Consumption, Liquidity and the Cross-Sectional Variation of Expected Returns

Elena Márquez

(Universidad Complutense de Madrid)

Belén Nieto

(Universidad de Alicante)

Gonzalo Rubio

(Universidad CEU Cardenal Herrera)

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Abstract

Recent papers in asset pricing have added a market-wide liquidity factor to traditional portfolio-based or factor models. None of these papers has reported any evidence on how aggregate liquidity behaves together with consumption growth risk as measured by ultimate consumption risk. This paper covers this gap by providing a comprehensive explanation of the cross-sectional variation of average returns under market-wide illiquidity shocks. It derives closed-form expressions for consumption-based stochastic discount factors adjusted by aggregate illliquidity shocks and tests alternative models specifications. A strongly negative and highly significant illiquidity risk premium is reported under recursive preferences only for the first quarter of the year suggesting a time-varying behavior of the market-wide illiquidity premium.

Key words: stochastic discount factor, ultimate consumption risk, market-wide liquidity.
JEL: G10, G12, E44

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Corresponding author: Gonzalo Rubio, Departamento de Economía y Empresa, Universidad CEU Cardenal Herrera, 03203 Elche, Alicante, Spain (34-965426486)

1. Introduction

Not surprisingly, asset pricing literature has been debating between reduced-form portfolio-based models or factor models, where marginal utility of consumption is directly measured by the returns on a few number of large portfolios, and macroeconomic models, where the focus is on understanding the marginal utility that drives asset prices. In other words, factor models take as given the risk premium associated with a set of large portfolios, and simply investigate whether expected returns can be explained in terms of a few sources of common movements captured by that (exogenous) set of large portfolios. On the other hand, macroeconomic-based asset pricing models are concerned about why those common factors represented by large portfolios are priced in financial markets. Hence, these models investigate whether the chosen stochastic discount factor –the chosen proxy for the marginal rate of intertemporal substitution of consumption- reflects macroeconomic conditions properly.[1]

The economic understanding of the behavior of stock markets must be based on the fact that investors fear stocks because they tend to do badly –ultimately reducing consumption- in economic downturns and especially on recessions. The systematic rejection of the basic consumption-based model has led to new models in which utility depends not only on consumption but also on other arguments which enter in the utility function in a non-separate fashion. This is the key success of models with non-separability over time and across states of nature. Well known models with habit persistence or recursive utility functions are good examples. Given non-separability, covariances of returns with state variables are priced because the derivatives of marginal utility of consumption with respect to the corresponding state variables are different from zero. The point is that these models retain aggregate consumption as the key source of macroeconomic risk but add non-separable arguments in the utility functions to increase the volatility of the stochastic discount factor (SDF).

Interestingly, non-separability of the marginal utility of consumption from state variables such as labor income growth, habits, housing collateral, the share of housing consumption in total consumption, and others are constrained to adjust slowly. This

[1] See Cochrane (2008) for a detailed and provocative discussion on these fundamental issues.

insight, together with the cost of adjusting consumption itself, suggests that even the basic consumption-based model may hold at long-horizons. Indeed, a recent line of research focuses on the treatment of consumption data. Even though, we should be aware of the measurement difficulties involved,[2] several authors attempt to find the most appropriate way to compute both the consumption variable and the consumption growth rate such that the investment timing decision of the representative agent is consistently matched with his desired pattern of consumption.

Jagannatan and Wang (2007) study the most appropriate sample frequency and the most appropriate moment of the year for measuring consumption growth rates. They argue that the consumption growth data which best reflects changes in agents decisions refer to the fourth quarter of each year. In fact, they show that the basic consumption-based model can account for the Fama-French 25 size and book-to-market portfolios.

Parker and Julliard (2005) discuss the optimal time interval when calculating consumption growth rates and consumption risk. They argue that changes in wealth have a delayed effect on consumption patterns. Hence, the covariance between portfolio returns and consumption growth over the quarter of the return and many following quarters (ultimate consumption) is needed for conciliating expected returns and consumption risk. They have some success in explaining the pricing of size and book-to-market portfolios by their exposure to ultimate consumption risk. In particular, they analyze whether multiple-time period returns obtained by buying stocks for one period and then bonds for S periods is priced by $S+1$ period consumption growth.

The authors argue that the slight forecasting ability of returns at time t of subsequent consumption growth accounts for the improved results relative to the traditional contemporaneous consumption-based specification.[3] The dynamics of consumption

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[2] For example, institutionally provided data refer to insufficiently representative consumption baskets, aggregation between all the individuals in the economy could compensate for individual consumption risk, the available data are updated with some delay, and so on.

[3] Hansen, Heaton and Li (2008), under the recursive preference framework of Epstein and Zin (1991) utility function with an intertemporal elasticity of substitution of one, show that revisions of expectations of the stochastic discount factor depends on the innovations to current consumption as well as on news about future consumption. They argue that this latter revision of expectations about future consumption growth explains the relative success of ultimate consumptions risk of Parker and Julliard (2005). See Bansal and Yaron (2004) and Bansal, Dittmar and Lundblad (2005) for additional evidence on the long-run covariances of cash flows with consumption.

growth provides the ultimate consumption risk SDF with a very strong business cycle pattern. Consumption falls through recessions, so that the ultimate risk SDF is highest right before recessions and lowest right before expansions. This strong counter-cyclical behavior is much more pronounced than the one observed for the contemporaneous consumption growth model. This is precisely what helps the model to present a modest success in explaining the value premium. Importantly, however, ultimate consumption risk does not seem to be able to price properly extreme size and book-to-market portfolios. All alternative specifications of the ultimate consumption risk model are in fact rejected by Parker and Julliard (2005).

Given this discussion, our conjecture is that the fundamental beauty of ultimate consumption risk may need to be accompanied by the recognition that small and value stocks may decline even further when the market as a whole becomes more illiquid. If this were indeed the case, these stocks should be compensated with extra returns which cannot be completely accounted for by the covariance between their returns and ultimate consumption growth risk. Therefore, our hypothesis is that two factors drive asset returns; ultimate consumption risk and aggregate liquidity risk.

It is also important to note that an increasing number of papers study whether liquidity risk as a market-wide factor is priced in stock markets. This literature starts with Chordia, Roll and Subrahmanyam (2000), and Hasbrouck and Seppi (2001), who show that the time-varying liquidity for individual stocks have common systematic components. More recently, Kamara, Lou and Sadka (2008) has shown that the cross-sectional variation of liquidity commonality has increased over the past three decades. This important result has the unfortunate implication that the ability to diversify systematic aggregate liquidity shocks has declined. These papers suggest the possibility that market-wide liquidity may be a priced aggregate factor. Amihud (2002) shows that the level of market-wide liquidity affects expected returns. Moreover, he shows that unexpected market liquidity (illiquidity) should be contemporaneously positively (negatively) correlated with stock returns because a shock to liquidity (illiquidity) raises (lowers) expected liquidity (illiquidity), which in turn lowers (raises) expected returns, and hence raises (lowers) prices. Pastor and Stambaugh (2003), Acharya and Pedersen (2005), Martínez, Nieto, Rubio and Tapia (2005), Sadka (2006), Liu (2006), and Korajczk and Sadka (2008) find that the covariance between returns and some measure of aggregate liquidity shocks is significantly priced in the market. Watanabe and Watanabe (2009) show that the liquidity risk premium is time varying. They report a large liquidity premium for states with particularly large liquidity betas and argue that their result is consistent with investors facing uncertainty about their trading counterparties´ preferences.

Rather surprisingly, however, all previous papers include an additional market-wide liquidity factor to traditional portfolio-based asset pricing models. None of these papers has reported any evidence on how aggregate illiquidity behaves together with consumption growth risk as measured by ultimate consumption risk. This paper fills this gap in literature by providing a comprehensive explanation of the cross-sectional variation of average returns using both ultimate consumption risk and market-wide illiquidity risk. Moreover, the second contribution of the paper is to develop closed-form expressions for consumption-based SDFs adjusted by aggregate illiquidity shocks. Our evidence suggests that aggregate illiquidity is indeed important in pricing risky stocks in models with ultimate consumption risk. This is particularly the case during the first quarter of the year.

2. The Consumption-based Liquidity-adjusted Stochastic Discount Factor

The empirical papers analyzing whether there is a liquidity market-wide factor are all based on the implicit assumption that there exists a SDF that depends on some measure of aggregate liquidity. To be explicit about a SDF with systematic liquidity is not an easy task. He and Modest (1995) argue that a combination of short-selling, borrowing and solvency constraints together with trading costs frictions can generate a large enough wedge between SDFs and asset prices so that well known puzzles may not be inconsistent with equilibrium in financial markets. More recently, Garleanu, Pedersen and Poteshman (2008) show explicitly how the SDF depends on demand pressure in a model with multiple assets. Finally, Lustig and Van Niewerburgh (2005) explore a model in which shocks in the housing market affecting housing collateral determine the size of the wedge between prices and the marginal rate of intertemporal substitution of consumption.[4]

[4] Others papers dealing with similar issues are those of Piazzesi, Schneider, and Tuzel (2007), and Duarte and Vergara-Alert (2008).

Rather than imposing non-separability in the arguments of the utility function, we assume that shocks to aggregate liquidity directly affect the intertemporal budget constraint in our framework. In that way, future liquidity conditions will affect future payoffs of investments of the representative agent and, therefore, its future consumption. More specifically, under our optimization problem specification, the same asset future payoff will have a higher value in terms of future consumption in moments in which the liquidity in the market is low. The result is a higher SDF just before recessions, when the market is more illiquid, than the one generated by the standard problem. This mechanism intensifies the desirable countercyclical time series property of this variable.

We assume two preferences specifications. First, we employ the commonly used power utility function under which the SDF is given by

$$M_{P,t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma}, \tag{1}$$

where β is the subjective discount factor, γ is the coefficient of relative risk aversion, and C_t is aggregate nondurable consumption at period *t*.

As usual, if future consumption is lower than current consumption, the investor has incentives to avoid consumption today and invest in financial assets, so that $M_{P,t+1}$ is high to give a larger weight to future payoffs. On the other hand, if future consumption is higher than current consumption, the investor will not have incentives to avoid consumption today for investing in financial assets. Hence, the investor penalizes future payoff, so that the SDF is low, and higher returns will be required to invest in financial asset. This implies that $M_{P,t+1}$ is high (low) at the very beginning of recessions (expansions).

Secondly, we consider recursive preferences, as in Epstein and Zin (1991). Under this specification, investors utility depend on time and also on states of nature resulting in the following SDF:

$$M_{R,t+1} = \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \right]^{\kappa} R_{Wt+1}^{\kappa-1}, \tag{2}$$

where R_{Wt+1} is the gross rate of return on the market portfolio or aggregate wealth, ρ is the inverse of the elasticity of intertemporal substitution, and $\kappa \equiv \frac{1-\gamma}{1-\rho}$.

Under systematic or market-wide liquidity risk, adverse liquidity shocks take place during recessions. Then we would expect that a liquidity-adjusted SDF should be even higher (lower) relative to the standard consumption-based SDF just before recessions (expansions). Incorporating illiquidity shocks to the budget constraint in the household problem, it is possible to obtain this effect. As shown in Appendix A1, the liquidity-adjusted SDFs under separable power utility ($M_{LAP,t+1}$) and recursive preferences ($M_{LAR,t+1}$), are as follow,

$$M_{LAP,t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \phi\left(L_{t+1}\right) \tag{3}$$

$$M_{LAR,t+1} = \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \phi\left(L_{t+1}\right) \right]^{\kappa} R_{Wt+1}^{\kappa-1}. \tag{4}$$

$\phi(L_{t+1})$ is a function that represent negative shocks on aggregate liquidity. It will be higher than one if the economy experiences an adverse aggregate liquidity shock, and lower than one if the economy experiences a positive aggregate liquidity shock. Hence, $M_{LA,t+1}$ will be higher than the correspondent non liquidity-adjusted SDF precisely in those time periods in which recessions are shortly expected. In other words, we obtain a SDF with the same counter-cyclical behavior but with a more accentuated increasing (and decreasing) cycle pattern.

Finally, we also consider the specification under *ultimate consumption risk* as in Parker and Julliard (2005). They propose to construct a SDF that relates marginal utility in period t with marginal utility in period $t+S+1$. In that way, investors use the expectation about far away future consumption when taking investment decisions today. Applying the same idea, we derive the liquidity-adjusted SDFs for both power and recursive preferences. The details are contained in Appendix A2, and the resulting SDFs are given by the following expressions,

$$M_{LAP,t+1}^{S} = \beta^{S+1}\left(\frac{C_{t+1+S}}{C_t}\right)^{-\gamma}\phi\left(L_{t+1,t+1+S}\right)R_{ft+1,t+1+S} \quad (5)$$

$$M_{LAR,t+1}^{S} = \left[\beta^{S+1}\left(\frac{C_{t+1+S}}{C_t}\right)^{-\rho}\phi\left(L_{t+1,t+1+S}\right)\right]^{\kappa} R_{Wt+1,t+1+S}^{\kappa-1}R_{ft+1,t+1+S}\,, \quad (6)$$

where $R_{Wt+1,t+1+S}$ and $R_{ft+1,t+1+S}$ denote the cumulative gross return on wealth and on the risk free asset, respectively, from period $t+1$ to period $t+1+S$.

It must be noted that equations (5) and (6) nest the correspondent standard (non liquidity-adjusted) SDFs under *ultimate* risk for power and recursive preferences, respectively, when $\phi\left(L_{t+1,t+1+S}\right)=1$.

These are the key consumption-based liquidity-adjusted models that we test in this paper. Once again, our conjecture is that this type of specifications should be able to price the extreme highly illiquid portfolios. Ultimately, the issue is to test whether expressions (5) and (6) mirror macroeconomic conditions correctly.

3. Data

For the period 1963:I to 2006:IV, we collect quarterly seasonally adjusted aggregate real per capita consumption expenditure of nondurables and services and total consumption from National Income and Product Accounts (NIPA) given in Table 7.1. Monthly value-weighted stock market return and risk-free rate are taken from Kenneth French´s web page, from which we compute quarterly returns. The price deflator from NIPA Table 2.8.4 is used to calculate real rates of returns. We also compute quarterly returns of 25 size/book-to-market value-weighted portfolio and 17 industry portfolio returns from the monthly figures available at Kenneth French´s web page.

Our liquidity measure is based on Amihud (2002) measure of individual stocks illiquidity, which is calculated as the ratio of the absolute value of daily return over the dollar volume, a measure that is closely related to the notion of price impact. Among others, this ratio has been used by Amihud (2002), Acharya and Pedersen (2005), Korajczyk and Sadka (2008), Kamara, Lou and Sadka (2008), and Watanabe and Watanabe (2009). The main advantage of Amihud´s illiquidity ratio is that can be computed using daily data and, consequently, allows us to study a long time period which is clearly relevant for testing asset pricing models. This illiquidity measure is estimated daily at the individual level as,

$$Illiq_{j,d} = \frac{\left|R_{j,d}\right|}{DVol_{j,d}} \tag{7}$$

where $\left|R_{j,d}\right|$ is the absolute return of asset *j* on day *d*, and $DVol_{j,d}$ is the dollar volume of asset *j* during day *d*.

This measure is aggregated over all days for each month in the sample period to obtain an individual illiquidity measure for each stock at month *t*,

$$Illiq_{j,t} = \frac{1}{D_{j,t}} \sum_{d=1}^{D_{j,t}} \frac{\left|R_{j,d}\right|}{DVol_{j,d}} \tag{8}$$

where $D_{j,t}$ is the number of days for which data are available for stock *j* in month *t*.[5]

Finally, using all *N* available stocks, we obtain the market-wide illiquidity measure as the cross-sectional average of expression (8) for each month in the sample period as,

$$ILLIQ_{m,t} = \frac{1}{N} \sum_{j=1}^{N} Illiq_{j,t} \tag{9}$$

Using the value of the aggregate illiquidity ratio given by equation (9) for the last month in each quarter, we compute our function representing market illiquidity shocks as the

[5] We thank Yakov Amihud for kindly provided his data until December 1996. We update his measure from January 1997 to December 2005 using daily data from CSRP on all individual stocks with enough data within a given month. At least 15 observations of the ratio within the considered month are required for asset *j* to be included in the sample. An exception has been made for September 2001 requiring at least 12 observations in this case.

residual from an *AR*(1) process.[6] Finally, $\phi(L_t)$ is the gross standardized residual from the autoregressive regression.[7] Figure 1 shows how our illiquidity function tends to jump either just before or during recessions suggesting a counter-cyclical time series behavior. The shaded regions of Figure 1 are U.S. macroeconomic recessions from peak to trough as defined by the National Bureau of Economic Research (NBER). It is also interesting to note the relatively much more stable behaviour of the illiquidity function after 1993. It should be recalled that a large number of empirical macroeconomic papers provide evidence of a striking decline in the volatility of U.S. macroeconomic time series since the end of the eighties. It is known as the "Great Moderation". It also reflects the well known period of high liquidity, both at the micro and macro levels that economies have experienced during the last years of our sample period.

4. Estimation and Tests

We estimate and compare the asset pricing models nested under the SDF specification given by equation (6):

$$M^{S}_{LAR,t+1} = \left[\beta^{S+1} \left(\frac{C_{t+1+S}}{C_t} \right)^{-\rho} \phi\left(L_{t+1,t+1+S}\right) \right]^{\kappa} R^{\kappa-1}_{Wt+1,t+1+S} R_{ft+1,t+1+S}$$

They are the following: the standard CCAPM, when *S*=0, $\gamma = \rho$ and $\phi(L) = 1$; the ultimate consumption risk version of the standard CCAPM, when *S>0*, $\gamma = \rho$ and $\phi(L) = 1$; the liquidity-adjusted CCAPM, when *S=0* and $\gamma = \rho$; the ultimate consumption risk version of the liquidity-adjusted CCAPM, when *S*>0 and $\gamma = \rho$; the Epstein-Zin model (recursive), when *S*=0 and $\phi(L) = 1$; the ultimate consumption risk specification of the Epstein-Zin model, when *S*>0 and $\phi(L) = 1$; the liquidity-adjusted Epstein-Zin model, when *S=0*; and the ultimate consumption risk version of the

[6] Unlike the *AR*(2) model usually employed in literature when using monthly data, we employ the *AR*(1) specification with quarterly data. The residuals from the *AR*(1) model, our illiquidity-shock measure, have a first-order autocorrelations of only -0.069. It should also be pointed out that the effect of detrending the autoregressive regressions using the ratio of market capitalizations between two adjacent periods is negligible.

[7] In order to have values of our illiquidity measure closely resembling units of rates of returns, the residuals have been standardized dividing by ten times its sample standard deviation. Then, we add up one in order to have the gross standardized residual.

liquidity-adjusted Epstein-Zin model, in which the SDF is given by equation (6) without restrictions.[8]

We employ two methodologies. The non-linear versions of the models are estimated by GMM, while the Fama-MacBeth (1973) procedure is used to estimate the corresponding beta specifications.

For the GMM estimation, we follow Parker and Julliard (2005) and Yogo (2006). The following vector defines the moment restrictions:

$$g_T\left(\theta,\alpha,\mu\right)=E\begin{bmatrix} R^e-\alpha 1_N+\dfrac{\left(M-\mu\right)R^e}{\mu} \\ M-\mu \end{bmatrix} \tag{10}$$

where R^e is the N x 1 vector of the excess return of the N assets, 1_N denotes the N x 1 vector of ones, $M\left(\theta\right)$ is one out of the eight specifications nested in equation (6), and θ is the vector of the preference parameters for each particular specification.

The inclusion of the parameter α enables to evaluate separately the ability of the model to explain the equity premium and the cross section of expected returns. So, if α is zero, we can conclude that the model does not present an equity premium puzzle. Moreover, the last condition forces the SDF to move back to its mean value (μ).

For the GMM estimation we employ a pre-specified weighting matrix that contains the matrix proposed by Hansen and Jagannathan (1997). As usual, it weights the moment conditions for the N testing assets using the (inverse) variance-covariance matrix of excess returns. Moreover, as in Parker and Julliard (2005), the weight of the last moment condition is chosen large enough to make sure that significant changes in that weight have no effects on the parameter estimates.[9] Given that this weighting matrix is not the optimal one, the quadratic form allowing us to test the performance of the model

[8] It should be noted that the general specification given by equation (6) also nests the four corresponding versions of the CAPM when the relative risk aversion equals one.
[9] In particular, we choose a weight of 1000 for the restriction on the mean of the SDF.

has an unknown distribution. We follow Hansen and Jaggannathan (1997) and Parker and Julliard (2005) to infer the *p*-value of the test. All details about the estimation and testing methodology are provided in Appendix B.

The second estimation procedure is based on the linear specification of the model. In Appendix C we derive the beta version of the model implied by the SDF given by equation (6). In this case, we estimate the following OLS cross-sectional regression at each moment of time:

$$R_{jt} - R_{ft} = \gamma_0 + \gamma_1 \beta_{jc,t} + \gamma_2 \beta_{j\phi,t} + \gamma_3 \beta_{jW,t} + e_{jt+1} \qquad (11)$$

where the explanatory variables are the sensitivities of the asset returns to changes in non-durable consumption growth, market illiquidity shocks and the return on aggregate wealth. These betas are estimated with a time-series regression using a moving-data set prior to each cross-sectional regression. When we test the linearized version of the models, we always express the three factors in logarithms.

5. Empirical Results

For both estimation methodologies described in Section 4, we employ two sets of test assets: the 25 size/book-to-market Fama-French portfolios and a set of 42 portfolios containing de 25 Fama-French portfolios and 17 industry portfolios. This second set of portfolios is used in order to mitigate the important concern raised by Lewellen, Nagel and Shanken (2008).

Moreover, the models have been estimated for different horizons (S=0, 3, 7, 11 and 15 quarters ahead). Consistently with Parker and Julliard (2005), the larger explanatory power for both methodologies is obtained for S=11. To save space, we just report the results based on S=11.[10] In order to have an homogeneous sample period when estimating all models to make the results comparable among them, and given that the long run specifications need growth rates of the risk factors from now to 3 years ahead, the sample period for the estimation ends at the first quarter of 2003.

[10] All results are available from authors upon request.

5.1. Portfolio Descriptive Statistics

Panel A of Table 1a contains the sample means and standard deviations of excess returns of the 25 Fama-French portfolios showing the well-known facts about these portfolios. Between 1963:II and 2003:I, both small and high book-to-market firms have larger average returns than other portfolios within the same category. The highest average return is obtained for portfolio *15* where we have simultaneously the smallest firms and the highest book-to-market stocks. However, the highest risk is found in the small but low book-to-market portfolio.

Panel B reports the return-based illiquidity betas of the 25 Fama-French portfolios. We run time series regressions of the return of each portfolio on our market-wide illiquidity factor. In particular, the regression is,

$$R_{jt} = \alpha_j + \beta_j \phi\left(L_t\right) + u_{jt} \tag{12}$$

As expected, given the economic implications of the market-wide illiquidity factor, we obtain negative and significant coefficients for all portfolios. All stock returns are negatively affected by adverse illiquidity shocks. By controlling for book-to-market, we report monotonically increasing (in absolute value) return-based illiquidity betas from big to small firms. On the contrary, when we control for size, we do not observe a monotonic pattern when moving from low to high-to-market firms. Interestingly, the illiquidity betas of the low book-to-market portfolios tend to be higher than those for high-book-to-market ones. The pattern closely follows the standard deviation of portfolios sorted by book-to-market contained in Panel A. Indeed the highest illiquidity beta is found for portfolio *11* which includes small but low book-to-market firms.

Although these previous results are interesting by themselves, it is clear that we should control for both the market portfolio return and non-durable consumption growth. Hence, in Panel C we report the results from the following time-series regressions,

$$R_{jt} = \alpha_j + \beta_{jL} \phi\left(L_t\right) + \beta_{jW} R_{Wt} + \beta_{jc} \Delta C_t + u_{jt} \tag{13}$$

We can now easily identify those portfolios more sensitive to illiquidity shocks over and above changes in market returns and consumption growth. Once again, controlling for book-to-market, we find a monotonically increasing (in absolute value) return-based illiquidity betas from big to small firms. In particular, small firms are negatively and significantly affected by illiquidity shocks. Similarly, and contrary to the results of Panel B, once we control for size, we report and increasing (in absolute value) pattern of illiquidity betas from low to high book-to-market. This suggests that firms which are simultaneously small and have high book-to-market ratios are those highly affected by illiquidity shocks. Indeed the highest (in absolute value) and significant illiquidity beta is obtained for portfolio *15* which comprises small and high book-to-market stocks. We conclude that small and value stocks are therefore negatively affected by market-wide illiquidity shocks. As we know from previous literature, this is one of the portfolios which are systematically hard to price using the three Fama-French factors or other well known asset pricing models. On the other hand, big firms with low book-to-market are not influenced by aggregate illiquidity shocks.[11]

Finally, Table 1b contains the same results for industry portfolios. As before, all industries are negatively affected by aggregate illiquidity shocks. However, once we control for the market portfolio return and consumption growth, we find that industries directly affected by market-wide illiquidity shocks are Durable Goods, Construction, Clothes, Retail Goods and Food.

5.2. GMM Estimation

Next we test the non-linear specification of consumption-based asset pricing models with and without illiquidity shocks. We therefore test equations (1) to (6) using both contemporaneous and ultimate consumption risk. We employ expression (10) with either 26 or 43 moment conditions depending upon we use the 25 Fama-French portfolios or the expanding set with additional 17 industry portfolios.

Table 2a contains the results for the power utility function. First, as in Parker and Julliard (2005), the risk aversion coefficient is always estimated with large standard errors due to the flatness of the GMM objective function with respect to γ. As expected, the estimation of risk aversion tends to be smaller for S=11 than for S=0. For

[11] According to the results reported by Kamara, Lou and Sadka (2008), our overall evidence might have changed in the last years of our sample period. In any case, it should be noted that they report illiquidity betas (from a regression of individual illiquidity changes on market-wide illiquidity changes) and not return-based market-wide illiquidity betas.

ultimate consumption risk and the expanding set of test assets, $\hat{\gamma}$ is either 2.76 or 2.48 depending upon we estimate the model with or without illiquidity shocks. Interestingly, even when we use contemporaneous consumption risk, the estimate of the risk aversion coefficient decreases from 13.93 to 8.76 by recognizing illiquidity shocks.

Second, given the estimated levels of either contemporaneous or ultimate consumption risk, the average excess returns of test assets is too large. In all cases, the estimated intercept is statistically significant, and implies that the average excess returns on a test asset from the expanding set exceeds that implied by consumption risk by 4.6 to 5.2 percent per year.[12] The lowest pricing error is obtained for contemporaneous consumption risk with illiquidity shocks in Panel B. This specification also presents the lowest H-J distance and a risk aversion of 8.76.[13]

Third, the model is always rejected by the data. The H-J distance is in all cases too large, so that the power utility function specification is rejected no matter the time horizon employed to calculate consumption growth or if illiquidity shocks are or not included. In any case, it is also true that within each panel, the distance is always reduced when illiquidity shocks are included in the asset pricing model specification.

Table 2b reports similar results for the recursive utility function. Overall, they tend to be slightly more encouraging than those for the power utility case. As before, Panel A uses only the 25 Fama-French portfolios, while Panel B employs the expanding set with industry portfolios. Once again, the estimates of risk aversion and the elasticity of intertemporal substitution contain large standard errors. They also become more reasonable when we use S=11 rather than S=0. The estimates of risk aversion and elasticity of intertemporal substitution are 2.57 and 0.40 respectively for ultimate consumption risk with illiquidity shocks. In Panel B, the pricing errors, which are systematically lower than in Panel A, are statistically different form zero suggesting large average returns for the implied consumption and illiquidity risk of the alternative specifications. The pricing errors for the expanding set of testing assets are

[12] It should be noted that the pricing error for contemporaneous consumption risk without illiquidity shocks and for the 25 Fama-French portfolios is 8.6 percent per year.

[13] Since we include a moment for the mean of the SDFs, the statistic is not just the squared root of the variance-covariance matrix of excess returns and, therefore, it is not strictly speaking the H-J distance. Appendix B obtains the correct p-value based on this adjusted distance.

approximately 5.2 percent per year for all pricing models. However, it must be pointed out the systematic reduction of the H-J distance when illiquidity shocks are included independently on considering S=0 or S=11. In particular, the contemporaneous consumption risk under recursive preferences with illiquidity shocks cannot be rejected by the over-identification test at the 5 percent significance level. We may conclude that, generally speaking, the recursive specification does present slightly better pricing results than the power case.

Figure 2.A shows the contemporaneous SDF under recursive preferences and no illiquidity shocks. As expected, it tends to move counter-cyclically but it presents a rather non-volatile behavior over time. On the other hand, Figure 2.B contains the contemporaneous SDF under recursive preference with market-wide illiquidity shocks. It becomes clear that we obtain a SDF with the same counter-cyclical behavior but with a more accentuated increasing (and decreasing) cycle pattern. This is precisely the time-varying behavior of any SDF potentially capable of explaining the cross-section and time-varying behavior of stock returns.

5.3. Fama-MacBeth Estimation

The non-linear pricing models tested above are linearized in Appendix C to obtain the general linear three-factor model given by equation (11),

$$R_{jt} - R_{ft} = \gamma_0 + \gamma_1 \beta_{jc,t} + \gamma_2 \beta_{j\phi,t} + \gamma_3 \beta_{jW,t} + e_{jt+1}$$

where betas are the sensitivities to consumption growth, illiquidity shocks and the market portfolio returns respectively, and the gammas are the risk premia associated to these aggregate risk factors.

The empirical results are reported in Tables 3a, 3b and 3c in which we respectively analyze the Consumption CAPM under power utility, the traditional (Wealth) CAPM under logarithmic utility, and the Consumption CAPM under recursive preferences. In all cases, we compare the results using either ultimate or contemporaneous consumption risk specifications, with or without illiquidity shocks. The most general linear three-factor model of equation (11) is a model under recursive preferences with illiquidity shocks. All other specifications are just special cases of expression (11). In Panel A of the three above mentioned tables we use the 25 Fama-French portfolios, while in Panel B we report the results employing the expanding set of 42 testing assets including the 17 industry portfolios.

Table 3a shows that the results for S=0 are very poor no matter if illiquidity shocks are or not included. As expected, the mean-squared errors diminish when we move from S=0 to S=11. As usual, ultimate consumption risk tends to improve the pricing fit of the traditional consumption pricing model. The inclusion of illiquidity shocks also improves the performance relative to the specification without market-wide illiquidity. However, once we include illiquidity, ultimate consumption risk does not improve the overall fit of the model. Both adjusted *R-square* and mean-squared errors tend to be very similar, although it must be pointed out that the risk premium of market-wide illiquidity is negative and significantly different from zero when the model is estimated under ultimate consumption risk. It is not the case when S=0. This suggests that both illiquidity shocks and ultimate consumption risk are important in pricing risky stocks, although it should be clear that the intercept of the second-pass cross sectional regressions are always statistically different from zero indicating the overall rejection of the model. The slight improvement in pricing assets can also be observed in Figure 3. It is clear that a better fit is obtained when we use S=11 and aggregate illiquidity shocks. As usual, portfolios *15* and, especially, *11* remain very problematic. It should be noted that portfolio *15* has a large and negative return-based illiquidity beta as shown in Table 1a, while portfolio *11* has an illiquidity beta much lower and even non-significant at the 5% level. In other words, it seems that the inclusion of a market-wide illiquidity factor alleviates the pricing model of portfolio *15*, but portfolio *11* is not sensitive enough to illiquidity shocks to be correctly priced by the model.

Table 3b contains even worse results. Indeed, the simultaneous use of market returns and illiquidity shocks does not help pricing data. None of the risk premia are significantly different from zero independently of using S=0 or S=11. As before, the intercept is always positive and statistically different from zero.

The two previous tables show that the average returns of alternative combinations of portfolios are too far to be explained by either ultimate consumption risk and illiquidity shocks or by market returns and market-wide illiquidity innovations. Table 3c contains further and more complete results. It reports the results from the model under recursive preferences with illiquidity shocks. It is our three-factor linear model in which we simultaneously take into account ultimate consumption risk, market risk and market-wide illiquidity risk. The results of Panel B with S=11 are the most promising results of the paper. They are also consistent with the results of Table 2b. As in Table 3a, the risk premium for ultimate consumption growth becomes positive relative to contemporaneous consumption. Moreover, the risk premium of aggregate illiquidity shocks is negative and significant, and the market risk premium has the expected sign. The mean-squared errors are also lower relative to Table 3a. In any case, the intercept remains positive and highly significant pointing out the overall misspecification of the model. As before, the improvement in the results can also be noted in Figure 4. Although, under recursive preferences, portfolio *11* remains far from the 45 degrees line, the inclusion of illiquidity and ultimate consumption risks improve the adjustment of the portfolio *15*.

5.4 Risk Premia Seasonality

Given the well known January seasonality of stock returns, we run the following OLS regressions using, as dependent variables, the four estimated risk premia from equation (11), for both the contemporaneous and ultimate consumption risk specifications with quarterly data:

$$\hat{\gamma}_{it} = a + bD_{RYt} + u_{it} \quad ; i = 1,2,3 \text{ and } 4 \tag{14}$$

The test assets include the 25 Fama-French portfolios and the 17 industry portfolios. D_{RYt} is a dummy variable which is equal to one if the observation belongs to either the second, third or fourth quarter of the year, and equals zero otherwise. The estimated intercept is therefore the average risk premia during the first quarter, while the slope coefficients represent the difference between the average risk premia during the rest of the year and the average risk premia during the first quarter.

The results reported in Table 4 show an extraordinary first quarter seasonality of the illiquidity risk premium for both the contemporaneous and ultimate risk consumption specifications. In particular, the negative and statistically significant risk premium reported in Table 3.c, for recursive preferences with long run consumption and illiquidity risk, is completely due to the first quarter of the year. The same result is obtained for the contemporaneous case; the negative but insignificant risk premium becomes strongly negative and highly significant. We can therefore conclude that the illiquidity risk premium seems to be negative and significant only during the first quarter of the year. In fact, the illiquidity premium for the rest of the year is positive and statistically different from the illiquidity risk premium during the first quarter. These results suggest a strong time-varying behavior of the illiquidity risk premium. Indeed, it might be the case that the time-varying behavior reported by Watanabe and Watanabe (2009) would be just a consequence of the striking seasonality found for the illiquidity risk premium during the first quarter of the year.

There is also some marginally significant evidence of seasonality of the consumption growth risk premium when the ultimate specification is employed. However, we find no evidence of market risk premium seasonality once we control for both consumption risk and illiquidity risk.

6. Conclusions

Given the pricing difficulties that previous research has found when explaining the behavior of small-low-book-to-market and small-high-book-to market portfolios, this paper proposes a model integrating ultimate consumption risk under recursive preferences with market-wide illiquidity shocks. Our research seems promising since we find negative and significant return-based illiquidity betas precisely for these extreme portfolios even controlling for consumption growth and the market portfolio return. However, using both the general non-linear specification of the model, and the linear three-factor cross-sectional version, we clearly tend to reject the expanding illiquidity asset pricing model. It is true that market-wide illiquidity is significantly priced in a model which integrates the illiquidity risk factor into the SDF, but the average excess returns of our testing assets remain too far of the estimated returns for the considered risk sources (ultimate consumption risk, market risk and aggregate illiquidity risk). There is however, a strong and highly significance evidence of a negative market-wide illiquidity premium during the first quarter of the year. Interestingly, the behavior of the illiquidity premium seems to change dramatically from a significant negative premium during the first quarter of the year to a positive and statistically different risk premium for the rest of the year.

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Appendix A: Derivation of the Model

1. Recursive Utility with Aggregate Liquidity Constraints

Assuming recursive preferences as in Epstein and Zin (1991), and considering that the aggregate illiquidity shocks affect future consumption throughout the budget constrain, the representative agent would solve the following problem,

$$\max_{\{z_j\}} \mathrm{U_t} = \left[(1-\beta) C_t^{1-\rho} + \beta \left[E_t \left(U_{t+1}^{1-\gamma} \right) \right]^{\frac{1-\rho}{1-\gamma}} \right]^{\frac{1}{1-\rho}}$$

$$s.a.$$

$$C_t = e_t - z_j p_{jt} \qquad \text{(A1)}$$

$$C_{t+1} = e_{t+1} + z_j X_{jt+1} \phi\left(L_{t+1}\right)$$

where C_t is the aggregate consumption at time *t*, β is the subjective discount factor, γ represents the coefficient of relative risk aversion, and ρ is the inverse of the elasticity of intertemporal substitution. Note that the expected utility adds over states which means that the recursive utility specification allows for non-separability across states of nature. The model, of course, disentangles risk aversion from the elasticity of intertemporal substitution. On the other hand, e_t is the consumption endowment, z_j is the amount invested today in asset *j*, p_{jt} is the price today of asset *j*, and X_{jt+1} is the payoff of the asset at *t+1*. Finally, $\phi(L_{t+1})$ is the aggregate illiquidity constraint shock affecting future consumption.

Introducing the budget restrictions on the objective function and taking the derivative with respect to the amount invested in asset *j*, z_j, we get the first order condition:

$$\frac{\partial U_t}{\partial z_j}=\frac{1}{1-\rho}U_t^{\rho}\left[\begin{array}{r}(1-\beta)(1-\rho)C_t^{-\rho}\left(-p_{jt}\right)+\beta\left(\frac{1-\rho}{1-\gamma}\right)\left[E_t\left(U_{t+1}^{1-\gamma}\right)\right]^{\frac{\gamma-\rho}{1-\gamma}}\\ x\;\; E_t\left[(1-\gamma)U_{t+1}^{-\gamma}\left(\frac{1}{1-\rho}\right)U_{t+1}^{\rho}(1-\beta)(1-\rho)C_{t+1}^{-\rho}X_{jt+1}\phi\left(L_{t+1}\right)\right]\end{array}\right]$$

$$=(1-\beta)U_t^{\rho}\left[-C_t^{-\rho}\left(-p_{jt}\right)+\beta\left[E_t\left(U_{t+1}^{1-\gamma}\right)\right]^{\frac{\gamma-\rho}{1-\gamma}}E_t\left[U_{t+1}^{\rho-\gamma}C_{t+1}^{-\rho}X_{jt+1}\phi\left(L_{t+1}\right)\right]\right]=0$$

Solving for the price today, we obtain,

$$p_{\mathrm{jt}}=\mathrm{E_t}\left[\beta\left(\frac{U_{t+1}}{\left[E_t\left(U_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}}\right)^{\rho-\gamma}\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\phi\left(L_{t+1}\right)X_{jt+1}\right] \tag{A2}$$

which can be written in a more compact way as,

$$p_{jt}=E_t\left[M_{R,t+1}\phi\left(L_{t+1}\right)X_{jt+1}\right]=E_t\left[M_{LAR,t+1}X_{jt+1}\right] \tag{A3}$$

where,

$$M_{R,t+1}=\beta\left(\frac{U_{t+1}}{\left[E_t\left(U_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}}\right)^{\rho-\gamma}\left(\frac{C_{t+1}}{C_t}\right)^{-\rho} \tag{A4}$$

Therefore, the liquidity adjusted SDF (M_{LAR}) is the standard SDF scaled by the function that picks up aggregate illiquidity shocks.

Now, we need to obtain an expression for $M_{R,t+1}$ as a function of the return on aggregate wealth. We divide the derivation in three parts.

a) By using the definition of recursive utility, we are able to solve first for $\left[E_t\left(U_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}$. In particular, note that

$$U_t=\left[(1-\beta)C_t^{1-\rho}+\beta\left[E_t\left(U_{t+1}^{1-\gamma}\right)\right]^{\frac{1-\rho}{1-\gamma}}\right]^{\frac{1}{1-\rho}}$$

which can be written as,

$$U_t^{1-\rho} - (1-\beta)C_t^{1-\rho} = \beta\left[E_t\left(U_{t+1}^{1-\gamma}\right)\right]^{\frac{1-\rho}{1-\gamma}}$$

$$\Rightarrow \left[E_t\left(U_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}} = \left(\frac{1}{\beta}\right)^{\frac{1}{1-\rho}}\left[U_t^{1-\rho} - (1-\beta)C_t^{1-\rho}\right]^{\frac{1}{1-\rho}} \tag{A5}$$

b) On the other hand, given that U_t depends on C_t and other future consumptions, we can write,

$$U_t = E_t\sum_{j=0}^{\infty}\frac{\partial U_t}{\partial C_{t+j}}C_{t+j} \Rightarrow \frac{U_t}{\partial U_t/\partial C_t} = E_t\sum_{j=0}^{\infty}\underbrace{\frac{\partial U_t/\partial C_{t+j}}{\partial U_t/\partial C_t}}_{M_{R,t+j}}C_{t+j} = W_t \tag{A6}$$

where W_t is aggregate wealth, and the last equality must hold by definition.[14]

c) We now derive U_t with respect to consumption,

$$\frac{\partial U_t}{\partial C_t} = \frac{1}{1-\rho}U_t^{\rho}(1-\beta)(1-\rho)C_t^{-\rho} = U_t^{\rho}(1-\beta)C_t^{-\rho} \tag{A7}$$

Combining (A6) and (A7),

$$W_t = \frac{U_t}{\partial U_t/\partial C_t} = \frac{U_t}{U_t^{\rho}(1-\beta)C_t^{-\rho}} = \frac{1}{1-\beta}U_t^{1-\rho}C_t^{\rho} \Rightarrow U_t = \left[(1-\beta)W_tC_t^{-\rho}\right]^{\frac{1}{1-\rho}} \tag{A8}$$

Finally, we plug (A5) and (A8) into $M_{R,t+1}$ and we operate to obtain,

[14] See Cochrane (2008).

$$M_{R,t+1} = \beta \left(\frac{U_{t+1}}{\left[E_t \left(U_{t+1}^{1-\gamma} \right) \right]^{\frac{1}{1-\rho}}} \right)^{\rho-\gamma} \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} = \beta \left[\frac{\left[(1-\beta) W_{t+1} C_{t+1}^{-\rho} \right]^{\frac{1}{1-\rho}}}{\left(\frac{1}{\beta} \right)^{\frac{1}{1-\rho}} \left[U_t^{1-\rho} - (1-\beta) C_t^{1-\rho} \right]^{\frac{1}{1-\rho}}} \right]^{\rho-\gamma} \left(\frac{C_{t+1}}{C_t} \right)^{-\rho}$$

$$= \beta (1-\beta)^{\frac{\rho-\gamma}{1-\rho}} \beta^{\frac{\rho-\gamma}{1-\rho}} \left[\frac{W_{t+1}^{\frac{\rho-\gamma}{1-\rho}} C_{t+1}^{-\rho\frac{(\rho-\gamma)}{1-\rho}}}{\left[(1-\beta) W_t C_t^{-\rho} - (1-\beta) C_t^{1-\rho} \right]^{\frac{\rho-\gamma}{1-\rho}}} \right] \left(\frac{C_{t+1}}{C_t} \right)^{-\rho}$$

$$= \beta^{\frac{1-\gamma}{1-\rho}} \left(\frac{W_{t+1}}{W_t - C_t} \right)^{\frac{\rho-\gamma}{1-\rho}} \left(\frac{C_{t+1}}{C_t} \right)^{-\rho\frac{(1-\gamma)}{1-\rho}}$$

(A9)

The intertemporal budget constraint for the representative agent is now given by,[15]

$$W_{t+1} = (W_t - C_t) R_{Wt+1} \phi(L_{t+1}) \quad \text{(A10)}$$

Therefore, equation (A9) is

$$M_{R,t+1} = \beta^{\kappa} \left[R_{Wt+1} \phi(L_{t+1}) \right]^{\kappa-1} \left(\frac{C_{t+1}}{C_t} \right)^{-\rho\kappa}, \quad \text{(A11)}$$

where we denote $\kappa = \dfrac{1-\gamma}{1-\rho}$.

The corresponding liquidity-adjusted SDF is as follow:

$$M_{LAR,t+1} = \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \phi(L_{t+1}) \right]^{\kappa} R_{Wt+1}^{\kappa-1} \quad \text{(A12)}$$

Note that when the relative risk aversion coeffcient equals the inverse of the elasticity of intertemporal substitution ($\gamma = \rho$), the optimization problem (A1) is based on the power utility. In that case, the liquidity-adjusted SDF with power utility function is given by:

$$M_{P,t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \phi(L_{t+1}) \quad \text{(A13)}$$

[15] This is of course equivalent to the budget constraint in (A.6).

2. Recursive Utility with Aggregate Liquidity Constraints under Ultimate Consumption Risk

The first order condition in equation (A2) can be written in terms of the gross return on asset *j* as follows.

$$C_t^{-\rho} = \mathrm{E_t}\left[\beta\left(\frac{U_{t+1}}{\left[E_t\left(U_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}}\right)^{\rho-\gamma} C_{t+1}{}^{-\rho}\phi\left(L_{t+1}\right)R_{jt+1}\right] \tag{A14}$$

Applying to the risk free rate between *t* and *t*+1,

$$C_t^{-\rho} = \mathrm{E_t}\left[\beta\left(\frac{U_{t+1}}{\left[E_t\left(U_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}}\right)^{\rho-\gamma} C_{t+1}{}^{-\rho}\phi\left(L_{t+1}\right)R_{ft,t+1}\right] \tag{A15}$$

and expanding forward such that the marginal utility in period *t*+*S* is linked to the marginal utility in period *t*,

$$C_t^{-\rho} = \mathrm{E_t}\left[\beta^S\prod_{\tau=1}^{S}\left(\frac{U_{t+\tau}}{\left[E_{t+\tau-1}\left(U_{t+\tau}^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}}\right)^{\rho-\gamma} \phi\left(L_{t+1,t+S}\right)C_{t+S}{}^{-\rho}R_{ft,t+S}\right] \tag{A16}$$

where $\phi\left(L_{t+1,t+S}\right)=\prod_{\tau=1}^{S}\phi\left(L_{t+\tau}\right)$ are the cumulative shocks in liquidity between periods $t+1$ and $t+S$.

Introducing (A16) into (A14),

$$C_t^{-\rho} = E_t\left[\beta^{S+1}\prod_{\tau=0}^{S}\left(\frac{U_{t+1+\tau}}{\left[E_{t+\tau}\left(U_{t+1+\tau}^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}}\right)^{\rho-\gamma} \phi\left(L_{t+1,t+1+S}\right)C_{t+1+S}{}^{-\rho}R_{ft+1,t+1+S}R_{jt+1}\right] \tag{A17}$$

As before, we incorporate equations (A5) and (A8) to equation (A17) to get

$$1 = E_t\left[\beta^{S+1}\prod_{\tau=0}^{S}\left(\beta^{\frac{\rho-\gamma}{1-\rho}}\left(\frac{W_{t+1+\tau}}{W_{t+\tau}-C_{t+\tau}}\right)^{\frac{\rho-\gamma}{1-\rho}}\left(\frac{C_{t+1+\tau}}{C_{t+\tau}}\right)^{-\rho\left(\frac{\rho-\gamma}{1-\rho}\right)}\right)\phi\left(L_{t+1,t+1+S}\right)\left(\frac{C_{t+1+S}}{C_t}\right)^{-\rho} R_{ft+1,t+1+S}R_{jt+1}\right]$$

(A18)

Finally, introducing the intertemporal budget constraint for aggregate wealth (A10), operating and denoting $\kappa = \dfrac{1-\gamma}{1-\rho}$, equation (A18) becomes,

$$1 = E_t\left[\beta^{(S+1)\kappa}\left(\frac{C_{t+1+S}}{C_t}\right)^{-\rho\kappa}\left(R_{Wt+1,t+1+S}\right)^{\kappa-1}\phi\left(L_{t+1,t+1+S}\right)^{\kappa}R_{ft+1,t+1+S}R_{jt+1}\right], \tag{A19}$$

where $R_{Wt+1,t+1+S}$ denotes the cumulative gross return on wealth from period t+1 to period t+1+S.

Therefore, the liquidity-adjusted SDF with recursive preferences under ultimate consumption risk is

$$M^{S}_{LAR,t+1} = \left[\beta^{S+1}\left(\frac{C_{t+1+S}}{C_t}\right)^{-\rho}\phi\left(L_{t+1,t+1+S}\right)\right]^{\kappa} R^{\kappa-1}_{Wt+1,t+1+S}R_{ft+1,t+1+S} \tag{A20}$$

As expected, equation (A20) nests equation (A12) when S=0.

As before, when $\gamma = \rho$, we obtain the liquidity-adjusted SDF with power utility function under ultimate consumption risk which is given by:

$$M^{S}_{LAP,t+1} = \beta^{S+1}\left(\frac{C_{t+1+S}}{C_t}\right)^{-\gamma}\phi\left(L_{t+1,t+1+S}\right)R_{ft+1,t+1+S} \tag{A21}$$

Appendix B: GMM Estimation and Tests

Let R_t^e be the *N x 1* vector of the excess return of the *N* assets at time *t* and $M_t(\theta)$ is one out of the eight specifications of the SDFs described in Section 2, where θ is the vector of the preference parameters for each particular specification.

Following Parker and Julliard (2005), we define an *(N*+1*) x* 1 vector containing the pricing errors generated by the model at time *t*. The first *N* conditions are the pricing errors of the model when explaining the *N* asset returns. The last condition forces the SDF to go to its mean value (μ).

$$f_t(\theta,\alpha,\mu)=\begin{bmatrix} R_t^e-\alpha 1_N+\dfrac{(M_t-\mu)R_t^e}{\mu} \\ M_t-\mu \end{bmatrix} \tag{B1}$$

where 1_N denotes an *N x 1* vector of ones. The inclusion of the parameter α enables to evaluate separately the ability of the model to explain the equity premium and the cross section of expected returns. That is, if α is zero we can conclude that the model has not an equity premium puzzle.

We also define a vector containing the sample averages corresponding to the elements of *f*.

$$g_T(\theta,\alpha,\mu)=\frac{\sum_{t=1}^{T} f_t(\theta,\alpha,\mu)}{T} \tag{B2}$$

Then, GMM minimizes the following quadratic form

$$g_T(\theta,\alpha,\mu)' W_T g_T(\theta,\alpha,\mu) \tag{B3}$$

where W_T is a weighting *(N*+1*) x (N*+1*)* matrix.

For the estimation, we could have used the optimal weighting matrix in the sense of Hansen (1982), S_T^{-1}, where

$$S_T=\frac{\sum_{t=1}^{T} f_t(\theta,\alpha,\mu)\, f_t(\theta,\alpha,\mu)'}{T}. \tag{B4}$$

Instead of that, we employ a pre-specified weighting matrix:

$$W_T = \begin{bmatrix} HJ & 0_N \\ 0_N\,' & 1000 \end{bmatrix}, \tag{B5}$$

where *HJ* is the matrix proposed by Hansen and Jagannathan (1997) containing the inverse of the variance-covariances matrix of the *N* asset excess returns

$$HJ = \left(\frac{\sum_{t=1}^{T} R_t^e R_t^e\,'}{T} \right)^{-1}, \tag{B6}$$

and 0_N is a *N*-dimensional vector of zeros.[16] A weight of 1000 for the last moment condition ensures the stability of the estimator for the mean of the SDF with respect to different initial conditions.

In general, the asymptotic variance-covariance matrix of the GMM estimates is given by

$$V = \frac{1}{T}\left(D_T\,'W_T D_T\right)^{-1} D_T\,'W_T S_T W_T D_T \left(D_T\,'W_T D_T\right)^{-1}, \tag{B7}$$

where D_T is a matrix of partial derivatives defined by

$$D_T = \frac{\sum_{t=1}^{T} \partial f_t(\theta,\alpha,\mu)/\partial(\theta,\alpha,\mu)}{T} \tag{B8}$$

Then, the standard errors of the estimated coefficients $\left(\hat{\theta},\hat{\alpha},\hat{\mu}\right)$ are computed from the estimated variance:

$$\hat{V} = \frac{1}{T}\left(\hat{D}_T\,'W_T \hat{D}_T\right)^{-1} \hat{D}_T\,'W_T \hat{S}_T W_T \hat{D}_T \left(\hat{D}_T\,'W_T \hat{D}_T\right)^{-1}, \tag{B9}$$

where $\hat{D}_T$ and $\hat{S}_T$ are obtained replacing (θ,α,μ) by $\left(\hat{\theta},\hat{\alpha},\hat{\mu}\right)$ in D_T and S_T, respectively.

The evaluation of the model performance is carried out by testing the following null hypothesis:

---

[16] A detailed description of the advantages of a pre-specified weighting matrix over the efficient GMM can be found in Cochrane (2005).

$$H_0 : T\left[Dist\left(\theta,\alpha,\mu\right)\right]^2 = 0\,, \tag{B10}$$

with $Dist = \sqrt{g_T\left(\theta,\alpha,\mu\right)' W_T g_T\left(\theta,\alpha,\mu\right)}$.

If the weighting matrix is optimal, $T\left[Dist\left(\hat{\theta},\hat{\alpha},\hat{\mu}\right)\right]^2$ is asymptotically distributed as a Chi-square with N+1-P degrees of freedom, where P is the number of parameters. However, for any other weighting matrix, the distribution of the test statistic is unknown. Hansen and Jagannathan (1997) and Parker and Julliard (2005) show that, in this case, $T\left[Dist\left(\hat{\theta},\hat{\alpha},\hat{\mu}\right)\right]^2$ is asymptotically distributed as a weighted sum of N+1-P independent Chi-squares random variables with one degree of freedom. That is

$$T\left[Dist\left(\hat{\theta},\hat{\alpha},\hat{\mu}\right)\right]^2 \xrightarrow{d} \sum_{i=1}^{N+1-P} \lambda_i \chi_i^2(1)\,, \tag{B11}$$

where λ_i, for $i = 1, 2, ..., \mathrm{N+1\text{-}P}$, are the positive eigenvalues of the following matrix:

$$A = S_T^{1/2} W_T^{1/2}\left[I_{N+1} - \left(W_T^{1/2}\right)^{-1} D_T\left(D_T' W_T D_T\right)^{-1} D_T' W_T^{1/2}\right]\left(W_T^{1/2}\right)'\left(S_T^{1/2}\right)' \tag{B12}$$

in which $X^{1/2}$ means the upper-triangular matrix from the Choleski decomposition of X, and I_{N+1} is a *(N*+1*)*-dimensional identity matrix.

Therefore, in order to test the different models we estimate, we proceed in the following way. First, we estimate the matrix A by

$$\hat{A} = \hat{S}_T^{1/2} W_T^{1/2}\left[I_{N+1} - \left(W_T^{1/2}\right)^{-1} \hat{D}_T\left(\hat{D}_T' W_T \hat{D}_T\right)^{-1} \hat{D}_T' W_T^{1/2}\right]\left(W_T^{1/2}\right)'\left(\hat{S}_T^{1/2}\right)' \tag{B13}$$

and compute its nonzero N+1-P eigenvalues. Second, we generate $\{v_{hi}\}$, h=1,2,…,100, i=1,2,…,N+1-P, independent random draws from a $\chi^2(1)$ distribution. For each h, $u_h = \sum_{i=1}^{N+1-P} \lambda_i v_{hi}$ is computed. Then we compute the number of cases for which $u_h > T\left[Dist\left(\hat{\theta},\hat{\alpha},\hat{\mu}\right)\right]^2$. Let p denote the percentage of this number.

We repeat this procedure 1000 times. Finally, the p-value for the specification test of the model is the average of the p values for the 1000 replications.

APPENDIX C: The linear factor model approximation

Now we obtain the beta (linear) version of the models analyzed in the paper. We do it for the most general case; this is, the recursive preferences with illiquidity shocks and ultimate consumption risk. The rest of the models are just special cases of our general specification.

The non-linear asset pricing model is given by

$$E_t\left[M^{S}_{LAR,t+1}R_{jt+1}\right]=1 \tag{C1}$$

Using the definition of the covariance, equation (C1) can be written as

$$E_t\left(R_{jt+1}-R_{ft+1}\right)=-\frac{Cov_t\left(M^{S}_{LAR,t+1},R_{jt+1}\right)}{E_t\left(M^{S}_{LAR,t+1}\right)} \tag{C2}$$

In Appendix A, the SDF based on recursive preferences with illiquidity shocks in the intertemporal budget constrain and ultimate risk is given by:

$$M^{S}_{LAR,t+1}=\left[\beta^{S+1}\left(\frac{C_{t+1+S}}{C_t}\right)^{-\rho}\phi\left(L_{t+1+S}\right)\right]^{\kappa}R^{\kappa-1}_{Wt+1,t+1+S}R_{ft+1,t+1+S} \tag{C3}$$

Taking logs in the SDF, we get

$$m^{S}_{LAR,t+1}=\kappa(S+1)log\left(\beta\right)-\kappa\rho\Delta c_{t,t+1+S}+\kappa\, log\left(\phi\left(L_{t+1+S}\right)\right)+(\kappa-1)r_{Wt+1,t+1+S}+r_{ft+1,t+1+S} \tag{C4}$$

where lowercase letters denote the logs of uppercase letters.

Assuming that the risk free rate is approximately constant over time, the covariance between the linear SDF in (C4) and the return on asset *j* is given by

$$\begin{aligned}&Cov_t\left(m^{S}_{LAR,t+1},R_{jt+1}\right)=\\&-\kappa\rho Cov_t\left(\Delta c_{t,t+1+S},R_{jt+1}\right)+\kappa Cov_t\left(log\left(\phi\left(L_{t+1+S}\right)\right),R_{jt+1}\right)+(\kappa-1)Cov_t\left(r_{Wt+1,t+1+S},R_{jt+1}\right)\end{aligned} \tag{C5}$$

Introducing (C5) in (C3) and operating, the beta version of the model is

$$E_t\left(R_{jt+1}-R_{ft+1}\right)\simeq\gamma_1\beta_{ct}+\gamma_2\beta_{\phi t}+\gamma_3\beta_{Wt}\,, \tag{C6}$$

where the risk premium associated to each beta is given by the following expressions

$$\gamma_1 = \frac{\kappa\rho Var_t\left(\Delta c_{t,t+1+S}\right)}{E_t\left(M^S_{LAR,t+1}\right)} \qquad \gamma_2 = \frac{-\kappa Var_t\left(log\left(\phi\left(L_{t+1+S}\right)\right)\right)}{E_t\left(M^S_{LAR,t+1}\right)} \qquad \gamma_3 = \frac{(1-\kappa)Var_t\left(r_{Wt+1,t+1+S}\right)}{E_t\left(M^S_{LAR,t+1}\right)}$$

And the risk factors are determined as follow:

$$\beta_{ct} = \frac{Cov_t\left(\Delta c_{t,t+1+S}, R_{jt+1}\right)}{Var_t\left(\Delta c_{t,t+1+S}\right)}$$

$$\beta_{\phi t} = \frac{Cov_t\left(log\left(\phi\left(L_{t+1+S}\right)\right), R_{jt+1}\right)}{Var_t\left(log\left(\phi\left(L_{t+1+S}\right)\right)\right)}$$

$$\beta_{Wt} = \frac{Cov_t\left(r_{Wt+1,t+1+S}, R_{jt+1}\right)}{Var_t\left(r_{Wt+1,t+1+S}\right)}.$$

Table 1a

Descriptive Statistics

25 *Fama and French Portfolios*

	Low	2	3	4	High	Mean
PANEL A: Mean return and Standard deviation						
	Low	2	3	4	High	Mean
Small	2.114	3.831	4.042	4.768	4.845	4.013
	(17.30)	(14.62)	(12.98)	(12.29)	(13.27)	(13.40)
2	2.605	3.455	4.105	4.297	4.368	3.861
	(15.45)	(12.88)	(11.38)	(11.05)	(11.84)	(11.75)
3	2.685	3.539	3.462	3.914	4.390	3.712
	(14.01)	(11.35)	(10.08)	(9.94)	(11.04)	(10.44)
4	3.013	2.836	3.489	3.758	3.847	3.518
	(12.64)	(10.50)	(9.51)	(9.39)	(10.53)	(9.68)
Big	2.643	2.826	2.710	3.057	2.985	2.915
	(9.73)	(8.71)	(7.65)	(7.85)	(8.82)	(7.62)
Mean	2.699	3.353	3.650	4.062	4.256	
	(12.90)	(10.77)	(9.52)	(9.41)	(10.28)	
PANEL B: Return-based illiquidity betas from the time series regression: $R_{jt} = \alpha_j + \beta_j \phi(L_t) + u_{jt}$						
	Low	2	3	4	High	Mean
Small	-0.762	-0.678	-0.627	-0.633	-0.660	-0.672
	(-6.38)	(-6.77)	(-7.20)	(-7.82)	(-7.41)	(-7.28)
2	-0.678	-0.598	-0.538	-0.517	-0.544	-0.575
	(-6.35)	(-6.83)	(-6.98)	(-6.88)	(-6.69)	(-7.05)
3	-0.619	-0.533	-0.447	-0.447	-0.469	-0.503
	(-6.43)	(-6.92)	(-6.40)	(-6.51)	(-6.05)	(-6.91)
4	-0.533	-0.476	-0.398	-0.378	-0.443	-0.445
	(-6.08)	(-6.64)	(-5.97)	(-5.64)	(-5.91)	(-6.52)
Big	-0.399	-0.323	-0.245	-0.293	-0.309	-0.314
	(-5.87)	(-5.22)	(-4.37)	(-5.23)	(-4.82)	(-5.68)
Mean	-0.598	-0.522	-0.451	-0.454	-0.485	
	(-6.58)	(-6.95)	(-6.75)	(-6.92)	(-6.73)	
PANEL C: Return-based illiquidity betas from the time series regression: $R_{jt} = \alpha_j + \beta_{jL} \phi(L_t) + \beta_{jW} R_{Wt} + \beta_{jc} \Delta C_t + u_{jt}$						
	Low	2	3	4	High	Mean
Small	-0.153	-0.166	-0.204	-0.250	-0.259	-0.206
	(-1.84)	(-2.36)	(-3.17)	(-4.03)	(-3.57)	(-3.16)
2	-0.099	-0.140	-0.129	-0.143	-0.153	-0.133
	(-1.61)	(-2.56)	(-2.68)	(-2.71)	(-2.47)	(-2.78)
3	-0.087	-0.104	-0.075	-0.102	-0.107	-0.095
	(-1.74)	(-2.58)	(-1.74)	(-2.16)	(-1.77)	(-2.52)
4	-0.034	-0.077	-0.038	-0.020	-0.072	-0.048
	(-0.83)	(-2.05)	(-1.00)	(-0.48)	(-1.32)	(-1.62)
Big	0.001	0.028	0.067	-0.006	0.001	0.018
	(0.02)	(1.03)	(2.08)	(-0.16)	(0.03)	(0.95)
Mean	-0.075	-0.092	-0.076	-0.104	-0.118	
	(-1.76)	(-2.57)	(-2.14)	(-2.60)	(-2.35)	

The sample period covers from 1963:II to 2003:I. Mean returns and standard deviations (in parenthesis) in Panel A are in percentages. In Panels B and C, numbers in parenthesis are *t*-statistics. In the three panels, last column refers to an average portfolio between the five groups of book-to-market for each size portfolio and the last row refers to an average portfolio between the five groups of size for each book-to-market portfolio. R_{jt} denotes the gross return on portfolio *j* at time *t*, $\phi(L_t)$ is the illiquidity function that depends on the Amihud ratio, R_{Wt} is the gross return on aggregate wealth, and ΔC_t is the non durable consumption growth rate.

Table 1b

Descriptive Statistics

17 *Industry Portfolios*

	Mean Return	Standard Deviation	Illiquidity beta Simple regression		Illiquidity beta Multiple regression	
Steel	1.933	11.43	-0.341	(-3.93)	0.075	(1.12)
Utilities	2.362	7.59	-0.182	(-3.10)	0.037	(0.70)
Durables	2.597	10.52	-0.508	(-6.93)	-0.148	(-2.93)
Chemicals	2.608	9.36	-0.343	(-4.95)	0.007	(0.14)
Other	2.688	9.62	-0.400	(-5.76)	0.014	(0.54)
Cars	2.767	11.56	-0.373	(-4.28)	0.038	(0.55)
Fab. Products	2.821	9.91	-0.404	(-5.61)	-0.050	(-0.98)
Mines	2.941	11.77	-0.292	(-3.22)	0.049	(0.57)
Transport	3.032	11.05	-0.465	(-5.82)	-0.053	(-0.97)
Machinery	3.038	12.25	-0.475	(-5.28)	0.028	(0.53)
Construction	3.105	11.82	-0.535	(-6.38)	-0.081	(-1.73)
Oil	3.117	8.60	-0.173	(-2.58)	0.103	(1.71)
Cloths	3.271	12.98	-0.631	(-6.99)	-0.215	(-3.08)
Finance	3.342	10.23	-0.441	(-6.00)	-0.054	(-1.28)
Retail	3.377	11.50	-0.517	(-6.33)	-0.099	(-1.85)
Food	3.506	9.04	-0.389	(-6.00)	-0.108	(-2.14)
Drugs	3.573	9.21	-0.342	(-5.03)	-0.060	(-1.07)

The sample period covers from 1963:II to 2003:I. Mean returns and standard deviations are in percentages. Numbers in parenthesis are *t*-statistics . Simple regression refers to equation in the top of Panel B of Table 1, and multiple regressions refer to equation in the top of Panel C of Table 1.

Table 2a

GMM Estimation

Power Utility

PANEL A: 25 Fama and French Portfolios						
	S=0			S=11		
	γ	α	$T\left(Dist\right)^2$	γ	α	$T\left(Dist\right)^2$
M_P	-21.21 (40.73)	0.0214 (0.0049)	50.922 (0.0701)	4.35 (8.03)	0.0205 (0.0050)	51.192 (0.2070)
M_{LAP}	-33.82 (40.96)	0.0206 (0.0050)	49.030 (0.0733)	9.68 (8.73)	0.0208 (0.0047)	50.219 (0.1796)
PANEL B: 25 Fama and French Portfolios and 17 Industry Portfolios						
	S=0			S=11		
	γ	α	$T\left(Dist\right)^2$	γ	α	$T\left(Dist\right)^2$
M_P	13.93 (34.31)	0.0123 (0.0041)	86.898 (0.2229)	2.48 (6.26)	0.0129 (0.0039)	87.185 (0.2637)
M_{LAP}	8.76 (35.07)	0.0116 (0.0042)	85.672 (0.2277)	2.76 (6.30)	0.0130 (0.0040)	86.391 (0.3131)

The sample period covers from 1963:II to 2003:I. M_P is the SDF based on power utility function and M_{LAP} is the SDF based on power utility function and illiquidity shocks affecting the intertemporal budget constrain. S=0 means that the marginal rate of substitution relates periods t+1 and t. S=11 means that the marginal rate of substitution relates periods t+12 and t. γ is the relative risk aversion coefficient, α is the mean error of the model in explaining the returns on the set of considered portfolios, and $T\left(Dist\right)^2$ is the measure to test the over-identifying restrictions (see Appendix B for details).

Table 2b

GMM Estimation

Recursive Utility

PANEL A: 25 Fama and French Portfolios								
	S=0				S=11			
	γ	ρ	α	$T\left(Dist\right)^2$	γ	ρ	α	$T\left(Dist\right)^2$
M_R	-11.53 (41.50)	-5.36 (22.54)	0.0245 (0.0066)	50.415 (0.0052)	12.42 (9.07)	9.05 (7.68)	0.0229 (0.0050)	50.211 (0.1084)
M_{LAR}	-37.25 (42.34)	-11.08 (14.37)	0.0254 (0.0062)	45.722 (0.0316)	10.81 (9.21)	6.89 (6.47)	0.0230 (0.0052)	49.259 (0.0642)
PANEL B: 25 Fama and French Portfolios and 17 Industry Portfolios								
	S=0				S=11			
	γ	ρ	α	$T\left(Dist\right)^2$	γ	ρ	α	$T\left(Dist\right)^2$
M_R	15.70 (34.84)	13.10 (28.69)	0.0128 (0.0048)	86.859 (0.0935)	2.57 (6.26)	2.53 (6.20)	0.013 (0.0041)	87.182 (0.2602)
M_{LAR}	13.13 (35.29)	6.99 (17.48)	0.0131 (0.0047)	84.778 (0.0460)	2.57 (5.97)	2.48 (5.73)	0.0131 (0.0040)	86.388 (0.0674)

The sample period covers from 1963:II to 2003:I. M_R is the SDF based on recursive preferences and M_{LAR} is the SDF based on recursive preferences and illiquidity shocks affecting the intertemporal budget constrain. S=0 means that the marginal rate of substitution relates periods t+1 and t. S=11 means that the marginal rate of substitution relates periods t+12 and t. γ is the relative risk aversion coefficient, ρ is the inverse of the elasticity of substitution, α is the mean error of the model in explaining the returns on the set of considered portfolios, and $T\left(Dist\right)^2$ is the measure to test the over-identifying restrictions (see Appendix B for details).

Table 3a

Fama-MacBeth Estimation

CCAPM and Illiquidity Shocks

PANEL A: 25 Fama and French Portfolios									
S=0					*S*=11				
γ_0	γ_1	γ_2	R^2_{adj}	$MSE^{1/2}$	γ_0	γ_1	γ_2	R^2_{adj}	$MSE^{1/2}$
0.0378	-0.0011		32.99	0.605	0.0301	0.0043		31.11	0.473
(5.12)	(-1.01)				(3.50)	(0.69)			
0.0299	-0.0033	-0.0011	57.13	0.459	0.0296	0.0061	-0.0493	52.42	0.458
(4.19)	(-2.97)	(-0.07)			(3.81)	(0.93)	(-2.05)		
PANEL B: 25 Fama and French Portfolios and 17 Industry Portfolios									
S=0					*S*=11				
γ_0	γ_1	γ_2	R^2_{adj}	$MSE^{1/2}$	γ_0	γ_1	γ_2	R^2_{adj}	$MSE^{1/2}$
0.0354	-0.0015		22.06	0.696	0.0311	0.0003		22.68	0.610
(5.21)	(-1.65)				(4.06)	(0.07)			
0.0247	-0.0021	-0.0070	39.43	0.614	0.0310	0.0033	-0.0455	36.79	0.615
(4.01)	(-2.65)	(-0.49)			(4.41)	(0.67)	(-2.68)		

Tables 3a, 3b and 3c provide estimators of the risk premia from different versions of the cross-sectional regression $R_{jt} - R_{ft} = \gamma_0 + \gamma_1 \beta_{jc,t} + \gamma_2 \beta_{j\phi,t} + \gamma_3 \beta_{jW,t} + e_{jt+1}$, for the period between 1971:I and 2003:I.

Results shown in Table 3a correspond to the power utility function estimation (in this case, γ_3 is zero as shown in Appendixes A and C). Table 3b presents the results from CAPM (in this case, γ_1 is zero as shown in Appendixes A and C). Finally, Table 3c shows results from the full regression.

β_{jc}, $\beta_{j\phi}$ and β_{jW} are the sensitivities of the return on asset *j* to changes into the three risk factors: non-durable consumption growth rate, unexpected aggregate illiquidity and the return on the aggregate wealth, respectively. They are estimated with a rolling window of data previous to each cross-sectional regression.

S=0 means that the risk factors are computed relating periods *t* and *t*+1. *S*=11 means that the risk factors are computed relating periods *t* and *t*+12.

R^2_{adj} is the adjusted determination coefficient, computed using the sum of the total sums and the sum of the residual sums from each cross-sectional regression and $MSE^{1/2}$ is square root of the mean square errors for the portfolios. Both are reported as percentages. *t*-statistics are in parenthesis.

Table 3b

Fama-MacBeth Estimation

CAPM and Illiquidity Shocks

PANEL A: 25 Fama and French Portfolios									
S=0					*S*=11				
γ_0	γ_2	γ_3	R^2_{adj}	$MSE^{1/2}$	γ_0	γ_2	γ_3	R^2_{adj}	$MSE^{1/2}$
0.0391 (4.02)		-0.0064 (-0.54)	49.20	0.634	0.0334 (4.01)		-0.0652 (-0.95)	33.86	0.799
0.0451 (4.37)	-0.0356 (-1.79)	-0.0172 (-1.36)	61.95	0.377	0.0384 (4.67)	-0.0139 (-0.55)	-0.0690 (-0.95)	50.87	0.578
PANEL B: 25 Fama and French Portfolios and 17 Industry Portfolios									
S=0					*S*=11				
γ_0	γ_2	γ_3	R^2_{adj}	$MSE^{1/2}$	γ_0	γ_2	γ_3	R^2_{adj}	$MSE^{1/2}$
0.0283 (3.80)		0.0008 (0.08)	32.73	0.629	0.0211 (3.01)		0.0052 (0.10)	23.32	0.790
0.0313 (3.92)	-0.0179 (-0.97)	-0.0045 (-0.41)	42.11	0.518	0.0261 (3.71)	-0.0280 (-1.57)	0.0165 (0.33)	35.71	0.656

See notes in Table 3a.

Table 3c

Fama-MacBeth Estimation

Recursive Preference and Illiquidity Shocks

PANEL A: 25 Fama and French Portfolios											
S=0						S=11					
γ_0	γ_1	γ_2	γ_3	R^2_{adj}	$MSE^{1/2}$	γ_0	γ_1	γ_2	γ_3	R^2_{adj}	$MSE^{1/2}$
0.0421	-0.0026		-0.0149	57.38	0.562	0.0251	0.0036		0.0812	51.03	0.453
(4.42)	(-2.64)		(-1.29)			(3.30)	(0.64)		(1.32)		
0.0437	-0.0021	-0.0190	-0.0184	69.73	0.385	0.0314	0.0066	-0.0450	0.0441	65.82	0.423
(4.58)	(-2.16)	(-1.05)	(-1.57)			(4.40)	(1.20)	(-2.01)	(0.85)		
PANEL B: 25 Fama and French Portfolios and 17 Industry Portfolios											
S=0						S=11					
γ_0	γ_1	γ_2	γ_3	R^2_{adj}	$MSE^{1/2}$	γ_0	γ_1	γ_2	γ_3	R^2_{adj}	$MSE^{1/2}$
0.0305	-0.0018		-0.0042	40.11	0.644	0.0241	-0.002		0.0773	38.01	0.573
(4.31)	(-2.46)		(-0.41)			(3.54)	(-0.34)		(1.61)		
0.0323	-0.0014	-0.0082	-0.0070	50.47	0.560	0.0302	0.0031	-0.0409	0.0595	48.46	0.559
(4.45)	(-2.03)	(-0.48)	(-0.69)			(4.32)	(0.65)	(-2.47)	(1.35)		

See notes in Table 3a.

Table 4

Fama-MacBeth Estimation: Risk Premia Seasonality
Recursive Preference and Illiquidity Shocks
25 Fama and French Portfolios and 17 Industry Portfolios

RISK PREMIA FIRST QUARTER: S = 0				RISK PREMIA FIRST QUARTER: S = 11			
γ_0	γ_1	γ_2	γ_3	γ_0	γ_1	γ_2	γ_3
0.0329	-0.0012	-0.0809	0.0105	0.0544	0.0156	-0.1168	0.0472
(2.28)	(-0.81)	(-2.42)	(0.52)	(3.99)	(1.65)	(-3.66)	(0.54)
DIFFERENCE BETWEEN RISK PREMIA DURING THE REST OF THE YEAR AND THE FIRST QUARTER: S = 0				DIFFERENCE BETWEEN RISK PREMIA DURING THE REST OF THE YEAR AND THE FIRST QUARTER: S = 11			
γ_0	γ_1	γ_2	γ_3	γ_0	γ_1	γ_2	γ_3
-0.0006	-0.0004	0.0976	-0.0236	-0.0326	-0.0167	0.1019	0.0166
(-0.04)	(-0.25)	(2.52)	(-1.01)	(-2.06)	(-1.53)	(2.76)	(0.16)

Table 4 provides estimators of the risk premia from different versions of the cross-sectional regression $R_{jt} - R_{ft} = \gamma_0 + \gamma_1 \beta_{jc,t} + \gamma_2 \beta_{j\phi,t} + \gamma_3 \beta_{jW,t} + e_{jt+1}$, for the period between 1971:I and 2003:I. In particular, the results show the risk premia estimators from the first quarter of the full sample period, and for the difference between the rest of the year and the first quarter. In particular, we run the following regression under recursive preferences with illiquidity shocks:

$$\hat{\gamma}_{it} = a + bD_{RYt} + u_{it} \quad \textit{for } i = 1,2,3 \textit{ and } 4$$

where D_{RYt} is a dummy variable which is equal to one for quarters two, three and four, and zero otherwise. The estimated intercept is the average risk premia for the first quarter, while the estimated slopes represent the difference between the risk premia during the rest of the year and the first quarter. We report the results for both the contemporaneous and ultimate risk specifications of the SDFs.

Figure 1
Aggregate Illiquidity Shocks and Recessions

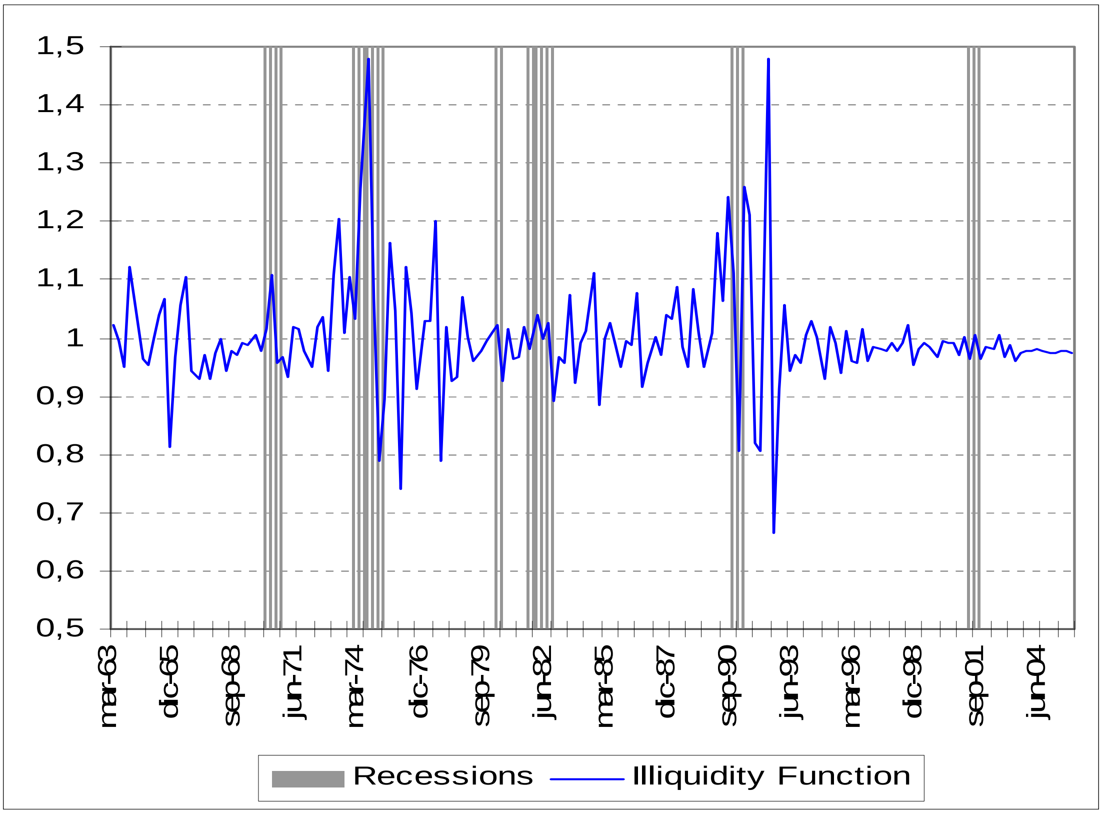

Figure 2.A

The Contemporaneous Stochastic Discount Factor Without Illiquidity Shocks

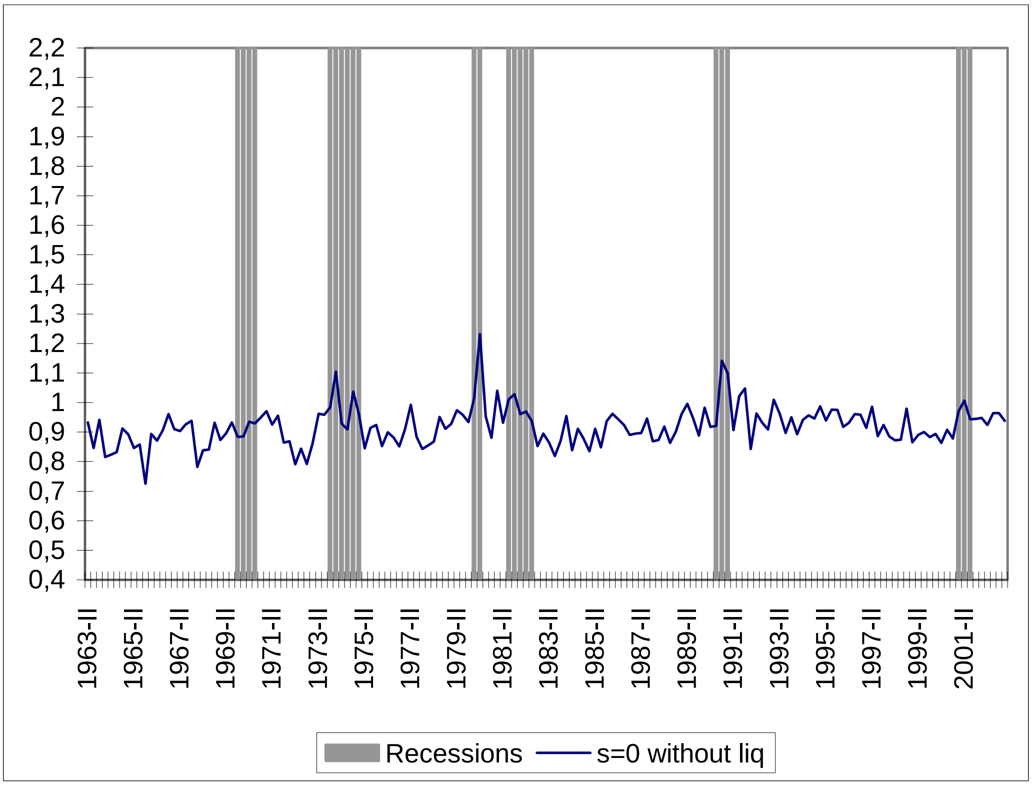

Figure 2.B

The Contemporaneous Stochastic Discount Factor With Illiquidity Shocks

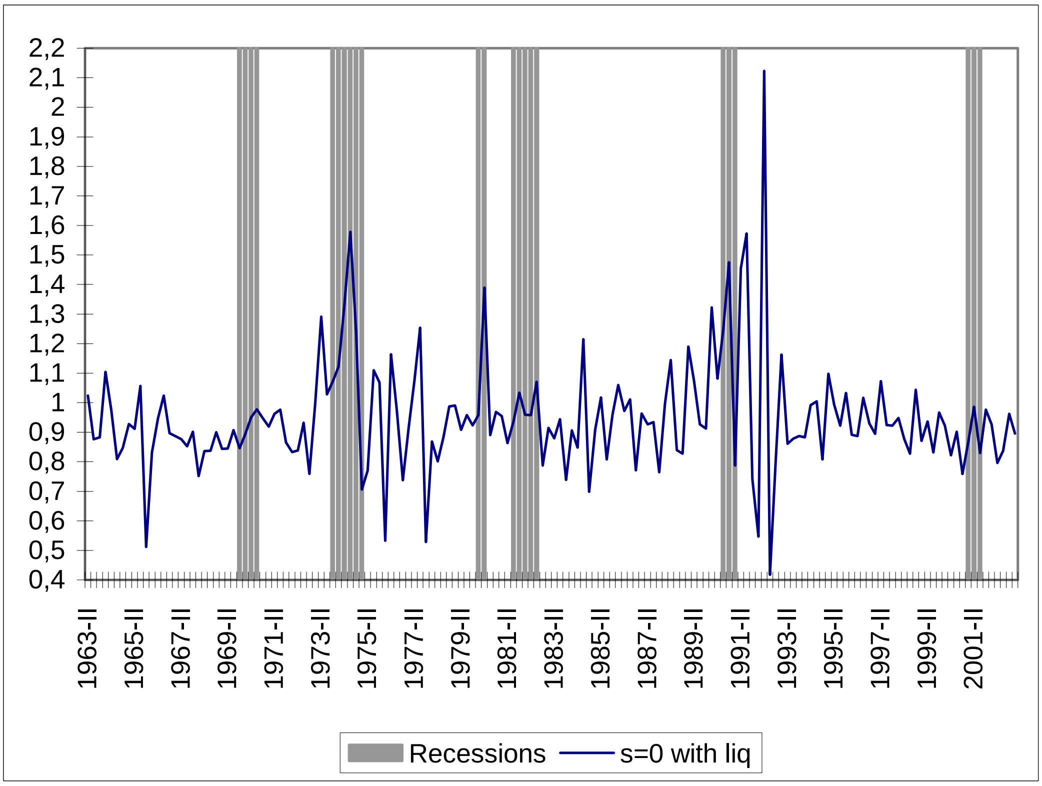

Figure 3
Mean Adjusted Returns versus Mean Observed Returns
Power Utility
Results from Fama-MacBeth Estimation with 25 Fama and French Portfolios

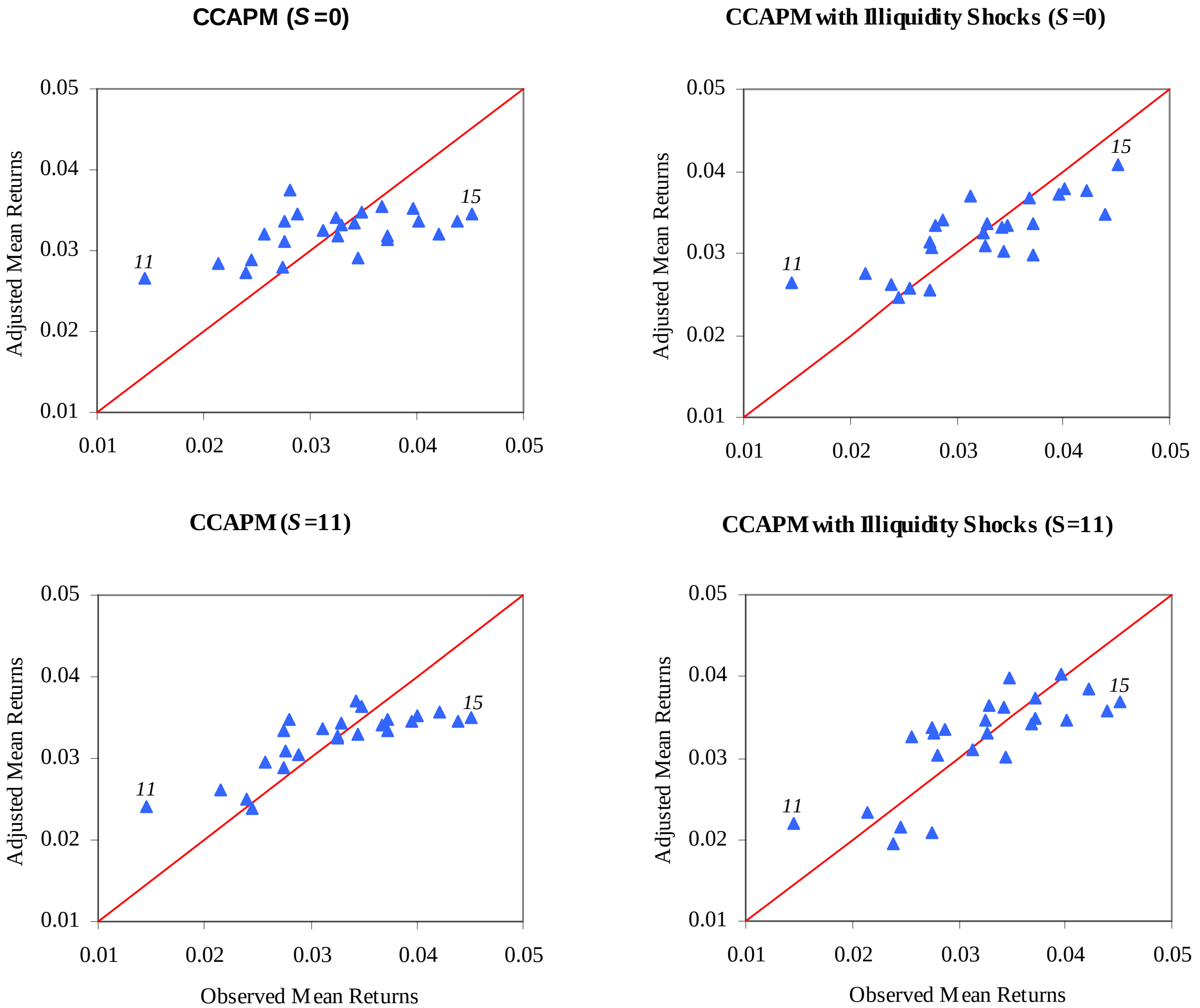

Figure 4

Mean Adjusted Returns versus Mean Observed Returns

Recursive Preferences

Results from Fama-MacBeth Estimation with 25 Fama and French Portfolios