Should Asset Managers Hedge their "Fees at Risk"?

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HULL (2007) writes: "For an asset manager the greatest risk is operational risk". In 2008 however asset management companies came under severe profitability pressure from market not operational risks. What has been seen as an annuity stream that was thought to expose firms to little or no earnings risk, materialized as directional stock market exposure combined with high operational leverage. Asset management companies should however hedge the risks of large swings in their P&L due to changes in their asset based fees in accordance with well established risk management principles. While alpha risks are regarded as core risks (it is the business of an asset management company to exploit theses risks in return for fees), beta risks arising from client benchmark exposure are incidental. We suggest both the hedging of production risks (fees at risk) as well as capital market related business risks (redemptions triggered by clients to either de-risk or raise cash).

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1 Introduction

"For an asset manager the greatest risk is operational risk".[1] In 2008 however asset management companies came under severe profitability pressure from market not operational risks. What has been seen as an annuity stream that was thought to expose firms to little or no earnings risk, materialized as directional stock market exposure combined with high operational leverage (high ratio of fixed to variable costs). While operational leverage led to what has been praised as a scalable business (low costs of taking on additional business) in good times it was always clear that this would lead to massive losses in bad times. In short asset managers partially share client's benchmark risks. As client benchmarks went down, so did asset based fees (percentage fee applied on average assets under management within a year) that still represent the bulk of fee agreements in the asset management industry.

At the same time operational leverage increased the downturn in profits. A small example should make the mechanics clear. Suppose an asset manager with $100\ billion$ € assets under management, $50bps$ fees and $35bps$ total costs and an operational leverage of 90% ($31.5bps$ in fixed costs). At the outset the expected profits for the year are $150\ million$ €. For a benchmark volatility of $\sigma = 30\%$ asset based fees will be exposed to a 17% variability (see the next section for the plausibility and calculation of these numbers). In other words, asset management revenue will be down by about 35% in a 2 sigma event. Should average assets under management fall by 35% all profits are wiped out and the company is left with a loss of $-12.75\ million$ €.[2] A 35% reduction in revenues led to a more than 100% reduction in profits. Operational leverage leads to a reduction in profits that is many times larger than the reduction in revenues (fees). Note that this number assumes zero redemptions and zero shifts from high fee equity products to low fee fixed income funds.

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[1] See HULL (2007, p. 372).

[2] At a lower operational leverage of 50% (with the same total costs at the beginning of the year) profits would have still been up to $36.25\ million$ €.

Given the size of this result it is surprising that asset managers did not undertake any effort to reduce a source of risk that is outside their control. In fact year after year careful business plans are drafted with detailed planning on new flows and revenues coming from existing and new clients, distribution channels, etc. while markets continue to make a complete mockery out of these exercises. Even a plus or minus one sigma event on market returns leads to windfall gains or losses that are outside the control of an asset management firm. While on could make the point that asset managers have a competitive advantage in assessing and taking stock market risks[3], this point would also have required them to actively manage these risks over time. They did not.

The paper is organized as follows. Section 2 introduces some closed form solutions to approximate the volatility of asset management fees as well as providing examples for the closeness of these approximations. Section 3 tries to explain the current failure of actively hedging fees at risk as a combination of misapplications of financial theory as well as corporate governance issues. Section 4 reviews the case for hedging and Sections 5 and 6 make some attempt to lay out what and how to hedge.[4] Two appendices contain technical material.

2 Fees at Risk

In order to asses the potential impact of market exposure in an asset managers P&L we need to find an expression for the volatility of asset management fees. Given asset based fees are calculated as a percentage on the average assets under management over a time period the calculations become slightly more involved than usual value at risk calculations. Note that asset based fees contain both benchmark (beta) as well as non-benchmark (alpha) exposure. We will always focus (unless stated otherwise) on the client imposed benchmark part of asset based fess. Implicitly we assume that the alpha part of asset based fees is negligible, which will be true for most mandates. Using standard

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[3] However, it must be said that the fact that market timing is an activity that few managers engage in would let us conclude that beta timing is not regarded as a core competency that asset management companies think they can earn economic profits from.

[4] Writing a paper on this subject might look procyclical or someone confusing hindsight knowledge with risk management. However all building blocks used in this paper have been readily available for quite some time but it always takes some crisis for people to listen.

results from derivatives pricing we can approximate the volatility of annual asset management revenues by[5]

(1) $$\sigma\left(\tilde{f}_{t+n}\right) \approx \theta \cdot A_t \sqrt{\left(\frac{2e^{\sigma^2}-2\left(1+\sigma^2\right)}{\sigma^4}\right)\left(\frac{2e^{\sigma^2}-2\left(1+\sigma^2\right)}{\sigma^4}-1\right)}$$

where A_t denotes assets under management at time t, θ reflects percentage fees and σ stands for the volatility of asset returns underlying the calculation of average assets under management. For those that prefer simpler formulas you could also use

(2) $$\sigma\left(\tilde{f}_{t+n}\right) \approx \theta \cdot A_t \frac{\sigma}{\sqrt{3}}$$

which will provide very similar results. Instead of using the arithmetic average it is based on the geometric average.[6] An example will both illustrate the quality of the approximations as well as the extent of "fees at risk" in asset management companies. Exhibit 1 calculates the volatility of average asset prices under approximations in (1) and (2) as well as the "true" bootstrapped volatility.

σ	Approx.: (1)	Approx.: (2)	Bootstrapping
50%	31,1%	28,9%	30,5%
40%	24,2%	23,1%	24,1%
30%	17,8%	17,3%	17,9%
20%	11,7%	11,5%	11,9%
10%	5.8%	5.8%	5.9%

Exhibit 1. Volatility of average stock prices.

Both approximations work remarkably well compared to the "true" value from bootstrapping. The simpler approximation in (2) seems however to persistently underestimate the volatility of asset management fees. For smaller volatilities the approximations become increasingly better. Fee volatility for example a European equity

[5] See Appendix A for a derivation of (1) and its underlying assumptions together with an expression for "Fees at Risk". Appendix B provides a brief simulation study on its approximating properties.
[6] See NELKEN (1996).

mandate with 30% volatility (not unusual in 2008) is around 18%. Too large to be left unmanaged.

3 Why Did Asset Managers Fail to Hedge Their P&L?

Given that *not* hedging the P&L seems to be very much an asset management industry wide practice it is easier to start with reviewing arguments put forward in favour of not hedging.

The first argument against hedging runs as follows. Given that holding financial assets requires an ex ante positive risk premium (markets go up on average), hedging asset based fees (that also would go up on average) will create a long run opportunity loss to shareholders. This argument addresses the basics of corporate finance: what projects should firms undertake? We know from corporate finance that companies need not worry about their shareholders as long as they engage in positive NPV projects. Is the beta part of collecting an asset based fees a net present value (NPV) positive project? The clear answer is no. This applies no matter how big the expected growth rate of assets might look like. The reason for this at glance unintuitive proposition is that asset based fees are the most simple form of a derivative contract (with assets under management as underlying). We know that the value of a derivative contract is independent of the real world growth rate of the underlying. As such the growth rate does not matter. This is just rephrasing the fact that capital market investments provide zero NPV, or as ROSS (2005, p. 71) put it: "…, since the fee is contingent on asset value, as a contingent claim its current value is independent of expected rates of returns."

The second argument is slightly more sophisticated as it relates to a cornerstone of financial economics: the famous Modigliani/Miller (MM) argument for hedging irrelevance. In frictionless markets, i.e. in the absence of (frictional) bankruptcy costs or taxes hedging would be irrelevant. Shareholders of an asset management company would simply undo the implicit beta exposure that comes with asset based fees in their private portfolios. The trouble with this argument is that it rests on unrealistic assumptions. In reality markets are not without frictions and hedging preserves costly liquidity while frictional bankruptcy costs are particularly high in the banking industry as 2008 has undoubtedly shown with an unprecedented number of bank runs. In other words the MM argument is itself particular irrelevant in the banking industry.[7]

The last argument that the author has been confronted with is what has been called intellectual risk. In other words a hedging policy might be intellectually sound, but as soon as the hedge makes losses corporate memory seems to fade and nobody sees the offsetting gains of the hedged position any more. As such the hedging policy might hedge corporate financial risk but not individual career risk. Just ask those airline treasurers that hedged their fuel costs at an oil price of 140$ per Barrel. This argument is particular relevant at the time of writing this paper where markets are perceived to be at their bottom. While the author has sympathy with the risk manager in an asset management organization with intellectual risks, this seems more rooted in positive (why things are done) rather than normative (how should things be done) theory. Intellectual risk seems more a corporate governance issue than a serious argument against hedging "fees at risk".

Finally it must be said, that it is certainly tempting for a CIO to be paid a large bonus on the basis of beta exposure and that the incentive to remove this exposure might be in the shareholders but not in the CIO's interest.[8] Managers that get paid with options (nonlinear exposure to earnings variability) on the underlying business will see an increase in the value of their executive options if they fail to hedge the P&L against market risks. Senior management hence often has good reason not to hedge fees at risk, even if (as we will argue in the next section) this will on average reduce shareholder value. Again this is a corporate governance issue. The opposite is true for private partnerships (manager and owner coincide) where managers have a linear exposure to

[7] Most asset managers are still owned by banks. However a similar argument applies to stand alone asset management companies. Asset managers that show large losses are likely to experience larger redemptions on client concerns on their ability to keep and recruit key staff etc.

[8] If in doubt, just recall what happened to CIO's of real estate asset management firms in 2006 and 2007. During the property bubble their management got highly paid for running a long beta business and often got promotions within a firms hierarchy. When the bubble burst they left the firm exposed. And of course the same applies to fixed income managers that invested into credit versus a government bond benchmark up to 2007.

earnings variability. Hedging here would reduce income volatility and hence increase manager (and owner) utility.

4 What are the Costs of Not Hedging the P&L?

After having identified some spurious arguments against hedging, we can show why hedging P&L risk can improve returns for an asset manager. In short, hedging is shareholder positive, if it creates a positive NPV project.[9]Quite generally we argue that while asset based fees are zero NPV projects they still create P&L risks, that in a world with capital market frictions and taxes are costly.

First not hedging asset based fees will remove positive NPV projects (investing in new products, people, IT platforms, …) by the necessity to hold cash against theses risks in order to maintain a target rating (crowding out). If alternatively no cash is held as risk capital, not hedging P&L risk will increase the expected value of frictional bankruptcy costs and simultaneously limit the ability to leverage (and hence the ability to reap a tax shield or to use operational leverage). Un-hedged swings in fee income will also increase the value of the tax option the government holds against the asset management company. Taxes have to be paid if profits are made, but with limited carry for and backwards no equal amount is received if losses are made. The larger these swings, the higher the value of this option. This argument obviously depends whether the tax option is at the money

Hedging P&L risk due to capital market movements should also allow an improved observability of effort in the principal agent relationship between firm management and shareholder. Not only might management compensation driven by windfall gains and

[9] All arguments used here have been available for many years in the corporate finance literature. A nice summary can be found in DOHERTY (2000). One contribution of this paper is to apply these thoughts into the business of managing client money.

losses attract the less skilled, it also discourages effort, as it is unlikely to be reliably observed by shareholders.

While we could think of many other channels through which risk management will increase value, 2008 taught the industry an unforgettable message. Losses on asset based fees are largest in a severe equity down markets, where almost all asset classes will fall in value, and clients will massively redeem assets to de-risk or raise much needed cash. And it is precisely these states of the world where bank funding will also dry up. In other words: a high correlation between revenue risks and funding risks clearly calls for hedging fees at risk. After all hedging protects costly liquidity as losses exhaust internal capital that is much needed (preferred) to finance new projects (pecking order theory).

5 What to Hedge?

So far, we talked about why hedging P&L risk is NPV positive for an asset manager, but we have been very narrow (we focused on the beta part of asset based fees, assuming no redemptions or client specific risks) about exactly what we should hedge. This section tries to provide a more context. Let us divide risks into production risks and business risks to get some further insight.

Production risks are risks that come as a by-product of managing client money. Both asset as well as performance based fees fall into this category. Production risks come both as alpha (outperformance versus a risk adjusted benchmark) and beta (economic facto exposure) risks. Taking (owning) alpha risks is one of the core competencies of an asset management firm. The asset manager is rewarded for its scarce and industry specific skills. Acquiring these risks is essentially a positive NPV projects as risks are more than compensated by the expected profits. Creating alpha is equivalent to creating positive NPV, both for the firm and its clients. Beta risks on the contrary are incidental risks that asset managers usually don’t engage in taking in, either because they think they lack the skills. Even if asset managers would exhibit skills in market timing the risk return ratio is likely to be so low that it seems wiser to hedge these risks to free up risk capacity for mor core, i.e. higher NPV generating risks.

While we argued very strongly in favour of hedging the beta part of asset based fees it should be clear that we should not hedge performance based fees. We would simply destroy the option value provided by the client. Here we want volatility. Also note that we can by definition only hedge systematic beta risks and that idiosyncratic risks are an asset managers core product, i.e. the core production risk to take. Practically it also would be (legally) difficult to hedge performance based fees as it would require the asset manager to hold offsetting positions (for his own P&L in a separate brokerage account) to those that he implements in a fiduciary function for a given client. Note that option pricing technology becomes a dangerous tool here as replication is impossible.

While it is not common for banks to also hedge business risks (risks that are common to a business model but not directly related to production) it is clear that some of these business risks are correlated with capital market risks. Business risks in the asset management industry are mainly related to systematic outflows affecting whole product lines. In a severe equity downmarket, retail investors will shift their asset allocation out of fee intensive equity funds into money market funds or government guaranteed deposits while institutional clients like insurance companies might also reduce their risk exposures due to their own (now binding) regulatory constraints. We could think of various ways to hedge client redemption risk. Redemptions in the retail sector are usually correlated with asset performance and we might want to hedge against extremely bad market scenarios under which redemptions are likely to be triggered. For institutional clients redemptions might additionally be motivated by clients own financial distress, i.e. the client might need to raise cash or de-risk its asset allocation. All instruments related to financial distress could be used. For example: an asset management company that is exposed to a weakly rated insurance company might want to buy puts or credit default swaps on the insurer to hedge parts of its fee income. A fund of hedge fund provider might want to buy puts on hedge fund replicating clones to isolate himself partially from client redemptions or fees at risk.

6 How to Hedge?

The easiest way to hedge asset based fees is not to offer them. This follows the idea of duality in risk management. We can either root out the cause (variability in markets) or the effect (offer fixed fees). Fixed fees have been discussed in institutional asset management for a while, but they are perceived to suffer from the obvious to renegotiate in a world with positive inflation (although they could be of course indexed). However the author anticipates an increase in these fee arrangements.

How do we practically implement a hedge program aimed at insulating an asset managers P&L from market induced variations of its average assets under management for a given time period? The naive proposition would be to sell futures with one year maturity on the underlying assets with a notional $\theta \cdot A_t$. If assets increase in value the hedge (ignoring carry) creates a loss of $-\theta \cdot \Delta A$ while asset management fees rise by an offsetting $+\theta \cdot \Delta A$. However this hedge will not generally work due to the path dependency of fees based on average assets under management. We can easily think of a situation were equity markets have been falling gradually over 11 month in a year but sharply recovering to its end compensating for more than its previous losses. Here we would loose money on the hedge (equity market is up on the year) and in revenues (average assets under management are down too). What we need is a hedging policy instrument that moves with average prices rather than year end prices. Fortunately all we need is to trade (or replicate) a forward contract on the average stock price, i.e. a contract that pays the average stock price at the end of the period (Asian forward, $F_{t,t+n}^{Asian}$). The price of such a contract to sell the average stock price (yet random) at a known price $\overline{S}$ is given by[10]

$$F_{t,t+n}^{Asian} = e^{-r}\left(S_t \sum_{i=1}^{n} \frac{e^{r\frac{i}{250}}}{n} - \overline{S}\right) \tag{3}$$

Note, that (3) is independent of the distribution that the average price process migh follow. We assume $n = 250$. Another way to think of (3) and to create a position a

[10] A replicating strategy would sell $\frac{1}{n}e^{-r}$ forward contracts for each of the n averaging points.

position in an Asian forward is to buy a long position into an Asian call with strike $\overline{S}$ and one year maturity and a short position in an Asian put with the same strike and maturity. Long call and short put provides a synthetic forward that comes at zero cost if the strike is at the money forward.[11]

How would this work? We assume current assets under management of 100 billion dollars with fees of 50 bps.[12] The current P&L to defend is 500 million € or 507.56 million € at current forward prices ($0.5\% \cdot 100\ billion \cdot \sum_{i=1}^{n} \frac{e^{r\frac{i}{250}}}{n} = 507.56\ million$). Suppose the asset manager wants to isolate the P&L at current rates of 3% per annum against market exposure. Suppose the benchmark asset trades at an index level of 4500, where each index point is valued at $2500€$. Here $S_t = 4500 \cdot 2500€ = 1125000 = 1.125$ milion €. A forward contract to sell the average index level at $\overline{S} = 1.142\ million\ €$ is valued at zero at time t. We need to sell

$$\#\,Asian\ Forwards = \frac{A_t \theta}{F_{t,t+n}^{Asian}} = 444.444$$

Now suppose the average index level drops at the end of the year to $\frac{1}{n}\sum_{i=1}^{n} S_{t+i} = 3800$
The payoff from our short forward position is

$$444.444 \cdot \left(\overline{S} - \tfrac{1}{n}\sum_{i=1}^{n} S_{t+i}\right) = 85.3533\ million\ €$$

Together with asset management fees of $\theta \cdot \frac{3800}{4500} \cdot 100\ billion\ €$ =422.22 $million\ €$ this amounts to total fees of $507.56\ million\ €$, which is exactly what we sold the fee income for the coming year for. We have insulated the P&L. One last objection against hedging asset based fees could creep in here. After all fixing your revenues when your input costs are variable (inflation, competitive pressure to hire talent etc.) does not sound like a good

[11] In other words if $\overline{S} = S_t \sum_{i=1}^{n} \frac{e^{r\frac{i}{250}}}{n}$.

[12] This example should be used for illustrative purposes only. As (all) asset managers manage various products with different benchmarks we need to hedge each product separately. As this is a straightforward extension of what is presented in this paper we continue to present the "single product" case.

idea. There are two answers to this. First, inflation expectations are incorporated in the forward curve for pricing $F_{t,t+n}^{Asian}$ (although we assumed for simplicity a flat curve above). Second, if input costs change any business needs to increase prises or increase efficiency. There is nothing special about asset management.

7 Summary

The existing risk management literature for asset managers narrowly focuses on measuring market risks that asset managers take on behalf on their clients. This paper in contrast looks at the impact of capital markets on asset management profitability. We argue that traditional benchmarked long only asset management companies take too much non-rewarded beta risk in their primarily asset based fee structures. Hedging these risks should create value for shareholders and insulate asset managers from swings in their fortune that are unrelated to their core skills, i.e. providing outperformance and suitable products to their client base.

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Appendix A – Approximate Distribution of Asset Based Management Fees

An asset based fee at time $t+n$, $\tilde{f}_{t+n}$, is a random variable (characterized by ~), that depends on the random realization of the path of future assets under management $\tilde{A}_{t+i}$ for $i = 1,...,n$. More precisely fees are calculated as a percentage θ (usually measured in basis points) on the average assets under management over a given time period. We assume assets under management are calculated daily (which is the case for all retail funds with daily liquidity) and we will usually assume that $n = 250$, i.e. we look at the distribution of annual fees with daily (almost continuous) averaging.

(4) $$\tilde{f}_{t+n} = \theta \cdot \left(n^{-1} \sum_{i=1}^{n} \tilde{A}_{t+i} \right)$$

Ignoring future in and outflows as well as active management returns, assets under management are tied to the evolution of benchmark returns, $\tilde{A}_{t+i} = A_t \frac{\tilde{S}_{t+i}}{S_t}$, where $\frac{\tilde{S}_{t+i}}{S_t}$ denotes the benchmark return for the period from t to $t+n$. In other words asset management companies share client's benchmark risks:

(5) $$\tilde{f}_{t+n} = \theta \cdot A_t \cdot \left(\frac{1}{n} \sum_{i=1}^{n} \frac{\tilde{S}_{t+i}}{S_t} \right)$$

The trouble with (5) is that even though $\frac{\tilde{S}_{t+i}}{S_t}$ is lognormal (with mean μ and variance σ^2) the sum of lognormal variables is not. However, HAUG (2006) provides an approximate formula for a process with zero drift[13]

(6) $$\ln\left(n^{-1} \sum_{i=1}^{n} \frac{\tilde{S}_{t+i}}{S_t} \right) \sim N\left(0, \sqrt{\ln\left(\frac{2e^{\sigma^2} - 2\left(1+\sigma^2\right)}{\sigma^4} \right)} \right)$$

[13] We use (6) for illustration given the communities obsession with closed from solutions. The interested reader might explore various approximations for the distribution of $\ln\left(n^{-1} \sum_{i=1}^{n} \frac{\tilde{S}_{t+i}}{S_t} \right)$ usually provided in the options pricing literature as for example in NELKEN (1996). However given that log returns themselves are neither normal nor uncorrelated nor independent all these expressions can only be seen as back of the envelope shortcuts. Given the low (computational) costs of bootstrapping the small sample distribution a simulation approach should always be the preferred route to a solution.

where σ^2 denotes the variance of benchmark asset returns. He argues that $\ln\left(n^{-1}\sum_{i=1}^{n}\frac{\tilde{S}_{t+i}}{S_t}\right)$ might very well be approximated by a normal distribution. The cumulative distribution for a lognormal variable is given by

$$P\left(n^{-1}\sum_{i=1}^{n}\frac{\tilde{S}_{t+i}}{S_t} \leq S\right) = \Phi\left(\frac{\ln(S)}{\sqrt{\ln\left(\frac{2e^{\sigma^2}-2(1+\sigma^2)}{\sigma^4}\right)}}\right) \tag{7}$$

where S denotes the average of rescaled (to 1) benchmark values and Φ stands for the cumulative density function of a standard normal. We can now easily calculate "Fees at Risk" (FaR) for alternative benchmark assets (i.e. volatilities σ) and confidence level $1-\alpha$ from (7) by solving for the required percentile. For $\Phi(z_\alpha) = 0.05$ we know that $z_\alpha = -1.64$. Given that expected returns are notoriously difficult to forecast we assume that benchmark assets exhibit zero drift.

$$FaR_{\alpha,t} = \theta \cdot A_t \cdot e^{-z_\alpha\sqrt{\ln\left(\frac{2e^{\sigma^2}-2(1+\sigma^2)}{\sigma^4}\right)}} \tag{8}$$

Also note that we can use (5) and (6) to calculate the variance of asset management fees. Applying again properties of the lognormal distribution we get

$$\begin{aligned} Var\left(\tilde{f}_{t+n}\right) &= (\theta \cdot A_t)^2\, Var\left(n^{-1}\sum_{i=1}^{n}\frac{\tilde{S}_{t+i}}{S_t}\right) \\ &= (\theta \cdot A_t)^2\left(\frac{2e^{\sigma^2}-2(1+\sigma^2)}{\sigma^4}\right)\left(\frac{2e^{\sigma^2}-2(1+\sigma^2)}{\sigma^4}-1\right) \end{aligned} \tag{9}$$

which is what we used in (1). Note that for deriving (9) we simply used that for a lognormal random variable X with variance σ^2 we know that $Var(X) = e^{\sigma^2}\left(e^{\sigma^2}-1\right)$.

Appendix B – Simulation Study

We want to test, whether $\ln\left(\tilde{f}_{t+n}\right)$ is approximately normally distributed, i.e. whether asset management fees can be approximated as a lognormal random variable. If we could a whole battery of approximations were available that could provide us with closed form solutions for "Fees at Risk" or fee volatility. In order to perform this task, we bootstrap 5000 annual time series for stock returns. To make the example realistic we use real world stock returns, i.e. we use daily returns on the S&P500 ranging from January 2000 to November 2008 to bootstrap from. It is well known, that classic bootstrapping destroys dependence structures in a time series as each draw is assumed to be drawn independently. Instead we try to maintain some of the dependence structure with a straightforward modification. After each return draw we draw a second random variable from a uniform distribution between 0 and 1. If the draw falls below $q = \frac{1}{2}$ we take the neighbouring entry from the original data series, otherwise we continue drawing randomly from the original time series.

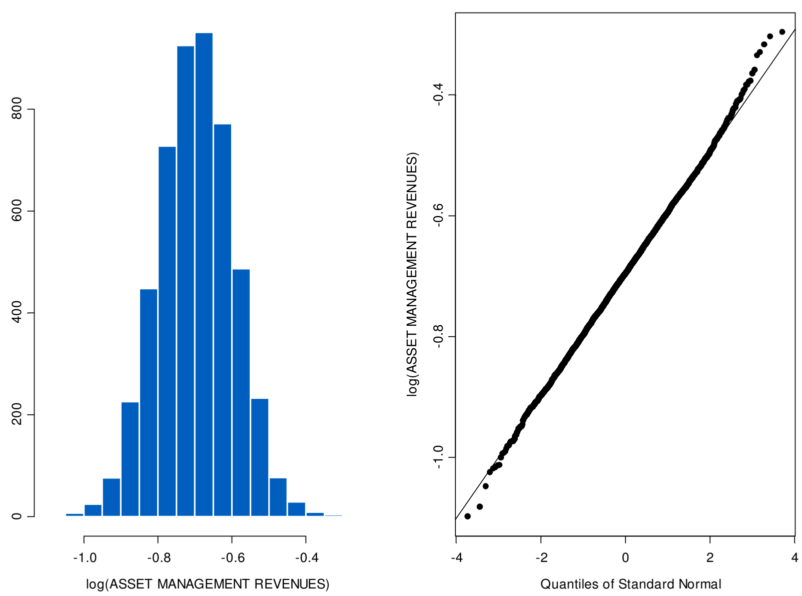

Figure 1. Histogram and quantile plot for Bootstrapped log asset management fees

The likelihood of drawing two consecutive entries is there 0.5, the likelihood of three consecutive entries is 0.25 and so on, i.e. the expected block length is 2. If q becomes larger, the expected block length increases. Figure 1 shows both histogram and quantile plot for $\ln(\tilde{f}_{t+n})$. Both figures confirm seem to our assumption that asset management fees can be approximated by a lognormal random variable. The QQ–plot shows a remarkable good fit, given that the underlying daily return series for the S&P 500 is far from being normal as we can see from Figure 2. Daily returns obviously exhibit large Kurtosis.

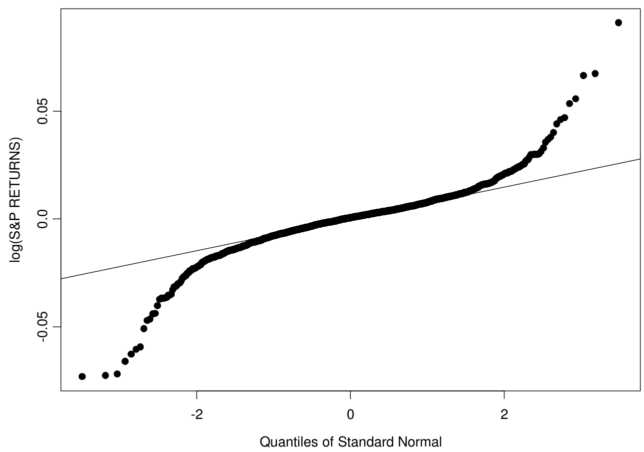

Figure 2. Quantile plot for Bootstrapped log S&P 500 returns

This is a remarkable result (that also holds for larger q). However, while Figure 1 suggests a perfect fit it is not. The term approximately lognormal still applies. Formal tests, like the Wilkinson/Shapiro test for normality of $\ln(\tilde{f}_{t+n})$ reject normality with a p-value of around zero. Even though skewness (0.0142) and kurtosis (0.143) are small, they are still significant given the large number of observations. While this result is encouraging, we are unable to generalize for other time series, time periods or data frequencies, i.e. if in doubt: bootstrap.