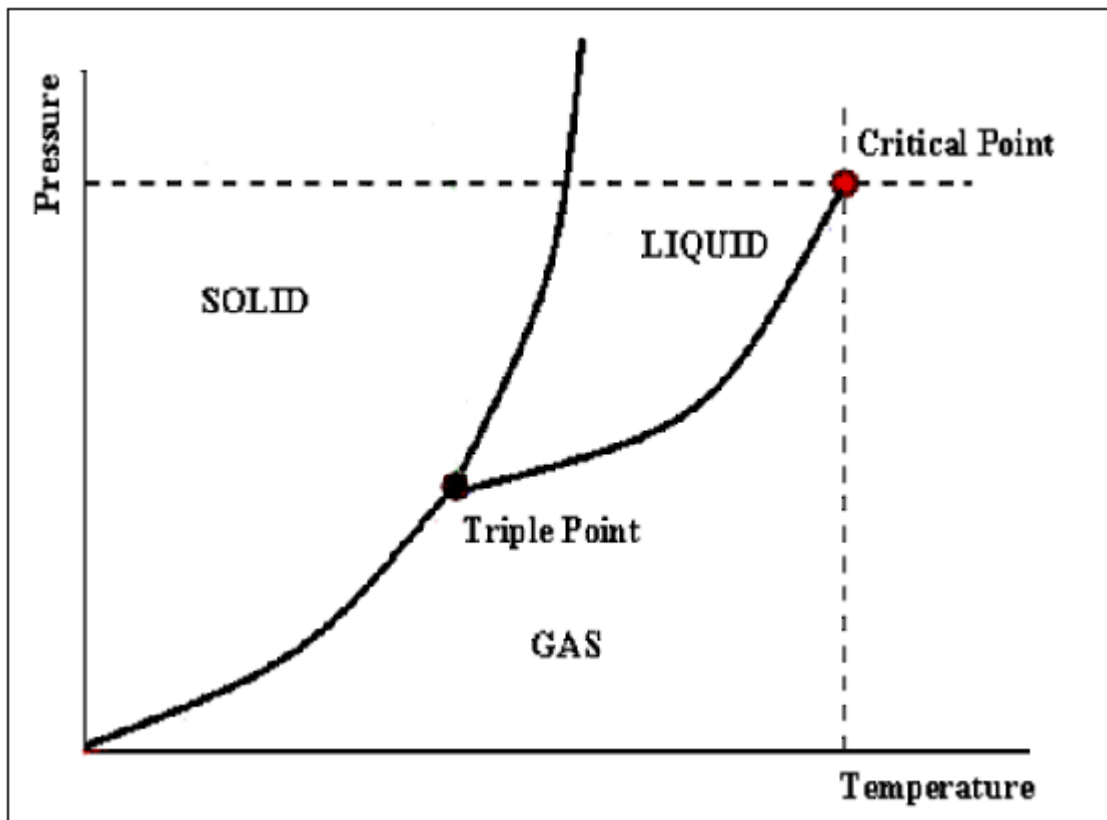
Market crashes have been explained theoretically and measured quantitatively in a variety of ways. Sornette et al. (1996) reveal log-periodic oscillations of index prices before significant drawdowns and suggest a phase transition model where market crashes appear at the **Critical Point**. The market crash as phase transition in Johansen and Sornette (1999) points at the analogy between the three states of a

physical system (solid, liquid and gas) and stock market dynamics at a "microscopic" level, where the individual trader has only three possible actions: selling, buying or waiting.

III. THE PHYSICAL BASIS OF PHASE TRANSITION (AND CRITICALITY?)

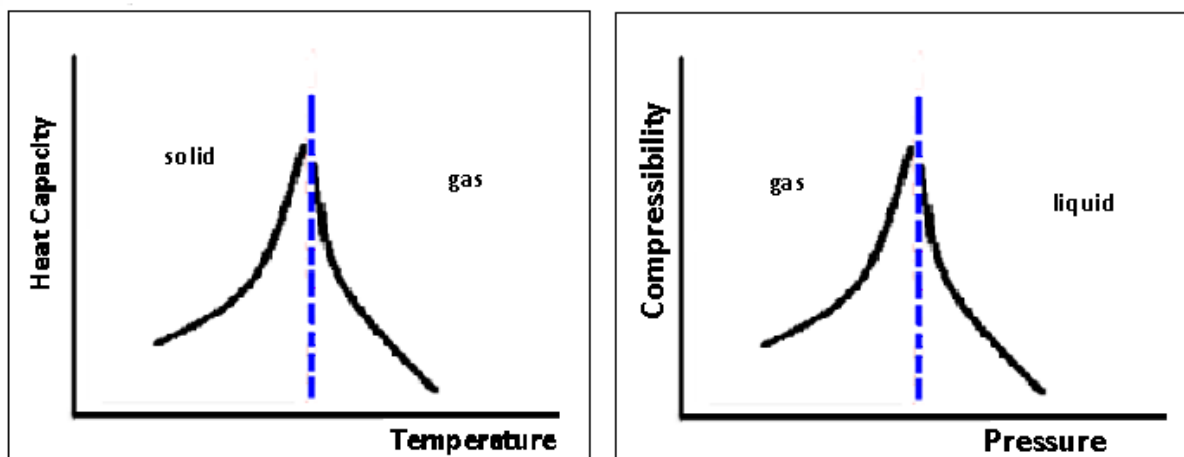
In the physical sciences, a phase space depicts the set of states of a macroscopic physical system that have relatively uniform chemical composition and physical properties. For water, as an example, the three phases (solid, liquid, and gas) are defined by temperature/pressure combinations. The different phases of a system may be represented using a phase diagram (Figure 1). The **Triple Point** is the combination of temperature and pressure that permits the co-existence of the three phases in dynamic equilibrium. A phase transition, (or phase change) is the transformation of a thermodynamic system from one phase to another. At the **Critical Point**, a second order phase transition occurs leading to the disappearance of the phase boundary and the presence of a super-critical liquid / gas state.

Figure 1: Phase Diagram



When a system transitions from one phase to another, there will generally be a stage where the free energy is non-analytic. Due to this non-analyticity, the free energies on either side of the transition are two different functions, so one or more thermodynamic properties will behave very differently after the transition. The property most commonly examined in this context is the heat capacity of the substance. During a transition, heat capacity may become infinite, jump abruptly to a different value, or exhibit a "kink" or discontinuity in its derivative, i.e. experience an abrupt sudden change in heat capacity with only a small change in temperature (Figure 2). Similarly, we can illustrate the change of compressibility along the changes in pressure.

Figure 2: Heat Capacity and Compressibility changes at the phase boundary



IV. FROM PHYSICS TO FINANCE: A PHASE-TRANSITION MODEL

In an auction, market price is determined by demand and supply, i.e. buy and sell orders. The balance between the two we define as net demand. Balanced net demand or "0" will determine a "wait" phase in the market. Transactions would not impact price. (As we are trying to explain market crashes as **Critical Points** we find that the analogy between sell/liquid and buy/gas would agree with Sornette (2003) description of critical market crashes. We could defend our choice of phase analogy but we'll spare technical details here in the initial draft.) When the demand is positive, the market is in "buy"

phase and conversely, a negative net demand indicates a “sell” market. Autonomous agents with heterogeneous information and bounded analytical abilities place their orders to buy and sell securities. At a macro level, the balance of these trading orders determines the phase of the market, i.e. the buy/sell decision ratio divides the plane. The phase diagram should be able to explain the origin of this balance with appropriately chosen variables on the x and y axis.

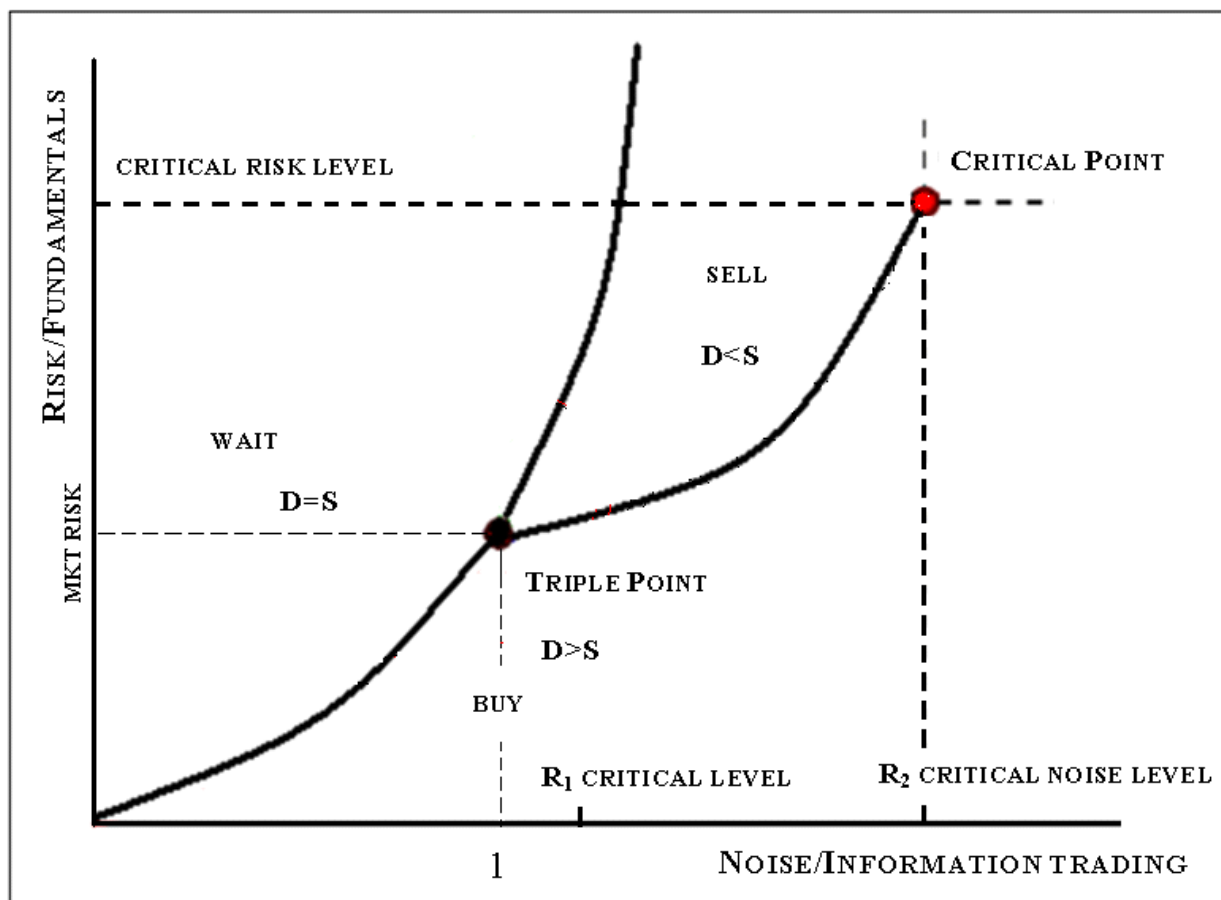
DEFINING THE AXES

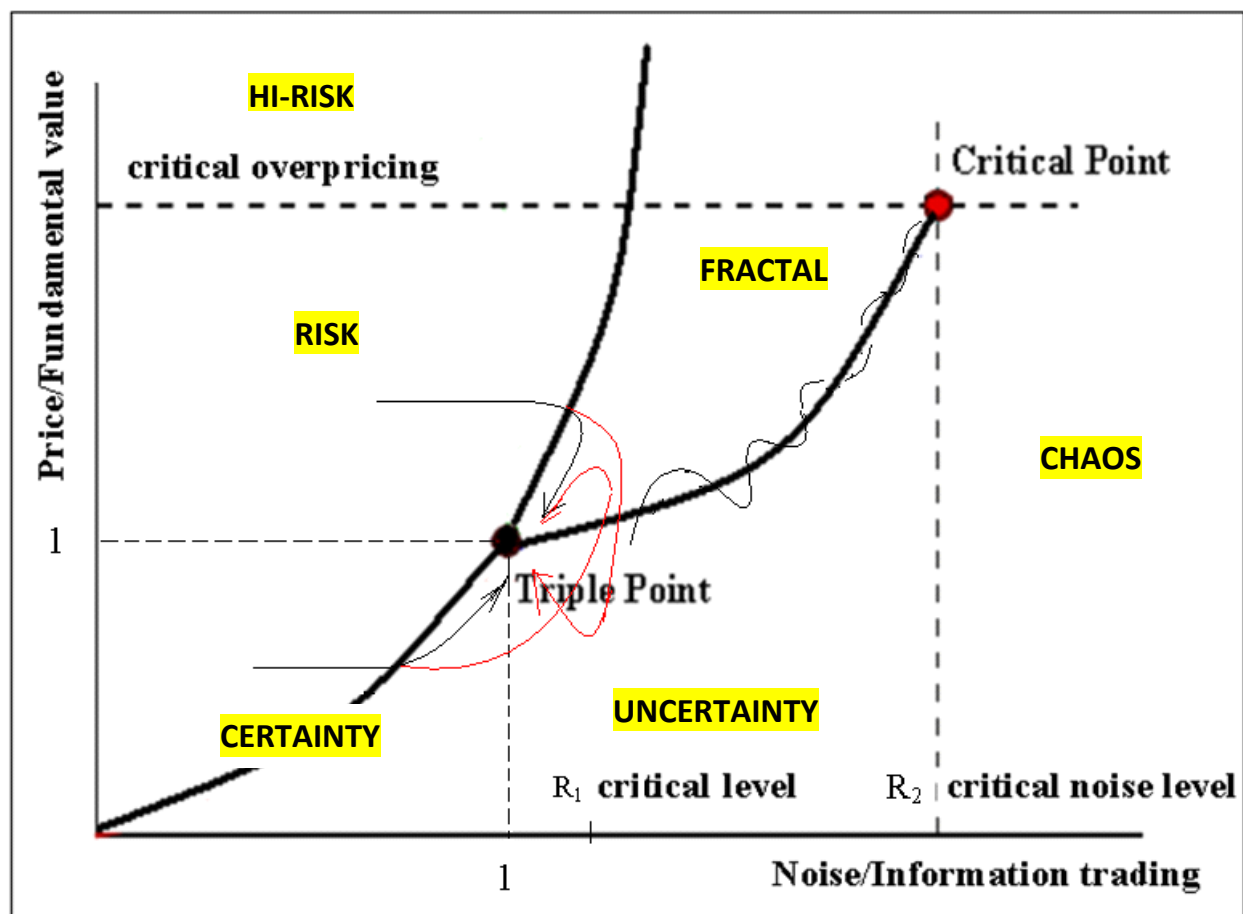
Returning to our two types of investors (rational and noise), rational investors evaluate all available information and make their trading decisions based on fundamental analysis of discounted future cash flows, defining underpriced (buy) and overpriced (sell) stocks. In the CAPM Return/Risk ratio is linear, that allows us to represent the same notion of ranking from under to overpriced using a risk/fundamentals ratio. Relative risk increase should suggest prevalence of “sell” decision under the efficient market hypothesis. Thus the y-axis of our financial markets phase diagram (Figure 3) measures a risk/fundamentals ratio.

On the x-axis, we replace temperature with the Noise to Information trading ratio. In the physical phase transition diagram, temperature was defined in terms of the 2nd Law of Thermodynamics, which deals with entropy. Since entropy is a measure of the disorder in a system, we believe that disorder in the market can be measured by the ratio of Noise to Information trading. Information trading implies investors can properly process information and act rationally. As we already mentioned if all investors are rational and all trading decisions are information based, we will have homogeneous agents, one sided trade orders and a “halt” in the market. Conversely, as noise trading becomes increasingly evident in the market, rationality recedes and disorder increases. In the base condition (where system complexity is minimal and rational traders hold uniform assumptions), the opposite side of a trade is attributed to liquidity traders in the EM paradigm.

The dimensions that we have chosen for our axes are also used in some non-linear financial models previously, e.g. the asset price diffusion process is explained by the ratio of noise to rational traders and the distance between fundamental to actual price in Chiarella 1992, Day and Huang 1990, and Lux 1995. These two dimensions will also allow us to place into the asset pricing framework the log periodic oscillations of index prices before crashes reported by physicists.

Figure 3: Financial Markets Phase Diagram





TRIPLE-POINT DYNAMICS

On the continuum of these two combinations, overall market state will be defined as "wait" when net demand is balanced, i.e. shares may exchange hands but this will not affect the price. In this area we have two subareas that are unstable and quickly lead to the **Triple Point**. This attracting basin is characterized by correctly priced securities (y-axis = market risk) and balance between noise and information trading (x-axis = 1). **RISK FRACTAL CHAOS**

We view the lower right corner as highly unstable as rational investors will be willing to buy the underpriced securities, but there are very few noise traders, who will sell at this price. As a result, the market moves to the right, crossing the phase boundary which results in falling liquidity as buy orders prevail and price goes up. Again the basin of attraction becomes apparent.

Note that this area is populated mainly by information traders. The upper left corner is an area of overpricing. Theoretically, if investors do not rush to sell overpriced securities, the market can be “suspended” in this area. Otherwise we need a high level of noise buying such that noise traders exceed the number of information traders and this is not the case in this area. If rational information traders "sell" to take profits, then an equivalent amount of buy orders from noise traders should be present. More traders on the sell side results in crossing the phase boundary change in liquidity (more “sell” orders) and the price will go down. The market again moves to the **Triple Point**.

Over- or under-pricing is a very short lived phenomenon after news (new relevant information) is released. In both cases moving from the “wait” phase either through sell or buy to reach the attracting point (Triple point = dynamic equilibrium) sees the market cross the phase boundary. If there is nonlinearity present, there should be a function that experiences abrupt change with a small change in the x-axis (noise/info trading). This also should be related to a jump in the price, as market quickly adjusts to new information and incorporates the news in the price. (Documented overreactions to news can be modeled with a longer route to the **Triple Point** as shown in Figure 4 in red.)

Figure 4: Dynamics

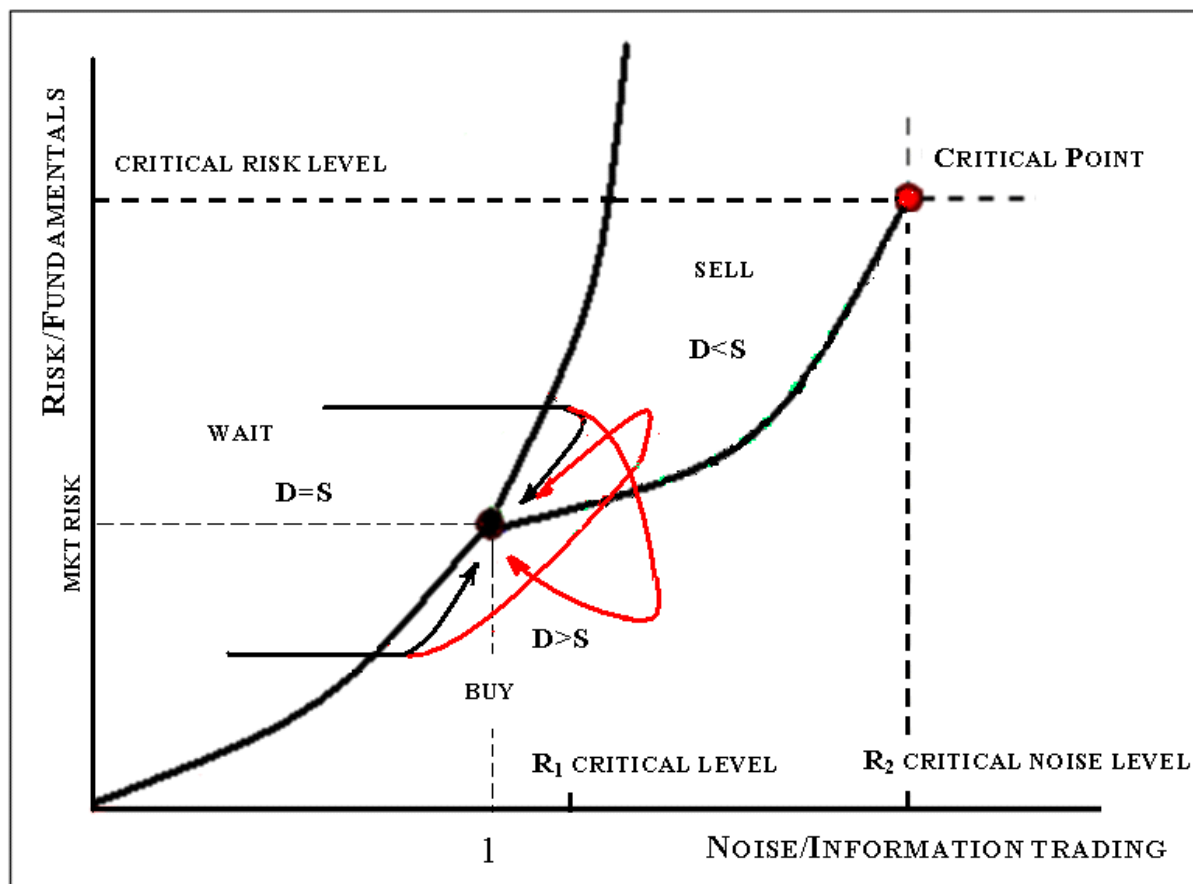
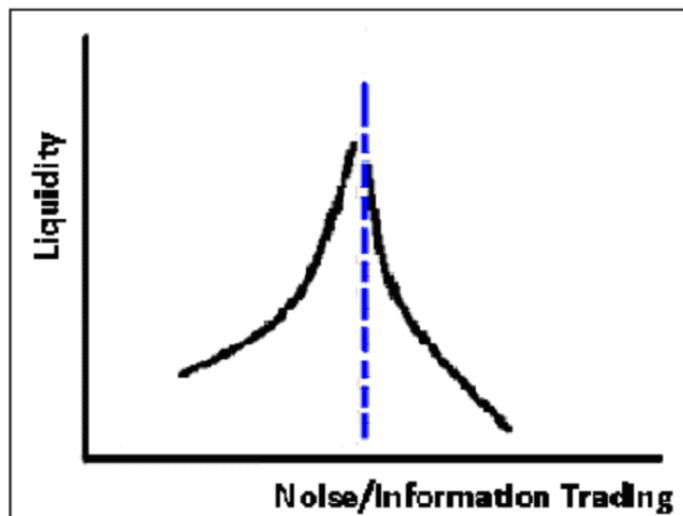


Figure4: Normal market dynamics with over-reaction in red.

In our model, at the phase boundary, liquidity¹ experiences an abrupt change with only a small change in the ratio of noise to rational trading (Figure 5). Lillo and Farmer (2004) empirically demonstrate that liquidity, not large volume, determines large price changes, therefore changed liquidity at the phase boundary will result in the jumps that are often observed in stock prices.

Figure 5: Liquidity increases until the phase boundary and experience discontinuity at phase transition

¹ Liquidity in finance, not to be confused with the liquid state in physics, is determined by the amount of orders on the opposite side of the trade. It means how “easily” you find a buyer or a seller at the price you are willing to trade.



As we move horizontally on the phase diagram (Figure 4) from “wait” to “buy” in the area of under pricing, the liquidity increases, as more noise traders are present. When we cross into the “buy” state characterized by positive demand, the liquidity function changes. (Please, note movements in the “wait” area could be only horizontal, by definition.)

Similarly, in the overpriced area, a rational investor wants to sell at x price, market should move horizontally to find a “noise” buyer. To the right the ratio of noise traders’ increases, increasing the ease of trade (liquidity), but after the phase boundary, the market is characterized by negative demand and a sell order faces lower liquidity, i.e. few buyers.

According to the efficient market hypothesis, these are short-lived anomalies that are arbitrated away. The simultaneous execution of a large number of trades produces efficient outcomes and a dynamic equilibrium between the three states is present. Also according to EMH, all investors are rational and base their decisions on fundamental values. Trading only occurs due to liquidity need investors, who take the opposite side of the trade. As noted earlier, we define those seeking liquidity as “noise” traders since they are not trading based on information.

THE CRITICAL POINT

Having shown the **Triple Point** to be an effective attracting basin, we still need to find a mechanism that will lead to the empirical record of extreme events evidenced by the stock market; one that far exceeds Gaussian distributed expectations (Baum and McKelvey, 2006). Sornette (2003) presents a general theory of financial crashes and stock market instabilities and asserts that markets exhibit complex organization and dynamics. Moreover, he suggests that large scale patterns of a catastrophic nature result from global co-operative process over the whole system by repetitive interaction. A power law distribution punctuated with log periodic oscillations in the index prices seems to be the signature of an impending crash. Among many other examples, Baum & McKelvey (2006) also show evidence of power law distribution in the daily log returns of Dow Jones and NASDAQ and argue that observed power laws stem from non-independent behavior that is ever present in social contexts (including stock markets).

“Critical Point” in sense of the stock market model is a combination of high level of noise trading based on imitation among traders and high level of risk. As Sornette (1998) suggested “order in the market attains”, all traders have the same opinion sell and the market collapses, the bubble bursts.

The EMH allows for temporary deviations/anomalies in the market that undergo corrections. The part that remains unexplained in that framework are the reported power law distributions in the returns and the long memory of volatility in such periods. EMH will not even admit the existence of log-periodic oscillations or other predictive patterns moreover, to attempt to explain their origin. We suggest not contradicting or refuting the EMH but simply append a regime when traders are not independent in their decisions. The level of imitation or impacting each other’s trading decision increases with the level of information complexity in the market. It translates to higher level of noise to information on the x-axis.

On the y-axis the feasibility of increasing the risk level above the normal market’s can be explained by increased rational speculation justified by increased levels of uncertainty. With this in mind we’ll

move to describe the “extreme market regime” of bubble build up and possibility of reaching the critical point followed by a market crash.

THE FOUR REGIONS

Having defined the Triple and Critical Points, we now have four Regions in Figure 4: Certainty, Uncertainty, Risk, and Fractal. We briefly define each of these Regions as follows:

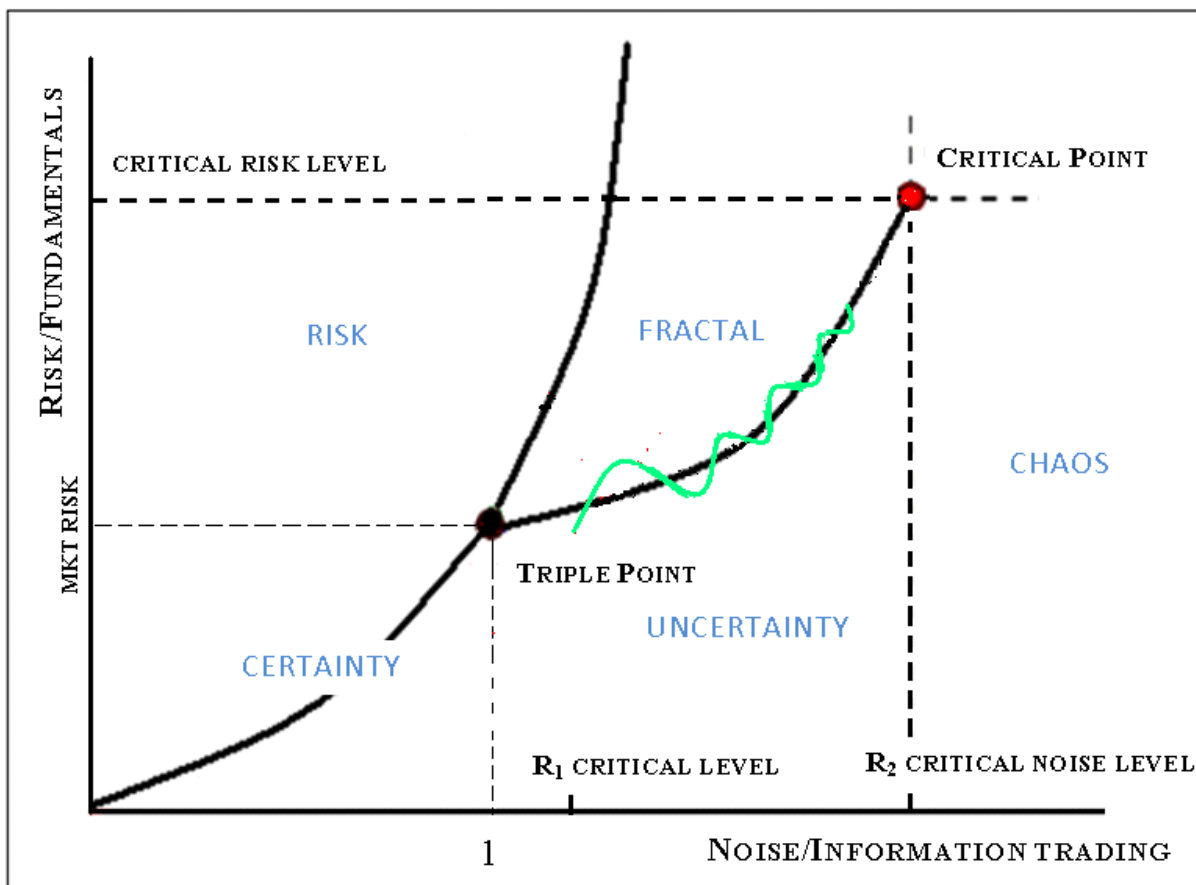


Figure 6: Four Regions and the origin of the log-periodic oscillations

1. *Certainty*. Most pronounced at the *Origin*. Note the points on each axis where the ratio of Risk/Fundamentals and Noise/Information equal 1. At this point on the X-axis, Fundamentals and accurate information dominate risk and noise. Traders in this Region approach perfect

information as the two ratios go to zero—when traders have perfect information about the fundamental value of a firm.

2. *Uncertainty*. To the right of the Triple Point location on the X-axis, traders lose contact with any reliable means of attaching true value to information about a particular stock/firm. This results in Noisy information, as traders guess one way or another. Uncertainty peaks at the location of the Critical Point on the X-axis.
3. *Risk*. Above the location of the Triple Point on the Y-axis, traders move away from simply trading based on knowledge about the *current* “fundamental” value of a stock/firm to start betting on *future* value. Risk increases up to the location of the Critical Point on the y-axis. Above this point we show “Chaotic Risk;” this is the point where risk-taking becomes vulnerable to chaos—bifurcations that can set off significant crashes.

Note that we show Knight’s (1921) risk, uncertainty, and certainty are juxtaposed at the Triple Point.

This is the core explanation underlying EMH—traders leaning toward all three situations trade concurrently with quick adjustments of the market shifting toward one or the other of the three conditions.

4. *Fractal*. The Region between the Triple and Critical Points is notable for increasingly dramatic volatility incidents. Since there is growing evidence that many of these incidents follow fractal patterns, we label the region Fractal, even though there undoubtedly is non-fractal volatility present as well.

V. NONLINEARITIES BETWEEN TRIPLE AND CRITICAL POINTS

SCALABILITY AND SCALE-FREE THEORIES

To begin, we recognize three basic Phases in the development of complexity science.

Phase 1 emphasizes critical values and dissipative structures. Is based on the works of Prigogine (1955, 1984, 1997), Haken (1977, 2004), and Mainzer (1994/2007), among many others. It begins with the Bénard (1901) process—an energy differential is set up between warmer and cooler surfaces of a container (measured as temperature, ΔT). In between the 1st and 2nd critical values (R_{c1} , R_{c2}), a region is created where the system undergoes a dramatic shift in the nature of fluid flow. For example, increasing the heat under water molecules in a vessel exposed to colder air above leads to geometric patterns of hotter and colder water—the chef’s “rolling boil” emerges; new order appears. The critical values define the “*melting zone*” (Kauffman, 1993; Stauffer, 1987), within which new structures spontaneously emerge; Prigogine (1955) termed these “*dissipative structures*” because they are pockets of order—governed by the 1st Law of Thermodynamics—that speed up the dissipation of the imposed energy toward randomness and entropy according to the 2nd Law (Swenson 1989).

Phase 2 emphasizes agent self-organization absent outside influence. It consists largely of scholars associated with the Santa Fe Institute (Pines, 1988; Arrow et al., 1988; Cowan et al., 1994; Arthur et al., 1997). While Phase 1 focuses mostly on dramatic phase transitions at R_{c1} ,—the *edge of order*, Phase 2 complexity scientists focus mostly on R_{c2} —the “*edge of chaos*” (Lewin, 1992; Kauffman, 1993). Focusing on living systems (Gell-Mann, 2002), Phase 2 emphasizes the spontaneous co-evolution of entities (i.e., the agents) in a CAS. Agents restructure themselves continuously, leading to new forms of emergent order consisting of patterns of evolved agent attributes and hierarchical structures displaying both upward and downward causal influences. Bak (1996) extends this treatment in his discovery of “*self-organized criticality*”, a process in which small initial events can lead to complexity cascades of avalanche proportions best described as an inverse power law. The signature elements

within the melting zone are self-organization, emergence and nonlinearity. Kauffman's "*spontaneous order creation*" begins when three elements are present: (1) heterogeneous agents; (2) connections among them; and (3) motives to connect —such as mating, improved fitness, performance, learning, etc. Remove any one element and nothing happens. According to Holland (2002) we recognize emergent phenomena as *multiple level hierarchies, bottom-up and top-down causal effects, and nonlinearities*. Nonlinearity often stems from scalability reflected as power laws.

Phase 3, Econophysics, is the most recent development. Its focus is on how order creation actually unfolds once the forces of emergent order creation by self-organizing agents—such as biomolecules, organisms, people, or social systems—are set in motion. Key parts of this third phase are *fractal structures, power laws, and scale-free theory*. In his opening remarks at the founding of the Santa Fe Institute, Gell-Mann (1988) emphasized the search for scale-free theories—simple ideas that explain complex, multi-level phenomena. Brock (2000) goes so far as to say that "*scalability*" is the core of the Santa Fe vision—no matter what the scale of measurement, the phenomena appear the same and result from the same causal dynamics. Gell-Mann (2002) concludes his chapter, "*What is Complexity?*" with a focus on scalability.

Fractals and power laws. Consider the cauliflower. Cut off a "floret"; cut a smaller floret from the first floret; then an even smaller one; and then even another, and so on. Despite increasingly small size, each lower-level component performs the same function and has roughly the same design as the floret above and below it in size. This feature defines it as fractal. Fractals can result from mathematical formulas—the very colorful ones figuring in Mandelbrot's "*Fractal Geometry*" (1982). We are more interested in fractal structures that stem from adaptive processes—like the cauliflower—in biological and social contexts. In fractal structures the same adaptation dynamics appear at multiple levels.

The econophysicist Barabási (2002) connects scalability, fractal structure, and power law findings to social networks. He shows how networks in the physical, biological and social worlds, are fractally structured such that there is a “rank/frequency” effect—an underlying Pareto distribution showing many sparsely connected nodes at one end and one very well connected node at the other. If plotted on a double-log graph, the Pareto-distributed progression of increasing numbers of connections from, say, small airports to giant ones like Heathrow and Atlanta, appears as a negatively-sloping straight line. This is the now famous power law “signature” dating back to Auerbach (1913) and Zipf (1949). Andriani and McKelvey (forthcoming) list 84 kinds of power laws—which are good indicators of fractal geometry—in social, and organizational phenomena. Stanley et al. (1996) find that manufacturing firms in the U.S. show a fractal structure, as does Axtell (2001). See also Newman (2005), Newman et al. (2006), and Clauset et al. (2007).

Scale-free theories explain why fractals appear as they do and behave as they do. Though scalability may have been at the core of the Santa Fe vision, scale-free theories have only recently begun to be consolidated and featured collectively by the econophysicists (West & Deering, 1995; Mantegna & Stanley, 2000; Newman, 2005). The key feature that sets scale-free theories apart from most social science theories is that they use a single cause to explain fractal dynamics at multiple levels. The earliest dates back to 1638—Galileo’s *Square-Cube Law*; the cauliflower keeps subdividing to keep its surface area at a constant ratio to its growing volume. Explanations for why some structures have adaptive success while others do not, range from biology to social science. If the same theory or principle applies to microbes and to organizations, it is assuredly scale-free. Andriani and McKelvey (forthcoming) describe 15 scale-free theories applying to firms.

At the end of each of the following sections, we add in descriptions quoted from the Andriani and McKelvey (forcoming) article applying scale-free theories to management and organizational research. We also comment on how they explain various kinds of trading behaviors.

THE CHARTISTS

Two studies, in particular, support the potential for the existence of co-operative behavior and increased number of noise traders to move the system to complex dynamics. Sethi's (1996) model shows local instability is possible if the adjustment of prices is rapid, chartist demand is highly sensitive to changes in expectations and the share of wealth in chartist hands is sufficiently large. In contrast, a unique equilibrium is formed when prices are equal to fundamental values and stability of price dynamics ensues when the chartist share of market wealth is sufficiently small. In another study, Corcos et al (2002), placed chartists into a simple model of repetitive interaction that led to hyperbolic bubbles, crashes and chaos. In this model a typical bubble starts at an exponential growth rate, crosses to non-linear power law leading to finite time singularity.

The market may thus move away from the attracting basin and tend toward extreme events when noise trading exceeds information trading. 'Noise' traders (including chartists) trade from misperceived information or for idiosyncratic reasons (e.g. liquidity). Black (1986) considers noise essential to the liquidity of financial markets. It is not reasonable to assume that differences in beliefs about prices are only the result of different information; noise is even produced by small events and the agents themselves. Noise trading is trading on noise *as if it were* information. Heiner (1983) argues that the difficulty involved in making an optimizing decision under conditions of complex dynamics leads to rule-governed behavior (e.g. technical analysis).

The above evidence explains how, under certain circumstances, noise trading in the market increases, breaks the symmetry in demand /supply and destabilizes prices. Below the first critical value (R_1 in Figure 3), the demand is roughly zero, neither buying nor selling predominates, which agrees with the dynamic stability in the basin of attraction. Above this critical noise level, two most probable values emerge that are symmetrical around zero demand as Plerou et al. (2003) describe the bi-modal distribution of demand above the first critical level of noise. Sethi (1996) shows that a large fraction of chartists tends to destabilize prices and can cause attracting periodic orbits to arise. The bimodal

distribution of demand reported by Plerou et al. (2003) may suggest oscillation of the market between negative and positive demand phases. The phase transition at this point is related to abrupt changes in the trading volume. The reversal frequency of the market sentiment is related to the increasing hazard rate of crash producing log periodicity in the oscillations. In the bubble build up, rational traders evaluate the increased hazard rate and adapt their speculative strategy. Sornette (2003) describes the build-up of cooperative speculation, which often translates into an accelerating rise of the market.

The following scale-free theories seem to fit the Chartists best.

Table 1: Scale Free Theories Explaining Nonlinear Trader Behaviors*

Phase transition	<i>Turbulent flows</i> : Exogenous energy impositions cause autocatalytic, interaction effects and percolation transitions at a specific energy level—the 1 st critical value—such that new interaction groupings form with a Pareto distribution (Prigogine, 1955; Nicolis & Prigogine, 1989).
Contagion bursts	<i>Epidemics; idea contagion</i> : Often, viruses are spread exponentially—each person coughs upon two others and the network expands geometrically. But, changing <i>rates</i> of contagious flow of viruses, stories, and metaphors, because of changing <i>settings</i> such as almost empty or very crowded rooms and airplanes, result in bursts of contagion or spreading via increased interactions; these avalanches result in the power-law signature (Watts, 2003; Baskin, 2005) due to the small-world structures of the underlying networks.

* **Material in this and the following Tables quoted from Table 2 in Andriani and McKelvey (forthcoming).**

Since our text uses the “critical value” phrase (see this and other underlined phrases), it is logical to suggest that scalability based on autocorrelation effects is present. Furthermore, the combination of “rule-governed behavior” and “cooperative speculation” fits the contagion burst theory—traders develop rules that spread more quickly because traders often communicate within groups, thus speeding up the rule-contagion process

HERDING

When consumers act sequentially rather than concurrently, herd-like behavior can impede the flow of information and a slight prevalence of public information (e.g. observing others' actions) is then sufficient to induce agents to ignore their private information and follow in the direction of the crowd. In this case, movement along the x-axis in Figure 3 shows increased “noise” trading.

According to Avery and Zemsky (1998) under informationally-efficient prices, herd behavior occurs when signals are non-monotonic and risk is multidimensional. In addition, Brunnermeier (2001), Bikhchandani and Sharma (2000) and Chamley (2004) concluded that herding does not involve violent price movements except in the most unlikely environments. Park and Sabourian (2006) though, argue that for financial-market herding one needs neither non-monotonic signal nor multidimensionality of risk. Instead, extreme price movements with herding are possible in variety of situations; herding often exacerbates price volatility. They require sufficient amount of noise and the existence of a signal with U-shaped conditional distribution (both extreme values generate this signal with large probabilities). The recipients of this signal are more volatile in their decisions, switching from selling to buying and back. We adopt this explanation for the bubble build up, where herding traders move through the buy/sell boundary back and forth (see Figure 3), as they believe more in extreme than in moderate values. Moreover, both types of herding (buy and sell) are possible in the same model if there are more than one middle type signals. This dynamical model should also examine the conditions when the trader changes his action to engage in contrarian behavior. While the large amount of noise is required again, i.e. proportion of information traders is not too large, now the signal with "hill-shaped" conditional distribution is necessary.

With respect to herd behavior in efficient markets, Park and Sabourian (2006) and Bikhchandani and Sharma (2000) suggest that the profit/utility maximizing investor may reverse their planned decision based on the belief that other investors are acting on information. While they are herding, they increase the number of noise traders when the price/fundamentals ratio stays the same. DeLong et al (1990) asserts that rational speculators' early buying triggers positive-feedback trading, which also increases the number of speculators. According to Avery and Zemsky (1998), herding in two dimensions of uncertainty may not distort prices. When the quality of traders' information uncertainty is added significant mispricing can occur. Therefore a "vertical move" with increase in price/fundamental ratio is

presumed. At the critical overpricing and level of noise the bubble bursts resulting in a second order phase transition (see below).

One obvious scalability fit is “preferential attraction.” Another is “coral growth.”

Table 2: Scale Free Theories Explaining Nonlinear Trader Behaviors

Preferential attachment	<i>Nodes; gravitational attraction:</i> Given newly arriving agents into a system, larger nodes with an enhanced propensity to attract agents will become disproportionately even larger, resulting in the power law signature (Barabási, 2002; Newman, 2005).
Irregularity generated gradients	<i>Coral growth; blockages:</i> Starting with a random, insignificant irregularity, coupled with positive feedback, the initial irregularity starts an autocatalytic process driven by emergent energy gradients, which results in the emergence of a niche. This explains the growth of coral reefs, innovation systems (Turner, 2000, Odling-Smee et al., 2003).

Preferential attachment is a power-law description of social ties or contacts. Perhaps a couple of traders begin with a small irregularity and then their group grows—the more followers they have, the more new ones they get. In the herding process a social network develops in which some traders have many contacts while other are isolates simply following the herd. This scale-free theory seems a good fit to herding. In addition, herding behavior is similar to coral growth; it begins with traders following insignificant cues and then positive feedback effects set in with the result that volatility increases.

COEVOLUTION

We suggest that breaking through the first critical level of noise the system enters into self-organizing dynamics or coevolutionary dynamics. There should be an initiating event such as new trading rules, hedging techniques, or the development of new derivative products. Individual traders and institutions engage in these initiating events and in the process of learning and adaptation, the noise level increases. Maruyama (1963) observes that initiating events may be random and insignificant. Arthur (1990) focuses on positive feedbacks stemming from initially small instigation events. Casti (1994) and Brock (2000), by continuing the focus on power laws, present a vision of co-evolution as a “driver” of complex system adaptation. McKelvey (2002) outlines the necessary and sufficient conditions for coevolution to occur. In addition to the initiating events the following four conditions must also exist:

1. Heterogeneous agents (I believe so far we convinced the reader in their existence).

2. Adaptive learning abilities

De Long et al. (1990), argue that if rational speculators purchase ahead of noise demand, this may trigger positive-feedback trading. An increase in the number of forward-looking speculators can increase volatility about fundamentals. Fundamentalists base their decision on the deviation of the asset prices from fundamentals and chartists on the trends they discern from past observations of the data. Their interaction is described by the disequilibrium models of Beja and Goldman (1982) and Chiarella (1992). The first model is linear and instability is global. The second one is a nonlinear version and prices oscillate around but never converge to fundamentals. The model agrees with the assertion of DeLong et al. (1990) that unboundedly rational traders take full account of the presence of noise traders, and destabilize prices to exploit the adaptive behavior of the latter.

3. Agents are able to interact and influence each other

4. Higher level constraint, adaptation to which motivates the coevolutionary process

The four conditions listed above fit our earlier introduction of self-organization-based complexity theory (see Kauffman, 1993; Holland, 1995, etc.). Several scalability theories fit coevolution.

Table 3: Scale Free Theories Explaining Nonlinear Trader Behaviors

Spontaneous order creation	<i>Heterogeneous agents</i> seeking out other agents to copy/learn from so as to improve fitness generate networks; there is some probability of positive feedback such that some networks become groups, some groups form larger groups & hierarchies (Kauffman, 1993; Holland, 1995).
Least effort	<i>Language; transition:</i> Word frequency is a function of ease of usage by both speaker/writer and listener/reader; this gives rise to Zipf's (power) Law; now found to apply to language, firms, and economies in transition (Zipf, 1949; Ishikawa, 2005; Podobnik et al., 2006).

The foregoing discussion zeros in on coevolution that is based on positive feedback. The “irregularity generated ingredients” (coral growth) theory fits here (but we don't repeat it in Table 3). In addition, the self-organization process is at the heart of the “spontaneous order creation” theory. The various studies listed in support of “least effort” theory suggest that freedom to self-organize without constraint will produce power-law distributed rank/frequency formations—in the foregoing, traders coevolve toward the most efficient set of trading rules, hedging techniques, derivative products, and so on.

VOLATILITY

In the EM paradigm volatility follows Brownian motion (*no memory*), while long range dependence (power law in the autocorrelation function) has been detected in financial time series. Research on the scaling behavior of volatility explains price changes at different horizons—hourly, daily, weekly monthly and reveals vertical dependence that is explained by the existence of traders with different time horizons. Coarse volatility at low frequency captures the views and actions of long-term traders while fine volatility at high frequency captures the views and actions of short-term traders. It has been shown in Müller et al. (1997) and Dacorogna et al. (2001) that there is an asymmetry in that coarse volatility predicts fine volatility better than the other way around.

Gençay and Selçuk (2004) show that in such heterogeneous markets, low-frequency shocks penetrate through all layers to the short-term traders, while high frequency shocks appear to be short lived. This explains the patterns of volatility observed in endogenous shocks as a result of self-organization (i.e., underlying chaotic dynamics—i.e., the period doubling, bifurcation state). Here also belongs the latest Sornette et al. (2002) article pointing to the cumulative effect of small shocks.

The following scale-free theories explain volatility best.

Table 4: Scale Free Theories Explaining Nonlinear Trader Behaviors

Self-organized criticality	<i>Sandpiles; forests; heartbeats:</i> Under constant tension of some kind (gravity, ecological balance, delivery of oxygen), some systems reach a critical state where they maintain adaptive stasis by preservative behaviors—such as sand avalanches, forest fires, changing heartbeat rate, species adaptation—which vary in size of effect according to a power law (Bak, 1996).
Interacting fractals	<i>Food web; firm & industry size:</i> The fractal structure of a species is based on the food web (S. Pimm quoted in Lewin 1992, p. 121), which is a function of the fractal structure of predators and niche resources (Preston, 1948; Pimm, 1982; Solé et al., 2001; West, 2006).

Volatility is simply prices changes that range from many small movements to a few large movements, with a crash being the largest. This process is exactly what Bak (1996) emphasizes in his “self-organized criticality” theory—the many small to few large change movements that keep the slope of a sandpile at a certain angle, are seen in all sorts of processes whereby a particular functionally adaptive position is maintained—sandpiles, species, markets, and so on. Since we know that U.S. manufacturing firms, for example, are power-law distributed, and that many industries are as well (Andriani and McKelvey,

forthcoming) the valuation basis of trading behavior occurs in the context of interacting fractal structures, such that the “predator-prey” relationships (otherwise called M&A and other competitive activities) underly the multifractal volatility incidents we see in market behavior.

VI. CONCLUSION

Under the efficient market paradigm, rational investors find undervalued stocks to buy and for the most part will exercise a buy and hold strategy. There will be very little trading in the undervalued zone because of a lack of supply at this price as traders would only sell in case of liquidity need. The price quickly adjusts to its fair value—i.e., around the **Triple Point**—where the coexistence of the three phases keeps normally functioning market in the basin of attraction. At this point fair value prevails, noise trading equals information trading and the average net demand is 0. At this level we have very simple rules, information is shared and investors are rational with unlimited abilities to process information. The market quickly adjusts to new information, anomalies are short lived and noise levels are low. We see this as the juxtaposition and rapid oscillation among Knight’s (1921) elements of risk, uncertainty, and certainty. Under these conditions, linear models will provide appropriate approximations. EMH also states that it is not possible to beat the market, since arbitrage opportunities are short lived and any anomalies are randomly distributed. Net demand is zero and the distribution of orders is symmetric around zero.

In reality, of course, institutions and individual investors, in an attempt to beat the market, introduce hedging strategies, trading rules, derivative securities, etc. and the complexity of the financial trading system increases. As complexity rises, heterogeneous agents switch increasingly from information trading to rule-governed behavior such that the ratio of noise traders to information trader increases. It has been shown that increased noise in the market leads to bimodal net demand distribution (bifurcation) and the buildup of bubbles. Bubbles are built when noise- and risk-based trading increases

as a result of an initiating event and the market moves away from the basin of attraction. However, the market may restore to the **Triple Point** (soft landing) or reach the **Critical Point** when the bubble bursts.

We suggest a number of scale-free theories assembled by Andriani and McKelvey (forthcoming) to explain the various trader-behaviors that serve to move trading behaviors from the Triple Point to, perhaps, the Critical Point. All of these serve to reduce trader heterogeneity. We know from LeBaron 's (2005) computational model that loss of heterogeneity results in market crashes. Since all of the scale-free theories explain Pareto-distributed, or in other words, power-law distributed, our contribution is to offer explanations for the various power-law distributed and multifractal volatility distributions that Sornette (2003 a,b), Sornette et al. (1996;2002), Sornette and Johansen(1998, 2001), Johansen, et al.(2000) and others find occurring between the Triple and Critical Points.

With the increase of noise trading, prices destabilize and periodic orbits emerge as demand distribution bifurcates. The boundary between buy and sell states is crossed multiple times forming log periodic oscillations in the price. Coevolution of trading rules towards one super rule (Lebaron, 2001) leads to order in the market as all traders adopt the same decision and the market collapses (Johansen and Sornette, 1997).

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