Returns and Volume: Between Information and Liquidity

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Abstract

This paper develops a model for stock trading which takes into account both information and liquidity shocks. First, we distinguish between two trading strategies, information-based and liquidity-based trading, and suggest that their respective impacts on returns and traded volume should be modelized differently. Second, we focus on the contemporaneous volatility-volume relationship to model impacts of information and liquidity. We relax the hypothesis of absence of liquidity problems and extend the standard mixture of distribution hypothesis (MDH) framework. This paper develops a modified MDH model which takes into account information and liquidity shocks. Third, we show how to use a structural model to exploit the volume-volatility relation in order to decompose the traded volume for a given stock into two components. Thus, we separate information from liquidity impact on the observed daily volume. This allows us to extract an average intra-day liquidity measure using daily data.

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Key words: Volatility-volume relationship, mixture of distribution hypothesis, liquidity shocks, information-based trading, liquidity arbitrage, GMM tests.

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1 Introduction

This paper develops a model for stock trading which takes into account both information and liquidity shocks. First, we distinguish between two trading purposes: the information-based and the liquidity-arbitrage trading motives. Then, we separate information from liquidity shocks by modeling their respective impacts on stock return variance and traded volume. To do so, we focus on the contemporaneous relationship between the daily stock returns and the trading volume and propose a modified mixture of distribution hypothesis model to account for both, information and liquidity shocks for a given stock.

Previous empirical studies [see Ying (1966), Crouch (1970), Clark (1973), Copeland (1976, 1977), Epps and Epps (1976), Westerfield (1977), Rogalski (1978), Tauchen and Pitts (1983), Harris (1983-86)] of both futures and equity markets find a positive association between price variability[1] and the contemporaneous trading volume[2]. The usual theoretical explanation of this positive volume-return volatility relation comes from microstructure models which analyze how information is disseminated and to which extent market prices convey information. Thus, several models predict a positive return volatility-volume relation that depends on the rate of information flow and the interaction between specialists, informed and liquidity traders [Kyle (1985), Glosten and Milgrom (1985), Easley and O'Hara (1987), Diamond and Verrecchia (1987), Admati and Pfleiderer (1988), Foster and Viswanathan (1990, 1993), Easley et al. (1996)], the market size [Gallant et al. (1992)], and the existence of short sales constraint [Diamond and Verrecchia (1987)].

The mixture of distribution hypothesis (MDH) models attempt to explore the microstructure framework in which information asymmetries and liquidity needs motivate trade in response to information arrivals. The MDH, pioneered by Clark (1973) and extended by Tauchen and Pitts (1983), Harris (1983), and Andersen (1996) among others, provides an explanation of the positive correlation between volume and the squared value of price change. For example, Clark's (1973) model assumes that events important to the pricing of a security occur at a random, not uniform, rate through time. It appears that price data are generated by a conditional normal stochastic process with a changing variance parameter that can be proxied by volume whose distribution is assumed to be lognormal. Clark (1973) shows that the lognormal-normal mixture outperforms several members of stable family. Using the same assumption, Harris (1982-1987) and Tauchen and Pitts (1983) show that the joint distribution of daily price changes and volume can also be modeled by a mixture of bivariate normal distributions. They

[1] As measured by either the square price change or the price change per se.

[2] See Karpoff (1987) for a detailed review of the literature.

assume that both variables (the daily price change and daily volume) are conditioned by the rate of information which is random and serially uncorrelated. Assuming a lognormal distribution for the mixing variable, the model can be estimated by maximum likelihood [see Tauchen and Pitts (1983) for further discussion]. As pointed out by Harris (1986, 1987), the MDH can explain the fat tailed probability distribution of the daily price change, and the positive correlation between return volatility and volume. It is important to note that the standard MDH models assume that only information arrivals drive the positive volatility-volume relationship. Securities are assumed to be liquid and the liquidity problem effects on volatility-volume relation are disregarded.

If earlier tests find evidence supportive of the MDH model [Clark (1973), Epps and Epps (1976), Tauchen and Pitts (1983), Harris (1986, 1987)], later studies are less favorable [Heimstra and Jones (1994), Lamoureux and Lastrapes (1994), Richardson and Smith (1994), and Andersen(1996)]. Different authors propose various extensions of the standard MDH model in order to improve its explicative power. Lamoureux and Lastrapes (1994) extension assumes that the information-arrival rate is serially correlated[3]. Andersen (1996) develops a modified MDH model that includes a conditional Poisson distribution for the trading process and a volume component that is not information sensitive. His tests suggest that the modified version significantly outperforms the standard MDH, which assumes that both returns and volume are normally distributed.

Note that, previous MDH tests are performed under the assumption that the market is perfectly liquid. Thus, they do not integrate the illiquidity dimension. However, several studies show that liquidity shocks are priced by the market [see Amihud (2002), and Acharya and Pedersen (2005) among others] and that they impact both returns and traded volume [see Chordia et al. (2001a), Chordia et al. (2000), and Darolles and Le Fol (2005)]. For example, Darolles and Le Fol (2005) point out that the liquidity problems cause price pressures which will increase the intraday price volatility[4]. These market imperfections will motivate the liquidity arbitragers to enter the market in order to provide the missing liquidity, and cash the liquidity premium. The presence of liquidity arbitragers raise the daily traded volume. It follows that the observed daily traded volume is the result of both information-based traders and liquidity arbitragers. It would be interesting to be able to decompose the volume into two components in order to isolate

[3]However their model fails to explain GARCH persistence in return variance. Their finding is consistent with the results of Richardson and Smith (1994), who used GMM to test the mixture model but did not account for time dependencies in the data. Thus, the evidence against the model isolates the inability of the model to jointly accommodate the dynamic properties of squared returns and volume.

[4]In particular, the liquidity pressures will rise the price for stocks facing sell-side liquidity lacks and lower the price of stocks facing buy-side liquidity lacks.

the impact of each type of trading on stock prices.

These observations motivate us to develop a modified MDH model which considers both, information and liquidity effects on individual stocks. We include an additive mixing variable L in the model of Tauchen and Pitts (1983) to take into account the random liquidity shocks. Our MDH model with two latent variables permits us to decompose the trading volume into two components driven by information and liquidity. Following Richardson and Smith (1994) we propose a direct test of the modified MDH model. In fact, because the model imposes restrictions on the joint moments of price changes and volume as a function of only a few parameters, it is possible to form overidentifying restrictions on the data. These restrictions can be tested using the generalized method of moments (GMM) procedure of Hansen (1982).

The contribution of this paper is threefold. First, it distinguishes between two trading strategies, information trading and liquidity arbitrage, and suggests that their respective impacts on returns and traded volume should be modelized differently. The former is incorporated into daily price changes and traded volume, and drives the positive volatility-volume relationship. The latter has intra-day effects on price variations but do not affect daily price changes. However, it increases the daily traded volume. Second, the paper relaxes the hypothesis of absence of liquidity problems and extends the MDH framework by developing a modified MDH model which integrates both information and liquidity shocks. Third, we use a structural model, the modified MDH model herein proposed, to exploit the volume-volatility relation in order to decompose the traded volume for a given stock into two components and thus separate the information from the liquidity trading impact in the observed volume. This allows us to extract an average intra-day liquidity measure using daily data.

The paper is organized as follows. In section 2, we briefly present the standard MDH model based on the Tauchen and Pitts (1983) framework. Section 3 develops our model. We first discuss the limits of the standard MDH model. Then, we present a mixture model which allows for the presence of both, information and liquidity shocks. In section 4, we present the GMM test and discuss the empirical application. Section 5 concludes.

2 The standard MDH model (Tauchen and Pitts, 1983)

This section summarizes the standard MDH model based on the theorical framework of Tauchen and Pitts (1983), henceforth TP. The model focuses on a single market for a traded asset with a random liquidation value of F at a (distant) point in the future. There are J active traders in the market who take long or short positions in a single asset. The authors assume that J is nonrandom and fixed for each day. Within the day the market passes through a sequence of distinct within-day equilibria. The movement from the $(i-1)$st equilibrium to the ith is initiated by the arrival of new information to the market. The intervals between successive equilibria are not necessarily of the same length. Note that , since buy or sell orders are executed sequentially, many actual transactions at the exchange can comprise what we think of as a single market clearing or transaction.

It is important to note that no assumptions are made concerning liquidity problems since, in the TP economy, assets are deemed perfectly liquid. At the ith equilibrium, the jth trader is willing to trade Q_{ij}, which is given by the linear relation:

$$Q_{ij} = \alpha[p^*_{ij} - p_i], \qquad (j = 1, 2, ..., J),$$

where $\alpha > 0$ is constant, p^*_{ij} is the trader's reservation price and p_i is the current market price. Note that a positive value for Q_{ij} represents a desired long position in the stock while a negative value represents a short position[5].

Traders react differently to new information and propose different reservation prices p^*_{ij}. The reservation price heterogeneity in the population of traders comes from expectation heterogeneity about the future liquidation value F, and from different needs to transfer the risk through the market. Equilibrium requires $\sum_{j=1}^{J} Q_{ij} = 0$. This implies that the reservation price average $p_i = \frac{1}{J}\sum_{j=1}^{J} p^*_{ij}$ clears the market. Using this equation and the equilibrium condition, the stock price change and the trading volume at the ith equilibrium can be written respectively as:

$$\Delta p_i = \frac{1}{J}\sum_{j=1}^{J} \Delta p^*_{ij}, \tag{1}$$

$$V_i \equiv \frac{1}{2}\sum_{j=1}^{J} |Q_{ij} - Q_{i-1,j}| = \frac{\alpha}{2}\sum_{j=1}^{J} |\Delta p^*_{ij} - \Delta p_i|, \tag{2}$$

where Δp^*_{ij} is the increment of the jth trader reservation (log) price. TP assume that traders update their reservation price as follows:

$$\Delta p^*_{ij} = \phi_i + \psi_{ij}, \tag{3}$$
$$with \quad \phi_i \sim N(0, \sigma^2_\phi), \;\; \psi_{ij} \sim N(0, \sigma^2_\psi),$$

[5]The demand equation abstracts from transaction costs and assumes that the traders differ only in their reservation prices.

where ϕ and ψ are mutually independent both across traders and through time[6]. They show that Δp_i and V_i are normally distributed[7]: $\Delta p_i \sim N(0, \sigma_p^2)$ and $V_i \sim N(\mu_v, \sigma_v^2)$.

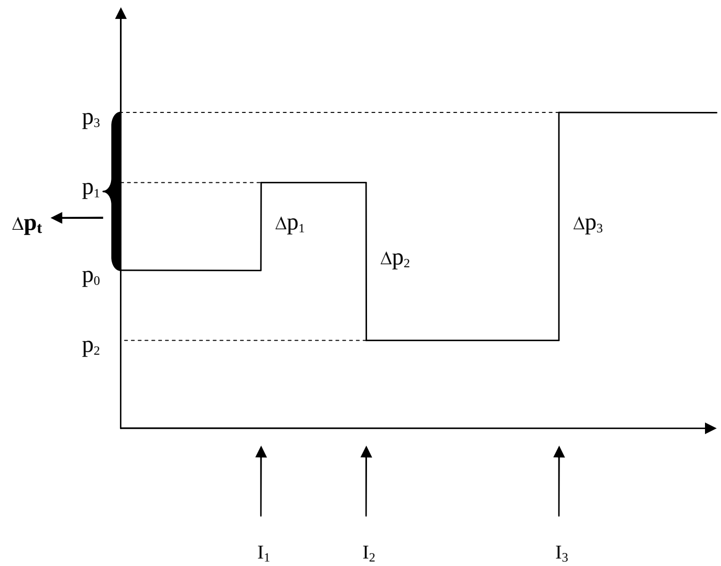

Figure 1: Day t price change as a function of intra-day price variations due to information shocks.

TP assume that the number of daily equilibria I_t is random because the number of new pieces of information hitting the market each day varies significantly. Figure 1 shows a simplified example of how intra-day price varies in response to the inflow of new information. Let p_0 be the equilibrium price at the beginning of the trading day t. We assume that only three pieces of information arrive at day t, I_1, I_2 and I_3.

I_1 is perceived as good news: trader expected value for the given asset is to increase, resulting in a new equilibrium price, $p_1 > p_0$, and the price increment due to the arrival of I_1, Δp_1, is positive. I_2 being seen as bad news, the next price increment, Δp_2 is negative. Lastly, I_3, which turns out to be good news, initiates the movement to the third intra-day equilibrium and Δp_3 is positive. At the end of day-t, we observe the daily price increment $\Delta p_t = p_3 - p_0$. Note that, the daily price change is the sum of intra-day price increments due to the arrival of the new information: $\Delta p_t = \Delta p_1 + \Delta p_2 + \Delta p_3$.

Summing the within-day price changes and trading volumes, we get the

[6]According to TP, the component ϕ_i is common to all traders while ψ_{ij} is specific to the jth trader. For example, a large absolute value of common component relative to the absolute values of the specific components represent a situation in which the traders react nearly unanimously to the new information.

[7]See TP (1983) proposition, page 490.

day- t price change, Δp_t, and traded volume, V_t:

$$\Delta p_t = \sum_{i=1}^{I_t} \Delta p_i, \qquad \Delta p_i \sim N(0, \sigma_p^2), \tag{4}$$

$$V_t = \sum_{i=1}^{I_t} V_i, \qquad V_i \sim N(\mu_v . \sigma_v^2), \tag{5}$$

Both Δp_t and V_t are mixtures of independent normals with the same mixing variable I. Conditional on I_t, the daily price change Δp_t is $N(0, \sigma_p^2 I_t)$ and the daily volume is $N(\mu_v I_t, \sigma_v^2 I_t)$. The bivariate normal mixture can also be written as:

$$\begin{aligned} \Delta p_t &= \sigma_p \sqrt{I_t} Z_{1t}, \\ V_t &= \mu_v I_t + \sigma_v \sqrt{I_t} Z_{2t}, \end{aligned} \tag{6}$$

where Z_{1t} and Z_{2t} are independent standard normal variables. At the end of the day t, all the incoming information is incorporated into the price change Δp_t. Following TP, the only reason for prices and volumes to change on an intra-day basis, as well as a day frequency, is information arrival. From equations given in (6), it follows that the contemporaneous relation between Δp_t^2 and V_t is:

$$\begin{aligned} Cov(\Delta p_t^2, V_t) &= E[\Delta p_t^2 V_t] - E[\Delta p_t^2] E[V_t] \\ &= \sigma_p^2 \mu_v E[I_t^2] - \sigma_p^2 \mu_v (E[I_t])^2 \\ &= \sigma_p^2 \mu_v Var[I_t] > 0. \end{aligned}$$

The volatility-volume relationship arises because Δp_t^2 and V_t are positively related to the unobserved mixing variable I_t, whose variance is different from zero. Note that when $Var[I_t] = 0$, the relationship vanishes.

Using maximum likelihood estimations[8], TP show that the standard MDH model captures the positive relationship between price change variance and volume on the 90-day T-bills futures market.

[8]Conditional on I_t, Δp_t and V_t are independent and so their joint conditional density is the product of marginals:

$$f_c(\Delta p_t, V_t | I_t; \sigma_p, \mu_v, \sigma_v) = n(\Delta p_t; 0, \sigma_p^2 I_t) n(V_t; \mu_v I_t, \sigma_v^2 I_t) \tag{7}$$

where $n(\cdot)$ is the normal density. The unconditional joint density is:

$$f(\Delta p_t, V_t; \sigma_p, \mu_v, \sigma_v, \theta) = \int f_c(\Delta p_t, V_t | I_t; \sigma_p, \mu_v, \sigma_v) G(I_t; \theta) \tag{8}$$

where $G(I_t; \theta)$ is the marginal distribution function of the mixing variable and θ is the vector of its parameters.

Richardson and Smith (1994) extend TP work by introducing a mean parameter for daily price change, and use GMM tests to validate the model. In this paper, we use Richardson and Smith (1994) version when estimating the standard MDH model for robustness checks (see section 4).

3 The modified MDH model with information and liquidity shocks

This section develops a modified MDH model integrating both information and liquidity shocks. TP model disregards the liquidity problem, and even if when J (number of traders in the TP model) is small some liquidity frictions may arise. In that case, the equilibrium price may differ greatly from the price revealing the information. In our model, the price distortion will be corrected by the intervention of a new type of trader: the liquidity arbitragers. We focus on a single market for a traded asset with random liquidation value F at a distant point in the future. We consider two kinds of traders: the active traders of TP, and the liquidity arbitragers. We assume that the number of each category of traders is nonrandom and fixed for each day. Based on TP, we make several assumptions concerning the information flow. Within the day, the market passes through a sequence of distinct equilibria. We assume that, for a particular stock, the movement from one equilibrium to the other is initiated by the occurrence of the new information, which modifies trader expectations about the future liquidation value, F, and motivates them to rebalance their positions. We assume that the information flow is random, which drives the randomness of the within-day equilibria:

A1 *The number of daily equilibria, I_t, is random because the number of new pieces of information reaching the market each day varies significantly.*

In addition, we assume that between two equilibria, or two successive information arrivals, informed traders may face liquidity problems. The lack of liquidity for a given stock can be either on the sell or the buy side. These liquidity problems cause price pressures which lift the price for stocks facing sell-side liquidity shortage and lower the price of stocks facing buy-side liquidity shortage. These market imperfections motivate the liquidity arbitragers to enter the market in order to provide the missing liquidity and cash the liquidity premium[9]. To formalize our reasoning and account for both information and liquidity shocks, we need to make additional assumptions:

A2 *Liquidity shocks arise only between two within-day equilibria (or two successive information arrivals), during the price adjustment process from*

[9]See Darolles and Le Fol (2005) for a more detailed discussion.

one equilibrium to the next.

A3 *The transaction prices when dealing with liquidity arbitragers are disadvantageous compared to prices that would prevail in the absence of liquidity problems.*

If the market faces sell (buy) side liquidity shortage, the transaction prices with a liquidity arbitrager are higher (lower) than those in absence of liquidity shocks, but this price would be even higher if the liquidity arbitrager was not participating. The presence of liquidity traders helps correct price imperfections: the sell-side prices decrease while the buy-side ones get higher. This motivates them to liquidate their initial positions and receive the liquidity premium. As a consequence, liquidity arbitragers react to market volatility rather than being responsible for it. The volume they trade adds to the volume that would be traded if there were no liquidity imperfection.

A4 *The liquidity arbitragers liquidate their positions and collect the liquidity premium, before the following information arrival.*

This hypothesis is set for simplification purpose. It can be relaxed without loss of generality.

A5 *The number of the within-day t liquidity shocks, L_t, is random and independent of I_t.*

Trading can occur either when information hits the market or when a liquidity event occurs. At each transaction date there is a probability q that an information occurs. The reservation prices of the traders are then updated such that:

$$\Delta p^*_{ij} = \phi_i + \psi_{ij}, \tag{9}$$

with $\psi \sim N(0, \sigma^2_\psi)$ as in TP, but:

$$\phi_i = \begin{cases} N(0, \sigma^2_\phi) & when \quad an \quad information \quad event \quad occurs \\ \\ 0 & otherwise. \end{cases} \tag{10}$$

Figure 2 is based on the example of the previous section (figure 1) while relaxing the hypothesis of the absence of liquidity imperfections. It shows how intra-day price increment behaves in response to both information flow and liquidity shocks. The intra-day price behavior in absence of liquidity shocks is visually described by the dashed lines. Let Δp_i be the price increment due to ith information inflow into the market in absence of liquidity shocks, and $\Delta p_i^{'}$ be the price variation due to liquidity shortage. Remember that in our example, only three flows of information reach the market: two positive ones, I_1 and I_3, and a negative one, I_2.

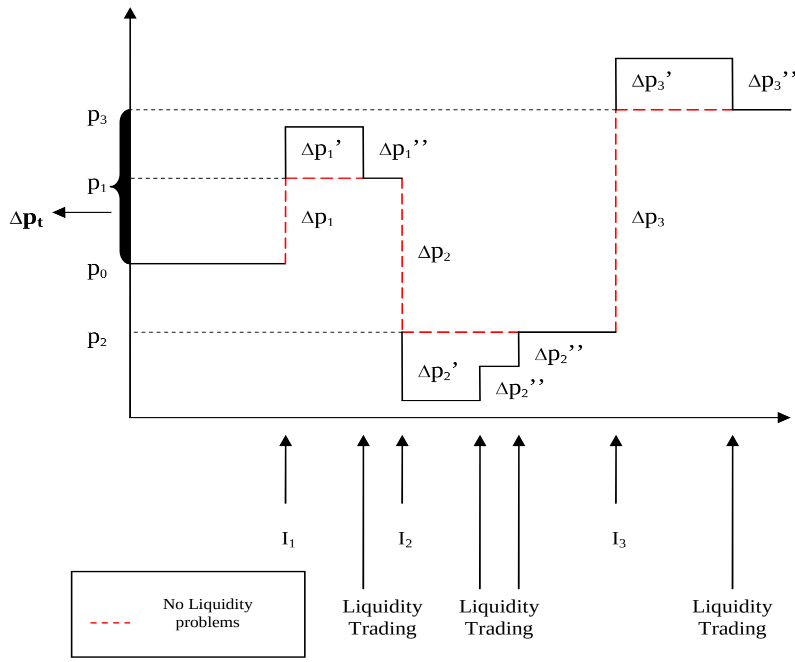

Figure 2: Day t price change as a function of intra-day price fluctuations due to information and liquidity shocks.

As for I_1 , the informed trader expectations concerning the future value of the asset will rise, resulting in a positive Δp_1. Should the active traders face sell-side liquidity shortage, the asset price will increase more than if there were no liquidity problems, say p_3. The liquidity arbitragers observing this price distortion will enter the market to provide the missing liquidity. They will sell at a price between p_1 and p_3. They will trade the stock at $p_0 + \Delta p_1 + \Delta p_1^{'}$. Later on, the liquidity arbitragers will enter the market to buy at p_1 (the information revelation price), and thus will cash the liquidity premium. We get $p_1 = p_0 + \Delta p_1 + \Delta p_1^{'} + \Delta p_1^{''}$, where $\Delta p_1^{''}$ and $\Delta p_1^{'}$ have opposite signs. Since $|\Delta p_1^{''}| = |\Delta p_1^{'}|$, the price returns to fully revealing equilibrium.

When I_2 hits the market, the price will be lessened by Δp_2. In addition, should the market face buy-side liquidity shortage, the price is to decrease by even more than $\Delta p_2^{'}$. As the liquidity arbitragers track these market imperfections, they will subsequently decide to enter the market and to purchase the stock at $p_1 + \Delta p_2 + \Delta p_2^{'}$. Thanks to liquidity arbitrager intervention, the price is to converge toward its fully revealing equilibrium level, $p_1 + \Delta p_2$. The liquidity trading may not be sufficient to immediately bring back prices to equilibrium. In this case, we observe a sequence of $\Delta p_2^{''}$ such as $\Delta p_2^{'} = -\sum \Delta p_2^{''}$. In any case, liquidity arbitragers will liquidate their positions in a sequence of prices which are higher than $p_1 + \Delta p_2 + \Delta p_2^{'}$, until the market comes back to $p_1 + \Delta p_2$.

In the third case, price increments will fluctuate in a similar way as in the first case. At the end of day t we observe $\Delta p_t = p_3 - p_0$. Since $\Delta p_i^{'}$ and $\Delta p_i^{''}$ offset one another, the daily price change is the sum of intra-day price increments due to the arrival of the new information: $\Delta p_t = \Delta p_1 + \Delta p_2 + \Delta p_3$.

Generally speaking, the sign of $\Delta p_i^{'}$ depends on the side of liquidity shortage, but $\Delta p_i^{'}$ and Δp_i have the same sign. From assumption A4, it follows that the liquidity arbitragers tend to correct prices before the next equilibrium by making $\Delta p_i^{'}$ vanish. Here, $\Delta p_i^{''}$ should be viewed as the price adjustment due to liquidity arbitrager activity. Summing the within-day price changes Δp_i, and the price imperfections due to lacks of liquidity $\Delta p_i^{'}$, and to liquidity adjustments $\Delta p_i^{''}$, yields the day t price change, Δp_t, as follows:

$$\Delta p_t = \sum_{i=1}^{I_t} \Delta p_i + \sum_{i=1}^{I_t} \Delta p_i^{'} + \sum_{l=1}^{L_t} \Delta p_l^{''} \qquad (11)$$

where L_t is the number of liquidity adjustments within a day and I_t is the number of information arrivals (or within-day equilibria) for day t.

Two cases can be observed: (i) If $|\Delta p_i^{''}| = |\Delta p_i^{'}|$, the price is immediately back to equilibrium i: $p_i = p_{i-1} + \Delta p_i$; (ii) If $|\Delta p_i^{''}| < |\Delta p_i^{'}|$, the price adjustment will not be immediate, but a convergence mechanism will take place. We assume that this mechanism will result in a price convergence to its equilibrium value, p_i. The lower the lack of liquidity, the higher the absolute value of the liquidity price adjustment and the faster the price adjustment.

The intra-day price and volume evolutions depend on both information as well as liquidity shocks. However, in the presence of liquidity arbitragers, price change between two consecutive equilibria due to liquidity shocks is not integrated into the daily price fluctuation. As a consequence, we have, $\sum_{i=1}^{I_t} \Delta p_i^{'} + \sum_{l=1}^{L_t} \Delta p_l^{''} = 0$. Following TP, we assume that the intra-day price change due to information imperfections is normally distributed with mean zero and variance σ_p^2, which yields:

$$\Delta p_t = \sum_{i=1}^{I_t} \Delta p_i, \quad \Delta p_i \sim N(0, \sigma_p^2) \qquad (12)$$

Note that liquidity problems increase intra-day price variance, while liquidity arbitrage transactions have a mean reverting effect which tends to correct intra-day price imperfections due to liquidity shocks. Consequently, only information flow impact is integrated in the daily price change.

However, the volume traded by liquidity arbitragers in order to liquidate their positions adds to the volume that would be traded if there were no liquidity imperfection. When the ith piece of information hits the market, the active traders revise their expectations and decide to rebalance their portfolios. Let V_i be the traded volume driven by the ith information arrival for a given asset in the absence of liquidity frictions. If the market faces liquidity problems, the active traders can not trade the requested amount. However, when assuming the presence of liquidity arbitragers, active traders will trade the remaining volume with them at a disadvantageous price, as described in the aforementioned example. From assumption A3, it follows that, in presence of liquidity arbitragers, the daily traded volume is not affected by liquidity problems. Active traders can exchange the amount they want, partly with the market, $V_i^{''}$, and partly with the liquidity arbitragers, $V_i^{'}$, such as: $V_i = V_i^{'} + V_i^{''}$. The more illiquid the market, the higher the part of volume traded with liquidity arbitragers, $V_i^{'}$. Since the latter will liquidate their positions[10], the total traded volume will be higher than if there were no liquidity problem. Let V_l be the traded volume due to arbitrager inventory purposes. Between two information arrivals we have: $V_i^{'} = \sum_l V_l$, where l denotes the number of liquidity shocks. Summing the within-day transaction volume motivated by information flow V_i, and the traded volume due to liquidity shocks V_l, gives the day t volume V_t as follows:

$$V_t \;=\; \sum_{i=1}^{I_t} V_i + \sum_{l=1}^{L_t} V_l, \qquad V_i \sim N(\mu_{v1}, \sigma_{v1}^2), \;\; V_l \sim N(\mu_{v2}, \sigma_{v2}^2) \qquad (13)$$

where L_t is the total number of within-day liquidity shocks at day t. We assume that V_i and V_l are independent normals with parameters μ_{v1}, σ_{v1}^2 and μ_{v2}, σ_{v2}^2. In our model, since trades are initially motivated by information flows, and the sum of V_l corresponds to part of V_i, we assume that L_t does not impact the within-daily volume variance, by imposing $\sigma_{v2}^2 = 0$, which means that $V_l = \mu_{v2}$. For notation simplicity, we replace σ_{v1}^2 by σ_v^2.

From equations (12) and (13), we can get a mixture of distribution model with two latent variables, I and L. From assumption $A5$, it follows that $Cov(I, L) = 0$ and $Cov(f(I), f(L)) = 0$, where $f(I)$ and $f(L)$ can be any

[10] Once the market is back to equilibrium.

function of I and L. The bivariate normal mixture can, then, be written:

$$\begin{aligned} \Delta p_t &= \sigma_p \sqrt{I_t} Z_{1t} \\ V_t &= \mu_{v1} I_t + \mu_{v2} L_t + \sigma_v \sqrt{I_t} Z_{2t} \\ with \quad & Cov(\Delta p_t, V_t \mid I_t, L_t) = 0 \end{aligned} \tag{14}$$

where Z_{1t} and Z_{2t} are mutually independent standard normal variables (and independent of I_t and L_t). Conditional on I_t, the daily price change, Δp_t, is $N(0, \sigma_p^2 I_t)$. Conditional on I_t and L_t, the daily volume, V_t, is $N(\mu_{v1} I_t + \mu_{v2} L_t, \sigma_v^2 I_t)$. Thus, our model implies that information flow impacts both, daily price change and traded volume, while only the daily volume is impacted by the random liquidity shocks.

From equations given in (14), it follows that the unconditional contemporaneous relation between Δp_t^2 and V_t is:

$$\begin{aligned} Cov(\Delta p_t^2, V_t) &= E[\Delta p_t^2 V_t] - E[\Delta p_t^2] E[V_t] \\ &= \sigma_p^2 \mu_{v1} Var(I_t). \end{aligned}$$

This relation is strictly positive and identical to that implied by the standard MDH model of Tauchen and Pitts (1983).

The model given in (14) is called the modified MDH model with liquidity (henceforth MDHL model), and forms the basis of our empirical work. The particularity of this model is that it takes into account information and liquidity shocks. While the former impacts volatility-volume relationship, the latter impacts the traded volume. Based on the MDHL model, we can exploit the volume-volatility correlation in order to decompose the traded volume for a given stock into two components and, thus, separate the information from the liquidity trading impact on the observed daily volume. More precisely, using the positive volume-volatility relationship, we can first filter the latent variable I_t. Then, we can isolate the component of traded volume driven by I_t, the rest being the volume component due to liquidity shocks. The model estimation will help identifying the presence of liquidity shocks for a given stock and extract an average intra-day liquidity measure using daily data.

4 Empirical application

Subsection 1 describes the data. The methodology used to test the model is developed in subsection 2. Finally, we discuss the empirical results in subsection 3.

4.1 The data

Our sample consists of the daily return and volume time series of all of the stocks listed on FTSE100 index at July 10, 2007. Note that, in our empirical work, we use the daily return R_t instead of the daily price change without loss of generality. Henceforth, Δp_t is replaced by R_t. The data history extends from January 4, 2005 to June 26, 2007. Over the sample period, we have 636 observations. We exclude stocks with missing observations ending up with 93 stocks. Daily returns and transaction volumes are extracted from Bloomberg databases. The float is calculated as annual common shares outstanding less closely held shares for the fiscal year for the dates requested. The common and closely held shares are extracted from Factset databases. Following Darolles and Le Fol (2005), and Bialkowski, Darolles and Le Fol (2007), we retain the turnover ratio as a measure for volume. Remember that turnover, as a measure of volume, was first introduced to account for the dependency between the traded volume and the total number of shares outstanding. The turnover ratio, that is the traded volume corrected by the number of shares outstanding, seems to be appropriate when studying the market volume [Smidt (1990), LeBaron (1992), Campbell, Grossman and Wang (1993)] or when comparing individual asset volumes [Morse (1980), Bamber (1986), Bamber (1987), Lakonishok and Smidt (1986), Richardson, Sefcik and Thompson (1986), Stickel and Verrechia (1994)]. However, Darolles and Le Fol (2005), Bialkowski, Darolles and Le Fol (2007) replace the number of shares outstanding by the float. Let q_{it} be the number of shares traded for asset i, $i = 1, ..., N$ on day t, $t = 1, ..., T$, and N_{it} the float for asset i on day t. The individual stock turnover for asset i on day t is $V_{it} = \frac{q_{it}}{N_{it}}$.

		Mean	**Standard Deviation**	**Skewness**	**Kurtosis**	**(Return)^2 with Volume Correlation**
Returns	**Mean**	0,0007	0,0137	0,2853	9,9205	0,42
	Std	0,0005	0,0031	0,9271	9,8313	0,14
	Min	-0,0005	0,0074	-4,0840	3,2134	0,17
	Max	0,0024	0,0263	3,1510	61,3788	0,75
Volume	**Mean**	0,0087	0,0065	3,4636	28,4178	-
	Std	0,0052	0,0062	1,7526	26,5025	-
	Min	0,0018	0,0011	1,0041	4,6813	-
	Max	0,0405	0,0545	9,8661	133,8895	-

Table 1: Summary statistics for return and turnover across securities.

For each of the 93 stocks, we compute the empirical first moments (mean, standard deviation, skewness and kurtosis) of volume and returns as well as the correlation between squared returns and volume. The cross-security distribution of these statistics are summarized in Table 1. The first column reports the average, the standard deviation, the min, and the max of the means of returns and volume across the 93 stocks. The second column gives the same cross-section statistics (the mean, the standard deviation, the min and the max) of the standard deviations of returns and volume, and so on for the skewness, kurtosis, and the correlation between squared returns and volume. We perform Pearson test to check the significance of the correlation coefficients. Our results show that the correlation coefficients are statistically significants for 92 over 93 stocks at the 95% confidence level. The statistics reported in the last column of Table 1 are computed using only the statistically significant correlations between squared returns and volume.

The implications of the Mixture of Distribution Hypothesis (MDH) for the joint distribution of daily returns and volume, are examined in details by Clark (1973), Westerfield (1977), Tauchen and Pitts (1983), and Harris (1986, 1987) among others. They assume that both variables (the daily price change and daily volume) are conditioned by the rate of information - called the mixing variable - which is random and serially uncorrelated. They show that the hypothesis can explain why the sample distribution of daily returns is kurtotic relative to the normal distribution, why the distribution of the associated traded volume is positively skewed and kurtotic relative to the normal distribution, and why squared returns are positively correlated with trading volume. The key to the mixture model is the randomness of the mixing variable. If the mixing variable were constant, there would be no reason to observe the above empirical patterns, and the daily returns and volume should be mutually independent and normally distributed.

The results reported in Table 1 are consistent with the mixture of distribution hypothesis. The mean and the min statistics of volume skewness and squared return correlation with volume are positive; and the averages and min statistics of return and volume kurtosis are superior to 3, as predicted by the mixture model. Moreover, the cross-security means and min statistics are all greater than their expected values if there were no variation in the mixing variable[11]. Note that, for the full sample and for various subdivisions of the sample, we found that returns have zero mean.

Finally, in figure 3, we present the scatter plots of returns and squared returns against turnover for two stocks, Anglo American (AAL LN) and AVIVA (AV LN), belonging to $FTSE$ 100. The upper (lower) graphs are pairwise scatter plots for AAL LN (AV LN) return-turnover on the left, and volatility-turnover on the right. The graphs highlight the well-documented positive[12] relation between the return volatility and volume.

[11]The expected value of the volume skewness and correlation coefficient is zero, and the expected value of return and volume kurtosis is 3 when the mixing variable is constant.

[12]Clark (1973), Copeland (1976, 1977), Tauchen and Pitts (1983), Harris (1983-86), Epps and Epps (1976), and Westerfield (1977) among others show a positive correlation between the variability of price change and volume.

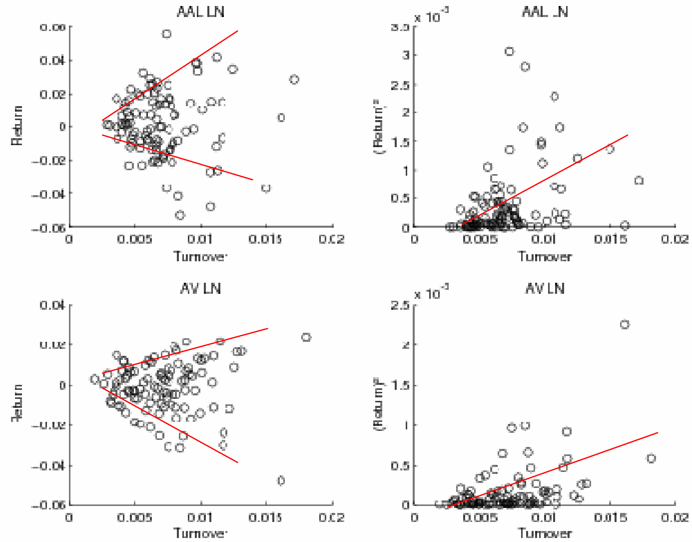

Figure 3: Scatter plots of returns and squared returns against turnover for stocks Anglo American (AAL LN), and AVIVA (AV LN) belonging to $FTSE$ 100.

4.2 The MDHL model test methodology

In this subsection, we develop a procedure which aims at testing whether time series of return and turnover stock-specific data are consistent with the MDHL model presented above. As discussed by Richardson and Smith (1994), since the model imposes restrictions on the joint moments of price changes and volume as a function of only a few parameters, it is possible to form overidentifying restrictions on the data. These restrictions can be tested using the generalized method of moments (GMM) procedure of Hansen (1982).

We first summarize the GMM test procedure developed by Hansen (1982), and then present the moment restrictions implied by our model.

4.2.1 The GMM procedure

Let $X_t = (R_t, V_t)$ be the vector of stock-specific return and volume in day t. If the bivariate series X_t conforms to MDHL model given in equation (14), its unconditional moments should also confirm those of MDHL model:

$$E[h(X_t, \theta)] = 0, \tag{15}$$

where $h(.)$ is an $(N_h \text{ x } 1)$ vector of unconditional moment conditions implied by the MDHL model, and θ is an $(N_p \text{ x } 1)$ vector of parameters governing the model. θ contains the mean and variance parameters of X_t related to both mixing variables I and L $(\sigma_{11}^2, \mu_{21}, \mu_{22}, \sigma_2^2)$, and the second central moments of I and L (m_{2I}, m_{2L}).

We do not observe the expectation of $h(.)$, and so we calculate its empirical counterpart g_T. In large samples, under the null hypothesis that X_t is distributed as MDHL model, the sample moments of (15) should be close to zero[13]:

$$g_T(\theta) \equiv \frac{1}{T}\sum_{t=1}^{T} h(X_t, \theta) \longrightarrow 0, \;\; when \;\; T \to \infty. \tag{16}$$

As a consequence, the idea behind the GMM procedure is to find the values of the unknown parameters θ that set the sample vector $g_T(\theta)$ equal to zero. This will not be possible if the system is overidentified, i.e., if $N_p < N_h$. In this case, we minimize a quadratic form, $Q_T(\theta)$, a weighted sum of squares and cross-products of the elements of $g_T(\theta)$:

$$Q_T(\theta) \equiv g_T(\theta)' W_T g_T(\theta), \tag{17}$$

where W_T is an $(N_h \text{ x } N_h)$ symmetric, positive definite weighting matrix. Since the problem is now nonlinear, this minimization must be performed numerically. The first order condition is:

$$D_T(\hat{\theta}_T)' W_T g_T(\hat{\theta}_T) = 0, \tag{18}$$

where $D_T(\hat{\theta})$ is a matrix of partial derivatives defined by[14]:

$$D_T(\hat{\theta}) = \partial g_T(\hat{\theta}) / \partial \hat{\theta}'. \tag{19}$$

Hansen (1982) shows that the optimal weighting matrix, W_T, that minimizes the variance of the estimation error, V, is any positive scalar times the inverted variance-covariance matrix of $h(X_t, \theta)$, S_0^{-1}. In practice, S_0 is approximated by $S_T(\hat{\theta})$ given by:

$$S_T(\hat{\theta}) = Var[T^{-1/2} g_T(\hat{\theta})]. \tag{20}$$

Hansen also provides the necessary distributional results for the parameter estimators, $\hat{\theta}$, and for the overidentifying test statistic, $J_T(\hat{\theta})$:

$$\begin{aligned} \sqrt{T}(\hat{\theta} - \theta) &\sim^{asy} N(0, [D_0' S_0^{-1} D_0]^{-1}), \\ J_T \equiv T g_T(\hat{\theta}) S_0^{-1} g_T(\hat{\theta}) &\sim^{asy} \chi^2_{N_h - N_p}. \end{aligned}$$

In practice, S_0 and D_0 are replaced by their consistent estimators, $S_T(\hat{\theta})$ and $D_T(\hat{\theta})$. The next subsection develops the moment restrictions implied by the MDHL model, as given by equation (14).

[13] See Hansen (1982).

[14] Note that $D_T(\hat{\theta})$ is the sample approximation of the true partial derivative matrix D_0.

4.2.2 The MDHL model moment restrictions

To test the validity of the MDHL model, we focus on the skewness and kurtosis of specific return and turnover time series and on some of their corresponding cross moments. We also include the covariance between the squared returns and turnover or squared turnover. In fact, under MDHL model in equation (14), it is possible to calculate the implied unconditional second, third and fourth moments and the corresponding cross-moments of the observable variables R_t and V_t. As Tauchen and Pitts (1983) point out, MDHL model is invariant with respect to scalar transformations of I_t and L_t. Thus, if a is any positive constant and $I_t^* \equiv I_t/a$ and $L_t^* \equiv L_t/a$, then the model

$$\begin{array}{rcl} R_t & \sim & N(0, [\sigma_p^2 a] I_t^* \mid I_t, L_t), \\ V_t & \sim & N([\mu_{v1} a] I_t^* + [\mu_{v2} a] L_t^*, [\sigma_v^2 a] I_t^* \mid I_t, L_t), \\ with & & Cov(R_t, V_t \mid I_t, L_t) = 0, \end{array} \tag{21}$$

cannot be differentiated empirically from MDHL model in equation (14). By normalizing $E[I_t^*] = E[L_t^*] = 1$, it is possible to identify the transformed parameters $\mu_{v1}^* = \mu_{v1} m_{1I}$, $\mu_{v2}^* = \mu_{v2} m_{1L}$, $\sigma_p^{*2} = \sigma_p^2 m_{1I}$, $\sigma_v^{*2} = \sigma_v^2 m_{1I}$, $m_{2I}^* = m_{2I}/m_{1I}^2$, $m_{2L}^* = m_{2L}/m_{1L}^2$, $m_{3I}^* = m_{3I}/m_{1I}^3$, $m_{3L}^* = m_{3L}/m_{1L}^3$, $m_{4I}^* = m_{4I}/m_{1I}^4$, and $m_{4L}^* = m_{4L}/m_{1L}^4$. Henceforth, we will consider only the transformed parameters. However, for notation simplicity, we omit the "$*$" symbol.

Moreover, we need to chose a distribution function for the latent variables I_t and L_t. Tauchen and Pitts (1983) assume that the mixing variable I_t follows a lognormal distribution[15]. This assumption has also been suggested by a number of authors such as Clark (1976) and Foster and Viswanathan (1993b). Richardson and Smith (1994) tested several distribution functions for information flow and conclude that the data reject the lognormal distribution less frequently than the other distribution candidates such as inverted gamma and Poisson distributions. These results motivate us to retain a lognormal distribution for both mixing variables I_t and L_t. As discussed by Richardson and Smith (1994), it is possible to show that the lognormality assumption implies the following moment restrictions:

$$\begin{array}{rcl} m_{3j} - m_{2j}^3 - 3m_{2j}^2 & = & 0 \\ m_{4j} + 4(1+m_{2j})^3 + 3 - (1+m_{2j})^6 - 6(1+m_{2j}) & = & 0 \end{array} \tag{22}$$

where $j = (I_t, L_t)$ and $m_{ij}, (i = (2,3,4))$ is the i^{th} centered moment for the mixing variable j.

Under the above transformations and the assumptions made here, the corresponding sample moment vector $g_T(\theta)$ is given by:

[15]Note that lognormality permits them to insure the positiveness of I_t.

$$
g_T(\theta) = \frac{1}{T}\sum_{t=1}^{T}\left(\begin{array}{cc}
(V_t - E(V_t)) & (1)\\
(R_t - E(R_t))^2 & (2)\\
(V_t - E(V_t))^2 & (3)\\
(R_t^2 - E(R_t^2))(V_t - E(V_t)) & (4)\\
(R_t^2 - E(R_t^2))(V_t^2 - E(V_t^2)) & (5)\\
(V_t - E(V_t))^3 & (6)\\
(R_t - E(R_t))^4 & (7)\\
(V_t - E(V_t))^4 & (8)\\
(R_t - E(R_t))^2(V_t - E(V_t))^2 & (9)
\end{array}\right) \qquad (23)
$$

where the sample moments (1) to (9) and $E(.)$ operators are functions of the vector of parameters ($\theta = (\mu_{v1}, \mu_{v2}, \sigma_p^2, \sigma_v^2, m_{2I}, m_{2L})$) governing the distribution of the observables R_t and V_t, and the latent variables I_t and L_t. Sample moments (1) to (9) are given in the Appendix A. The above restrictions represent a system of nine equations and six parameters to be estimated. This leaves us with three overidentifying restriction to test. By applying the GMM procedure to the sample moment vector in (23), we can estimate θ and test the MDHL model jointly. Let $\hat{\theta}$ be the vector of the estimated parameters. Then, the resulting $J_T(\hat{\theta})$ will have an asymptotic χ_3^2 distribution. If, for example, $J_T(\hat{\theta})$ exceeds 7.8, then we can reject the MDHL model at the 95-percent level of significance. Finally, note that the weighting matrix is estimated using Newy and West (1987) procedure.

4.3 Empirical results

We applied the GMM procedure to the sample moment vector in (23) in order to estimate θ and test the MDHL model jointly. Tables 2 and 3 in the Appendix B.1 present the test of the moment restrictions implied by the MDHL model, applied to the $FTSE$ 100 stocks. The test statistics are given in the column 9. With three overidentifying restrictions, they are asymptotically distributed as a χ_3^2. For 94% of the stocks of our sample, the test statistic values do not exceed their critical value of $7,815$. Consequently, we can not reject the MDHL model at the 95 level of significance. For robustness checks, we also estimate the standard MDH model using the procedure of Richardson and Smith (1994)[16]. Results are presented in tables 4 and 5 in the Appendix B.2. The model is accepted by the data for 89% of stocks. In terms of global validity, our model outperforms the standard MDH model.

Columns 2 to 5 in tables 2 and 3 provide the parameter estimates for returns and volume, while columns 6 and 7 provide the estimated variances for

[16] To estimate the standard MDH model we use the implied unconditional means, variances, skewness, and corresponding cross-moments of the observable variables, R_t and V_t. See Richardson and Smith (1994) for more detailed calculations.

the latent variables I_t and L_t. We have denoted by "*" or "**" the statistically significant parameters at respectively 90% and 95% level of confidence. It is important to comment the behavior of the mean parameters for volume. Since we work with normalized data in order to identify and estimate the parameters ($E(I_t) = E(L_t) = 1$), the estimated μ_{v1} and μ_{v2} can be interpreted as the average, across days, of respectively the amplitude of intra-day information flow and liquidity shocks. The more illiquid the market for a given stock, the higher is the volume traded with liquidity arbitragers, which results in higher volumes and thus, higher μ_{v2}. The model helps identifying the presence of liquidity shocks and their extent by decomposing the average volume into two components, and separate the information from the liquidity trading impact.

Since our model implies that information guides the market from one equilibrium to the next, and liquidity shocks appear between these equilibria, we should expect to observe a statistically significant μ_{v2} parameter only for stocks having also a significant μ_{v1}. The results exposed in the Appendix B.1 confirm our intuitions. 25 stocks have both mean parameters statistically significants and there is no stock with only μ_{v2} significant. Three additional remarks can be made: (i) A μ_{v2} significantly positive suggests that the stock faces, on average, intra-day lacks of liquidity, which motivate the liquidity arbitragers to enter the market and increase the average traded volume. Since we do not observe liquidity shocks, we can infer their occurrence from liquidity arbitrage trading, which directly impacts the volume. Our model helps identifying the intra-day impact of this type of traders in the traded volume using daily data. Using our data, we can identify 23 stocks facing liquidity problems. (ii) If μ_{v2} is nonsignificant, our model corresponds to that of Tauchen and Pitts (1983) which assumes that traded volume is explained by information flow. Empirically, we conclude that, for half of the stocks in our sample, volume can be explained by information; (iii) When comparing mean volume parameters obtained by both models, we observe that μ_v is equal to the sum of μ_{v1} and μ_{v2}. The result is intuitive and shows that the MDHL model succeeds in decomposing the average traded volume into information and liquidity-based components.

We use a structural model to separate the respective impacts of the two latent variables I an L on the trading volume. To do so, our model exploits the positive volatility-volume relation driven by I to extract the impact of I on volume, μ_{v1}. Then, the remaining volume is due to liquidity shocks. The model is particularly interesting in practice since it provides a measure for average intra-day liquidity shocks using daily data.

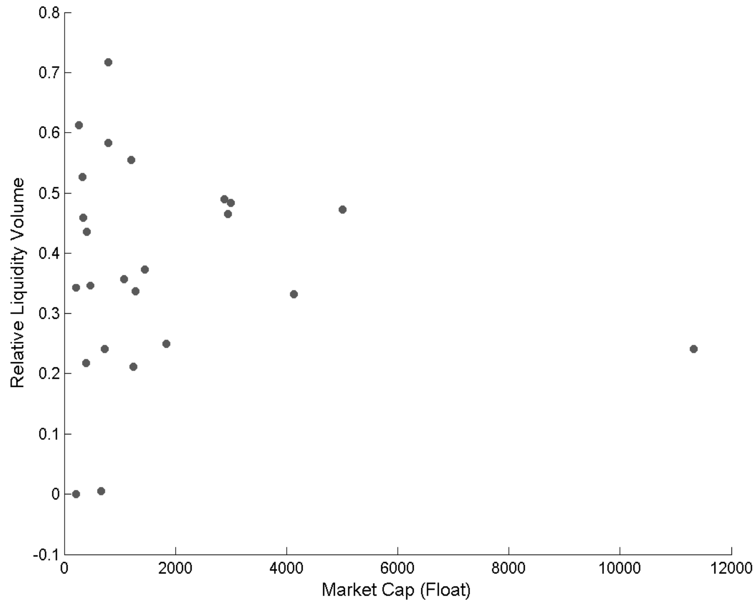

Figure 4: Relative liquidity volume versus average market cap measured by the float.

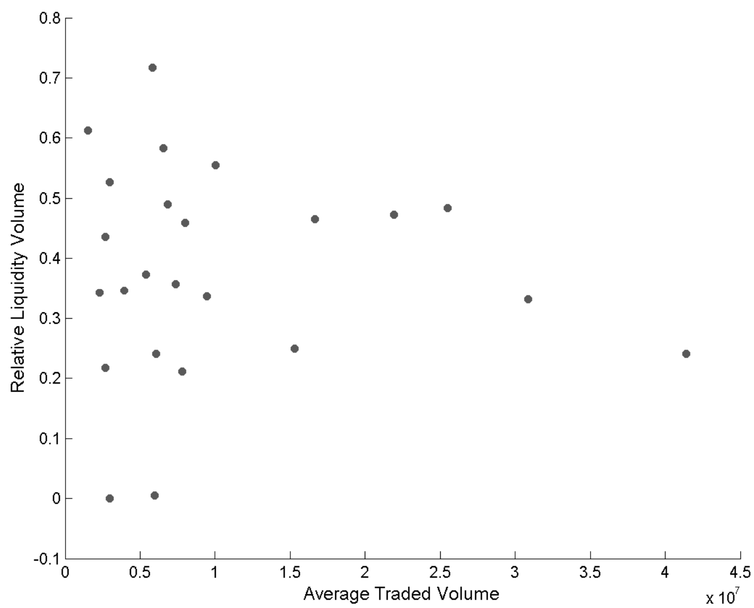

Figure 5: Relative liquidity volume versus average daily traded volume.

Previous literature relates stock liquidity to traded volume, and suggests that illiquid stocks have low traded volume or turnover[17]. It follows that the

[17]See Datar, Naik and Radcliffe (1998), and Chordia, Subrahmanyam and Anshuman (2000) ammong others.

traded volume is a good proxy for liquidity. Moreover using market capitalization as a proxy for stock liquidity is a current practice in the financial markets: small stocks are assumed to face more liquidity problems than large ones. We focus on the estimated μ_{v2} to investigate the relationship between the liquidity shocks as measured by μ_{v2}, and market capitalization or traded volume. Our results show that size and traded volume are not good proxies for liquidity problems. For example, the three smallest[18] stocks of our sample, Lonmin (LMI), Vedanta (VED), and Whitbread (WTB), have μ_{v2} not significant. In addition, some other stocks, such as GlaxoSmithKline (GSK), Old Mutual (OML), HSBC Holdings (HSBA), having significant μ_{v2} belong to the two highest deciles of average market capitalization.

Figures 4 and 5 focus on the 23 stocks of our sample having μ_{v2} statistically significant, and plot the relative liquidity volume with respectively average market capitalizations and average daily traded volume. The relative liquidity volume for a given stock is the estimated μ_{v2} parameter divided by the sum of μ_{v1} and μ_{v2}. The average market capitalization and traded volume for a given stock are calculated over the estimation period. Both graphs point out that the relation between liquidity problems and average market capitalization (or traded volume) is not very clear. In fact, for low values of average market capitalization (or traded volume), we observe a high heterogeneity of μ_{v2} relative values. Stocks having important μ_{v2} relative values as well those having relative μ_{v2} close to zero rank on the lowest average market capitalization (or traded volume) deciles. Vice-versa, some stocks having important relative values of μ_{v2}, have hight values of market capitalization (or traded volume). These results suggest that size (or traded volume) is not always a good proxy for liquidity shocks: stocks with higher market capitalizations (or traded volume) may be less liquid than small (or less traded) ones.

Finally, table 6 given in the Appendix B.3 compares our paper contributions to previous research results. Three additional remarks can be made: (i) As previously observed, the MDHL model implies a positive volatility-volume relation driven by the dependence of both variables in the information flow. In our model, this positive correlation is function of the part of volume due to information-based trading, μ_{v1}, after controlling for the impact of liquidity shocks, μ_{v2}. Here, the standard MDH model appears to be a special case of the MDHL model in absence of liquidity shocks; (ii) While previous literature considers liquidity consumers versus liquidity providers, we are the first to distinguish two types of liquidity providers, the strategic (the liquidity arbitragers) and the non strategic traders. The intervention of liquidity arbitragers rises the daily traded volume; (iii) The interest of our approach is that we use daily data to estimate a liquidity measure related to intra-day phenomena, such as the flow of liquidity shocks. This allows us

[18]Stocks having the lowest average market cap over the sample period.

to identify illiquid stock for a given period.

5 Concluding remarks

In this paper, we develop a model for stock trading which considers both information and liquidity shocks. We first distinguish between two trading strategies, information-based and liquidity-based trading, and suggest that their respective impacts on returns and traded volume should be modelized differently. The former is incorporated into daily price changes and traded volume, and drives the positive volatility-volume relationship. The latter consists of strategic (liquidity arbitrage) and non-strategic trading. Liquidity arbitrage trading has mean reverting intra-day effects on price change but do not affect daily price fluctuations. However, it raises the daily traded volume.

Second, we focus on the contemporaneous relationship between return variance and volume to modelize the impacts of information and liquidity. The paper relaxes the hypothesis of absence of liquidity imperfections and extends the standard MDH framework by developing a modified MDH model, which incorporates both information and liquidity shocks.

Third, we show how to use a structural model, to exploit the volume-volatility relation in order to decompose the traded volume for a given stock into two components, and separate the information from the liquidity impact in the observed daily volume. In other words, the increase of volume due to liquidity arbitragers helps inferring the presence of liquidity shocks for a given stock. To do so, we first need to estimate the normal volume that would prevail if there were no liquidity shortage. This information is contained in the volatility-volume positive relation. Once I filtered from volume, we know that the remaining part is related to liquidity shocks. The model is particularly interesting in practice since it provides a measure for average intra-day liquidity shocks using daily data. It would be interesting to confront this measure to other liquidity intra-day measures such as price spreads or trade impact.

Finally, the MDHL model developed in this paper can be extended to allow for serial dependence in L. Several studies actually indicate that liquidity shocks are not isolated events in time but rather seem to be persistent[19]. We state that persistence in L_t may explain the presence of serial correlation in volatility and volume. It would be interesting to use signal extraction methods to filter the latent variables I_t and L_t. This point is to be investigated in a forthcoming paper.

[19] See, for example, Acharya and Pedersen (2005).

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APPENDIX A

Sample moment conditions for the MDHL model

The sample moment conditions in equation (23) are given as follows:

$$
\begin{aligned}
(V_t - E(V_t)) &= \mu_{v1} + \mu_{v2}, && (24)\\
(R_t - E(R_t))^2 &= \sigma_p^2, && (25)\\
(V_t - E(V_t))^2 &= \mu_{v1}^2 m_{2I} + \mu_{v2}^2 m_{2L} + \sigma_v^2, && (26)\\
(R_t^2 - E(R_t^2))(V_t - E(V_t)) &= \mu_{v1}\sigma_p^2 m_{2I}, && (27)\\
(R_t^2 - E(R_t^2))(V_t^2 - E(V_t^2)) &= \mu_{v1}^2\sigma_p^2(m_{3I} + 2m_{2I}) && (28)\\
&\quad +2\mu_{v1}\mu_{v2}\sigma_p^2 m_{2I} + \sigma_v^2\sigma_p^2 m_{2I}, &&\\
(V_t - E(V_t))^3 &= 3\mu_{v1}\sigma_v^2 m_{2I} + \mu_{v1}^3 m_{3I} + \mu_{v2}^3 m_{3L}, && (29)\\
(R_t - E(R_t))^4 &= 3\sigma_p^4(m_{2I} + 1), &&\\
(V_t - E(V_t))^4 &= 3\sigma_v^4(m_{2I} + 1) + \mu_{v1}^4 m_{4I} && (30)\\
&\quad +\mu_{v2}^4 m_{4L} + 6\mu_{v1}^2\mu_{v2}^2 m_{2I} m_{2L} &&\\
&\quad +6\mu_{v1}^2\sigma_v^2(m_{3I} + m_{2I}) &&\\
&\quad +6\mu_{v2}^2\sigma_v^2 m_{2L}, &&\\
(R_t - E(R_t))^2(V_t - E(V_t))^2 &= \mu_{v1}^2\sigma_p^2(m_{3I} + m_{2I}) && (31)\\
&\quad +\mu_{v2}^2\sigma_p^2 m_{2L} + \sigma_v^2\sigma_p^2(m_{2I} + 1). && (32)
\end{aligned}
$$

where equations (24) to (31) correspond respectively to sample moment conditions (1) to (9) in equation (23), and where the third and fourth central moments of I and L, $(m_{3I}, m_{3L}, m_{4I}, m_{4L})$, are functions of their respective second central moments, (m_{2I}, m_{2L}), as given in equation (22), and where E(.) operators are also functions of θ:

$$\begin{aligned} E(R_t) &= 0, \\ E(V_t) &= \mu_{v1} + \mu_{v2}, \\ E(R_t^2) &= \sigma_p^2, \\ E(V_t^2) &= \sigma_v^2 + 2\mu_{v1}\mu_{v2} \\ &\quad + \mu_{v1}^2(m_{2I}+1) + \mu_{v2}^2(m_{2L}+1). \end{aligned} \tag{33}$$

APPENDIX B.1

GMM estimation results for MDHL model

ID	μ_{v1}	μ_{v2}	σ_p^2	σ_v^2	m_{2I}	m_{2L}	χ_1^2
1	0,004592**	0,002557**	0,000353**	0,000003	0,356521**	0,100684	0,43
2	0,007168*	-0,000986	0,000077**	0,000003	0,367095*	1,581157	2,95
3	0,005085**	0,001414*	0,000143**	0,000001	0,809817**	0,000038*	4,04
4	0,072554	-0,056201	0,000371**	0,000218	0,054197	0,000137	3,64
5	0,004189**	0,000792	0,000130**	0,000003	0,282547**	2,900634**	2,70
6	0,015721	-0,010089	0,000112	0,000012	0,074901	0,000936	9,08
7	0,004356**	0,004089**	0,000202**	0,000009*	0,823090**	0,077773	1,51
8	0,005958**	-0,000228	0,000142**	0,000002	0,314040**	0,000266**	3,32
9	0,005805**	-0,000413	0,000106**	0,000000	0,216152**	0,001290**	6,95
10	0,009839*	0,003839	0,000242	0,000004	0,568549	0,494827	11,91
11	0,006748**	0,003519*	0,000240**	0,000021**	0,338145**	0,343929	3,23
12	0,006007**	-0,001428	0,000203**	0,000001	0,152702**	0,091857	1,90
13	0,041900	-0,034455	0,000162**	0,000019	0,036970	0,022144	5,39
14	0,008303**	-0,003144	0,000378**	0,000007*	0,170230**	0,083028	2,15
15	0,014961*	-0,009137	0,000134**	0,000010	0,066888	0,007497	0,92
16	0,015013**	-0,002631	0,000117**	0,000019*	0,475838**	2,111389	1,09
17	0,015933	-0,010203	0,000128**	0,000018	0,098846	0,006966	5,75
18	0,006421**	-0,000114	0,000103**	0,000000	0,312727*	0,000502**	1,10
19	0,002351**	0,000162	0,000172**	0,000000	0,537135**	0,001305	1,11
20	0,013597**	-0,007242*	0,000152**	0,000007	0,131178**	0,086226	2,02
21	0,032198	-0,022711	0,000156**	0,000093	0,134339	0,009017	3,98
22	0,035934	-0,029251	0,000120**	0,000011	0,046330	0,037325	3,28
23	0,054873	-0,042099	0,000205**	0,000238	0,101230	0,000140	3,13
24	0,003045**	0,002650**	0,000069**	0,000006**	0,480282**	0,336686**	0,26
25	0,013947	-0,008718	0,000124**	0,000020	0,172881	0,016039	3,16
26	0,021351	-0,010105	0,000141**	0,000035	0,144205	0,050033	6,25
27	0,014124*	-0,003456	0,000248**	0,000012	0,239652*	0,019187	5,84
28	0,008981**	0,000050*	0,000137**	0,000001	0,365326**	0,000495**	0,97
29	0,006198**	0,002057**	0,000177**	0,000012	1,225154**	2,477023	0,48
30	0,002285**	0,002052**	0,000112*	0,000000	0,774375**	0,075987	0,61
31	0,004133*	0,000001	0,000102	0,000001	0,307006*	0,000359**	5,83
32	0,006426	-0,000302	0,000166	0,000000	0,363608	0,001853	10,44
33	0,005551**	0,001758*	0,000185**	0,000013	0,669770**	1,824690**	1,78
34	0,006700**	0,002500**	0,000156**	0,000040*	0,760000**	0,670000**	1,89
35	0,005362**	0,002834**	0,000295**	0,000031**	0,982038**	1,420236*	0,82
36	0,006316	0,002451	0,000209**	0,000003	0,673408	0,638940	6,51
37	0,008268**	0,001223	0,000150**	0,000005	0,501508**	1,636350	3,40
38	0,004326**	0,003339**	0,000137**	0,000015**	0,649970**	0,397851**	1,84
39	0,006440**	-0,000879	0,000092**	0,000001	0,309875**	4,093407**	1,98
40	0,026387	-0,017941	0,000199*	0,000035	0,080861	0,004867	8,54
41	0,018477**	-0,007157	0,000197**	0,000048**	0,290554**	0,040000	1,19
42	0,014043**	-0,004334	0,000293**	0,000023	0,294781**	0,000113	2,42
43	0,007237**	-0,000479	0,000158**	0,000001	0,367964**	6,304101**	2,33
44	0,006176**	-0,000019	0,000121**	0,000000	0,283954**	0,001691**	6,60
45	0,012112**	-0,002277	0,000171**	0,000020*	0,414964**	0,004042	2,55

Table 2: MDHL model estimated parameters (1).

ID	μ_{v1}	μ_{v2}	σ_p^2	σ_v^2	m_{2I}	m_{2L}	χ_1^2
46	0,018687	-0,012749	0,000148**	0,000023	0,102885	0,010298	3,84
47	0,017643	-0,012088	0,000160**	0,000006	0,072632	0,049380	2,85
48	0,002142**	0,003388**	0,000144**	0,000009**	1,024586**	0,159036*	2,06
49	0,015456	-0,009203	0,000087**	0,000009	0,098522	0,035609	4,60
50	0,008890**	0,004488	0,000430**	0,000020	0,958863**	0,523920	0,99
51	0,007465*	0,004276	0,000179**	0,000074	1,105972	2,180080	1,98
52	0,010325**	-0,001414	0,000134**	0,000008	0,495946**	2,666157	2,87
53	0,011136**	-0,001733	0,000152**	0,000024	0,543685**	0,771515	4,72
54	0,005144	-0,000520	0,000096**	0,000000	0,225200	0,000951	6,80
55	0,006715	0,002134	0,000132**	0,000005	0,458861	0,490909	7,07
56	0,038058	-0,024071	0,000130**	0,000097	0,107051	0,002521	8,03
57	0,004692**	0,002332**	0,000216**	0,000001	0,523044**	1,599162**	3,57
58	0,011572	-0,004839	0,000186**	0,000007	0,158543*	0,100663	4,50
59	0,006421**	0,001363	0,000263**	0,000004	0,399654**	0,661273	0,99
60	0,012708**	-0,004618	0,000112	0,000003	0,171040*	0,200807	3,53
61	0,013023**	0,000005**	0,000177**	0,000014**	0,477992**	44,548389*	2,32
62	0,010417*	-0,004819	0,000102**	0,000005	0,135650*	0,038057	4,82
63	0,053063	-0,048449	0,000083**	0,000057	0,021441	0,000152	7,24
64	0,004278*	-0,002492	0,000121**	0,000001	0,117000*	0,101503	1,81
65	0,006781*	0,000071	0,000101**	0,000003	0,329437**	0,000282**	4,56
66	0,015129**	-0,007553*	0,000108**	0,000010	0,185757**	0,130840	3,43
67	0,017310	-0,010584	0,000297**	0,000010	0,057213	0,002983	11,59
68	0,008915**	0,001087	0,000195**	0,000004	0,286098**	0,280562	1,98
69	0,015410**	-0,006659	0,000184**	0,000020	0,201118*	0,022876	4,03
70	0,006848**	0,002909	0,000168**	0,000060*	0,237499**	1,839111**	6,62
71	0,017676	-0,007115	0,000180**	0,000075	0,445070	0,124465	1,66
72	0,002303**	0,001375**	0,000162**	0,000000	0,489226**	0,355318**	2,47
73	0,009789**	0,001865	0,000100**	0,000017	0,714350**	2,885819**	3,78
74	0,003229**	0,004530**	0,000126**	0,000030**	1,321688**	0,516989**	1,50
75	0,003300**	0,000875	0,000224**	0,000002	0,498127**	1,441435	1,66
76	0,004904**	0,002485**	0,000188**	0,000005	0,695894**	0,218002	0,65
77	0,030266	-0,024020	0,000153**	0,000014	0,056328	0,035361*	1,75
78	0,008342**	0,002784	0,000223**	0,000003	0,947708**	0,361908	1,96
79	0,004100**	0,004564**	0,000137**	0,000023**	0,652324**	0,336706**	2,39
80	0,014816	-0,006544	0,000131**	0,000032	0,251056	0,001193	5,16
81	0,004453**	0,001321	0,000110**	0,000008**	0,256106**	1,678379	2,73
82	0,004621**	0,001241**	0,000176**	0,000005	0,543131**	2,076741**	4,08
83	0,006758**	0,000965	0,000121**	0,000006	0,292637**	1,904712**	2,91
84	0,006561**	-0,001419	0,000099**	0,000005	0,262634**	0,104353	2,18
85	0,001196*	0,001151*	0,000099**	0,000000	0,715224**	0,490877	2,10
86	0,002067**	0,005255**	0,000083**	0,000001	1,137846**	0,327187**	0,50
87	0,067322	-0,047323	0,000518**	0,000322	0,106073	0,004482	4,89
88	0,013626	-0,008762	0,000160**	0,000015	0,105255	0,010584	3,53
89	0,380956	-0,341413	0,000175**	0,000829	0,048733	0,030764	7,64
90	0,003705**	0,004630**	0,000126**	0,000010**	0,752620**	0,097367	0,23
91	0,009685**	-0,000671	0,000141**	0,000006	0,578370**	4,920062	2,99
92	0,015439**	0,013117**	0,000455**	0,000415**	0,385198*	0,781706	4,52
93	0,040238	-0,030511	0,000153**	0,000180	0,161703	0,020837	3,42

Table 3: MDHL model estimated parameters (2).

APPENDIX B.2

Results for Richardson and Smith (1994) MDH model GMM estimation

ID	μ_p	μ_v	σ_p^2	σ_v^2	m_{2I}	m_{3I}	χ_3^2
1	0,001576**	0,006974**	0,000341**	0,000000	0,164821**	0,076466**	4,43
2	0,000293	0,006213**	0,000082**	0,000007	0,537526**	0,771725**	4,04
3	0,000459	0,006450	0,000124	0,000003	0,615128	1,182884	9,18
4	0,002793**	0,016191**	0,000348**	0,000015	0,168119**	0,086267**	7,89
5	0,000593	0,004944**	0,000117**	0,000000	0,251352**	0,200954*	6,16
6	0,000648	0,005961**	0,000141**	0,000013**	0,623407**	0,916119**	1,89
7	0,000992**	0,008348**	0,000198**	0,000008	0,370009**	0,002626	2,39
8	0,000304	0,005915**	0,000136**	0,000000	0,335995**	0,431232**	2,61
9	0,001127**	0,005421**	0,000104**	0,000001	0,273959**	0,000798	4,37
10	0,000924*	0,014019**	0,000277**	0,000043**	0,506739**	0,544113**	0,85
11	0,001128**	0,010572**	0,000247**	0,000020**	0,209113**	0,504891**	4,44
12	0,001227**	0,004564**	0,000203**	0,000000	0,201264**	0,081835	2,78
13	0,000790*	0,007445**	0,000185**	0,000008*	0,236387**	0,547731**	3,55
14	0,002101**	0,005181**	0,000354**	0,000002	0,218648**	0,422673**	7,01
15	0,000725	0,005650	0,000122	0,000000	0,142258	0,053039	8,15
16	0,000204	0,012450**	0,000116**	0,000020	0,626882**	1,485704**	2,77
17	0,000516	0,005783**	0,000136**	0,000004**	0,369925**	0,364957**	3,74
18	0,000308	0,006282**	0,000104**	0,000002	0,383053**	0,491836**	1,15
19	-0,000209	0,002463**	0,000163**	0,000001	0,462963**	0,457725**	2,70
20	0,000640	0,006393**	0,000152**	0,000002	0,261004**	0,290055**	2,57
21	0,000582	0,009301**	0,000138**	0,000003	0,358138*	0,402375**	7,50
22	0,000851**	0,006702**	0,000119**	0,000006	0,268992**	0,385132	3,77
23	0,001061*	0,012701**	0,000209**	0,000028	0,574036**	0,816102	7,33
24	0,000560**	0,005666**	0,000067**	0,000000	0,254055**	0,296119**	1,31
25	-0,000340	0,005180**	0,000103**	0,000018*	-0,213526	1,376168	4,97
26	0,000337	0,011794**	0,000171**	0,000062**	0,791139**	1,474336**	4,09
27	0,001466**	0,011202**	0,000271**	0,000015**	0,468301**	0,549208**	0,32
28	0,000753	0,008973**	0,000135**	0,000000	0,348388**	0,386472**	1,62
29	0,000160	0,008094**	0,000171**	0,000009	0,742010**	2,013111*	1,78
30	0,000213	0,004189**	0,000101**	0,000001	0,253895*	0,083571	4,78
31	0,000403	0,004333**	0,000110**	0,000001	0,338555**	0,400176**	1,81
32	0,000780	0,007169**	0,000187**	0,000011*	0,417406**	1,046547**	2,13
33	0,001097**	0,007001**	0,000178**	0,000018	0,360728*	10,347836	2,76
34	0,000092	0,003656**	0,000052**	0,000001	0,342485**	0,366668**	1,22
35	0,000013	0,008095	0,000271	0,000025	0,181488	0,632359	8,64
36	0,000806*	0,008736**	0,000207**	0,000013	0,505544**	0,636415*	5,52
37	0,000832*	0,009746**	0,000156**	0,000006	0,484987**	0,656846**	1,42
38	0,000898**	0,007722**	0,000143**	0,000002	0,315477**	0,334730**	3,73
39	0,000501	0,005558**	0,000091**	0,000001	0,392350**	0,757375**	3,18
40	0,001586**	0,008806**	0,000226**	0,000011**	0,448834**	0,447771**	0,68
41	-0,000652	0,010979	0,000175	0,000007	0,297440	0,761764	8,64
42	0,000931	0,009666**	0,000311**	0,000025	0,646960**	1,074764	1,94
43	0,000782*	0,006418**	0,000147**	0,000000	0,326012**	0,549908**	4,60
44	0,000667*	0,006578**	0,000119**	0,000005*	0,249873**	0,414520**	3,13
45	-0,001165**	0,009964**	0,000168**	0,000004	0,464029**	1,221262**	5,37

Table 4: MDH model estimated parameters (1).

ID	μ_p	μ_v	σ_p^2	σ_v^2	m_{2I}	m_{3I}	χ_3^2
46	0,000308	0,006052**	0,000170**	0,000002	0,369355**	0,421853**	3,22
47	0,000359	0,005642**	0,000158**	0,000006	0,218251**	1,077850	2,74
48	0,000282	0,005360**	0,000126**	0,000002	0,304649**	0,321132**	6,57
49	0,000184	0,006046	0,000083	0,000001	0,235209	0,195631	9,36
50	0,002334**	0,013313**	0,000435**	0,000039*	0,612897**	1,071623**	1,61
51	0,001268**	0,012308**	0,000185**	0,000009	0,896280**	3,419218**	2,99
52	0,001181**	0,008821**	0,000129**	0,000015*	0,648002**	1,009654**	7,64
53	0,000696	0,009805**	0,000161**	0,000044**	1,140210**	3,849740**	7,01
54	0,000398	0,004967**	0,000107**	0,000003	0,266530**	0,480544**	3,24
55	0,000037	0,009627**	0,000139**	0,000010	0,518172**	0,794861*	3,86
56	0,000210	0,014942**	0,000170**	0,000108**	0,855629**	1,725279**	1,08
57	0,000429	0,007450**	0,000179**	0,000017**	0,429604**	2,473198**	12,86
58	0,000385**	0,006954**	0,000189**	0,000010**	0,491210**	0,718748**	6,44
59	0,000551	0,007730**	0,000253**	0,000004	0,229570**	0,237471**	2,13
60	0,000270	0,008247**	0,000112**	0,000001	0,327870**	0,370144**	2,49
61	0,000975*	0,013523**	0,000176**	0,000025	0,483996**	2,539710**	1,21
62	0,000763**	0,005644**	0,000105**	0,000001	0,310431**	0,257974**	6,28
63	0,000080	0,004747**	0,000093**	0,000004*	0,453089**	0,922374**	4,41
64	0,000421	0,001753**	0,000122**	0,000000	0,256621**	0,317692**	2,41
65	0,000399	0,007069**	0,000121**	0,000025**	0,807689**	1,646781**	1,44
66	0,000322	0,007495**	0,000113**	0,000006	0,532255**	0,735960**	3,05
67	0,002328	0,006563	0,000259	0,000004	0,081158	0,034710	13,13
68	0,001318*	0,010106**	0,000195**	0,000003	0,328610**	0,391733**	1,66
69	0,001144**	0,009141**	0,000197**	0,000016**	0,569769**	0,797812**	2,70
70	-0,000217**	0,011143**	0,000159**	0,000214**	0,015643	16,593616*	3,31
71	0,000831*	0,010479**	0,000168**	0,000007	0,539331**	1,714445	2,91
72	0,000684	0,003646**	0,000158**	0,000000	0,264416**	0,196433**	6,06
73	0,000830**	0,012073**	0,000104**	0,000082**	1,263259**	2,909492	2,95
74	0,000318	0,007707**	0,000123**	0,000005	0,415593**	0,990330**	2,07
75	0,000801	0,004110**	0,000216**	0,000001	0,329465**	0,565500**	3,21
76	0,000259	0,007227**	0,000179**	0,000003	0,358411**	0,352879	3,54
77	0,000297	0,005903**	0,000144**	0,000006	0,280831**	0,527107**	5,27
78	0,001124*	0,011166**	0,000217**	0,000021	0,626097**	1,111593	3,63
79	0,000378	0,008791**	0,000141**	0,000003	0,315523**	0,413985**	2,90
80	-0,000031	0,008275	0,000108	0,000022	0,003433	0,377380	10,43
81	0,000828**	0,005752**	0,000110**	0,000005**	0,165209**	0,225023**	1,37
82	0,000016	0,006422**	0,000177**	0,000012**	0,338610**	1,834843**	6,33
83	0,000415	0,007782**	0,000124**	0,000002	0,296313**	0,011812	3,64
84	0,000401	0,005160**	0,000101**	0,000004**	0,426593**	0,517322**	1,30
85	0,000494	0,002321**	0,000093**	0,000000	0,275580**	0,259745**	5,16
86	0,000398	0,007114**	0,000072**	0,000003	0,150148**	0,093268**	3,25
87	0,003429**	0,020237**	0,000541**	0,000001	0,418237**	0,420424**	4,32
88	0,000316	0,004931**	0,000184**	0,000011*	0,698633**	1,195637	2,12
89	0,000265	0,038437**	0,000218**	0,001783**	0,602895**	2,741132*	1,17
90	-0,000004	0,008261**	0,000115**	0,000006*	0,159434**	0,250851**	5,56
91	0,001053**	0,008996**	0,000137**	0,000025**	0,889680**	2,407741**	7,87
92	0,002545**	0,029124**	0,000449**	0,000298**	0,188494**	0,762121**	2,06
93	0,000493	0,010147**	0,000180**	0,000029**	1,005279**	3,151976**	1,73

Table 5: MDH model estimated parameters (2).

APPENDIX B.3

Summary results

	Data	MDH extension	Model validity	Contributions
Tauchen and Pitts (1983)	90-day T-bills futures market	–	Favorable	Explains $Cov(R_t^2, V_t) > 0$
Richardson and Smith (1994)	Dow Jones30 stocks	$E(R_t) \neq 0$	Less favorable	GMM test
Lamoureux and Lastrapes (1994)	10 NYSE stocks	$Cov(I_t, I_{t-1}) \neq 0$	Unfavorable	MDH explanation for GARCH effects?
Andersen (1996)	IBM common stocks	Non-informed part of volume	Unfavorable to standard MDH; Modified MDH does better.	Volume decomposition: informed versus unindormed part of volume with market maker
Roskelley (2001)	Dow Jones30 stocks	$Cov(I_t, I_{t-1}) \neq 0$	Unfavorable	Moment simplification
Li and Wu (2006)	Dow Jones30 stocks	Extend Andersen (1996): Non-informed part of return volatility	Rejection of Andersen (1996); Validation of their model.	Non-informed traders have negative impact on $Cov(R_t^2, V_t)$
MDHL model	FTSE 100 Stocks	Extend TP (1983): Information and Liquidity shocks	Favorable to standard MDH and to MDHL	Liquidity arbitragers are strategic agents and not noisy traders; Extends standard MDH by accounting for liquidity shocks; Volume decomposition; Proposes a new liquidity measure.

Table 6: Paper contributions compared to previous literature.