Optimal Takeover Contests with Toeholds*

by

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Abstract

This paper characterizes how a target firm should be sold when raiders have prior stakes in its ownership (toeholds). We find that the optimal mechanism needs to be implemented by a non-standard auction which imposes a bias against buyers with high toeholds. This discriminatory procedure is so that the target's average sale price is increasing in both the size of the common toehold and the asymmetry in these stakes. Furthermore, a simple negotiation-based mechanism replicates the main properties of the optimal procedure and outperforms, in terms of average selling price, the standard auctions commonly used in takeover battles.

Keywords: optimal auctions, takeovers, toeholds, asymmetric auctions

JEL Classification: C72, D44, D82, G32, G34

1 Introduction

Target firms often face a takeover threat from raiders with prior stakes in its ownership (toeholds). This rises interesting questions regarding the selling procedure that the board of directors -or any special committee on behalf of non-bidding shareholders - should use to extract the highest price from the potential buyers. These concerns arise because, unlike standard auctions, the presence of toeholds introduces countervailing incentives on bidders, as they can get a payoff not only when they win, but also when they lose the takeover contest. In fact, since the losing bidder owns a proportion of the seller's surplus, he cares about the payment *received* by the seller. As the winner bidder must buy all the shares, losing transforms a bidder with a toehold into a *minority seller*.

This implies that, conditional on losing, a toehold induces a *more* aggressive bidding behavior. In addition, holding stakes in the target firm also means, by comparison with the outside bidders, lower costs of overbidding when winning as the amount of shares to be bought is smaller. Consequently, toeholds strengthen the standard incentive to increase bids present in any auction, but now with the intention of selling at a higher price.

Previous literature (see below) has concluded that, in the context of takeover battles, this more aggressive bidding behavior leads to the break-down of the equivalence of standard auctions in terms of the target selling price that they can attain.[1] Further, this selling price equivalence no longer holds even when raiders possess symmetric stakes. Nonbidding shareholders -by means of the board of directors or a special committee- should therefore pay special attention to the mechanism used to sell a target company.

From this, the current paper deals with the issue of how to run a takeover battle in the presence of toeholds. Consequently, we analyze how the maximizing target price mechanism should be and how it could be implemented. In sharp contrast with the existing literature, our work is, to the best of our knowledge, pioneering in that it adopts a *normative* approach rather than a positive one. Thus, instead of taking a particular auction format as given for exogenous reasons, we characterize how the optimal selling procedure should be. To this end, our methodology follows the mechanism design approach, introduced by Myerson (1981), assuming that each potential buyer has a particular synergy-based value associated to run the firm.[2]

[1] This is a classical result in auction theory: the so-called Revenue Equivalence Theorem (Myerson 1981, and Riley and Samuelson 1981).

[2] In the auction literature this framework is called the independent private values (IPV) setting. In the context of takeovers, this setting is applying to *trade* buyers, who derive idiosyncratic gains from taking over a company. Alternatively, one may consider *financial* buyers, who derive gains from restructuring strategies that their rival bidders would also implement after taking over the firm. In the auction theory terminology, the last situation is better modeled under the so-called common value (CV) setting.

Two main features of our model are the possibility of asymmetry among bidders' toeholds and the existence of a bidder without toeholds (outside bidder). In this setting, we find that the optimal allocation rule imposes no bias against any bidder as the presence of toeholds only links bidders' *payments*, but *not* bidders' *valuations*. As a consequence, a maximizing revenue seller prefers a symmetric equilibrium even though buyers hold asymmetric stakes. It is shown, however, that this optimal rule needs to be implemented by a non-standard auction. In particular, we prove that the implementation is possible through a second-price auction augmented with a reserve price and a scheme of asymmetric payments. The latter includes a penalty against the winner (with respect to the non-toehold case) and a payment by the loser whenever he is a toehold bidder.[3]

Our discriminatory policy has the following rationale. By imposing a heavier bias against the toehold bidder, the optimal mechanism extracts more surplus from the strongest player in the game. In the context of takeovers, this advantaged player corresponds to the raider who bids more aggressively due to his larger stake in the target. As a result, it pays the seller to adopt this discriminatory rule, as we show that

[3] Technically speaking, the reason for this apparent contradiction between the allocation rule and the scheme of payments is the same as that behind the failure of the Revenue Equivalence Principle. That is, the presence of toeholds implies the impossibility of fully characterizing the revenues based only on the allocation rule and the payment made by the lowest-type bidder. Thus, with toeholds it is the *entire* system of transfers which plays a role when it becomes to characterizing revenues.

the expected selling price is increasing in both the common toehold (the symmetric case), and the degree of asymmetry in these stakes (the asymmetric case).[4]

In addition, we show that a sequential negotiation procedure replicates the main properties of the optimal mechanism. This negotiation-based procedure sets an agenda of take-it-or-leave-it offers that gives priority to the more aggressive bidder, i.e., the toeholder. Moreover, it yields a higher expected sale price than the formats commonly used in real takeover battles as the first-price and the second-price auctions. It is worthy to note that this alternative procedure has some appealing and realistic properties.[5] First, in practice we observe rounds of one-to-one negotiations in which the board of directors attempts to close an exclusive deal. Also, it is usual that the board of directors conducts this process under the threat of excluding its counterpart in subsequent negotiations if there is no agreement.

Second, the use of this negotiation-based procedure may shed light on the puzzle regarding the low frequency of toeholds observed in real takeover processes (see Betton et al. 2008, and Bradley et al. 1988). Prior explanations to this puzzle and the optimal acquisition of these ownership stakes are based upon managerial resistance costs (Betton et al. 2008); managerial entrenchment and costs of toeholds when a

[4]This revenue-increasing property of an optimal discriminatory policy has also been found in contests with asymmetric informed buyers (see Povel and Singh 2004, and Povel and Singh 2006).

[5]See in Appendix A a real-world takeover contest with toeholds that illustrates how our proposed mechanism could substantially improve the outcome of the selling process.

takeover fails (Goldman and Quian 2005); and optimal surplus extraction from subsequent raiders by the target firm and an alliance partner (Mathews 2007). As an alternative hypothesis, our results suggest that bidders should not buy toeholds at all if they anticipate that the selling procedure will be close to the optimal one (a negotiation procedure) and thereby, far away from standard auction formats. Accordingly, they would prefer to preserve a symmetric environment in order to avoid being discriminated with a higher target price. Our model then generates a clear testable prediction on how the size of the toehold should vary with the procedure used to sell a target company. Interestingly, Betton et al. (2008) report that the frequency of zero-toehold bidding is greater in friendly than in hostile takeovers, and greater in process initiated by merger negotiations than by a tender offer. Although they do not specify the mechanism used in each of these cases, one can conjecture that negotiations are more frequent than auctions in friendly and merger processes. If this is true, empirical evidence would then be consistent with our prediction about the optimal size of toeholds.

1.1 Related Literature

The auction literature has studied takeovers using different valuation environments, but assuming always that signals are independently distributed. As was commented above, one of the main conclusions of this approach is that the more aggressive bidding behavior induced by toeholds breaks a classical result in auction theory, the so-called Revenue Equivalence Theorem (Myerson 1981, and Riley and Samuelson 1981). As a consequence, the equivalence in terms of target's average sale price between standard auctions no longer holds, as several papers have shown. In particular, Singh (1998), when analyzing a game in which a toehold bidder and an outside bidder compete to gain control of a company in a private values framework, has shown the superiority of the second-price auction over the first-price auction. The major insight stemming from his model is what he calls the *owner's curse*. According to this phenomenon, the higher aggressiveness of the toeholder is so that in the second-price auction he is (rationally) willing to bid more than his valuation. Since such an overbidding behavior is absent in the first-price auction due to the traditional trade-off present in this mechanism, the non-equivalence between both standard auctions emerges.[6] Ettinger (2002) confirms this result in a contest in which buyers have *symmetric* stakes in the seller's surplus, finding the same sale price dominance of the second-price auction over the first-price format.

Bulow et al. (1999) also study a two-bidder takeover contest, but under a common value set-up.[7] They compare the sealed-bid first-price and the ascending-price (equiv-

[6] This is the trade-off that bidders face between payoff from and chances of winning when forming their bids in a first-price auction.

[7] They study takeovers among financial bidders for which, as the authors point out, the common values environment seems more appropriate.

alent to the second-price one) auctions in both the symmetric and the asymmetric cases. They show that with symmetric toeholds, the ascending auction performs better than the first-price auction in terms of the expected selling price per share. In contrast, when analyzing the asymmetric case, they find the opposite result whenever toeholds are very asymmetric and sufficiently small.[8]

The features of our proposed procedure are in line with the established superiority of sequential mechanisms which give priority to stronger bidders. Povel and Singh (2006), for instance, analyze takeover contests under a general value setting that allows both private and common value environments. They characterize the optimal selling procedure that a target company should design when it faces two outside bidders (without toeholds) who are asymmetrically informed. Interestingly, Povel and Singh also conclude about the optimality of imposing a heavier bias against the strongest bidder (the better-informed one) by means of a two-stage procedure. Similarly, Dasgupta and Tsui (2003) examine in an interdependent value setting the properties of the "matching auction", a sequential procedure where the first mover is also the strong bidder. In their model, the strong player can be either the larger-toehold bidder or the better-informed one. As with our sequential procedure, Dasgupta and Tsui also find that the matching auction allows the target's seller to obtain

[8] These contrasting findings rest on two facts. First, the negative effect of the winner's curse on bidders' aggressiveness is larger in asymmetric ownership structures. Second, the first-price auction involves an allocation rule that is less sensitive to the distortions caused by the presence of toeholds.

a higher expected transaction price than with the standard auctions, but only when asymmetry is sufficiently large. An important difference between the last two papers and ours, apart from the valuation environment adopted, lies in the mechanism itself, which implies bidders' participation strategies of different nature. Povel and Singh (2006) propose a hybrid sequential procedure that combines standard auctions and exclusive deals. Similarly, in the auction-based mechanism studied by Dasgupta and Tsui (2003), the bidder moving first *actively* follows a bid strategy, whereas the one moving second only decides whether or not to match this bid. In contrast, our procedure is based upon a scheme of take-it or leave-it offers made by the seller so that all bidders are in some sense *passive* players.

This paper proceeds as follows. Section 2 sets up a model of takeover contests in the presence of toeholds. Under this framework, Section 3 characterizes the optimal selling mechanism, and establishes its main properties. The next section proposes a simple negotiation procedure that replicates most of these properties. Section 5 compares this negotiation-based mechanism with the auction formats commonly used in practice. Finally, Section 6 concludes and stresses some policy implications. All the proofs and an illustrative real-world example are collected in the Appendix.

2 The Model

The nonbidding shareholders of a target company (the *seller*), represented by the board of directors or a special committee, face a takeover threat from two possible risk-neutral buyers (the *bidders*). The value of the target to bidder i is t_i, which is private information, but it is common knowledge that it is independently and identically drawn according to c.d.f. F with support $\left[\underline{t}, \bar{t}\right]$, density f and hazard rate $H(t_i) = f(t_i)/(1 - F(t_i))$.[9] We denote the value that the initial shareholders assign to the target company by t_0, which is common knowledge and is here normalized to zero.[10]

A toehold of bidder i is defined as a partial participation of this bidder in the seller's surplus, or, equivalently, a partial participation in the ownership of the target company. We assume that bidder 1 has a larger initial stake in the seller's surplus than bidder 2. The parameter ϕ_i represents the share of bidder i in the seller's surplus. Thus, $(1 - \phi_1 - \phi_2)$ represents the participation of the seller in her surplus. Toeholds are assumed common knowledge, with $1/2 > \phi_1 \geq \phi_2 \geq 0$.[11]

[9] As it is standard in auction theory, we concentrate on the regular case, that is, increasing hazard rates.

[10] As we will see below, the seller may not be an exclusive initial owner.

[11] Notice that this formulation allows the presence of an outside bidder (non-toeholder), which is precisely the case analyzed in Section 4, given its predominance in actual takeovers. Bradley, et al. (1988) find that 66% of the bidders in their sample of 236 successful tender offers have zero toeholds,

We will also refer to the players as follows: a bidder with toehold as a *bidding shareholder* (or *toehold bidder*), a bidder without toehold as an *outside bidder* (or *non-toehold bidder*) and the seller as the *nonbidding shareholder*. Given this ownership structure, we interpret t_0 as the common value that all shareholders assign to the firm when they own it partially. In other words, t_0 represents the value that all shareholders assign to the firm under the current management, i.e., either before the takeover takes place or when this process is finally unsuccessful. In contrast, we understand t_i to be the private value that bidder i assigns to the target when he owns it fully. In consequence t_i can be interpreted as a private synergy that bidder i can exploit when he wins the contest and obtains absolute control of the company. It is also called the value "to run the firm".[12] Implicit in this interpretation is the assumption that the takeovers modeled in the present paper are not partial. That is, all shareholders must sell their stakes to the winning contestant (and he must buy it) according to the price stated by the contest's rules.

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while Betton and Eckbo (2000) find that 47% of initial bidders in their sample of over 1,300 tender offers (including failed ones) have zero toeholds (see Goldman and Qian 2005).

[12] Alternatively, since we have normalized $t_0 = 0$, t_i can be interpreted as an incremental cash flow generated by the new control and management under bidder i (See Singh 1998).

3 The Optimal Mechanism

Due to the revelation principle, we only need to focus on direct revelation mechanisms. We denote the vector of signal realizations of all bidders by t, i.e., $t = (t_1, t_2)$, and similarly, denote by t_{-i} the vector of signal realizations of all bidders except bidder i. Let T and T_{-i} be the support of t and t_{-i}, respectively.[13] Let us define $p_i(t)$ as the probability with which the optimal mechanism allocates the target company to bidder i, conditional on the vector of reported signal realizations t, and, define $x_i(t)$ as the transfer from bidder i to the seller, conditional on the same vector. Let $Q_i(t_i)$ be bidder i's conditional probability of winning given that his type is t_i, i.e., $Q_i(t_i) \equiv \int_{T_{-i}} p_i(t_i, t_{-i}) f(t_{-i}) dt_{-i}$. Bidder i's expected payoff, conditional on signal t_i and announcement $\widehat{t_i}$, is then given by[14]

$$U_i(\widehat{t_i}/t_i) \equiv \int_{T_{-i}} \left[(t_i p_i - (1 - \phi_i) x_i) + \phi_i x_j \right] f(t_{-i}) dt_{-i}$$

for all $t_i, \widehat{t_i} \in \left[\underline{t}, \overline{t}\right]$ and for $i, j = 1, 2, i \neq j$. We define bidder i's truthtelling payoff as $V_i(t_i) \equiv U_i(t_i/t_i)$ and the seller's expected revenue when all bidders report their true

[13] In our set-up t_{-i} is just t_j. We have opted for the notation t_{-i} since the characterization of the optimal mechanism can be easily extended to the case of more than two bidders. For the three-bidder case (two asymmetric toeholders and one outside bidder) the characterization can be obtained from the author upon request.

[14] For the sake of presentation, we have omitted the arguments of p_i and x_i, but it should be clear that $p_i = p_i(\widehat{t_i}, t_{-i})$ and $x_i = x_i(\widehat{t_i}, t_{-i})$, for all i.

type as follows[15]

$$U_0 \equiv \sum_{i=1}^{2} \int_T (1 - \phi_1 - \phi_2) x_i(t) f(t) dt. \tag{1}$$

Let us define $c_i(t_i)$, bidder i's marginal revenue,[16] as

$$c_i(t_i) \equiv t_i - \frac{1}{H(t_i)} \text{ for all } i.$$

Following Myerson (1981) (see more details in the Appendix), it can be shown that the optimal mechanism solves the following program:[17]

$$\max_{p_i, V_i(\underline{t})} \sum_{i=1}^{2} \left[-V_i(\underline{t}) + \int_T c_i(t_i) p_i(t) f(t) dt \right] \tag{2}$$

$$s.t.$$

$$V_i(\underline{t}) \geq 0, \text{ for all } i. \tag{3}$$

$$Q_i'(t_i) \geq 0 \text{ for all } t_i \in \left[\underline{t}, \bar{t}\right] \text{ and for all } i. \tag{4}$$

$$\sum_{i=1}^{2} p_i(t) \leq 1 \text{ and } p_i(t) \geq 0, \text{ for all } i \text{ and for all } t \in T, \tag{5}$$

where (3) and (4) are sufficient conditions for the participation and incentive compatibility constraints of bidders to hold, respectively, and (5) represents the feasibility

[15] This function is similar to that defined for the nonbidding shareholders by Bulow, et al. (1999) in the context of a takeover contest with *common* values.

[16] Bulow and Roberts (1989) provide an interpretation of $c_i(t_i)$ as bidder i's *marginal revenue*, instead of the *virtual valuation* concept defined by Myerson (1981).

[17] Notice that this problem is identical to the optimization program in Myerson (1981), who does *not* consider the presence of toeholds.

constraints of this problem.

3.1 Optimal allocation rule

Lemma 1. *The optimal mechanism sets* $V_i(\underline{t}) = 0$ *and*

$$p_i(t) = \begin{cases} 1 & \text{if } c_i(t_i) > \max\{0, \max_{j \neq i} c_j(t_j)\} \\ 0 & \text{otherwise} \end{cases}$$

for all i, *and for all* $t \in T$.

Note that bidder i's marginal revenue is larger than bidder j's if and only if $t_i > z_{ij}(t_j) \equiv c_i^{-1}(c_j(t_j))$ for all $i \neq j$. In addition, let us define $t_i^* \equiv c_i^{-1}(0)$ as the threshold signal for which bidder i's marginal revenue is larger than the seller's. Since c_i is well-behaved, so it is its inverse function, it is thus equivalent to say that the optimal mechanism sets $V_i(\underline{t}) = 0$ and

$$p_i(t) = \begin{cases} 1 & \text{if } t_i > \max\{t_i^*, \max_{j \neq i} z_{ij}(t_j)\} \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

for all i, and for all $t \in T$.

Lemma 1 establishes that, in the presence of toeholds, the optimal allocation rule is *not* a discriminatory one as the policy function satisfies that $z_{ij}(t_j) = t_j$ as $c_i(.) = c(.)$ for all bidders.[18] This implies that even though bidders possess asymmetric toeholds, it is revenue maximizing for the nonbidding shareholders to offer them

[18] In the terminology introduced by Bulow and Roberts (1989), all bidders exhibit the same marginal revenue function for the seller, who is interpreted as a monopolist.

the same chances of winning whenever they report the same signal value. This result is surprising because one would expect that, since a toehold induces a more aggressive bidding behavior, the seller should take it into account to design the optimal rule. Our interpretation is that, as opposed to horizontal crossholdings (see Loyola 2007), toeholds only impose links between bidders' *payments*, but *not* between bidders' *valuations*. Consequently, in the terminology of Bulow and Roberts (1989), the marginal revenue function (which depends only on valuations) is the same for all bidders. This implies that the seller perceives all bidders as symmetric players, and hence, it is optimal to impose no bias and to attain a symmetric equilibrium.

However, as we will see in the next subsection, this optimal symmetric equilibrium requires the seller to introduce an asymmetry into the payment scheme. The underlying rationale for this apparent contradiction between the allocation rule and the scheme of transfers is the same as the one behind the break-down of the Revenue Equivalence Principle. That is, when toeholds exist, revenues *do* depend on the entire payment scheme, not only on the transfers made by the lowest type bidder. As a result, it does not suffice to examine only the allocation rule to state the properties of the optimal mechanism. In fact, one needs to characterize the payment scheme fully as this is crucial in order to recognize the non-standard and discriminatory nature of the optimal selling procedure.

3.2 Implementation

Since all bidders provide the same marginal revenue, the implementation of the optimal allocation rule requires a scheme of payments that induce an efficient allocation, that is, one which guarantees that the target firm be awarded to the bidder who values it the most. Since we have assumed that players are asymmetric in their toeholds, and thus in their expected payoff functions, the only way to attain an efficient allocation is to design a scheme of "*personalized*" payments. This implies that we must rule out any standard auction, as it imposes symmetric payments on the players and thus results in an asymmetric and inefficient equilibrium. This fact is formalized in the next corollary.

Corollary 1. *A standard auction cannot implement the optimal selling mechanism.*

From the incentive compatible constraint, we show next that the optimal allocation rule can be implemented by a selling mechanism with an asymmetric scheme of transfers.

Proposition 1. *In the presence of toeholds, the optimal mechanism can be implemented by a modified second price auction with a reserve price and a scheme of payments that includes a penalty against the winner and a payment by the loser. The* *scheme is the following one:*

$$x_i(t) = \begin{cases} z_i(t_{-i}) + [\delta_i - 1]\, z_i(t_{-i}) & \text{if } p_i(t) = 1 \\ \pi_i z_j(t_{-j}) & \text{if } p_i(t) = 0 \text{ and } p_j(t) = 1 \\ 0 & \text{otherwise} \end{cases}$$

for all $i, j = 1, 2$, $i \neq j$, *and for all* $t \in T$, *where*

$$\delta_i \equiv \frac{1 - \phi_j}{(1 - \phi_i - \phi_j)}, \quad \pi_i \equiv \frac{\phi_i}{(1 - \phi_i - \phi_j)},$$

and $z_i(t_{-i}) = \inf \{s_i : c_i(s_i) \geq 0 \text{ and } c_i(s_i) \geq c_j(t_j)\}$.

This scheme of payments has the following properties.

Discriminatory policy with winning penalties and losing payments. First, since $z_i(t_{-i}) > 0$ and $\delta_i \geq 1$, this implies that when the winner is a bidder with toeholds, his payment has a *penalty* when compared to the payment he would make in case of holding no toeholds. This penalty is given by $[\delta_i - 1]\, z_i(t_{-i})$. Second, since $z_j(t_{-j}) > 0$, and $\pi_i \geq 0$, this means that when the loser is a bidder with an initial stake, his payment is *positive*. Third, from $\phi_1 > \phi_2$, it follows that $\delta_1 > \delta_2$ and $\pi_1 > \pi_2$. Thus, it is clear that the scheme of transfers proposed imposes a discriminatory policy with a bias against the bidder with the largest initial stake.[19]

Truthtelling and efficient mechanism. The discriminatory scheme of winning penal-

[19] Moreover, this discriminatory policy gets exacerbated with the degree of asymmetry, as the gaps of both winning penalties and losing payments are increasing with the difference in toeholds.

ties and losing payments implies that bidder i's payoff simplifies to

$$\pi_i(t_i) = \begin{cases} t_i - z_i(t_{-i}) & \textit{if } p_i(t) = 1 \\ 0 & \textit{otherwise} \end{cases}$$

The scheme of transfers therefore induces symmetric objective functions for all bidders, as in the standard problem when there are no toeholds (see Myerson 1981). In this context, the rules of a second-price auction with reserve price guarantee that the proposed mechanism is *truthtelling* (all possible raiders bid their true valuations) and *efficient* (the target firm is sold to the raider who values it the most).

Average sale price increasing with common toeholds and asymmetry. First, let Π_0^* be the seller's expected revenue under the optimal mechanism, and hence, define $\rho_0^* \equiv \Pi_0^*/(1-\phi_1-\phi_2)$, the average sale price under the same procedure. From (1) and Proposition 1, it follows directly that Π_0^*, and thus ρ_0^*, are increasing with both the winning penalty and the losing payment. Second, consider the symmetric toeholds case (i.e. $\phi_1 = \phi_2 = \phi > 0$). In this case, both the winner's penalty and the loser's payment are increasing in the common toehold, as it is easy to check that $\partial\delta_i/\partial\phi > 0$ and $\partial\pi_i/\partial\phi > 0$ for all i. All of this implies that, at the optimal mechanism, the seller's expected revenue (and thereby, the average sale price) is *increasing* with the size of common toeholds. Finally, consider the asymmetric toeholds case (i.e. $\phi_1 > \phi_2 > 0$). Let us define $\varepsilon \equiv \phi_1 - \phi_2$ so that the parameters of the winning penalty and the losing payment can be rewritten as

$$\delta_1 = \frac{1-\phi_2}{1-2\phi_2-\varepsilon}, \ \ \delta_2 = \frac{1-\phi_2-\varepsilon}{1-2\phi_2-\varepsilon},$$

$$\pi_1 = \frac{\phi_2+\varepsilon}{1-2\phi_2-\varepsilon}, \ \ \pi_2 = \frac{\phi_2}{1-2\phi_2-\varepsilon}.$$

Hence, it is easy to verify that $\partial\delta_i/\partial\varepsilon > 0$ and $\partial\pi_i/\partial\varepsilon > 0$ for all i. Therefore, the optimal mechanism is such that the seller's expected revenue (and thus, the average sale price) is *increasing* with the degree of asymmetry in toeholds. All of this implies that it pays the seller to impose a discriminatory policy.

4 A Sequential Negotiation Procedure

In this section we show that a simple sequential negotiation procedure replicates the main properties of the optimal mechanism. The negotiation procedure works as follows:

Stage I

I.1. The seller makes a take-it-or-leave-it offer ρ_1 to bidder 1, where the offer ρ_i is the price to be paid by bidder i for the target shares.

I.2. Bidder 1 accepts or rejects this offer. If he accepts, the target is sold to him and the game is over. If bidder 1 rejects the exclusive deal, negotiation moves to the next round.

Stage II

II.1. The seller makes a new take-it-or-leave-it offer ρ_2 to bidder 2.

II.2. Bidder 2 accepts or rejects this offer. If he accepts, the target is sold to him. Otherwise, the target company remains under the current ownership structure and management.

The next proposition illustrates the discrimination policy resulting from the negotiation procedure when valuations are uniformly distributed.[20]

Proposition 2. *Suppose that t_i is uniformly distributed on the interval $[0,1]$ for all $i = 1, 2$. At the Subgame Perfect Nash equilibrium of the game induced by the sequential negotiation procedure, it is optimal for the seller to set $\rho_1^* > \rho_2^*$ for all $\phi_1 \geq \phi_2 \geq 0$.*

With sequential negotiations the sale price charged to the first bidder is higher than the one charged to the second bidder. As the first-mover is the buyer with the highest toehold, the sequential mechanism discriminates against him. Moreover, the degree of this bias *increases* with the degree of asymmetry in the toeholds. More precisely, if we define the degree of asymmetry by $\varepsilon \equiv \phi_1 - \phi_2$, then the difference in prices offered by the seller, i.e., $\Delta\rho(\phi_2, \varepsilon) \equiv \rho_1^* - \rho_2^*$, is increasing in ε. To see this

[20] For simplicity and without loss of generality, all the results in the paper are henceforth stated assuming uniformly distributed valuations on the unitary interval.

note that

$$\Delta\rho(\phi_2, \varepsilon) \equiv \rho_1^* - \rho_2^* = \frac{1 - 2\phi_2 + 4\varepsilon}{8(1 - (\phi_2 + \varepsilon))(1 - \phi_2)},$$

so that $\partial\Delta\rho(\phi_2, \varepsilon)/\partial\varepsilon > 0$. Notice also that $\Delta\rho(\phi_2, \varepsilon)$ is strictly increasing in ϕ_1 and strictly decreasing in ϕ_2, with $\Delta\rho(\phi_2, \varepsilon)$ strictly increasing in ϕ_2 for fixed and given ε. Hence, the negotiation procedure highlights the importance of establishing an asymmetric scheme of payments, as the price charged to the high-toehold bidder exceeds that of the low-toehold one, and this bias is larger when the ownership stakes become more asymmetric.

To analyze whether it pays the seller to adopt this price discrimination policy, we must look at the average sale price delivered by the equilibrium of the sequential procedure. Let Π_0^{SN} be the seller's expected revenue under the sequential procedure, and consequently, define $\rho_0^{SN} \equiv \Pi_0^{SN}/(1 - \phi_1 - \phi_2)$, the average sale price under the same mechanism.[21] Rewriting ρ_0^{SN} in terms of $\varepsilon = \phi_1 - \phi_2$, it follows that

$$\rho_0^{SN} = \frac{1}{16(1 - \phi_2)^2} \left[\frac{(5 - 6\phi_2)^2}{4(1 - \phi_2 - \varepsilon)} + \phi_2 + \varepsilon \right].$$

It is easy to verify that $\partial\rho_0^{SN}/\partial\varepsilon > 0$ for all $\phi_2, \varepsilon \in (0, 1/2)$ so that the average sale price is *increasing* in the *degree of asymmetry*. This result is displayed in Figure 1.

Furthermore, similarly to the optimal mechanism in the *symmetric* case, the afore-defined sequential procedure yields an average sale price which is also *increasing* in the

[21] See the Appendix (Proof of Proposition 2) for details on how this average price is computed.

common toehold. In fact, when $\phi_1 = \phi_2 = \phi$, it is possible to check that $\partial \rho_0^{SN} / \partial \phi > 0$ for all $\phi \in (0, 1/2)$, as it is illustrated in Figure 2.

Notice however, that unlike the optimal mechanism described in the previous section, the sequential procedure always discriminates against the bidder moving first, even if the toeholds are symmetric or zero. In fact, as the proof of Proposition 2 establishes, the prices charged to both players in the symmetric case (i.e., $1/2 > \phi_1 = \phi_2 = \phi \geq 0$) satisfy the following inequality

$$\rho_1^* = \frac{5 - 6\phi}{8(1-\phi)^2} > \frac{1}{2(1-\phi)} = \rho_2^*.$$

In addition, the different priorities given by the negotiation timetable to different buyers implies that, unlike the optimal procedure, the sequential mechanism may be ex post *inefficient*.

In sum, and despite these differences, our sequential procedure replicates the two most important properties of the optimal mechanism: the expected selling price is increasing both in the common toehold and in the degree of asymmetry in the initial stakes held by bidders.

5 Sequential Procedure vs. Auctions

Although there is not a specific practice to sell a company, sometimes the legal framework implicitly induces the board of directors to conduct an auction among the raiders.[22] The underlying rationale behind this recommendation is the idea that an auction run with several bidders at once offers a more competitive environment than a negotiation held with a single buyer at each round. Nevertheless, and despite this idea, the coexistence of both types of mechanisms in real world takeover processes has been widely documented.[23] In this section we compare the sequential procedure to the auction formats commonly used in practice from the nonbidding shareholders' point of view. We show that the nonbidding shareholders benefit from the discrimination policy to the extent that the sequential procedure generates a higher expected selling price than both the first-price and the second-price auctions.

We analyze here two ownership structures in which this result holds: (i) the symmetric case, i.e. $\phi_1 = \phi_2 = \phi \geq 0$, and (ii) a particular asymmetric case in which there are two classes of bidders: one toeholder and one outsider, i.e., $\phi_1 > \phi_2 = 0$.[24] For both of these ownership structures, the literature provides a ranking between the

[22] For instance, the Delaware law in the US establishes that the board must act as "*auctioneers* charged with getting the best price for the stock-holders at a sale of the company". See also Cramton and Schwartz (1991).

[23] See the evidence provided by Boone and Mulherin (2003), Boone and Mulherin (2007), Povel and Singh (2006), and Bulow and Klemperer (2007).

[24] As the evidence presented by Bradley et al. (1988), Betton and Eckbo (2000), and Betton et al. (2008) suggests, the presence of an outside bidder is very common in actual takeovers. The symmetric case seems to be relevant as well, as it is usual that initial toehold bidders are challenged by rivals who on average have toeholds of a similar size (see Betton and Eckbo 2000).

first and second price formats. In the second-price auction, under both ownership environments, the toehold bidder exhibits the *owner's curse,* an overbidding behavior according to which his equilibrium bid exceeds his valuation. This overbidding phenomenon is however not present in the case of the outside raider, as bidding his true valuation continues to be a dominant strategy for him. In contrast, given the traditional bidding trade-off present in the first-price auction, the owner's curse is absent in this selling format. Because of this, the second-price auction outperforms the first-price auction in terms of revenue, in both the symmetric and asymmetric structures.[25] As a result, it suffices to compare the selling price generated by the sequential mechanism with that generated by the second-price auction.

The following auxiliary result characterizes the expected selling price in the second-price auction.

Lemma 2. *Let ρ_0^{SPA} be the average sale price resulting from the second-price auction. Then,*

(1) In the symmetric case, this price is given by

$$\rho_0^{SPA} = \frac{(1+2\phi)(1-\phi)}{(1-2\phi)(1+\phi)} - \frac{2}{3(1-2\phi)}.$$

(2) In the asymmetric case, it corresponds to

$$\rho_0^{SPA} = \frac{1}{1-\phi_1}\left[\frac{\phi_1}{\phi_1+1} - \frac{5}{6}\phi_1 - \frac{1}{2\phi_1+2} + \frac{2}{3\phi_1+3} + \frac{1}{6}\right].$$

[25] Ettinger (2002) performs this comparison for the symmetric case, and Ettinger (2005) does so for the specific asymmetric environment analyzed here.

Now, we establish the predominance of our sequential mechanism over the auction formats commonly used in practice, irrespective of the degree of symmetry in toeholds.

Proposition 3. *The sequential procedure generates a higher average sale price than both the first-price and the second-price auctions, no matter the degree of asymmetry.*

As mentioned in the previous section, the sequential procedure always discriminates against the first-mover bidder. This fact implies that it yields a larger expected sale price than both auction formats in the symmetric case, even when there are no toeholds at all. The average sale price comparison for the symmetric case between the second-price auction and our sequential mechanism is depicted in Figure 3. Note from the figure that the second-price auction induces a *concave* average sale price whereas the negotiation procedure exhibits a *convex* one. As a result, the price gap between both mechanisms is larger when the toehold becomes sufficiently low or sufficiently high. The difference attains its minimum for values around .25.

Furthermore, the superiority of our sequential mechanism over auctions is exacerbated in the asymmetric case, as the discriminatory policy involves a sequence of negotiations with a pecking order consistent with the aggressiveness of each buyer (see Figure 4).

Notice that the clear advantage of the sequential mechanism becomes larger when the degree of asymmetry (represented in this case by ϕ_1) goes up. This is a consequence of the fact that whereas ρ_0^{SN} is always an increasing and convex function in

ϕ_1, ρ_0^{SPA} is a concave function and an increasing one *only* for a sufficiently low degree of asymmetry (for all $\phi_1 < .38$).

This last result is formalized in the following statement.

Corollary 2. *The larger the degree of asymmetry, the better the sequential procedure when compared with both the first-price and the second-price auctions.*

Finally, let us mention that our results here are in line with the well-established supremacy of sequential mechanisms which give higher priority to stronger bidders.[26] Accordingly, and in contrast with the standard auction formats, the particular order of negotiations involved in our procedure allows to exploit the higher aggressiveness of raiders with larger stakes.

6 Concluding Remarks

We have characterized how a target firm should be sold when bidders possess prior stakes in its ownership. This optimal mechanism corresponds to a non-standard auction with a scheme of asymmetric payments that imposes a bias against toeholders. The rationale of such a discriminatory policy is the fact that a standard mechanism is unable to induce a symmetric and efficient allocation rule, as it preserves the initial advantage of toehold bidders. In contrast, a scheme of asymmetric winning penalties and losing payments allows both to take advantage of the higher aggressiveness of

[26] See Povel and Singh (2006), and Dasgupta and Tsui (2003).

toeholders and to go back to a symmetric environment.

The presence of losing payments in the optimal procedure is in line with similar results found in the literature devoted to characterizing optimal auctions when externalities exist. For instance, Goeree et al. (2005) show that the positive externalities present in fund-raising activities lead to discarding winner-pay auctions in favor of all-pay formats. In a result reminiscent of ours, they establish the optimality of an auction with a reserve price and payments by the losers - a mix between participation fees and an all-pay auction run in a subsequent stage-, which depend on the degree of the externality. Moreover, Goeree et al. (2005) emphasize that some characteristics of this optimal procedure are present in the procedures used for raising funds in the real world. As a consequence, the characteristics of our non-standard auction in the takeover case are not far from those exhibited by the optimal procedure in other contests with externalities.

We have also demonstrated that the nonbidding shareholders benefit from the discriminatory mechanism, as the target average sale price is increasing both in the common toehold and in the degree of asymmetry in these stakes. The latter finding is in sharp contrast with the properties of standard auction formats in takeover battles, which then leads to opposite policy implications. For instance, Bulow et al. (1999) show that in general the asymmetry in toeholds lowers prices in common-value ascending auctions. As a result, they recommend the "level the playing field" practice, according to which it may be revenue increasing to sell toeholds very cheaply to the buyer with the smaller stake in the target. On the contrary, our normative approach suggests that the seller should follow strategies with the aim of preserving this asymmetry. Accordingly, the board of directors should block or discourage the entrance of new shareholders suspected of becoming competitors against the incumbent toeholder in a future takeover battle.

As an alternative to the optimal non-standard auction-based mechanism, we have proposed a simpler and realistic negotiation procedure that replicates the main properties of the first one. This mechanism contains a timetable that gives priority to the higher-toehold bidders, but charges higher prices to them. Such a negotiation-based procedure shares some features of other selling procedures already considered in the literature. In particular, it balances out properly the trade-off between creation and extraction of value caused by the implicit *threats* involved in the sequential nature of the negotiation process. This characteristic is also present in the posted-price rule discussed by Campbell and Levin (2006) in an environment with interdependent valuations. These authors find conditions under which a hybrid mechanism of a posted-price rule and a random rationing may outperform standard auctions. This fact occurs essentially when the increase of all buyers' willingness to pay offsets the losses stemming from ex post inefficient allocations. Similarly, in the context of our paper, the individual and sequential feature of the negotiation scheme imposes costs and benefits on nonbidding shareholders. On the one hand, the expected target price decreases due to both less competition and less efficiency. On the other hand, the higher priority given to the high-toehold bidder increases his willingness to pay, as the opportunity of winning the contest emerges even though his valuation may be lower than the small-toehold bidder's one. We have proved that the last effect dominates the shortcomings, therefore keeping open the ongoing debate on auctions versus negotiations in takeover wars.

7 Appendix

Appendix A: A Real-World Example

In order to illustrate some of the features of takeover battles with toeholds, consider the next real life example. In 2006, the Spanish tollway operator Europistas was the target of a takeover battle between two bidders. Firstly, the group Isolux submitted an offer for 100% of the ownership. At this stage, Cintra, one of the principal block shareholders of the target firm, attained an agreement with Isolux. According to the deal, Cintra committed itself to participate in this tender offer and sell irrevocably its 27.1 per cent stake for a price of 5.13 euros per share. In less than 24 hours, a second buyer emerged: a bidding consortium formed by the constructor conglomerate Sacyr and three Basque saving banks grouped in the society Telekutxa. While Isolux was an outside bidder, Telekutxa held a 32.4 per cent stake in the capital of Europistas. The final tender offer of this consortium rose to 9.15 euros per share, that is, an improvement of 78.36% with respect to the first offer. This implied that Cintra was trapped in the pre-sale agreement reached with Isolux, which impeded it from taking advantage of the substantially better tender offer made by the consortium led by Sacyr. Finally, Cintra paid 131 million euros to Isolux as a compensation to recover its freedom to sell its stake to the bidding consortium, which was the winner of the contest and thus, took over Europistas.

This case highlights some interesting issues. First, it allows us to conjecture about the source of the large price difference observed between the two offers. It seems plausible to argue that this gap reflected not only a higher valuation from the *toehold bidder* (the consortium headed by Sacyr), but also a more aggressive bidding behavior than that exhibited by the *outside bidder* (Isolux). Second, this case illustrates the large costs that an incorrect choice of selling procedure may impose on the nonbidding shareholders' wealth, and thus, one of the main motivations of the paper. In fact, the price gap of both tender offers meant a total difference of 147 millions of euros, which represented about eight times the annual net profits of Cintra. Lastly, this case provides a clear example of how things should *not* be done when selling a target firm in which one of the shareholders could become a bidder. Of course, in this case the nonbidding shareholder (Cintra) chose incorrectly to negotiate and close a deal first with the outside bidder (Isolux) instead of doing it previously with the toehold bidder (the consortium). In this paper we show that an appropriate sequential negotiation mechanism should precisely take the *opposed* order of negotiations.

Appendix B: The Optimal Mechanism Problem

The optimal mechanism solves the following problem:

$$\max_{x_i \in \mathbb{R},\ p_i \in [0,1]} \ U_0 \tag{A1}$$

$$s.t.$$

$$V_i(t_i) \ \geq 0 \quad \forall t_i \in \left[\underline{t}, \overline{t}\right], \ i = 1, 2 \tag{A2}$$

$$V_i(t_i) \ \geq U_i(\widehat{t_i}/t_i) \quad \forall t_i, \widehat{t_i} \in \left[\underline{t}, \overline{t}\right], \ i = 1, 2 \tag{A3}$$

$$\sum_{i=1}^{2} p_i(t) \leq 1 \text{ and } p_i(t) \geq 0, \ i = 1, 2, \forall t \in T \tag{A4}$$

where (A1) is the seller's expected revenue, (A2) is bidder i's participation constraint, (A3) represents the incentive compatibility constraints of the bidders and (A4) corresponds to the feasibility constraints.[27] From Myerson (1981), standard substitutions and computations lead to state the equivalence between the incentive compatibility constraints and the following two conditions:

(i) $\frac{\partial V_i(t_i)}{\partial t_i} = Q_i(t_i)$

(ii) $\frac{\partial Q_i(t_i)}{\partial t_i} \geq 0$

[27] Following Jehiel, et al. (1996) and Jehiel, et al.(1999), it is possible to show that the optimal threat for the non-participating bidder is that the target remains under the current management and control. As a result, the outside utility for the lowest-type bidder is the same for all buyers (toeholders and outsiders), and so, it can be normalized to zero (see Loyola 2007, Section 3).

These conditions allow to replace (A3) by (ii) and

$$V_i(t_i) \;= V_i(\underline{t}) + \int_{\underline{t}}^{t_i} Q_i(s_i) ds_i. \tag{A5}$$

Similarly, (A2) is guaranteed to hold if $V_i(\underline{t}) \geq 0$ for all i. Hence, straightforward computations allow us to rewrite the seller's expected payoff and simplify the maximization problem as presented in Section 3.

Appendix C: Proofs

Proof of Lemma 1. From (2), it is in the seller's interest to make $V_i(\underline{t}) = 0$ for all i because $V_i(\underline{t}) > 0$ is suboptimal and setting $V_i(\underline{t}) < 0$ violates the participation constraint. Moreover, $H'(t_i) > 0$ implies that $c_i'(t_i) > 0$ and thereby $\partial p_i(t)/\partial t_i \geq 0$, so that $Q_i'(t_i) \geq 0$ is satisfied for all i. Finally, since $t_0 = 0$, the optimal allocation rule is found by comparing for a given $t = (t_1, t_2)$ the terms $c_i(t_i)$, whenever they are positive. The solution sets then $p_i(t) = 1$ iff $c_i(t_i) > \max\{0, \max_{j \neq i} c_j(t_j)\}$. ■

Proof of Proposition 1. For any vector t_{-i} consider

$$z_i(t_{-i}) = \inf\{s_i : c_i(s_i) \geq 0 \text{ and } c_i(s_i) \geq c_j(t_j) \text{ for all } j \neq i\}$$

for all i, i.e., the infimum of all winning values for i against t_{-i}. Then, in equilibrium

$$p_i(s_i, t_{-i}) = \begin{cases} 1 & \text{if } s_i > z_i(t_{-i}) \\ 0 & \text{if } s_i < z_i(t_{-i}) \end{cases} \tag{A6}$$

and

$$\int_{\underline{t}}^{t_i} p_i(s_i, t_{-i}) ds_i = \begin{cases} t_i - z_i(t_{-i}) & \text{if } t_i \geq z_i(t_{-i}) \\ 0 & \text{if } t_i < z_i(t_{-i}) \end{cases} \tag{A7}$$

for all i. Substitute $Q_i(s_i)$ into (A5), change the order of integration and substitute $V_i(t_i)$. After rearranging, we obtain that the truthtelling payoff of the bidder with the lowest signal can be written as

$$V_i(\underline{t}) = \int_{T_{-i}} \{t_i p_i(t) - [1-\phi_i]\, x_i(t) + \phi_i \sum_{j \neq i} x_j(t) - \int_{\underline{t}}^{t_i} p_i(s_i, t_{-i}) ds_i\} f(t_{-i}) dt_{-i} \tag{A8}$$

for all i and $t_i \in \left[\underline{t}, \overline{t}\right]$. Since it is optimal $V_i(\underline{t}) \ = 0$ for all i, then sufficient conditions for (A8) to hold are:

$$t_i p_i(t) - [1-\phi_i]\, x_i(t) + \phi_i \sum_{j \neq i} x_j(t) = \int_{\underline{t}}^{t_i} p_i(s_i, t_{-i}) ds_i$$

for all i and for all state $t = (t_i, t_{-i})$. If we fix a particular state $t = (t_i, t_{-i})$, two cases are possible: (i) a winning bidder exists, or (ii) the target company is not awarded to any bidder. Applying (A6) and (A7), the solution of this system of equations for these both cases yields the desired scheme of asymmetric payments. ■

Proof of Proposition 2. Using backward induction, we first characterize the Nash equilibrium resulting from Stage II. In this stage, bidder 2 accepts the offer if $t_2 - (1-\phi_2)\rho_2 > 0$, i.e., if $t_2 > (1-\phi_2)\rho_2$, and rejects otherwise. The seller's problem is hence

$$\max_{\rho_2} \left[(1-\phi_1-\phi_2)\rho_2\right]\left[1-(1-\phi_2)\rho_2\right],$$

whose solution is given by $\rho_2^* = 1/2(1-\phi_2)$. The optimal seller's expected revenue from this stage is equal to $(1-\phi_1-\phi_2)/4(1-\phi_2)$.

In stage I.2, bidder 1 accepts any seller's offer if his expected payoff is larger than the expected payoff at the equilibrium of stage II. That is, if $t_1 - (1-\phi_1)\rho_1 > E_{t_2}[\phi_1\rho_2^*] = \phi_1/4(1-\phi_2)$, which is equivalent to the condition $t_1 > (\phi_1/4(1-\phi_2)) + (1-\phi_1)\rho_1$. Thus, the seller's optimal offer is characterized by

$$\begin{aligned}\rho_1^* &= \arg\max_{\rho_1}(1-\phi_1-\phi_2)\rho_1\left[1-\frac{\phi_1}{4(1-\phi_2)}-(1-\phi_1)\rho_1\right] \\ &\quad +\frac{1-\phi_1-\phi_2}{4(1-\phi_2)}\left[\frac{\phi_1}{4(1-\phi_2)}+(1-\phi_1)\rho_1\right].\end{aligned}$$

The solution is given by $\rho_1^* = (5-6\phi_2)/8(1-\phi_1)(1-\phi_2)$, which yields an optimal seller's expected revenue equal to

$$\Pi_0^{SN} = \frac{(1-\phi_1-\phi_2)}{16(1-\phi_2)^2}\left[\frac{(5-6\phi_2)^2}{4(1-\phi_1)}+\phi_1\right],$$

and an average sale price equal to

$$\rho_0^{SN} \equiv \Pi_0^{SN}/(1-\phi_1-\phi_2) = \frac{1}{16(1-\phi_2)^2}\left[\frac{(5-6\phi_2)^2}{4(1-\phi_1)}+\phi_1\right]. \tag{A9}$$

Since $1/2 > \phi_1 \geq \phi_2 \geq 0$, it is simple to verify that

$$\rho_1^* = \frac{5-6\phi_2}{8(1-\phi_1)(1-\phi_2)} \geq \frac{5-6\phi_2}{8(1-\phi_2)^2} > \frac{1}{2(1-\phi_2)} = \rho_2^*,$$

which proves the statement of the proposition. ∎

Proof of Lemma 2. Since the *asymmetric* case is the most general one, we first prove the second part of the proposition. In the second-price auction, bidder 2's payoff function, when his signals is t_2 and he behaves as if it were $\widehat{t_2}$, is given by

$$\pi_2(t_2, \widehat{t_2}) = \max_{\widehat{t_2}} \int_0^{b_1^{-1}(b_2(\widehat{t_2}))} (t_2 - b_1(t))dt, \tag{A10}$$

that is, the traditional payoff function in a second-price auction without toeholds. Consequently, it follows that $b_2(t_2) = t_2$. Given the bid strategies $b_1(.)$ and $b_2(t_2) = t_2$, bidder 1's optimal choice of $\widehat{t_1}$ when he observes t_1 is obtained by maximizing his expected profits

$$\pi_1(t_1, \widehat{t_1}) = \max_{\widehat{t_1}} \int_0^{b_1(\widehat{t_1})} (t_1 - (1 - \phi_1)t)dt + \phi_1 \int_{b_1(\widehat{t_1})}^1 b_1(\widehat{t_1})dt. \tag{A11}$$

From Ettinger (2005), bidder 1's equilibrium bid is given by

$$b_1(t_1) = \frac{\phi_1}{1 + \phi_1} + \frac{t_1}{1 + \phi_1}.$$

Now, in order to compute the seller's revenues, let us define $\psi_j(t_i)$, the equilibrium correspondence function, such that $b_i(t_i) = b_j(\psi_j(t_i))$ for all $i, j = 1, 2$. Applying the definition of $\psi_j(.)$ to the equilibrium bid strategies yields

$$\psi_2(t_1) = \frac{\phi_1}{1 + \phi_1} + \frac{t_1}{1 + \phi_1}, \tag{A12}$$

$$\psi_1(t_2) = -\phi_1 + t_2(1 + \phi_1). \tag{A13}$$

Appealing to the Envelope Theorem, and using the fact that $\psi_2(.) = b_1(.)$ and $\psi_1(.) =$

$b_1^{-1}(b_2(.))$, it can be verified that $\frac{d\pi_i(t_1,\widehat{t_i})}{dt_i} = \psi_j(t_i)$, which implies

$$\pi_i(t_i) = \pi_i(1) - \int_{t_i}^{1} \psi_j(t)dt \tag{A14}$$

for all $i, j = 1, 2$. Evaluating $t_i = 1$ in (A10) and (A11), and using the fact that in equilibrium $\psi_j(\widehat{t_i}) = \psi_j(t_i)$ and $\psi_j(1) = 1$, it can be shown that

$$\pi_1(1) = 1 - \frac{1-\phi_1}{2}, \tag{A15}$$

$$\pi_2(1) = \frac{1}{2(1+\phi_1)}. \tag{A16}$$

Substituting (A12), (A13), and the results (A15) and (A16) into (A14), bidder i's interim payoff becomes

$$\pi_1(t_1) = 1 - \frac{(1-\phi_1)}{2} - \frac{(1-t_1^2)}{2(1+\phi_1)} - \frac{\phi_1(1-t_1)}{(1+\phi_1)},$$
$$\pi_2(t_2) = \frac{1}{2(1+\phi_1)} - 1 + \frac{1+\phi_1}{2} - \phi_1 t_2 + \frac{(1+\phi_1)t_2^2}{2}.$$

After taking expectations, bidder i's ex-ante payoff is given by

$$\Pi_1 = 1 - \frac{(1-\phi_1)}{2} - \frac{1}{3(1+\phi_1)} - \frac{\phi_1}{2(1+\phi_1)},$$
$$\Pi_2 = \frac{1}{2(1+\phi_1)} + \frac{(1+\phi_1)}{6} - \frac{1}{2}$$

The nonbidding shareholders' expected revenues are then given by

$$\begin{aligned}
\Pi_0^{SPA} &= \left[\int_0^1 t_1 \int_0^{\psi_2(t_1)} dt_2 dt_1 + \int_0^1 t_2 \int_0^{\psi_1(t_2)} dt_1 dt_2\right] - \Pi_1 - \Pi_2 \\
&= \left[\int_0^1 t_1\psi_2(t_1)dt_1 + \int_0^1 t_2\psi_1(t_2)dt_2\right] - \Pi_1 - \Pi_2 \\
&= \frac{\phi_1}{\phi_1+1} - \frac{5}{6}\phi_1 - \frac{1}{2\phi_1+2} + \frac{2}{3\phi_1+3} + \frac{1}{6}
\end{aligned}$$

and the average selling price is

$$\rho_0^{SPA} \equiv \Pi_0^{SPA}/(1-\phi_1) = \frac{1}{1-\phi_1}\left[\frac{\phi_1}{\phi_1+1} - \frac{5}{6}\phi_1 - \frac{1}{2\phi_1+2} + \frac{2}{3\phi_1+3} + \frac{1}{6}\right].$$

We now turn to demonstrate the statement for the *symmetric* case. From Proposition 1 in Ettinger (2002), the second-price auction equilibrium bid is given by

$$b_i(t_i) = \frac{\phi}{1+\phi} + \frac{t_i}{1+\phi}$$

for all i. Hence, $\psi_2(t) = \psi_1(t) = t$ for all t. Applying the same line of reasoning used in the asymmetric case, it can be verified that the seller's expected revenues are given by

$$\begin{aligned}\Pi_0^{SPA} &= \left[\int_0^1 t_1^2 dt_1 + \int_0^1 t_2^2 dt_2\right] - 2\left[\frac{2}{3} - \frac{(1+2\phi)(1-\phi)}{2(1+\phi)}\right] \\ &= \frac{(1+2\phi)(1-\phi)}{(1+\phi)} - \frac{2}{3}\end{aligned}$$

and the corresponding average sale price becomes

$$\rho_0^{SPA} = \frac{(1+2\phi)(1-\phi)}{(1-2\phi)(1+\phi)} - \frac{2}{3(1-2\phi)},$$

which completes the proof. ■

Proof of Proposition 3. Consider the symmetric case. Substituting $\phi_1 = \phi_2 = \phi$ into (A9), and using Lemma 2, we can state that

$$\rho_0^{SN} = \frac{32\phi^2 - 56\phi + 25}{64(1-\phi)^3} > \rho_0^{SPA} \geq \rho_0^{FPA}$$

where the second inequality is strict when $\phi > 0$, and follows from Proposition 3 in Ettinger (2002).

Consider now the asymmetric case. Lemma 2 and the substitution of $\phi_1 > \phi_2 = 0$ into (A9) yields

$$\rho_0^{SN} = \frac{1}{16}\left[\frac{25}{4(1-\phi_1)} + \phi_1\right] > \rho_0^{SPA} > \rho_0^{FPA}$$

where the last inequality holds as overbidding is not present in the first-price auction.■

8 References

Bradley, M., A. Desai, and E.H. Kim, 1988, "Synergistic Gains from Corporate Acquisitions and Their Division between The Stockholders of Target and Acquiring Firms", *Journal of Financial Economics*, 21, 3–40.

Betton, S. and B. E. Eckbo, 2000, "Toeholds, Bid Jumps, and Expected Payoff in Takeovers", *Review of Financial Studies*, 13, 841-882.

Betton, S., B. E. Eckbo and K. S. Thorburn, 2008, "Merger Negotiations and The Toehold Puzzle", *Journal of Financial Economics*, forthcoming.

Boone, A. and H. Mulherin, 2003, "Corporate Restructuring and Corporate Auctions", mimeo, University of Kansas and University of Georgia.

Boone, A. and H. Mulherin, 2007, "How Are Firms Sold?", *Journal of Finance*, 62, 847-875(29).

Bulow, J., M. Huang and P. Klemperer, 1999, "Toeholds and Takeovers", *Journal of Political Economy*, 107, 427-454.

Bulow, J. I. and P. Klemperer, 2007, "When are Auctions Best?", Discussion Paper 6393, Discussion Paper Series, CEPR.

Bulow, J. I. and J. Roberts, 1989, "The Simple Economics of Optimal Auctions", *Journal of Political Economy*, 97, 1060-90.

Campbell, C.M. and D. Levin, 2006, "When and Why Not to Auction", *Economic Theory*, 27, 583-596.

Cramton, P. and A. Schwartz, 1991, "Using Auction Theory to Inform Takeover Regulation", *Journal of Law, Economics, and Organization*, 7, 27-53.

Dasgupta, S. and K. Tsui, 2003, "A 'Matching Auction' for Targets with Heterogeneous Bidders", *Journal of Financial Intermediation*, 12, 331-364.

Ettinger, D., 2002, "Auctions and Shareholdings", working paper, CERAS, Paris, France.

Ettinger, D., 2005, "Takeover Contests, Toeholds and Deterrence", working paper, THEMA, Université de Cergy-Pontoise, France.

Goeree, J. K., E. Maasland, S. Onderstal and J. L. Turner, 2005, "How (Not) to Raise Money", *Journal of Political Economy*, 113, 897-926.

Goldman, E. and J. Qian, 2005, "Optimal Toeholds in Takeover Contests", *Journal of Financial Economics*, 77, 321-346.

Jehiel, P., B. Moldovanu, and E. Stachetti, 1996, "How (Not) to Sell Nuclear Weapons", *American Economic Review*, 86, 814-829.

Jehiel, P., B. Moldovanu, and E. Stachetti, 1999, "Multidimensional Mechanism Design for Auctions with Externalities", *Journal of Economic Theory*, 85, 258-293.

Loyola, G., 2007, "How to Sell to Buyers with Crossholdings", Working Paper 07-50, Universidad Carlos III, Madrid, Spain.

Mathews, R. D., 2007, "Optimal Equity Stakes and Corporate Control", *Review of Financial Studies*, 20, 1059-1086.

Myerson, R., 1981, "Optimal Auction Design", *Mathematics of Operations Research*, 6, 58-73.

Povel, P. and R. Singh, 2004, "Using Bidder Asymmetry to Increase Seller Revenue", *Economics Letters*, 84, 17-20.

Povel, P. and R. Singh, 2006, "Takeover Contests with Asymmetric Bidders", *Review of Financial Studies*, 19, 1399-1431.

Riley, J. G. and W. F. Samuelson, 1981, "Optimal Auctions", *American Economic Review*, 71, 381-392.

Singh, R., 1998, "Takeover Bidding with Toeholds: The Case of the Owner's Curse", *Review of Financial Studies*, 11, 679-704.

9 Figures

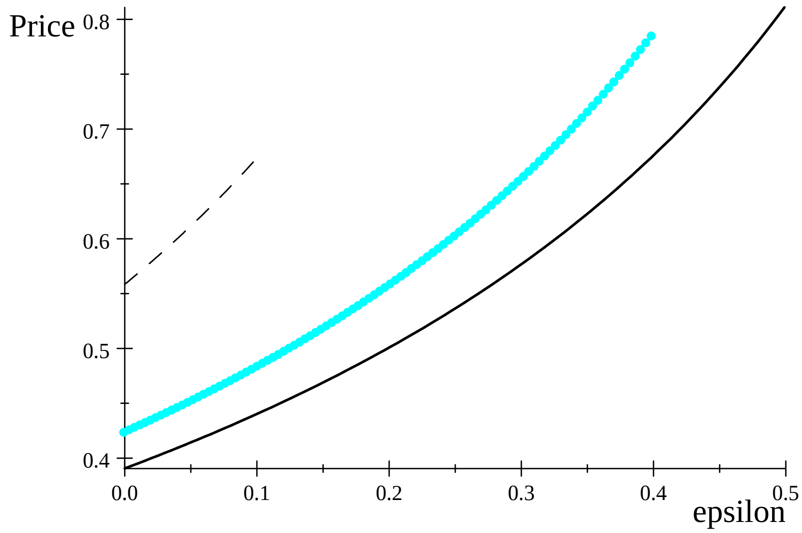

Figure 1. Average sale price from the sequential negotiation mechanism with two bidders and $\phi_1 > \phi_2 \geq 0$, for $\phi_2 = 0$ (solid line), $\phi_2 = .1$ (dotted line) and $\phi_2 = .4$ (dash line).

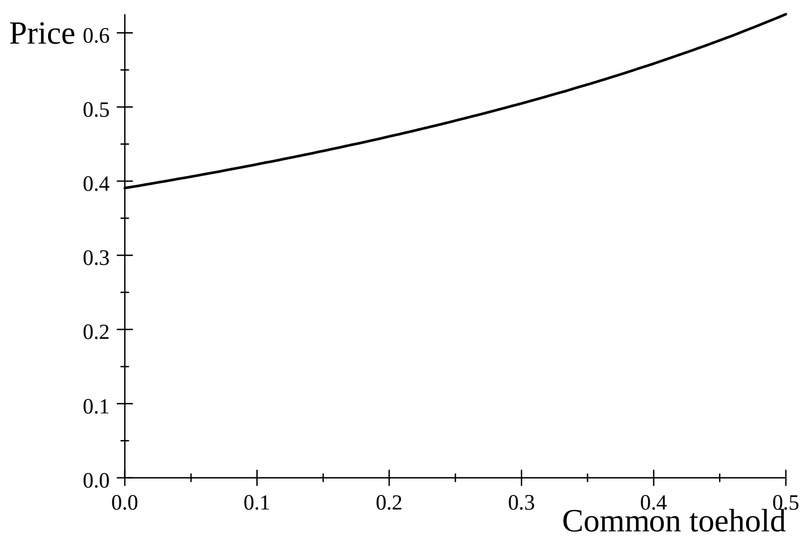

Figure 2. Average sale price from the sequential negotiation mechanism with two bidders for $\phi_1 = \phi_2 = \phi \geq 0$.

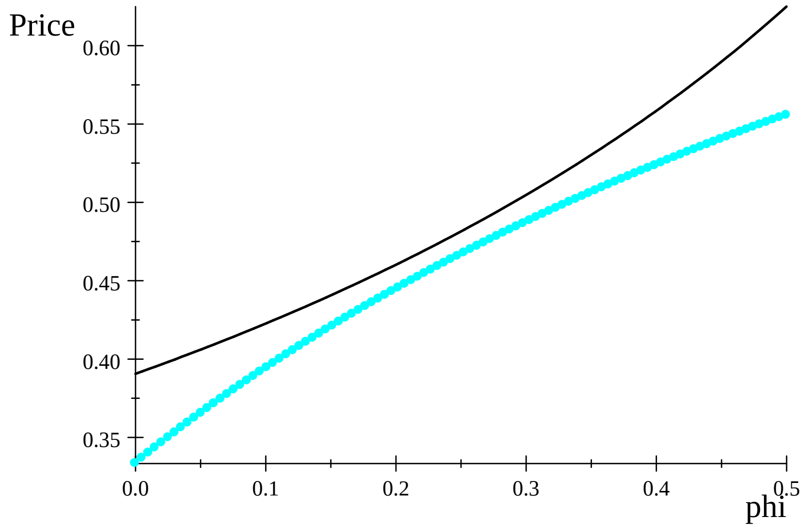

Figure 3. Average sale price from the sequential negotiation mechanism (solid line) and the SPA (dotted line) with two bidders for $\phi_1 = \phi_2 = \phi \geq 0$.

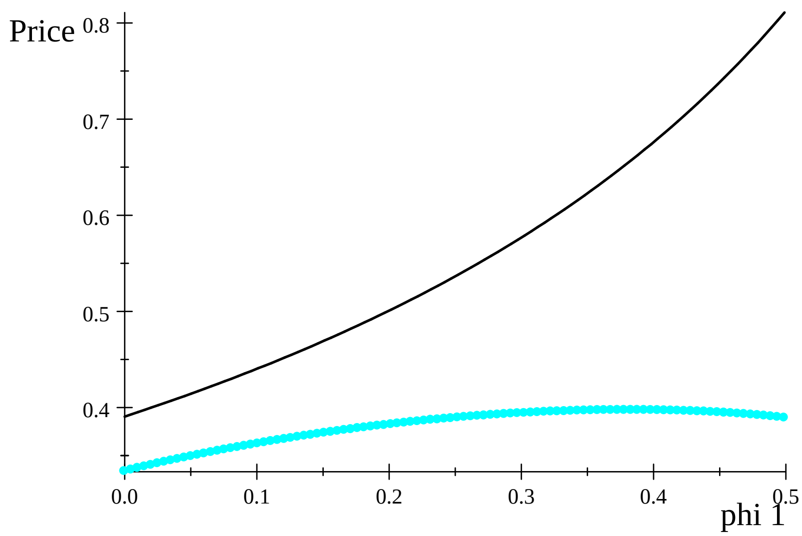

Figure 4. Average sale price from the sequential negotiation mechanism (solid line) and the SPA (dotted line) with two bidders for $\phi_1 > \phi_2 = 0$.