Interest Rate Risk Estimation: A New Duration-Based Approach

EMANUELE BAJO, MASSIMILIANO BARBI, AND DAVID HILLIER

Abstract

Duration is widely used by fixed income managers to proxy the interest rate risk of their assets and liabilities. However, it is well known that the convexity of the price-yield relationship introduces approximation errors that grow with changes in yield. In this paper we suggest a new approach, discrete duration, which significantly improves upon the accuracy of traditional duration methods and achieves a level of accuracy close to the more complex duration plus convexity. In particular, discrete duration performs particularly well for long dated and low coupon rate bonds, where the estimation error is impressively close to zero.

JEL classification: G10

Keywords: Duration; Interest Rate Risk; Hedging; Fixed Income.
1. Introduction

This paper contributes to the fixed income literature by proposing a new method of interest rate sensitivity, which we call *discrete duration*. Discrete duration significantly improves upon the accuracy of the classical duration measure and matches or betters the more complex *duration plus convexity* extension.

Duration hedging is a major building block of interest rate risk management. A number of fixed income sensitivity indicators, including the “price value of a basis point” (PV01) and the “yield value of a price change” are situated within the duration framework [see Fabozzi, 2005]. Furthermore, the simplicity and intuitive interpretation of duration has made it a popular interest rate risk management tool for treasury managers. In particular, duration can be used to complement earnings at risk (EaR) and gap analysis, since it summarizes, in a single number, the relevant characteristics of future cash flows (size and timing). Finally, duration can be used to measure the impact of an interest rate change on net shareholder wealth [see Crouhy, Galai, and Mark, 2001].

Unfortunately, given its linearity property and the number of restrictive assumptions in calculating duration, duration-based forecast errors of bond price movements can be substantial. In response, two separate approaches have been developed to minimize approximation errors arising from using duration alone.

The first approach, which assumes perfect correlation between prevailing rates at different maturities, introduces higher order terms into the classical duration formula [for example, the *duration plus convexity* method]
of Redington, 1952]. Alternatively, the efficiency of the original duration formula may be improved [see for example, exponential duration (Livingston and Zhou, 2005)].

The second approach removes the assumption of a parallel shift in the yield curve and introduces a multifactor structure to represent the discount rate function [see Litterman and Scheinkman, 1991; Nelson and Siegel, 1987; and Svensson, 1994], or adds additional risk factors to the standard duration approach [Ho, 1992].

Generally, the second approach offers more realistic models. However, the effort required to estimate a large number of parameters makes it less convenient in most cases. It is therefore not surprising that methodologies based on the first approach are still used in spite of their lower accuracy.

This paper builds on existing duration-based techniques by introducing a simple but highly effective transformation that significantly improves upon the base measure and matches more complex methods. We carry out a number of simulations to assess the robustness of the new measure and stress test it for different types of bonds.

For example, when classical duration is used to predict bond price changes, a 1 percent drop in bond yields produces a 0.39 percent approximation error for a ten year coupon bond trading at par and a 4.68 percent error for a thirty year zero-coupon bond. Our method slashes the respective estimation errors to 0.04 percent and 0.01 percent. In contrast, duration plus convexity produces comparable results for the ten year
coupon bond (0.01 percent error), but undoubtedly a worse performance in the case of the thirty year zero-coupon bond (0.46 percent).

In the next section, we review existing duration-based approaches and introduce our discrete duration measure. Section 3 presents our main simulation results, and Section 4 concludes.

2. Duration-Based Measures

In this section, we review the traditional and exponential measures of duration. The new discrete duration measure is then introduced.

2.1 Traditional and exponential Duration

We consider the price of a bond as a function of its yield-to-maturity, \( y \), defined as the internal rate of return that discounts the bond’s cash flows back to the current price. Hence, the bond price, \( P_0 \), can be expressed as

\[
P_0 = \sum_{n=1}^{T} \text{coupon} \times (1 + y_n)^{-n} + \text{face value} \times (1 + y_0)^{-T} \tag{1}
\]

By classical Taylor expansion we can evaluate the impact of a parallel shift in the yield curve, from \( y_0 \) to \( y_0 + \Delta y \), on the bond value. The exact new price is given by

\[
P_1 = P_0 \sum_{n=0}^{\infty} \frac{\Delta y^n}{n!} \left( \frac{d^n P(y_n)}{dy^n} \right) \frac{1}{P_0} \Delta y^n \tag{2}
\]

Truncating this series at a given order yields an approximate bond price and, limited to the first or second order, is represented by equations [3] and [4], respectively.
where \( D \) and \( C \) define Macaulay’s (1938) modified duration, equation [5], and Redington’s (1952) modified convexity, equation [6].

\[
D = -\frac{\mathrm{d}P(j_0)}{\mathrm{d}y} \frac{1}{P_0} = \frac{1}{P_0} \left[ \sum_{i=1}^{T} \frac{t \cdot \text{coupon}_i}{(1 + j_0)^{t+i}} + \frac{T \cdot \text{face value}_i}{(1 + j_0)^{T+i}} \right] \tag{5}
\]

\[
C = \frac{\mathrm{d}^2P(j_0)}{\mathrm{d}y^2} \frac{1}{P_0} = \frac{1}{P_0} \left[ \sum_{i=1}^{T} \frac{t \cdot (t+1) \cdot \text{coupon}_i}{(1 + j_0)^{t+i+1}} + \frac{T \cdot (T+1) \cdot \text{face value}_i}{(1 + j_0)^{T+i+1}} \right] \tag{6}
\]

Clearly, the approximation error is progressively reduced by adding further terms to the Taylor expansion in equation [2]. In other words, the difference between the true bond price (equation [2]) and the approximated bond price obtained through the classical duration approximation (equation [3]) is always greater than the difference between the true bond price and the duration-plus-convexity estimated price (equation [4]).

Recently, Livingston and Zhou (2005) have proposed a new measure, called exponential duration, which improves upon the accuracy of the first-order price approximation formula. With exponential duration, the bond price in response to a change in its yield-to-maturity is approximated by equation [7].

\[
P_{i}^{\text{ED}} = P_0 e^{D \Delta y} \tag{7}
\]

1 Note that the latter difference requires the absolute value since duration-plus-convexity overestimates (underestimates) the true bond price for positive (negative) interest rate changes. This is different from the classical duration approximation error which is always negative (Fabozzi, 2005).
The authors prove that this solution is more accurate than traditional duration (equation [3]). In fact, their exponential bond price function lies, for any condition (time-to-maturity, coupon rate and yield-to-maturity change), between the classical duration estimate and the true price function, thus reducing the net approximation error. In some circumstances, exponential duration also produces estimates close to duration plus convexity.

2.2 Discrete duration

Although exponential duration produces smaller approximation errors relative to the classical duration approach, in the event of significant interest rate changes or when applied to low-coupon bonds, it generates unsatisfactory forecasts.

For example, a ten year 5 percent coupon bond trading at par generates forecast errors of -8 basis points, -36 basis points, and -88 basis points for a 1 percent, 2 percent and 3 percent fall in bond yields when using exponential duration. Similarly, for a 1, 2, and 3 percent increase in bond yields, exponential duration produces forecast errors of −6 basis points, −26 basis points, and −52 basis points.

For a 30 year zero-coupon bond, errors are even larger. A fall in the bond yield of 1, 2, and 3 percent leads to forecast errors of -18 basis points, -98 basis points, and -296 basis points, respectively. Bond yield increases incur less of an error, but are significant nonetheless. For the same bond, a 1, 2, and 3 percent jump in bond yields results in forecast errors of -10 basis points, -30 basis points, and -51 basis points, respectively.
As these examples show, the exponential approximation always produces an underestimation of the actual bond price. In fact, much of the error in exponential duration is caused by a mismatch between the discrete compounding used to construct duration and the continuous transformation used in equation [7]. A more appropriate transformation is give by equation [8], which uses discrete compounding, hence the name *discrete duration*.

\[ D_{DD} \approx P \left(1 + \Delta r\right)^{-\Delta t} \]  

It can be shown (the proof is presented in the appendix) that the absolute value of the approximation error stemming from equation [8] is always smaller than that of exponential duration. In the next section, we use simulation methods to demonstrate the dominance of discrete duration over other first-order solutions for a variety of bond types.

3. **Simulations**

3.1 **Base analysis**

To assess the accuracy of our method we first take into consideration a ten year, five percent, coupon bond, trading at par. Figure 1 presents the approximation error (in basis points) of the different duration-based approaches.

Ignoring classic duration, which always leads to large forecast errors for significant jumps in bond yield, discrete duration always performs better than exponential duration and matches (and frequently betters) duration plus convexity. For the simulation domain, the error obtained through exponential approximation is roughly twice as large as discrete duration.
For a 1, 2, and 3 percent drop in yield-to-maturity, discrete duration underestimates the true bond price by 4, 18, and 43 basis points compared to exponential duration, which underestimates the true bond price by 8, 36, and 88 basis points, respectively.

Discrete duration has comparable performance to that of duration plus convexity. Duration plus convexity produces better forecasts for drops in yield to maturity of less than 3 percent. However, for large drops (greater than 3 percent) and any increase in yield to maturity, discrete duration performs markedly better. Moreover, the dominance of discrete duration over all other approximation methods becomes greater as the jumps in yield get higher. Finally, while duration plus convexity overestimates bond values for any growth in bond yield, discrete duration always remains below zero. This is a particularly important characteristic for prudent fixed income managers.

3.2 **The effect of term to maturity**

In this, and the next, subsection, we extend the core analysis to consider the impact of yield to maturity and coupon rate on the performance of discrete duration.

Figure 2 shows the difference between the discrete duration and exponential duration approximation errors for a 5 percent coupon par bond with different terms to maturity. For any yield change and term to maturity combination, discrete duration always performs better than exponential duration. For small changes in the bond yield, both approaches are comparable, but not the case for large yield jumps.
For example, with a 30 year, 5 percent, par bond, discrete duration guarantees an increase in accuracy equal to 10 basis points for a 1 percent yield drop, but four and ten times as much for 2 percent and 3 percent falls in yield to maturity. Figure 2 markedly shows the improvement in performance of discrete duration for drops in the bond yield. However, the most important insight is that, regardless of the change in yields or term to maturity, discrete duration always provides a more efficient approximation of the bond price function compared to exponential duration for any term to maturity and yield change.

Figure 3 presents a comparison of the performance of discrete duration against duration plus convexity for bonds of different terms to maturity. For small yield changes, the difference is virtually zero. For larger jumps, the behavior of the two methods is widely divergent and strengthens as term to maturity gets bigger.

Specifically, discrete duration generally leads to higher accuracy for upward interest rate movements, whilst it underperforms duration plus convexity when the bond yield falls. Similar to Figure 2, the difference gets more pronounced as term to maturity grows. However, this difference tends to remain steady and only peaks up (for about 400 basis points) or down (for about 200 basis points) as a response to a jump of yield to maturity above or below 3 percent.

### 3.3 Premium and Discount Bonds

Another important bond characteristic that affects the curvature of the bond price function is the coupon rate relative to the yield to maturity. We now compare the precision of discrete duration, exponential duration,
and duration plus convexity for a premium bond (10 percent coupon rate) and a discount (zero coupon) bond. The yield to maturity is still assumed to be 5 percent.

A 10 percent coupon rate may appear unrealistic if compared to the yield to maturity of 5 percent. However, our objective is to show the performance of duration-based approximation on a bond that is far from parity, and once we establish the error pattern for this and a par bond, the approximation error of any other bond can be inferred.

Table 1 reports the error difference (in basis points) between exponential duration, discrete duration, and duration plus convexity, for a par, zero-coupon, and premium bond for different levels of time-to-maturity. Starting with the latter, our simulation results clearly indicate the higher accuracy of discrete duration relative to exponential duration.

For example, in the event of a 1 percent drop in yield to maturity, the error difference between the two methods\(^2\) for the premium bond is 2 basis points with five years to maturity, 4 basis points with ten years to maturity and 8 basis points with thirty years to maturity. The par bond leads to a similar relative performance of 2, 4, and 9 basis points.

Larger changes bring relatively greater differences. For example, assuming a 3 percent drop in yield to maturity, the same errors for a premium (par) bond becomes 21 (23), 39 (45) and 95 (112) basis points for maturities of 5, 10 and 30 years, respectively.

\(^2\) Measured as the exponential duration forecast error minus discrete duration forecast error.
More interesting is the scenario for zero-coupon bonds. For positive changes in interest rates, discrete duration is slightly more accurate than exponential duration, particularly for long-dated bonds. The performance of discrete duration improves by 3 basis points (from 5 basis points to 8 basis points) when there is a 1 percent increase in yield to maturity for a thirty year zero-coupon bond. A 2 percent increase in yield to maturity improves the relative performance between discrete duration and exponential duration by 7 basis points (from 22 to 29 basis points).

With a drop in yield to maturity, the precision of discrete duration is notably better, especially for long dated zero-coupon bonds where the forecast error of discrete duration is half that of exponential duration.

### 3.3.1 A closer look at zero-coupon bonds

The base duration model is unable to precisely estimate the interest-rate risk for low coupon bonds with high term to maturity. A simple numeric example, based on a 30 year zero-coupon bond will help to illustrate the magnitude of the potential problem. A 1 percent drop in yield to maturity causes a duration approximation error equal to -468 basis points. Even with duration plus convexity, the approximation error is -46 basis points.

Hence, even the use of a two-factor approximation does not fully mitigate the approximation error, potentially leading to a misspecification and underperformance of hedging strategies based on the duration method. For such reason, we now focus on a long-dated zero-coupon bond to illustrate how the advantage of discrete duration.
Figure 4 illustrates the approximation error produced by different duration-based approximation methods for a 30 year zero coupon bond around a narrower yield change window of (-100, +100) basis points. It is immediately visible that, with the exception of discrete duration, all the duration-based methods are unsuccessful in measuring the price change in response to a fluctuation of interest rates.

The performance of discrete duration is fairly impressive since the continuous error line in Figure 4 lies just slightly above the x-axis. The data reported below the graph also indicate that, for any yield change within the considered interval, the discrete duration approximation error is contained between 0 and 1 bps.

Recalling the previous example, a 1 percent yield drop would lead to a 46 basis point error if duration-plus-convexity is used, 18 basis points with exponential duration and only 1 basis point with discrete duration method. Furthermore, the higher accuracy of discrete duration is not constrained to only negative yield changes.

4. Conclusions

Money and bond market managers use duration to implement trading and hedging strategies. However, duration provides only an approximate measure of interest-rate risk and generates significant estimation bias for large yield shifts, particularly with long-dated and low-coupon bonds.

The use of second-order Taylor approximation (convexity) leads to a substantial reduction in estimation error, but requires the computation of an additional risk factor. Moreover, duration plus convexity can lead to significant errors for particular yield change-bond characteristic
combinations. For instance, in the event of a 1 percent fall in bond yields, the price estimation via duration plus convexity for a 30 year zero-coupon bond is about fifty basis points, which cannot be considered a negligible error.

Recently, Livingston and Zhou (2005) have proposed a new duration-based approach for the estimation of bond price. The authors show how an exponential transformation of classical duration produces a more accurate price forecast. However, as the authors point out, the exponential approximation always underestimates the actual bond price and frequently, duration plus convexity, leads to lower forecast errors.

We introduce and test a new duration measure, which we call discrete duration, and show that it outperforms all other methods for a large set of different bond characteristic and yield change scenarios. Discrete duration dominates exponential duration in all simulations, and performs exceptionally well against duration plus convexity. Specifically, discrete duration is generally more accurate than duration plus convexity for upward yield curve shifts and, with low coupon bonds, for any interest rate movement.
Appendix

In this appendix we prove that the discrete duration measure given in Equation [8] outperforms Livingston and Zhou’s (2005) exponential duration (Equation [7]). Specifically, we demonstrate that the discrete duration price forecast always lies within the interval of the true bond price +/− the exponential duration forecast error.

For the sake of simplicity we split the proof into two propositions. First, we demonstrate that the discrete duration estimated bond price, \( P_{1DD} \), is always greater than the exponential approximation, \( P_{1ED} \). Next, we show that the discrete duration estimated bond price never exceeds the sum of the true price and the exponential duration approximation error.

Combining the two propositions, the proof follows directly.

Proposition 1: \( P_{1DD} \) is always greater than \( P_{1ED} \).

We first expand the discrete duration approximated bond price in a Taylor series expansion around the current yield to maturity, obtaining [A1].

\[
P_{1DD} = P_0 \left[ 1 - D \cdot \Delta y + \sum_{n=2}^{\infty} \frac{1}{n!} (-1)^n \Delta y^n \prod_{m=0}^{n-1} (n + D) \right] \tag{A1}
\]

Equation [A2] presents a similar Taylor series expansion of exponential duration.

\[
P_{1ED} = P_0 \left[ 1 - D \cdot \Delta y + \sum_{n=2}^{\infty} \frac{1}{n!} (-1)^n \Delta y^n D^n \right] \tag{A2}
\]

Subtracting \( P_{1ED} \) from \( P_{1DD} \) always results in a positive value.
\[ P_{t}^{DD} - P_{t}^{ED} = P_{t} \sum_{n=0}^{\infty} \frac{(-1)^n}{(n+D)} - D^{n} \]
\[ = P_{t} \left[ (1+\Delta y)^{-D} - \exp(-\Delta y \cdot D) \right] \]  

**Proposition 2:** \( P_{t}^{DD} \) never exceeds the actual \( P_{t} \) plus the exponential duration approximation error.

First, we need to determine the condition that guarantees \( P_{t}^{DD} \) exceeding the true price, \( P_{t} \). In fact, for \( P_{t}^{DD} < P_{t} \), proposition 2 is necessarily true. By comparing, in second-order terms, the discrete duration approximated price,\(^3\) we find that \( P_{t}^{DD} \) exceeds the true price if and only if

\[ \frac{1}{2}(D^2 + D)\Delta y^2 > \frac{1}{2} \left( D^2 - \frac{dD}{dy} \right)\Delta y^2 \]  

This yields the inequality \([A5]\).

\[ D > -\frac{dD}{dy} \]  

We now show that the difference between \( P_{t}^{DD} \) and \( P_{t} \) is always smaller than the difference between \( P_{t} \) and \( P_{t}^{ED} \) to complete proposition 2. From \([A1]\) and \([A2]\), we arrive at the inequality, \([A6]\).

\[ \frac{1}{2}(D^2 + D)\Delta y^2 - \frac{1}{2} \left( D^2 - \frac{dD}{dy} \right)\Delta y^2 < \frac{1}{2} \left( D^2 - \frac{dD}{dy} \right)\Delta y^2 - \frac{1}{2}D^2\Delta y^2 \]  

Combining \([A6]\) with \([A5]\), yields \([A7]\).

\[ D < -2\frac{dD}{dy} \]  

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\(^3\) Recall that bond (modified) convexity can also be written as the difference between the (modified) duration squared and the first derivative of the bond (modified) duration with respect to the YTM.
It is straightforward to show that this inequality holds for any level of interest-rate change, time-to-maturity and coupon rate; hence proposition 2 is true.
References


Figure 1 - Approximation accuracy for different duration-based approaches: 10 year, 5% par bond

The figure shows the approximation errors (in basis points) produced by the different duration-based formulas. Below the graph, estimated prices and approximation errors are reported for +/- 5% yield to maturity (YTM) shifts. Approximation errors are computed as (estimated price - actual price) / actual price. The simulation is based on 10 year and 5% coupon rate bond.

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Figure 2 - Difference between exponential duration approximation and discrete duration error.

The figure plots the error difference between the exponential duration and the discrete duration produced on a 5% coupon par bond. A positive error denotes a higher accuracy for the discrete duration method. The simulation is run for +/- 5% yield to maturity (YTM) shifts and for 1 to 30 years bond’s time-to-maturity. Approximation errors (reported in basis points) are computed as (estimated price – actual price) / actual price.
Figure 3 - Difference between duration-plus-convexity and discrete duration error.

The figure plots the error difference between the classical duration-plus-convexity and the discrete duration methods produced on a 5% coupon par bond. A positive error denotes a higher accuracy for the discrete duration method. The simulation is run for +/- 5% yield to maturity (YTM) shifts and for 1 to 30 years term to maturity. Approximation errors (reported in basis points) are computed as (estimated price – actual price) / actual price.
Figure 4 - Approximation accuracy for different duration-based approaches: 30 year zero coupon bond

The figure shows the approximation errors (in basis points) produced by the different duration-based formulas. Below the graph, estimated prices and approximation errors are reported for +/- 1% yield to maturity (YTM) shifts. Approximation errors are computed as (estimated price - actual price) / actual price. The simulation is based on 30 year zero-coupon bond.

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Table 1 - Comparison of Approximation Errors

Panel A shows the difference between exponential and discrete approximation error (in basis points); Panel B the difference between classical duration plus convexity and discrete approximation error.

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