Measuring Volatility Regime Switching and Volatility Contagion: A Range-based Volatility Approach

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December 2008

Abstract

This article proposes a new approach to evaluate volatility regime switching and volatility contagion in financial markets. A time-varying conditional autoregressive range (TVCARR) model is proposed to capture possible regime switching in the range process. A misspecification test for the conditional autoregressive range (CARR) model against the TVCARR model is introduced. The finite-sample properties of the test are discussed by simulation. Copula functions are used to construct the bivariate TVCARR model. The approach is applied to the stock markets of the G7[1] in order to investigate the impact of the subprime mortgage crisis. The evidence shows that volatility regime switching and volatility contagion occurred in most of the seven markets.

Keywords: Burr; CARR; Conditional autoregressive range; Copula; G7; Range-based volatility; Regime switching; Subprime mortgage crisis; TVCARR; Volatility contagion

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[1]The G7 refers to the seven leading industrial nations: the United States, Germany, Japan, France, the United Kingdom, Canada and Italy.

1. Introduction

As financial markets becoming increasingly integrated, the linkages of financial markets are believed to become an important mechanism for the transmission of financial shocks across countries. This dynamic linkage is of importance in risk management, asset pricing, and portfolio allocation. Most recent research on cross-country linkages has been carried out under the concept of contagion[2] via modeling the temporal dependence of financial returns to investigate the co-movement of asset prices between financial markets. Although contagion necessarily entails a substantial change in the market linkages before and after a financial crisis, the linkages do not necessarily connect through asset returns. Research on the dependence structure of return-based volatility has also caused significant attention, as the relation between volatilities gives an interesting perspective with which to interpret causation in variance between financial markets.

Even though return-based volatility is a well applied volatility measure in time series analysis, especially after the appearance of the (G)ARCH model, one drawback of this measure is that it has to be estimated via the return process since it is by nature unobserved property. As we know, the accuracy of estimated variables could be influenced by the estimation method and data quality. Another problem is less sensitivity when compared to the observed volatility measure, range-based volatility. Chou (2005) first proposed the concept of modeling range dynamically via the ACD framework and named the model “Conditional Autoregressive Range” (CARR). He shows that the CARR model provides sharper volatility estimates when compared with a standard GARCH model.

This study aims at two main purposes. First, we will propose a time-varying CARR (TVCARR) model to investigate the regime switching dynamic volatility structure. By using time as the transition variable in the smooth transition function, this model can identify the location of regime switching for a range-based volatility process. In addition, the causality relationship between each process can also be investigated by a VAR framework. Therefore the behavior in each regime can be studied separately. Second, in order to investigate the contemporaneous dependence structure between different volatility processes, we construct a bivariate TVCARR (BTVCARR) model via a copula approach. Consequently, we can evaluate the dependence change of two

[2] The most accepted definition of contagion is defined as a significant increase in cross-market linkages after a shock to one country (or a group of countries) (Forbes and Rigobon 2002).

range-based volatility processes in different regimes and discuss the existence of volatility contagion.

Aside from a variety of studies that focused on the presence of spillovers of shocks to market returns, there is a growing body of literature that considers the possibility of contagion taking place through spillovers of volatility across financial markets. These volatility spillover related studies aim at understanding how information is transmitted across assets and markets. Cheung and Ng (1996) indicated that changes in variance reflect the arrival of information, and the relation between information flow and volatility gives an interesting perspective from which to interpret the causation in variance between a pair of financial time series. Engle and Susmel (1993) argued that international markets might be uncorrelated in returns but related through their volatility. Engle et al. (1990) applied the GARCH model with a VAR framework to test the spillovers in daily exchange rate volatility across Japanese and American foreign exchange markets. They found that the intra-daily exchange rates across market segments present the meteor shower phenomenon, that is, intra-daily volatility spillovers from one market to the next. Hong (2001) indicated that the presence of volatility spillover means that the impact of one large shock not only increases the volatility of its own market but also other markets as well. If information comes in clusters, asset prices may exhibit volatility even if the market adjusts perfectly and instantaneously to the news.

All of these papers show that market volatility is transmitted across countries. However, they do not explicitly test whether the extent of transmission changes significantly after a financial crisis. Lin et al. (1994) indicated that when volatility is high, the price changes in major markets tend to become highly correlated. Edwards and Susmel (2001) proposed a bivariate SWARCH model to investigate the behavior of volatility in Latin American markets and found that there is more evidence of volatility co-movement across countries than there is evidence of contagion stories. Unlike the aforementioned studies, Diebold and Yilmaz (2008) proposed a quantitative measure that directly used the opening, closing, highest, and lowest prices[3], instead of using return-based volatility, in order to investigate the volatility spillover during both non-crisis and crisis periods. They found strong contagion in volatility but not in returns.

In this study, the daily range series is used to measure volatility regime switching and

[3] They use weekly high, low, opening and closing prices to obtain an efficient estimate of the weekly unconditional variance of stock returns.

volatility contagion by the proposed TVCARR model. A misspecification test against the TVCARR model is also introduced. The results of the size and power simulation for the test show that the proposed test performs well when the sample size is large enough. In the empirical study, we apply the proposed approach to the daily ranges of seven developed stock market (the G7) indices in order to test the volatility regime switching and volatility contagion caused by the ongoing subprime mortgage crisis that has occurred since 2007. The evidence shows that the presence of causality in volatility between the U.S. market and other indices exists during the data period. A misspecification test indicates that five of the seven markets experience volatility regime switching caused by the subprime mortgage crisis. The copula approach also indicates contemporaneous dependence changes of volatility between the U.S. and the other five markets before and after the crisis. According to the evidence of contemporaneous dependence changes, we conclude that the volatility contagion occurs due to the subprime mortgage crisis.

The article is organized as follows: In Section 2, we introduce the CARR model and derive the misspecification test against its time-varying counterpart, TVCARR model. Size and power simulations are also performed. In Section 3, properties of conditional copula are introduced. In Section 4, seven indices of the G7 are used to test the volatility regime switching and volatility contagion caused by the subprime mortgage crisis originating from the United States. We conclude in Section 5.

2. Univariate dynamic volatility model

Unlike the return-based volatility estimated by the GARCH-type models used in most volatility contagion studies, we resort to observable range-based volatility as the volatility measurement. In this section, the original CARR model with Burr error distribution will be introduced first and followed by the introduction of a time-varying CARR model.

2.1. Burr-CARR model

The Conditional Autoregressive Range (CARR) model was first proposed by Chou (2005). Let P_t be the logarithmic price of an asset at time t. The observed range is defined as $R_t = \max\{P_\tau\} - \min\{P_\tau\}$, where $\tau = t-1, t-1+\frac{1}{n}, t-1+\frac{2}{n}, ..., t.$

Therefore, the range R_t is modeled as

$$R_t = \lambda_t \varepsilon_t , \tag{1}$$

$$\lambda_t = \omega + \sum_{i=1}^{p} \alpha_i R_{t-i} + \sum_{j=1}^{q} \beta_j \lambda_{t-j} + \sum_{l=1}^{L} \phi_l X_{t-1,l} \tag{2}$$

$$\varepsilon_t \mid I_{t-1} \overset{iid}{\sim} f(\varepsilon_t ; \vartheta) .$$

This model is called the CARR model with exogenous variables (CARRX) model, where $X_{t,l}$, for $l = 1,2,\cdots, L$, are the exogenous variables, $f(\cdot)$ is a general distribution over $(0,\infty)$ with parameter vector ϑ . I_{t-1} is the information set available up to time $t-1$.

Since the range is non-negative, and the distribution is highly right skewed, there are several distributions to be chosen. The literature regarding ACD models provides a great number of options, such as exponential, Weibull, Burr, and Generalized gamma distributions, of which all are potential candidates. In the study, we use the Burr distribution as the conditional error term distribution for its flexibility with regard to capturing the distribution of daily range data. The correct specification of error distribution is important in fitting the CARR model since the misspecification of error distribution would subsequently influence the estimation accuracy of the copula approach. The density of Burr distribution is as follows:

$$Burr(\varepsilon_t ; \kappa, \sigma^2) = \frac{\kappa \varepsilon_t^{\kappa-1}}{(1+\sigma^2 \varepsilon_t^{\kappa})^{\frac{1}{\sigma^2}+1}} , \tag{3}$$

where κ and σ^2 are the shape parameters, which adjust the shape of the Burr distribution according to the behavior of the range data.
The conditional density of range R_t is

$$g(R_t ; \mathbf{\theta}) = \frac{\kappa \cdot R_t^{\kappa-1} \cdot \lambda_t^{-\kappa}}{\left(1+\sigma^2 \cdot R_t^{\kappa} \cdot \lambda_t^{-\kappa}\right)^{\frac{1}{\sigma^2}+1}} , \tag{4}$$

where $\mathbf{\theta} = (\alpha_1, \cdots, \alpha_p, \beta_1, \cdots, \beta_q, \phi_1, \cdots \phi_L, \kappa, \sigma^2)'$[4].

2.2. Time-varying CARR model

When modeling a financial series using a CARR model, it is unrealistic to assume that

[4] Readers are advised to refer to Grammig and Maurer (2000) for the details of the Burr distribution.

the parameters of the conditional mean function are constant since the possibility of regime shifts caused by certain events occurred during the study period. In order to accommodate this situation, we propose a CARR model allowing for time-varying parameters and having the capacity to identify the location of structural shifts. This time-varying CARR (TVCARR) model[5] is constructed as follows:

$$R_t = \xi_t \varepsilon_t , \quad \xi_t = \frac{\exp(\lambda_t)}{\mu} , \tag{5}$$

$$\lambda_t = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i} + \sum_{j=1}^{q} \beta_j \lambda_{t-j} + \left(\omega^* + \sum_{i=1}^{p} \alpha_i^* \varepsilon_{t-i} + \sum_{j=1}^{q} \beta_j^* \lambda_{t-j} \right) \cdot \overline{G}(s_t ; \gamma, \mathbf{c}) , \tag{6}$$

where

$$\mu = \frac{\Gamma\left(1+\frac{1}{\kappa}\right)\Gamma\left(\frac{1}{\sigma^2}-\frac{1}{\kappa}\right)}{\sigma^{2\left(1+\frac{1}{\kappa}\right)}\Gamma\left(\frac{1}{\sigma^2}+1\right)} ,$$

$$G(s_t ; \gamma, \mathbf{c}) = \left(1+\exp\left(-\gamma \prod_{k=1}^{m}(s_t - c_k)\right)\right)^{-1} ,$$

$$\overline{G}(s_t ; \gamma, \mathbf{c}) = G(s_t ; \gamma, \mathbf{c}) - 1/2 ,$$

$s_t = t$,
$\mathbf{c} = (c_1, \cdots, c_m)$, $c_1 \le \cdots \le c_m$, and
$\gamma > 0$.

The transition variable s_t in the logistic transition function $G(s_t ; \gamma, \mathbf{c})$ is defined as the day t from the beginning of the sample to the end. The size of γ governs the transition speed, and $\mathbf{c}$ represents the potential switching time location.

The log-likelihood function for observation R_t is

$$l_t(\boldsymbol{\theta}) = \ln\kappa - \kappa \ln \xi_t + (\kappa - 1)\ln R_t - \left(\frac{1}{\sigma^2}+1\right)\ln(1+\sigma^2 \xi_t^{-\kappa} R_t^{\kappa}) \tag{7}$$

2.3. Misspecification test for the TVCARR model

2.3.1. Linearity approximation for the TVCARR model

Before estimating the parameters of the TVCARR model, the first step is to verify the adequacy for this alternative by conducting a misspecification test. The transition

[5] The log-ACD type mean function is used in the TVCARR model to solve a potential problem where the negative sign of exogenous variables may cause a negative conditional range.

function is first approximated by its first order Taylor expansion around $\gamma = 0$ since the adoption of the linear approximation in TVCARR model avoids the problem of nuisance parameters under the null.

By the Taylor expansion, $\overline{G}(s_t;\gamma,\mathbf{c})$ in (6) can be approximated as

$$\overline{G}(s_t;\gamma,\mathbf{c}) \cong \frac{1}{4}\gamma\prod_{k=1}^{m}(s_t - c_k) + \overline{G}(s_t;\tilde{\gamma},\mathbf{c}) = \sum_{l=0}^{m}\gamma\tilde{c}_l(s_t)^l + \overline{G}(s_t;\tilde{\gamma},\mathbf{c}) \quad (8),$$

where $\tilde{\gamma} \in [0,\gamma]$. Then the conditional mean function can be reformulated as follows:

$$\begin{aligned}
\lambda_t &\cong \omega + \sum_{i=1}^{p}\alpha_i\varepsilon_{t-i} + \sum_{j=1}^{q}\beta_j\lambda_{t-j} \\
&\quad + \left(\omega^* + \sum_{i=1}^{p}\alpha_i^*\varepsilon_{t-i} + \sum_{j=1}^{q}\beta_j^*\lambda_{t-j}\right)\cdot\left(\sum_{l=0}^{m}\gamma\tilde{c}_l(s_t)^l + \overline{G}(s_t;\tilde{\gamma},\mathbf{c})\right) \\
&= \left(\omega + \omega^*\cdot\sum_{l=0}^{m}\gamma\tilde{c}_l(s_t)^l\right) \\
&\quad + \sum_{i=1}^{p}\left(\alpha_i + \alpha_i^*\cdot\sum_{l=0}^{m}\gamma\tilde{c}_l(s_t)^l\right)\varepsilon_{t-i} \\
&\quad + \sum_{j=1}^{q}\left(\beta_j + \beta_j^*\cdot\sum_{l=0}^{m}\gamma\tilde{c}_l(s_t)^l\right)\lambda_{t-j} \\
&= (\omega + \omega^*\gamma\tilde{c}_0) + \sum_{l=1}^{m}\omega^*\gamma\tilde{c}_l(s_t)^l \\
&\quad + \sum_{i=1}^{p}\left(\alpha_i + \alpha_i^*\gamma\tilde{c}_0\right)\varepsilon_{t-i} + \sum_{i=1}^{p}\sum_{l=1}^{m}\alpha_i^*\gamma\tilde{c}_l(s_t)^l\varepsilon_{t-i} \\
&\quad + \sum_{j=1}^{q}\left(\beta_j + \beta_j^*\gamma\tilde{c}_0\right)\lambda_{t-j} + \sum_{j=1}^{q}\sum_{l=1}^{m}\beta_j^*\gamma\tilde{c}_l(s_t)^l\lambda_{t-j}
\end{aligned}$$

Let $w = \omega + \omega^*\gamma\tilde{c}_0$, $a_i = \alpha_i + \alpha_i^*\gamma\tilde{c}_0$, $b_j = \beta_j + \beta_j^*\gamma\tilde{c}_0$, $d_l = \omega^*\gamma\tilde{c}_l(s_t)^l$, $e_{il} = \alpha_i^*\gamma\tilde{c}_l$,

and $f_{jl} = \beta_j^*\gamma\tilde{c}_l$, the formula above can be written as

$$\lambda_t \cong w + \sum_{i=1}^{p}a_i\varepsilon_{t-i} + \sum_{j=1}^{q}b_j\lambda_{t-j} + \sum_{l=1}^{m}d_l(s_t)^l + \sum_{i=1}^{p}\sum_{l=1}^{m}e_{il}(s_t)^l\varepsilon_{t-i} + \sum_{j=1}^{q}\sum_{l=1}^{m}f_{jl}(s_t)^l\lambda_{t-j} \quad . (9)$$

Using the linear approximation above, we have transformed the original testing problem into testing the CARR(*p*, *q*) against the approximate alternative

$$R_t = \xi_t\varepsilon_t, \quad \xi_t = \frac{\exp(\lambda_t + \varphi_t)}{\mu}, \quad (10)$$

$$\lambda_t + \varphi_t = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i} + \sum_{j=1}^{q} \beta_j (\lambda_{t-j} + \varphi_{t-j})$$

$$+ \sum_{l=1}^{m} d_l (s_t)^l + \sum_{i=1}^{p} \sum_{l=1}^{m} e_{il} (s_t)^l \varepsilon_{t-i} + \sum_{j=1}^{q} \sum_{l=1}^{m} f_{jl} (s_t)^l \lambda_{t-j},$$

$$\varphi_t = \sum_{j=1}^{q} \beta_j \varphi_{t-j} + \sum_{l=1}^{m} d_l (s_t)^l + \sum_{i=1}^{p} \sum_{l=1}^{m} e_{il} (s_t)^l \varepsilon_{t-i} + \sum_{j=1}^{q} \sum_{l=1}^{m} f_{jl} (s_t)^l \lambda_{t-j}.$$

According to the new expression, this model reduces to the null model when $d_l = 0$, $(l = 1, \cdots, m)$, $e_{il} = 0$, $(i = 1, \cdots, p, l = 1, \cdots, m)$, and $f_{jl} = 0$, $(j = 1, \cdots, q, l = 1, \cdots, m)$, and all parameters are identified under the null hypothesis.

The next step of the misspecification test is to provide an accurate and powerful test statistic. Because of the structural similarity between the CARR model and the ACD model, we follow a procedure similar to that used in the theorem in Meitz and Terasvirta's (2006) time-varying ACD model, and introduce the LM test statistic for the TVCARR model.

2.3.2. LM test statistic

Consider the generalized expression model (10). Let

$$\xi_t = \frac{\exp(\lambda_t + \varphi_t)}{\mu}, \quad \lambda_t = \lambda_t(\varepsilon_{t-1}, \cdots, \varepsilon_1; \boldsymbol{\theta}_1), \quad \varphi_t = \varphi_t(\varepsilon_{t-1}, \cdots, \varepsilon_1; \boldsymbol{\theta}_1, \boldsymbol{\theta}_2),$$

where

$$\boldsymbol{\theta}_1 = (\omega, \alpha_1, \cdots, \alpha_p, \beta_1, \cdots, \beta_q)',$$

$$\boldsymbol{\theta}_2 = (d_1, \cdots, d_p, (vec\mathbf{E})', (vec\mathbf{F})')',$$

$\mathbf{E} = [e_{il}]$ $(i = 1, \cdots, p, l = 1, \cdots, K)$ is an $(p \times K)$ matrix,

$\mathbf{F} = [f_{jl}]$ $(j = 1, \cdots, q, l = 1, \cdots, K)$ is an $(q \times K)$ matrix,

and the vec-operator stacks the columns of the matrix.

The null hypothesis is:

$$H_0 : \boldsymbol{\theta}_2 = \boldsymbol{\theta}_2^0, \quad \varphi_t = \varphi_t(\varepsilon_{t-1}, \cdots, \varepsilon_1; \boldsymbol{\theta}_1, \boldsymbol{\theta}_2^0) \equiv 0.$$

Hence the equation $\xi_t = \dfrac{\exp(\lambda_t)}{\mu}$ holds under the null.

Let $\hat{\boldsymbol{\theta}} = (\hat{\boldsymbol{\theta}}_1', \boldsymbol{\theta}_2^{0}{}', \hat{\kappa}, \hat{\sigma}^2)'$, $\hat{\boldsymbol{\theta}}_1 = (\hat{\omega}, \hat{\alpha}_i, \hat{\beta}_j)$ and $\hat{\kappa}, \hat{\sigma}^2$ are estimated parameters. Also let $\hat{\mathbf{a}}_t = \left(\frac{\partial l_t(\hat{\boldsymbol{\theta}})}{\partial \boldsymbol{\theta}_1}, \frac{\partial l_t(\hat{\boldsymbol{\theta}})}{\partial \kappa}, \frac{\partial l_t(\hat{\boldsymbol{\theta}})}{\partial \sigma^2} \right)$, $\hat{\mathbf{b}}_t = \frac{\partial l_t(\hat{\boldsymbol{\theta}})}{\partial \boldsymbol{\theta}_2}$.

Then, under the null hypothesis, the LM statistic

$$LM = \left\{ \sum_{t=1}^{T} \hat{\mathbf{b}}_t' \right\} \times \left\{ \sum_{t=1}^{T} \hat{\mathbf{b}}_t \hat{\mathbf{b}}_t' - \left(\sum_{t=1}^{T} \hat{\mathbf{b}}_t \hat{\mathbf{a}}_t' \right) \left(\sum_{t=1}^{T} \hat{\mathbf{a}}_t \hat{\mathbf{a}}_t' \right)^{-1} \left(\sum_{t=1}^{T} \hat{\mathbf{a}}_t \hat{\mathbf{b}}_t' \right) \right\}^{-1} \times \left\{ \sum_{t=1}^{T} \hat{\mathbf{b}}_t \right\}, \quad (11)$$

has an asymptotic chi-squared distribution with $(1 + p + q)K$ degrees of freedom.

For the proof, see Appendix A.

2.3.3. Size simulations

The data generating process used in the size simulation has the form

$$R_t = \frac{\exp(\lambda_t)}{\mu} \varepsilon_t, \quad \lambda_t = \omega + \alpha \varepsilon_{t-i} + \beta \lambda_{t-j}, \quad \varepsilon_t \mid I_{t-1} \sim Burr(\kappa, \sigma^2).$$

Two parameter settings are used in the simulation,

Case 1: $(\omega, \alpha, \beta, \kappa, \sigma^2) = (-0.1, 0.1, 0.95, 4, 1)$,

Case 2: $(\omega, \alpha, \beta, \kappa, \sigma^2) = (-0.15, 0.15, 0.95, 5, 1)$

Sample sizes are set as $T = 1{,}000, 2{,}000, 5{,}000$; the replication is 10,000. The results of the experiments shown in Table 1 indicate that the proposed misspecification test performs well for $T = 5{,}000$. Some distortions are present when $T = 1{,}000$ and when $T = 2{,}000$. In general, a larger sample size is required for adopting the asymptotic chi-squared distribution.

2.3.4. Power simulations

To evaluate whether the testing performance is affected by the transition speed γ and the switching location c, two parameter specifications are used according to the estimation results from the empirical study. The data generating process has the following form:

$$R_t = \frac{\exp(\lambda_t)}{\mu} \varepsilon_t, \quad \lambda_t = \omega + \alpha \varepsilon_{t-1} + \beta \lambda_{t-1} + \left(\omega^* + \alpha^* \varepsilon_{t-1} + \beta^* \lambda_{t-1} \right) \cdot \overline{G}(s_t; \gamma, \mathbf{c})$$

The parameter settings of the two cases are as follows[6]:

Case 3: $(\omega, \alpha, \beta, \omega^*, \alpha^*, \beta^*, \kappa, \sigma^2) = (-0.15, 0.16, 0.94, -0.07, 0.1, -0.02, 3.7, 0.7)$

Case 4: $(\omega, \alpha, \beta, \omega^*, \alpha^*, \beta^*, \kappa, \sigma^2) = (-0.10, 0.11, 0.95, 0.07, -0.02 - 0.02, 4.7, 1.2)$

[6] The parameter settings of Case 3 and Case 4 are the parameter estimates of SP and CAC in fitting TVCARR(1,1) model, respectively.

The sample sizes are $T = 1{,}000, 1{,}500, \text{and } 2{,}000$; with each experiment having 10,000 replications. Three different settings of transition speed ($\gamma = 0.1, 0.5, \text{and } 1.0$) and three switching locations ($c = 0.25T, 0.5T, \text{and } 0.75T$) are considered for the experiments. The results in Table 2 show that the performance is good and has no relevance with γ and c as $T = 2{,}000$. A relatively slight power loss occurs when $T = 1{,}000 \text{ and } 1{,}500$.

3. Bivariate time-varying CARR model

3.1. The copula function

The selection of a bivariate distribution for modeling bivariate financial time series data is essential but sometimes difficult. In this subsection, in order to handle the modeling in the bivariate time series flexibly, the copula function methodology is introduced[7].

Consider the two continuous random variables X and Y have marginal distributions $F_X(x) = \Pr(X \le x)$, and $G_Y(y) = \Pr(Y \le y)$. According to Sklar's theorem, for continuous distribution, the joint distribution, $H(x, y) = \Pr(X \le x, Y \le y)$, can be uniquely constructed by a copula C such that, for all real number x and y, one has the equality

$$H(x, y) = C(F_X(x), G_Y(y)). \tag{12}$$

Note that the copula does not constrain the choice of marginal distributions. Since the copula is based on ranks, the dependence is invariant under strictly increasing transforms. That is, the copula extracts the way in which x and y co-move, regardless of the scale used to measure them. Besides, copula is a convenient way of creating modeling dependence between random variables because it separates the marginal and the association part of a multivariate distribution.

In order to apply the copula function to a financial time series area, Patton (2004) introduced the concept of the conditional copula, which relaxes the i.i.d. assumption in Sklar's theorem. Let H_t be a conditional bivariate distribution function with continuous marginal distributions F_t and G_t, and let I_{t-1} be some conditional set. Then there exists a unique conditional copula C_t: $[0,1] \times [0,1] \to [0,1]$ such that

$$H_t(x_t, y_t \mid I_{t-1}) = C_t(F_t(x_t \mid I_{t-1}), G_t(y_t \mid I_{t-1}) \mid I_{t-1}), \quad \forall x_t, y_t \in \overline{R}, \tag{13}$$

[7] A thorough introduction to copula is presented in Joe (1997), and Nelsen (2006).

where $\overline{R} = R \bigcup \{\pm\infty\}$. $F_t(x_t \mid I_{t-1})$ and $G_t(y_t \mid I_{t-1})$ are the conditional probability integral transforms of X and Y given the past information set I_{t-1}. In this study, x_t and y_t represent the residuals of two TVCARR models.

Because the distribution of range data is highly right skewed, the joint distribution of two or more range series is far from the elliptical distribution. Therefore, the traditional dependence measure, the linear correlation coefficient, is no longer a satisfactory measure of the dependence in this study. The alternative selection is the use of non-parametric dependence measure, Kendall's τ. The Kendall's τ can be calculated as the following formulas:

$$\tau_C = 4\int_0^1\int_0^1 C(u,v)dC(u,v) - 1, \tag{14}$$

where $dC(u,v) = \dfrac{\partial^2 C(u,v)}{\partial u \partial v} dudv$. This rank based dependence measure τ can be directly derived by the estimated dependence parameters in copula. The relationship will be introduced later.

Another important feature of copula is the tail dependence. The tail dependence coefficient is defined as the probability that a random variable is higher (lower) than a certain threshold value, given that another random variable is higher (lower) than the threshold value. Supposing U and V are two random variables, the lower tail dependence, τ^L, and upper tail dependence, τ^U, are defined as follows:

$$\tau^L \equiv \lim_{\varepsilon \to 0} \Pr(U \le \varepsilon \mid V \le \varepsilon) = \lim_{\varepsilon \to 0} \Pr(V \le \varepsilon \mid U \le \varepsilon) = \lim_{\varepsilon \to 0} \frac{C(\varepsilon,\varepsilon)}{\varepsilon}, \tag{15}$$

$$\tau^U \equiv \lim_{\varepsilon \to 1} \Pr(U > \varepsilon \mid V > \varepsilon) = \lim_{\varepsilon \to 1} \Pr(V > \varepsilon \mid U > \varepsilon) = \lim_{\varepsilon \to 1} \frac{1 - 2\varepsilon + C(\varepsilon,\varepsilon)}{1 - \varepsilon}. \tag{16}$$

Two random variables exhibit lower tail dependence if $\tau^L > 0$, and they exhibit upper dependence if $\tau^U > 0$. This indicates that a positive τ^U would imply a non-zero probability of observing an extremely large volatility of one series together with an extremely large volatility of the other series in this study.

We consider two copulas which have different characteristics in terms of tail dependence. The Gumbel copula has upper tail dependence, while the Gumbel-Clayton copula has both upper and lower tail dependence since it mixes the Gumbel and Clayton copulas[8]. For notational convenience, set $u = F_X(x)$, and

[8] Table B1 presents the Kendall's τ and tail dependence information about the two copulas referenced herein.

$v = G_Y(y)$. The Gumbel copula is defined as

$$C_G(u,v;\eta) = \exp\{-[(-\ln u)^{\eta} + (-\ln v)^{\eta}]^{1/\eta}\}, \tag{17}$$

where the parameter η determines the dependence and tail dependence. The range of η is $[1,+\infty)$. The Gumbel-Clayton copula, called BB1 in Joe (2001), is defined as

$$C_{GC}(u,v;\delta_1,\delta_2) = \left\{\left[(u^{-\delta}-1)^{\eta} + (v^{-\delta}-1)^{\eta}\right]^{1/\eta} + 1\right\}^{-1/\delta}. \tag{18}$$

If $\delta \to 0$, the Gumbel-Clayton copula becomes the Gumbel copula.

One main purpose of this paper is to measure the volatility contagion effect. First, the breakpoint of the volatility structural shift can be identified by the TVCARR model. Then, the regime variation dependence between low and high regimes can be captured by the following regime switching setting in a conditional copula:

Let

$$C_t(u,v\,|\,\boldsymbol{\kappa}_L,\boldsymbol{\kappa}_H;I_{t-1}) = \begin{cases} C_t(u,v\,|\,\boldsymbol{\kappa}_L;I_{t-1}), \text{if } t < t_b \\ C_t(u,v\,|\,\boldsymbol{\kappa}_H;I_{t-1}), \text{if } t \geq t_b \end{cases}, \tag{19}$$

where $\boldsymbol{\kappa}_L$ and $\boldsymbol{\kappa}_H$ are the dependence parameters in the low and high regimes, respectively. t_b is the break time regarding the structural shift. Corresponding to the constant version of Gumbel and Clayton-Gumbel copulas given in (17) and (18), the same copulas with regime switching setting are called the regime switching version of the Gumbel and Clayton-Gumbel copulas. The four copulas (Gumbel and Clayton-Gumbel with constant and regime switching versions) are fitted in the empirical study. The AIC (Akaike's information criterion) is used in measuring the goodness-of-fit of the four copulas. If a constant version of the copulas is formally selected, then no dependence changes between two volatility regimes. On the other hand, the selection of a regime switching version of the copulas indicates that the series experiences dependence change before and after time t_b, and this can be inferred as volatility contagion.

3.2. Two-stage maximum likelihood estimation

The density function of $H_t(x_t, y_t)$ can be decomposed as the product of marginal densities and copula density.

$$h_t(x_t, y_t\,|\,I_{t-1}) = f_t(x_t\,|\,I_{t-1}) \cdot g_t(y_t\,|\,I_{t-1}) \cdot c_t(u_t, v_t\,|\,I_{t-1}), \quad \forall x_t, y_t \in \overline{R}, \tag{20}$$

where $u_t = F_t(x_t\,|\,I_{t-1})$, $v_t = G_t(y_t\,|\,I_{t-1})$, and $c_t(u_t, v_t\,|\,I_{t-1})$ is the density of $C_t(u_t, v_t\,|\,I_{t-1})$. The joint log-likelihood function is then

$$L = \sum_{t=1}^{T} \left(\ln f_t(x_t \mid I_{t-1}) + \ln g_t(y_t \mid I_{t-1}) + \ln c_t(u_t, v_t \mid I_{t-1}) \right), \tag{21}$$

which permits an application of the maximum likelihood estimation in two stages: first the parameters of the marginals, f_t and g_t, and then the parameters of the copula c_t.

4. Empirical application

To illustrate the application of the bivariate TVCARR model discussed above, this section explores the turmoil in equity markets resulting from the subprime mortgage crisis originating from the U.S. since 2007. The subprime mortgage crisis is an ongoing economic event which became more apparent after mid-2007. It began with the bursting of the U.S. housing bubble and high default rates on subprime mortgages.

The interest of this paper aims to study how this crisis influences the stock markets in the world. The purposes will be carried out by investigating the volatility regime switching and the volatility contagion between stock indices of seven major countries, the G7. In the following analyses, we use abbreviations to represent the seven indices. The labels are SP for the S&P 500, TSX for the S&P/Toronto Composite Index, FTSE for the Financial Times 100 stock index, DAX for the Deutsche Aktien Index, CAC for the French Cotation Automatique Continue index, MIB for the Milano Italia Borsa 30 Index, and N225 for the NIKKEI 225 index. For simplicity, SP is also called the volatility "originator", and the other six indices are also called the volatility "recipients"[9]. The daily ranges series used in this article are from Datastream, sampled at a daily frequency during periods from January 2, 2004 to September 30, 2008. Fig. 1 displays the daily ranges of the seven stock market indices.

Since the U.S. is serving as the numeraire country, the observations of each index are first extracted according to the trading days of the SP. The records with at least one missing value in the other six indices are then eliminated from the dataset. To allow for differences in the time zones between the stock markets in North America, Europe, and Asia, according to Bae et al. (2003), the log ranges[10] of the N225 index are dated at calendar time t, while those of the other six indices are dated at calendar time $t-1$.

4.1. Univariate analysis: preliminary results

[9] The terminologies "originator" and "recipient" are borrowed from Edwards and Susmel (2001).
[10] The log range means the difference between the maximum and minimum of the logarithmic price of an index, as defined in Section 2. We call it "range" for the purpose of saving space hereafter.

Table 3 provides sample statistics of range observations, including dependence measures Kendall's τ for the full sample period – January 2, 2004, to September 30, 2008 (1086 observations). In Panel A, the averages and standard deviations of daily ranges in all seven indices are quite similar. The DAX has the highest average daily range (1.28%), and the FTSE has the highest daily range standard deviation (0.808%). The values of skewness and kurtosis indicate the nature of the right skewness and the leptokurtosis of the daily range. The Ljung-Box (LB) statistics suggest significant autocorrelation in all seven indices. Panel B presents the dependence matrix between the seven indices measured by Kendall's τ. The dependence measure demonstrates that the SP is highly positively dependent in volatility with the other six indices. Not surprisingly, the SP has the strongest positive dependence with the TSX, while has the weakest dependence with the N225. In addition, dependence within regions (North America, Europe) is higher than dependence across regions.

As the first step in our analysis of stock market volatility, we fit the CARR model for each one of the series. The results of the AIC, SBC, and likelihood ratio test in Table 4 indicate that the CARR(1,1) model is adequate with regard to the SP, TSX, DAX, and CAC, while the CARR(2,2) is adequate with regard to the FTSE, MIB, and N225. Table 5 represents the parameter estimates and the p-values of the misspecification LM test for each indices[11]. The estimates of β_1 (or $\beta_1 + \beta_2$) indicate high-volatility persistence over time for all of the series. Hamilton and Susmel (1994) indicated that the observed high persistence of shocks to conditional volatility implies a potential structural break in volatility. We will discuss this property hereafter.

Before the discussion of the TVCARR model and its application to the subprime mortgage crisis, we first investigate the causality for the volatility of the seven indices. Following the definition of causality in Granger (1969), causality in volatility is defined as the effects of one foreign market's daily ranges on the subsequent domestic daily ranges. Therefore, to analyze the direction of causality in volatility, the exogenous variable, lagged ranges of the other (R'_{t-1}), is included in the CARRX models to examine the causality from originator to recipients, and vice versa.

Table 6 presents the parameter estimates of six originator-recipient pairs of CARRX models. For the four VAR-type pairs, SP—TSX, SP—CAC, SP—FTSE, and SP—MIB, the estimates of ϕ_R with respect to SP are significant, which indicate that the four indices—TSX, CAC, FTSE, and MIB, cause SP in volatility through their

[11] Quasi maximum likelihood estimation is used in the parameters estimation in this study.

lagged ranges R_{t-1}. On the other hand, the estimates of ϕ_R with respect to CAC, FTSE, and N225 of the three VAR-type pairs, SP—CAC, SP—FTSE, and SP—N225, are significant. This indicates that SP causes CAC, FTSE, and N225 in volatility. SP and DAX do not present the causality in volatility for each other.

4.2. Regime switching analysis: time-varying CARRX (TVCARRX) model

The results from the preceding section provide some preliminary evidence of daily ranges fitted by the CARRX model. According to the aforementioned high volatility persistence results, the regime switching model may be another good choice to model these daily range series. In this subsection, we first evaluate the structural break effect for each range series by using the misspecification test proposed in section 2[12]. Then the TVCARR models are applied to the series rejected by the test. The results of the misspecification test are shown in Table 5. Five of the seven indices with the exception of DAX and N225 are formally rejected by the misspecification test and identified to have one regime shift during the sample period under the linearity hypothesis. Table 7 presents the parameter estimates of the five TVCARR and of the two CARR models.

The first regime with parameter estimates of $\omega, \alpha_1, \alpha_2, \beta_1, \beta_2$ represents the dynamic status of conditional mean ranges at the location time c since the transition function is $\overline{G}(s_t;\gamma,c)$, not $G(s_t;\gamma,c)$. The dynamic behavior of conditional ranges departed from location time c would be adjusted by the second regime with parameter estimates of $\omega^*, \alpha_1^*, \alpha_2^*, \beta_1^*, \beta_2^*$ as well as the parameter estimates of γ, c in the transition function. The transition variable s_t, representing the trading days from the beginning of the sample to the end, is first standardized to obtain values between 0 and 1 for numerical stability when conducting estimation. Hence the estimates of cs are all between 0 and 1. As the major purpose of this study is to measure volatility contagion from SP to recipient markets, we focus on the c estimate of the SP. The estimate of c in the SP market is 0.749, represented by the 813$^{\text{th}}$ observation, which also corresponds to the trading day July 18, 2007. Therefore, the day July 18, 2007 is identified as the breakpoint of volatility regime switching. Daily ranges before this day are regarded as low volatility regime, and those after are regarded as high volatility regime.

The same exogenous variables fitted in CARRX models are included again in

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[12] The number for the breakpoint is set as one ($m=1$) to precisely capture the subprime mortgage crisis event even though the proposed method is able to detect one more regime switching breakpoint.

modeling the six originator-recipient VAR-type pairs of TVCARRX models to investigate the causality in volatility. The parameter estimates are given in Table 8. The significant estimates of ϕ_R with respect to the SP of the SP—TSX and SP—N225 pairs indicate that the TSX and the N225 cause SP in volatility. On the other hand, the significant estimates of ϕ_R with respect to the FTSE, MIB, and N225 of the SP—FTSE, SP—MIB and SP—N225 pairs indicate that the SP causes the FTSE, MIB, and N225 in volatility. The differences of causality in volatility with respect to the SP between low and high volatility regimes is pronounced (ϕ_R^*) for the SP—TSX pair. This positively estimated value of ϕ_R^* indicates that the strength of causality in volatility of the TSX to the SP is stronger in the high volatility regime than it is in the low volatility regime. Similarly, we find that the strength of causality in volatility of the SP to the MIB has the same result.

4.3. Bivariate analysis: A Copula approach

Because the conditional copula is a function of conditional probability integral transforms in residuals of the TVCARRX model, the accuracy of dependence measured by the copula method is based on the correctness of the density specification of the TVCARRX models. The misspecification of marginal distributions would lead to incorrect probability integral transforms, thus affecting the accuracy of the estimated dependence. Hence, two essential examinations to evaluate the goodness-of-fit of CARRX (or TVCARRX) model are performed before the copula approach. The first one is the Lagrange Multiplier (LM) test provided by Diebold et at. (1998) and applied by Patton (2004) for testing serial dependence in the probability integral transforms of the AR-GARCH model. In this study, the test is performed by examining the serial correlation of $(u_t - \bar{u})^k$ and $(v_t - \bar{v})^k$, for $t = 1, \cdots, T$, $k = 1, \cdots, 4$, where u_t and v_t represent the probability integral transforms of residuals of originator and recipient models, separately. This test is carried out by regressing $(u_t - \bar{u})^k$ and $(v_t - \bar{v})^k$ on their own 15 lags. The test statistic, $(T-15)R^2$, a function of coefficient of determination R^2 then follows $\chi^2(15)$ distribution under null hypothesis. The results (labeled $indep(k)$ in p-values) in Table 8 indicate almost all the first four moments are serially uncorrelated for all of the six pair models[13]. The second is the density specification test. If the error term distribution in the CARRX (or TVCARRX) model is correctly specified, the distribution of probability integral transforms will be Uniform(0,1). The Kolmogorov-Smirnov (KS) test is used to test the density specification. Each pair in the CARRX (or TVCARRX) model passes the test, and the p-values of the KS test are

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[13] Only one statistic, $indep(2)$ with respect to TSX in SP—TSX pair is significant at 5% level.

given in Table 8.

Two copulas, Gumbel and Clayton-Gumbel with constant and regime switching versions, are used to capture the dependence structure of the bivariate TVCARR model. Tables 9 and 10 present the separate estimated results of the Gumbel and Clayton-Gumbel copulas. According to the AIC statistic, regime switching Gumbel copula is adequate for the SP—FTSE model; the constant version of the Clayton-Gumbel copula is adequate for the SP—DAX model; while the regime switching version of the Clayton-Gumbel copula is adequate for the SP—TSX, SP—CAC, SP—MIB, and SP—N225 models. Hence, the dependence structure changes before and after the breakpoint of the SP's volatility regime shift for five pairs except for the SP—DAX. The estimates of dependence parameters in high volatility regimes ($\eta_{G,RSH}$ and $\eta_{CG,RSH}$) are larger than estimates in low volatility regimes ($\eta_{G,RSL}$ and $\eta_{CG,RSL}$) for the five dependence change pairs. The estimates of Kendall's τ present the same results since τ is an increasing function of η. Because the originator of the subprime mortgage crisis is known as beginning in the U.S., the dependence changes indicate that the volatility is contagious from SP to TSX, CAC, FTSE, MIB, and N225. In addition, the association between the SP and the five markets are stronger in a high volatility regime. The estimates of tail dependence in Clayton-Gumbel copulas indicate that a lower tail dependence is weak for daily range data, while upper tail dependence is pronounced and larger in a high volatility regime. Table 11 presents a summary of the estimates of Kendall's τ and the tail dependence. The high and low volatility regimes are separated by the estimated breakpoint of the SP index. Kendall's τ and the tail dependence are the estimated results extracted from Tables 9 and 10 according to the selected models.

5. Conclusions

In this paper, we propose a new approach to the study of volatility regime switching and volatility contagion. A Lagrange multiplier test has been developed for testing a linear CARR model against a time-varying CARR model. The Burr distribution is used as the error term distribution of the CARR/TVCARR model because of its superior flexibility in capturing the distribution of range data by two shape parameters. The dependence measurement resorts to copula functions for modeling the dependence structure between range series since bivariate distribution of range data cannot be easily specified.

The results of the size simulation show that the proposed misspecification test performs well for a large sample size; some slight distortions are present otherwise. In the power simulation, the transition speed does not influence the performance, while time location causes some performance loss in the case of a small sample size.

In the empirical study, we apply the proposed approach to the daily ranges of seven developed stock market indices. The evidence of causality in volatility analysis shows that the SP and other indices cause each other in volatility with the exception of the DAX. Five indices are tested to be adequate for the TVCARR model with the exception of the DAX and the N225. Therefore, the SP, TSX, CAC, FTSE, and MIB experience volatility regime switching caused by the subprime mortgage crisis. In the contemporaneous dependence analysis, the regime switching version of Gumbel copula is adequate for the SP—FTSE; the constant version of Clayton-Gumbel copula is adequate for the SP—DAX, while the regime switching version of Clayton-Gumbel copula is adequate for the other four pairs. According to the evidence of contemporaneous dependence, we conclude that volatility contagion occurs due to the subprime mortgage crisis.

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Appendix A: Proof of LM test statistic

$\varepsilon_t \sim Burr(\kappa, \sigma^2)$,

$R_t = \xi_t \varepsilon_t$, $\xi_t = \dfrac{\exp(\lambda_t + \varphi_t)}{\mu}$,

$$\lambda_t = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i} + \sum_{j=1}^{q} \beta_j \lambda_{t-j} + \left(\omega^* + \sum_{i=1}^{p} \alpha_i^* \varepsilon_{t-i} + \sum_{j=1}^{q} \beta_j^* \lambda_{t-j} \right) \cdot \overline{G}(s_t; \gamma, \mathbf{c})$$
$$= (\omega + \omega^* \cdot \overline{G}(s_t; \gamma, \mathbf{c})) + \sum_{i=1}^{p} (\alpha_i + \alpha_i^* \cdot \overline{G}(s_t; \gamma, \mathbf{c}))\varepsilon_{t-i} + \sum_{j=1}^{q} (\beta_j + \beta_j^* \cdot \overline{G}(s_t; \gamma, \mathbf{c}))\lambda_{t-j}$$

where

$$\mu = \frac{\Gamma\left(1 + \frac{1}{\kappa}\right)\Gamma\left(\frac{1}{\sigma^2} - \frac{1}{\kappa}\right)}{\sigma^{2\left(1+\frac{1}{\kappa}\right)}\Gamma\left(\frac{1}{\sigma^2} + 1\right)} = E(\varepsilon_t)$$

$$G(s_t; \gamma, \mathbf{c}) = \left(1 + \exp\left(-\gamma \prod_{k=1}^{m} (s_t - c_k)\right)\right)^{-1} ,$$

$\overline{G}(s_t; \gamma, \mathbf{c}) = G(s_t; \gamma, \mathbf{c}) - 1/2$,,

$s_t = t$,
$c_1 \le \cdots \le c_m$, and
$\gamma > 0$

Likelihood function for observation R_t is

$$l_t(\boldsymbol{\theta}) = \ln \kappa - \kappa \ln \xi_t + (\kappa - 1) \ln R_t - \left(\frac{1}{\sigma^2} + 1\right) \ln(1 + \sigma^2 \xi_t^{-\kappa} R_t^{\kappa})$$

$\boldsymbol{\theta} = (\boldsymbol{\theta}_1', \boldsymbol{\theta}_2', \kappa, \sigma^2)'$,

$$\frac{\partial l_t(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}_1} = \left(-\kappa - \left(\frac{1}{\sigma^2} + 1\right) \cdot \frac{-\kappa \xi_t^{-\kappa} \sigma^2 R_t^{\kappa}}{(1 + \sigma^2 \xi_t^{-\kappa} R_t^{\kappa})} \right) \cdot \left(\frac{\partial \lambda_t}{\partial \boldsymbol{\theta}_1} + \frac{\partial \varphi_t}{\partial \boldsymbol{\theta}_1} \right),$$

$$\frac{\partial l_t(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}_2} = \left(-\kappa - \left(\frac{1}{\sigma^2} + 1\right) \cdot \frac{-\kappa \xi_t^{-\kappa} \sigma^2 R_t^{\kappa}}{(1 + \sigma^2 \xi_t^{-\kappa} R_t^{\kappa})} \right) \cdot \left(\frac{\partial \varphi_t}{\partial \boldsymbol{\theta}_2} \right),$$

$$\frac{\partial l_t(\boldsymbol{\theta})}{\partial \kappa} = \frac{1}{\kappa} - \frac{\kappa}{\xi_t} \cdot \frac{\partial \xi_t}{\partial \kappa} + \ln\left(\frac{R_t}{\xi_t}\right) - \left(\frac{1}{\sigma^2} + 1\right) \cdot \frac{\sigma^2}{(1 + \sigma^2 \xi_t^{-\kappa} R_t^{\kappa})} \cdot \left(\frac{R_t}{\xi_t}\right)^{\kappa} \cdot \left(\ln\left(\frac{R_t}{\xi_t}\right) - \frac{\kappa}{\xi_t} \cdot \frac{\partial \xi_t}{\partial \kappa} \right)$$

$$\frac{\partial \xi_t(\boldsymbol{\theta})}{\partial \kappa} = f(\boldsymbol{\theta},\kappa,\sigma^2) = \exp(\lambda_t + \varphi_t)\cdot$$

$$\frac{-\sigma^{2\left(1+\frac{1}{\kappa}\right)}\Gamma\left(\frac{1}{\sigma^2}+1\right)\left(\ln\sigma^2 + \left(\Gamma\left(\frac{1}{\sigma^2}-\frac{1}{\kappa}\right)\right)^{-1}\Gamma'\left(\frac{1}{\sigma^2}-\frac{1}{\kappa}\right) - \left(\Gamma\left(1+\frac{1}{\kappa}\right)\right)^{-1}\Gamma'\left(1+\frac{1}{\kappa}\right)\right)}{\kappa^2\Gamma\left(1+\frac{1}{\kappa}\right)\Gamma\left(\frac{1}{\sigma^2}-\frac{1}{\kappa}\right)}$$

$$\frac{\partial l_t(\boldsymbol{\theta})}{\partial \sigma^2} = -\frac{\kappa}{\xi_t}\cdot\frac{\partial \xi_t}{\partial \sigma^2} + \left(\frac{1}{\sigma^2}\right)^2\cdot\ln(1+\sigma^2\xi_t^{-\kappa}R_t^{\kappa}) - \frac{\left(\frac{1}{\sigma^2}+1\right)}{(1+\sigma^2\xi_t^{-\kappa}R_t^{\kappa})}\cdot\left(\frac{R_t}{\xi_t}\right)^{\kappa}\cdot\left(1-\sigma^2\cdot\frac{\kappa}{\xi_t}\cdot\frac{\partial \xi_t}{\partial \sigma^2}\right)$$

$$\frac{\partial \xi_t(\boldsymbol{\theta})}{\partial \sigma^2} = g(\boldsymbol{\theta},\kappa,\sigma^2) = \exp(\lambda_t + \varphi_t)\cdot$$

$$\frac{\sigma^{\frac{2}{\kappa}}\Gamma\left(\frac{1}{\sigma^2}+1\right)\left(\left(1+\frac{1}{\kappa}\right) - \frac{1}{\sigma^2}\left(\Gamma'\left(\frac{1}{\sigma^2}+1\right)\left(\Gamma\left(\frac{1}{\sigma^2}+1\right)\right)^{-1} - \Gamma'\left(\frac{1}{\sigma^2}-\frac{1}{\kappa}\right)\left(\Gamma\left(\frac{1}{\sigma^2}-\frac{1}{\kappa}\right)\right)^{-1}\right)\right)}{\Gamma\left(\frac{1}{\sigma^2}-\frac{1}{\kappa}\right)\Gamma\left(1+\frac{1}{\kappa}\right)}$$

Let $\boldsymbol{\theta}^0 = (\boldsymbol{\theta}_1^{0\prime},\boldsymbol{\theta}_2^{0\prime},\kappa^0,(\sigma^2)^0)'$ be the parameter vector under the null hypothesis, the score for range R_t evaluated at the true parameter values equals

$$\frac{\partial l_t(\boldsymbol{\theta}^0)}{\partial \boldsymbol{\theta}_1} = \left(-\kappa^0 - \left(\frac{1}{(\sigma^2)^0}+1\right)\cdot\frac{-\kappa^0\left(\xi_t(\boldsymbol{\theta}^0)\right)^{-\kappa^0}(\sigma^2)^0R_t^{\kappa^0}}{\left(1+(\sigma^2)^0\left(\xi_t(\boldsymbol{\theta}^0)\right)^{-\kappa}R_t^{\kappa^0}\right)}\right)\cdot\left(\frac{\partial \lambda_t(\boldsymbol{\theta}_1^0)}{\partial \boldsymbol{\theta}_1}\right)$$

$$\frac{\partial l_t(\boldsymbol{\theta}^0)}{\partial \boldsymbol{\theta}_2} = \left(-\kappa^0 - \left(\frac{1}{(\sigma^2)^0}+1\right)\cdot\frac{-\kappa^0\left(\xi_t(\boldsymbol{\theta}^0)\right)^{-\kappa^0}(\sigma^2)^0R_t^{\kappa^0}}{\left(1+(\sigma^2)^0\left(\xi_t(\boldsymbol{\theta}^0)\right)^{-\kappa^0}R_t^{\kappa^0}\right)}\right)\cdot\left(\frac{\partial \varphi_t(\boldsymbol{\theta}_1^0,\boldsymbol{\theta}_2^0)}{\partial \boldsymbol{\theta}_2}\right)$$

$$\frac{\partial l_t(\boldsymbol{\theta}^0)}{\partial \kappa} = \frac{1}{\kappa^0} - \frac{\kappa^0}{\xi_t(\boldsymbol{\theta}^0)}\cdot\frac{\partial \xi_t(\boldsymbol{\theta}^0)}{\partial \kappa} + \ln\left(\frac{R_t}{\xi_t(\boldsymbol{\theta}^0)}\right)$$

$$-\left(\frac{1}{(\sigma^2)^0}+1\right)\cdot\frac{(\sigma^2)^0}{(1+(\sigma^2)^0\cdot\xi_t(\boldsymbol{\theta}^0)^{-\kappa^0}R_t^{\kappa^0})}\cdot\left(\frac{R_t}{\xi_t(\boldsymbol{\theta}^0)}\right)^{\kappa^0}\cdot\left(\ln\frac{R_t}{\xi_t(\boldsymbol{\theta}^0)} - \frac{\kappa^0}{\xi_t(\boldsymbol{\theta}^0)}\frac{\partial \xi_t(\boldsymbol{\theta}^0)}{\partial \kappa}\right)$$

$$\frac{\partial l_t(\boldsymbol{\theta}^0)}{\partial \sigma^2} = -\frac{\kappa^0}{\xi_t(\boldsymbol{\theta}^0)}\cdot\frac{\partial \xi_t(\boldsymbol{\theta}^0)}{\partial \sigma^2} + \left(\frac{1}{(\sigma^2)^0}\right)^2\cdot\ln(1+(\sigma^2)^0\xi_t(\boldsymbol{\theta}^0)^{-\kappa^0}R_t^{\kappa^0})$$

$$-\frac{\left(\frac{1}{(\sigma^2)^0}+1\right)}{(1+(\sigma^2)^0\xi_t(\boldsymbol{\theta}^0)^{-\kappa^0}R_t^{\kappa^0})}\cdot\left(\frac{R_t}{\xi_t(\boldsymbol{\theta}^0)}\right)^{\kappa^0}\cdot\left(1-(\sigma^2)^0\cdot\left(\frac{\kappa^0}{\xi_t(\boldsymbol{\theta}^0)}\right)\cdot\frac{\partial \xi_t(\boldsymbol{\theta}^0)}{\partial \sigma^2}\right)$$

where

$$\mu^0 = \frac{\Gamma\left(1+\frac{1}{\kappa^0}\right)\Gamma\left(\frac{1}{(\sigma^2)^0}-\frac{1}{\kappa^0}\right)}{(\sigma^2)^{0\left(1+\frac{1}{\kappa^0}\right)}\Gamma\left(\frac{1}{(\sigma^2)^0}+1\right)}$$

$$\xi_t(\boldsymbol{\theta}^0) = \frac{\exp(\lambda_t(\boldsymbol{\theta}_1^0))}{M^0} = \xi_t(\boldsymbol{\theta}_1^0, \kappa^0, (\sigma^2)^0)$$

$$\frac{\partial \xi_t(\boldsymbol{\theta}^0)}{\partial \kappa} = f(\boldsymbol{\theta}_1^0, \kappa^0, (\sigma^2)^0)$$

$$\frac{\partial \xi_t(\boldsymbol{\theta}^0)}{\partial \sigma^2} = g(\boldsymbol{\theta}_1^0, \kappa^0, (\sigma^2)^0)$$

$$\frac{\partial l_t(\boldsymbol{\theta}^0)}{\partial \boldsymbol{\theta}} = \begin{bmatrix} \frac{\partial l_t(\boldsymbol{\theta}^0)}{\partial \boldsymbol{\theta}_1} \\ \frac{\partial l_t(\boldsymbol{\theta}^0)}{\partial \kappa} \\ \frac{\partial l_t(\boldsymbol{\theta}^0)}{\partial \sigma^2} \\ \frac{\partial l_t(\boldsymbol{\theta}^0)}{\partial \boldsymbol{\theta}_2} \end{bmatrix} = \begin{bmatrix} \mathbf{a}_{1t}(\boldsymbol{\theta}_1^0, \kappa^0, (\sigma^2)^0) \\ a_{2t}(\boldsymbol{\theta}_1^0, \kappa^0, (\sigma^2)^0) \\ a_{3t}(\boldsymbol{\theta}_1^0, \kappa^0, (\sigma^2)^0) \\ \mathbf{b}_t(\boldsymbol{\theta}_1^0, \boldsymbol{\theta}_2^0, \kappa^0, (\sigma^2)^0) \end{bmatrix} = \begin{bmatrix} \mathbf{a}_t^0 \\ \mathbf{b}_t^0 \end{bmatrix}, \text{ where } \left[\mathbf{a}_t^0\right] = \begin{bmatrix} \mathbf{a}_{1t}(\boldsymbol{\theta}_1^0, \kappa^0, (\sigma^2)^0) \\ a_{2t}(\boldsymbol{\theta}_1^0, \kappa^0, (\sigma^2)^0) \\ a_{3t}(\boldsymbol{\theta}_1^0, \kappa^0, (\sigma^2)^0) \end{bmatrix}$$

Let $\hat{\boldsymbol{\theta}} = (\hat{\boldsymbol{\theta}}_1', \boldsymbol{\theta}_2^{0}{}', \hat{\kappa}, \hat{\sigma}^2)'$ be the vector of maximum likelihood estimates (MLEs) under the null hypothesis. Then the score evaluated at the MLEs is

$$\frac{\partial l(\hat{\boldsymbol{\theta}})}{\partial \boldsymbol{\theta}} = \sum_{t=1}^{T} \frac{\partial l_t(\hat{\boldsymbol{\theta}})}{\partial \boldsymbol{\theta}} = \begin{bmatrix} \mathbf{0} \\ \sum_{t=1}^{T} \mathbf{b}_t(\hat{\boldsymbol{\theta}}_1, \boldsymbol{\theta}_2^0, \hat{\kappa}, \hat{\sigma}^2) \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \sum_{t=1}^{T} \hat{\mathbf{b}}_t \end{bmatrix},$$

Then,

$$T^{-1/2} \frac{\partial l(\boldsymbol{\theta}^0)}{\partial \boldsymbol{\theta}} = T^{-1/2} \sum_{t=1}^{T} \frac{\partial l_t(\boldsymbol{\theta}^0)}{\partial \boldsymbol{\theta}} \xrightarrow{d} N\left(\mathbf{0}, T^{-1} \sum_{t=1}^{T} E\left(\frac{\partial l_t(\boldsymbol{\theta}^0)}{\partial \boldsymbol{\theta}} \frac{\partial l_t(\boldsymbol{\theta}^0)}{\partial \boldsymbol{\theta}'}\right)\right),$$

$$E\left(\frac{\partial l_t(\boldsymbol{\theta}^0)}{\partial \boldsymbol{\theta}} \frac{\partial l_t(\boldsymbol{\theta}^0)}{\partial \boldsymbol{\theta}'}\right) = E\left(\begin{bmatrix} \mathbf{a}_t^0 \mathbf{a}_t^{0}{}' & \mathbf{a}_t^0 \mathbf{b}_t^{0}{}' \\ \mathbf{b}_t^0 \mathbf{a}_t^{0}{}' & \mathbf{b}_t^0 \mathbf{b}_t^{0}{}' \end{bmatrix}\right),$$

Thus, $T^{-1/2} \frac{\partial l(\boldsymbol{\theta}^0)}{\partial \boldsymbol{\theta}} \xrightarrow{d} N\left(\mathbf{0}, T^{-1} \sum_{t=1}^{T} E\left(\begin{bmatrix} \mathbf{a}_t^0 \mathbf{a}_t^{0}{}' & \mathbf{a}_t^0 \mathbf{b}_t^{0}{}' \\ \mathbf{b}_t^0 \mathbf{a}_t^{0}{}' & \mathbf{b}_t^0 \mathbf{b}_t^{0}{}' \end{bmatrix}\right)\right)$.

The quadratic form

$$\left\{T^{-1/2}\frac{\partial l(\boldsymbol{\theta}^0)}{\partial\boldsymbol{\theta}'}\right\}\times\left\{T^{-1}\sum_{t=1}^{T}E\left(\begin{bmatrix}\mathbf{a}_t^0\mathbf{a}_t^{0\,\prime} & \mathbf{a}_t^0\mathbf{b}_t^{0\,\prime}\\ \mathbf{b}_t^0\mathbf{a}_t^{0\,\prime} & \mathbf{b}_t^0\mathbf{b}_t^{0\,\prime}\end{bmatrix}\right)\right\}^{-1}\times\left\{T^{-1/2}\frac{\partial l(\boldsymbol{\theta}^0)}{\partial\boldsymbol{\theta}}\right\}$$

follows an asymptotic chi-squared distribution $\chi^2(df)$, df is the dimension of $\boldsymbol{\theta}_2$.
If the Burr distribution is the true distribution for the model, $\hat{\boldsymbol{\theta}}$ is a consistent estimator of $\boldsymbol{\theta}^0$ and

$$T^{-1}\sum_{t=1}^{T}\begin{bmatrix}\mathbf{a}_t(\hat{\boldsymbol{\theta}}_1,\hat{\kappa},\hat{\sigma}^2)\mathbf{a}_t'(\hat{\boldsymbol{\theta}}_1,\hat{\kappa},\hat{\sigma}^2) & \mathbf{a}_t(\hat{\boldsymbol{\theta}}_1,\hat{\kappa},\hat{\sigma}^2)\mathbf{b}_t'(\hat{\boldsymbol{\theta}}_1,\boldsymbol{\theta}_2^0,\hat{\kappa},\hat{\sigma}^2)\\ \mathbf{b}_t(\hat{\boldsymbol{\theta}}_1,\boldsymbol{\theta}_2^0,\hat{\kappa},\hat{\sigma}^2)\mathbf{a}_t'(\hat{\boldsymbol{\theta}}_1,\hat{\kappa},\hat{\sigma}^2) & \mathbf{b}_t(\hat{\boldsymbol{\theta}}_1,\boldsymbol{\theta}_2^0,\hat{\kappa},\hat{\sigma}^2)\mathbf{b}_t'(\hat{\boldsymbol{\theta}}_1,\boldsymbol{\theta}_2^0,\hat{\kappa},\hat{\sigma}^2)\end{bmatrix}$$
$$=T^{-1}\sum_{t=1}^{T}\begin{bmatrix}\hat{\mathbf{a}}_t\hat{\mathbf{a}}_t' & \hat{\mathbf{a}}_t\hat{\mathbf{b}}_t'\\ \hat{\mathbf{b}}_t\hat{\mathbf{a}}_t' & \hat{\mathbf{b}}_t\hat{\mathbf{b}}_t'\end{bmatrix}$$

is a consistent estimator of

$$T^{-1}\sum_{t=1}^{T}E\left(\begin{bmatrix}\mathbf{a}_t^0\mathbf{a}_t^{0\,\prime} & \mathbf{a}_t^0\mathbf{b}_t^{0\,\prime}\\ \mathbf{b}_t^0\mathbf{a}_t^{0\,\prime} & \mathbf{b}_t^0\mathbf{b}_t^{0\,\prime}\end{bmatrix}\right).$$

Therefore, the LM statistic

$$LM=\left\{T^{-1/2}\frac{\partial l(\hat{\boldsymbol{\theta}})}{\partial\boldsymbol{\theta}'}\right\}\times\left\{T^{-1}\sum_{t=1}^{T}\begin{bmatrix}\hat{\mathbf{a}}_t\hat{\mathbf{a}}_t' & \hat{\mathbf{a}}_t\hat{\mathbf{b}}_t'\\ \hat{\mathbf{b}}_t\hat{\mathbf{a}}_t' & \hat{\mathbf{b}}_t\hat{\mathbf{b}}_t'\end{bmatrix}\right\}^{-1}\times\left\{T^{-1/2}\frac{\partial l(\hat{\boldsymbol{\theta}})}{\partial\boldsymbol{\theta}}\right\}$$
$$=\begin{bmatrix}\mathbf{0}\\ \sum_{t=1}^{T}\hat{\mathbf{b}}_t\end{bmatrix}'\left\{\begin{bmatrix}\sum_{t=1}^{T}\hat{\mathbf{a}}_t\hat{\mathbf{a}}_t' & \sum_{t=1}^{T}\hat{\mathbf{a}}_t\hat{\mathbf{b}}_t'\\ \sum_{t=1}^{T}\hat{\mathbf{b}}_t\hat{\mathbf{a}}_t' & \sum_{t=1}^{T}\hat{\mathbf{b}}_t\hat{\mathbf{b}}_t'\end{bmatrix}\right\}^{-1}\begin{bmatrix}\mathbf{0}\\ \sum_{t=1}^{T}\hat{\mathbf{b}}_t\end{bmatrix}$$
$$=\left\{\sum_{t=1}^{T}\hat{\mathbf{b}}_t'\right\}\times\left\{\sum_{t=1}^{T}\hat{\mathbf{b}}_t\hat{\mathbf{b}}_t'-\left(\sum_{t=1}^{T}\hat{\mathbf{b}}_t\hat{\mathbf{a}}_t'\right)\left(\sum_{t=1}^{T}\hat{\mathbf{a}}_t\hat{\mathbf{a}}_t'\right)^{-1}\left(\sum_{t=1}^{T}\hat{\mathbf{a}}_t\hat{\mathbf{b}}_t'\right)\right\}^{-1}\times\left\{\sum_{t=1}^{T}\hat{\mathbf{b}}_t\right\}$$

follows an asymptotic chi-squared distribution $\chi^2(df)$ under the null hypothesis, and df is the dimension of $\boldsymbol{\theta}_2$.
where

$$\hat{\mathbf{a}}_{1t}=\frac{\partial l_t(\hat{\boldsymbol{\theta}})}{\partial\boldsymbol{\theta}_1}=-\hat{\kappa}\cdot\left(\frac{\partial\lambda_t(\hat{\boldsymbol{\theta}}_1)}{\partial\boldsymbol{\theta}_1}\right)-\left(\frac{1}{\hat{\sigma}^2}+1\right)\cdot\frac{-\hat{\kappa}\left(\xi_t(\hat{\boldsymbol{\theta}})\right)^{-\hat{\kappa}}\hat{\sigma}^2R_t^{\hat{\kappa}}}{\left(1+\left(\xi_t(\hat{\boldsymbol{\theta}})\right)^{-\hat{\kappa}}\hat{\sigma}^2R_t^{\hat{\kappa}}\right)}\cdot\left(\frac{\partial\lambda_t(\hat{\boldsymbol{\theta}}_1)}{\partial\boldsymbol{\theta}_1}\right)$$

$$\hat{a}_{2t}=\frac{\partial l_t(\hat{\boldsymbol{\theta}})}{\partial\kappa}=\frac{1}{\hat{\kappa}}-\frac{\hat{\kappa}}{\xi_t(\hat{\boldsymbol{\theta}})}\cdot\frac{\partial\xi_t(\hat{\boldsymbol{\theta}})}{\partial\kappa}+\ln\left(\frac{R_t}{\xi_t(\hat{\boldsymbol{\theta}})}\right)$$
$$-\left(\frac{1}{\hat{\sigma}^2}+1\right)\cdot\frac{\hat{\sigma}^2}{(1+\hat{\sigma}^2\cdot\xi_t(\hat{\boldsymbol{\theta}})^{-\hat{\kappa}}R_t^{\hat{\kappa}})}\cdot\left(\frac{R_t}{\xi_t(\hat{\boldsymbol{\theta}})}\right)^{\hat{\kappa}}\cdot\left(\ln\frac{R_t}{\xi_t(\hat{\boldsymbol{\theta}})}-\frac{\hat{\kappa}}{\xi_t(\hat{\boldsymbol{\theta}})}\frac{\partial\xi_t(\hat{\boldsymbol{\theta}})}{\partial\kappa}\right)$$

$$\hat{a}_{3t} = \frac{\partial l_t(\hat{\boldsymbol{\theta}})}{\partial \sigma^2} = -\frac{\hat{\kappa}}{\xi_t(\hat{\boldsymbol{\theta}})} \cdot \frac{\partial \xi_t(\hat{\boldsymbol{\theta}})}{\partial \sigma^2} + \left(\frac{1}{\hat{\sigma}^2}\right)^2 \cdot \ln(1+\hat{\sigma}^2 \xi_t(\hat{\boldsymbol{\theta}})^{-\hat{\kappa}} R_t^{\hat{\kappa}})$$

$$-\frac{\left(\frac{1}{\hat{\sigma}^2}+1\right)}{(1+\hat{\sigma}^2 \xi_t(\hat{\boldsymbol{\theta}})^{-\hat{\kappa}} R_t^{\hat{\kappa}})} \cdot \left(\frac{R_t}{\xi_t(\hat{\boldsymbol{\theta}})}\right)^{\hat{\kappa}} \cdot \left(1-\hat{\sigma}^2 \cdot \left(\frac{\hat{\kappa}}{\xi_t(\hat{\boldsymbol{\theta}})}\right) \cdot \frac{\partial \xi_t(\hat{\boldsymbol{\theta}})}{\partial \sigma^2}\right)$$

$$\hat{\mathbf{a}}_t = (\hat{\mathbf{a}}_{1t}', \hat{a}_{2t}', \hat{a}_{3t}')'$$

$$\hat{\mathbf{b}}_t = \frac{\partial l_t(\hat{\boldsymbol{\theta}})}{\partial \boldsymbol{\theta}_2} = \left(-\hat{\kappa} - \left(\frac{1}{\hat{\sigma}^2}+1\right) \cdot \frac{-\hat{\kappa}\left(\xi_t(\hat{\boldsymbol{\theta}})\right)^{-\hat{\kappa}} \hat{\sigma}^2 R_t^{\hat{\kappa}}}{\left(1+\left(\xi_t(\hat{\boldsymbol{\theta}})\right)^{-\hat{\kappa}} \hat{\sigma}^2 R_t^{\hat{\kappa}}\right)}\right) \cdot \left(\frac{\partial \varphi_t(\hat{\boldsymbol{\theta}}_1, \boldsymbol{\theta}_2^0)}{\partial \boldsymbol{\theta}_2}\right)$$

$$= \left(-\hat{\kappa} - \left(\frac{1}{\hat{\sigma}^2}+1\right) \cdot \frac{-\hat{\kappa}\left(\xi_t(\hat{\boldsymbol{\theta}})\right)^{-\hat{\kappa}} \hat{\sigma}^2 R_t^{\hat{\kappa}}}{\left(1+\left(\xi_t(\hat{\boldsymbol{\theta}})\right)^{-\hat{\kappa}} \hat{\sigma}^2 R_t^{\hat{\kappa}}\right)}\right) \cdot \left((s_t, \cdots, (s_t)^l, (vec\mathbf{X}_{t,1})', (vec\mathbf{X}_{t,2})')' + \sum_{j=1}^{q} \hat{\beta}_j \frac{\partial \hat{\varphi}_{t-j}}{\partial \boldsymbol{\theta}_2}\right)$$

Appendix B: The two copulas used in the study

Table B1. The properties of Gumbel and Clayton-Gumbel copulas

Copula	$C(u,v)$	Kendall's τ	τ^L	τ^U
Gumbel	$\exp\{-[(-\ln u)^{\eta}+(-\ln v)^{\eta}]^{1/\eta}\}$	$1-\eta^{-1}$	0	$2-2^{1/\eta}$
Clayton-Gumbel	$\left\{\left[(u^{-\delta}-1)^{\eta}+(v^{-\delta}-1)^{\eta}\right]^{1/\eta}+1\right\}^{-1/\delta}$	$\frac{(\delta+2)\eta-2}{(\delta+2)\eta}$	$2^{-1/\delta\eta}$	$2-2^{1/\eta}$

Table 1. Size simulation of the Lagrange multiplier test for misspecification test

	Case 1			Case 2		
	Significance Level			Significance Level		
T	1%	5%	10%	1%	5%	10%
1,000	0.01	0.07	0.12	0.02	0.07	0.13
2,000	0.01	0.06	0.11	0.01	0.06	0.11
5,000	0.01	0.05	0.11	0.01	0.05	0.10

Table 2. Power simulation of the Lagrange multiplier test for misspecification test

		Case 3			Case 4		
			c			*c*	
T	γ	0.25T	0.5 T	0.75 T	0.25 T	0.5 T	0.75 T
1,000	0.1	0.842	0.967	0.960	0.993	0.968	0.821
	0.5	0.836	0.965	0.953	0.993	0.962	0.812
	1.0	0.835	0.958	0.960	0.995	0.958	0.815
1,500	0.1	0.953	0.998	0.997	1.000	0.996	0.968
	0.5	0.951	0.998	0.998	1.000	0.997	0.958
	1.0	0.959	0.997	0.997	1.000	0.996	0.964
2,000	0.1	0.991	1.000	1.000	1.000	1.000	0.996
	0.5	0.991	1.000	1.000	1.000	1.000	0.996
	1.0	0.992	1.000	1.000	1.000	1.000	0.995

Note: Significance level is 5%.

Table 3. Summary statistics of daily ranges on G7 stock market indices, January 2, 2004, to September 30, 2008

Panel A. Descriptive statistics							
	SP	*TSX*	*DAX*	*CAC*	*FTSE*	*MIB*	*N225*
Mean (%)	1.125	1.116	1.280	1.185	1.197	1.035	1.271
Std Dev (%)	0.715	0.695	0.686	0.726	0.808	0.635	0.677
Skewness	3.029	2.958	2.287	3.016	3.118	2.893	1.511
Kurtosis	21.806	20.090	14.667	20.864	21.430	18.229	6.372
Min (%)	0.247	0.262	0.302	0.297	0.232	0.282	0.299
Max(%)	8.872	8.200	7.547	8.483	9.688	6.515	5.112
T	1,086	1,086	1,086	1,086	1,086	1,086	1,086
LB(15)	2,289	2,128	1,747	2,136	2,974	1,769	1,412

Panel B. Dependence matrix: Kendall's τ							
τ	*SP*	*TSX*	*DAX*	*CAC*	*FTSE*	*MIB*	*N225*
SP	1.00						
TSX	0.43	1.00					
DAX	0.39	0.28	1.00				
CAC	0.41	0.30	0.64	1.00			
FTSE	0.41	0.37	0.47	0.55	1.00		
MIB	0.40	0.32	0.54	0.62	0.51	1.00	
N225	0.21	0.21	0.21	0.24	0.23	0.22	1.00

Table 4. Model selection by AIC, SBC, and likelihood ratio test

	Models	*AIC*	*SBC*	*LL*	*LR test*
SP	CARR(1,1)	1.085	1.108	-584.21	0.252
	CARR(2,2)	1.086	1.118	-582.83	
TSX	CARR(1,1)	1.014	1.037	-545.39	0.174
	CARR(2,2)	1.014	1.046	-543.64	
DAX	CARR(1,1)	1.240	1.263	-668.27	0.121
	CARR(2,2)	1.240	1.272	-666.16	
CAC	CARR(1,1)	1.070	1.093	-576.18	0.608
	CARR(2,2)	1.073	1.105	-575.68	
FTSE	CARR(1,1)	1.011	1.034	-543.92	**<0.001**
	CARR(2,2)	0.992	1.024	-531.57	
MIB	CARR(1,1)	0.825	0.848	-442.96	**<0.001**
	CARR(2,2)	0.812	0.845	-434.10	
N225	CARR(1,1)	1.351	1.374	-728.64	**0.002**
	CARR(2,2)	1.344	1.376	-722.54	

Note: AIC represents the Akaike Information Criterion. SBC represents the Schwarz Bayesian Criterion. *LL* denotes the log-likelihood function. LR test represents the likelihood ration test. The values present in LR test are p-values. The CARR(3,3) model is also considered but yields insignificant results. The p-values less than 0.1 are shown in boldface.

Table 5. Parameter estimates of CARR(1,1) and CARR(2,2) models and the p-values of misspecification LM test

	SP		*TSX*		*DAX*		*CAC*		*FTSE*		*MIB*		*N225*	
	Coef	Std Err	Coef	Std Err	Coef	Std Err	Coef	Std Err	Coef	Std Err	Coef	Std Err	Coef	Std Err
ω	**-0.150**	0.017	**-0.181**	0.017	**-0.129**	0.022	**-0.123**	0.025	**-0.322**	0.055	**-0.266**	0.062	**-0.029**	0.030
α_1	**0.145**	0.016	**0.166**	0.016	**0.119**	0.020	**0.112**	0.024	**0.152**	0.026	**0.114**	0.027	**0.188**	0.027
α_2									**0.151**	0.028	**0.117**	0.028	**-0.161**	0.032
β_1	**0.982**	0.008	**0.977**	0.009	**0.987**	0.008	**0.985**	0.009	-0.014	0.014	-0.013	0.013	**1.661**	0.226
β_2									**0.979**	0.013	**0.976**	0.011	**-0.663**	0.223
κ	**3.760**	0.169	**4.354**	0.200	**4.729**	0.214	**4.749**	0.226	**4.506**	0.212	**5.081**	0.188	**4.316**	0.251
σ^2	**0.752**	0.112	**1.061**	0.108	**1.159**	0.124	**1.201**	0.145	**1.000**	0.135	**1.438**	0.132	**1.092**	0.155
LL	-584		-545		-668		-576		-544		-443		-723	
Misspecificantion LM test	**<0.001**		**<0.001**		0.337		**0.001**		**<0.001**		**<0.001**		0.098	

Note: 1. Model: $R_t = \dfrac{\exp(\lambda_t)}{\mu}\varepsilon_t, \quad \lambda_t = \omega + \sum_{i=1}^{2}\alpha_i\varepsilon_{t-i} + \sum_{j=1}^{2}\beta_j\lambda_{t-j}$.

2. κ and σ^2 represent the parameters of Burr distribution. *LL* denotes the log-likelihood function. The values of misspecification LM test are p-values. The estimated parameters with p-values less than 0.05 are shown in boldface.

Table 6. Parameter estimates of CARRX(1,1) and CARRX(2,2) models with the exogenous variables, lagged range of the other (R'_{t-1}), for each pair indices

	SP		*TSX*		*SP*		*DAX*		*SP*		*CAC*	
	Coef	Std Err	Coef	Std Err	Coef	Std Err	Coef	Std Err	Coef	Std Err	Coef	Std Err
ω	**-0.162**	0.021	**-0.193**	0.030	**-0.162**	0.023	**-0.136**	0.028	**-0.168**	0.019	**-0.134**	0.029
α_1	**0.115**	0.013	**0.162**	0.024	**0.135**	0.015	**0.115**	0.021	**0.127**	0.013	**0.098**	0.023
α_2												
β_1	**0.939**	0.027	**0.956**	0.027	**0.962**	0.017	**0.967**	0.021	**0.939**	0.022	**0.951**	0.034
β_2												
ϕ_R	**0.042**	0.017	0.017	0.015	0.017	0.010	0.016	0.013	**0.033**	0.015	**0.028**	0.022
κ	**3.732**	0.204	**4.325**	0.248	**3.806**	0.249	**4.629**	0.184	**3.785**	0.197	**4.658**	0.230
σ^2	**0.708**	0.123	**1.038**	0.153	**0.778**	0.161	**1.088**	0.144	**0.759**	0.125	**1.129**	0.148
LL	-575		-544		-582		-666		-579		-571	
	SP		*FTSE*		*SP*		*MIB*		*SP*		*N225*	
	Coef	Std Err	Coef	Std Err	Coef	Std Err	Coef	Std Err	Coef	Std Err	Coef	Std Err
ω	**-0.161**	0.023	**-0.263**	0.063	**-0.171**	0.025	-0.018	0.240	**-0.177**	0.031	**-0.279**	0.048
α_1	**0.123**	0.017	**0.157**	0.021	**0.124**	0.018	**0.135**	0.033	**0.149**	0.017	**0.145**	0.025
α_2			0.023	0.043			-0.123	0.122			**0.082**	0.021
β_1	**0.934**	0.022	**0.555**	0.100	**0.933**	0.025	1.766	1.893	**0.962**	0.017	0.058	0.042
β_2			**0.345**	0.090			-0.771	1.790			**0.865**	0.043
ϕ_R	**0.031**	0.013	**0.074**	0.030	**0.044**	0.018	0.004	0.067	0.019	0.013	**0.040**	0.012
κ	**3.725**	0.207	**4.376**	0.255	**3.783**	0.186	**5.119**	0.591	**3.772**	0.198	**4.304**	0.254
σ^2	**0.717**	0.127	**0.884**	0.147	**0.756**	0.114	**1.440**	0.398	**0.755**	0.126	**1.069**	0.152
LL	-579		-530		-579		-433		-582		-718	

Note: 1. Model: $R_t = \frac{\exp(\lambda_t)}{\mu}\varepsilon_t, \quad \lambda_t = \omega + \sum_{i=1}^{2} \alpha_i \varepsilon_{t-i} + \sum_{j=1}^{2} \beta_j \lambda_{t-j} + \phi_R R'_{t-1}$.

2. κ and σ^2 represent the parameters of Burr distribution. *LL* denotes the log-likelihood function. The estimated parameters with p-values less than 0.05 are shown in boldface.

Table 7. Parameter estimates of TVCARR(1,1) and TVCARR(2,2) models

	SP		*TSX*		*DAX*		*CAC*		*FTSE*		*MIB*		*N225*	
	Coef	Std Err	Coef	Std Err	Coef	Std Err	Coef	Std Err	Coef	Std Err	Coef	Std Err	Coef	Std Err
ω	**-0.148**	0.021	**-0.187**	0.020	**-0.129**	0.022	**-0.101**	0.027	**-0.165**	0.065	**-0.246**	0.040	**-0.029**	0.030
α_1	**0.160**	0.014	**0.187**	0.019	**0.119**	0.020	**0.110**	0.036	**0.169**	0.027	**0.113**	0.018	**0.188**	0.027
α_2									0.065	0.036	**0.111**	0.020	**-0.161**	0.032
β_1	**0.935**	0.022	**0.939**	0.019	**0.987**	0.008	**0.945**	0.047	**0.430**	0.036	**-0.062**	0.031	**1.661**	0.226
β_2									**0.406**	0.042	**0.923**	0.028	**-0.663**	0.223
ω^*	-0.066	0.046	-0.066	0.048			0.065	0.056	-0.045	0.119	0.151	0.088		
α_1^*	**0.111**	0.033	**0.101**	0.046			-0.019	0.072	0.001	0.071	-0.023	0.039		
α_2^*									**0.215**	0.057	-0.042	0.042		
β_1^*	-0.022	0.048	-0.026	0.035			-0.020	0.092	**-1.066**	0.113	0.036	0.066		
β_2^*									**0.870**	0.074	0.024	0.059		
γ	**5.089**	0.755	2.283	1.190			**1.364**	0.636	0.663	1.183	**1.054**	0.165		
c	**0.749**	0.003	**0.750**	0.005			**0.811**	0.008	**0.746**	0.004	**0.742**	0.004		
κ	**3.746**	0.147	**4.379**	0.170	**4.729**	0.214	**4.735**	0.191	**4.594**	0.211	**5.114**	0.250	**4.316**	0.251
σ^2	**0.714**	0.101	**1.053**	0.108	**1.159**	0.124	**1.168**	0.113	**1.000**	0.124	**1.421**	0.145	**1.092**	0.155
LL	-571		-536		-668		-568		-524		-428		-723	

Note: 1. Model: $R_t = \frac{\exp(\lambda_t)}{\mu}\varepsilon_t,\ \ \lambda_t = \omega + \sum_{i=1}^{2}\alpha_i\varepsilon_{t-i} + \sum_{j=1}^{2}\beta_j\lambda_{t-j} + \left(\omega^* + \sum_{i=1}^{2}\alpha_i^*\varepsilon_{t-i} + \sum_{j=1}^{2}\beta_j^*\lambda_{t-j}\right)\cdot\overline{G}(s_t;\gamma,c)$

2. κ and σ^2 represent the parameters of Burr distribution. *LL* denotes the log-likelihood function. The estimated parameters with p-values less than 0.05 are shown in boldface.

Table 8. Parameter estimates of TVCARRX(1,1) and TVCARRX(2,2) models with the exogenous variables, lagged range of the other (R'_{t-1})

	SP		*TSX*		*SP*		*DAX*		*SP*		*CAC*	
	Coef	Std Err	Coef	Std Err	Coef	Std Err	Coef	Std Err	Coef	Std Err	Coef	Std Err
ω	**-0.117**	0.026	**-0.182**	0.028	**-0.163**	0.034	**-0.136**	0.028	**-0.158**	0.026	**-0.097**	0.026
α_1	**0.083**	0.023	**0.183**	0.026	**0.142**	0.028	**0.115**	0.021	**0.145**	0.023	**0.085**	0.023
α_2												
β_1	**0.857**	0.036	**0.928**	0.067	**0.892**	0.043	**0.967**	0.021	**0.898**	0.040	**0.904**	0.058
β_2												
ϕ_R	**0.058**	0.016	0.000	0.025	0.027	0.016	0.016	0.013	0.027	0.016	0.031	0.019
ω^*	0.009	0.053	-0.080	0.057	-0.034	0.055			-0.043	0.054	**0.104**	0.047
α_1^*	-0.030	0.047	0.091	0.055	**0.135**	0.053			**0.109**	0.040	-0.054	0.047
α_2^*												
β_1^*	**-0.163**	0.069	-0.065	0.135	0.039	0.078			-0.026	0.081	-0.057	0.106
β_2^*												
ϕ_R^*	**0.087**	0.030	0.030	0.050	-0.042	0.031			-0.013	0.031	0.007	0.035
γ	5.089		2.283		5.089				5.089		1.364	
c	0.749		0.750		0.749				0.749		0.811	
κ	**3.711**	0.171	**4.386**	0.206	**3.815**	0.156	**4.629**	0.184	**3.773**	0.180	**4.661**	0.155
σ^2	**0.662**	0.108	**1.057**	0.121	**0.744**	0.091	**1.088**	0.144	**0.724**	0.116	**1.111**	0.103
LL	-561		-535		-564		-666		-568		-564	
KS	0.260		0.218		0.363		0.374		0.283		0.402	
indep(1)	0.331		0.623		0.233		0.871		0.243		0.435	
indep(2)	0.596		**0.044**		0.563		0.504		0.461		0.614	
indep(3)	0.722		0.183		0.655		0.290		0.642		0.464	
indep(4)	0.438		0.109		0.407		0.563		0.348		0.356	

Note: 1. Model:

$$R_t = \frac{\exp(\lambda_t)}{\mu}\varepsilon_t, \quad \lambda_t = \omega + \sum_{i=1}^{2}\alpha_i\varepsilon_{t-i} + \sum_{j=1}^{2}\beta_j\lambda_{t-j} + \phi_R R'_{t-1} + \left(\omega^* + \sum_{i=1}^{2}\alpha_i^*\varepsilon_{t-i} + \sum_{j=1}^{2}\beta_j^*\lambda_{t-j} + \phi_R^* R'_{t-1}\right)\cdot \overline{G}(s_t;\gamma,c)$$

Table 8. Continuous

	SP		*FTSE*		*SP*		*MIB*		*SP*		*N225*	
	Coef	Std Err	Coef	Std Err	Coef	Std Err	Coef	Std Err	Coef	Std Err	Coef	Std Err
ω	**-0.154**	0.026	**-0.119**	0.046	**-0.163**	0.040	**-0.229**	0.004	**-0.184**	0.034	**-0.279**	0.048
α_1	**0.152**	0.018	**0.072**	0.036	**0.149**	0.022	**0.107**	0.005	**0.153**	0.020	**0.145**	0.025
α_2			0.009	0.043			**0.080**	0.003			**0.082**	0.021
β_1	**0.916**	0.037	**0.480**	0.122	**0.905**	0.044	0.005	0.014	**0.861**	0.052	0.058	0.042
β_2			**0.247**	0.094			**0.792**	0.000			**0.865**	0.043
ϕ_R	0.015	0.015	**0.103**	0.027	0.029	0.022	**0.031**	0.004	**0.049**	0.022	**0.040**	0.012
ω^*	-0.052	0.050	**0.338**	0.088	-0.037	0.074	**0.225**	0.012	-0.102	0.070		
α_1^*	**0.122**	0.039	**-0.170**	0.066	**0.127**	0.042	**-0.073**	0.010	**0.094**	0.040		
α_2^*			-0.072	0.075			**-0.081**	0.006				
β_1^*	-0.007	0.069	0.121	0.234	0.017	0.082	**0.184**	0.017	-0.136	0.110		
β_2^*			-0.341	0.178			**-0.225**	0.010				
ϕ_R^*	-0.023	0.030	0.084	0.056	-0.046	0.035	**0.037**	0.008	0.072	0.043		
γ	5.089		0.663		5.089		1.054		5.089			
c	0.749		0.746		0.749		0.742		0.749			
κ	**3.756**	0.167	**4.518**	0.229	**3.791**	0.168	5.151	0.333	**3.782**	0.185	**4.304**	0.254
σ^2	**0.715**	0.109	**0.908**	0.134	**0.734**	0.108	1.417	0.035	**0.723**	0.106	**1.069**	0.152
LL	-569		-506		-567		-419		-566		-718	
KS	0.281		0.310		0.307		0.600		0.326		0.184	
indep(1)	0.274		0.999		0.202		0.998		0.098		0.133	
indep(2)	0.420		0.632		0.615		0.306		0.414		0.248	
indep(3)	0.693		0.929		0.626		0.996		0.422		0.278	
indep(4)	0.320		0.670		0.365		0.244		0.300		0.578	

Note: 2. κ and σ^2 represent the parameters of Burr distribution. *LL* denotes the log-likelihood function. *KS* denotes the Kolmogorov-Smirnov statistic. Statistics $indep(k)$, $k=1,2,3,4$ denote the independence test. The estimated parameters with p-values less than 0.05 are shown in boldface.

Table 9. Parameter estimates of constant and regime switching versions of Gumbel copula

Gumbel copula	*SP—TSX*		*SP—DAX*		*SP—CAC*		*SP—FTSE*		*SP—MIB*		*SP—N225*	
	Coef	Std Err	Coef	Std Err	Coef	Std Err	Coef	Std Err	Coef	Std Err	Coef	Std Err
$\eta_{G,CONST}$	**1.451**	0.032	**1.356**	0.030	**1.331**	0.030	**1.302**	0.028	**1.318**	0.030	**1.069**	0.020
$\eta_{G,RSL}$	**1.399**	0.036	**1.323**	0.032	**1.281**	0.031	**1.265**	0.029	**1.273**	0.030	**1.035**	0.013
$\eta_{G,RSH}$	**1.623**	0.072	**1.450**	0.053	**1.484**	0.062	**1.417**	0.064	**1.455**	0.064	**1.181**	0.046
$\tau_{G,CONST}$	**0.311**	0.015	**0.263**	0.016	**0.249**	0.017	**0.232**	0.017	**0.242**	0.018	**0.064**	0.018
$\tau_{G,RSL}$	**0.285**	0.019	**0.244**	0.018	**0.219**	0.019	**0.210**	0.018	**0.214**	0.018	**0.034**	0.012
$\tau_{G,RSH}$	**0.384**	0.027	**0.310**	0.025	**0.326**	0.028	**0.294**	0.032	**0.313**	0.030	**0.153**	0.033
$\tau^U_{G,CONST}$	**0.387**	0.017	**0.333**	0.019	**0.317**	0.020	**0.297**	0.020	**0.308**	0.021	**0.087**	0.023
$\tau^U_{G,RSL}$	**0.359**	0.021	**0.311**	0.021	**0.282**	0.022	**0.270**	0.021	**0.276**	0.022	**0.046**	0.017
$\tau^U_{G,RSH}$	**0.467**	0.029	**0.387**	0.028	**0.405**	0.031	**0.369**	0.036	**0.390**	0.034	**0.201**	0.041
$LL_{G,CONST}$	150.0		105.2		95.8		84.2		88.4		7.1	
$LL_{G,RS}$	153.8		106.8		99.9		86.6		91.7		11.2	
$AIC_{G,CONST}$	-0.274		-0.192		-0.175		-0.153		-0.161		-0.011	
$AIC_{G,RS}$	-0.279		-0.193		-0.180		-0.156		-0.165		-0.017	

Note: LL denotes the log-likelihood function. AIC represents the Akaike Information Criterion. The first subscript of each coefficient notation G denotes the Gumbel copula, while the second subscript denotes constant version ($CONST$), regime switching version (RS), regime switching version in low volatility regime (RSL), and regime switching version in high volatility regime (RSH). The standard errors of τ and τ^U are calculated by delta method. The estimated parameters with p-values less than 0.05 are shown in boldface.

Table 10. Parameter estimates of constant and regime switching versions of Clayton-Gumbel copula

Clayton-Gumbel copula	*SP—TSX*		*SP—DAX*		*SP—CAC*		*SP—FTSE*		*SP—MIB*		*SP—N225*	
	Coef	Std Err	Coef	Std Err	Coef	Std Err	Coef	Std Err	Coef	Std Err	Coef	Std Err
$\delta_{CG,CONST}$	**0.135**	0.070	**0.293**	0.078	**0.186**	0.067	0.033	0.079	0.123	0.067	0.117	0.064
$\eta_{CG,CONST}$	**1.382**	0.046	**1.223**	0.036	**1.246**	0.038	**1.287**	0.051	**1.261**	0.039	**1.032**	0.022
$\delta_{CG,RSL}$	0.104	0.081	**0.292**	0.079	**0.187**	0.082	0.030	0.082	**0.153**	0.072	0.104	0.065
$\eta_{CG,RSL}$	**1.348**	0.051	**1.196**	0.040	**1.198**	0.044	**1.252**	0.048	**1.202**	0.042	**1.011**	0.021
$\delta_{CG,RSH}$	0.219	0.146	**0.292**	0.134	0.211	0.151	0.033	0.151	0.070	0.127	0.094	0.139
$\eta_{CG,RSH}$	**1.499**	0.093	**1.304**	0.076	**1.381**	0.090	**1.400**	0.102	**1.421**	0.076	**1.144**	0.061
$\tau_{CG,CONST}$	**0.322**	0.030	**0.287**	0.030	**0.266**	0.030	**0.235**	0.039	**0.253**	0.031	**0.085**	0.032
$\tau_{CG,RSL}$	**0.295**	0.035	**0.270**	0.033	**0.236**	0.037	**0.213**	0.040	**0.227**	0.035	0.060	0.033
$\tau_{CG,RSH}$	**0.399**	0.051	**0.331**	0.051	**0.345**	0.058	**0.297**	0.068	**0.320**	0.052	**0.165**	0.066
$\tau^{U}_{CG,CONST}$	**0.349**	0.027	**0.238**	0.030	**0.256**	0.030	**0.286**	0.037	**0.267**	0.029	0.043	0.028
$\tau^{U}_{CG,RSL}$	**0.328**	0.033	**0.215**	0.035	**0.216**	0.038	**0.260**	0.037	**0.220**	0.035	0.015	0.029
$\tau^{U}_{CG,RSH}$	**0.412**	0.045	**0.299**	0.053	**0.348**	0.054	**0.359**	0.059	**0.371**	0.042	**0.167**	0.059
$\tau^{L}_{CG,CONST}$	0.024	−	**0.144**	0.068	**0.050**	0.011	0.000	−	0.011	−	0.003	−
$\tau^{L}_{CG,RSL}$	0.007	−	**0.137**	0.065	0.045	−	0.000	−	0.023	−	0.001	−
$\tau^{L}_{CG,RSH}$	0.121	0.148	0.162	0.124	0.092	0.118	0.000	−	0.001	−	0.002	−
$LL_{CG,CONST}$	151.8		114.8		99.7		84.3		90.2		9.4	
$LL_{CG,RS}$	155.8		116.3		104.1		86.7		94.0		13.2	
$AIC_{CG,CONST}$	-0.276		-0.208		-0.180		-0.152		-0.162		-0.014	
$AIC_{CG,RS}$	-0.279		-0.207		-0.184		-0.152		-0.166		-0.017	

Note: LL denotes the log-likelihood function. AIC represents the Akaike Information Criterion. The first subscript of each coefficient notation CG denotes the Clayton-Gumbel copula, while the second subscripts denote constant version ($CONST$), regime switching version (RS), regime switching version in low volatility regime (RSL), and regime switching version in high volatility regime (RSH).The standard errors of τ, τ^{L}, and τ^{U} are calculated by delta method; notation " − " means the calculated Std Err is not a real value. The estimated parameters with p-values less than 0.05 are shown in boldface.

Table 11. Summary of the estimates of Kendall's τ, and tail dependence for each selected model

	SP—TSX	*SP—DAX*	*SP—CAC*	*SP—FTSE*	*SP—MIB*	*SP—N225*
τ_{CONST}		**0.287**				
τ_{RSL}	**0.295**		**0.236**	**0.210**	**0.227**	0.060
τ_{RSH}	**0.399**		**0.345**	**0.294**	**0.320**	**0.165**
τ^{U}_{CONST}		**0.238**				
τ^{U}_{RSL}	**0.328**		**0.216**	**0.270**	**0.220**	0.015
τ^{U}_{RSH}	**0.412**		**0.348**	**0.369**	**0.371**	**0.167**
τ^{L}_{CONST}		**0.144**				
τ^{L}_{RSL}	0.007		0.045		0.023	0.001
τ^{L}_{RSH}	0.121		0.092		0.001	0.002
The selected model	$C_{CG,RS}$	$C_{CG,CONST}$	$C_{CG,RS}$	$C_{G,RS}$	$C_{CG,RS}$	$C_{CG,RS}$

Note: $C_{G,RS}$, $C_{CG,CONST}$, and $C_{CG,RS}$ denote the regime switching Gumbel copula, constant Clayton-Gumbel copula, and regime switching Clayton-Gumbel copula, respectively. The subscripts of the estimated parameters denote constant version ($CONST$), regime switching version in low volatility regime (RSL), and regime switching version in high volatility regime (RSH).The estimated parameters with p-values less than 0.05 are shown in boldface.

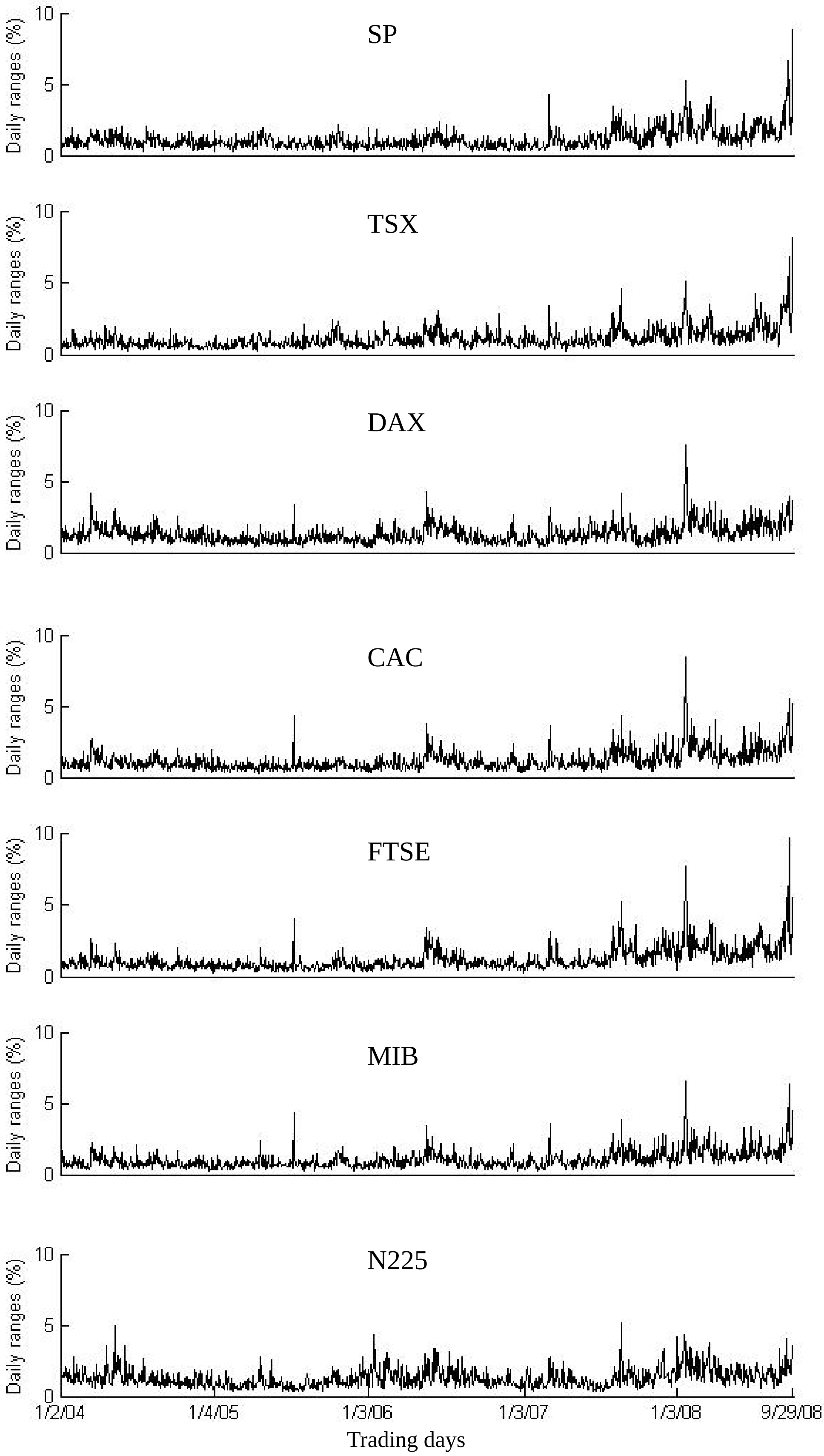

Figure 1. Daily ranges of the seven stock market indices