An Analysis of Government Loan Guarantees and Direct Investment through Public-Private Partnerships *

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Abstract

This paper compares two forms of government support: loan guarantee and direct investment through public-private partnerships (PPPs). With loan guarantee, government provides financial guarantees to enhance project creditworthiness. With direct investment, government invests capital in return for shares in the project. We find that loan guarantees are more effective in reducing project borrowing costs. In an informationally asymmetric environment, where the government knows less about project quality than do private partners, in other words the so-called plum problem rather than the familiar lemon problem, the project sponsors should seek a loan guarantee from the government, unless they are willing to give up control over the project. We show how the portion of shares given to the government can be a bargaining tool and can mitigate information asymmetry when structuring PPPs.

Keywords: Government loan guarantee, financial guarantees, government direct investment, public-private partnerships (PPPs).

1 Introduction

The last few decades have seen an unprecedented increase in capital-intensive projects worldwide that are financed mainly through special-purpose vehicles or entities (SPV/SPE).[1] The literature points to many advantages of SPVs or SPEs such as mitigation of underinvestment, lower agency costs of free cash flows, less information asymmetry and signal costs, better structuring of debt, containment of risk, improved corporate organization and management compensation, and better corporate governance (e.g., Finnerty (2007), Gatti (2008) and Subramanian et al. (2007)). Since capital-intensive investments, such as public infrastructures, green energy, and other forms of sustainable development, involve huge amounts of financing and are highly levered (e.g., Esty (2003, 2004)), project sponsors usually resort to government participation to share project risk and improve their creditworthiness.

Government participation can take many forms, such as guaranteeing loans, entering into a public-private partnership (PPP), and aiming for better cost management and resource allocation (e.g., Grimsey and Lewis (2002) and Grout (2003)).[2] At close quarters, recently, the Obama Administration proposed public-private partnership investment programs as part of their rescue plan to tackle the current financial crisis.[3]

[1] A well-known form of capital-intensive financing is project finance. Project finance is an arrangement where one or more sponsors (shareholders) create a project company with a view to repaying the lender largely out of the project's future cash flow. According to Esty (2004), project-financed investments have grown at a compound rate of almost 20 percent in recent years. In 2004, project finance fueled a total of $234 billion in capital spending, up from $172 billion in 2003.

[2] Public-private partnerships (PPPs) can take different forms such as build-operate-transfer (BOT), build-transfer-operate (BTO), build-own-operate (BOO), buy-build-operate (BBO), and design-build-operate (DBO), among others (e.g., Esty (2004), Finnerty (2007), FitchRatings (2004), Yescombe (2007)). Since the focus is not to study the different forms of PPPs per se, we leave these interesting issues for future research.

[3] Indeed, on March 23rd, 2009, the New York Times Quoted "Administration officials outlined a three-part Public-Private Investment Program that offers private investors vast amounts of cheap, taxpayer-supported financing for every dollar they put up of their own money." in U.S. Expands Plan to Buy Banks' Troubled Assets, by Edmund L. Andrews and Eric Dash. The same day, ABC News reported "The plan aims to remove so-called toxic assets – many of them bad mortgage investments – from the banks' balance sheets through a private-public partnership. The program will rely heavily on private investors, such as hedge funds and private-equity firms, to buy up $500 billion to $1 trillion of assets with the government providing incentives such as low interest loans and sharing in both the risk and possible profits." in Obama's $1 Trillion Plan to End Bank Crisis: Treasury Secretary Tim Geithner Hopes to Attract Private Investors to Buy Toxic Assets, by Matthew Jaffe, Scott Mayerowitz and Jake Tapper. Interested readers can find more documents introducing the Obama Administration Public-Private Partnership Investment Program at

This paper compares two general forms of government support: loan guarantees and direct investment through public-private partnerships (PPPs). A loan guarantee is a promise from a government or public institution, like export credit agencies (ECA) or multilateral organizations, to make good on loan payments if the project company defaults.[4] With direct investment, the government participates directly in the project by investing an amount of capital in return for shares in the project, thereby, sharing in the profits. In other words the government and the private partners operate as a PPP. With loan guarantees, the government reduces the tax deductible interest payments and thus creates more taxable income for itself. With direct investment, the government receives a share of the profit, in addition to the tax revenue. Both forms of support are viable ways to assist a project that otherwise may be abandoned for lack of financing due to institutional constraints and credit rationing that pervade a capital-intensive environment.

We extend previous studies on the role of government support for project development (e.g., Chaney and Thakor (1985), Galai and Wiener (2003)) by explicitly including information asymmetry between the contracting parties. Indeed, with many capital-intensive investments, the entrepreneurs are better informed than the host government. This is referred to in the literature as the plum problem (e.g., Chen (2005)) as opposed to the lemon problem (e.g., Akerlof (1970)).[5] We have included this feature by introducing information asymmetry between the government and the other stakeholders. We assume that the government knows the distribution of project risk but not its estimated value, while the lenders and the project sponsors are perfectly informed about project risk. When the government and private partners enter into a PPP, the government is granted part of the project profit, which in a perfect world should be highly correlated with the government's contribution. We have studied an agency conflict between the government

http://www.criminallawlibraryblog.com/2009/03/documents_introducing_the_obam_1.html.

[4]Export credit agencies (e.g., US Ex-Im Bank, UK Export Credits Guarantee Department (ECGD), Export Development Canada (EDC), COFACE-France) and multilateral development banks (e.g., African Development Bank, Asian Development Bank, Inter-American Development Bank, World Bank MIGA) are some of the main providers of financial guarantees, especially for large-scale projects (see for example Dailami and Leipziger (1998), Ehrhardt and Irwin (2004), Garcia-Alonso et al. (2004)).

[5]Here, the lemon problem is due to the local government knowing more about the project than do the private partners.

and the private partners by analyzing the difference between the government's actual share of the total profit and its deemed fair share.

We find that, as expected, both forms of government support enhance a project's creditworthiness. All else being equal, a loan guarantee directly reduces the probability of loss for lenders and thus the project's borrowing costs. With an asymmetric information environment in which the government knows less about the project quality than do private partners, i.e., the plum problem, private partners should seek a loan guarantee from the government, unless they are willing to give up more control over the project. With perfect information or a weakly asymmetric information environment, they may gain more from a PPP arrangement than from a loan guarantee. If the government does not receive a large enough share of the profits, the cost of the direct investment will exceed the earnings (e.g., tax revenue, its share of the total profit, and other social benefits). Thus, the government can use its portion of the shares as a bargaining tool, thus mitigating the information asymmetry. It should require more control over the project when information is asymmetrical, especially for very capital-intensive projects.

The remainder of our paper is structured as follows. Section 2 presents the model. In this section, we derive the payoffs to the government and to the project sponsors and we introduce the different forms of government support and their potential impacts on all stakeholders. Section 3 provides a general discussion of the findings through several numerical experiments. Section 4 is the conclusion. The proofs are presented in the appendix.

2 The model

We consider a single project implemented through a special-purpose vehicle (SPV) as a stand-alone firm, i.e., the project is an independent and separate entity. It is owned by sponsors and its cash flows are used to pay off the stakeholders. In this framework, lenders depend on project performance for repayment rather than on the sponsors as such. The principal commitment from the sponsors is their capital contribution.

The project requires an initial investment I. Cash flow at time t is A_t and is characterized by a risk-neutral[6] stochastic process

$$\frac{dA_t}{A_t} = (r_f + g - \delta)dt + \sqrt{V} dZ_{1t}, \tag{1}$$

where r_f is the continuous risk-free interest rate in the economy assumed to be non-stochastic, g the externally financed project asset growth, δ the asset payout rate, $\sqrt{V}$ the volatility of project assets, and Z_{1t} the Wiener process with risk-neutral probability. Project cash flows are thus represented as the present value of all expected cash flows (e.g., Lucas and McDonald (2006)). One concern here is the uncertainty surrounding valuation of future cash flows. We are aware of this point, but since not the main focus of our study, we will assume that the present value of total expected cash flows follows a geometric Brownian motion process with a risk level $\sqrt{V}$ that the project manager has chosen or knows.

We will assume a simple capital structure of single loan and equity contracts. There will be neither dividend payments nor intermediate payments on the debt before it matures. The project will mature when the debt matures, i.e., T. We will assume the existence of corporate taxes. We will also assume intervention by government or an equivalent public authority as with most project financing (see Kleimeier and Megginson (2001), Esty (2003, 2004), Finnerty (2007) for extensive reviews on project finance).

The government may intervene in several ways. Here we will study two forms of government intervention: (i) loan guarantee and (ii) direct investment. Loan guarantee support consists of insuring the project's debt. We will assume that the loan guarantee agreement compels the government to cover debt up to haircut level H. Direct investment support, by comparison, consists of contributing directly to the initial investment in the project. With direct investment support, we will assume the government contributes amount K to the project. Government direct investment may be either investment subsidy or equity participation or both. With equity participation, the government receives α share of net revenue after taxes and debt repayment and the project sponsors keep the residual, i.e., $1 - \alpha$ of the profit.

Therefore, the project is financed with equity (owner-contributed capital), debt (out-

[6]See Harrison and Kreps (1979) and Merton (1973) for the use of contingent claims analysis (CCA) in pricing assets.

side financing), and government's contribution, if any. We denote by C the total capital contribution from the project sponsors, D the total amount borrowed, and K the government's direct investment, if any. Total investment is $I = C + K + D$. Note that K will equal zero with loan guarantee support only, even if the government can obtain warrants or equity shares as compensation for part of its support.

We will assume imperfect information, in the sense that the government knows less about the project than do the sponsors. This is known in the literature as the plum problem, e.g., Chen (2005). As in Mason (1998), we will model information asymmetry using a stochastic variance process for project assets. Unlike Mason (1988), who uses a geometric Brownian motion process, we will adopt the following mean-reverting variance process:

$$dV_t = \nu(\beta - V_t)dt + \xi\sqrt{V_t}dZ_{2t}, \tag{2}$$

where Z_{2t} is the Brownian motion that represents the uncertainty of the volatility process, ν the speed of adjustment, β the long-run mean variance, and ξ the variation of the diffusion volatility that captures the degree of information asymmetry. For small values of ξ, the distribution has little variance and therefore less information asymmetry (e.g., Innes (1991)). Unlike Mason (1998), who follows the Hull and White (1987) in assuming zero correlation between asset return and volatility diffusion, we will instead use the Hull and White (1988) stochastic volatility process while assuming non-zero correlation as follows:

$$\text{corr}(Z_{1t}, Z_{2t}) = \rho dt. \tag{3}$$

Based on this assumption, we provide in the appendix, Hull and White (1988)-derived closed-form solutions for call and put options.

At debt maturity, if the project is successful, the net profit, P, will be total cash flow minus corporate taxes, debt principal, and interest payments:

$$P \;=\; E\Big[e^{-r_f T}\max\big(A_T - (1+R)D - \tau_c \max(A_T - RD - \lambda I, 0), 0\big)\Big], \tag{4}$$

where $E[.]$ is the expectation with risk-neutral probability, λ the depreciation of the initial investment, and R the financing cost of the debt. The first term $A_T - (1+R)D$ is the net profit before taxes and the second term $\tau_c \max(A_T - RD - \lambda I, 0)$ the adjustment for taxes with τ_c the corporate tax rate. The total tax is equivalent to a call option held by the government where the underlying asset is the asset value of the project and the exercise price is the total interest payment plus capital depreciation (e.g., Green and Talmor (1985)). The project benefits from tax shields on interest payments and for depreciation of the initial investment. It is well known that loan guarantees lower borrowing costs R. Since the project benefits from tax shields on interest payments, i.e., RD, the loan guarantee will reduce tax write-offs according to the magnitude of reduction in R. The explicit closed-form expression of equation (4) is in the appendix.

2.1 Lender payoff and participation constraint

The lenders have a participation constraint when the loan is worth less than or as much as the present value of future payments they will receive. Similar to Merton (1974), the value of non-guaranteed risky debt is equal to the value of risk-free debt minus a put option. Thus, with a loan guarantee, maximum debt is worth non-insured risky debt plus the value of the guarantee:

$$D = e^{-r_f T}(1+R)D - E\left[e^{-r_f T}\max((1+R)D - A_T, 0)\right] + G. \tag{5}$$

The first two terms of expression (5) are the value of the risky debt without a loan guarantee. Indeed, D is the market value of the debt. Since the debt is a zero coupon bond, its face value is equal to the total interest plus the principal, i.e., $(1+R)D$. The third term, G, is equal to the value of the loan guarantee. If the guarantee is duly large, the debt becomes risk-free, i.e., $1+R = e^{r_f T}$. Optimally, the lenders' participation constraint is

$$D = I - C - K. \tag{6}$$

Since the project sponsors and lenders are assumed to be perfectly informed,they know the true value of SPV cash flow risk $\sqrt{V}$. By converting the borrowing rate, R, into a continuous rate, r, following $1+R = e^{rT}$, and by combining expressions (5) and (6), we obtain the following expression for the credit spread:

$$r - r_f = \frac{1}{T}\ln\Big(\frac{1 - N(-x)A_0 e^{(g-\delta)T}/(I-C-K) - G/(I-C-K)}{N(x-\sqrt{VT})}\Big), \tag{7}$$

where

$$x = \frac{\ln(A_0/(1+R)D) + (r_f + g - \delta + V/2)T}{\sqrt{VT}}.$$

All else being equal, the credit spread $r - r_f$ decreases when K and/or G increase. Therefore, government support will alleviate the debt burden.

2.2 Government net earnings and participation constraint

As mentioned above, government can support the project by providing a loan guarantee and/or direct investment. In return it will receive tax revenues and a share of the net profit through joint equity participation as incremental earnings. We assume that with direct investment and joint equity participation the α portion of the shares goes to the government and the residual $1-\alpha$ goes to the project sponsors. Therefore, the net earnings to the government is

$$W = \tau_c E\left[e^{-r_f T}\max(A_T - RD - \lambda I, 0)\right] + \alpha P + B - G - (1+r_g)K, \tag{8}$$

where τ_c is the corporate tax rate, G the cost of the loan guarantee, K the amount of the government's direct investment, and α the government's portion of the shares. B captures the other social benefits (or costs) created by positive (or negative) externalities. The rate r_g is the government cost of raising the upfront amount K for direct investment. For the rest of our analysis, we set $B = 0$ and focus on other quantifiable variables.[7] We also set $r_g = 0$ as it does not impact the qualitative results. With a strict subsidy, $\alpha = 0$ since no shares are given to the government. The special case of loan guarantee only exists when $K = 0$ and $\alpha = 0$; and that of direct investment only exists when $H = 0$ implying $G = 0$. In general, the government and the SPV will agree on the appropriate sharing rule $(\alpha, 1-\alpha)$ on the basis of their respective bargaining power. In the appendix, we show how we derive the payoff function (8) where information asymmetry is defined by the above stochastic volatility process.

Since both a loan guarantee and direct investment are costly for the government, the government's constraint is calculated when its net gain is positive, i.e., $W > 0$. Ideally,

[7] This equation simplifies government's social objectives. Indeed, there are intangible social benefits. We assume the tax rate τ_c to be a proxy for many or all these features. We leave the issue of public good externalities for future research (e.g., Grimsey and Lewis (2002) and Grout (2003)).

the guarantee haircut H and/or direct investment K will be chosen by the government to maximize its net earnings W from the project. This will not always happen since the government will make compromises in order for the project to go ahead. Indeed, if the sponsors make no net earnings from the project, they will abandon it. There may also be financial constraints caused by credit rationing, thus making external debt financing unfeasible. In this case, government financial intervention may help overcome these constraints and makes the project worthwhile.

To obtain the cost G of the loan guarantee for the government, we use Merton (1977)'s approach (e.g, Schich (1997)). Merton (1977) draws a parallel between a financial guarantee and a put option written by a guarantor and granted to a bondholder. Since we are considering a zero coupon bond with bankruptcy at the debt maturity, the guarantee costs as much as the present value of the expected claim payments to be paid by the government when default occurs:[8]

$$G = E\left[e^{-r_f T}\max(\min(H,(1+R)D) - A_T, 0)\right]. \tag{9}$$

When default occurs, the government will make good on residual payment on the debt up to haircut level H. Therefore, its maximum payment at debt maturity is $\min(H,(1+R)D)$. As assumed in Chaney and Thakor (1985), the government is benevolent in the sense that it requires no explicit fee for a loan guarantee. The rationale for government intervention is to enable the project to go ahead; otherwise it may be abandoned through lack of financing due to strict credit rationing, as is often the case with this type of capital-intensive investment (e.g., Stiglitz and Weiss (1981) and Smith and Stutzer (1989)). If the project is undertaken and successful, the government will gain through tax revenues and other social benefits, such as job creation and/or job saving, which will compensate for the guarantee cost. By guaranteeing the loan, the government expects to benefit. In the government's net earning equation (8), higher interest rate, R, or larger debt, D, implies less tax revenue. With a loan guarantee, the interest rate falls and, all else being equal, so does the total interest on the debt, RD. The government thus ensures more

[8] Instead of a haircut amount, the government could guarantee part of the face value of the loan, i.e., it would cover up to percentage ω of the debt face value: $\omega(1+R)D$. In that case, the guarantee would cost the government $G = E\left[e^{-r_f T}\max(\omega(1+R)D - A_T, 0)\right]$. Without loss of generality, such a formula will have qualitative implications similar to the ones obtained with a haircut guarantee contract.

future tax revenue for itself. With direct investment, the project needs less external financing; therefore less tax shields from interest payments and more tax revenue goes to the government.

Later, we will compare the impacts of the two forms of government support (loan guarantee versus direct investment) and evaluate their effects on payoffs, especially to the sponsors and to the government.

2.3 Value to the project sponsors

As mentioned above, with direct investment and joint equity participation (or PPP), the government and the project sponsors share the net profit after taxes and debt repayment in the proportions α and $1-\alpha$, respectively. Therefore, the net gain to the project sponsors is

$$S = (1-\alpha)P - C. \tag{10}$$

This expression states that the net earnings for the project sponsors are equal to their share of the residual profit minus their initial capital investment.

The value of guarantee haircut H and/or investment amount K reflects the government's participation constraint. The government chooses H and/or K in order to generate net earnings. The borrowing cost R (or rT in continuous time) reflects the lenders' participation constraint. Therefore, the project's manager makes financing and investment decisions under the participation constraints of both the lenders and the government, i.e., $W \geq 0$ for the government and $D = I - C - K$ for the lenders.

3 Analysis and general discussion

3.1 Baseline parameter estimation

The parameter values are based on data from project finance for capital-intensive investments (e.g., Kleimeier and Megginson (2001), Esty (2003, 2004), Finnerty (2007)). Baseline values are: $r_f = 5\%$ for the risk-free interest rate, $\delta = 0\%$ for the payout rate, $\lambda = 0.95$ for the depreciation code allowance, and $\tau_c = 40\%$ for the corporate tax rate. Debt maturity is $T = 8$ years, which corresponds to an observed average of 8 to

12 years. We assume constant returns to scale for the total asset value of the project $A_t = p_t I^\gamma$, where $\gamma = 1$ and p_t is a random variable that captures the stochastic nature of the assets. Here, the investment amount is exogenously given and remains constant over the life of the project; only the output price varies. Therefore, total assets (1) vary only with output price p_t. We use starting value $p_0 = 1.5$, which yields $A_0 = 1.5I$. The externally financed project asset growth rate is $g = 0$. The other social benefits B for the government are set to zero. The government's financing cost r_g is also set to zero. For stochastic volatility, we set the baseline parameters as follows: annualized initial asset volatility is $\sqrt{V_0} = 40\%$, mean long-run volatility $\beta = 0.40^2 = 0.16$, speed of adjustment $\nu = 0$, and volatility diffusion coefficient $\xi = 0$. We also assume the correlation between asset returns and their volatility $\rho = 0$. These baseline values assume perfect information. Tables 1 and 2 summarize respectively the baseline values and the endogenous variables from the optimization exercises.

We will next run our numerical experiments and analyze the results. We will also conduct several sensitivity analyzes with respect to our parameter values.

3.2 Government and sponsor earnings sensitivity to H and K

In Panel (a) of Figure 1, we plot the loan guarantee cost to the government (left) and the borrowing rate (right) as a function of H (guarantee haircut) and K (government's direct investment). Not surprisingly, the loan guarantee cost, G, increases with H and decreases with K. Indeed, since $D = I - C - K$, when K increases, the demand for debt financing decreases, as does the need for a loan guarantee. The borrowing interest rate, obtained from the lenders' participation constraint, decreases faster with H than with K. To make a fair comparison of the two forms of government support, we use the same cost base, i.e., $G = K$. We will explore this point in more detail in the next section.

In Panels (b) and (c) of Figure 1, we plot government and sponsor earnings from the project as a function of H and K. In Panel (b), the government invests through an investment subsidy (i.e. $K \neq 0$ and $\alpha = 0$), while in Panel (c), it invests through equity participation with $\alpha = K/I$ being the portion of the profits that goes to the government. When $\alpha = 0$, i.e., the government receives no share of the profits, we observe from Panel

(b) that government earnings W decrease with both H and K. The reason is that such government support is costly and is not compensated by the increase in tax revenue. For the sponsors, we observe the reverse, i.e., their net earnings S increase with both H and K since they keep all after-tax profit.

When the government receives $\alpha = \frac{K}{I}$ of the profits, from Panel (c), its net earnings, W defined in (8), still decreases with guarantee haircut level H since the loan guarantee is costly. With direct investment, however, the trend is ambiguous and depends on the level of H. For example, when H is small, W follows a U-shaped curve as a function of K, i.e., W is high when K is low or high, and is low when K is medium. When H is large, W increases with K since the government's share of the profits increases, and that gain outweighs the cost of the government's support. The sponsor's wealth also increases with H. For K, the impact is ambiguous on both the sponsor and the government. At small values of H, the sponsor's earnings S follow an inverse U-shaped curve as a function of K, there thus being an optimum value for K. We will explore this point later in the following sections. When H is large, S decreases with K as a greater share of the profits goes to the government.

As shown by the graphs, when $\alpha = K/I$ the sponsors reach their optimal earnings with small values of H. When H is relatively large, their earnings decrease with K because part of the after-tax profits are transferred to the government in proportion to its direct investment. Therefore, when the loan guarantee is small, there is an optimal level of K from the sponsors' viewpoint if the government is sharing the profits with them. We will explore this point in the next section.

3.3 Sponsor earnings optimization as a function of K

In this paper we focus on variables K and α. The sponsor's goal is to maximize its net earnings from the project by deciding how much government direct investment K is needed and the proportion of shares going to the government α.[9] The sponsors then

[9]There are other significant policy variables worth investigating. We will nonetheless leave them for future study.

perform the following optimization:

$$\{K^*, \alpha^*\} = \arg\{\max S(K, \alpha)\}, \tag{11}$$

under the participation constraints of both the government and the lenders, i.e., $W \geq 0$ and $D = I - C - K$, and the constraints $0 \leq \alpha \leq 1$. We allow for different values of α, instead of initially imposing $\alpha = K/I$, the portion of the government's investment in the project. As we will discuss later, the level of α will be a good indication of the government's bargaining power (ownership control, balance of power, etc.).

When we run this optimization, as shown by Panels (b) and (c) of Figure 1, the sponsors attain their optimal earnings when α is near zero. This case is straightforward and less interesting. We now consider the case where the government receives a portion of the after-tax profits in proportion to its direct investment, i.e., we impose an additional constraint $\alpha = K/I$. The results are on Figure 2.

As shown by Figure 2, the sponsors should want more direct investment K^* when project cash flow is less risky. When cash flow is risky, sponsors should want the government to invest less directly in the project and instead provide a loan guarantee. Indeed, when H is relatively large, the sponsors' earnings are higher with high-risk cash flow, and lower when H is small. For the government, we observe the opposing trend, i.e., the government's earnings are lower when risk increases for high values of H. Indeed, the cost of the loan guarantee to the government being equivalent to an option, its value is high when volatility is high and H is large (because the strike price of the option is $\min(H, (1 + R)D)$). We also observe that the borrowing interest rate decreases quickly with H, especially when the project is risky. We will explore in more detail the impact of the two forms of government support on the borrowing interest rate when we compare loan guarantee with direct investment.

3.4 Comparing forms of government support

We will now compare the two forms of government support (loan guarantee versus direct investment) using the same cost comparison base. To obtain the same cost base, we proceed as follows. With a loan guarantee, for a given value of H, we compute the corresponding cost G and generate the borrowing interest rate, the government's earnings and the sponsor's earnings. With direct investment, we assume the same cost for the government as with a loan guarantee. Therefore $K = G$.

In Table 3, we give our results for each form of government support. Panel (a) gives our results when the government supports the project by loan guarantee alone. To obtain the results, we varied H from 0 to 140 and calculated the other quantities. Panel (b) gives the results when the government supports the project by direct investment alone. For each column, we set K equal to the corresponding G in order to have the same cost comparison base. We consider two cases, $\alpha = 0$ (subsidy) and $\alpha = K/I$ (equity participation), and calculate the government's earnings and sponsor's earnings accordingly.

We observe that a loan guarantee is more effective in reducing the SPV's cost of borrowing (see Figure 3). Indeed, if there is credit rationing with a cap $\bar{R}$ (or $\bar{r}$ in continuous compounding) on the borrowing interest rate, i.e., lenders are unwilling to lend beyond $\bar{R}$, the government can overcome this credit rationing constraint at less expense with a loan guarantee than with direct investment. For example, to maintain a maximum borrowing cost of $\bar{r} = 10\%$, the guarantee haircut must be $H = 80$ at a cost of $G = 9$ if a loan guarantee is provided. The same interest rate could be maintained through direct investment support at a cost of $K = 20$, more than twice the cost of the loan guarantee. To maintain an interest rate below $\bar{r} = 9\%$, through a loan guarantee, the minimum guarantee haircut level must equal $H = 90$, which today corresponds to a loan guarantee cost of at least $G = 12$. The same interest rate could be maintained through direct investment only at a cost of more than 26. So, to reduce the interest rate by 1%, we need an additional loan guarantee costing 3 units or an additional direct investment costing more than 6 units, i.e., over twice the cost. Therefore, to maintain debt financing at some desired interest rate, a loan guarantee is more cost-effective for the government than direct investment. A further advantage of a loan guarantee is that it is an off-budget item, being nonetheless a future contingent claim (e.g., Brixi and Irwin (2004)).

We also observe that sponsors prefer direct investment with $\alpha = 0$ (subsidy) since naturally they do not wish to share the after-tax profits with the government. This is, conversely, the worst-case scenario for the government. Direct investment with a share of the profits $\alpha = K/I$ transfers wealth from the sponsors to the government and is the preferred scenario from the government standpoint, followed by loan guarantee support. This is because direct investment lowers the financing needs of the project, the interest rate on the loan remains relatively high. The project will have tax write-offs for interest payments, and since it does not explicitly pay the government intervention, the sponsors will have more net earnings. A loan guarantee, however, will considerably decrease the interest payments; therefore, with less tax write-off for interest payments the project will pay more tax to the government. Of course, the government may prefer a loan guarantee and refuse to invest directly and receive a share of the earnings because the revenue gain alone would not compensate for the cost of direct investment.

3.5 Impact of information asymmetry on government earnings estimate

As mentioned above, we have used a stochastic volatility process to capture the information asymmetry between the government and the other stakeholders. To model the plum problem as opposed to the more familiar lemon problem (e.g., Chen (2005)), we assume an asymmetric information context in which the government knows the distribution of project cash flow risk but not its value, whereas debtholders and shareholders have a better estimate of project risk level. The degree of information asymmetry is measured by ξ with more asymmetry when ξ is high. Perfect information corresponds to $\nu = 0$ and $\xi = 0$.

We have analyzed the impact of information asymmetry on the government's own estimate of its earnings by varying ξ and ρ. The results are on Figure 4. As we can see from the graphs, the degree of information asymmetry has an impact on the government's estimated earnings and, thereby on its decision making. The curves are similar in shape no matter what level of correlation we choose. We will therefore focus on the level of ξ. Note that $\xi = 0$ corresponds to an exact estimate of government earnings. For the loan guarantee, we observe two areas with respect to the level of H. The first area corresponds to small values of guarantee haircut (i.e., H roughly less than 60), where the government underestimates its own wealth when information is highly asymmetric. In other words, when information is less asymmetric, ξ is close to zero, and the government will prefer less guarantee haircut. For larger values of guarantee haircut (i.e., H roughly more than 60), the government overestimates its gain from the project, the implication being that the government tends to provide more guarantee when information is highly asymmetric. The reason : when the true value of the volatility is less certain, the government tends to overestimate its tax revenue and thus its earnings. With direct investment, we observe that the government's estimated earnings are higher when information is less asymmetric. When $\alpha = 0$, the gap between the government's estimated and real earnings increases with K. When $\alpha = K/I$, the degree of information asymmetry has less impact on the government's estimate of its earnings when K becomes very large since the government has become a majority shareholder, thereby eliminating most of the conflict between itself and the project sponsors. Thus, the degree of asymmetry faced by the government will cause it to under/over-estimate its net earnings from the project, thereby affecting the level of support it would be willing to provide.

For ease of comparison, we have also plotted government earnings from different forms of support: loan guarantee, direct investment with $\alpha = 0$ and direct investment with $\alpha = K/I$ (see Figure 5). With perfect information, direct investment with $\alpha = K/I$ is the best option for the government, followed by loan guarantee. Direct investment with $\alpha = 0$ is the worst option. For all forms of support, information asymmetry causes the government to misestimate its gain from the project. With severe information asymmetry, the government will prefer a loan guarantee to all forms of direct investment, since the estimated yield is higher. This definitely is a perception by the government, since the true value of its earnings is known only when $\xi = 0$. Nevertheless, this perception will drive its decision making.

3.6 Alpha indifference curves for the government

Since the degree of information asymmetry impacts the government's estimated earnings, we want to know the value of α that will make its estimated earnings with a loan guarantee equal to its estimated earnings with direct investment for the same cost. In other words, what value of α will make the government indifferent to choosing a loan guarantee or direct investment with $K = G$? The results are on Figure 6. We also report the value of the K/I ratio for comparison. With less information asymmetry (low ξ), α is below the K/I ratio. With more information asymmetry (high ξ), α is above K/I when K roughly exceeds seven. In all cases, if the government receives more than α shares, it will from its own viewpoint prefer investing directly in the project to guaranteeing the loan on the same cost base. If it receives fewer than α shares, a loan guarantee will be preferred.

These findings lead to some policy implications. Indeed, when information is highly asymmetric, when the government knows less about project quality than do the private partners, the so-called plum problem, the project sponsors should seek a loan guarantee from the government, unless they are willing to give up more control over the project and decrease their wealth. For the sponsors, the α value that equalizes direct investment with loan guarantee only, corresponds to a project with no information asymmetry, $\xi = 0$. Unfortunately, the sponsors' estimate of the government earnings does not match those of the government. If the government's equity exceeds α (with $\xi = 0$), the sponsors will have lower earnings than with a loan guarantee. Therefore, a PPP will be preferred to a loan guarantee if there is less information asymmetry. We see that α is used as a bargaining tool and can mitigate information asymmetry, since the government should require more control over the project when information is asymmetric, especially for very capital-intensive projects.

Assume that the project cannot go ahead without government participation. In that case, as mentioned above, the government will participate in the project if there is at least a positive net benefit (including a social benefit). To analyze the agency conflict (e.g., Jensen and Meckling (1976)) associated with this participation constraint, we solve for the minimum level of α for government participation in the project, i.e., the minimum α such that government earnings $W \geq 0$. We denote it by α_1.

Now, assume that the project can go ahead regardless of the amount of government involvement. The government will then receive any way tax revenue from the project. If

$K = 0$ (no direct investment) and $H = 0$ (no loan guarantee), government earnings are $\bar{W}_{(H=0,K=0)}$. If the government wishes to increase its earnings, it can directly participate in the project by investing K amount of money. We then solve for the minimum value of α that provides the government with $W \geq \bar{W}_{(H=0,K=0)}$. We denote this second value of α by α_2.

The results are in Table 4 and on Figure 7. As we can see, α_1 and α_2 increase with the degree of information asymmetry with $\alpha_1 \leq \alpha_2$ and follow a concave curve as a function of direct investment K. We also provide K/I values for comparison. We observe that α_1 values are always below the K/I curve, with the two curves becoming closer as ξ increases. The α_2 curve, however, lies above the K/I curve up to a certain investment level (for example $K = 60$ when $\xi = 0.50$). Beyond that level, the α_2 curve falls under the K/I curve. Intuitively, if not enough shares are given to the government, the cost of direct investment (K) will exceed government earnings from the project (e.g., tax revenue, its share of the total profits, and other social benefits). Therefore, to support the project, the government will require a minimum level of equity α.

4 Conclusion

This paper compares two general forms of government support, loan guarantee and direct investment through public-private partnerships (PPPs). Through a loan guarantee, the government enhances project creditworthiness. Through direct investment, the government invests an amount of capital in return for shares in the project, thereby sharing in the profits. In other words, the government and the private partners operate as a PPP. With the first form of support, the government reduces the tax deduction for interest payments, thereby making more taxable income available for the government. With the second form, the government receives a share of the profits in addition to tax revenue. The government may prefer a loan guarantee to direct investment if the marginal gain will not compensate for the cost of the money invested.

All else being equal, a loan guarantee is more effective in reducing the project's borrowing rate. If information is highly asymmetric, and the government knows less about project quality than do the private partners, i.e., the so-called plum problem rather than the common encountered lemon problem (Akerlof (1970)), the project sponsors should seek a loan guarantee from the government, unless they are willing to give up more control over the project and thus lower their earnings. If information is less asymmetric, they should prefer a PPP to an equivalent project with a loan guarantee. It is shown that the government will need enough equity in the project to offset the cost of its direct investment and generate net earnings from the project (e.g., tax revenue, its share of the total profit, and other social benefits). This degree of ownership and participation in the profits can be a bargaining tool and can mitigate information asymmetry. The government should require more ownership when it faces severe information asymmetry, especially for highly capital-intensive projects.

Appendix

Appendix 1: Closed-form solutions for earnings, assuming constant volatility

Assuming constant volatility $\sqrt{V}$, we can use the Black-Scholes-Merton (1973) option pricing formula for calculation of earnings. For the net-profit expression P given in (4), we have

$$\begin{aligned} P &= \big[A_0 e^{(g-\delta)T} N(x) - e^{-r_f T}(1+R)DN(x-\sqrt{VT})\big] \times 1_{\{D\leq\lambda I\}} \\ &\quad -\tau_c\big[A_0 e^{(g-\delta)T} N(y) - e^{-r_f T}(RD+\lambda I)N(y-\sqrt{VT})\big] \times 1_{\{D\leq\lambda I\}}, \end{aligned} \tag{12}$$

where

$$x = \frac{\ln(A_0/(1+R)D) + (r_f+g-\delta+V/2)T}{\sqrt{VT}}$$

and

$$y = \frac{\ln(A_0/(RD+\lambda I)) + (r_f+g-\delta+V/2)T}{\sqrt{VT}}.$$

For the debt expression (5), we have

$$D = e^{-r_f T}(1+R)DN(x-\sqrt{VT}) + A_0 e^{(g-\delta)T} N(-x) + G, \tag{13}$$

with x given above. Combining this expression of D with the equilibrium condition $D = I - C - K$, we obtain the expression for credit spread:

$$r - r_f = \frac{1}{T}\ln\Big(\frac{1 - N(-x)A_0 e^{(g-\delta)T}/(I-C-K) - G/(I-C-K)}{N(x-\sqrt{VT})}\Big). \tag{14}$$

For the value of the guarantee in (9), we have

$$G = e^{-r_f T}\min(H,(1+R)D)N(-z+\sqrt{VT}) - A_0 e^{(g-\delta)T} N(-z), \tag{15}$$

where

$$z = \frac{\ln(A_0/\min(H,(1+R)D)) + (r_f+g-\delta+V/2)T}{\sqrt{VT}}.$$

Appendix 2: Hull and White (1988) derivation of option prices, assuming stochastic volatility

We assume the following processes for the project's total assets A_t and volatility V_t

$$\begin{aligned}\frac{dA_t}{A_t} &= (r_f + g - \delta)dt + \sqrt{V_t}dZ_{1t},\\ dV_t &= \nu(\beta - V_t)dt + \xi\sqrt{V_t}dZ_{2t},\end{aligned}$$

where r_f, g, δ, ν, β, ξ, and $\text{corr}(Z_{1t}, Z_{2t}) = \rho$ are constant parameters. Using these specifications, Hull and White (1988) propose approximate closed-form solutions for call and put options with underlying assets A and stochastic volatility. The value of the option is the Black-Scholes formula augmented by a Taylor series expansion terms that contains the stochastic volatility diffusion parameters.

Assuming constant variance $\bar{V}$, the Black-Scholes pricing formula for call and put options with underlying asset A and exercise price X is

$$\begin{aligned}\text{Call}_{BS}(A, X, \bar{V}) &= e^{(g-\delta)T}AN(d_1) - e^{-rT}XN(d_2),\\ \text{Put}_{BS}(A, X, \bar{V}) &= e^{-rT}XN(-d_2) - e^{(g-\delta)T}AN(-d_1),\end{aligned}$$

with $d_1 = \frac{\ln(A/X)+(r+g-\delta+\bar{V}/2)T}{\sqrt{\bar{V}T}}$ and $d_2 = d_1 - \sqrt{\bar{V}T}$.

Assuming stochastic volatility and $\nu \neq 0$, Hull and White (1988) show that the value of a call option is

$$\text{Call}(A, X, V) = \text{Call}_{BS}(A, X, \bar{V}) + f_1\xi + f_2\xi^2 + ...$$

with

$$\begin{aligned}f_1 &= \frac{\rho}{\nu^2 h}\{\nu(\beta - V)(1 - e^h + he^h) + \nu\beta(1 + h - e^h)\}A\frac{\partial^2\text{Call}_{BS}(A, X, \bar{V})}{\partial A\partial\bar{V}},\\ f_2 &= \frac{\phi_1}{T}A\frac{\partial^2\text{Call}_{BS}(A, X, \bar{V})}{\partial A\partial\bar{V}} + \frac{\phi_2}{T^2}\frac{\partial^2\text{Call}_{BS}(A, X, \bar{V})}{\partial\bar{V}^2}\\ &\quad + \frac{\phi_3}{T^2}A\frac{\partial^3\text{Call}_{BS}(A, X, \bar{V})}{\partial A\partial\bar{V}^2} + \frac{\phi_4}{T^3}\frac{\partial^3\text{Call}_{BS}(A, X, \bar{V})}{\partial\bar{V}^3},\end{aligned}$$

where

$$
\begin{aligned}
\phi_1 &= \frac{\rho^2}{\nu^4}\Big\{\nu(\beta-V)[e^h(\frac{1}{2}h^2-h+1)-1]+\nu\beta[e^h(2-h)-(2+h)]\Big\},\\
\phi_2 &= 2\phi_1+\frac{1}{2\nu^4}\Big\{\nu(\beta-V)(e^{2h}-2he^h-1)-\frac{\nu\beta}{2}(e^{2h}-4e^h+2h+3)\Big\},\\
\phi_3 &= \frac{\rho^2}{2\nu^6}\Big\{\nu(\beta-V)(e^h-he^h-1)-\nu\beta(1+h-e^h)\Big\}^2,\\
\phi_4 &= 2\phi_3,\\
h &= -\nu T,
\end{aligned}
$$

the partial derivatives are

$$
\begin{aligned}
\frac{\partial \text{Call}_{BS}(A,X,\bar V)}{\partial A} &= e^{(g-\delta)T}N(d_1),\\
\frac{\partial \text{Call}_{BS}(A,X,\bar V)}{\partial \bar V} &= Ae^{(g-\delta)T}N'(d_1)\frac{1}{2\sqrt{\bar V}}\sqrt{T},\\
\frac{\partial^2 \text{Call}_{BS}(A,X,\bar V)}{\partial A\partial \bar V} &= -e^{(g-\delta)T}N'(d_1)\frac{d_2}{2\bar V},\\
\frac{\partial^2 \text{Call}_{BS}(A,X,\bar V)}{\partial \bar V^2} &= Ae^{(g-\delta)T}N'(d_1)\frac{(d_1d_2-1)}{4\bar V^{3/2}}\sqrt{T},\\
\frac{\partial^3 \text{Call}_{BS}(A,X,\bar V)}{\partial A\partial \bar V^2} &= e^{(g-\delta)T}N'(d_1)\frac{d_1^2+2d_2-d_1d_2^2}{4\bar V^2},\\
\frac{\partial^3 \text{Call}_{BS}(A,X,\bar V)}{\partial \bar V^3} &= Ae^{(g-\delta)T}N'(d_1)\frac{(d_1d_2-1)(d_1d_2-3)-(d_1^2+d_2^2)}{8\bar V^{5/2}}\sqrt{T},
\end{aligned}
$$

and $\bar V$ is the average expected variance rate in the interval $[0,T]$ given by

$$\bar V = \frac{1}{T}\int_0^T E[V_t]dt.$$

To obtain the explicit expression for $\bar V$, we need to compute $E[V_t]$. Since V_t is mean-reverting, applying Ito's lemma to $e^{\nu t}V_t$ yields

$$e^{\nu t}V_t - V \;=\; e^{\nu t}\beta - \beta + \int_0^t e^{\nu s}\xi\sqrt{V_s}dZ_{2s},$$

which implies

$$V_t = e^{-\nu t}(V-\beta)+\beta+e^{-\nu t}\int_0^t e^{\nu s}\xi\sqrt{V_s}dZ_{2s}.$$

Making this expression conditional on V yields

$$E[V_t] = \beta + e^{-\nu t}(V-\beta).$$

Now we can compute $\bar{V}$ by integrating this expression over the interval $[0, T]$ as follows:

$$\begin{aligned}
\bar{V} &= \frac{1}{T}\int_0^T (\beta + (V-\beta)e^{-\nu t})dt \\
&= \beta + (V-\beta)\frac{1-e^{-\nu T}}{\nu T}.
\end{aligned}$$

When $\nu = 0$, $\bar{V} = V$ and the above expressions become

$$f_1 = \rho V \frac{T}{2} A \frac{\partial^2 \mathrm{Call}_{BS}(A, X, V)}{\partial A \partial \bar{V}},$$

the expression of f_2 remains the same with

$$\begin{aligned}
\phi_1 &= \rho^2 V \frac{T^3}{6}, \\
\phi_2 &= (2 + \frac{1}{\rho^2})\phi_1, \\
\phi_3 &= \rho^2 V^2 \frac{T^4}{8}, \\
\phi_4 &= 2\phi_3.
\end{aligned}$$

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Table 1: **Baseline values**

This table summarizes the baseline values of our optimization exercises. These values are used in our optimization program unless stated otherwise.

Variable	**Description**	**Value**
r_f	Risk-free interest rate	0.05
g	Externally financed project asset growth rate	0.00
δ	Project payout rate	0.00
τ_c	Corporate tax rate	0.40
λ	Depreciation code allowance	0.95
T	Project debt maturity (in years)	8.00
γ	Coefficient of the production technology (elasticity)	1.00
p_0	Initial output price	1.50
B	Other social benefits for the government	0.00
r_g	Government cost of capital	0.00
V_0	Annualized initial project asset variance	0.16
β	Long run mean of the variance	0.16
ν	Speed of adjustment of the volatility process	0.00
ξ	Diffusion coefficient of the volatility process	0.00
ρ	Correlation between the asset and its volatility	0.00
I	Total investment in the project	100
C	Sponsor's investment in the project	10

Table 2: **Derived variables from the optimization**

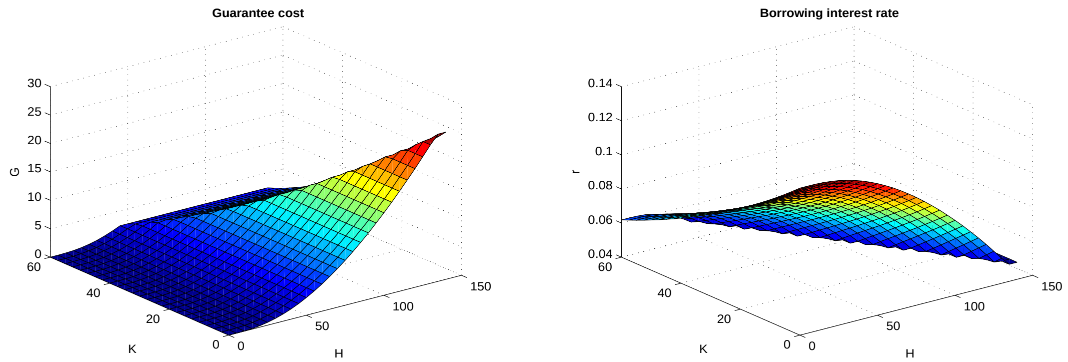
This table summarizes the endogenous variables generated by our optimization.

Variable	**Description**
K	Government direct investment
G	Cost of the government guarantee
D	Market value of debt
r	Borrowing rate
W	Government's net earnings from the project
S	Project sponsors' net earnings
α	Portion of shares given to the government

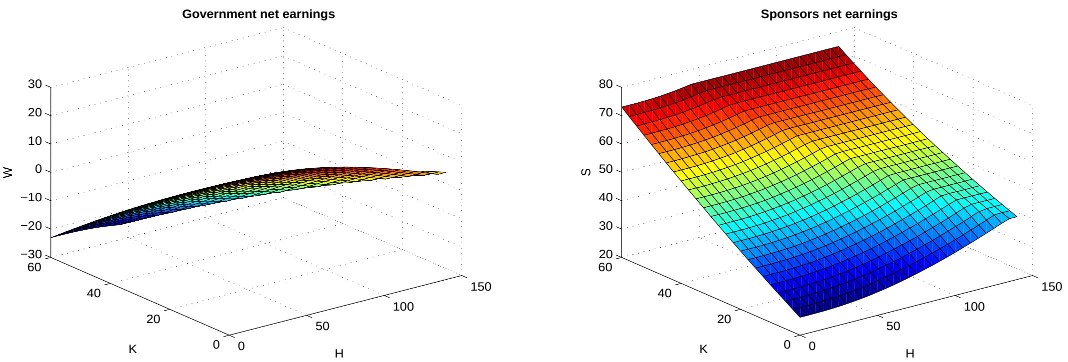
Figure 1: **Guarantee cost, interest rate, and government and sponsor earnings sensitivity to H and K**

In Panel (a), we plot the cost of guarantee G and borrowing interest rate r as a function of loan guarantee haircut H and direct investment K. In Panel (b) and (c), we plot government and sponsor earnings (W and S respectively) for combinations of loan guarantee (H) and direct investment (K) with number of shares $\alpha = 0$ (no shares) or K/I (ratio of government investment to total investment), respectively. We use the following values: $I = 100$, $C = 10$, $A_0(I) = 1.5I$, $\tau_c = 0.40$, $\lambda = 0.95$, $r_f = 5\%$, $g = 0\%$, $\delta = 0\%$, $\sqrt{V_0} = 0.40$. We assume perfect information between the stakeholders, i.e., $\nu = 0$ and $\xi = 0$.

(a)- Guarantee cost and borrowing interest rate



(b)- Government and sponsor earnings with $\alpha = 0$



(c)- Government and sponsor earnings with $\alpha = \frac{K}{I}$

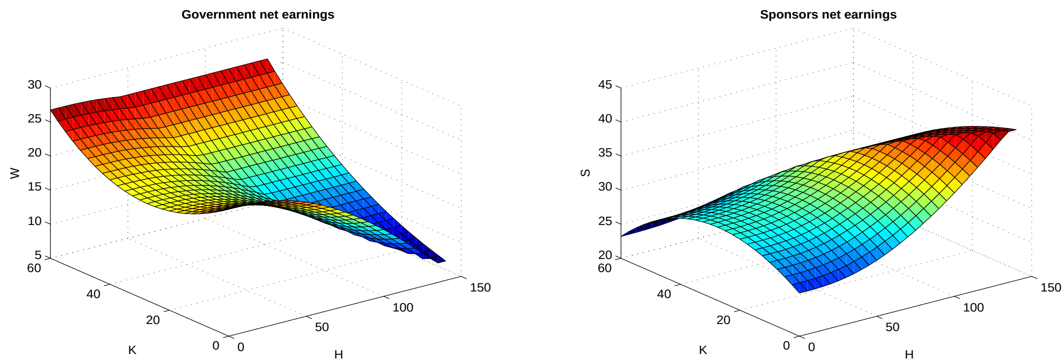


Figure 2: **Optimal K for the project sponsors**

We plot the optimal value of government direct investment K^* for the sponsors. This value optimizes their net gain S, the borrowing interest rate r^* (top graphs), and government and sponsors net earnings (W and S respectively) (bottom graphs) as a function of loan guarantee haircut H. These amounts apply to the project sponsors for H varying from 0 to 120. $\{K^*\} = \arg\{\max S(K, \alpha)\}$, i.e., optimal direct investment K^*, with the participation constraints of both the government and the lenders (i.e., government's estimated earnings $W \geq 0$ and project's total debt $D = I - C - K$). The constraint on the portion of shares given to the government $\alpha = K/I$. We use the following values: $I = 100$, $C = 10$, $A_0(I) = 1.5I$, $\tau_c = 0.40$, $\lambda = 0.95$, $r = 5\%$, $g = 0\%$, $\delta = 0\%$, $\sqrt{V_0} = 0.40$. We assume perfect information between the stakeholders, i.e. $\nu = 0$ and $\xi = 0$.

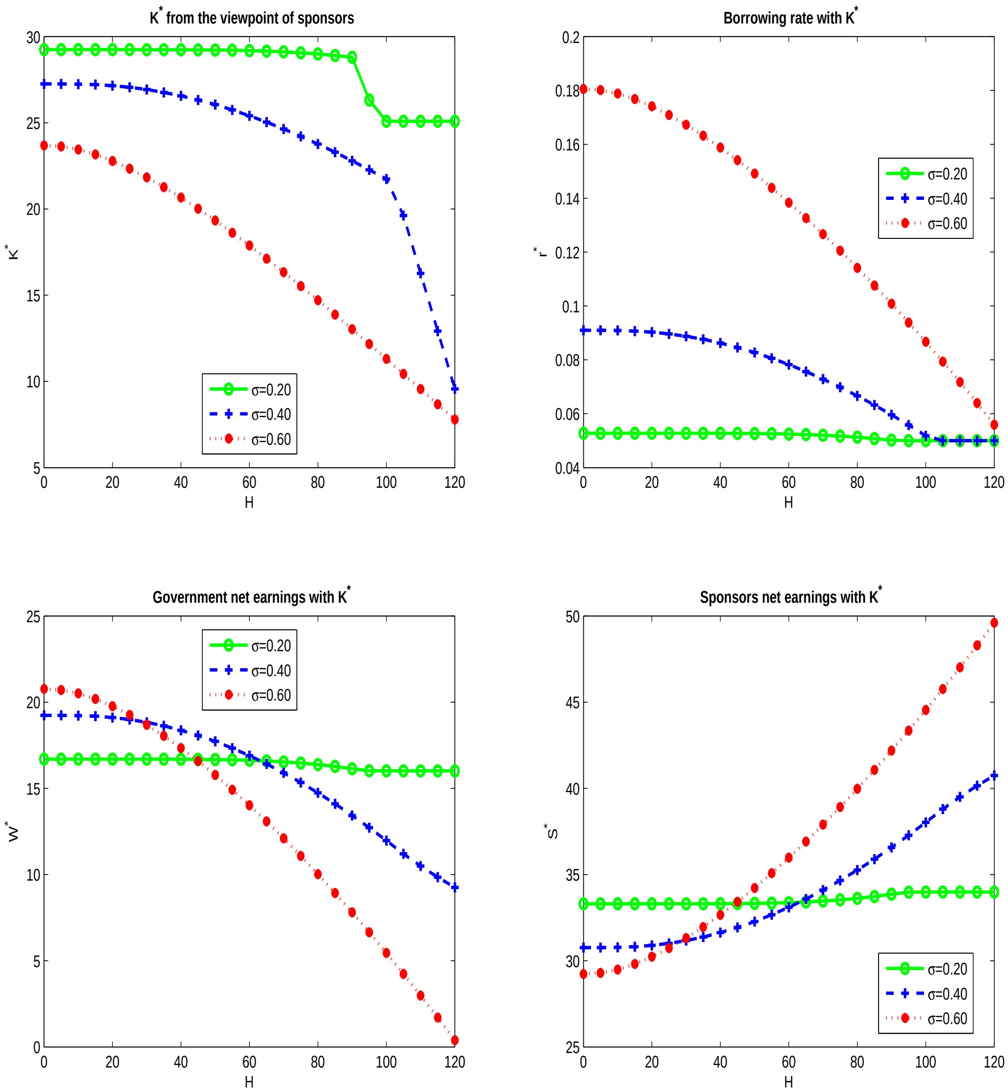


Table 3: **Comparisons of the forms of government support**

These tables show guarantee cost G, debt value D, interest rate r, and government and sponsor net earnings from the project (respectively W and S) as a function of cost to the government G (loan guarantee) or K (direct investment) for different values of loan guarantee haircut H. To compare the impact of these two forms of support, their costs are assumed to be equal. We vary H from 0 to 140 and compute G, the cost of a loan guarantee. We use G as the value of direct investment K, i.e., $K = G$, and calculate the other amounts. For direct investment, we consider two scenarios: $\alpha = 0$ (no shares) and $\alpha = K/I$ (ratio of government investment to total investment). We use the following values: $I = 100$, $C = 10$, $D = I - C - K$ for debt value, $A_0(I) = 1.5I$, $\tau_c = 0.40$, $\lambda = 0.95$, $r_f = 5\%$, $g = 0\%$, $\delta = 0\%$, $\sqrt{V_0} = 0.40$. We assume perfect information between the parties, i.e., $\nu = 0$ and $\xi = 0$.

(a)- Loan guarantee only

H	0	20	40	60	80	100	120	140
G	0	0.26	1.78	4.78	9.09	14.52	20.88	25.91
D	90	90	90	90	90	90	90	90
$r(\%)$	12.63	12.55	12.11	11.23	9.98	8.40	6.52	5.00
W	23.66	23.50	22.58	20.75	18.12	14.81	10.93	7.85
S	26.34	26.50	27.43	29.25	31.88	35.19	39.07	42.15

(b)- Direct investment

K	0	0.26	1.78	4.78	9.09	14.52	20.88	25.91
D	90	89.74	88.22	85.22	80.91	75.48	69.12	64.09
$r(\%)$	12.63	12.59	12.36	11.92	11.31	10.60	9.82	9.24
				$\alpha = 0$				
W	23.66	23.48	22.45	20.39	17.38	13.49	8.80	4.99
S	26.34	26.52	27.55	29.61	32.62	36.51	41.20	45.01
				$\alpha = \frac{K}{I}$				
W	23.66	23.58	23.12	22.28	21.25	20.24	19.49	19.25
S	26.34	26.42	26.88	27.72	28.75	29.76	30.51	30.75

Figure 3: **Borrowing interest rates with the different forms of government support**

This graph plots borrowing interest rate r as a function of G (cost of loan guarantee) or K (cost of direct investment). To compare the impacts of these two forms of support, their costs are assumed to be equal. We vary loan guarantee haircut H from 0 to 140 and compute G, the cost of a loan guarantee. We use G as the value of direct investment amount K, i.e., $K = G$, and calculate the other amounts. We use the following values: $I = 100$, $C = 10$, $A_0(I) = 1.5I$, $\tau_c = 0.40$, $\lambda = 0.95$, $r = 5\%$, $g = 0\%$, $\delta = 0\%$, $\sqrt{V_0} = 0.40$. We assume perfect information between the parties, i.e., $\nu = 0$ and $\xi = 0$.

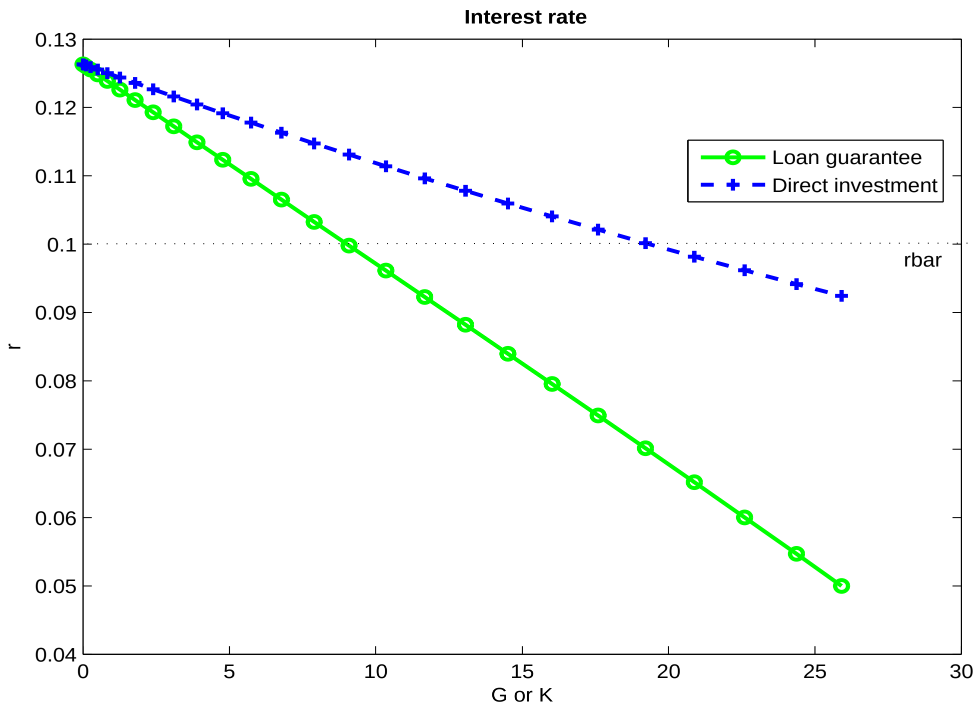
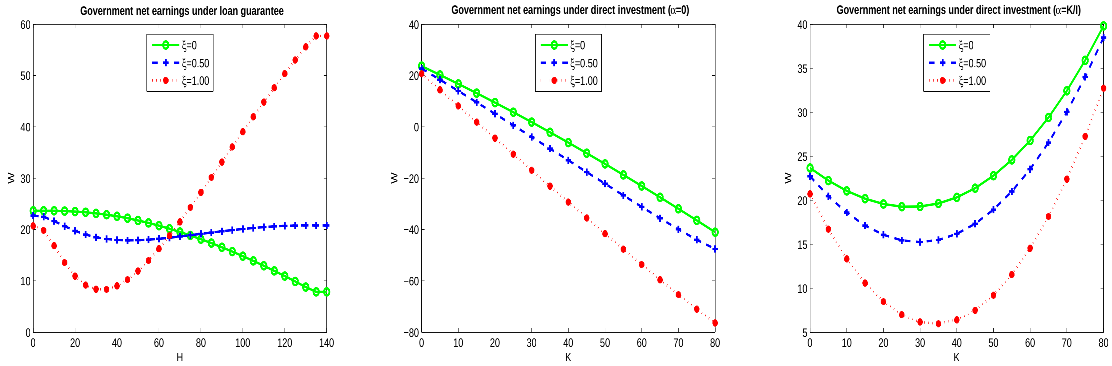


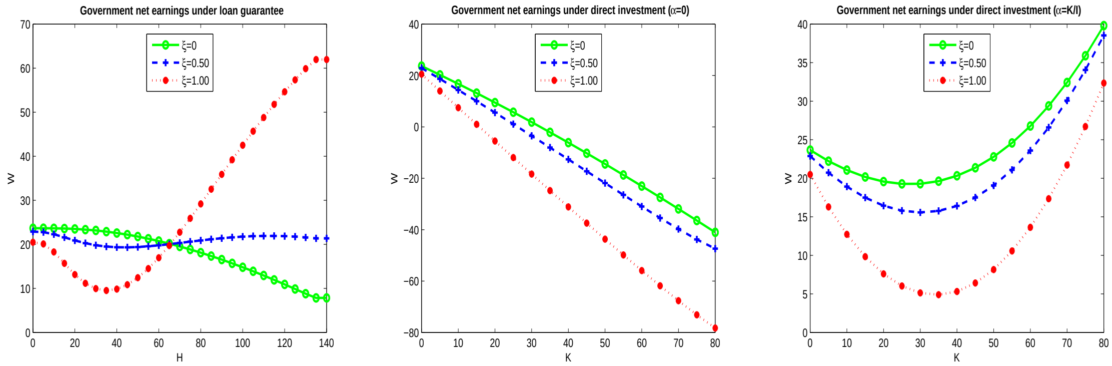
Figure 4: **Impact of information asymmetry on the government's expected earnings**

These graphs plot the government's expected earnings W as a function of G (cost of loan guarantee) or K (cost of direct investment) as estimated by the government for three degrees of information asymmetry. The degree of information asymmetry is captured by volatility coefficient ξ with $\xi = 0, 0.50, 1.00$. For the loan guarantee, we vary guarantee haircut H from 0 to 140. For direct investment, we vary the cash amount K from 0 to 80. We consider two direct investment scenarios: $\alpha = 0$ (no shares) and $\alpha = K/I$ (ratio of government investment to total investment). We use the following values: $I = 100$, $C = 10$, $A_0(I) = 1.5I$, $\tau_c = 0.40$, $\lambda = 0.95$, $r = 5\%$, $g = 0\%$, $\delta = 0\%$, $\sqrt{V_0} = 0.40$. For the volatility process, we set $\beta = 0.16$, $\nu = 0$, $\xi \in \{0, 0.5, 1\}$ and the correlation between asset volatility and its return $\rho \in \{-0.5, 0, 0.5\}$. $\xi = 0$ corresponds to perfect information and gives the true earnings for the government.

(a)- Estimated earnings when $\rho = -0.50$



(b)- Estimated earnings when $\rho = 0.00$



(c)- Estimated earnings when $\rho = 0.50$

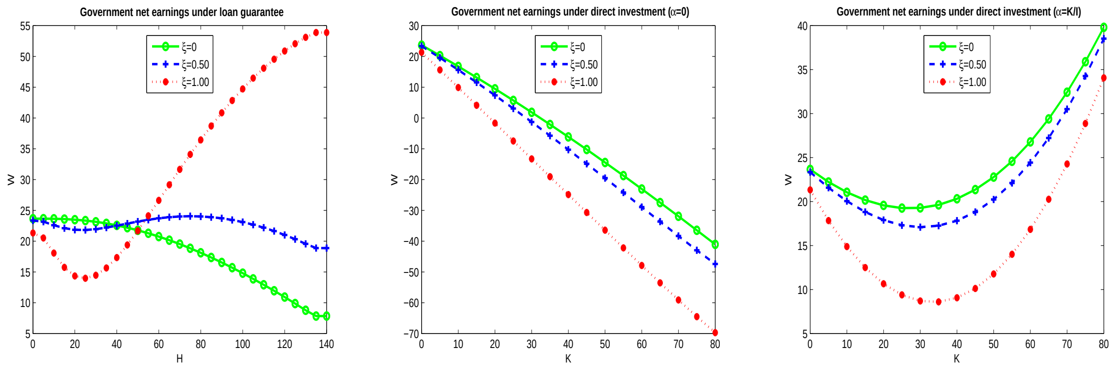


Figure 5: **Government's expected earnings as a function of G or K**

These graphs plot the government expected earnings W as a function of G (cost of loan guarantee) or K (cost of direct investment) as estimated by the government. To compare the impacts of these two forms of support, their costs are assumed to be equal. We vary loan guarantee haircut H from 0 to 140 and compute G, the cost of a loan guarantee. We use G as the value of direct investment K, i.e., $K = G$, and calculate the other amounts. We consider two direct investment scenarios: $\alpha = 0$ (no shares) and $\alpha = K/I$ (ratio of government investment to total investment). We use the following values: $I = 100$, $C = 10$, $A_0(I) = 1.5I$, $\tau_c = 0.40$, $\lambda = 0.95$, $r = 5\%$, $g = 0\%$, $\delta = 0\%$, $\sqrt{V_0} = 0.40$. For the volatility process, we set $\beta = 0.16$, $\nu = 0$, ξ is chosen from the set $\{0, 0.25, 0.50\}$ and the correlation between asset volatility and its return $\rho = 0$. $\xi = 0$ corresponds to perfect information (no asymmetry). The first graph assumes perfect information ($\xi = 0$) with no difference between expected and actual earnings for the government. The second and third graphs assume asymmetric information as estimated by the government.

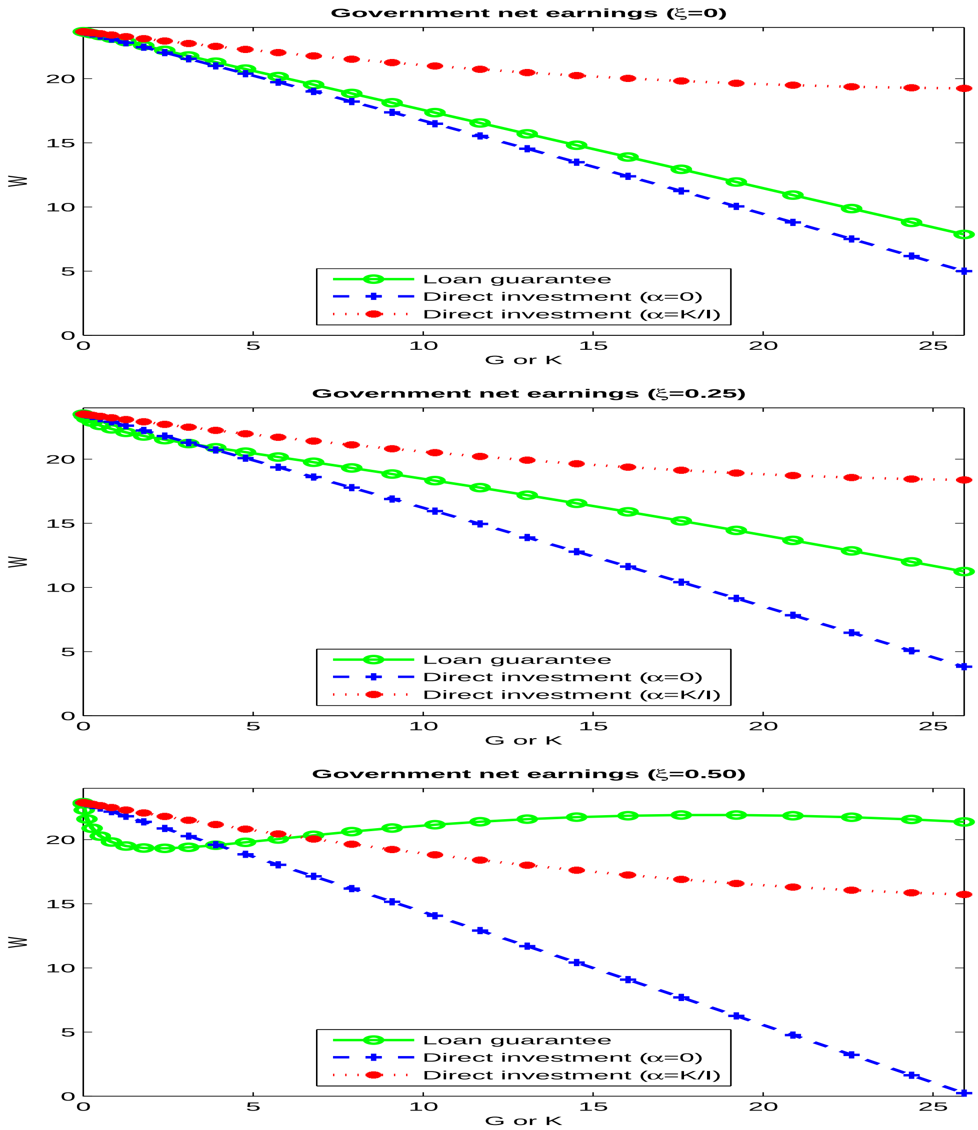


Figure 6: **Alpha indifference curves of equivalent government earnings for loan guarantee scenario and direct investment scenario**

We plot the proportion of shares given to the government α and K/I the ratio of government investment to total investment (black line) as a function of $K = G$. α is the government's portion of the projet's profits in a direct investment scenario with $K = G$, i.e., the government's earnings are equal to its earnings with a loan guarantee only and cost G. G is today's guarantee cost for haircut H. To obtain all values of G, we vary guarantee haircut H from 0 to 140. We use the following values: $I = 100$, $C = 10$, $A_0(I) = 1.5I$, $\tau_c = 0.40$, $\lambda = 0.95$, $r = 5\%$, $g = 0\%$, $\delta = 0\%$, $\sqrt{V_0} = 0.40$. For the volatility process, we set $\beta = 0.16$, $\nu = 0$, ξ is chosen from the set $\{0, 0.25, 0.50\}$, and the correlation between asset volatility and its return $\rho = 0$. $\xi = 0$ corresponds to perfect information (no asymmetry).

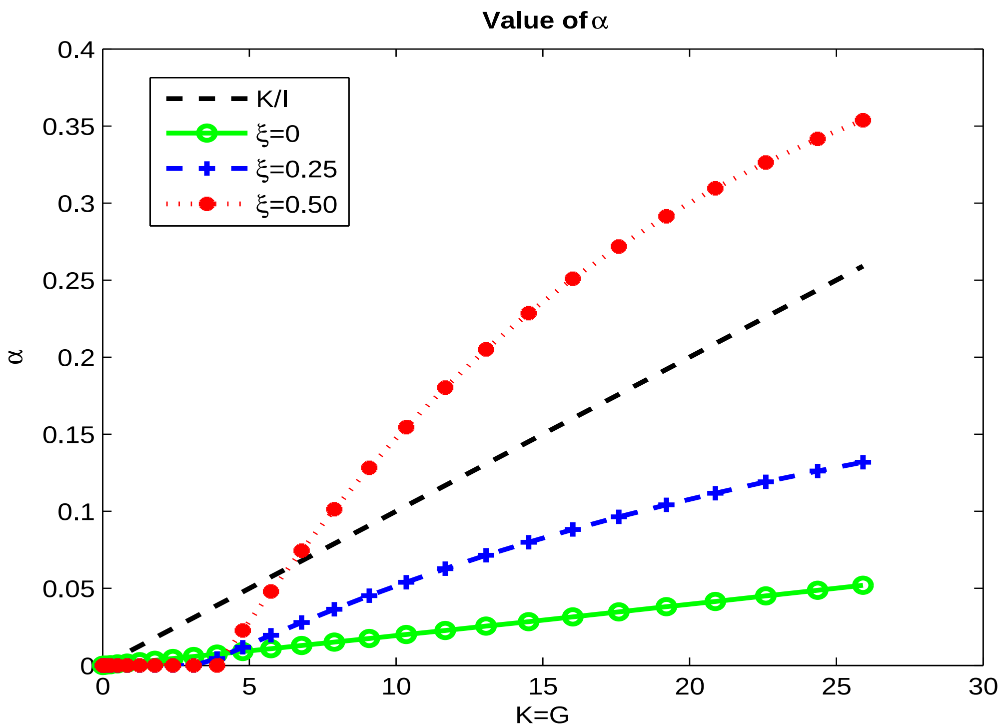


Table 4: **Alpha isocurve for the government's participation constraint**

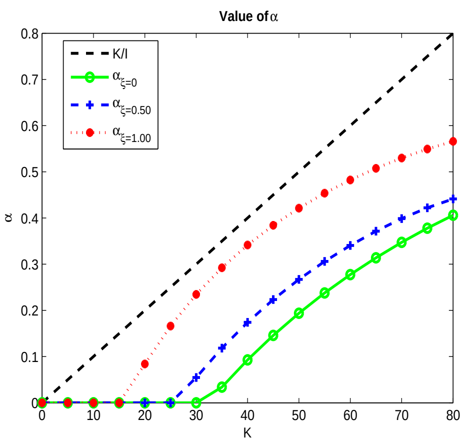
This table shows the values of profit shares α_1 and α_2, where $\alpha_1 = \arg\{W(\alpha) = 0\}$ and $\alpha_2 = \arg\{W(\alpha) = \bar{W}_{(H=0,K=0)}\}$, H is the loan guarantee haircut and K the direct investment. α_1 is the value of α shares in a direct investment scenario where the government's earnings equal zero. With α_2, the government is indifferent to choosing either direct investment or no support of the project at all, assuming the project goes ahead. We use the following values: $I = 100$, $C = 10$, $A_0(I) = 1.5I$, $\tau_c = 0.40$, $\lambda = 0.95$, $r_f = 5\%$, $g = 0\%$, $\delta = 0\%$, $\sqrt{V_0} = 0.40$. For the volatility process, we set $\beta = 0.16$, $\nu = 0$, ξ is chosen from the set $\{0, 0.50, 1.00\}$, and the correlation between asset volatility and its return $\rho = 0$. $\xi = 0$ corresponds to perfect information (no asymmetry).

K	0	10	20	30	40	50	60	70	80
$\frac{K}{I}$	0	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80
					$\alpha_1 = \arg\{W(\alpha) = 0\}$				
$\alpha_1(\xi = 0)$	0	0	0	0	0.0928	0.1939	0.2775	0.3473	0.4061
$\alpha_1(\xi = 0.50)$	0	0	0	0.0548	0.1741	0.2671	0.3406	0.3986	0.4413
$\alpha_1(\xi = 1.00)$	0	0	0.0839	0.2345	0.3416	0.4212	0.4823	0.5298	0.5660
					$\alpha_2 = \arg\{W(\alpha) = \bar{W}_{(H=0,K=0)}\}$				
$\alpha_2(\xi = 0)$	0	0.1601	0.2810	0.3752	0.4505	0.5118	0.5624	0.6047	0.6403
$\alpha_2(\xi = 0.50)$	0	0.1866	0.3182	0.4152	0.4890	0.5466	0.5921	0.6279	0.6543
$\alpha_2(\xi = 1.00)$	0	0.2478	0.3968	0.4959	0.5664	0.6189	0.6591	0.6904	0.7142

Figure 7: **Alpha isocurve for the government's participation constraint**

These graphs plot the values of profit sharing α_1 (left) and α_2 (right), where $\alpha_1 = \arg\{W(\alpha) = 0\}$ and $\alpha_2 = \arg\{W(\alpha) = \bar{W}_{(H=0,K=0)}\}$, H is the loan guarantee haircut and K the direct investment. α_1 is the value of α shares in a direct investment scenario where the government's earnings equal zero. With α_2, the government is indifferent to choosing either direct investment or no support of the project at all, assuming the project goes ahead. We use the following values: $I = 100$, $C = 10$, $A_0(I) = 1.5I$, $\tau_c = 0.40$, $\lambda = 0.95$, $r_f = 5\%$, $g = 0\%$, $\delta = 0\%$, $\sqrt{V_0} = 0.40$. For the volatility process, we set $\beta = 0.16$, $\nu = 0$, ξ is chosen from the set $\{0, 0.50, 1.00\}$, and the correlation between asset volatility and its return $\rho = 0$. $\xi = 0$ corresponds to perfect information (no asymmetry).

Value of α_1



Value of α_2