How Does Bankruptcy Punishment Impact on Renegotiable Debt Contracts?

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Abstract:

This research investigates how legal sanctions prevailing under bankruptcy impact on debt contracting and on investing decision, when companies may engage faulty management. Unlike most papers considering a passive behavior of the bank in case of default of the borrower, the creditor and the debtor actively trade off between private renegotiation and costly bankruptcy procedure.

The model focuses on three possible equilibriums. The derived propositions are linked to empirical findings from a French database on 240 distressed firms. The first equilibrium encompasses situations when the firms behave honestly (economic efficiency) and the bankruptcy costs are avoided through private renegotiation (legal efficiency): yet, the legislator cannot directly implement this equilibrium as it does not depend on the level of legal sanctions. A second equilibrium cover situations when the firms turn to the less profitable and riskiest project (economic inefficiency) and the default is still privately solved (legal efficiency): a minimal level of sanctions may prevent the occurrence of such equilibrium. Last, we consider mixed strategies on the investment policy (partial economic efficiency): in case of financial distress, two bargains prevail (pooling or separating) and costly bankruptcy may occur (legal inefficiency).

Simulations illustrate how the bank finally chooses between these equilibriums while the legal environment becomes more severe. For moderate values of legal sanctions, banks may accept a certain level of moral hazard from their debtors, expecting to take advantage of bankruptcy punishment. An increase of sanctions changes the story, as it incites the companies to respect more their commitments. But once the optimal equilibrium prevails, any additional increase of sanctions is worthless as the decision variables do not depend on the legal environment anymore. As a result, extreme severity is not needed to ensure both economic and legal efficiency. In addition, an increase of legal sanctions is likely to reduce the contractual interest rate, as the bank is more protected by the law. A noteworthy consequence is the debtors may benefit of increased severity. Last, we find a slight modification of the law may involve a drastic adjustment of financial variables and lead to financial instability.

Keywords: Corporate Bankruptcy, Credit Lending, Interest Rate, Moral Hazard, Legal Sanctions

JEL Classification: G33, D82, D21

Acknowledgements: we would like to thank, for their comments, support and precious advices: Bertrand Chopard, Nicolas Eber, Jean-Daniel Guigou, Nicolas Jonard, and Gwenaël Piaser. All remaining errors are ours.

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INTRODUCTION

Bankruptcy law has received considerable attention due to its implications regarding the financing and investing decisions of companies. Two complementary aspects of the "efficient bankruptcy" have been separately investigated. On the one hand, *ex-post* efficiency of bankruptcy law focuses on the maximization of value of distressed company by considering all the stakeholders (managers, creditors): default is considered as given. On the other hand, *ex-ante* efficiency analyses the effects of bankruptcy code on the incentives of all stakeholders.

The literature on *ex-post* efficiency of bankruptcy codes mainly discusses the tradeoff between rival ways of resolving financial distress: following the Coase theorem, Haugen and Senbet [1978] and [1988] prove the superiority of the market solution over the legal one, through a mechanism of internalization of bankruptcy costs. On the contrary, but following the same *ex-post* perspective, other authors discuss the advantages of implementing particular procedures to distressed firms (away from simple renegotiation): auctions and options (Bebchuk [1988] and [2000]), or procedures allowing deviations from the absolute priority rule (Jackson [1986], Baird and Picker [1991], Blazy and Chopard [2004]). Nevertheless, the major drawback that can be addressed to the *ex-post* view is that it ignores the impact of such procedures – whatever their design – on the strategies taking place before default. Turning to the literature on *ex-ante* efficiency of bankruptcy provides interesting views on how the legal environment influences managers' and creditors' behavior in the presence of information asymmetries (Aghion and Bolton [1992]; Berkovitch and *al.* [1998], Kolecek [2008]). However, it rarely provides an explicit explanation of the influence of bankruptcy law on the design of debt contracts. Namely, as former papers such as Cornelli and Felli [1997] show the influence of bankruptcy law on creditors' behavior in terms of monitoring firms or granting loans, bankruptcy law may also influence the design of debt contracts through the recovery process (Gorton and Kahn [2000], Jappelli, Pagano, and Bianco [2005]).

Our paper aims at filling the gap between the *ex-post* and *ex-ante* approaches to bankruptcy. We provide a theoretical framework for post-default renegotiation under asymmetric information, and link this framework to the pre-default investing and financing decisions. Our approach suggests that the design of bankruptcy code play a key role in the process, as it may impact on the cost of debt when debt contracts are renegotiable. In return, this impact indirectly affects the firms' investment policy. As a consequence, we can wonder to which extent the legislator can use the *ex-post* bankruptcy rules in order to improve the *ex-ante* behaviors and/or affect profits sharing.

When focusing on the fundamentals of bankruptcy codes, the literature isolates three major functions of the "Court solution". First, bankruptcy codes help in coordinating interests between diverse claimants: without any coordination between creditors, distressed firms are likely to be dismantled through an anarchic creditors' run, which eventually reduces the value of the firm. This common pool problem has been widely addressed by Bulow and Shoven [1978], Gertner and Scharfstein [1990], Asquith, Gertner and Scharfstein [1994], and more recently by Longhofer and Peters [2004]. Through specific legal mechanisms (stay of claims, specific voting procedure, and/or Court enforcement, *etc.*), the design of bankruptcy codes helps in solving the lack of coordination between the creditors.

Second, bankruptcy codes produce information, through the implementation of audit procedures, monitored – directly or not – by the Court. A similar issue is addressed by the literature on the theoretical justification of standard debt contracts (Townsend [1979], Gale and Hellwig [1985]): such contracts are efficient as they limit the occurrence of situations when the creditors have to check the actual value of the debtor's assets. Here, the costly state verification process takes place only when the debtor cannot repay its debt anymore, which is the most common triggering criterion of formal bankruptcy.

Third, bankruptcy codes are superior to the out-of-court solution, in the sense they help in assessing the assets and claim's value. By forcing or deviating from the absolute priority order (White [1989], Hart [2000]), by helping in the verification of claims, and/or in the distinction between anterior, posterior, junior, and senior claims, or by transferring the management from the previous directors to the creditors (Harris and Raviv [1991]),

bankruptcy codes settle specific rules which reduce uncertainty. In a sense, this third function of bankruptcy can be viewed as a mix of the two previous functions (coordination and information).

We focus here on a fourth function of bankruptcy, which is the sanction of faulty management. Indeed, this feature is the angular stone of the modern approach to bankruptcy: until the middle of the twentieth century, most of bankruptcy codes did not distinguish the firm's fate from the manager's one. Financial distress had to be punished and the punishment of decision makers was a consequence of the non-respect of previous financial commitments. This view has been evolving as most modern economies now admit that default may be due to bad luck or unfavorable environment. This perspective is fundamental: legal sanctions should apply to faulty managers only, whose bad or tricky behaviors increase the financial consequences of default. Following Bester [1985], one could argue that implementing personal guarantees on the manager's wealth reduces incentives to moral hazard. Indeed, personal collateralization is a good way of discriminating between good and bad risks. Yet, the systematic use of such collateral, by breaking limited liability, may lead to under-investment.

We rather focus on the role of legal sanctions, which have the advantage on personal guarantees to be enforceable each time moral hazard is discovered. Of course, this implies a costly state verification process, which is one of the fundamental functions of modern bankruptcy codes. For instance, in France, legal administrators have to engage a costly audit of the firm as soon as bankruptcy is triggered (*"période d'observation"*): since 1985 (Code n°85-98, 25th of January 1985, Title V, Art. 180 to 182), the Court can sanction managers if the administrator's report reveals faulty management. The "fault" covers asset substitution, tricky behavior, and, more generally, any action that might have worsened the financial situation of the firm. Sanctions are either criminal and/or pecuniary. The latter makes the manager pay for the firm's debt using his own personal wealth.

In this paper, we model a three stages lending relationship between a monopolistic bank[4] and a small firm, directed by a shareholder-manager. The bank proposes a contractual interest rate to the firm, which directly affects its probability of default[5]. The firm's manager-shareholder has initial incentives to substitute assets at the time of investment: once funds are leveraged, he can undertake a much riskier and slightly less profitable investment project, contrary to the one announced to the bank (this remains the manager's private information). In case of default, a bankruptcy procedure may be triggered off: a costly state verification process takes place and legal sanctions may apply against the manager, if it appears he previously performed moral hazard. Costly bankruptcy can be avoided yet, if the firm achieves a private agreement with the bank.

The structure of the paper is the following. Section 1 presents some empirical facts out of two samples of distressed firms (faulty managers and honest ones). Section 2 presents the general structure of the model. Section 3 computes the equilibriums and derives the related propositions. Finally, some simulations and results are discussed in section 4. The last section gathers our main results and concludes.

1. EMPIRICAL PUZZLE

The mechanism that mainly affects the agent's *ex-ante* strategies is the way default is *ex-post* resolved: default may lead to either private renegotiation, or formal bankruptcy. Provided bankruptcy costs are less than expected legal sanctions, one could argue faulty managers should *always* end up in bankruptcy (so that sanctions can apply), whereas honest ones should *always* escape bankruptcy through renegotiation. Actually, the tradeoff between both outcomes is much more complex as it depends on various features: the quality of the information at the time of the tradeoff, the internalization of bankruptcy costs through renegotiation, the number of competing claimants, and the length and complexity of bargaining between the creditor(s) and the debtor, (…).

[4] As shown in Section 1, 28% up to 55% of the French distressed SMEs rely on one banker only.
[5] Contrary to many other models, the probability of default is not constant here, but directly depends on the level of the interest rate, which we consider as a much more realistic description of the bankruptcy process. This feature directly stems from the way we model earnings, as a continuous random variable.

However, empirical observations confirm that choosing the way of resolving default is not a straightforward decision: we discuss here some summary statistics[6] from an original database we collected under the *S&P Risk Solution* supervision. Our sample is the French part of the European sample designed, built, and used by Davydenko and Franks [2007]. The data encompasses 240 small and medium French distressed firms on a period of time covering 20 years (1985-2005), whose debt exposure exceeds € 100 000. Loans were granted from 1984 to 2001. The "default" event follows the Basel 2 definition: a firm defaults as soon as delays on its financial commitments exceed 90 days. Data come from 5 major French commercial banks[7], and were gathered manually from their internal recovery unit. When the client defaults, several practical ways of resolving financial distress may apply: [1] some of these companies directly go to formal bankruptcy; [2] other firms undergo a workout (this one may be either a private renegotiation with the Bank or a "*Règlement Amiable*" procedure, monitored by a legal supervisor); [3] others face a mixture of solutions, in the sense that the initial negotiation process ends up with formal bankruptcy.

Our database gathers information on the origin of default (47 codes[8]), the way of resolving financial distress (private renegotiation, formal bankruptcy, or mixed solution), and the type of banking relationship. Other data are not discussed here but statistics are available on request[9]. Based on this information, Table I provides several statistics on [1] the presence / absence of faulty management prior to default[10], [2] the quality of the information (proxied by the length of the credit relationship, and the company's rating just before it defaults), [3] the strength of the banking relationship (is the bank the main creditor?), [4] the year of the default.

[6] Additional statistics are available on request.
[7] Banks are : Crédit Agricole (CA), Crédit Commercial de France (CCF), Union de Banques à Paris (UBP), Société Marseillaise de Crédit (SMC), Banque Hervet.
[8] We use the literal description of the origin of default: this compulsory information is available in the administrator's report. This information is coded into 47 causes covering the following areas: outlets, strategy, production, finance, management, accident, and external environment. We highlight here all the causes reflecting faulty management, that is: *asset substitution, voluntary excessive risk taking, private abuse of the company's assets, tricky behavior and swindle, accounts falsification and financial fraud*. According to the French legislation (Code n°85-98, 25th Jan. 1985, Title V, Art. 180-182), all these actions are subject to pecuniary sanctions ("*action en comblement de passif*, "*extension de procedure*").
[9] Additional data are: recovery rates, interest rates and collaterals (firm and credit line levels).
[10] The information on the origin of default is usually disclosed after certain a period of time, either during bankruptcy, or at the end of renegotiation. It is unlikely to be available at the beginning of the process.

Remarkable features arise from Table I. First, the percentage of faulty companies entering directly to bankruptcy is strongly below expectations (18.5%). We argue that one reason for this relies in the relative poor information the bank owns at the time of default: compared to the other outcomes, reliable ratings are less often available (23.5%, against 17.5% for renegotiations). This may be explained by the relative youth of the credit relationship, inferior to 2 years in 25.3% of cases, compared to 18.4% and 12.5% for the other outcomes). As a consequence, bankruptcy may be triggered even if the firm did not engage any faulty management. This is more likely to happen either, when the bank is suspicious or lacks information on his/her debtor, or when honest managers consider renegotiation as a too expensive way of resolving default.

Table 1. Faulty Management and Default Resolution

Sample : 240 French distressed firms (1985-2005, 5 banks)	**Direct Bankruptcy**	**Pure Renegotiation**	**Renegotiation and Bankruptcy**	**Total of Defaults**	
				%	**Number**
Faulty management (subject to legal sanctions) ? ** ***(asset subtitution, private benefits, account falsification, voluntary excessive risk taking…)***					
No	81,5%	90,0%	68,4%	80,8%	194
Yes	18,5%	10,0%	31,6%	19,2%	46
Last firm's rating ***("Banque de France" rating, as collected by the bank itself)***					
Unknown	23,5%	17,5%	23,7%	22,5%	54
Group: "Risky"	38,9%	47,5%	36,8%	40,0%	96
Group: "Safe"	37,7%	35,0%	39,5%	37,5%	90
Length of the credit relationship					
< 2 years	25,3%	12,5%	18,4%	22,1%	53
2-5 years	30,3%	32,5%	42,1%	32,5%	78
5-10 years	27,2%	32,5%	15,8%	26,3%	63
10 years +	17,3%	22,5%	23,7%	19,2%	46
Is the bank the main creditor?**					
Info. not available	11,1%	12,5%	2,6%	10,0%	24
No	38,9%	60,0%	42,1%	42,9%	103
Yes	50,0%	27,5%	55,3%	47,1%	113
Year of default *** ***("default" is: one credit line at least is classified as "doubtful")***					
1985-1996	34,6%	10,0%	39,5%	31,3%	75
1996-2000	32,1%	35,0%	47,4%	35,0%	84
2000-2005	33,3%	55,0%	13,2%	33,8%	81

*Variables whose Chi-2 statistic is significant at the 1%, 5% and 10% levels are indicated by ***, **, * respectively.*

Data source : Recovery units of Crédit Agricole, Crédit Commercial de France, Union des Banques à Paris, Société Marseillaise de Crédit, and Banque Hervet (default files were randomly collected for the years 1985-2005). This dataset is the French subsample of the European sample used by Franks and Davydenko (2007). Data are available on request.

The second feature is more in accordance with expectations: in 90% of cases, honest firms resolve financial distress through private renegotiation. This seems logical, first, as the bank's information is relatively good (17.5% of ratings only are unknown, and the length of the credit relation is superior to 2 years in 87.5% of cases), and second, as the bank is less often the main creditor (27.5% of cases): the bank has strong incentives to avoid bankruptcy and privately renegotiate with the firm.

The third feature is contrary to expectations and encompasses more complex defaults, which are resolved through renegotiation *leading to* bankruptcy. These defaults show the highest levels of faulty management (31.6%). In addition, these cases are characterized by quite heterogeneous information: on one hand, 23.7% of firms have no rating at all, but on the other hand, the bulk of them (42.1%) have a recent existing relation with the bank (between 2 and 5 years). We suggest the bank has incentives to initially renegotiate, as the renegotiation process is a good way of completing his/her initial information. If this additional piece of information indicates faulty management, the bank is inclined to trigger bankruptcy, in order to take advantage of legal sanctions.

These empirical features suggest that the tradeoff between renegotiation and formal bankruptcy heavily depend on the quality of the information at the time of the default, and on the legal environment of bankruptcy. We argue that the consequences of this complex tradeoff does not affect *ex-post* strategies only, but may impact on the *ex-ante* decisions taking place before any default. More precisely, the design of bankruptcy law is likely to affect the post-default bargaining, and in return, the pre-default investing and financing strategies. In the next section, we provide a model of such an environment, and analyze the impact of bankruptcy punishment on the investment strategies and on the contractual interest rate, as a component of the debt contract design.

2. THE MODEL: HYPOTHESES AND GENERAL STRUCTURE

We describe the general structure of the model and the basic hypotheses. The model analyses a single lending relationship between a small firm managed by a shareholder-manager (named "the Firm", in the rest of the paper) and a monopolistic bank (named "the Bank") who decides the level of the contractual interest rate. As described below, the Firm may perform asset substitution at the time of investment, after having been financed by the Bank. Such a moral hazard behavior is sanctioned by the Law in case of default leading to formal bankruptcy. Such a costly way of resolving financial distress can be avoided yet, if both parties achieve an informal agreement and turn to private renegotiation. In the rest of the paper, we adopt a specific distinction between "economic efficiency" and "legal efficiency" (see definitions D1 and D2).

Definition D1. *Economic efficiency*. A firm's strategy is said to be economically efficient if and only if it leads to the project which has the maximum expected value compared to any other rival project. ¦

Definition D2. *Legal efficiency*. A way of resolving default is said to be legally efficient as soon as the chosen solution maximizes the value of the firm or, equivalently here, avoids costs related to the resolution of financial distress (as mentioned by Haugen and Senbet [1978 and 1988], bankruptcy costs reduce the overall value of the firm's project, even if they help in revealing public information). ¦

The model relies on five sets of hypotheses: these cover [H1] the lending relationship under risk neutrality; [H2] the firms' initial incentive to asset substitution; [H3] the renegotiation process taking place after default; [H4] players' rationality, strategies, and equilibriums; [H5] the Bayesian revision process under mixed strategies; [H6] the reservation amounts.

Hypotheses H1. *The lending relationship under risk neutrality*

All agents are risk neutral. At time (t), a monopolistic creditor[11] (the Bank) trades with a debtor (the Firm) for a loan amount \$1, aimed at financing an investment project. The

[11] This assumption is in accordance with the observed imperfect competition on banking markets (De Bandt and Davis, [2000]). It fits well intermediated economies composed of numerous SMEs heavily financed by a main bank, such as in Europe (France, Belgium, Italy, Spain or Germany). Section 1 confirms this view.

project is fully leveraged and the Bank is the sole claimant. The manager – who has a personal specific wealth (w) for an amount of \$1 (his/her house) – owns 100% of the Firm, so that we can indifferently talk of "manager" or "shareholder". The project's earnings equal \$(1+x) at time (t+2) (x takes non-strictly positive values and is the realization of a continuous random variable X). At time (t), the Bank defines a unique debt contract, characterized by the interest rate (i). Repayment is scheduled for time (t+2), so the firm has to repay \$(1+i). Under H1, the worst case for the bank occurs when x equals zero, so that all interests (i) are completely lost[12].

Hypotheses H2. *The Firm's initial incentives to asset substitution*

As Gorton and Kahn [2000], we consider moral hazard stems from a risk of asset substitution by the Firm[13]. At time (t), when the debt contract is effectively signed, the Firm declares to the Bank the \$1 amount is to be invested into a project (j) starting at date (t+1). In real, at this time, the shareholder-manager may swap projects, turning to another (j'). The action of swapping projects remains the Firm's private information. Yet, the uninformed Bank knows moral hazard is likely to happen. This standard asset substitution issue happens as soon as the alternative project leads to a strictly[14] higher level of expected profits. Compared to (j), project (j') is riskier and less profitable, or equivalently, less economically efficient. We denote (X|J) the random earnings conditioned by any generic project (J, $\forall J \in (j,j')$). Both variables (X|j) and (X|j') follow the same probability density function, but with differences in the two first moments:

$$E(X|j) > E(X|j') \quad (1a)$$

$$\sigma_j < \sigma_{j'} \quad (1b)$$

Where E(X|J) and σ_J ($\forall J \in (j,j')$) are respectively the expectation and the standard deviation operators.

[12] The bank recovers the capital part only. That assumption is made for simplification purpose. Another presentation – where default affects not only interests but also the principal share of the debt – is possible: this would not affect our propositions, but lead to a more complex modeling. Indeed, the whole model can be reproduced through a variable change on variables (i) and (x) ($x \in [-1,\infty)$).

[13] In their paper, Gorton and Kahn [2000] add a second source of moral hazard on the bank's side.

[14] In case both projects lead to identical levels of expected profit, we suppose the firm respects its commitments, and chooses project (j). This assumption is not only made for simplification purpose: when two projects have the same expected value, it is natural to turn to the project with the minimum standard deviation, which is the case for project (j), less risky than project (j').

Inequality (1a) shows project (j') is sub-optimal compared to project (j'), leading to lower expected earnings. This gives the Legislator a rationale to punish moral hazard, due to its negative effect on global welfare. Inequality (1b) is necessary so that the manager has initial incentives to perform asset substitution. More precisely, the debtor's risk inclination stems from the limited responsibility which is contained in any standard debt contract: from the 3rd theorem of Stiglitz and Weiss [1981], we know that any increase of risk – preserving the level of profitability – leads to a rise (respectively reduction) of the debtor's (resp. creditor's) expected profit. In addition here, we assume that asset substitution involves a slight reduction of profitability: thus, a technical condition on E(X|j') is needed so that the firm has initial incentives to turn to a riskier but less profitable project (in other words, moral hazard should be possible in such a context only if the earnings expectation attached to project (j') is not *too small*). Inequality (2) reflects such a condition on E(X|j'): there is initial incentive to asset substitution from project (j) to project (j') if and only if the following condition[15] prevails:

$$E(X|j') > E_0^i(X|j') + E_i^{\infty}(X|j) - i \cdot \left(F_{X|j'}(i) - F_{X|j}(i)\right) \qquad (2)$$

Where: $E_a^b(X|J)$ is the truncated expectation operator on the interval [a,b], for any continuous random variable conditional to project J ((X|J), $\forall J \in (j,j')$) (i.e. the integral of expectation, restricted to interval [a,b]). $F_{X|J}(.)$ is the distribution function for (X|J).

Proof [Inequality (2)] *See Appendix A1.* ¦

Inequality (2) provides the initial condition which should hold so that the implementation of legal sanctions is justified in order to reduce the incentives to asset substitution. Yet, once legal sanctions are implemented, the rules of the game change actually, and condition (2) is not needed anymore: the players compute their new programs based on these new rules (see the game structure described in Figure 1).

[15] Notice this inequality involves the level of (i): in other terms, the bank can contractually define an interest rate giving (or not) incentives to behave honestly. Which is of high interest here, is the situation where – without any legal context – inequality (2) applies (i.e. the standard debt contract leads to asset substitution): so the question becomes, does the introduction of legal sanctions reduces (or not) this risk? This shall be illustrated by Section 4.

Hypotheses H3. *The renegotiation process after default: bankruptcy and legal sanctions*

Default is fully observable[16] and depends on the level of the interest rate[17]: when it occurs ($x < i$), and whatever its initial choice ((j) or (j')), the Firm may intend to avoid bankruptcy, offering a renegotiation amount (R) to the Bank[18]. Of course, Tricky Firms (those having chosen project (j')) have incentives to offer a higher amount to the Bank so that they may avoid legal sanctions prevailing under formal bankruptcy. We consider a "one shot" renegotiation process: the Bank accepts or declines the firm's offer[19]. In case (R) is accepted, the story ends, and the Firm's debt is forgiven. Otherwise, a bankruptcy procedure is triggered off: bankruptcy costs (c) are paid out of the Firm's assets ($\$1+x$)[20], so that the Court can audit the Firm and discover any previous asset substitution. Such a production of information is one of the basic functions of bankruptcy: as mentioned by Webb [1987], bankruptcy costs are basically revelation costs. They are paid first and foremost in comparison to other payments. The amount (c) is expressed in percent of the size of loan ($1), as a proxy of the firm's size. In case the wrong project (j') was undertaken at time (t+1), the manager-shareholder's wealth (w, normalized to 1) is subject to legal sanctions (s). The Law settles their amount as a percentage of the manager's personal wealth ($w = \$1$, $s \in [c,1]$). In addition, (s) is higher than (c), so that it is worthwhile to trigger a costly bankruptcy. It has to be stressed these are financial sanctions only (no jail or interdiction to manage other firms, here): the manager-shareholder is sanctioned by breaking in some extent limited responsibility.

[16] Some authors consider companies may hide financial distress (see the recent paper of Kolecek [2008]). While interesting, we consider this view as rather unrealistic, as [a] accountancy is subject to regular compulsory verification procedures, and [b] banks have a direct and permanent access to their customer's cash account: this is especially true for SMEs financed by one main bank.

[17] Some papers assume the probability of default remains constant whatever the level of interest rate. While interesting, we prefer to follow the Stiglitz and Weiss [1981]'s approach, so that the probability of default increases with the cost of capital.

[18] We restrict ourselves to a decrease of the claim value under renegotiation. Indeed, increases in the value of the creditor's claims are much more observed for big failures, more specifically in the United-States (Chapter 11): see for instance, James [1995], Asquith, Gertner and Scharfstein [1994]. Most of the European studies on recovery rates (Davydenko and Franks [2007], Armour, Hsu, and Walters [2006]) estimate the bank's recovery rates (inside or outside bankruptcy) to be far less than one, so that banks rather decrease the value of their claims when their debtors default.

[19] The transaction costs of renegotiation are normalized to zero (see Gilson [1997] for a study of the out-of-court transaction costs). There is no room for counter-proposals from the Bank. This hypothesis leads to simple properties which have the advantage to reflect the short delays characterizing the bargaining period through most of the European countries. For instance, under the French code, the bankruptcy procedure must be triggered within 15 days after default.

[20] Remember there is not initial contribution from shareholders.

Hypotheses H4. *Players' rationality, strategies, and equilibriums*

The rules of the game are common knowledge and all agents behave in an absolute rational manner. When choosing between rival projects, the Firm may adopt either pure strategies or mixed strategies (in case of indifference between actions)[21]. We derive four[22] equilibriums:

[a] one pure equilibrium (E1): the firm undertakes the contractual project (j) (time t+1) and an informal agreement is reached under default (time t+2);

[b] one pure equilibrium (E2): the firm undertakes the rival project (j') (time t+1) and an informal agreement is reached under default (time t+2);

[c] two mixed equilibriums (E3a and E3b): the firm undertakes project (j) with probability (p), and (depending on the bank's beliefs at time t+2[23]), the bargaining process leads either to a pooling equilibrium, where private agreements prevail (E3a), or to a separating equilibrium, where Honest Firms trigger bankruptcy, and Tricky Firms privately renegotiate, at the highest cost (E3b).

Hypotheses H5. *The Bayesian revision process under mixed strategies*

Under mixed strategies (see H4), the bank has priors on the firm's choice: (p) is the *a priori* probability of undertaking project (j). Such prior is public information. Next, the Firm successively conveys two signals to the bank. These signals are: [a] the realized earnings, $p \to p(j|x)$; [b] the renegotiation amount (R) proposed to the Bank under default, $p(j|x) \to p(j|x,R)$. In other words, the observed earnings and the firm's willingness to renegotiate can be used as signals by the bank to update its beliefs on the project choice (using the Bayes rule). After (x) is realized and before (R) is disclosed, the Bank computes the revised probability, given by equation (3).

$$p(j|x) = \frac{f(x|j)p}{f(x|j)p + f(x|j')(1-p)} \qquad (3)$$

[21] We focus on mixed strategies taking place at time (t+1) for this reason: at time (t), the Bank may define an interest rate inciting the firm to play mixed strategies at (t+1), providing then a higher expected profit for the bank. This specificity of the model cannot appear if we do not study the occurrence of mixed strategies at time (t+1). This is the reason why both pure and mixed equilibriums are described here.

[22] In this paper, we do not study "double-mixed" equilibriums, where [a] the firm chooses (j) with probability (p) and (j') with probability (1-p), and [b] the bank accepts the renegotiation amount (R) with probability q(x) (private agreement) and rejects it with probability 1-q(x) (formal bankruptcy).

[23] These revised beliefs depend on the realized value (x), which a inferior to (i) under default.

Hypotheses H6. *The reservation profits*

All agents have access to alternative contracts guaranteeing reservation profits. On one side, we assume the shareholder-managers' reservation profit is null: i.e. if the debt contract offered by the Bank is not signed, the manager-shareholder simply closes the business. On the other side, in case the debt contract is not signed with the Firm, the Bank allocates the \$1 amount to a risk-free activity (we assume the risk-free rate equals zero).

Figure 1 displays the general structure of the model. The decisions of agents are successively made through a three time sequence model. At time (t), the bank defines the contractual level of interest rate (i). At time (t+1), the firm chooses the project to undertake ((j) or (j')). The project leads to random earnings (x) at time (t+2): all parties observe the success or the failure of the project. In case of success ($x \geq i$), all payments are made and the game ends: the Firm's earnings \$(1+x) are the basis of a full payment to the bank \$(1+i), so we have:

$$\text{Gain of the Firm (success):} \quad x - i \tag{4a}$$

$$\text{Gain of the Bank (success):} \quad 1+i \tag{4b}$$

Contrary to the success event, default ($x < i$) is complicated by the possible firm's endeavor to renegotiate the debt contract: the Firm may try to avoid bankruptcy by proposing a renegotiation amount (R) to the Bank. An agreement is reached when each party earns as much as [24] or more under private renegotiation than under formal bankruptcy. This tradeoff stems from a comparison between the expected gains under each rival solution[25]: this leads to definition D3.

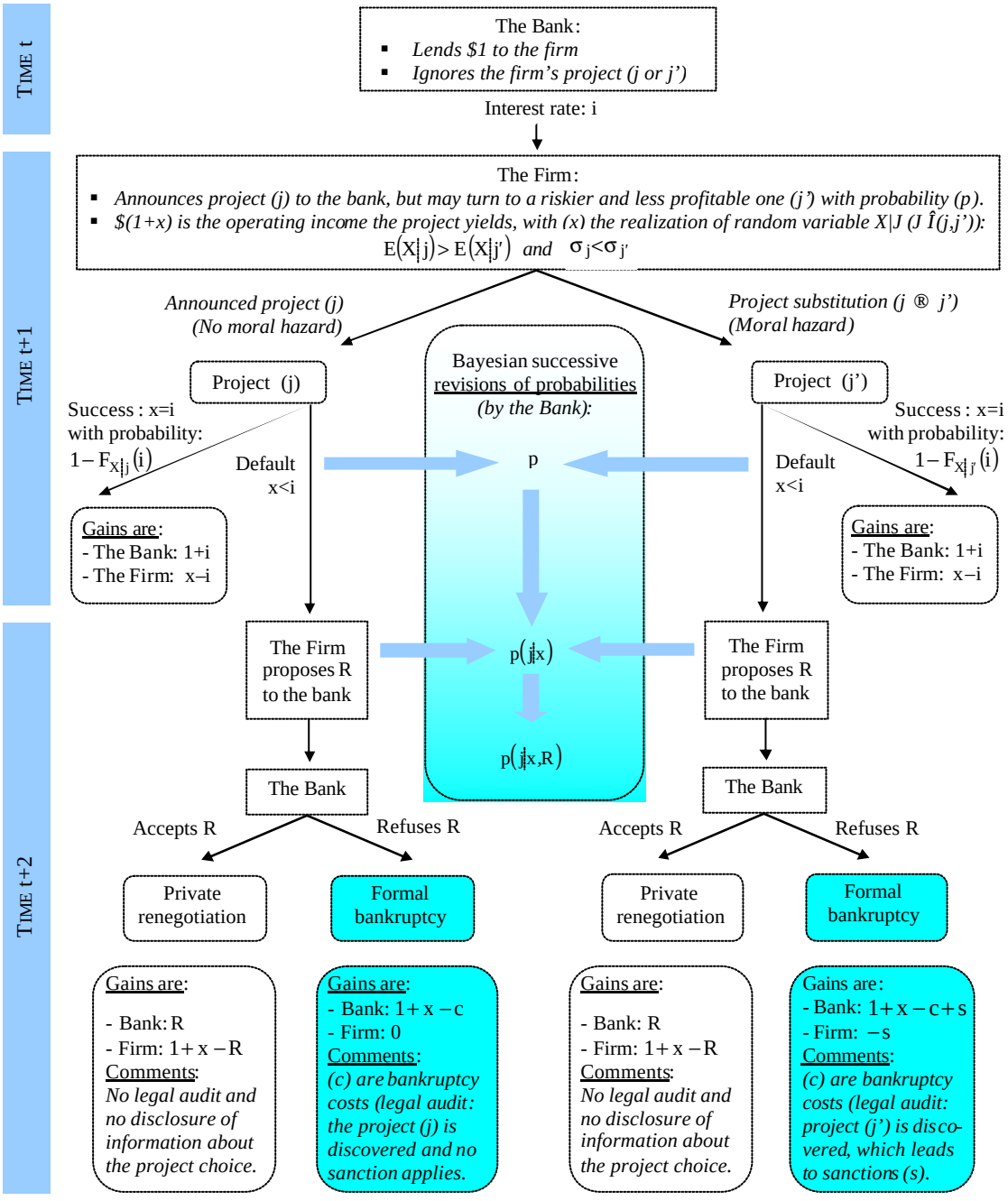
Definition D3. *Acceptance thresholds*

"Acceptance thresholds" (denoted AT) are the minimum levels of (R) each party wants (respectively accepts) to receive (respectively grant) outside bankruptcy. We respectively denote as "BankAT(x)", "jAT(x)", and "j'AT(x)" the Bank's, the Honest Firm's, and the Tricky Firm's thresholds. ¦

[24] We suppose that all parties privately renegotiates when their gains under formal bankruptcy or under private renegotiation happen to be equal.

[25] Thus, the legal environment concerning legal sanctions (s) exerts an impact on the resolution of default. Consequently, decisions made at times (t) and (t+1) will be changed.

Figure 1. The General Structure of the Model



3. THE RESOLUTION OF THE MODEL

A first set of propositions is obtained through the identification of equilibriums: the current section discusses these propositions (Section 4 shall illustrate which equilibrium can prevail once the Bank settled an incentive debt contract at the beginning of the game (time t)). Section 3.1 focuses on equilibrium E1, under which the firm plays (j) and a private agreement prevails in case of default. Section 3.2 deals with equilibrium E2, under which the firm plays (j') and a private agreement prevails. Section 3.3 focuses on both equilibriums E3a and E3b, which apply depending on the realized value (x): under E3a, the firm plays (j) with probability (p) and the bargaining process is pooling (private agreement); under E3b, plays (j) with probability (p) and the bargaining process separates both investing strategies (projects (j) and (j') respectively lead to formal bankruptcy and private agreement).

3.1. Equilibrium E1: the Firm purely chooses project (j)

Under equilibrium E1, all firms respect their contractual commitments and choose project (j): all are "honest" players. Section A describes the renegotiation process taking place in case of default. The associated expected gains are incorporated by the Bank, at the time he/she grants credit (section B).

A. The renegotiation process (equilibrium E1)

In case of default, Honest Firms prefer to privately renegotiate if their earnings \$(1+x) net of the amount granted to the Bank (R) are equal or greater than the net amount the manager-shareholder recovers under formal bankruptcy: i.e. simply \$0 here. The acceptance threshold for Honest Firms, (named "jAT(x)": see definition D3) is given by relation (5): Honest Firms prefer private renegotiation to bankruptcy if and only if:

$$1+x-R \geq 0 \Leftrightarrow R \leq \underbrace{1+x}_{jAT(x)} \qquad (5)$$

At equilibrium E1, the Bank prefers to privately renegotiate if the proposed amount (R) equals or exceeds the expected gains if bankruptcy is triggered: under bankruptcy, the recovered amount equals earnings \$(1+x) net of bankruptcy costs ($c \times \$1$) (notice that , as

all firms choose project (j) at equilibrium E1, there is no room for any legal sanction). Thus, the Bank prefers private renegotiation if and only if condition (6) prevails[26]:

$$R \geq \underbrace{1+x-c}_{\text{BankAT(x)}} \tag{6}$$

As the Bank's threshold ("BankAT(x)") is inferior to the Firm's one ("jAT(x)"), an agreement is always reachable, whilst all Firms propose the minimum amount needed to ensure private renegotiation: bankruptcy costs are fully internalized, as predicted by Haugen and Senbet [1978] and [1988], so that:

$$R^{*} = 1+x-c \tag{7}$$

This amount affects the expected gains under default: using the truncated expectation operator, the expected profits are given by equations (8a) and (8b). We respectively denote $E(\Pi|i,j)$ and $E(\Pi^{B}|i,j)$ the Firm's and the Bank's expected profits, when the contractual interest rate equals (i) and project (j) is chosen.

$$\begin{aligned} E(\Pi|i,j) &= c\cdot\int_{0}^{i} f(x|j)dx + \int_{i}^{\infty}(x-i)f(x|j)dx \\ &= (c+i)\cdot F_{X|j}(i) + E_{i}^{\infty}(X|j) - i \end{aligned} \tag{8a}$$

$$\begin{aligned} E(\Pi^{B}|i,j) &= \int_{0}^{i}(1+x-c)f(x|j)dx + \int_{i}^{\infty}(1+i)f(x|j)dx \\ &= 1+i-(c+i)\cdot F_{X|j}(i) + E_{0}^{i}(X|j) \end{aligned} \tag{8b}$$

When profitable ($x \geq i$), the Firm recovers earnings (1+x) net of interest charges (1+i). When financially distressed, at equilibrium E1, the bargaining process leads to a private agreement: following equation (7), bankruptcy costs are fully internalized and the firm recovers (c), whereas the bank receives R^{*}. To be stable, equilibrium E1 must respect condition (9) ($\Leftrightarrow$ (9a) or (9b)): if the expected gains of project (j) are higher than the ones associated to project (j'), no firm has any incentive to deviate, switching from (j) to (j')[27].

[26] In case legal sanctions (s) are very high, this can lead to an expected recovery rate superior to 100% for the bank. This in not an issue, considering sanctions as dissuading tools only: their purpose is to give the right incentives to the firms – even if this is paid in disproportionate proportions by faulty managers.

[27] Following the Nash approach, this deviation takes place while the Bank's beliefs and strategy are given. Suppose the firm deviates from (j) to (j'), the Bank still believes that project (j) was chosen. Thus, if financial distress happens, the Tricky Firm can renegotiate the repayment at the same advantageous conditions than for Honest Firms.

$$E(\Pi|i,j) > E(\Pi|i,j')$$

$$\Leftrightarrow c\cdot\int_0^i f(x|j)dx + \int_i^\infty (x-i)\cdot f(x|j)dx > c\cdot\int_0^i f(x|j')dx + \int_i^\infty (x-i)\cdot f(x|j')dx \quad (9)$$

$$\Leftrightarrow E_i^\infty(X|j) + (c+i)\cdot F_{X|j}(i) > E_i^\infty(X|j') + (c+i)\cdot F_{X|j'}(i)$$

If $F_{X|j'}(i) > F_{X|j}(i)$: $\quad c < \dfrac{E_i^\infty(X|j) - E_i^\infty(X|j')}{F_{X|j'}(i) - F_{X|j}(i)} - i \quad (9a)$

If $F_{X|j'}(i) < F_{X|j}(i)$: $\quad c > \dfrac{E_i^\infty(X|j) - E_i^\infty(X|j')}{F_{X|j'}(i) - F_{X|j}(i)} - i \quad (9b)$

Inequalities (9a) and (9b) lead to lemma 1 and propositions 1.1 and 1.2. Lemma 1 shows that the stability constraints of equilibrium E1 depend on a peculiar relation between the levels of interest rate and of bankruptcy cost. Proposition 1.1 discusses the nature of equilibrium E1, which is economically *and* legally efficient, as initially suggested by Haugen and Senbet [1978] and [1988]. Proposition 1.2 shows that the occurrence of such equilibrium is independent from legal severity, and only depends on the Bank's behavior, so that the legislator cannot *directly* implement the best equilibrium: his/her action may be restricted to the avoidance of sub-optimal equilibriums, as shown later in the paper.

Lemma 1. *Under H1 to H4, the economically efficient equilibrium E1 is stable, provided conditions on bankruptcy costs (9a) and (9b) prevail:*

[1] *When the contractual interest rate is « low » (i.e.* $F_{X|j'}(i) > F_{X|j}(i)$ *under H2), the legal environment should not be too costly: i.e. bankuptcy costs should be less than the threshold*[28] *given by relation (9a).* This proposition is due to the fact that, under E1, bankruptcy costs are fully internalized by the Firm (i.e. they are additional gains, thanks to the renegotiation process). Given that – when (i) is low – the probability of default with project (j'), $F_{X|j'}(i)$, is higher than with project (j), $F_{X|j}(i)$, bankruptcy costs should not be too high so that swapping assets is not attractive enough.

[2] *For symmetrical reasons, when the contractual interest rate is « high » (i.e.* $F_{X|j'}(i) < F_{X|j}(i)$ *under H2), equilibrium E1 applies, provided the bankruptcy process is*

[28] Notice this threshold is always positive, whatever the level of interest rate.

relatively costly: i.e. bankuptcy costs should exceed the threshold given by relation (9b). Indeed, as the probability of default under project (j) is higher than with project (j') when (i) is high, bankruptcy costs have to be sufficiently important, so that the Firms accepts a higher probability of default, staying with project (j).

From lemma 1, we derive propositions 1.1 and 1.2.

Proposition 1.1. Under H1 to H4, when equilibrium E1 applies (no firm undertakes tricky projects), all defaults lead to private agreements, so that the bankruptcy procedure is never triggered off: thus, equilibrium E1 is not only economically efficient, but it ensures legal efficiency too. This result is close to the Haugen and Senbet's [1978] and [1988] predictions: when the firms purely respect their commitments, the ability of internalizing bankruptcy costs through renegotiation incites both parties to avoid bankruptcy. The question is: will this result hold when turning to other equilibriums?

Proposition 1.2. Under H1 to H4, the stability of equilibrium E1 does not depend on the level of legal sanctions: this directly stems from conditions (9a) and (9b). Contrary to bankruptcy costs, which can be considered as exogenous, legal sanctions cannot be used by the legislator to implement equilibrium E1. Thus, banks only decide if equilibrium E1 should prevail. The question is: while powerless regarding the implementation of the best equilibrium, can the legislator use sanctions to avoid other sub-optimal equilibriums?

B. The debt contract's design (equilibrium E1)

All firms stay with project (j) under equilibrium E1. At time (t), the Bank designs a debt contract compatible with this equilibrium: condition (9) ($\Leftrightarrow$ (9a) or (9b)) is the corresponding incitation constraint. Besides, under H6, two participation constraints prevail in the Bank's program: the contractual interest rate must lead to expected profits equal or greater than the respective reservation profits. We denote as i_1^*, the optimal interest rate related to equilibrium E1, coming from the resolution of the Bank's program (10) (where the expected profits are given by (8a) and (8b)) :

$$
\begin{aligned}
& i_1^* = \underset{i}{arg\,max}\, E\left(\Pi^B \middle| i, j\right) \\
& \text{u.c. Firms's participation}: E\left(\Pi \middle| i, j\right) \geq 0 \\
& \qquad \text{Bank's participation}: E\left(\Pi^B \middle| i, j\right) \geq 1 \\
& \qquad \text{Incitation to (E1)}: E_i^{\infty}\left(X \middle| j\right) + (c+i)\cdot F_{X|j}(i) > E_i^{\infty}\left(X \middle| j'\right) + (c+i)\cdot F_{X|j'}(i)
\end{aligned}
\tag{10}
$$

The optimal interest rate (i_1^*) is compatible with both economic and legal efficiencies (see proposition 1.1). Yet, the prevalence of E1, through the contractual implementation of (i_1^*) is not guaranteed at all: other equilibriums may prevail, so that different kinds of inefficiency appear. Sections 3.2 and 3.3 focus on such sub-optimal equilibriums[29].

3.2. Equilibrium E2: the Firm purely chooses project (j')

Under equilibrium E2, all firms perform asset substitution: "Tricky Firms" compute a similar tradeoff as for equilibrium E1, but now their manager-shareholder has to pay $(s) under bankruptcy. As for the previous section, the post-default renegotiation process and the Bank's program are described respectively in sections A and B.

A. The renegotiation process (equilibrium E2)

The acceptance threshold for these firms (named "j'AT(x)": see definition D3) is given by relation (11a). Under equilibrium E2, the bank knows it recovers legal sanctions if bankruptcy is triggered off. As a consequence, his/her acceptance threshold ("BankAT(x)") is superior to the one that prevailed under E1 (see relation (11b)).

$$
1 + x - R \geq -s \Leftrightarrow R \leq \underbrace{1 + x + s}_{j'AT(x)} \qquad \forall s \in [c, 1] \tag{11a}
$$

$$
R \geq \underbrace{1 + x - c + s}_{BankAT(x)} \qquad \forall s \in [c, 1] \tag{11b}
$$

As the Firm's threshold is less than the Bank's one, a private agreement is always reachable under equilibrium E2 (as for E1): all Firms propose the minimum acceptable amount (that is: 1+x–c+s), and bankruptcy costs are fully internalized: as predicted by

[29] Section 4 provides several simulations where – depending on the level of legal sanctions (s) – the Bank prefers sub-optimal equilibriums to E1.

Haugen and Senbet [1978] and [1988], the renegotiation process ensures legal efficiency. The Firm's and the Bank's expected profits are respectively given by equations (12a) and (12b): when profitable ($x \geq i$), the Firm and the Bank recover identical earnings to equilibrium E1. When financially distressed, the bargaining process leads to a private agreement: the Firm's gains are negative (c–s), and the bank receives R^* (=1+x–c+s).

$$\begin{aligned} E(\Pi|i,j') &= (c-s)\cdot\int_0^i f(x|j')dx + \int_i^\infty (x-i)f(x|j')dx \\ &= (c-s+i)\cdot F_{X|j'}(i) + E_i^\infty(X|j') - i \end{aligned} \tag{12a}$$

$$\begin{aligned} E(\Pi^B|i,j') &= \int_0^i (1+x-c+s)f(x|j')dx + \int_i^\infty (1+i)f(x|j')dx \\ &= 1+i-(c-s+i)\cdot F_{X|j'}(i) + E_0^i(X|j') \end{aligned} \tag{12b}$$

Condition (13) must prevail so that E2 is a stable equilibrium: The firm keeps choosing (j'), provided the expected gains are higher with this project (left-hand side of (13)) than with the rival one (right-hand side of (13)). Notice the right-hand side of condition (13) shows null gains under default: indeed, if the firm deviates from (j') to (j) (with no change of the Bank's beliefs[30]), the Firm directly triggers bankruptcy, without any prior proposal, as the private agreement is too expensive ($c-s \leq 0$). Condition (13) follows:

$$\begin{aligned} & E(\Pi|i,j') > E(\Pi|i,j) \\ \Leftrightarrow\ & (c-s)\cdot\int_0^i f(x|j')dx + \int_i^\infty (x-i)\cdot f(x|j')dx > \int_i^\infty (x-i)\cdot f(x|j)dx \\ \Leftrightarrow\ & s < f_1(i) \quad \text{with} \quad f_1(i) = c + \frac{E_i^\infty(X|j') - E_i^\infty(X|j)}{F_{X|j'}(i)} + i\cdot\left(1 - \frac{F_{X|j}(i)}{F_{X|j'}(i)}\right) \end{aligned} \tag{13}$$

Condition (13) leads to lemma 2 and propositions 2.1 and 2.2. Lemma 2 discusses the condition and shows the stability of equilibrium E2 partially depends on the legal environment (s). Again, this dependence is linked to the initial level of interest rate. Proposition 2.1 discusses the efficiency of equilibrium E2. Proposition 2.2 is derived from lemma 2 and highlights the fact the role of the legislator is indirect only, as it depends on the *ex-ante* financial decisions.

[30] Under equilibrium E2, the bank believes that all firms perform moral hazard.

Lemma 2. Under H1 to H4, equilibrium E2 is stable, provided condition (13) prevails. As function $f_1(i)$ *is lower (respectively higher) than (c) for extreme (resp. central) values of (i), we have:*

[1] *When the level of the contractual interest rate is extreme (either very low or very high), condition (13) is impossible (as* $f_1(i)$ *is lower than (c), and legal sanctions cannot be less than bankruptcy costs, under H3): thus, the inefficient equilibrium E2 never applies, whatever the level of legal sanctions, provided they exceed at least the bankruptcy costs*[31] . This proposition can be explained as follows: when (i) is extremely low, the probability of default is near zero, so that substituting projects is not attractive enough as is reduces the expected gains, as $E(X|j') < E(X|j)$. For extremely high values of (i), the event of default is nearly certain, and the firm expects it has good chances to pay (c–s) through renegotiation: again, but for different reasons, E2 does not prevail.

[2] *When the contractual interest rate takes central values, condition (13) may apply, depending on the level of legal sanctions (s): an increase of these may prevent the inefficient E2 equilibrium to apply.* Indeed, for moderate values of (i), the firm's rewards stemming from the riskier projects may overcompensate the risk of paying (c–s) through renegotiation. To avoid this, the legislator should increase legal sanctions. Yet, as (s) cannot exceed one, E2 may prevail anyway.

Proof [Lemma 2.] See Appendix A2. ¦

From lemma 2, we derive propositions 2.1 and 2.2.

Proposition 2.1. Under H1 to H4, when equilibrium E2 applies (all firms substitute projects), only private agreements prevail after default. Distressed firms fully internalize bankruptcy costs through renegotiation, but paying the highest price (c–s). *Thus, equilibrium E2, even if economically inefficient, still ensures legal efficiency, as bankruptcy costs are internalized.* As for proposition 1.1, this result is close to the Haugen and Senbet's [1978] and [1988] prediction. Yet, we shall see this result does not hold anymore when turning to mixed strategies.

[31] In Germany, the bankruptcy procedure cannot be triggered off, if it appears that the debtor's expected gains will not cover the bankruptcy costs.

Proposition 2.2. *The enforcement power of the legislator is indirect only*. Anytime (s)he increases the level of sanctions (s) in order to avoid the prevalence of the inefficient equilibrium E2, his/her action must take into consideration the Bank's strategy, through the level of (i). In other terms, the legal policy is not independent from the financial environment. When comparing proposition 2.2 with proposition 1.2, the legislator has some enforcement power here, and his/her action helps in avoiding sub-optimal equilibrium E2. Yet, for some peculiar values of the interest rate (see lemma 2), his/her action may be unsuccessful.

B. The debt contract's design (equilibrium E2)

At time (t), the Bank settles a debt contract compatible with equilibrium E2, under which all firms substitute projects: condition (13) gives the incitation constraint associated to this equilibrium. As usual, two participation constraints are added to the Bank's program: one for the Firm, and one for the Bank itself. The optimal interest rate related to equilibrium E2, i_2^*, stems from the resolution of the Bank's program (14) (where the expected profits are given by (12a) and (12b)).

$$\begin{aligned} & i_2^* = \underset{i}{arg\,max} E\left(\Pi^B \middle| i, j'\right) \\ & \text{u.c. Firms's participation: } E\left(\Pi \middle| i, j'\right) \geq 0 \\ & \qquad \text{Bank's participation: } E\left(\Pi^B \middle| i, j'\right) \geq 1 \\ & \qquad \text{Incitation to (E2): } s < f_1(i); \text{ where } f_1(i) \text{ is given by equation (13)} \end{aligned} \tag{14}$$

Under E2, the optimal interest rate (i_2^*) ensures legal efficiency only: all firms substitute assets, which decreases the global wealth (see proposition 2.1).

Until now, we have studied pure strategy equilibriums. Yet, in case of indifference between projects (j) and (j'), the firm may adopt mixed strategies: this situation is discussed in section 3.3.

3.3. Equilibrium E3: mixed strategy on the Firm's investment choice

When both projects reward identical expected amounts, the firm may adopt mixed strategies, at time (t+1), that is respectively choose project (j) or (j') with probability (p) and (1-p). This probability is the basis of the Bayesian revision process described in H5. At time (t+1), the observed earnings and the distressed firm's willingness to renegotiate are signals to the Bank: once (x) is realized and before (R) is disclosed, the Bank updates its beliefs on (p): $p \to p(j|x)$. Then, once (R) is proposed to the Bank, this probability is revised for a second time: $p(j|x) \to p(j|x,R)$. Section A describes the post-default renegotiation process. This one may lead to either pooling or separating bargains (section A). The derived Bank's program is described in section B.

A. The renegotiation process (equilibrium E3)

We describe below the conditions under which all parties prefer to privately renegotiate *just before signal (R) is released*: i.e. when beliefs are based on $p(j|x)$ (see equation (3)). We split the description of this post-renegotiation process into two successive parts: first, sub-section a) explains how the Bank's beliefs impact on renegotiation. Second, sub-section b) analyzes how the resulting expected gains and corresponding profits depends on the level of the realized earnings.

a. The impact of the Bank's beliefs on renegotiation

As defined in D3, the individual "acceptance thresholds" (named "AT") are the minimum levels of (R) each party wants (respectively accepts) to receive (respectively grant) outside bankruptcy. Of course, because amount (R) is a new signal for the Bank, these thresholds will change as soon as (R) is proposed. Let us consider first the Honest Firms: their acceptance threshold is identical to the one which prevailed under E1, and they prefer private renegotiation to bankruptcy if and only if:

$$1 + x - R \geq 0 \Leftrightarrow R \leq \underbrace{1 + x}_{jAT(x)} \qquad (15)$$

Second, Tricky Firms compute a similar tradeoff: they prefer private renegotiation to bankruptcy if and only if:

$$1 + x - R \geq -s \Leftrightarrow R \leq \underbrace{1 + x + s}_{j'AT(x)} \qquad (16)$$

The Bank's tradeoff between the rival ways of resolving default is complicated by the fact that the project choice is random under E3. The Bank prefers to privately renegotiate if the proposed amount (R) equals or exceeds the expected gains if bankruptcy is triggered: in the latter case, the recovered amount equals earnings \$(1+x) net of bankruptcy costs ($c \times \$1$), plus – possibly – legal sanctions ($s \times \$1$), in case moral hazard prevailed at time (t+1): this happens with probability $1-p(j|x)$. Namely, just before (R) is disclosed, the Bank prefers private renegotiation if and only if condition (17) prevails:

$$\begin{aligned} & R \geq \underbrace{1+x-c+\left(1-p\left(j|x\right)\right)\cdot s}_{\text{BankAT(x)}} \\ & \text{with} \quad p\left(j|x\right)=\frac{f\left(x|j\right)\cdot p}{f\left(x|j\right)\cdot p+f\left(x|j'\right)\cdot\left(1-p\right)} \end{aligned} \tag{17}$$

The Bank's acceptance threshold defined in equation (17) depends on the probability of choosing project (j), $p(j|x)$. This probability represents the beliefs of the Bank after earnings (x) are disclosed and just before receiving signal (R). Depending on the level of $p(j|x)$, the Bank is said to be either "suspicious" or "confident" (see definition (D4).

Definition D4. *Suspicion and confidence (pivot value $\hat{p}$)*

Parameter ($\hat{p}$) is the pivot value taken by probability $p(j|x)$, so that the minimum amount required by the Bank equals the maximum amount payable by the Honest firm. Using (15) and (17), we obtain equation (18):

$$\begin{aligned} & 1+x-c+\left(1-\hat{p}\right)\cdot s=1+x \\ & \Leftrightarrow \hat{p}=1-\frac{c}{s}\in\left[0;1\right] \text{ when } s\geq c \end{aligned} \tag{18}$$

The Bank is said to be "suspicious" any time $p(j|x)$ is lower than ($\hat{p}$). The bank is "confident" otherwise. Under suspicion, the intersection between the Firm (j) and the Bank's acceptance thresholds do not overlap. ¦

Before the amount (R) is disclosed, and depending on the Bank's updated beliefs, two rival bargains may arise from renegotiation (see proposition 3 $\Rightarrow$ 3.1 and 3.2). The consequences on efficiency strongly differ from the previous equilibriums E1 and E2, so

that the Haugen en Senbet's view does not hold anymore under mixed strategies: some players may choose costly bankruptcy instead of private renegotiation: this is likely to happen when the Bank is initially suspicious.

Proposition 3. Under H1 to H5, the post-default bargaining process may lead to two exclusive bargains (either pooling or separating); each of them is attached to a peculiar way of resolving financial distress (either private renegotiation or formal bankruptcy: see below, propositions 3.1. and 3.2. and their proofs).

> *Proposition 3.1. If the bank is "suspicious" before signal (R) is disclosed (here, suspicion prevails when* $p(\theta|x)<\hat{p}$*, where threshold* $\hat{p}=1-c/s$*: see equation (18)), the bargaining process leads to a separating equilibrium and the initial project choice is discovered. Honest Firms go to formal costly bankruptcy, whereas Tricky Firms privately renegotiate (at a high price). We consider such equilibrium as legally inefficient because bankruptcy costs are not fully internalized.*
>
> *Proposition 3.1. Corollary 1.* Contrary to Haugen and Senbet [1978, 1988], costly bankruptcy may be preferred by Honest Firms over private negotiation. This is due to asymmetric information and happens when the separating equilibrium prevails, i.e. when the Bank is suspicious before signal (R) is disclosed. Here, "suspicion" depends on the position of the bank's beliefs $p(\theta|x)$ relative to the amount $\hat{p}$: this amount varies with the legal environment, (c) and (s): *thus, some legal environments may lead to a sub-optimal solution, so that costly bankruptcy cannot be avoided by Honest Firms, which can be interpreted as a legal inefficiency of bankruptcy law.*
>
> *Proposition 3.2. If the bank is "confident" before signal (R) is disclosed (here,* $p(\theta|x)\geq\hat{p}$*), the bargaining process leads to a pooling equilibrium: the initial project choice is not discovered and private renegotiation prevails. We consider such equilibrium as legally efficient because bankruptcy costs are fully internalized.*

The rationale of proposition 3 is given below: it plays a key role in the writing of the expected profits, and thus, in the associated programs, at the time of the credit-lending decision.

Proof [Proposition 3]. The bargaining equilibrium comes from the comparison of all individual thresholds. Figure 2 represents all possible configurations given by equations (15) to (17), depending on the initial level of probability $p(j|x)$, which can be viewed as the Bank's beliefs on the project choice after (x) is disclosed and before the firm proposes (R).

Figure 2. Individual Acceptance Thresholds and Bargaining Tradeoffs

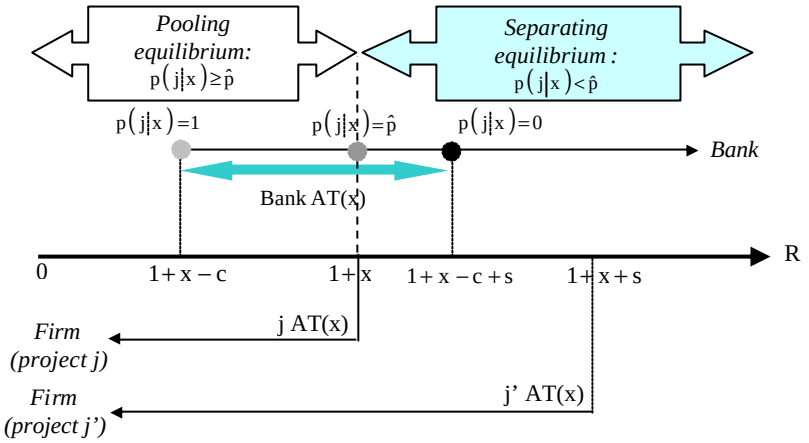


Figure 2 (interpretation): The bold and black arrow indicates all possible values of (R). Plots ● ● ● are the Bank's minimum thresholds when probability p(j|x) respectively takes value 0, $\hat{p}$ and 1. The direction of small black arrows indicates that the more (the less) (R) is, the more the Bank (the Firm) prefers to informally renegotiate.

- If the Bank is "confident" ($p(j|x) \geq \hat{p}$): the Bank requires a relatively low minimum amount, anticipating that bankruptcy has little chance to involve legal sanctions. All firms (honest and tricky ones) prefer to turn to private renegotiation, because debt forgiveness allows the manager to internalize bankruptcy costs. Here, the Bank minimum requirement is always lower than the amount both types of firms accept to pay: threshold values jAT(x) and j'AT(x) totally overlap with BankAT(x). We then obtain a pooling equilibrium, so that the signal (R) is "empty", meaning it does not provide any additional information to the Bank:

$$p(j|x) \geq \hat{p} \overset{R>0}{\rightarrow} p(j|x,R) = p(j|x) \qquad (19)$$

All firms propose the minimum amount required by the Bank with an unchanged probability $p(j|x)$: all firms avoid bankruptcy by offering the amount R^* (equation (17) leads to (20)):

$$R^* = 1 + x - c + (1 - p(j|x)) \cdot s \tag{20}$$

- <u>If the Bank is "suspicious"</u> ($p(j|x) < \hat{p}$): the Bank believes there is a good chance that project (j') was previously selected and, is more incline to trigger bankruptcy in order to receiver the proceeds from legal sanctions. As a consequence, the Bank requires a rather high minimum amount to accept renegotiation. On the other side, any Honest Firm prefers bankruptcy, whereas any Tricky Firm prefers private renegotiation. Then, any strictly positive offered amount (R) can only come from Tricky Firms because only BankAT(x) and j'AT(x) overlap. In that case, signal (R) perfectly reveals the project initial choice, which is (j') only:

$$p(j|x) < \hat{p} \overset{R>0}{\rightarrow} p(j|x,R) = 0 \tag{21}$$

As shown in relation (21), the Bank internalizes this update and replaces $p(j|x,R)$ by zero. Replacing this value in equation (17) leads to a revised value for the bank's minimum required amount, BankAT(x) (which equals $1 + x - c + s$ from now on). Tricky Firms finally propose this amount ($R^* = 1 + x - c + s$), which is always accepted by the Bank. Honest Firms do not propose anything and trigger bankruptcy, because renegotiation is too expensive[32]:

$$\begin{cases} \text{Honest Firm : does not propose anything (formal bankruptcy)} & \text{(22a)} \\ \text{Tricky Firm : proposes } R^* = 1 + x - c + s \text{ (private renegotiation)} & \text{(22b)} \end{cases}$$

Hence, the bargaining taking place at time (t+2) leads to two equilibriums: a separating one and a pooling one. Each case relies on the comparison between probability $p(j|x)$ and the pivot value given by relation (19).

End of proof (Proposition 3.) ¦

[32] It is essential to notice here that Tricky Firms are not incited to bluff, by proposing nothing to the bank, so that they appear as Honest Firms. Indeed this would imply automatic bankruptcy triggering, and the bluff would be directly discovered (remember bankruptcy costs are revealing costs).

b. The impact of the realized earnings on the expected profits

As shown in proposition 3, the expected gains vary with the Bank's belief $p(j|x)$, once the value of earnings (x) becomes public information. Now, from hypotheses H2, we know the distribution of values (x) depends on the initial choice (j) or (j'). In other words, the project choice at time (t+1) affects the distribution of (x), whose realized value modifies the bank's beliefs at time (t+2), which – in return – may change the bargain (either pooling or separating), and consequently the expected gains under default. Given this, the computation of all expected gains, for time (t+2), requires the definition of two sets $\{S_i\}$ and $\{\overline{S}_i\}$ of all possible values for variable (x) so that conditions $p(j|x) \geq \hat{p}$ and $p(j|x) < \hat{p}$ are respectively verified or not (see below: definitions (D5a) and (D5b)).

Definition D5a. Under default ($x < i$), $\{S_i\}$ is the set of all possible realizations (x) so that the Bank is "confident" before receiving any proposal (R), and the resulting bargaining equilibrium is a pooling one. This applies when inequality (14a) is verified:

$$p(j|x) \geq \hat{p} \Leftrightarrow \frac{f(x|j)\cdot p}{f(x|j)\cdot p + f(x|j')\cdot(1-p)} \geq \hat{p}$$

$$\Leftrightarrow \underbrace{\frac{f(x|j)}{f(x|j')}}_{\varphi(x)} \geq \left(\frac{1}{p}-1\right)\cdot\left(\frac{s}{c}-1\right) \Leftrightarrow \varphi(x) \geq \left(\frac{1}{p}-1\right)\cdot\left(\frac{s}{c}-1\right) \qquad (23a)$$

Definition D5b. Under default ($x < i$), $\{\overline{S}_i\}$ is the set of all possible realizations (x) so that the Bank is "suspicious" before receiving any proposal (R), and the resulting bargaining equilibrium is a separating one. This applies when inequality (14b) is verified:

$$\varphi(x) < \left(\frac{1}{p}-1\right)\cdot\left(\frac{s}{c}-1\right) \qquad (23b)$$

Notice that[33]: $S_i \cup \overline{S}_i = [0;i)$

[33] As inequalities (1a) and (1b) apply, $\varphi(x)$ is a non-monotonous function so that the $\{S_i\}$ is a contiguous set, whereas $\{\overline{S}_i\}$ is non-contiguous, and covers extreme values of (x), either very low, or very high (provided these values are less than (i): remind the firm is supposed to be in default here).

Outside default ($x \geq i$), the Firm and the Bank are rewarded normally. Under default, their respective gains depend on the bargain prevailing after renegotiation. As shown in proposition 3, contingently to the realized earnings (x) ($\in S_i$ or $\in \overline{S}_i$), the value of (R^*) differs: it is given by equation (20), when $x \in S_i$ (i.e. the Bank is confident), or by equations (22a) and (22b), when $x \in \overline{S}_i$ (i.e. the Bank is suspicious). Equation (24a) to (24c) provides the corresponding expected profits at time (t+1), for the Honest Firm, the Tricky Firm, and the Bank: these are denoted respectively $E(\Pi | i,j,p(j|x))$, $E(\Pi | i,j',p(j|x))$, and $E(\Pi^B | i,p(j|x))$[34].

- The Honest Firm's expected profit: Under default, two cases may arise, depending on the value of (x): the Bank is either confident ($x \in S_i$), or suspicious ($x \in \overline{S}_i$). In the Former case, the Firm escapes bankruptcy and pays R^* (given by equation (20)). In the latter case, the Firm triggers directly bankruptcy, so that the expected default-gain is null. This leads to equation (24a).

$$\begin{aligned} E\left(\Pi | i,j,p(j|x)\right) &= \underbrace{\int_{S_i} \left(c - \left(1 - p\left(j|x\right)\right)\cdot s\right)\cdot f\left(x|j\right)dx}_{\textit{Pooling bargain}} + \int_i^{\infty} (x-i)\cdot f\left(x|j\right)dx \\ &= \int_{S_i} \left(c - \left(1 - p\left(j|x\right)\right)\cdot s\right)\cdot f\left(x|j\right)dx + i\cdot F_{X|j}(i) + E_i^{\infty}\left(X|j\right) - i \end{aligned} \tag{24a}$$

- The Tricky Firm's expected profit: Under default, if $x \in S_i$, the equilibrium is pooling and the Firms pays R^* (given by equation (20)); if $x \in \overline{S}_i$, the equilibrium is separating, and the Firm escapes bankruptcy paying the highest price (given by (22b)). This leads to equation (24b).

$$\begin{aligned} E\left(\Pi | i,j',p(j|x)\right) &= \underbrace{\int_{S_i} \left(c - \left(1 - p\left(j|x\right)\right)\cdot s\right)\cdot f\left(x|j'\right)dx}_{\textit{Pooling bargain}} + \underbrace{\int_{\overline{S}_i} (c-s)\cdot f\left(x|j'\right)dx}_{\textit{Separating bargain}} \\ &\quad + \int_i^{\infty} (x-i)\, f\left(x|j'\right)dx \\ &= s\cdot \int_{S_i} p\left(j|x\right)\cdot f\left(x|j'\right)dx + (c-s+i)\cdot F_{X|j'}(i) + E_i^{\infty}\left(X|j'\right) - i \end{aligned} \tag{24b}$$

[34] Where the Bank's belief $p(j|x)$ is given by equation (3).

- The Bank's expected profit: At time (t+1), the Bank still does not know which project is currently undertaken: its priors are equal to (p): the Bank's expected profits are weighted by these priors. Besides, as the density function of (x) is public information, the Bank can compute at time (t+1) its future updated beliefs, $p(j|x)$, which are contingent to the realization of (x): this affects the expected gains, when $x \in S_i$ or when $x \in \overline{S}_i$. This leads to equation (24c).

$$\begin{aligned} E\left(\Pi^B \left|_{i,p(j|x)}\right.\right) &= p\cdot\left(\begin{array}{l}\int_{S_i}(1+x-c+(1-p(j|x))\cdot s)\cdot f(x|j)dx+\\ \int_{\overline{S}_i}(1+x-c)\cdot f(x|j)dx+\int_i^{\infty}(1+i)\cdot f(x|j)dx\end{array}\right)+\\ &(1-p)\cdot\left(\begin{array}{l}\int_{S_i}(1+x-c+(1-p(j|x))\cdot s)\cdot f(x|j')dx+\\ \int_{\overline{S}_i}(1+x-c+s)\cdot f(x|j')dx+\int_i^{\infty}(1+i)\cdot f(x|j')dx\end{array}\right) \qquad (24c)\\ &= 1+i+p\cdot\left(E_0^i(X|j)-(c+i)\cdot F_{X|j}(i)+s\cdot\int_{S_i}(1-p(j|x))\cdot f(x|j)dx\right)\\ &+(1-p)\cdot\left(E_0^i(X|j')-(c+i-s)\cdot F_{X|j'}(i)-s\cdot\int_{S_i}p(j|x)\cdot f(x|j')dx\right)\end{aligned}$$

Based on these expected profits (24a), (24b), and (24c), condition (25) must prevail so that equilibrium E3 is stable: at time (t+1), the firm adopts mixed strategies provided his/her expected profits are equal, whatever the undertaken project. The left (respectively right) hand of condition (25) gives the expected gains of project (j) (resp. (j')). Condition (25) can be written as follows:

$$E\left(\Pi\left|_{i,j,p(j|x)}\right.\right) = E\left(\Pi\left|_{i,j',p(j|x)}\right.\right) \qquad (25)$$

Introducing expressions (3), (24a), and (24b) in (25), we finally have:

$$\begin{aligned} &s = f_2(i,p)\\ &\text{with}\quad f_2(i,p) = \frac{E_i^{\infty}(X|j') - E_i^{\infty}(X|j) + i\cdot\left(F_{X|j'}(i)-F_{X|j}(i)\right)+c\cdot\left(F_{X|j'}(i)-\int_{S_i}f(x|j)dx\right)}{F_{X|j'}(i)-\int_{S_i}\frac{f(x|j)\cdot f(x|j')}{p\cdot f(x|j)+(1-p)\cdot f(x|j')}dx} \qquad (26)\end{aligned}$$

Proof [equality (26)] *See Appendix A3.* ¦

Equality (26) leads to proposition 4. This proposition is of key importance: indeed, it shows that legal environment, financial and economic decisions are linked together, at equilibrium when strategies are mixed. The apparent independence shown in proposition 1.2 does not hold anymore. Simulations (see section 4) shall illustrate this strong dependence between the three types of variables.

Proposition 4. *Under H1 to H5, equilibrium E3 is stable provided condition (26) prevails. This condition defines a required level of legal sanctions, equal to* $f_2(i,p)$*, so that the Firm respectively chooses project (j) or project (j') with probability (p) and (1-p). Looking at (26), the contractual interest rate (i) is linked to the expected Firm's investment policy (p), which depends on the level of legal sanctions (s).* In other terms, any change in the legal environment affects the cost of credit and the firms' mixed investment policy.

Proposition 4. Corollary. *When strategies are mixed, the legislator is able to drive the investment choice. Yet, this enforcement power is constrained and indirect only.* Designing the Law, the legislator has to take into account the financial adjustments from the Banks, captured here by the level of the contractual interest rate.

B. The debt contract's design (equilibrium E3)

Similarly to the other equilibriums, the Bank settles a debt contract at time (t), so that the proposed contract is compatible with equilibrium E3. Condition (26) is the incitation constraint associated to E3. Again, two participation constraints apply. We denote as i_3^* the optimal interest rate associated to this program (27) (where the individual expected profits are given by (24a) to (24c)):

$$\begin{aligned} & i_3^* = \underset{i}{arg\,max}\, E\left(\Pi^B \middle| i,p(j|x)\right) \\ & \text{u.c. Firms's participation : } E\left(\Pi \middle| i,j,p(j|x)\right) = E\left(\Pi \middle| i,j',p(j|x)\right) \geq 0 \\ & \qquad \text{Bank's participation : } E\left(\Pi^B \middle| i,p(j|x)\right) \geq 1 \\ & \qquad \text{Incitation to (E3) : } s = f_2(i,p); \text{ where } f_2(i,p) \text{ is given by equation (26)} \end{aligned} \tag{27}$$

Under E3, the optimal interest rate (i_3^*) ensures legal efficiency provided the future realized earnings belong to the set S_i ($x \in S_i$), so that all firms privately renegotiate under default. Economic efficiency is not guaranteed as the firm randomly chooses between both projects. In section 4, we use simulated results for answering two issues: [a] among all possible equilibriums, which one prevails, depending on the level of legal sanctions? [b] If equilibrium E3 applies, how the legislator can increase probability (p), hence enhancing economic efficiency?

4. SIMULATIONS

From section 3, we know the Bank derives from the three equilibriums E1, E2, and E3 three levels of contractual interest rates: i_1^*, i_2^*, and i_3^*. Actually, comparing the corresponding expected profits, the Bank finally chooses the optimal interest rate (denoted i^{**}) which leads to the highest expected profit (see equation (28)):

$$i^{**} = \underset{i^*}{arg\,max}\left[E\left(\Pi_B \middle| i^* = i_1^*\right); E\left(\Pi_B \middle| i^* = i_2^*\right); E\left(\Pi_B \middle| i^* = i_3^*\right)\right] \qquad (28)$$

Any equilibrium may arise, as E1, E2 and E3 respectively apply when i^{**} equals i_1^*, i_2^*, or i_3^*. We provide[35] simulations based on the programs (10), (14), and (27), assuming variables (X|j) and (X|j') follow a Gaussian distribution law[36], and bankruptcy costs equal 5%. The mean is set to 0.50 and 0.49 for projects (j) and (j')[37]. For each level of the legal sanctions (s), Graphs 1 to 6 respectively show: [1] the evolution of the optimal interest rate i^{**}; [2] the corresponding probability of default $F_{X|J}(i^{**})$ $\forall J \in (j,j')$; [3] the probability of choosing project (j) p^{**} (if E3 prevails); [5] the ratio between i^{**} and p^{**}; [5] the Bank's

[35] The simulations were made with Maple© software: our programs are available on request.

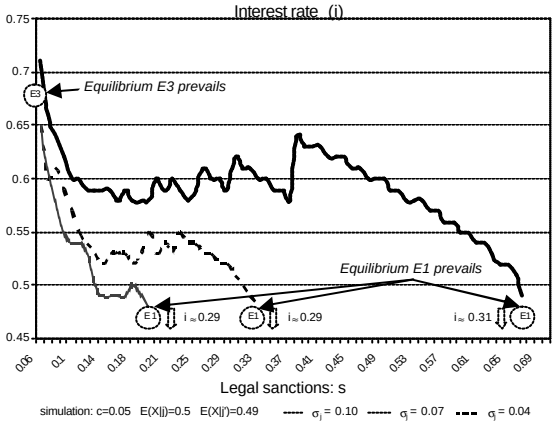
[36] The attentive reader may notice Gaussian distribution law allows for both negative and positive values, whereas variable (x) (the realization of (X|J), $\forall J \in (j,j')$) is supposed to take positive values only (see Hypotheses H1). Using other distribution law, such as Log-Normal or Exponential laws may avoid this problem. However, we still use the Gaussian law in our simulations, because it allows the two first moments (mean and standard deviation) to vary in opposite directions. This is not the case for other standard statistical laws, for which the mean and the standard deviation are positively linked together: this would be inconsistent with hypotheses H2, so that asset substitution leads to a decrease of profitability and to an increase of risk. In order to be sure that negative values are unlikely to happen in our simulations, we choose rather high values for the mean – whatever the project (j) or (j') – so that the probability of getting negative values for (x) is close to zero (inferior to 3×10^{-4}).

[37] We computed additional simulations with values of 0.48 (mean for (j)) and of 0.51 (mean for (j')): these values lead to similar results (the main difference with the current simulation is that equilibrium E1 is reached for lower values of legal sanctions. All the other discussed features remain identical.

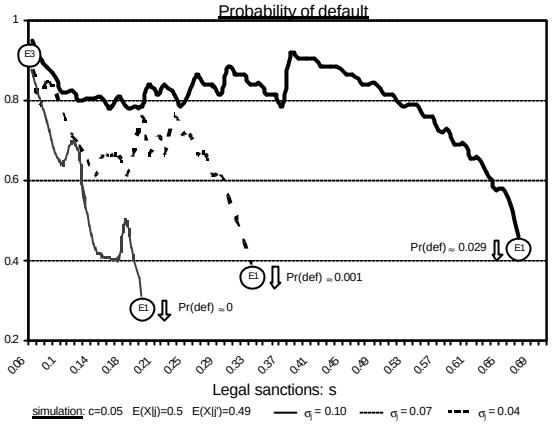
expected profit; and [6] the Firm's expected profit[38]. We consider the following cases: the standard deviations (σ_j and $\sigma_{j'}$) are set to: 0.04 and 0.14 for project (j) and (j') [curve 1]; 0.07 (σ_j) and 0.14 ($\sigma_{j'}$) [curve 2]; 0.10 (σ_j) and 0.14 ($\sigma_{j'}$) [curve 3]. Plots (E1) and (E3) indicate which equilibrium prevails[39]. Appendix A4 provides an example of the evolution of the expected profits with (i) for any equilibrium.

Graphs 1 to 6. Simulated Results for Different Values of the Legal Sanctions (s)

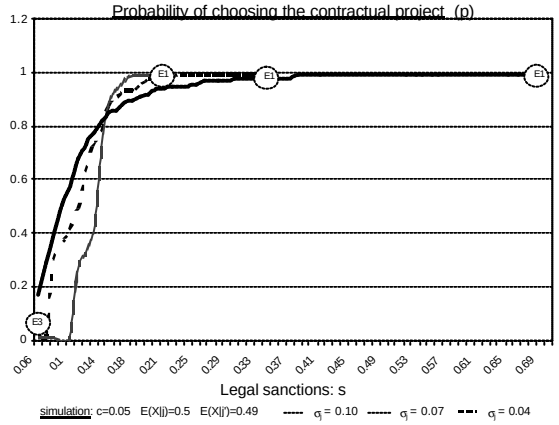
Graph 1



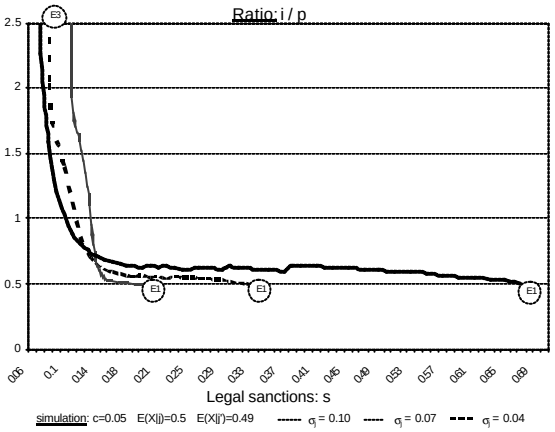
Graph 2



Graph 3



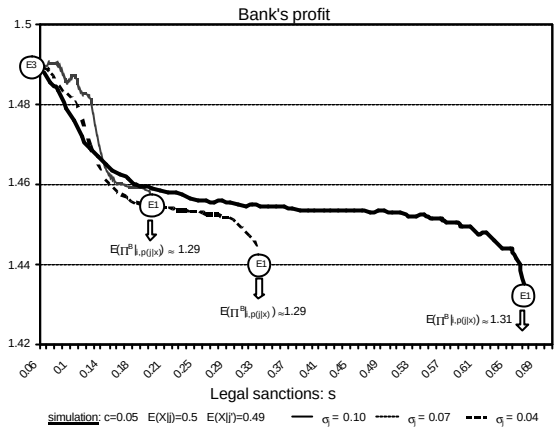
Graph 4



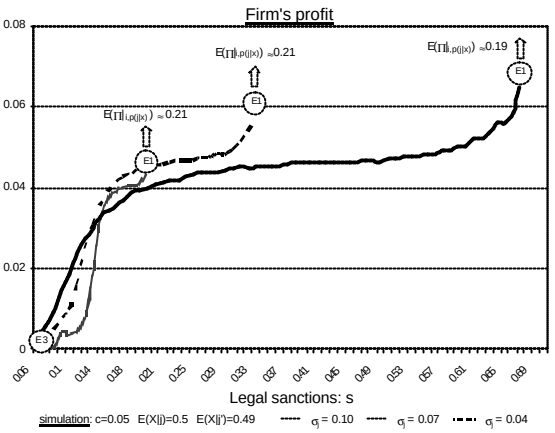
[38] Given the initial values, the global surplus – defined as the sum of both Firm's and Bank's expected profits, respectively equal 1.50 and 1.49 when the project is (j) or (j').

[39] For the very first lowest values of (s), equilibriums E2 and E3 lead to close levels of Bank's expected profits, so that both equilibriums may apply for the same value of (s). Yet, equilibrium E2 disappears for higher values of (s) (more than 10% on average).

Graph 5



Graph 6



Graphs 1-6 show **four interesting specificities of the model**. [a] The first illustrates the impact of a modification in the Law (sanctions) on financial and economic variables; [b] the second shows that, depending on the exogenous level of sanctions, the Bank may voluntarily accept a certain level of moral hazard at equilibrium; [c] the third shades light on the resulting change in the profits sharing: contrary to the Bank, the Firm may benefit from a more severe legal environment; [d] the fourth suggests that a change in the legal environment may involve more financial than economic instability.

A change of the legal environment effectively affects the investment policy (p) (Graph 2), the design of the debt contract (i) (Graph 1), and the resulting probability of default (Graph 3): more precisely, as expected, an increase of sanctions (the legal environment turns to be severe) leads to a higher probability of choosing project (j). For each level of σ_j (0.04, 0.07, and 0.10), *there is an optimal level of sanctions* (21%, 35%, and 69%) *which ensure equilibrium E1, so that both economic and legal efficiencies are preserved*. Any additional increase of (s) is not needed, as equilibrium E1 does not depend on legal sanctions (as suggested by proposition 1.2): beyond these values, the economic and financial variables do not change anymore. As a result, an extreme severity is not needed to ensure economic and legal efficiency, and is only justified in case future changes affect the economy, and the nature of the investment projects (mean and variance).

As legal sanctions are increasing, the interest rate (i) tends to decrease, after some erratic changes; the decrease is much more regular and pronounced when focusing on the ratio

(i/p) (Graph 4). This result is coherent with the empirical findings from Qian and Strahan [2007], whose cross-country analysis confirm that low interest rates are related to codes with a strong creditor protection. In our model, this effect reflects the following mechanism: *when sanctions are low or medium, the Bank accepts some moral hazard from the firm* (p is low) leading to moderate sanctions in case of default (s): E3 prevails and the Bank can charge more the Firm, applying a risk premium into the debt contract. The story changes as the legal environment becomes more severe: as a high level of (s) gives the firm the right incentives to reduce moral hazard (p increases), the Bank cannot charge a high interest rate anymore, as it is more protected by the Law: (i) decreases.

An interesting consequence is that *the sharing between profits goes into a rather noteworthy direction* (Graphs 5 and 6): whereas the rationale behind the implementation of legal sanctions is to protect creditors against a risk of moral hazard from their debtors, the Firm's profit surprisingly increases with sanctions (s) (Graph 5), contrary to the Bank (Graph 6). Indeed, a quadruple mechanism stems from an increase of (s): all thing remaining equal, on one hand, the Firm's profit declines as [1] it turns toward a less risky project ($\sigma_j < \sigma_{j'}$); [2] higher sanctions apply, in case it still chooses project (j'); on the other hand, the Firm's profit augments as [3] it turns toward a more profitable project ($E(X|j) > E(X|j')$); [4] it pays a lower level of interest rate, as the risk premium declines. According to the simulation, mechanisms [3] and [4] over-compensate mechanisms [1] and [2]. As a consequence, *a more severe legal environment is likely to involve less costly financial resources: the Firm finally benefits of a more severe environment, and expects a higher profit.*

Third, as the model switches from equilibrium (E3) to (E1), decision variables are subject to discontinuous changes: the discontinuity is moderate regarding the investment policy (before project (j) is surely chosen under E1, it was decided 99% of times under E3). On the opposite, the changes are quite marked for the financial variables: the interest rate and the resulting profits dramatically decrease once the Bank turns to equilibrium E1. As a consequence, *a slight change of the legal environment may involve a drastic adjustment of financial variables*. This last issue raises opportunities for further empirical works.

CONCLUSIONS

This research investigates how bankruptcy law influences the design of renegotiable debt contracts. This is a contribution to the literature that usually disconnects *ex-post* and *ex-ante* efficiencies of bankruptcy law. In our view, the legal environment plays a key role onto the financial decisions which are made at the time of credit granting. More precisely, a modification of bankruptcy punishment – through legal sanctions – is likely to affect the price of debt and the subsequent investing decision of the managers. Unlike most papers considering a passive behavior of the bank when the borrower defaults, we assume an active renegotiation process between the debtors and creditors: when facing financial distress, they decide either to renegotiate or to turn to a Court solution. Indeed, stylized facts show that the tradeoff between both types of solutions (agreement *vs*. bankruptcy) is not straightforward, as it depends on the lending relation, the asymmetries of information between the creditor and the debtor, and on the legal environment of bankruptcy. We suggest this tradeoff anticipated by both parts, and thus affect their respective strategies: following that perspective, we model the debtor's investment policy – with a risk of asset substitution – and the creditor's financial policy – with an endogenous interest rate.

Our model leads to several propositions. Each of them is related to a peculiar equilibrium of the model and can be linked to some empirical findings: we use summary statistics from an original database we collected under the *S&P Risk Solution* supervision: it is the French part of the European sample designed, built, and used by Davydenko and Franks [2007]. A first equilibrium encompasses situations when the firms do not perform moral hazard at all (economic efficiency) and the default is privately solved (legal efficiency): empirical findings confirm the existence of such equilibrium, as 90% of private renegotiations deal with non-faulty managers. This equilibrium interestingly requires a double condition on bankruptcy costs and on interest rates only, but does not depend on the level of legal sanctions. In other terms, the legislator cannot implement directly economic *and* legal efficiency. A second equilibrium covers situations where the firms turn to the less profitable and riskiest project (economic inefficiency) and the default is privately solved (legal efficiency). This equilibrium is coherent in some extent with our empirical findings: a little set of faulty firms (18.5%) go to formal bankruptcy only. This equilibrium can be avoided through a minimal level of legal sanctions, whose threshold

value depends on the contractual interest rate: in that view, the legislator policy is indirect only, as it depends on the banks' individual strategy. The last equilibrium encompasses mixed strategies on the investment policy (partial economic efficiency): when financial distress occurs, two bargains may prevail – pooling or not: under pooling bargains, all firms privately renegotiate with the bank, but under separating ones, honest firms directly go to bankruptcy whereas faulty ones renegotiate at the highest price. Empirical findings partially reflect this situation, as the bulk of direct bankruptcy triggering deals with non-faulty managers (81.5%). Yet, the model needs additional features to explain why a significant part of faulty managers (31.6%) still go to bankruptcy after a renegotiation attempt, whereas renegotiation could have saved bankruptcy costs. This may reflect some uncertainty on the legal environment and open an avenue for additional research.

Some simulations illustrate how the bank finally chooses between the alternative equilibriums, as the legal environment becomes more severe. Our findings reflect several normative specificities: First, for moderate values of legal sanctions, banks may accept a certain level of moral hazard from their debtors, expecting to take advantage of bankruptcy punishment (even if moderate). As expected, an increase of legal sanctions changes the story, as it incites the companies to respect more their commitments: our simulations find a minimal level of sanctions preventing sub-optimal equilibrium to prevail. But any additional increase of sanctions is worthless, as – once the optimal equilibrium prevails – the decision variables do not depend on the legal environment anymore. As a result, extreme severity is not needed to ensure both economic and legal efficiency, and is only justified in case of future economic changes. Second, when strategies are mixed, an increase of legal sanctions is likely to involve a reduction of the contractual interest rates: as the banks are more protected by the law, they cannot charge a risk premium anymore. An interesting consequence is that the sharing between profits goes into a rather surprising direction: whereas the rationale behind legal sanctions is to protect the creditors against assets substitution, the debtors benefit in some extent of increased severity, as they are inclined to invest in the most profitable projects and, consequently, pay a lower interest rate. Third, we finally find that a slight change of the legal environment may involve a drastic adjustment of financial variables, so that small changes in the law may create financial instability.

APPENDIXES

A1. Proof of inequality (2)

Considering a standard debt contract[40], the Firm's expected profit depends on the chosen project ((j) or (j')). Remembering the Firm is in default as soon as the realized earnings (x) are inferior to the interest rate (i), we have (where f(x|J) is the density function of the random variable (X|J), $\forall J \in (j,j')$):

$$E\left(\Pi \middle| \text{project (j); standard debt}\right) = \int_i^{\infty} (x-i)\cdot f\left(x\middle|j\right)dx = E_i^{\infty}\left(X\middle|j\right) - i\cdot\left(1-F_{X|j}(i)\right) \qquad \text{(A1-1)}$$

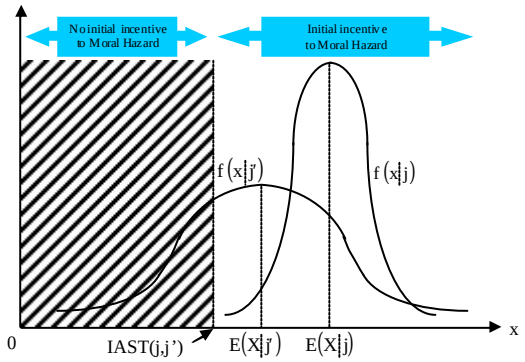
$$E\left(\Pi \middle| \text{project (j'); standard debt}\right) = \int_i^{\infty} (x-i)\cdot f\left(x\middle|j'\right)dx = E_i^{\infty}\left(X\middle|j'\right) - i\cdot\left(1-F_{X|j'}(i)\right) \qquad \text{(A1-2)}$$

The firm has initial incentive to undertake project (j') instead of project (j) as soon as profit shown in equation (A1-2) leads to a higher profit compared with equation (A1-1). This lead to inequality (A1-3), so that the tradeoff between projects (j) and (j') leads to a minimum value for E(X|j'), as described in Figure I:

$$\begin{aligned} & E_i^{\infty}\left(X\middle|j'\right) + i\cdot F_{X|j'}(i) > E_i^{\infty}\left(X\middle|j\right) + i\cdot F_{X|j}(i) \\ \Leftrightarrow\ & E\left(X\middle|j'\right) > \underbrace{E_0^{i}\left(X\middle|j'\right) + E_i^{\infty}\left(X\middle|j\right) - i\cdot\left(F_{X|j'}(i) - F_{X|j}(i)\right)}_{\text{Incentive to Asset Substitution Threshold: ``IAST(j, j)''}} \end{aligned} \qquad \text{(A1-3)}$$

Figure I. Condition for having initial incentive to moral hazard: two projects (j) and (j')

The expected profitability for project (j') must exceed the "IAST(j,j')" threshold



End of proof [Inequality (2)] ¦

[40] Following Stiglitz and Weiss [1981], a debt contract is said to be "standard" as soon as it preserves limited responsibility and the derived expected profits (Firm and Bank) do not take into account nor private renegotiation in case of default, neither the contingent appliance of legal sanctions.

A2. Proof of lemma 2

As shown in relation (13), the position of function $f_i(i)$ (relatively to (c)) depends on the sum of $A = E_i^{\infty}(X|j') - E_i^{\infty}(X|j)$ and $B = i \cdot (F_{X|j'}(i) - F_{X|j}(i))$. Under H2, the amounts (A) and (B) are of opposite signs. First, for extreme-low values of (i), (A) is negative and higher than (B), in absolute values: A+B is negative, so we have $f_i(i)<c$. Second, for extreme-high values of (i), (A) is positive and lower than (B), in absolute values: A+B<0, and $f_i(i)<c$. Third, for moderate-low values of (i), (A) is negative and lower than (B), in absolute values: A+B>0, and $f_i(i)>c$. Fourth, for moderate-high values of (i), (A) is positive and higher than (B) , in absolute values: A+B>0, and $f_i(i)>c$.

End of proof [Proposition 2.1.] ¦

A3. Proof of equality (26)

When equality (25) prevails, we have (using the profit expressions from (24a) and (24b)):

$$\begin{aligned} & E(\Pi|i,j,p(j|x)) = E(\Pi|i,j',p(j|x)) \\ & \Leftrightarrow \int_{S_i} (c-(1-p(j|x))\cdot s)\cdot f(x|j)dx + i\cdot F_{X|j}(i) + E_i^{\infty}(X|j) - i = \\ & \quad s\cdot \int_{S_i} p(j|x)\cdot f(x|j')dx + (c-s+i)\cdot F_{X|j'}(i) + E_i^{\infty}(X|j') - i \end{aligned} \qquad \text{(A2-1)}$$

With (from equation (3)): $p(j|x) = \dfrac{f(x|j)\cdot p}{f(x|j)\cdot p + f(x|j')\cdot(1-p)}$

Which can be rewritten as follows (using truncated moments):

$$\begin{aligned} & s\cdot\left(F_{X|j'}(i) + \int_{S_i} p(j|x)\cdot(f(x|j) - f(x|j'))dx - f(x|j)dx\right) = \\ & E_i^{\infty}(X|j') - E_i^{\infty}(X|j) + i\cdot\left(F_{X|j'}(i) - F_{X|j}(i)\right) + c\cdot\left(F_{X|j'}(i) - \int_{S_i} f(x|j)dx\right) \end{aligned} \qquad \text{(A2-2)}$$

Replacing the value of $p(j|x)$ by its Bayesian expression (equation (3)), we finally obtain:

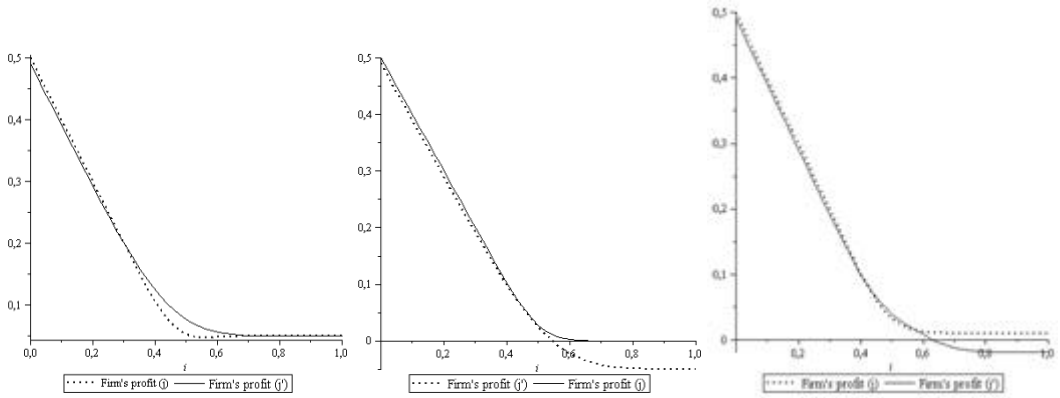
$$s = \frac{E_i^{\infty}(X|j') - E_i^{\infty}(X|j) + i\cdot\left(F_{X|j'}(i) - F_{X|j}(i)\right) + c\cdot\left(F_{X|j'}(i) - \int_{S_i} f(x|j)dx\right)}{F_{X|j'}(i) - \int_{S_i} \dfrac{f(x|j)\cdot f(x|j')}{p\cdot f(x|j) + (1-p)\cdot f(x|j')}dx} \qquad \text{(A2-3)}$$

End of proof [Equality (26)] ¦

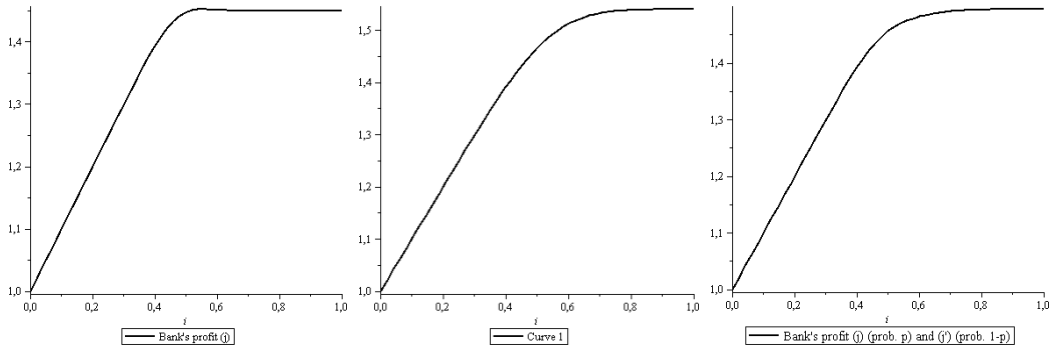
A4. Evolution of the expected profits with (i): an example

For each value of the interest rate (i), graphs I to III provide the evolution of the Firm's expected profit [a] at equilibrium and [b] after deviation[41]: Graphs I, II, III are respectively for equilibriums E1, E2, E3. Graphs IV to VI show the corresponding evolution of the Bank's expected profits. The considered values are: $c = 0.05$; $E(X|j) = 0.50$; $E(X|j') = 0.49$; $\sigma_j = 0.07$; $\sigma_{j'} = 0.14$; $s = 10\%$; $p = 50\%$.

Graph I: Firm's profits (E1) **Graph II: Firm's profits (E2)** **Graph III: Firm's profits (E3)**



Graph IV: Bank's profit (E1) **Graph V: Bank's profit (E2)** **Graph VI: Bank's profit (E3)**



[41] When strategies are mixed (E3), both profits should be equal.

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