Ambiguity, Information Quality and Credit Risk

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Abstract

This paper studies the implications of ambiguity for the credit spreads. We consider two ways of incorporating ambiguity into the Duffie and Lando [2001] model of credit spreads under incomplete information. We begin by examining the credit spreads in a setting where news about the asset value are of an uncertain quality. In this setting, ambiguity-averse investors act as if they take the worst-case assessment of quality. Thus, they react more strongly to bad news than to good news. As a result, changes in the information environment can lead to a widening of credit spreads and implied default intensities, even if firm fundamentals do not change. If the underlying asset value dynamics are also ambiguous, then the ambiguity-averse investors in the secondary debt market act as if the drift rate of the underlying asset value is lower than it actually is. We calibrate the two competing models to match the observed increase in credit spreads after August 9, 2007. Comparing the implications of the two models for return volatilities, we find that ambiguity about the underlying asset value dynamics is better able to match the patterns observed in the data.

1 Introduction

In the summer of 2007, the world financial markets entered into a severe liquidity crisis. On August 9, 2007, France's largest bank BNP Paribas announced that it was having difficulties because two of its off-balance-sheet funds had loaded up on securities based on American subprime mortgages. But Paribas was not alone in its troubles: a month before, the German bank IKB announced similar difficulties, and the Paribas announcement was followed the next day by Northern Rock's revelation that it had only had enough reserve cash to last until the end of the month. These and other similar announcements lead to a freeze of the credit markets as banks lost faith in each other's balance sheets. The situation was particular surprising considering the market conditions shortly before the crisis began. At the beginning of 2007, financial markets were liquidity-unconstrained and credit spreads were at historical lows (see Fig. 1). Even as late as May 2007, it would have been hard to predict the magnitude of the response that the losses on subprime mortgages had generated. Compared to the total value of financial instruments traded worldwide, the subprime losses were relatively small: even the worst-case estimates put them at around USD 250 billion [1]. Further, for investors familiar with the instruments, the losses were not unexpected. By definition, the subprime mortgages were part of the riskiest segment of the mortgage market, so it was hardly surprising some borrowers would default on the loans. Yet, despite their predictability, the defaults had precipitated the current liquidity crisis that spread between the credit markets.

As Caballero and Krishnamurthy [Forthcoming] argue, the crisis was primarily caused by the rise in ambiguity that followed the subprime defaults. That is, although the value of the underlying assets did not necessarily change, the credit market participants were taken by surprise at how the complex credit derivatives were reacting and became uncertain about the quality of their investments. The rise of the complex credit derivatives in the previous five years proliferated the problem. Because investors had no prior experience with how these instruments would behave in a time of crisis, they did not know how to interpret the securities' response to mortgage failure. The complexity of the instruments involved made it almost impossible to calculate the proper response to changes in the underlying. Thus, when even AAA-rated subprime tranches suffered

[1] Source: Caballero and Krishnamurthy [2008]

losses, the resulting increase in ambiguity about the quality of investments in place lead market participants to make decisions based on their perceived worst-case scenario. The revelations by major banks about their exposure to subprime mortgages through off-balance sheet funds only served to reinforce the increase in ambiguity about signals from the major market participants.

The current paper takes the Caballero and Krishnamurthy [Forthcoming] argument – that the credit freeze was precipitated by a sudden increase in ambiguity – as given and considers the implications of an increase in ambiguity for credit spreads. We extend the model of Duffie and Lando [2001] by introducing ambiguity into the secondary debt market in two different ways. First, we assume that the participants in the secondary debt market do not know the information quality of the signal about the underlying asset value. This assumption generates an asymmetric response to positive and negative news, with negative news being interpreted by the agents as having the highest possible precision and positive as having the lowest possible. As an alternative specification, we also consider the case when investors are ambiguous about the asset value dynamics itself. To capture the Caballero and Krishnamurthy [2008] observation that the increase in ambiguity has lead investors to evaluate investment under the worst-case scenario, we specify the debt holder's aversion to ambiguity by a max-min expected utility optimization problem. The axiomatic basis for preferences of this type was first introduced by Gilboa and Schmeidler [1989] and extended to the case of intertemporal utility maximization by Epstein and Schneider [2003].

A rapidly growing literature studies the behavior of asset prices in the presence of ambiguity in dynamic economies. A substantial part of this literature considers investor ambiguity about the data-generating model. Anderson et al. [2003] derive the pricing semigroups associated with robust perturbations of the true state probability law. Trojani and Vanini [2002] use their framework to address the equity premium and the interest rate puzzles, while Leippold et al. [Forthcoming] consider also the excess volatility puzzle. Gagliardini et al. [Forthcoming] study the term structure implications of adding ambiguity to a production economy. This setting has also been used to study the portfolio behavior of ambiguity-averse investors and the implications for the options markets (see e.g. Trojani and Vanini [2004] and Liu et al. [2005]).

The second strand in the literature, however, assumes that, although the agents in the economy know the "true" data-generating model, they face uncertainty about the quality of the observed signal about an unobservable underlying. Chen and Epstein [2002] study the equity premium and the interest rate puzzles in this set-up, and Epstein and Schneider [2008] consider the implications for the excess volatility puzzle. The portfolio allocation implications of this setting have also been studied extensively in e.g. Uppal and Wang [2003] and Epstein and Miao [2003].

However, none of these papers study the relationship between ambiguity aversion and the term structure of credit spreads. We begin by introducing ambiguity aversion using the simple setting of Epstein and Schneider [2008], who measure ambiguity by the size of the set of possible signal precisions. In this way, we can easily study the impact of an increase in the signal quality ambiguity on the term structure of credit spreads, in dependence on the boundaries of the set of possible signal precisions. The preferences underlying the max-min expected utility problem solved by our representative agent are of the recursive multiple priors utility type, which, in the terminology of Epstein and Schneider [2003], implies a "rectangular" set of relevant likelihoods and admits an axiomatic foundation.

Using this setting to extend the model of Duffie and Lando [2001], we find that, under the max-min preferences, the ambiguity-averse participants in the secondary debt market will always choose the lowest signal precision in evaluating positive news about the corporate bonds and the highest signal precision when evaluating negative news. This is exactly in line with the observation of Caballero and Krishnamurthy [2008] that the current credit crisis prompted investor to consider the worst-possible case. Notice also that this coincides with the result of Epstein and Schneider [2008] who find that, when evaluating signals about a company's cash-flows, equity investors interpret negative signals as very precise and positive signals as having low precision. Hence, in our case, since the accounting signals received were negative, the lower bound of signal variance needs to increase to generate changes in the credit spreads. Using the calibration of Duffie and Lando [2001] as the parameters of the economy before August 9, 2007, we find that, in order to generate the observed changes in the credit spreads, the upper bound of signal precision had to decrease to at least 60%. Further, the decrease in signal quality was asymmetric between the Aaa and Baa bonds, with the quality of information decreasing more for the Aaa bonds. Finally, we consider the behavior of the market-perceived default intensities and find that, although there was a substantial increase in the perceived default intensities, the levels remained low for both Aaa and Baa bonds.

Next, we consider the setting where investors know the signal precision but are unsure about the model for the underlying asset value dynamics. In this setting, participants in the secondary debt market will always choose the likelihood corresponding to the lowest possible expected drift rate. As before, we use the calibration of Duffie and Lando [2001] as the base case, and find that the ambiguity tolerance (that is, the size of the set of likelihoods considered plausible by the debt holders) of the debt holders has to increase substantially to match the observed changes in the credit spreads. Unlike the case of ambiguity about the signal quality, however, the corresponding increase in default intensities is much smaller: around 600% for Aaa-rated firms and 200% for Baa-rated firms. This suggests that a way to distinguish these two types of ambiguity in the data is by considering changes in the default intensities that are contemporaneous to changes in credit spreads.

To further distinguish between the two models, we consider their implications for the instantaneous volatility of the zero-coupon bond returns. We find that, although both models generate plausible magnitudes for the volatility, ambiguity about information quality leads to an implausible response of volatility to increases in ambiguity. More specifically, in the presence of ambiguity about the information quality of accounting signals, an increase in ambiguity leads to a decrease in the instantaneous volatility. In the presence of ambiguity about the underlying asset dynamics, however, an increase in ambiguity leads to an increase in volatility, which is what we observe in the data. Thus, we conclude that ambiguity about the underlying is more suitable in this setting.

The rest of the paper is organized as follows. In Section 2, we recall the setting of Duffie and Lando [2001] and provide some necessary definitions. Section 3 presents the problem in the presence of ambiguity about signal quality and Section 4 in the presence of ambiguity about asset value dynamics. We discuss the implications of the increase in ambiguity associated with the current crisis for credit spreads in Section 5 and provide an estimate of how much the ambiguity in the credit markets had to increase to generate the observed movements in the credit spreads. Section 6 concludes. Technical details are relegated to the Appendix.

2 Model

2.1 The firm

The model considered in this paper extends the model of Duffie and Lando [2001] to allow for ambiguity aversion on the part of the participants in the secondary debt market. As in Duffie and Lando [2001], denote by V_t the stock of assets of a given firm and assume that it evolves according to a geometric Brownian motion. In particular, let $V_t = e^{Z_t}$ and assume:

$$dZ_t = mdt + \sigma dW_t, \qquad t \geq 0, \tag{2.1}$$

where W_t is a standard Wiener process. The above specification implies that the stock of assets grows at an expected rate of $\mu = m + \frac{1}{2}\sigma^2$.

While the firm is in operation, it generates cash flows equal to a constant fraction, δ, of the asset value, so that at time t the cash-flows of the firm are given by δV_t. At time 0, the firm issues debt for some amount D with coupon rate $C > 0$. As in Duffie and Lando [2001], we assume that the coupon rate C is chosen to maximize the total value of the firm (the market value of equity plus the market value of debt) at time 0. The amount of debt issued, D, is then calculated as the expected present value of the cash flows to the bond. To simplify the analysis, we assume that the debt is non-callable and that the tax rate is a constant $\theta \in (0, 1)$.

2.2 Equity owners' problem

The equity owners' problem considered in this paper is the same as in Duffie and Lando [2001]. Here, we recall the main points of the set-up and provide the optimal decision rules.

The firm is operated by risk-neutral equity owners, who discount the future at a constant rate r. The equity owners are assumed to have complete information about the value of the firm's assets, V_t. That is, the equity owners' information set is $\mathcal{F}_t = \sigma\{V_s : s \leq t\}$. Given this information, the equity owners decide when to liquidate the firm at an $\mathcal{F}_t$ - stopping time $\tau : \Omega \to [0, \infty]$. The firm is assumed to be liquidated at the expected present value of the discontinued cash flows:

$$\frac{\delta V_\tau}{r - \mu},$$

but a fraction $\alpha \in [0,1]$ of the assets is lost in liquidation. The cash generated by the liquidation, $(1-\alpha)\delta V_\tau/(r-\mu)$, is distributed among the debt holders. Finally, it is assumed that the proceeds from the initial sale of debt are paid as a cash distribution to the initial shareholders. Thus, the equity holders solve:

$$\sup_{\tau} F(V_0, C, \tau) = \mathbb{E}\left[\int_0^\tau e^{-rt}\left(\delta V_t + (1-\theta)C\right) dt\right]. \tag{2.2}$$

Duffie and Lando [2001] show the following result.

Proposition 2.1. *(Proposition 2.1 of Duffie and Lando [2001])*
Suppose $r > \mu$. Let $v_B(C)$ and $d(V_0, C)$ be defined by:

$$v_B(C) = \frac{(1-\theta)C\gamma(r-\mu)}{r(1+\gamma)\delta} \tag{2.3}$$

$$d(v, C) = \frac{(1-\alpha)v_B(C)\delta}{r-\mu}\left(\frac{v}{v_B(C)}\right)^{-\gamma} + \frac{C}{r}\left[1 - \left(\frac{v}{v_B(C)}\right)^{-\gamma}\right], \tag{2.4}$$

where

$$\gamma = \frac{m + \sqrt{m^2 + 2r\sigma^2}}{\sigma^2}.$$

Then the optimal liquidation problem (2.2) is solved by the first time $\tau\left(v_B(C)\right)$ that V is at or below $V_B = v_B(C)$. The associated initial values of equity and debt are $w(V_0)$ and $d(V_0, C)$, respectively, with w given by:

$$w(v) = \left\{\frac{\delta v}{r-\mu} - \frac{v_B(C)\delta}{r-\mu}\left(\frac{v}{v_B(C)}\right)^{-\gamma} + (\theta-1)\frac{C}{r}\left[1 - \left(\frac{v}{v_B(C)}\right)^{-\gamma}\right]\right\}\mathbf{1}_{\{v \geq v_B(C)\}}, \tag{2.5}$$

where $\mathbf{1}_{\{\cdot\}}$ is the indicator function.

2.3 Debt holder's problem

Unlike the equity owners, the participants in the secondary debt market do not observe the asset valuation process, V_t, directly. Instead, they receive imperfect accounting signals, $\hat{V}_t$, at selected times $t_1 < t_2 < \ldots$ More specifically, we assume that $Y(t) \equiv \log \hat{V}_t = Z(t) + U(t)$, where $U(t)$ is normally distributed and independent of $Z(t)$. In the following sections, we examine the debt owners' problem in the presence of ambiguity about the signal quality (Section 3) and about the underlying asset dynamic (Section 4). Below, we present the problem of participants in the secondary debt market for a single likelihood, over both the signal quality and the asset dynamics. Denote by $\sigma_u^2 = \text{var}(U_t)$ the variance of the signal error; the quality of signal is thus given by $1/\sigma_u^2$. We assume that the agents believe the accounting signal to be unbiased, so that $\mathbb{E}\left[e^{U(t)}\right] = 1$. This implies that, for a given level of signal variance, σ_u^2, the mean error is $\overline{u} \equiv \mathbb{E}[U_t] = -\sigma_u^2/2$.

The participants in the secondary debt markets understand that the optimizing equity owners will force liquidation if the equity value falls below V_B. Further, in addition to the accounting signals, they observe at each $t \in [0, \infty)$ whether the firm has been liquidated. Thus, the information set of the debt holders is given by:

$$\mathcal{H}_t = \sigma\left\{Y(t_1), \ldots, Y(t_n), \mathbf{1}_{\{\tau \leq s\}} :\ 0 \leq s \leq t\right\}, \tag{2.6}$$

for the largest n such that $t_n \leq t$ and $\tau = \tau(V_B)$. As in Duffie and Lando [2001], we assume that equity is not traded in the public market and that equity owners are precluded from trading in the public debt markets. This assumption allows us to keep the simple structure (2.6) for the information set of the secondary market and avoid solving the rational-expectations equilibrium problem with asymmetric information.

Denote: $\underline{v} = \log V_B$, $\tilde{y} = y - \underline{v} - \overline{u}$, $\tilde{z} = z - \underline{v}$, and $\tilde{z}_0 = z_0 - \underline{v}$. Then, from Duffie and Lando [2001], we know that, conditionally on having observed a single noisy observation at time t_1, y, and on Z starting at some given level z_0 at time 0, the time t distribution of z is given by:

$$g(z|y, z_0, t) = \frac{\sqrt{\frac{\beta_0}{\pi}}e^{-J(\tilde{y},\tilde{z},\tilde{z}_0)}\left[1 - \exp\left(\frac{-2\tilde{z}_0\tilde{z}}{\sigma^2 t}\right)\right]}{\exp\left(\frac{\beta_1^2}{4\beta_0} - \beta_3\right)\Phi\left(\frac{\beta_1}{\sqrt{2\beta_0}}\right) - \exp\left(\frac{\beta_2^2}{4\beta_0} - \beta_3\right)\Phi\left(\frac{-\beta_2}{\sqrt{2\beta_0}}\right)}, \tag{2.7}$$

where

$$J(\tilde{y}, \tilde{z}, \tilde{z}_0) = \frac{(\tilde{y} - \tilde{z})^2}{2\sigma_u^2} + \frac{(\tilde{z}_0 + mt - \tilde{z})^2}{2\sigma^2 t} \tag{2.8}$$

$$\beta_0 = \frac{\sigma_u^2 + \sigma^2 t}{2\sigma_u^2\sigma^2 t} \tag{2.9}$$

$$\beta_1 = \frac{\tilde{y}}{\sigma_u^2} + \frac{\tilde{z}_0 + mt}{\sigma^2 t} \tag{2.10}$$

$$\beta_2 = -\beta_1 + 2\frac{\tilde{z}_0}{\sigma^2 t} \tag{2.11}$$

$$\beta_3 = \frac{1}{2}\left(\frac{\tilde{y}^2}{\sigma_u^2} + \frac{(\tilde{z}_0 + mt)^2}{\sigma^2 t}\right). \tag{2.12}$$

Notice that we can interpret $\tilde{y}$ as the new information contained in the signal y. In particular, since we do not update the mean of the accounting error, $\overline{u}$, and since the liquidation threshold $\underline{v}$ is given exogenously, $\tilde{y}$ measures the expected distance to the liquidation barrier, conditional on the signal y. Thus, the sign of $\tilde{y}$ indicates the "direction" of the news. If $\tilde{y} < 0$, then the expected distance to the liquidation threshold is negative so the news contained in the signal is bad. Similarly, if $\tilde{y} > 0$, then the signal carries good news as the expected distance to liquidation is positive.

2.4 Credit spreads, volatility and default intensity

We turn now to the implications of the above model for the term structures of credit yield spreads of the modeled firm. For a given time T to maturity, the yield spread on a given zero-coupon bond selling at a price $\varphi > 0$ is the real number η such that $\varphi = e^{-(r+\eta)T}$. We assume that a bond with maturity $s > t$ issued by the modeled firm recovers some fraction $R(s) \in [0, 1]$ of its face value at default; then the secondary market price $\varphi(t, s)$ at time t of such a bond is given by:

$$\varphi(t,s) = e^{-r(s-t)}p(t,s) - R(s)\int_t^s e^{-r(u-t)}p(t,du), \tag{2.13}$$

where $p(t, s)$ is the probability of survival to time s for a given σ_u^2:

$$p(t,s) = \int_{\underline{v}}^{+\infty} [1 - \pi(s-t, x-\underline{v})]\, g(x|Y_t, z_0, t)dx. \tag{2.14}$$

Here, $\pi(t, x)$ denotes the probability of the first passage of a Brownian motion with drift m and volatility parameter σ from initial condition $x > 0$ to a level below 0 at time t. The expression for π is known in closed form, which we provide in the appendix; for a derivation, see e.g. Harrison [1985] Chapter 1. Using the definition of the credit spread, we can rewrite (2.13) as:

$$e^{-\eta(s-t)} = p(t,s) - R(s)\int_t^s e^{-r(u-s)}p(t,du).$$

As in Duffie and Lando [2001], we assume that the recovery rate $R(s)$ for a bond with maturity s is proportional to the default-free discount $e^{-r(s-\tau)}$ for that maturity. In that case, we can rewrite the above as:

$$e^{-\eta(s-t)} = p(t,s) + (1-\nu)(1-p(t,s)), \tag{2.15}$$

where ν is the proportionality parameter. Thus, the calculation of the credit spread requires a single numerical integration, which can be easily done using the importance sampling method.

Consider now the instantaneous volatility of the return on the zero-coupon bond at time t with maturity s. Applying Ito's Lemma, we have that this is given by $\sqrt{\sigma_\varphi \sigma_\varphi'}$, where

$$\sigma_\varphi = \frac{\nu}{\nu p(t,s) + 1 - \nu} \frac{\partial p}{\partial y} [\sigma, \ \sigma_u]' . \tag{2.16}$$

Thus, the instantaneous volatility of the zero-coupon bond return can be calculated as:

$$\frac{\nu}{\nu p(t,s) + 1 - \nu} \left| \frac{\partial p}{\partial y} \right| \sqrt{\sigma^2 + \sigma_u^2}. \tag{2.17}$$

Notice that, even though the bond price is independent of the actual realization of z, the volatility of the true asset value still enter into the instantaneous volatility of the bond returns.

We now turn our attention to the calculation of the corresponding default intensity. Recall that the default intensity λ_t gives a local default rate, in that:

$$\mathbb{P}(\tau \in (t, t+dt] | \mathcal{H}_t) = \lambda_t dt.$$

More formally, the non-negative $\mathcal{H}_t$-measurable process λ is called the intensity process of the default stopping time τ if $\int_0^t \lambda_s ds < \infty$ a.s. for all t and $\mathbf{1}_{\{\tau \le t\}} - \int_0^t \lambda_s ds$ is an $\mathcal{H}_t$-martingale.

From the results of the previous section, we know that the $\mathcal{H}_t$-conditional distribution of Z_t has a continuously-differentiable density $f(t, \cdot, \omega)$ at any (ω, t) such that $0 < t < \tau(\omega)$. Further, this density is zero at the boundary $\underline{v}$ and it's right derivative $f_x(t, \underline{v}, \omega)$ exists and non-zero. Then, from Duffie and Lando [2001], we have:

Proposition 2.2. *(Proposition 2.2 of Duffie and Lando [2001])*
Define a process λ by $\lambda(t) = 0$ for $t > \tau$ and

$$\lambda_t(\omega) = \frac{1}{2}\sigma^2 f_x(t, \underline{v}, \omega), \qquad 0 < t \le \tau. \tag{2.18}$$

Then λ is an $\mathcal{H}_t$-intensity process of τ.

We provide an explicit formula for λ_t in the Appendix.

3 Ambiguity about Information Quality

In this section, we consider the implications of investor ambiguity about the accounting signal quality for credit spreads and perceived default intensities. In particular, assume that in addition to the uncertainty about the true realization of the asset value, V_t, the participants in the secondary market are uncertain about the quality about the quality of the accounting signal and, thus, treat it as ambiguous. That is, the secondary debt market assumes that the signal $\hat{V}_t$ is related to the true realization V_t by a family of likelihoods, parametrized by $\text{var}(U_t) \equiv \sigma_u^2 \in \left[\underline{\sigma}_u^2,\ \overline{\sigma}_u^2\right]$. Notice that, although the signal quality is unknown, we still assume the debt holders believe the accounting report to be unbiased. Thus, the information quality of the accounting signal is captured by a range of precisions $\left[1/\overline{\sigma}_u^2,\ 1/\underline{\sigma}_u^2\right]$.

We assume that, unlike the equity owners, the debt holders have the intertemporal version of the Gilboa and Schmeidler [1989] preferences axiomatized by Epstein and Schneider [2003]. In particular, keeping the assumption that the debt holders are risk-neutral (but ambiguity-averse), the representative agent's utility from a consumption stream C, conditional on having observed a single noisy observation at time t, is given by:

$$U(C_t, t) = \min_{\sigma_u^2 \in [\underline{\sigma}_u^2,\ \overline{\sigma}_u^2]} \mathbb{E}_t \left[\int_0^{+\infty} e^{-ru} C_{t+u} du \right]. \tag{3.1}$$

Since, in equilibrium, the representative participant in the secondary debt market holds all of the secondary market debt, it follows that the worst-case conditional probability minimizes the conditional expected payoffs. Thus, in the presence of ambiguity about signal quality, the time t secondary debt market price of a zero-coupon bond with maturity s is given by:

$$\varphi(t, s) = \min_{\sigma_u^2 \in [\underline{\sigma}_u^2,\ \overline{\sigma}_u^2]} \left\{ e^{-r(s-t)} p(t, s; \sigma_u^2) - R(s) \int_t^s e^{-r(u-t)} p(t, du; \sigma_u^2) \right\}, \tag{3.2}$$

where $p(t, s; \sigma_u^2)$ is the probability of survival to time s for a given $\sigma_u^2 \in [\underline{\sigma}_u^2,\ \overline{\sigma}_u^2]$. Notice that we can rewrite (2.13) as:

$$\varphi(t, s) = e^{-r(s-t)} \min_{\sigma_u^2 \in [\underline{\sigma}_u^2,\ \overline{\sigma}_u^2]} \left\{ p(t, s; \sigma_u^2) - R(s) \int_t^s e^{-r(u-s)} p(t, du; \sigma_u^2) \right\},$$

so that the credit spread is given by:

$$e^{-\eta(s-t)} = \min_{\sigma_u^2 \in [\underline{\sigma}_u^2,\ \overline{\sigma}_u^2]} \left\{ p(t, s; \sigma_u^2) - R(s) \int_t^s e^{-r(u-s)} p(t, du; \sigma_u^2) \right\}. \tag{3.3}$$

We have the following result:

Proposition 3.1. *The worst-case likelihood solving the problem (3.3) is given by:*

$$\tilde{\sigma}_u^2 = \begin{cases} \overline{\sigma}_u^2, & y - \overline{u} > \underline{v} \\ \underline{\sigma}_u^2, & y - \overline{u} < \underline{v} \end{cases} \tag{3.4}$$

Proof. See the Appendix. □

Thus, participants in the secondary debt market treat positive signals (i.e. signals which are above the default threshold even after accounting for the mean signal error) as very imprecise and negative signals as very precise. This result is in line with the result of Epstein and Schneider [2008], who find that, when evaluating signals about a company's cash flows in an uncertain informational environment, equity investors believe negative signals to contain the highest possible informational content and positive signals to have the lowest. Notice that this results differs fundamentally from the standard Bayesian learning. In the presence of multiple priors, the agent who uses Bayesian updating would update the probability of each prior using Bayes' rule. Thus, no one prior would receive full probability weight with finite amount of data, unless the initial probability distribution over possible priors had the same property. Further, with Bayesian updating, negative and positive signals would not necessarily be perceived to have radically different informational quality.

The asymmetric evaluation of signal quality implies that participants in the secondary debt markets underreact to positive news (as defined above) and overreact to negative news. This observation provides the following empirical implication.

Empirical Implication 3.1. *In the presence of ambiguity about signal quality, the participants in the secondary markets underreact to positive news and overreact to negative news. Thus, after a negative accounting report is released, the credit spreads increase more than they decrease following a positive accounting report of similar magnitude. Further, as ambiguity increases, the disparity between these movements in the credit spreads increases.*

4 Ambiguity about Asset Dynamics

Unlike the previous section, assume now that the information quality, σ_u^2, of the accounting report is known but, instead, the participants in the secondary debt market fear misspecifications of the asset value transition law which are sufficiently small so that they are difficult to detect because they are obscured by the random noise, U_t. In particular, we consider the specification in (2.1) as the reference model for the asset value dynamics and assume that the equity owners make the liquidation decisions based on the belief that this is the correct specification. The representative participant in the secondary debt market, however, assumes that the true asset value dynamics are of the form:

$$dZ_t = (m + \sigma h(m,t))\, dt + \sigma dW_t, \tag{4.1}$$

for all $t \geq 0$ and some $h(m) \in \Xi(m)$. Similarly to Leippold et al. [Forthcoming], we assume that the set of admissible disturbances, $\Xi(m)$, is given by:

$$\Xi(m) \equiv \left\{ h(m) : \ \frac{1}{2} h^2(m,t) \leq \eta \ \forall\ t \geq 0 \right\}, \tag{4.2}$$

where $\eta \geq 0$ is the ambiguity tolerance of the debt holder. The above assumption implies that, for finite values of the ambiguity tolerance, the discrepancy between the reference model-implied distributions and those implied by the h-likelihood is constrained to be statistically small. Recall that Anderson et al. [2003] define:

$$\epsilon(h)(m) = \frac{1}{2} h^2(m,t) \tag{4.3}$$

to be the relative entropy between the two probability laws and show this to bound model detection probabilities.

Since we assume that the drift rate of the log asset value, m, is known to the debt holders, some interpretation of the above setting is in order. In this section, we assume that m is a parameter reported to the secondary market by the equity owners. Thus, although the participants in the secondary debt market directly observe m, they do not know for sure that the underlying asset value grows at the rate $\mu = m + \frac{1}{2}\sigma^2$, as reported by the equity owners. Instead, they assume that the asset value evolves according to one of the models parametrized by the set of admissible likelihoods $\Xi(m)$. The underlying asset value then grows at the rate $\mu^h = m + \sigma h(m) + \frac{1}{2}\sigma^2$ for a given likelihood $h(m) \in \Xi(m)$. Notice also that, in some situations, it might be plausible to allow the ambiguity tolerance of the debt holders to depend on the reported value m. For example, it is plausible to assume that η is (weakly) increasing in m. That is, if the equity owners report that the asset value is growing at a low rate, the secondary debt market will consider this to be more likely to be the true rate of asset value growth than if the equity holders report a high rate of growth. In the following, however, we derive the worst-case likelihood without imposing any assumptions on the form of η.

Notice that, since the expected asset growth rate under the perturbed model is still independent of the asset value realization, the filtering solution from Section 2 carries through. That is, for a given likelihood $h(m) \in \Xi(m)$, the conditional distribution of z at time t is given by the expression in (2.7) but with m replaced by $m + \sigma h(m)$. It is worth noting that, unlike the case of ambiguity about signal quality, ambiguity about the asset dynamics impacts the debt holders' perceptions about the liquidation decision rule of the equity owners. In particular, in the presence of ambiguity about the underlying, the (log) liquidation boundary, $\underline{v}$, depends on the likelihood chosen:

$$v_B^h(C) = \frac{(1-\theta)C\gamma^h(r-\mu^h)}{r(1+\gamma^h)\delta}, \tag{4.4}$$

where we now have:

$$\begin{aligned} \gamma^h &= \frac{m+\sigma h(m)+\sqrt{(m+\sigma h(m))^2+2r\sigma^2}}{\sigma^2} \\ \mu^h &= m+\sigma h(m)+\frac{1}{2}\sigma^2. \end{aligned}$$

Similarly to the previous section, we assume that the participants in the secondary debt market have the intertemporal version of Gilboa and Schmeidler [1989] preferences, so that the secondary debt market price at time t of a zero-coupon bond with maturity s is given by:

$$\varphi(t,s) = \inf_{h(m)\in\Xi(m)} \left\{ e^{-r(s-t)}p^h(t,s) - R(s)\int_t^s e^{-r(u-t)}p^h(t,du) \right\}, \tag{4.5}$$

where $p^h(t,s)$ is the probability of survival to time s for a given $h(m) \in \Xi(m)$. We have the following result.

Proposition 4.1. *The worst-case likelihood solving problem (4.5) is given by:*

$$h^*(m,t) = -\sqrt{2\eta}. \tag{4.6}$$

Proof. See the Appendix. □

Thus, in the presence of ambiguity about the asset value dynamics, the participants in the secondary debt markets choose the lowest possible drift rate for the underlying. We have:

Empirical Implication 4.1. *In the presence of ambiguity about the underlying asset value dynamics, participants in the secondary debt markets evaluate assets under the lowest feasible drift rate for the underlying. Thus, all else equal, ambiguity about asset dynamics will lead to high m firms being evaluated as if the true asset value drift is much smaller, leading to the credit spreads on some AAA securities behaving as if the underlying securities have a much lower credit rating. Further, as ambiguity about AAA-rated firms (relatively) increases, the credit spreads on these firms will also increase.*

5 Credit Crisis

In this section, we examine the behavior of credit spreads and default intensities in response to a change in the information quality in a market. As Caballero and Krishnamurthy [2008] argue, when France's largest bank BNP Paribas revealed that two of its off-balance-sheet funds had loaded up on sub-prime mortgage securities, it not only sent a negative signal about its own performance, but also increased the ambiguity present in the market as a whole. Similar revelations by IKB, a German bank, and Northern Rock, a British bank, reinforced the shock to the information quality that the credit market received. Because these revelations dealt with off-balance-sheet debt, banks reacted by losing trust in each other's balance sheets, thereby freezing the credit markets. The effect of the Paribas announcement is well illustrated in the behavior of the credit spread of both Aaa and Baa above the 1 month T-bill rate. From Fig. 1 we can see that both credit spreads increased dramatically immediately following the announcement. A similar effect is observed following the bail-out of Bear Stearns on March 16, 2008. Both of these occurrences suggest a strong response of the credit spreads to deterioration in the market information quality.

Notice that the revelation that banks were exposed to defaults in the sub-prime mortgage market through their off-balance-sheet debt increased ambiguity in the credit markets in two ways. First, these revelations made financial reports seem less plausible, thereby increasing ambiguity about the information quality. The complexity of the securities involved, however, also lead to an increase in the ambiguity about the true “asset” value growth rates for financial institutions. In the following subsections, we consider separately the two sources of ambiguity and examine how much ambiguity would have needed to increase to generate the observed changes in credit spreads.

5.1 Base Calibration

As the base calibration, we use the parameters from Duffie and Lando [2001]. In particular, we assume that, before August 9, 2007, for Aaa-rated firms we have:

$$\theta = 0.35; \qquad \sigma = 0.05; \qquad r = 0.06; \qquad m = 0.01; \qquad \delta = 0.05; \qquad \alpha = 0.3. \tag{5.1}$$

For the Baa-rated firms, we assume that $\sigma = 0.1$ and the remaining parameters are unchanged. Since the solutions for the optimal total coupon rate $C^*(V_0)$ and the optimal liquidation boundary $V_B = v_B(C^*(V_0))$ are linear in V_0, we may assume without loss of generality that $V_0 = 100$. For these parameters, we have that for the Aaa-rated firms, the coupon rate is $C_{Aaa} = 8.00$, the liquidation boundary is $V_{Aaa} = 78.0$, and the initial par debt level $D_{Aaa} = d(V_0, C) = 129.4$. For the Baa-rated firms these values are, respectively, $C_{Baa} = 7.91$, $V_{Baa} = 63.37$ and $D_{Baa} = 121.75$. With these values, we have that the yield to debt for Aaa-rated firms is $C/D = 6.18\%$ and for Baa-rated firms 6.50%. Recovery of the debt at default, as a fraction of face value, is $\delta(r-\mu)^{-1}(1-\alpha)V_B/D = 43.3\%$ for Aaa and 40.5% for Baa bonds. For comparison, the corresponding average recovery of all defaulted bonds monitored by the rating agency Moody's, for 1920 to 1997, is 41%. Finally, we assume that, initially, the participants in the secondary debt market assumed that the standard deviation of the error in the accounting signal was $\sigma_u = 10\%$, for both the Aaa- and the Baa-rated firms and that, initially, the debt owners had no ambiguity about the asset dynamics (so that $\eta = 0$). While there is no direct empirical evidence for these parameters, Duffie and Lando [2001] demonstrate that this parameter combination generates plausible credit spreads for various levels of the initial asset value and observed signals. The full list of calibrated parameters is presented in Table 1.

5.2 Information Quality Ambiguity

In this subsection, we consider the increase in ambiguity about the quality of the accounting reports necessary to generate the increase in credit spreads following the August 9, 2007, Paribas announcement and the March 16, 2008, announcement of the Bear Stearns bail-out. First, consider the observed changes in the credit spreads. From Fig. 2, we see that after August 9, 2007, the Aaa spread increased by 100% in the first three days and the Baa spread increased by 50%. Following the Bear Stearns bailout, the spreads increased by a further 20% for Aaa bonds and 15% for Baa bonds. Consider now the changes in the calculated spreads, presented in Fig. 3. We can see that, to generate the 100% change in the Aaa spread, we need to increase the lower bound of signal volatility, σ_u, to more than 60%. To match the increase 50% in the Baa spreads, we need to increase the lower bound of signal volatility to 60%. Thus, not only did the ambiguity increase after August 9, 2007, but the change in the ambiguity was not the same for Aaa and Baa bonds. Notice that the greater increase in ambiguity about the Aaa bonds corresponds to the fact that the defaults in subprime mortgages brought the stability of Aaa-rated securities into question. In Fig. 4, we plot the change in the Baa-Aaa spread as we increase the ambiguity about the signal quality, assuming that the ambiguity is the same in both cases. To match the 10% in the spread following the Paribas announcement, the common ambiguity has to increase to 15%. Thus, in terms of our model, the Baa-Aaa spread did not increase enough after August 9, 2007.

We also consider the impact of the increase in ambiguity on the default intensities. In Fig. 6, we plot the percent change in the default intensity relative to that corresponding to $\sigma_u = 10\%$. We see that, based on the calibrated the default intensity for Aaa bonds increased by more than 2000% following August 9, 2007; the default intensity for Baa bonds increased by more than 150,000%. Notice, however, that although we observe these dramatic increases, the default intensity for both types of bonds remains small. From Fig. 5, we see that, even for $\sigma_u = 60\%$, the default intensity for the Aaa bonds is 0.005% and for Baa bonds is 0.02%. This suggests that, although the credit crunch precipitated an increase in the ambiguity about the quality of accounting signals, the market-perceived default intensities remain small.

5.3 Asset Dynamics Ambiguity

In this subsection, we calibrate the increase in ambiguity about asset value dynamics that matches the increase in credit spreads during the current credit crisis. We assume that the debt holders' ambiguity tolerance is given by:

$$\eta = \eta m^2. \tag{5.2}$$

In particular, we have that, before the current crisis began, $\eta = 0$. Consider once again the changes in the credit spreads over the risk-free rate first. From Fig. 8, we can see that to match the 100% change in the Aaa spread after the Paribas announcement, η needs to increase to more than 10; to match the 50% increase in the Baa spread, η needs to increase to around 3. Thus, just as with ambiguity about signal quality, the increase in the ambiguity about asset value dynamics for Aaa-rated bonds is much higher than the increase in the ambiguity about Baa-rated bonds. Intuitively, because the current credit crisis has brought into question the Aaa rating of complex securities, debt holders' would have higher ambiguity about the true growth rate of the asset value of such securities. Looking at the Baa-Aaa spread as a function of η, from Fig. 4, we can see that the observed 10% increase can be generated by an increase in η to just 0.5. This implies that, if we were to assume that the ambiguity about Aaa- and Baa-rated bonds increased by equal amounts, then the Baa-Aaa spread did not increase enough in relation to the increase in Aaa and Baa spreads.

Consider now the impact of the increase of ambiguity about asset value dynamics on default intensities. From Fig. 11, we see that corresponding to the calibrated increase in ambiguity about Aaa-rated bonds to $\eta = 10$, the default intensity increased by 600% to 0.002%. The corresponding increase for Baa-rated bonds is around 200%, with the new default intensity around 0.005%. Notice that these changes in default intensities are much lower than those implied by the increase in ambiguity about signal quality. This finding suggests that a way to empirically differentiate between the different sources of ambiguity is to consider observed changes in default intensities for securities with different credit ratings.

5.4 Return volatility

The results of the previous two subsections suggest that both ambiguity about the signal quality and about the underlying are plausible model. In order to distinguish between these two ways of introducing ambiguity into the model, we turn our attention now to the implications for the instantaneous volatility of returns on the zero-coupon bonds. Compare first the magnitudes of volatility generated by these models. From Fig. 7, we see that, in the presence of ambiguity about the information quality, the volatility is around 1-4.5% for Aaa-rated bonds and 1.5-5% for Baa-rated bonds, while, in the presence of ambiguity about the dynamics of the underlying asset value, the volatility is around 4.5-5.5% for Aaa-rated bonds and 5-6% for Baa-rated bonds. For comparison, notice that Kiesel et al. [2003] find an average volatility of about 2.1% for both Aaa- and Baa-rated bonds in the data. Thus, the magnitudes of volatilities suggest that the model with ambiguity about the information quality fits the data better.

Consider, however, the behavior of the instantaneous volatility in the presence of ambiguity about the information quality. From Fig. 7, we can see that, as the signal quality decreases, so that σ_u increases, the volatility decreases as well. This suggests that, during periods when the ambiguity about information quality increases, volatility of bond returns decreases. We find the opposite effect in the data: as with other financial markets, the volatility of returns in the credit markets increases whenever prices decrease (and credit spreads increase).

Consider now the behavior of instantaneous volatility in the presence of ambiguity about the evolution of the underlying asset value. From Fig. 12, we can see that an increase in the ambiguity tolerance of the participants in the secondary debt market (so that the set of feasible mean drift rates for the underlying asset value increases) is accompanied by an increase in the volatility of bond returns. That is, during periods when ambiguity about the underlying increases, the volatility of bond returns increases as well. This behavior better corresponds qualitatively to the previous observation of an increase in the observed volatility of bond returns.

6 Conclusion

In this paper, we have considered the implications of ambiguity for credit spreads. Using the Duffie and Lando [2001] model of credit spreads under incomplete information as a starting point, we introduced ambiguity in two different ways. First, we examined the behavior of credit spreads in the presence of ambiguity about the information quality of the accounting signals that the participants in the secondary debt market receive. We find that, since the ambiguity-averse investors take the worst possible assessment of signal quality, they react more strongly to bad news than to good news. Thus, a deterioration of the information environment leads to a widening of credit spreads, even if the firm fundamentals do not change. Further, even if the informational environment remains constant, ambiguity about signal quality causes an asymmetry in the response of credit spreads to negative and positive signals, with the increase in credit spreads following negative news larger than the corresponding decrease following positive news of similar magnitude.

We continued by introducing ambiguity about the dynamics of the underlying asset value. In this setting, participants in the secondary market as if they believe the asset value growth rate to be the lowest feasible one. Thus, in this setting, it is possible for the credit spreads on Aaa-rated bonds to behave similarly to the credit spreads on Baa-rated bonds, if there is relatively greater ambiguity about the Aaa-rated firm.

We then proceeded to calibrate these two competing models to match the observed increase in credit spreads after August 9, 2007. We find that, for both models, ambiguity about Aaa-rated firms had to increase more than the ambiguity about Baa-rated firms. Further, both models are able to match the increases in credit spreads for reasonable values of model parameters. Next, we compare the implications of the two models (and respective calibrations) for bond return volatilities and find that, although both models generate reasonable magnitudes, ambiguity about the underlying asset value matches better the qualitative patterns found in the data. In particular, ambiguity about information quality causes volatility to decreases when ambiguity increases while ambiguity about the underlying has the opposite effect. We conclude that ambiguity about the underlying asset value dynamics is a better model of ambiguity in the context of credit spreads.

In future research, we plan to examine the implications of having both sources of ambiguity at the same time. This requires finding the right criterion for admissible combinations of the underlying drift rates and signal volatility so that the models are indistinguishable in the data.

A Technical Appendix

A.1 One-sided first passage time distribution

Recall from Harrison [1985], that if we define $T(x)$ the first time at which a random variable Y_t reaches $x > 0$ from below, with $Y_t = 0$, then we have:

$$\mathbb{P}\{T(x) > t\} = \Phi\left(\frac{x - \mu t}{\sigma\sqrt{t}}\right) - e^{2\mu x/\sigma^2}\Phi\left(\frac{-x - \mu t}{\sigma\sqrt{t}}\right),$$

where μ and σ^2 are the drift and variance of the Brownian motion, Y_t, respectively. In our case, we are interested in the mirror-opposite of the above problem: we would like to know the distribution of the first passage hitting time of when a process Y_t, with initial condition $x > 0$, reaches 0 from above. Notice, however, that the one-dimensional Brownian motion is time-reversible. Hence, in the notation of the paper, we have:

$$\pi(t,x) = \Phi\left(\frac{x - mt}{\sigma\sqrt{t}}\right) - e^{2mx/\sigma^2}\Phi\left(\frac{-x - mt}{\sigma\sqrt{t}}\right). \tag{A.1}$$

A.2 Default intensity λ_t

From Harrison [1985], we know that the $\mathcal{H}_t$-conditional distribution of Z_t is given by:

$$f(t,z,\omega) = \frac{1}{\sigma}\exp\left(\frac{mz}{\sigma^2} - \frac{m^2 t}{2\sigma^2}\right) g(z,\omega). \tag{A.2}$$

Thus, taking the derivative with respect to z, we obtain:

$$f_z(t,z,\omega) = \frac{m}{\sigma^3}\exp\left(\frac{mz}{\sigma^2} - \frac{m^2 t}{2\sigma^2}\right) g(z,\omega) + \frac{1}{\sigma}\exp\left(\frac{mz}{\sigma^2} - \frac{m^2 t}{2\sigma^2}\right) g_z(z,\omega).$$

From the definition of g, (2.7), we have:

$$g_z(z,\omega) \propto -\left(\frac{\tilde{y} - \tilde{x}}{\sigma_u^2} + \frac{\tilde{z}_0 + mt - \tilde{z}}{\sigma^2 t}\right) e^{-J(\tilde{y},\tilde{z},\tilde{z}_0)}\left[1 - \exp\left(\frac{-2\tilde{z}_0\tilde{z}}{\sigma^2 t}\right)\right] + \frac{2\tilde{z}_0}{\sigma^2 t} e^{-J(\tilde{y},\tilde{z},\tilde{z}_0)}\exp\left(\frac{-2\tilde{z}_0\tilde{z}}{\sigma^2 t}\right).$$

Evaluating the above at $z = \underline{v}$, we obtain:

$$\lambda = \frac{1}{2}\frac{2\tilde{z}_0}{\sigma t}\frac{\sqrt{\frac{\beta_0}{\pi}}e^{-J(\tilde{y},0,\tilde{z}_0)}}{\exp\left(\frac{\beta_1^2}{4\beta_0} - \beta_3\right)\Phi\left(\frac{\beta_1}{\sqrt{2\beta_0}}\right) - \exp\left(\frac{\beta_2^2}{4\beta_0} - \beta_3\right)\Phi\left(\frac{-\beta_2}{\sqrt{2\beta_0}}\right)}. \tag{A.3}$$

A.3 Proof of Proposition 3.1

Recall that the time t credit spread on a zero-coupon bond with maturity s is given by:

$$\begin{aligned} e^{-\eta(s-t)} &= \min_{\sigma_u^2 \in [\underline{\sigma}_u^2, \overline{\sigma}_u^2]} \left\{ p(t,s;\sigma_u^2) + (1-\nu)[1-p(t,s;\sigma_u^2)] \right\} \\ &= \nu \min_{\sigma_u^2 \in [\underline{\sigma}_u^2, \overline{\sigma}_u^2]} p(t,s;\sigma_u^2) + (1-\nu). \end{aligned}$$

Thus, to find the worst-case likelihood, we need to solve:

$$\min_{\sigma_u^2 \in [\underline{\sigma}_u^2, \overline{\sigma}_u^2]} p(t,s;\sigma_u^2).$$

We have:

$$p(t,s) = \int_{\underline{v}}^{+\infty} [1-\pi(s-t, x-\underline{v})]\, g(x|Y_t, z_0, t) dx.$$

Taking the derivative w.r.t. σ_u^2, we obtain:

$$\begin{aligned} \frac{\partial p}{\partial \sigma_u^2} &= \int_{\underline{v}}^{+\infty} \left[-\frac{\partial \pi(s-t, x-\underline{v})}{\partial \sigma_u^2} g(x|Y_t, z_0, t) + [1-\pi(s-t, x-\underline{v})] \frac{\partial g(x|Y_t, z_0, t)}{\partial \sigma_u^2} \right] dx \\ &= \int_{\underline{v}}^{+\infty} [1-\pi(s-t, x-\underline{v})] \frac{\partial g}{\partial \sigma_u^2}(x|Y_t, z_0, t) dx. \qquad \text{(A.4)} \end{aligned}$$

Recall:

$$g(x|y, z_0, t) = \frac{h(x|y, z_0, t)}{\int_0^{+\infty} h(x|y, z_0, t)},$$

where:

$$h(x|y, z_0, t) = \exp\left[-\frac{(y-\overline{u}-x)^2}{2\sigma_u^2} - \frac{(z_0+mt-z)^2}{2\sigma^2 t} \right] \left[1 - \exp\left(-\frac{2(z_0-\underline{v})(z-\underline{v})}{\sigma^2 t} \right) \right].$$

Hence:

$$\begin{aligned} \frac{\partial g}{\partial \sigma_u^2} &= \frac{\partial h / \partial \sigma_u^2}{\int_0^{+\infty} h dx} - \frac{h}{\left(\int_0^{+\infty} h dx \right)^2} \int_0^{+\infty} \frac{\partial h}{\partial \sigma_u^2} dx \\ &= \frac{1}{2(\sigma_u^2)^2} g(x|y, z_0, t) \left[(y-\overline{u}-x)^2 - \int_0^{+\infty} (y-\overline{u}-x)^2 g(x|y, z_0, t) dx \right] \\ &= \frac{1}{2(\sigma_u^2)^2} g(x|y, z_0, t) \left[(y-\overline{u}-x)^2 - \mathbb{E}_t[(y-\overline{u}-x)^2] \right]. \end{aligned}$$

Notice that:

$$\mathbb{E}_t[(y-\overline{u}-x)^2] = (y-\overline{u}-\underline{v})^2 - 2(y-\overline{u}-\underline{v})(x-\underline{v}) + (x-\underline{v})^2,$$

so, substituting into (A.4), we obtain:

$$\begin{aligned}\frac{\partial p}{\partial \sigma_u^2} &= -\frac{1}{(\sigma_u^2)^2}\int_{\underline{v}}^{+\infty}\left[1-\pi(s-t,x-\underline{v})\right]g(x|Y_t,z_0,t)(Y_t-\overline{u}-\underline{v})\left(x-\underline{v}-\mathbb{E}_t[x-\underline{v}]\right)\\ &\quad +\frac{1}{2(\sigma_u^2)^2}\int_{\underline{v}}^{+\infty}\left[1-\pi(s-t,x-\underline{v})\right]g(x|Y_t,z_0,t)\left((x-\underline{v})^2-\mathbb{E}_t[(x-\underline{v})^2]\right)dx.\end{aligned}$$

Form the above, we can see that, if $y-\overline{u}-\underline{v}>0$, the derivative is negative for all values of σ_u^2, so the worst-case likelihood is given by $\sigma_u^2=\overline{\sigma}_u^2$. Similarly, if $y-\overline{u}-\underline{v}<0$, the derivative is positive for all value of σ_u^2, so the worst-case likelihood is given by $\sigma_u^2=\underline{\sigma}_u^2$.

A.4 Proof of Proposition 4.1

Recall that, in the presence of ambiguity about the asset value dynamics, the time t credit spread on a zero-coupon bond with maturity s is given by:

$$\begin{aligned}e^{-\eta(s-t)} &= \min_{h(m)\in\Xi(m)}\left\{p^h(t,s)+(1-\nu)[1-p^h(t,s)]\right\}\\ &= \nu\min_{h(m)\in\Xi(m)}p^h(t,s)+(1-\nu).\end{aligned}$$

Thus, to find the worst-case likelihood, we need to solve:

$$\min_{h(m)\in\Xi(m)}p^h(t,s).$$

We have:

$$p(t,s)=\int_{\underline{v}}^{+\infty}\left[1-\pi(s-t,x-\underline{v})\right]g(x|Y_t,z_0,t)dx.$$

Notice that, since the choice of h is equivalent to the choice of m (with the feasible set appropriately defined), we can solve the above problem by minimizing over m. Taking the derivative w.r.t. m, we obtain:

$$\begin{aligned}\frac{\partial p}{\partial m} &= \left[1-\pi(s-t,0)\right]g(\underline{v}|Y_t,z_0,t)\frac{\partial \underline{v}}{\partial m}+\\ &\quad +\int_{\underline{v}}^{+\infty}\left[-\frac{\partial\pi(s-t,x-\underline{v})}{\partial m}g(x|Y_t,z_0,t)+\left[1-\pi(s-t,x-\underline{v})\right]\frac{\partial g(x|Y_t,z_0,t)}{\partial m}\right]dx\\ \text{(A.5)}\quad &= \int_{\underline{v}}^{+\infty}\left[-\frac{\partial\pi(s-t,x-\underline{v})}{\partial m}g(x|Y_t,z_0,t)+\left[1-\pi(s-t,x-\underline{v})\right]\frac{\partial g(x|Y_t,z_0,t)}{\partial m}\right]dx.\end{aligned}$$

Similarly to the previous subsection, we have:

$$\begin{aligned}
\frac{\partial \pi}{\partial m} &= -\sqrt{t}\left[\phi\left(\frac{x-mt}{\sigma\sqrt{t}}\right) - e^{2mx/\sigma^2}\phi\left(\frac{-x-mt}{\sigma\sqrt{t}}\right)\right] - 2\frac{x}{\sigma}e^{2mx/\sigma^2}\Phi\left(\frac{-x-mt}{\sigma\sqrt{t}}\right) \\
&= -2\frac{x}{\sigma}e^{2mx/\sigma^2}\Phi\left(\frac{-x-mt}{\sigma\sqrt{t}}\right) \\
\frac{\partial g}{\partial m} &= -\frac{1}{\sigma^2}g(x|y,z_0,t)\left[(z_0+mt-z) - \mathbb{E}_t[z_0+mt-z]\right] \\
&= \frac{1}{\sigma^2}g(x|y,z_0,t)\left[z - \mathbb{E}_t[z]\right].
\end{aligned}$$

Substituting into (A.5), we obtain:

$$\frac{\partial p}{\partial m} = \int_{\underline{v}}^{+\infty}\left[2\frac{x}{\sigma}e^{2mx/\sigma^2}\Phi\left(\frac{-x-mt}{\sigma\sqrt{t}}\right) + [1-\pi(s-t,x-\underline{v})]\frac{1}{\sigma^2}\left[z-\mathbb{E}_t[z]\right]\right]g(x|Y_t,z_0,t)dx,$$

which is positive for all values of m. Thus, p is increasing in m and the minimum is achieved by taking the smallest feasible value of the drift rate. Hence, the minimization problem is solved by

$$h^* = -\sqrt{2\eta}.$$

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Common Parameters							
θ	r	m	δ	α	ν	σ_u^0	η
35%	6%	1%	5%	30%	50%	10%	0
Aaa parameters				Baa parameters			
σ	C	V_B	D	σ	C	V_B	D
5%	8	78	129.4	10%	7.91	63.37	121.75

Table 1: Reference Parameters and Some Base Quantities

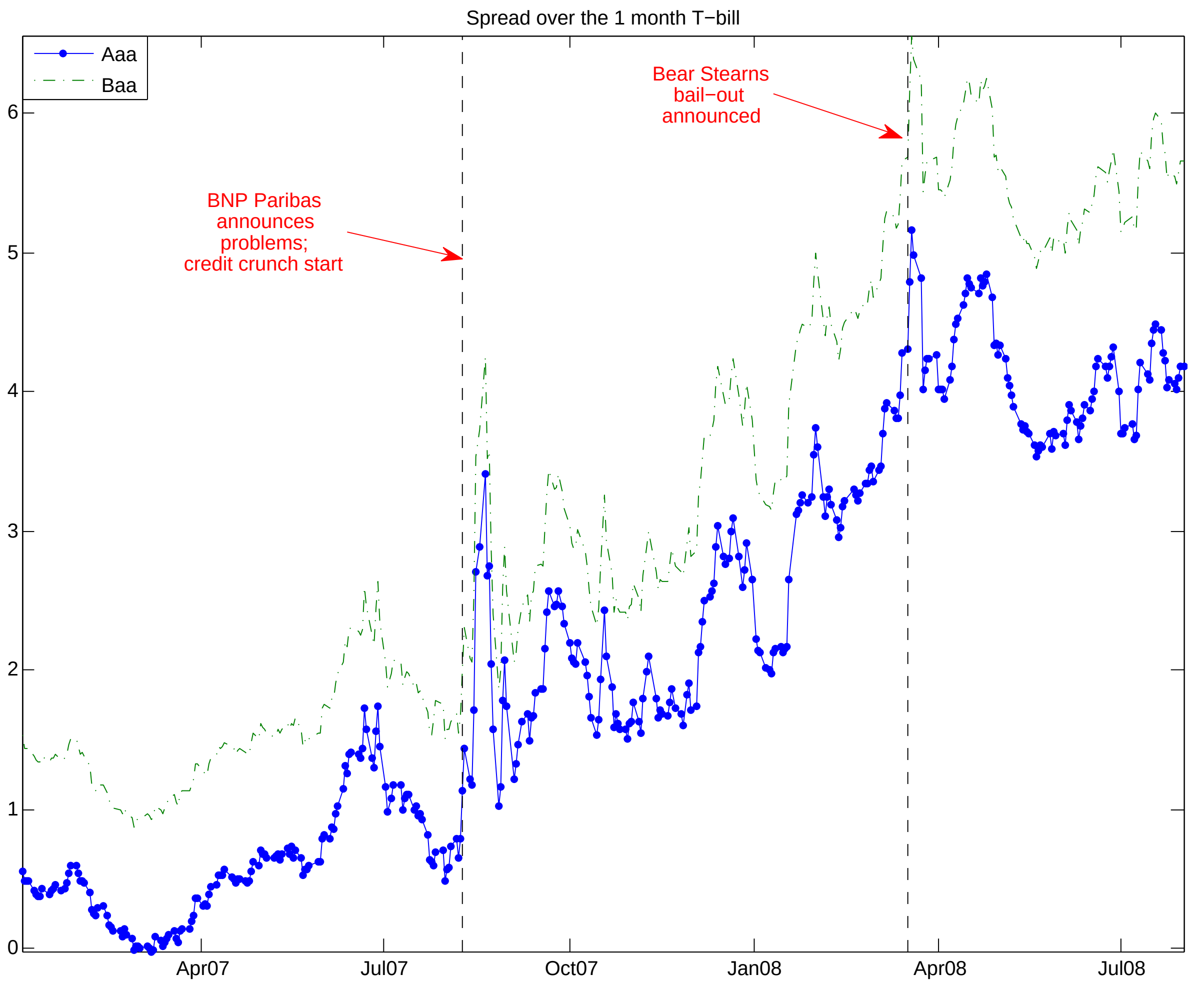

Figure 1: The daily credit spread for Aaa (dash-dot) and Baa (dash) bonds over the one month T-bill rate for the time period January 2007 - August 2008. Source: Moody's.

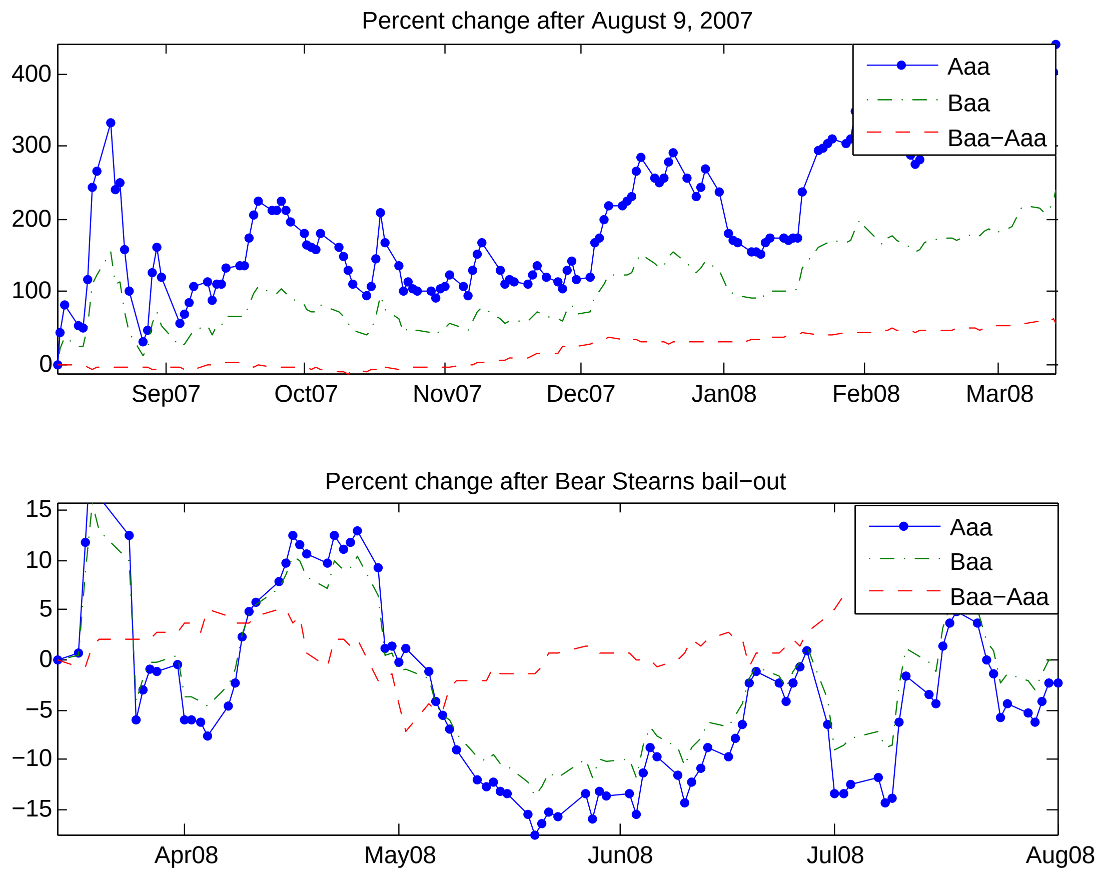

Figure 2: The percent change in the credit spreads after August 9, 2007 (upper panel) and after March 16, 2008 (lower panel). In each case, the change is calculated relative to the level on the previous available day. Source: Moody's.

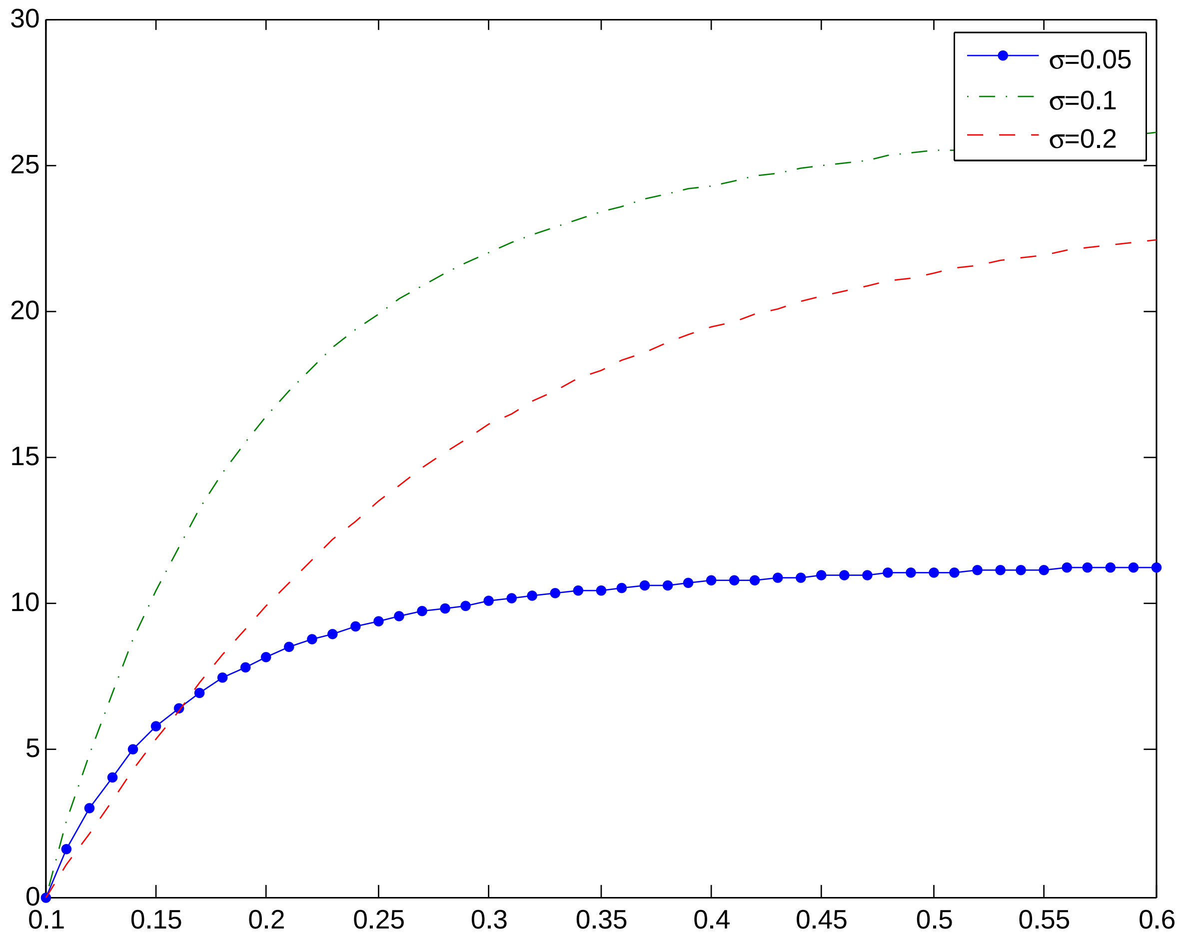

Figure 3: Percent change in the credit spreads relative to the level corresponding to $\sigma_u = 10\%$ for different levels of the asset value volatility, σ. Following Duffie and Lando [2001], the time since the debt valuation is known for sure, t, is set to 1 and the signal value to V_0. The maturity of the bond is set $T = 5$ to match the duration of the Moody reported bonds. The other values are as reported in Table 1.

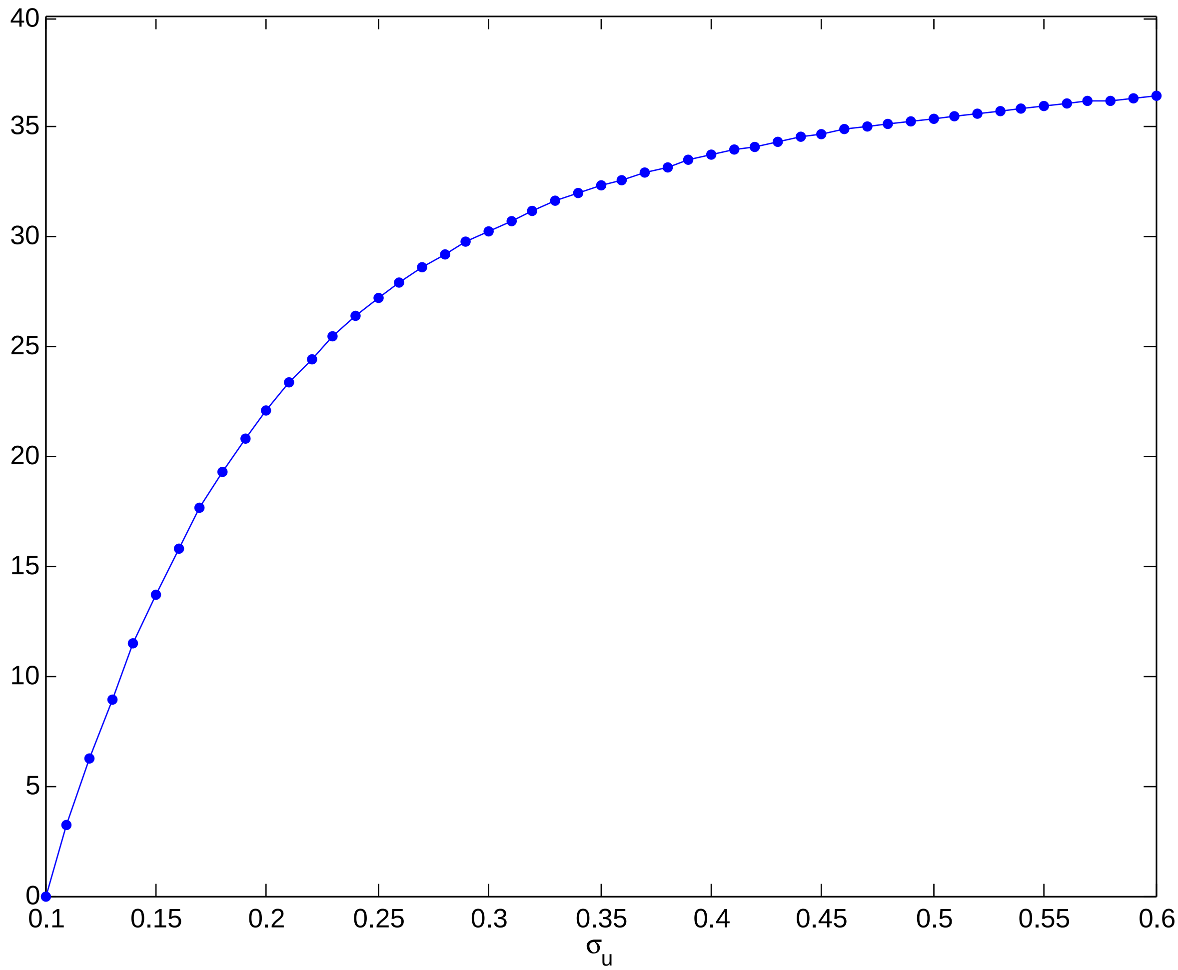

Figure 4: Percent change in the spread between $\sigma = 5\%$ and $\sigma = 10\%$ relative to the level corresponding to $\sigma_u = 10\%$. Following Duffie and Lando [2001], the time since the debt valuation is known for sure, t, is set to 1 and the signal value to V_0. The maturity of the bond is set $T = 5$ to match the duration of the Moody reported bonds. The other values are as reported in Table 1.

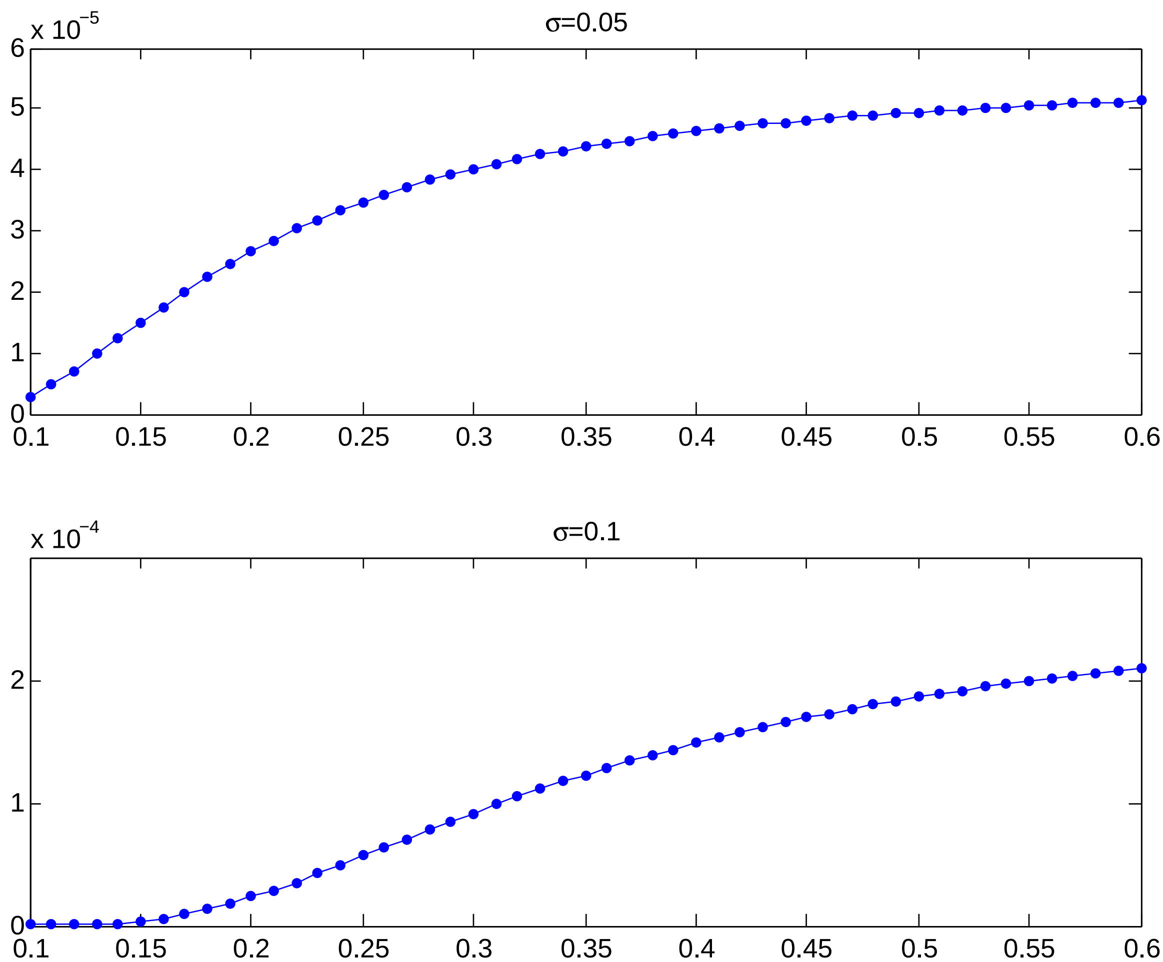

Figure 5: Default intensity, λ, for different levels of the asset value volatility, σ. Following Duffie and Lando [2001], the time since the debt valuation is known for sure, t, is set to 1 and the signal value to V_0. The maturity of the bond is set $T = 5$ to match the duration of the Moody reported bonds. The other values are as reported in Table 1.

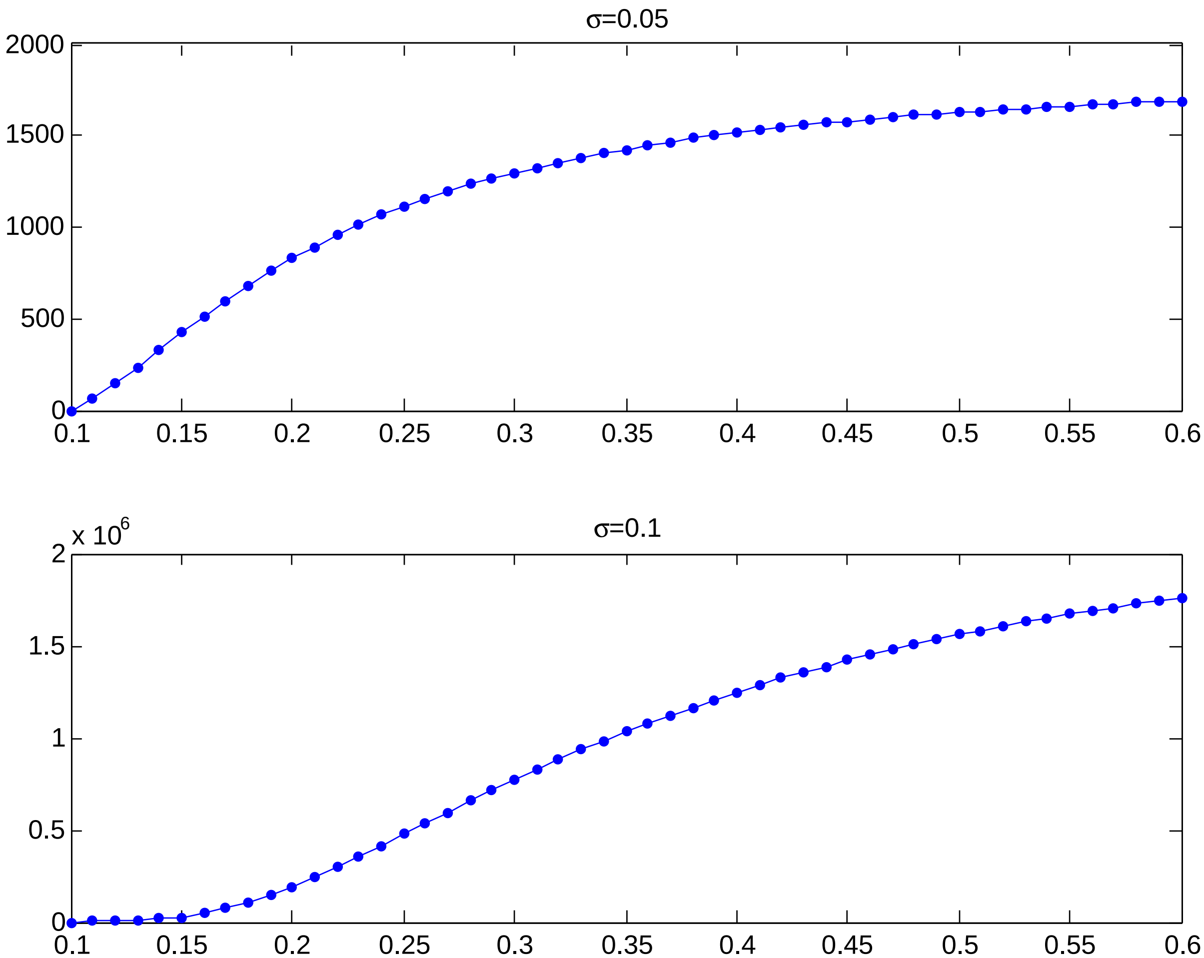

Figure 6: Percent change in the default intensities relative to the level corresponding to $\sigma_u = 10\%$ for different levels of the asset value volatility, σ. Following Duffie and Lando [2001], the time since the debt valuation is known for sure, t, is set to 1 and the signal value to V_0. The maturity of the bond is set $T = 5$ to match the duration of the Moody reported bonds. The other values are as reported in Table 1.

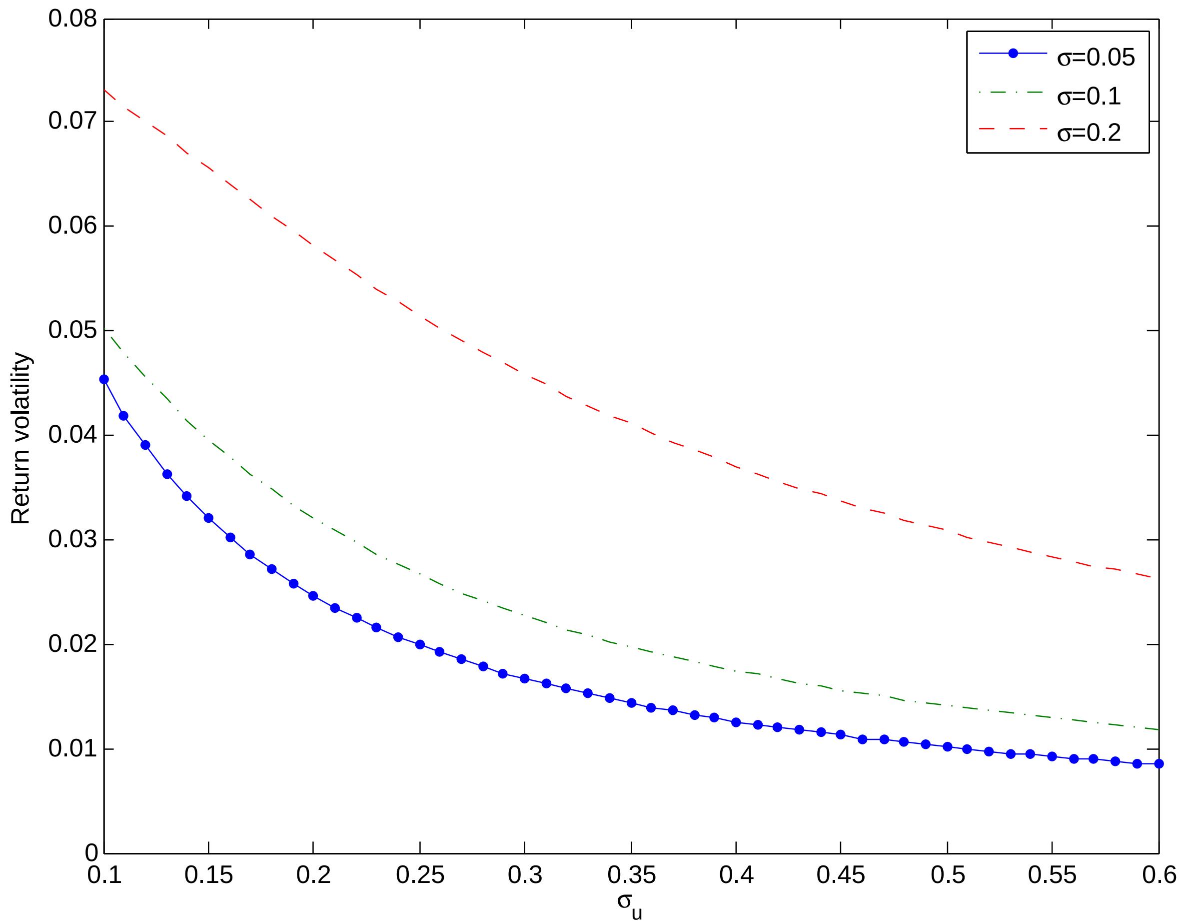

Figure 7: Instantaneous volatility of bond returns for different levels of the asset value volatility, σ. Following Duffie and Lando [2001], the time since the debt valuation is known for sure, t, is set to 1 and the signal value to V_0. The maturity of the bond is set $T = 5$ to match the duration of the Moody reported bonds. The other values are as reported in Table 1.

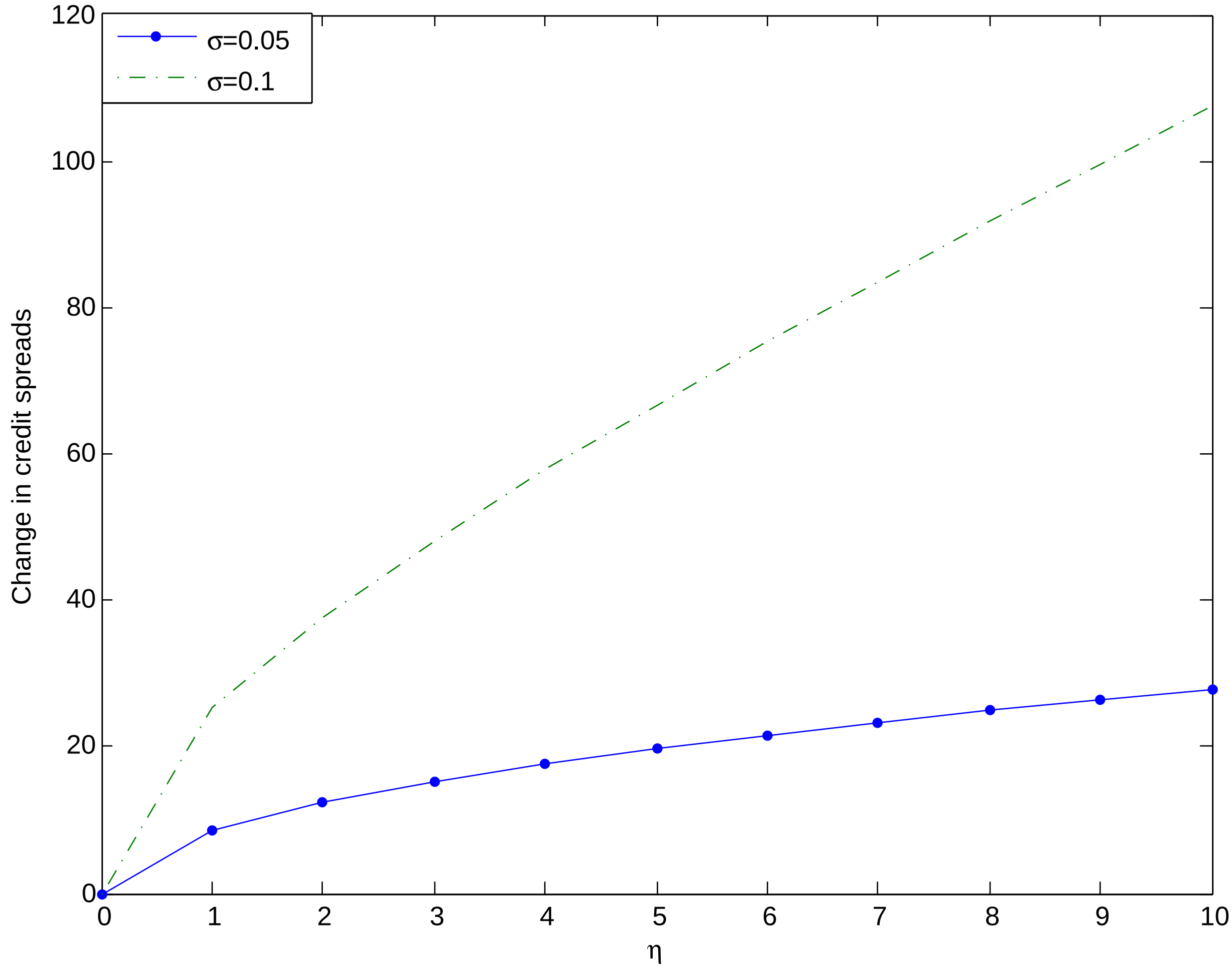

Figure 8: Percent change in the credit spreads relative to the level corresponding to $\eta = 0$ for different levels of the asset value volatility, σ. The standard deviation of the observation error is assumed to be $\sigma_u = 10\%$. Following Duffie and Lando [2001], the time since the debt valuation is known for sure, t, is set to 1 and the signal value to V_0. The maturity of the bond is set $T = 5$ to match the duration of the Moody reported bonds. The other values are as reported in Table 1.

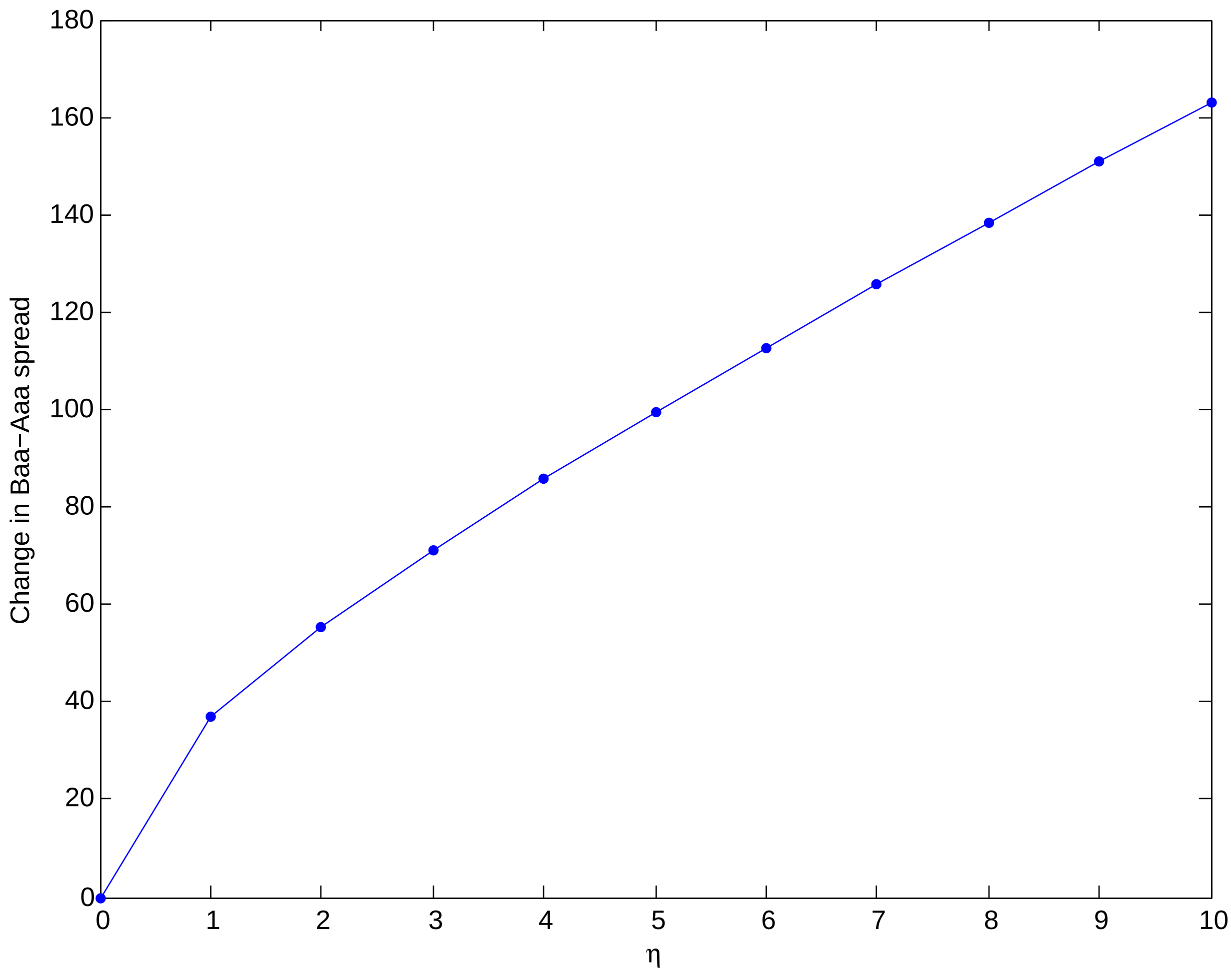

Figure 9: Percent change in the spread between $\sigma = 5\%$ and $\sigma = 10\%$ relative to the level corresponding to $\eta = 0$. The standard deviation of the observation error is assumed to be $\sigma_u = 10\%$. Following Duffie and Lando [2001], the time since the debt valuation is known for sure, t, is set to 1 and the signal value to V_0. The maturity of the bond is set $T = 5$ to match the duration of the Moody reported bonds. The other values are as reported in Table 1.

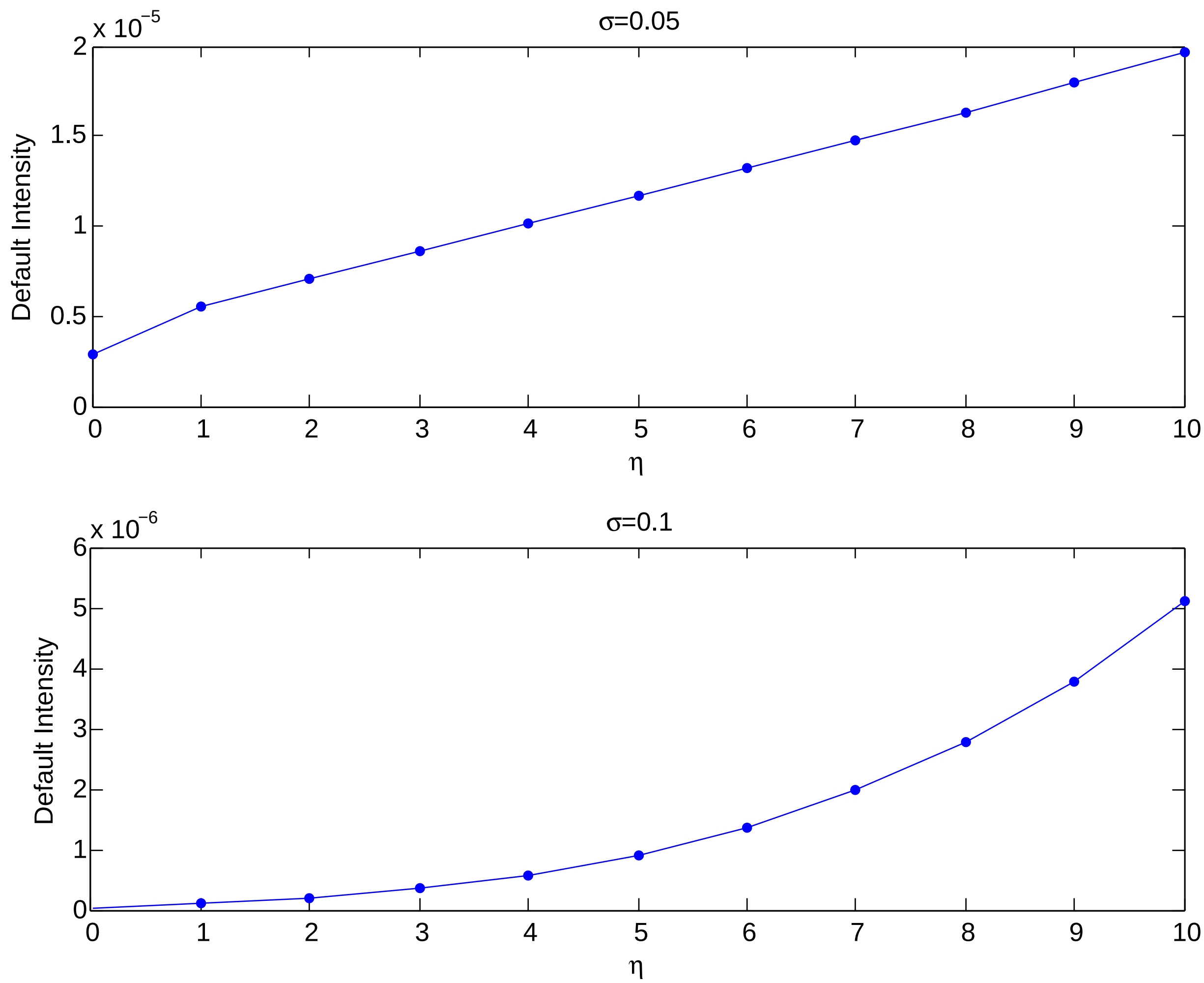

Figure 10: Default intensity, λ, for different levels of the asset value volatility, σ. The standard deviation of the observation error is assumed to be $\sigma_u = 10\%$. Following Duffie and Lando [2001], the time since the debt valuation is known for sure, t, is set to 1 and the signal value to V_0. The maturity of the bond is set $T = 5$ to match the duration of the Moody reported bonds. The other values are as reported in Table 1.

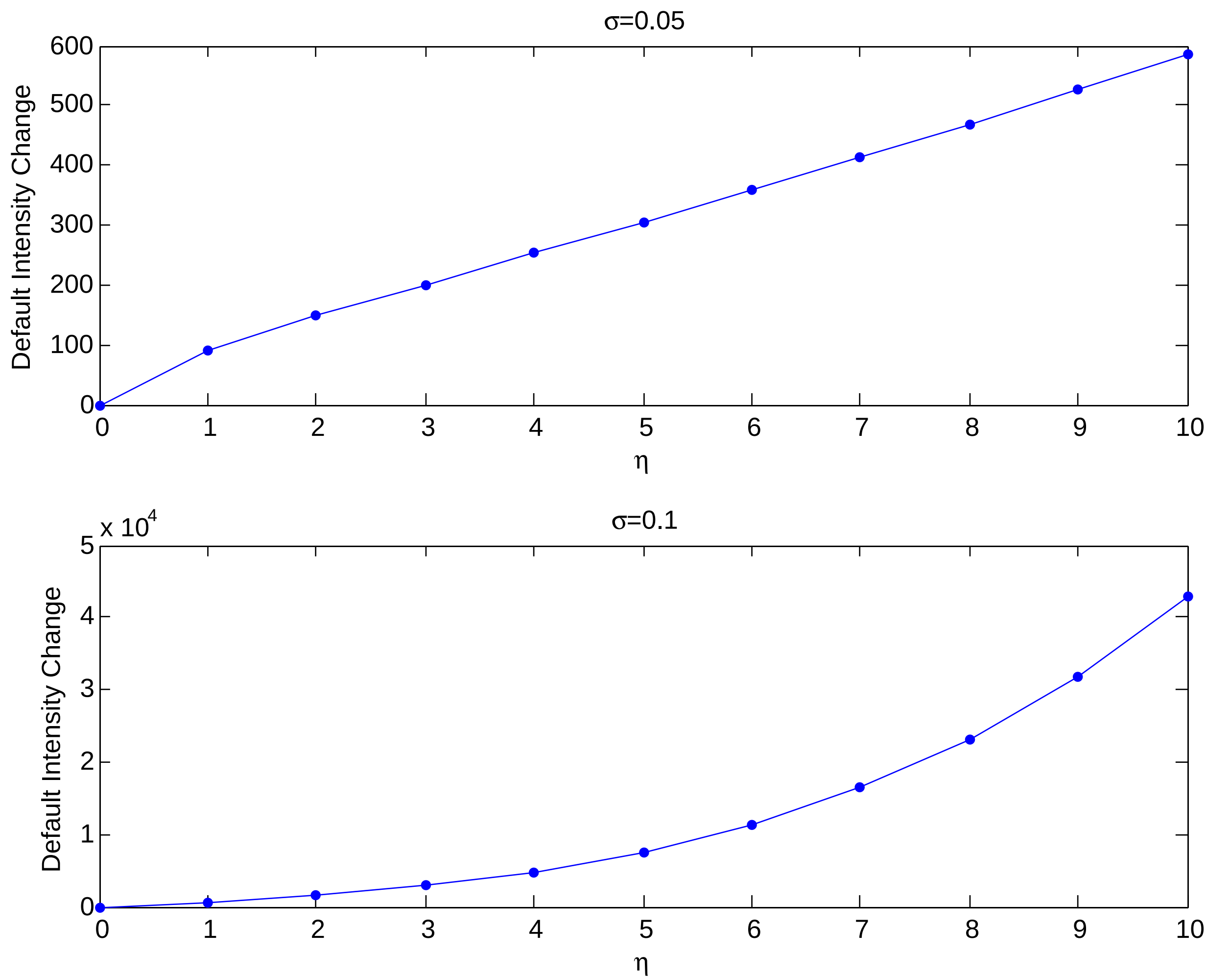

Figure 11: Percent change in the default intensities relative to the level corresponding to $\eta = 0$ for different levels of the asset value volatility, σ. The standard deviation of the observation error is assumed to be $\sigma_u = 10\%$. Following Duffie and Lando [2001], the time since the debt valuation is known for sure, t, is set to 1 and the signal value to V_0. The maturity of the bond is set $T = 5$ to match the duration of the Moody reported bonds. The other values are as reported in Table 1.

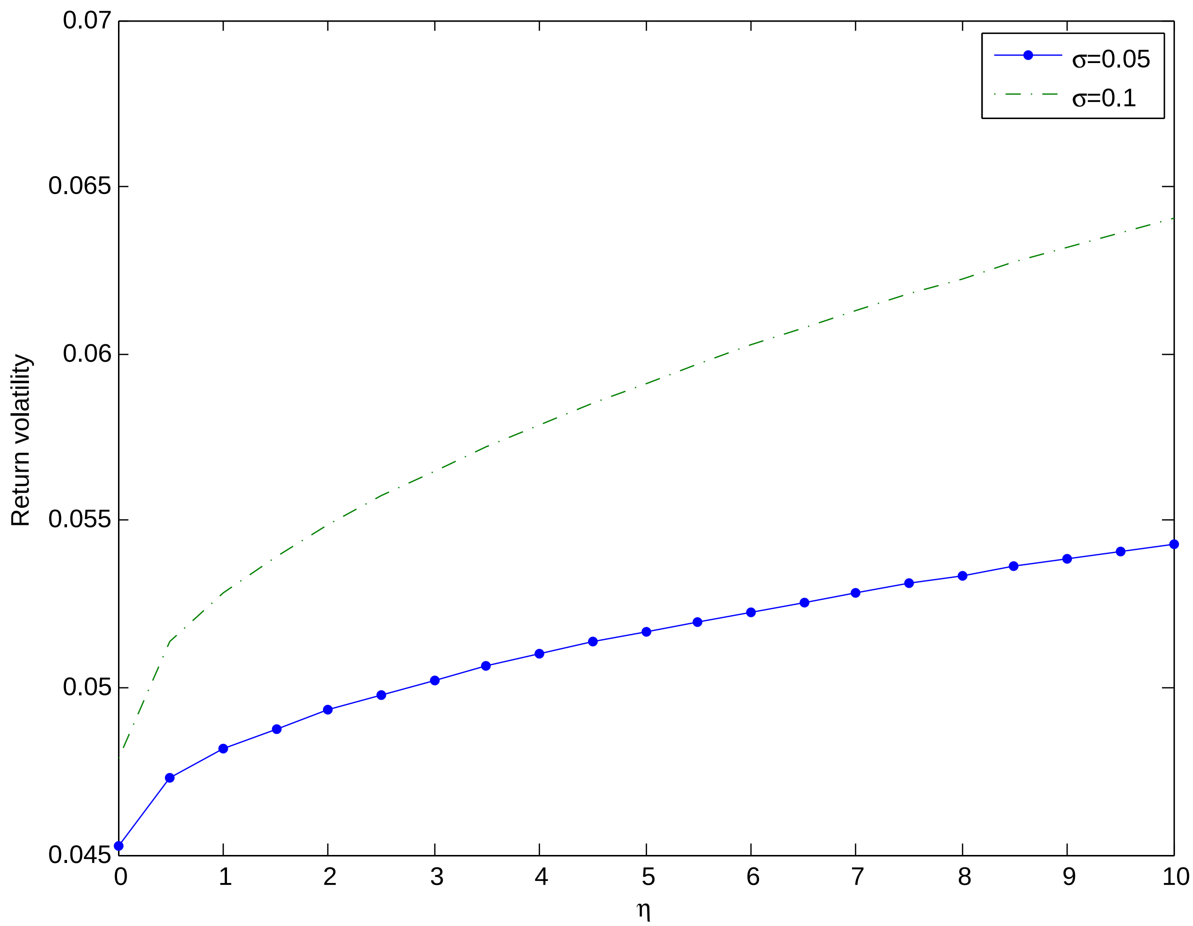

Figure 12: Instantaneous volatility of bond returns for different levels of the asset value volatility, σ. The standard deviation of the observation error is assumed to be $\sigma_u = 10\%$. Following Duffie and Lando [2001], the time since the debt valuation is known for sure, t, is set to 1 and the signal value to V_0. The maturity of the bond is set $T = 5$ to match the duration of the Moody reported bonds. The other values are as reported in Table 1.