What drives value creation in large projects? An application of sensitivity analysis to project finance transactions

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Abstract

The valuation of the economic convenience and the financial sustainability of large projects requires the development of large complex models for capital budgeting purposes [Kleijnen and Van Groenendaal (1997).]. However, complex models challenge the management in the identification of the key value drivers of the performance of the new project, particularly if management needs to know which, among a large set of inputs, influences to a larger extent the valuation criteria used in capital budgeting. Sensitivity analysis (SA) plays a central role in explaining the model results but recent literature [Saltelli (1999) and (2002), Borgonovo and Peccati (2006)] has shown that SA techniques must be quantitative, model independent and must minimize the need for qualitative statements. In this paper, we develop an approach based on the differential importance measure aimed at evaluating model correctness, response to changes in the input factors and identification of key performance drivers in a systematic way. We apply the methodology to the special case of project finance where a new project is incorporated in a specially created vehicle (a Special Purpose Vehicle or SPV) whose sole objective is the design, construction and management of a single, high capital intensive venture for a given number of years. In our case, the project is represented by the construction of a parking lot for which we have available the evaluation model prepared by the mandated lead arranging bank in charge of the organization of the funding. The model requires a vector of 428 inputs. Using such a complex model enables us to reach two important results: (1) we introduce a computational algorithm and derive the sensitivity of the model to each factor in order to identify model response and key individual performance drivers and (2) we group the inputs in categories so as to offer a synthetic view of the problem exploiting the additivity property of the differential importance measure which streamlines the assessment of

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the impact of joint input changes, and gives flexibility in combining parameters in any group and at the desired aggregation level. Results indicate that sponsors and lenders are exposed to exogenous factor variations in a different way, both when exogenous variables are considered individually and in groups.

Keywords: Sensitivity Analysis, Investment Planning, Large Projects, Valuation, Project Financing, Differential Importance Measure; Comparative Statics.

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1 Introduction

Investment planning is central to the growth and expansion of a corporation. The valuation of new industrial opportunities is often accompanied by a sophisticated modeling exercise that leads to the creation of financial models aimed at reproducing the investment economics.

The use of financial models plays a crucial role in project finance transactions. According to Esty and Sesia (2007), project finance is a transaction that "[...] *involves the creation of a legally independent project company financed with nonrecourse debt (and equity from one or more corporations known as sponsoring firms) for the purpose of financing investment in a single purpose capital asset, usually with a limited life*". Project finance is usually associated with large capital intensive ventures (for example power plants, transportation infrastructure, telecom projects) with low redeployability value and limited recovery value in case of project default. Under these circumstances, lenders pay particular attention to the performance of the project on a going concern basis since the possibility to repay principal and interests lays on the ability of the project to generate sufficient cash flows. When the project is presented to potential lenders, the model becomes the shared platform for the negotiation between debt- and share-holders. To accompany negotiation, the modeling effort requires extreme accuracy, becoming both costly and time consuming, as it involves the interaction and contribution of fiscal, technical, legal and financial consultants.

In this paper, our focus is dedicated to such transactions for a number of reasons: *i*) project finance is a no-recourse form of financing, so that lenders must dedicate time and resources to the careful estimation of the future performance of the project via complex models; *ii*) project finance is associated with the creation of a special purpose vehicle (SPV) so that we can concentrate our evaluation exercise only on one single venture with limited economic life. No other case in standard corporate finance settings could enable us to separate the fate of the venture from any other existing project carried out at the same time by the sponsoring firm; *iii*) since financial models used for project finance evaluation are particularly complex, they challenge the Sensitivity Analysis exercise both from the methodological and numerical viewpoints.

This last item is crucial for our paper since the lack of an analytical expression and the dimensions of the model make it a black box (Diaconis (1988)). The model behavior as a function of the exogenous variables is, then, unknown to the financial analyst.

A further problem generated by the presence of a large number of exogenous variables is that SA is usually not performed on all exogenous variables. Conversely, for reasons of time and cost, the attention is restricted to a subset of inputs usually pre-selected based on experience or qualitative statements. However, especially when the investment setting is new, such an approach may lead to the a-priori exclusion of relevant parameters from the analysis.

An additional issue is the selection the SA method itself. Clemen (1997) (ch.5) sets forth the central role of SA in the decision making process and proposes a series of simple questions that can be answered through the use of one-parameter-at-a-time SA[1]. However, this type of SA has been decidedly improved in the subsequent literature. The recent development of SA techniques [Saltelli (1999), Borgonovo and Peccati (2004) and (2006)] provides analysts and decision makers with new tools capable of enabling management to fully exploit the information contained in the model [Koltai and Terlaki (2000), Koltai and Terlaki (2000), Van Groenendaal (1998), Saltelli (2002); Frey (2002); Van Groenendaal and Kleijnen (2002), Borgonovo and Peccati (2004), Borgonovo and Peccati (2006)]. In particular, for applications in investment evaluation the SA method should allow a decision maker to:

Insight (1): test the robustness and correctness of the model;

Insight (2): detect model response to changes in the parameters;

Insight (3): determine the influence of each of the assumptions on the valuation criterion.

Insight (1) is necessary in the decision making process since in the case model results do not comply with the underlying theory they should not be used to make decisions [for

[1] In the literature, a criticism to the use of one-parameter-at-a-time SA in investment evaluation can be found in Kleijnen and Van Groenendaal (1997), Van Groenendaal (1998), Kelijnen and Van Groenendaal (2002), Borgonovo and Peccati (2004) and then Borgonovo and Peccati (2006).

a discussion on modeling risk see Fabozzi (2000).] Insight (2) is in the line of Samuelson's classical statement of comparative statics (see Borgonovo (2008)), An analyst must gather the understanding of how a change in an assumption affects the valuation criteria. Insight (3) accomplishes the task of avoiding to screen-out relevant factors based on an a-priori qualitative statements without a quantitative support. We remark that obtaining these insights is particularly relevant for large models, since an analyst has no other way of dissecting the model results[2]. The absence of this information would prevent analysts and decision makers from fully exploiting the information contained in the (complex and costly) model. As a result, one runs the risk of undermining the modeling effort and the valuation process. This issue is particularly relevant in the case of complex transactions financed on a project-finance base.

In this work, we aim at making the acquisition of these insights systematic. Our approach is based on the use of the differential importance measure (D) [Borgonovo and Peccati (2004), Borgonovo and Peccati (2006) and Borgonovo and Apostolakis (2001)]. D extends comparative statics [Borgonovo (2008)] and elasticities, overcoming their limitations, especially with reference to dimensionality issues [Borgonovo (2008)]. In this respect, we note that the presence of a high number of parameters denominated in different units poses two questions to traditional comparative statics methods. The first is the fact that partial derivatives cannot be used as sensitivity measures to identify key project drivers [Borgonovo (2008)]. The second is that both analysts and decision makers feel the need to synthesize results assessing not only the influence of individual inputs (the list would bo too long), but of categories (e.g. revenue, fiscal, technical assumptions etc.). One is then facing a joint sensitivity analysis problem. We show that the first issue is solved by exploiting the definition of D and the second issue by exploiting its additivity. Additivity, in fact allows to obtain joint sensitivities without additional model runs. Thus, analysts are free in setting the level of detail in result communication.

Our first step is to allow the estimation of D in the context of large spreadsheet models, i.e., in the absence of a closed-form expression of the valuation criterion. We note that in previous literature applications of D, analytical expressions of the valuation criteria were available [see, for instance, Borgonovo and Peccati (2004) and Borgonovo and Peccati (2006)]. We then adapt and apply a numerical estimation algorithm whose mathematical aspects are set forth in Borgonovo and Apostolakis (2001).

We next discuss the financial and managerial interpretations of the results, i.e., how to gain insights (1), (2) and (3). As far as insight (1) is concerned, we shall see that, by

[2] Saltelli (2002) utilizes the following metaphor: "Sensitivity analysis for modelers? Would you go to an orthopaedist who didn't use X-ray?"

application of the algorithm, it is possible to create an automated model correctness test. As far as insight (2) is concerned, we show that the sign of D completely reveals the dependence of the valuation criterion on the exogenous variables. As far as the identification of the Key Performance Drivers (KPD) is concerned, the method allows to consider all exogenous variables and to rank them according to their influence. Furthermore, the case of project finance is particularly interesting in this respect, because the viability of the initiative must satisfy the valuation criteria set by banks and sponsors simultaneously (`Yescombe, 2002`; Gatti (2007), ch. 5). These criteria are, in general, conflicting each other. For this reason, when considering KPD, we cannot limit ourselves to the shareholder valuation perspective – based essentially on equity NPV – but also on the lenders' viewpoint focused on debt service coverage ratios (DSCR) and loan life coverage ratios (LLCR) – by performing the SA on the valuation criteria utilized by the two sides. We are then lead to investigate whether KPD are the same for sponsors and lenders. This analysis is complicated by the presence of a large number of parameters. We then introduce a methodology to obtain a quantitative indication of the ranking agreement by synthesizing individual results. The methodology is based on the use of Savage scores, a statistical technique introduced in Iman and Conover (1987), which has found wide application in sensitivity analysis of large models (Borgonovo (2006)).

We illustrate the methodology through its application to a full-fledged financial model developed for the evaluation of an infrastructure investment project (namely a parking lot) financed with no-recourse debt. The model was prepared and agreed by the mandated lead arranging bank and the project sponsoring firms in order to decide about the financial viability of the parking facility. The model has been implemented on a series of Excel spreadsheets requiring a set of 428 inputs parameters .

For the investment case study at hand, as far as individual contributions are concerned, results show that on average KPD tend to be the same, both for sponsor and lender valuation criteria with higher agreement on the most relevant factors. However, in a few but significant cases parameters that are influential on sponsor's criteria are not on lender's criteria. A notable example is the cost of capital (k_e), which is a significant contributor of the equity net present value but has null influence on the debt service coverage ratio (DSCR) (See Section 2). For communication purposes, not only the results for individual exogenous variables are discussed, but also the results for groups with two levels of detail. First, the 428 parameters are grouped in the 6 main categories. Revenue assumptions turn out to be the most relevant ones both on the NPV and the minimum DSCR (mDSCR), followed by investment costs and with operational costs playing a minor role. Then, the results for 17 groups, where each main category is split into subcategories selected by the analysts, providing one with a further dissection of the results.

The remainder of the paper is organized as follows. In Section 2, we discuss the specific features of project finance deal valuation, focusing on sponsors and lenders criteria. In Section 3, we present the SA method we use in this work illustrating its mathematical properties and computational aspects. In Section 4, we presents the application of the method to the financial analysis of an industry-used financial model created for measuring the financial viability of an investment in the construction and operation of a parking lot. Conclusions are offered in Section 5.

2 Financial Valuation: the Case of Project Finance Transactions

This section deals with the characteristic features of project finance transactions and their implication in the valuation of the economic convenience and the financial sustainability of these types of deals. The analysis also aims at pointing out the differences in the sponsor and lender valuation perspectives.

Project finance is an important part of the international syndicated loans market. Heinz and Kleimeier (2003) underline that the value of project finance deals closed in the January 1980-March 2003 period was about USD 960bn and Esty and Sesia (2007) report that the size of project finance loans market is larger than the IPO market in the USA. Corielli et al (2008) find that the average value of a project finance investment is about 512 million US$, with an average Debt-to-equity ratio of 4.23x.

Project finance originated in the energy generation sector and nowadays is widely used to fund oil & gas, power and telecom projects [Gatti et al (2007)] and the preferred way for firms willing to enter in foreign risky markets limiting balance sheet exposure. In addition, project finance is used more intensively in developing countries as an efficient solution for a quick recovery of the infrastructure gap (Hammami et al (2006)). More recently, project finance schemes have been sought to fund internet and e-commerce projects.

The nature of project finance is to be a nexus of contracts (Jensen and Meckling (1976)) revolving around a specially incorporate entity known as Special Purpose Company (SPV) which becomes the counterpart for all the operating and financial contracts (Vinter (1998)). Money needed to design, build and operate the new projects is provided by a group of sponsoring firms (the SPV 's shareholders) and to a larger extent by a bank syndicate headed by a Mandated Lead Arranger. Loans are fully guaranteed by all the assets of the company and supplemented by a large set of covenants that aims at imposing restrictions on the uses of funds by the SPV (Smith and Warner (1979)). Very often, the loans are granted on a no-recourse basis meaning sponsors limit their responsibility toward the project performance up to the original equity injection. In other words, project finance allows sponsors to fund the venture "off-balance sheet".

The success of a project finance transaction lays on the capacity of the project to generate sufficient cash during its operating phase in order to match the cash needed for debt service – interest and principal repayment – and the payment of dividends to the project sponsors. The operating phase is usually a very long but finite period of time implying that – contrarily to what happens in standard corporate finance settings – the SPV will not reinvest cash flows for further development of the initiative but will distribute all the available cash to all the participating counterparts.

Given the importance of cash flow generation in project finance deals, it is not surprising that extensive negotiations involve the estimation of the SPV cash flows. This estimation is performed by means of a financial model, which is aimed at recreating the financial statements of the SPV , so as to be able to accurately forecast the SPV economic and financial performance (Benninga (2000)). The model aims at adhering as much as possible to what would be the actual reported statements of the project company and, therefore, full consideration is devoted to accounting and fiscal rules in the Country or Region where the SPV operates. The ultimate goal is that if the assumptions stated at the moment of the evaluation were realized, then the model would produce what originally expected.

Core of the model is the cash flow statement, from which lenders and investors cash flows are estimated. The first cash flow of interest is the project free cash flow defined as revenues less operating expenses, less correction for changes in working capital and taxes. Tax outflows are estimated via an income statement built in compliance with the fiscal and accounting rules of the country where the SPV operates. The income statement is also necessary to estimate profits, which turn into dividends being the SPV retention ratio equal to zero after having satisfied lenders' requests about the set up of a minimum level of cash reserves (DSRA or debt service reserve account). The project represents the cash available before debt service and cash remittance to shareholders. FCF is then disgorged to interests, principal repayment and debt related reserves. Once all debt-holders cash flows are subtracted, the remaining cash constitutes the free cash flow to equity (FCE). Once identified and estimated, the debt and equity cash flows feed into the valuation criteria. The criteria used by sponsoring firms and banks in order to decide whether or not to move forward with project implementation are different.

From the point of view of SPV 's shareholders, sponsors base their decision on standard NPV, which becomes an adjusted present value when third party financing is present [Myers (1974)]. Since project finance is characterized by a closed life cycle without possibility of scope changes or reinvestment for expansion, real options are not a concern because flexibility is practically absent [Dixit and Pyndyk (1994), Zettl (2002)]. The financial model utilized for the SA of this work is based on the assumption of no possibility of delays in the investment decision and no abandonment or expand options. Under these conditions [Dixit and Pyndyk (1994)] an investor should apply the net present value (NPV) rule , i.e. undertake the project if $NPV > 0$.

The perspective of lenders is different [Gatti (2007)] In particular, due to the peculiar investment structure, lenders focus on the project debt repayment capability [Navitt and Fabozzi (1995), Gatti (2007).] Hence, criteria utilized by financial institutions to investigate lending decisions to industrial projects look at debt service. The two most often encountered in the practice of project finance are the debt service coverage ratio (DSCR) and the loan life coverage ratio (LLCR). DSCR is a period-on-period (typically year-on-year) measure, which quantifies the capacity of the operating cash flows to service the debt. It is defined as follows [Gatti (2007)]:

$$DSCR_t = \frac{FCF_t}{P_t + I_t} \; t = 1, 2, ..., T_L \tag{1}$$

where: FCF_t is the free cash flow generated by the project at time t, P_t is the principal repayment for period t, I_t is the interest repayment for period t and T_L is the loan tenor, e.g., the length of the repayment period. Loan contract default clauses require the SPC to maintain the minimum value of DSCR over time greater than a predetermined threshold. We write:

$$\min_t DSCR_t > DSCR_{Th} \tag{2}$$

where $DSCR_{Th}$ is a number greater than 1, whose magnitude depends on the Bank's risk perception of the PF deal. In the practice, $DSCR_{Th}$ range from 1.2 to 1.9. If the SPC fails in maintaining such $DSCR_t$ at any period in which the loan is present, then default may be triggered.

The LLCR is a project-life measure of debt repayment capability and is defined as (Gatti (2007)):

$$LLCR_t = \frac{\sum_{s=t}^{T_{Debt}} \frac{FCF_t}{(1 + k_d)^{s-t}}}{D_t} \tag{3}$$

where t is the time of interest, T_{Debt} is the debt tenor, D_t the debt outstanding at time t. The numerator in eq. (3) represents the present value at time t of the free cash flows generated by the project from t to T_{Debt} discounted at k_d.

The cash flows (both FCF and CFE) depend upon the several factors influencing the investment performance, as macroeconomic parameters (future inflation), market driven parameters (demand, price of goods sold, raw material costs), financial aspects (leverage, spreads, currency), technical aspects (plant efficiency), investment costs and many others.

Correspondingly,the valuation criteria depend on the same exogenous factors. An analytical exemplification of the dependency of valuation criteria on the exogenous variables can be found in eq. (21) of Borgonovo and Peccati (2006), which represents the present value of an investment in the energy sector. The analytical expression is, however, made possible, by the assumptions of a perfectly efficient financial structure and a simplification in the timing of the cash in- and out-flows, with the cash outflows concentrated at $t = 0$. For complex projects in an advanced phase of the valuation process, such assumptions are usually not realistic. In fact, monthly forecasts are utilized during the cash-outflow period. The first is the need of financing institution to accurately compute the total amount of debt to be disbursed to the project. Since the amount of debt at the end of construction contains capitalized interests that are computed on a monthly or daily basis, utilizing a yearly approximation could lead to misleading estimates. If one adds that often project costs are an itemized list that can count more than a-hundred items and that the model elaborates numerous other intermediate calculations to account for escalation, compute appropriate depreciation amounts, estimate debt outflows reflecting loan agreement repayment schedule and interest calculation rules, include tax and accounting rules, etc., one easily grasps that analytical approaches are not practical. Hence, the calculations are implemented on large spreadsheets.

The above discussion can be summarized as follows. Project finance valuation models are the result of professional efforts devoted to accurately include all factors (fiscal, accounting rules, investment costs, macroeconomic conditions, technical aspects, etc.) concerning the investment life. Building accurate financial models is both time and resource consuming - the cost of technical consultants and financial advisors hired to provide inputs or financial modeling as well as model auditing can be non negligible. - As a result models are complicated, large, and usually not analytically known (Yescombe (2002), Finnerty (2007)). All this leads to the impossibility for the decision maker to have a closed-form expression for the NPV or the mDSCR as a function of the exogenous variables. Due to these features, the model runs the risk to become a black box that processes a vector of inputs and estimates valuation criteria.

Gaining insights on the model behavior and understanding the influence of exogenous variables on the investment performance is, however, crucial in adding value to the modeling exercise. The technical aspects of how to accomplish this task are the subject of the next section.

3 Sensitivity Analysis of Financial Models: Estimating the Differential Importance Measure

In Section 1, we have synthesized some of the insights that a decision maker derives from SA as: (1) information on the model correctness, (2) information on the model response to input changes and (3) relevance of parameters with respect to the value assumed by the valuation criterion. In this section, we explain how these insights can be derived for large models by making use of an SA approach based on the differential importance measure.

Let

$$V = v(\boldsymbol{\lambda}) \quad v : \Lambda \subseteq \mathbb{R}^n \to \mathbb{R} \tag{4}$$

denote the relationship that links the valuation criterion (V) to the exogenous variables $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, ..., \lambda_n)$. (We recall that V depends on the decision-maker; for instance, it is an NPV for a shareholder or an mDSCR for a lender.) We let $\boldsymbol{\lambda}^0$ denote the reference (base case) value of the exogenous variables on which the valuation criterion depends. The numerical values of the exogenous variables reflect the current assumptions and state-of-knowledge of the decision-maker concerning $\boldsymbol{\lambda}$. We assume that $v(\boldsymbol{\lambda})$ is differentiable at $\boldsymbol{\lambda}^0$ and that ∇v is not orthogonal to $\mathrm{d}\boldsymbol{\lambda} = [\mathrm{d}\lambda_1, \mathrm{d}\lambda_2, \ldots, \mathrm{d}\lambda_n]^T$ at $\boldsymbol{\lambda}^0$.

The sensitivity of V on exogenous variable λ_s at λ_0 can be defined as [Borgonovo and Apostolakis (2001), Borgonovo and Peccati (2004), Borgonovo and Peccati (2006)]:

$$D_s(\boldsymbol{\lambda}^0, \mathrm{d}\boldsymbol{\lambda}) = \frac{v_s(\boldsymbol{\lambda}^0)\mathrm{d}x_s}{\sum_{j=1}^n v_j(\boldsymbol{\lambda}^0)\mathrm{d}x_j} \tag{5}$$

where $v_s(\boldsymbol{\lambda}^0)$ is the partial derivative of F w.r.t λ_s at $\boldsymbol{\lambda}^0$. $D_s(\boldsymbol{\lambda}^0, d\boldsymbol{\lambda})$ measures the parameter importance as the change in F provoked by a change in λ_s, over the sum of the changes in F provoked by changes in all the input parameters. $D_s(\boldsymbol{\lambda}^0, \mathrm{d}\boldsymbol{\lambda})$ is the fractional change in V that follows a (small) change in λ_s. In fact, the numerator in eq. (5), is the change provoked by a variation in λ_s, while the denominator is $\mathrm{d}V$, i.e., the differential of V, which equals the change in V provoked by a simultaneous change in all the parameters.

It can be shown that D [eq. (5)] shares the following properties [Borgonovo and Apostolakis (2001), Borgonovo and Peccati (2004)]:

- D generalizes partial derivatives if one assumes a uniform change in the parameters. In fact, if one assumes

$$\mathrm{d}\lambda_j = \mathrm{d}\lambda_s \ \forall s, j = 1, 2, ..., n \tag{6}$$

then it holds that [Borgonovo and Peccati (2004) and Borgonovo and Peccati (2006)]:

$$D_s(\boldsymbol{\lambda}^0, \mathrm{d}\boldsymbol{\lambda}) = D1_s^0 = \frac{v_s(\boldsymbol{\lambda}^0)}{\sum_{j=1}^n v_j(\boldsymbol{\lambda}^0)} \tag{7}$$

Eq. (7) implies that $D1_s^0 \propto v_s(\boldsymbol{\lambda}^0)$, i.e., under the assumption of equal variations in the exogenous variables, the differential importance of a parameter is proportional to the corresponding partial derivative. This implies that measuring sensitivity based on partial derivatives is equivalent to state an assumption of uniform changes in the parameters. In Borgonovo and Peccati (2004), it is shown that, if V is the net present value of an investment and $\boldsymbol{\lambda}$ the vector of the expected cash flows, then the (vector of) $D1_s^0$ is the cash flow profile. However, Borgonovo and Peccati (2006) note that if one moves at the level of the parameters that determine the cash flows, then this conclusion does not hold anymore. In fact, when the exogenous variables are denominated in different units, then eq. (6) cannot hold and the uniform change assumption cannot be adopted.

- D generalizes Elasticity if one assumes a proportional change in the parameters. In fact, if

$$\frac{\mathrm{d}\lambda_j}{\lambda_j^0} = \omega = \frac{\mathrm{d}\lambda_s}{\lambda_s^0} \ \forall s, j = 1, 2, ..., n \tag{8}$$

it turns out that [Borgonovo and Peccati (2006)]:

$$D_s(\boldsymbol{\lambda}^0, \mathrm{d}\boldsymbol{\lambda}) = D2_s^0 = \frac{v_s(\boldsymbol{\lambda}^0)\lambda_s^0/V^0}{\sum_{j=1}^n v_j(\boldsymbol{\lambda}^0)\lambda_j^0/V^0} = \frac{E_s^0}{\sum_{j=1}^n E_j^0} \tag{9}$$

where E_s^0 is the elasticity of V with respect to λ_s at $\boldsymbol{\lambda}^0$. Eq. (9) implies that measuring sensitivity based on elasticity is equivalent to stating an assumption of proportional changes in the parameters [Borgonovo and Peccati (2004) and Borgonovo and Peccati (2006).] In fact, if eq. (9) implies that $D2_s^0 \propto E_s^0$, which indicates that D and Elasticity differ only for a normalization factor if one assumes proportional parameter variations. In Borgonovo and Peccati (2004), it is shown that if V is an NPV and $\boldsymbol{\lambda}$ the vector of expected cash flow, then is the fraction of the NPV associated with $\boldsymbol{\lambda}$. In Borgonovo and Peccati (2006) it is shown that even when $\boldsymbol{\lambda}$ represents the parameters that determine the cash flows, then $D2$ can be still computed (as opposite to D1), and has the interpretation of the fraction of the change in NPV related to a change in λ_s.

- D shares the additivity property. Let λ_{i_1},λ_{i_2},..., λ_{i_k} be a set of k input factors. The sensitivity of V on λ_{i_1},λ_{i_2},..., λ_{i_k} is related to the individual sensitivities as [Borgonovo and Apostolakis (2001)]:

$$D_{i_1, i_2, \ldots, i_k} = \sum_{i=1}^{k} D_{i_k} \tag{10}$$

i.e., the differential importance of a group of parameters is equal to the sum of the differential importance of each of the parameters in the group. The additivity property [eq. (10)] allows to directly obtain the sensitivity of V on any parameter sets from the individual sensitivities. In the remainder, we shall see that this property is the key to synthesize results and reduce the computational effort in the SA of complex financial models.

- The immediate consequence of the additivity property is that the sum of the D_s of all parameters equals unity [Borgonovo and Apostolakis (2001)]:

$$\sum_{i=1}^{n} D_i = 1 \tag{11}$$

In the remainder of this section, we discuss the technical aspects of the application of D to the SA of large project valuation models. The first key-feature is that, in real life applications, financial models are characterized by a large number of input variables. This makes it impossible to utilize an analytical approach for the computation of D, that must be accomplished via a numerical algorithm. This presents a first novel feature of the present work as, in previous applications of D in the industrial investment realm, analytical expressions of the valuation criteria were available. Thus, the sensitivity measures must be estimated numerically. We make use of the estimation algorithm D developed by Borgonovo and Apostolakis (2001), after having adapted it to spreadsheet modeling. The algorithm is based on the steps presented in Table 1 (see Borgonovo and Apostolakis (2001)).

For the sake of methodological completeness, we describe the rationale underlying Table 1, with the purpose of illustrating the features of its application to financial models. The rationale underlying the algorithm is the operational definition of D offered in Borgonovo and Apostolakis (2001):

$$D_s(\boldsymbol{\lambda}^0, \mathrm{d}\boldsymbol{\lambda}) = \lim_{\Delta\boldsymbol{\lambda}\to\mathbf{0}} \frac{\Delta V_s}{\sum_{i=1}^{n} \Delta V_i} = \lim_{\Delta\boldsymbol{\lambda}\to\mathbf{0}} \frac{V(\lambda_s^0 + \Delta\lambda_s; \boldsymbol{\lambda}_{(-s)}^0) - V(\boldsymbol{\lambda}^0)}{\sum_{i=1}^{n} V(\lambda_i^0 + \Delta\lambda_i; \boldsymbol{\lambda}^0) - V(\boldsymbol{\lambda}^0)} \tag{12}$$

where $\Delta\boldsymbol{\lambda}$ is the vector of all parameter changes, ΔV_s is the change in the differentiable function V due to the change in exogenous variable λ_s while the other exogenous variables are kept at $\boldsymbol{\lambda}^0$ (denoted in eq. (12) as $\boldsymbol{\lambda}_{(-s)}^0$).

Table 1: Steps for the numerical estimation of D (from Borgonovo and Apostolakis (2001))

Loop	**nr.**	**Step**
External	1	Define the $\Delta\lambda_s^j$ $j=1,2,...,m$, $s=1,2,...,n$ sequences
	2	Perform the steps of the internal loop (steps 2.1 and 2.2 below)
	3	Set a discrepancy
	4	Test convergence and eventually stop iterations
Internal	2.1	Compute V with all the parameters at $\boldsymbol{\lambda}^0$
	2.2	For a given j and for $s=1,2,...,n$ compute: $\Delta V_s^j = V(\lambda_s^0+\Delta\lambda_s^j;\boldsymbol{\lambda}_{(-s)}^0) - V(\boldsymbol{\lambda}^0)$ $\Delta V^j = \sum_{s=1}^n \Delta V_s^j$ $r_s^j = \frac{\Delta V_s^j}{\sum_{s=1}^n \Delta V_s^j}$

One considers then the incremental rations in eq. (12) as functions of $\Delta\boldsymbol{\lambda}$, and introduce the functions:

$$r_s(\Delta\boldsymbol{\lambda}) = \frac{V(\lambda_s^0+\Delta\lambda_s;\boldsymbol{\lambda}_{(-s)}^0) - V(\boldsymbol{\lambda}^0)}{\sum_{i=1}^n V(\lambda_i^0+\Delta\lambda_i;\boldsymbol{\lambda}_{(-s)}^0) - V(\boldsymbol{\lambda}^0)} \tag{13}$$

Each function in [eq. (13)] is continuous in $\Delta\boldsymbol{\lambda}$. This fact, combined with a well-known multivariate calculus result (see Burkill and Burkill, 1970, pp.33-34) guarantees that $r_s(\Delta\boldsymbol{\lambda})$ converges to $D_s(\boldsymbol{\lambda}_0)$ as $\Delta\boldsymbol{\lambda}$ tends to zero continuously. In a numerical calculation, one can then exploit this result. In fact, numerical estimations are necessarily made up by discrete steps. If a function $f(\Delta\boldsymbol{\lambda})$ tends to a limit L for $\Delta\boldsymbol{\lambda}$ tending to an accumulation point (say 0) on a continuous basis, then it will converge to the same limit L for every discrete sequence such that $\Delta\boldsymbol{\lambda}^j$ tends to the same accumulation point. Hence, by building appropriate (discrete) sequences of values $\Delta\boldsymbol{\lambda}^j$ such that $\lim_{j\to+\infty}\Delta\boldsymbol{\lambda}^j = \mathbf{0}$ one is assured that the n sequences $r_s^j(\Delta\boldsymbol{\lambda})$ tend to $D_s(\boldsymbol{\lambda}_0)$ $(s=1,2,...,n)$. Observe that each element of the sequence $r_s^j(\Delta\boldsymbol{\lambda})$ can be interpreted as an approximation of D_s, where j is the j^{th} step of the algorithm. A way of defining the discrete sequence of $\Delta\boldsymbol{\lambda}^j$ (Step 1 in Table 1) is to set:

$$\Delta\lambda_s^j = \frac{\lambda_s}{\omega_s^j} \qquad s=1,2,...,n \tag{14}$$

where ω_s^j is a diverging and increasing sequence of integers, such that $\lim_{j\to+\infty}\omega_s^j = +\infty$. With these definitions, $\lim_{j\to+\infty}\Delta\lambda_s^j = 0$, $\forall s=1,2,...,n$. One can then test the convergence of the algorithm as j progresses.

We need two additional observations about implementation. Regarding Step 1 of Table 1, we note that, if arbitrary relative parameters changes are allowed, then one needs to define a distinct sequence ω_s^j for each parameter. However, in the case of proportional changes, one needs to introduce only one sequence, since eq. (8) implies

$$\omega_s^j = \omega_l^j \quad \forall s = 1, 2, ..., n \tag{15}$$

Also in the case of uniform changes, it is sufficient to define a single sequence, since, eq. (6) implies that once $\Delta\lambda_1^j$ is determined, the other parameter variations are are equal. However, in the case of financial models and, more in general, in the case of economics models, parameters have different dimensions [Borgonovo and Peccati (2004) and (2006).] For example, inflation indices are "pure numbers" being the ratios of homogeneous quantities, while costs are denominated by the corresponding currency. Due to this reason, one cannot compare a change of one inflation unit (a pure number) with a unit change in investment costs (denominated in EUR or USD or BRL). Hence an assumption of uniform parameter changes does not hold for most financial models. Instead, one can compare proportional changes in the factors. A typical question would be: is it more important a 1% change in inflation or the corresponding change in investment costs? The natural sensitivity measure to answer this question is $D2$, eq. (9) [Borgonovo and Peccati (2006).] Thus, the numerical estimation will foresee to perform the steps in Table 1, with the sequence generated by eq. (15).

The second observation concerns the convergence test utilized in our implementation (Step 4 in Table 1). The last task is namely to determine the value of j such that convergence is reached (numerically). One notes that, as any convergent sequence is a Cauchy's sequence, Cauchy's convergence criterion can be applied (Burkill and Burkill (1970), pp.47-49; Borgonovo and Apostolakis (2001)). Thus, for every small number ε there will exist an index $j_s^*(\varepsilon)$, such that for all m and k greater than $j_s^*(\varepsilon)$:

$$\left| r_s^m(\Delta\boldsymbol{\lambda}) - r_s^k(\Delta\boldsymbol{\lambda}) \right| < \epsilon \quad \forall m, k > j_s^*(\varepsilon) \tag{16}$$

From the numerical viewpoint, then, when j is greater than m or k, D_s is estimated with an error smaller than ϵ.

We use a percentage test of the form:

$$\max_s \left| \frac{r_s(\Delta\boldsymbol{\lambda}^j) - r_s(\Delta\boldsymbol{\lambda}^{j+1})}{r_s(\Delta\boldsymbol{\lambda})} \right| < \varepsilon \tag{17}$$

i.e. the algorithm stops when the percentage discrepancy in the estimation of $D2$ in two consecutive steps is lower than a small pre-determined positive number. Note that, if eq. (17) is satisfied at $j = j^*$, then Cauchy's convergence criterion assures that this difference shall remain lower than ε for $j > j^*$.

In Section 4, we shall discuss the numerical results in the case of a real-life valuation model, in which the SA of the valuation criteria over 428 input variables is explored. In the numerical case study, we have set $\varepsilon = 10^{-3}$. The previous algorithm has been implemented on a Visual Basic subroutine that has enabled the application of D to the SA of a financial model developed for the valuation of an investment in a parking lot. The next section presents the application of the method in obtaining financial decision-making insights in the valuation of a large project in an advanced negotiation and modelling phase.

4 Application: Valuing a Project Finance Investment in a Parking Lot

The purpose of this Section is to illustrate information and insights gained by the application of the SA method proposed in Section 3 in the planning and evaluation of an infrastructure initiative. The project consists in the construction and operation of a parking lot through a project finance scheme. There is a single sponsor and the sale is by definition a merchant one (i.e. the project cannot count on one single buyer (an off-taker) for all the production available during the operating phase).

The financial model has been developed by the sponsor in the initial due diligence phase. At the moment of requesting financing to the Mandated Lead Arranging Bank, the Bank took over the financial modeling exercise. The resulting financial model parallels the investment timing. It foresees a 2 year construction period, in which cash outflows are modeled monthly. The operation period is modeled annually over a 20 year time horizon. The total investment cost is estimated at around $40MEUR$. The financial structure of the SPV foresees a 70% third party financing, and a 30% equity further split in equal portions of ordinary shares and shareholder subordinated loan. The model contains 40 calculation worksheets and requires a set of $n = 428$ inputs to be supplied by the analysts to estimate the valuation criteria. The model provides a very detailed estimation of the project cash flows. From these, all the necessary valuation criteria can be obtained. We focus on equity NPV and mDSCR as representative of the sponsors and lenders viewpoints, respectively. The base case assumptions lead to a positive NPV and to a value around 1.3 for the mDSCR.

The SA has been performed by implementing the computational algorithm proposed in Section 2 on a Visual Basic subroutine. Convergence was obtained after 10 iterations (i.e. $j = 10$ using the notation of Section 3) and with a total computational time of around 20 min. The importance of each of the 428 factors, D_s ($s = 1, 2, \ldots, 428$), has been estimated with an accuracy of 10^{-5} (Section 3).

In addition, at each iteration a correctness test has been implemented as follows. Given their complexity, financial models are usually equipped with warning or error messages to help analysts correct eventual faults. The most diffuse one is an error message signalling unbalance between assets and liabilities in any year. As in each iteration of the proposed algorithm all inputs are varied, if some of the changes provokes an erroneous model response one can register the corresponding warning signal, thus detecting which input causes the fault. Eventual inconsistencies can then be corrected. Thus, an automated model correctness test is a first benefit coming from the proposed SA method.

68 inputs registered a value $D_s = 0$ in all iterations. This result implies that this subset of inputs does not play any role in the financial calculations. Further examination of the model structure enabled to realize that these inputs were indeed disconnected from the financial calculations in the evolution of the model, but still considered active by the modelers. They were therefore excluded from further analysis.

The above two results can be ascribed as insights of type (1), in so far they have contributed to corroborate the model and test its correctness thus increasing the degree of confidence in its results.

The identification of the direction of change in the valuation criteria and the influence of each parameter [insights (2) and (3) of Section 1] can be deducted at the same time from the sign and magnitude of D_s. In fact, from eqs. (5) or (9), it is easy to see that the numerator of D_s is the change in V provoked by a change in λ_s. Hence its sign indicates whether the project value or debt service capability is impacted positively or negatively by a change in the assumption. The ranking and direction of change of the 10 most influential factors is reported in Table 2. The last two rows of Table 2 report the least influential inputs on the NPV and mDSCR, respectively. As one readily notes, these last two parameters correspond to very detailed assumptions and reflect the extreme accuracy of the model.

Table 2: Parameter ranking for sponsors (NPV) and lenders (mDSCR)

Sign	**Rank** NPV	**Parameter**	**Sign**	**Rank** $mDSCR$	**Parameter**
+	1	Nr of parking slots	+	1	Nr. of parking slots
+	1	Daily occupation	+	1	Daily occupation
+	3	Tariff for first two hours	+	3	First 2 hs tariff
+	3	Number of rotations in the first 2 hs	+	3	First 2 hs Rotation number
+	5	Occupation % of first 2 hours	+	3	First 2 hs Occupation %
-	6	k_e (cost of equity)	-	6	k_d (cost of debt)
+	7	Tariff after the first 2 hs	-	7	Rooms construction costs
+	7	Rotation number after first 2 hours	-	8	Leverage
-	9	VAT on Revenues	+	9	Tariff after first 2 hs
-	10	k_d (cost of debt)	+	9	Rotation number after first 2 hs
+	11	Night tariff	+	9	Occupation % after 2hs
+	11	Number of rotations per hr after first 2 hs	-	12	VAT on Revenues
+	11	Nightly occupation %	+	13	Night Tariff
-	14	Rooms construction costs	+	13	Number of Night Rotation
-	367	Geological Inspection Cost	+	13	Night occupation %
-	368	Days payables for electricity connection costs	-	368	Workplace set up cost

As far as the direction of impact is concerned (insight (2)), columns 1 and 4 in Table 2, results are consistent with expectations, since one notes that, for example, increases in k_d and k_e lead to decrease in mDSCR and NPV and that a positive change in tariffs would lead to an increase in NPV and mDSCR.

From Table 2, one notes that the five most influential parameters on the NPV and the mDSCR are the same, and all concern revenue assumptions. The parameter ranking sixth w.r.t. the NPV is k_e, the cost of capital, denoting that the assumption related to the discount factor is a very relevant one for investors adopting the NPV as a valuation criterion. On the contrary k_e is non influential on mDSCR, since is that it does not play any role in eq. (1). At the same time, the parameter that ranks 6^{th}on the mDSCR is the cost of debt, k_d. k_d is, however, also relevant for the NPV ranking 10^{th}. We note that leverage ranks 8^{th} w.r.t. mDSCR, while it is the 30^{th} most important parameter w.r.t. the NPV. The reason lies in the fact that leverage is highly linked to the project debt service capability, in that it determines the total amount of the funds disbursed by lenders. As such, it has a strong direct impact on $I_t + P_t$, the denominator of the DSCR, while its impact is not as strong on the NPV. Among fiscal assumptions, the income tax rate ranks 45^{th} w.r.t. the NPV, but 358^{th} (i.e. almost non influential) w.r.t. the mDSCR. One can note that the least ranked parameters concern very detailed assumptions, as the on-site geological inspection cost, or the number of days required before the investment vehicle paid connection costs to the local electricity provider.

We now want to take a step back and establish a synthetic way of indicating whether parameters influential on the NPV tend to maintain their influence also on mDSCR. To do so, we start with studying the set of the ranking shift for the parameters (Figure 1).

Figure 1 reports the distribution of the ranking shifts. Excluding the 68 non influential factors, 30 parameters rank the same, while a total of 338 factors rank differently when one considers their influence on the NPV or on mDSCR. The maximum jump registered is of -362 positions corresponding to k_e, which ranks 6^{th} w.r.t. the NPV and 368^{th} w.r.t. mDSCR. The average shift is of 9 positions. A synthetic way of expressing whether the ranking agreement/discrepancy is high or low is provided for by Iman and Conover (1987). For clarity, let R_i^{NPV} and R_i^{mDSCR} denote the rank of factor λ_s^0 w.r.t. the NPV and to mDSCR respectively. Then, R_i^{NPV} and R_i^{mDSCR} are two vectors with 428 components. The approach consists in the calculation of the correlation coefficient between $\mathbf{R}^{NPV}$ and $\mathbf{R}^{mDSCR}$ and on the corresponding Savage scores [Iman and Conover (1987), Campolongo and Saltelli (1997).] Savage scores have been introduced with the purpose of emphasizing the agreement among the top ranked factors and are defined as follows. Let n be the number of factors under consideration (428 in our case) and R_s denotes the rank of parameter λ_s^0. Then the

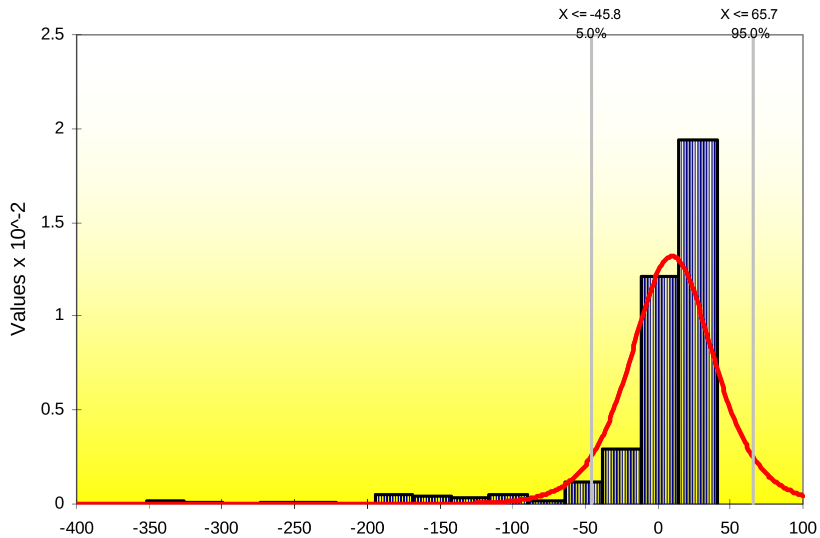

Figure 1: Histogram of Ranking Shifts

Savage score of λ_s^0 is computed as:

$$SS_i = \sum_{h=R_i}^{n} \frac{1}{h} \tag{18}$$

Just as an example, the Savage Score of input "Nr. Of parking slots from 5 year on" is equal to 6.64, while the Savage Score of input "Cost for workplace set up" (least ranked w.r.t. the DSCR) is equal to 0.16.

As Borgonovo (2006) underlines, computing correlation coefficients on ranks (in our case one would compute $(\rho_{\mathbf{R}^{NPV},\mathbf{R}^{mDSCR}})$ one gathers information on the overall ranking agreement, while computing the correlation on the Savage scores $(\rho_{\mathbf{SS}^{NPV},\mathbf{SS}^{mDSCR}})$ one gathers information on the agreement among the top ranked parameters. Applying the definitions to the SA results of the financial model at hand leads to $\rho_{\mathbf{R}^{NPV},\mathbf{R}^{mDSCR}} = 0.88$ and $\rho_{\mathbf{SS}^{NPV},\mathbf{SS}^{mDSCR}} = 0.93$. The 0.88value of indicates an overall ranking agreement. The fact that $\rho_{\mathbf{SS}^{NPV},\mathbf{SS}^{mDSCR}} = 0.93 > \rho_{\mathbf{R}^{NPV},\mathbf{R}^{mDSCR}} = 0.88$ indicates that discrepancies lie mostly in the ranking of the least influential factors.

To facilitate the analysis and the input in the financial model, investment parameters are usually classified in the revenue, operating expenses, investment cost, financial„ fiscal and macroeconomic assumption categories. The categories reported in Table 3.

The second column in Table 3 displays the number of parameters in each category. The investment cost category encompasses 219 inputs. This number reflects the high level of accuracy utilized by the Mandated Lead Arranging Bank in estimating investment costs.

Table 3: Ranking of the six assumption categories w.r.t. the NPV and mDSCR.

Category	**Parameters**	$\mathbf{R}^{NPV}$	$\mathbf{R}^{mDSCR}$
Revenue Assumptions	135	1	1
Investment Cost Assumptions	219	2	2
Financial Assumptions	29	3	5
Fiscal Assumptions	19	4	4
Operating Expenses	25	5	3
Macroeconomic (Inflation)	1	6	6

The accuracy is called for by the fact that investment costs determine of the amount of debt to be disbursed to the project. The revenue assumption category contains 130 parameters. This high number is a consequence of the sophisticated revenue calculation method, based on the number of parking slots, rotations, tariffs and occupation times, which are allowed intra-day variations. The category with the lowest number of parameters is Macroeconomic assumptions with only one input, since a unique inflation index has been used for escalation purposes at all instances.

For result communication purposes, management finds it desirable to obtain the importance of the investment categories. The question which is addressed is, then, whether it is revenues or investment costs that drive the valuation results. The additivity property of D [eq. (10)] allows to streamline this analysis, as the importance of a category is the sum of the importances of the parameter in the category. Note that no further model runs are necessary to estimate the importance of groups, with notable savings in computational cost.

The third and fourth columns in Table 3 report the category ranking with respect to the NPV and the mDSCR, respectively. Revenue assumptions are the most important group, followed by investment costs. Financial assumptions rank third for the NPV and 5^{th} for the mDSCR, fiscal assumptions rank 4^{th}w.r.t. both criteria, operating expenses rank 5^{th} for the NPV and 3^{rd} for the mDSCR. Macroeconomic assumptions are the least relevant.

Figure 2 shows the magnitude and the direction of impact of each category.

While there is one discrepancy on the ranking, the direction of the impact is the same w.r.t. the NPV and mDSCR for all groups i.e. groups whose positive change leads to an increase the NPV would also increase the mDSCR. More in detail, an increase in revenue and macroeconomic assumptions improves the economic performance of the project both from the sponsor and lender perspectives. The negative sign related to fiscal assumptions is immediately explained by the fact that an increase in taxes would lead to a decrease in the project economic performance. An increase in construction costs affects project performance negatively both from the sponsor and lender perspectives. However, lenders are more exposed than sponsors (Figure 2). This is due to the fact that, as mentioned above,

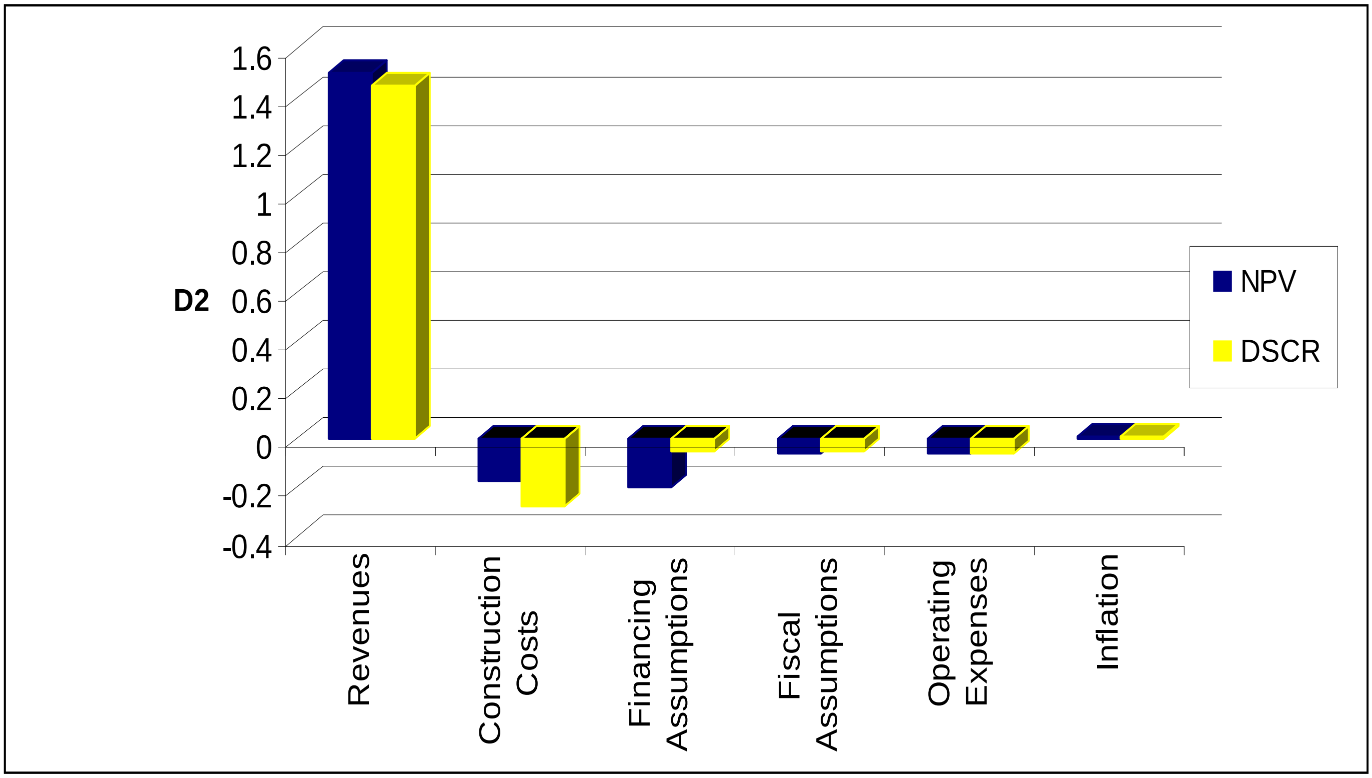

Figure 2: $D2$ for each of the assumption categories. The sensitivity measure of each group is found directly as the sum of the sensitivity measures of each of the input parameters in the group.

construction costs are the basis used by lenders to estimate the amount of debt to be disbursed, and, thus, have a direct impact in determining lenders' exposure. The explanation of the negative sign of financial assumptions is as follows. An increase in the cost of money (debt/capital) has a negative effect on both NPV and mDSCR. On the other side, an increase in leverage increase has a negative effect on the DSCR but a positive one on the NPV. The negative sign of the category then suggests that the impact of k_e (ranking 6^{th}) is not compensated by a proportional increase in leverage (ranking 45^{th}). Note also that sponsors are more exposed to changes in financial structure than lenders, since for sponsors this category includes parameters as shareholder loans percentage and interest rate which do not influence mDSCR.

In the presentation of the results, management found it informative to further analyze the results by splitting each main category into subcategories. Revenue assumptions are further subdivided in their main components: tariffs (45 inputs), occupation days (20), number of rotations (20), percentage of occupation (20), available car parking slots (10), occupation time (10), number of motorbike slots (10). Within the financial assumptions, the cost of capital is separated from the rest of assumptions to isolate its influence. Construction costs are also split into 5 subcategories. The results of the analysis are reported in Figure 3.

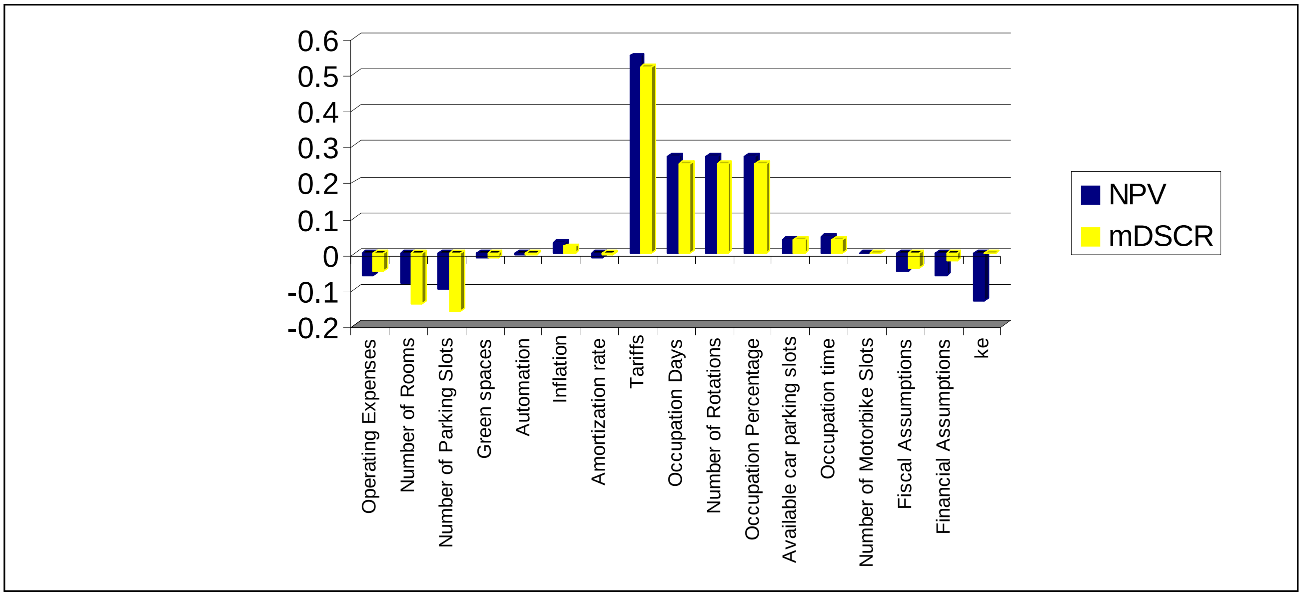

Figure 3: Influence of parameters grouped in 17 categories.

Figure 3 shows that assumptions on tariffs are the most important ones in determining both the project NPV and mDSCR. Assumptions concerning slots occupation (occupation days per year, percentage of occupation, number of rotations) follow, and are relevant both from the debt and equity criteria viewpoints. In agreement with Figure 2, construction costs tend to be more relevant for lenders than for sponsors. Again financial assumptions are more relevant from the equity than from the debt perspective and one notes the high impact of k_e.

Overall, Figures 2 and 3 show that fiscal and financial assumptions play a less relevant role than revenue or investment cost assumptions. Also, operational costs are not a main concern to the project.

We note that the flexibility in choosing the level of detail (Figures 2 and 3 and Table 3) is advantageous also for result communication purposes, when the number of assumptions is large. For example, we have seen that construction costs or revenues calculation were broken down at a very high level of detail, while a decision maker might want to understand their influence as a whole.

5 Conclusions

In this work, we have illustrated a method to better exploit the informative content of financial models in the valuation of industrial investments funded on a project finance base. The high level of detail and the complexity of the models used in such large and often complex initiatives gives SA an essential role in deepening the understanding of the model results. We formalized a systematic approach based on the Differential Importance Measure (D). The enables one to obtain the following insights: 1) . The approach also avoids the risk of an ex-ante exclusion of important factors from the analysis.

In the SA of complex models, two main problems have to be coped with: the high number of inputs and the need of assessing the sensitivity on factor groups. The first problem has been solved by the implementation of an algorithm based on Cauchy's convergence criterion which allows an accurate estimation of D. The second problem has been solved by utilizing the additivity property of D. By additivity, in fact, an analyst assesses the joint relevance of parameters without additional model runs. This grants analysts full flexibility both in the choice of groups and aggregation levels.

We have implemented the algorithm on a financial model for the evaluation of a project financed parking lot. The model realistically reproduces the investment settings. 428 input parameters are processed by the model. The approach has allowed us to obtain the sensitivity measures of all the exogenous. We have then identified the key drivers and screen out non relevant parameters in a rigorous way, without a-priori selecting the relevant parameters. The utilization of a warning signal has also enabled us to test model correctness in an automated fashion.

For the interpretation of results we have not only considered the equity or shareholder perspective, synthesized in an equity NPV, but also the lender perspective on debt performance, synthesized in the minimum DSCR. We have discussed individual parameter ranking and notable discrepancies. For example, we have seen that while the cost of capital plays a relevant role on the NPV, it does not affect the minimum DSCR. Similarly, the income tax rate impacts the NPV more decidedly than the minimum DSCR. Conversely, leverage is one of the most significant parameters for lenders, but is not as significant for sponsors. The introduction of a comparison method based on Savage scores has enabled us to obtain quantitative measures of the ranking agreement/discrepancy. Results show that differences are attributable mostly to the least relevant factors.

We have further explored the ranking agreement by analyzing the importance of categories. Parameters have been first grouped into the six standard categories in investment project financial analysis. Results show that revenue assumptions are the drivers of the economic performance both from the sponsor and lender perspectives, followed by investment costs. Further breaking down the revenue assumption group in its subcategories, we have seen that tariffs are the key performance driver of the project. As far as the direction of change is concerned (insight (2) in Section 2), results have shown that the direction of change of the of NPV and the minimum DSCR was the same in response to changes in the groups. I.e., a group influencing NPV positively would also influence the minimum DSCR in the same way. Instead, individually, factors can affect the NPV and mDSCR in different ways.

We finally note the flexibility in assessing the combined effect of factors, responds to the need of decision makers to understand the influence of factors aggregated by categories with different level of details.

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