Intertemporal effects of capital requirements on risk taking behavior of banks

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Abstract

This paper analyses the impact of regulatory capital requirements on the risk taking behavior of value maximizing banks using a dynamic financial intermediation model. It investigates several cases of intertemporal effects of capital regulation on risk choices when banks face different regulatory conditions. The results reveal differences in a bank's risk taking behavior based on profit, multiplier, and leverage effects. The relationships between retention rate, discount factor, and risky asset return have important implications for first and/or second best regulation. Optimal regulatory rules are derived for different scenarios.

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1. Introduction

Recent banking crises, combined with the vital role of banks in the economy, have strengthened the traditionally strong interest in regulatory policies capable of constraining bank risks. Financial deregulation of banking has increased competition, resulting in lowered profits and decreased banks' franchise or charter values (Keeley 1990; Strahan 2003; Repullo 2004). This change has contributed to the frequency of banking crises worldwide due to increased moral hazard at the bank level (Hellmann *et al.* 2000). Because of the limited liability of equity, banking shareholders have increasingly engaged in risky gambles to enhance banks' franchise values, because they are not liable for lost debt capital.

The negative externalities of bank failures are perceived to be large and hence warrant a degree of regulatory intervention. Capital requirements serve as a major building block in modern banking regulation because they increase the bank's financial cushion to absorb losses and the shareholders' capital stake at risk. They add security for depositors and, where applicable, the deposit insurance system (Spong 2000).

This paper addresses the ability of basic capital regulation to constrain banks' risk-taking choices in a dynamic framework and uncovers several important properties that have eluded

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prior analysis. Unlike previous dynamic models of banking, our framework quantifies the role of earnings retention rates in the tradeoff perceived by shareholders between current and future returns, which affects not only a bank's preferred choice of risk but also its response to regulatory capital requirements. Our main findings include a non-neutral effect of bank size on optimal regulation, contrasting effects of regulation on old versus new banks, an effect of regulation on the future value of equity, and interrelated effects of dividend payments, discounting, and risky asset returns. In addition, the analysis suggests possible new regulatory instruments that may help improve the efficacy of regulatory incentives to mitigate banks' risk-taking.

The remainder of this paper is organized as follows. Section 2 reviews prior studies, section 3 presents our theoretical model and the risk levels chosen by a social planner and by an unregulated bank, section 4 characterizes the risk of banks face new or ongoing regulation, and section 5 presents the conclusion and a discussion of potential extensions.

2. Related literature

Early studies of capital requirements such as Kahane (1977) and Koehn and Santomero (1980) applied portfolio-based utility-maximization models to conclude an inverse relationship between regulation and bank risk level for a sufficiently risk avers bank shareholder. Most recent work has focused on informational asymmetries between banks and borrowers. In a static model, Besanko and Kanatas (1996) demonstrate that conflicts of interest between shareholders and bank management help capital requirements reduce the bank's risk of default. A similar result is shown by Santos (1999) who models the relationship between bank and borrower in a principal agent fashion.

Dynamic models emphasize important intertemporal consequences of changes in the bank's capital and portfolio choices. Hellmann et al. (2000) analyze the intertemporal relationships between financial liberalization and prudential regulation with moral hazard at the bank level. The effect of capital requirements with safe and risky assets depends on the magnitude of two opposing effects. The "capital-at-risk effect" reduces risk taking while the "future-franchise value effect" increases the desire to gamble. Two extensions that introduce spatial monopolistic competition (Repullo 2004) and imperfect competition in the deposit markets (Repullo & Suarez 2004) show that capital requirements can control bank risk taking.

However, there are important intertemporal issues regardless of issue of moral hazard at the bank level. Calem & Rob (1999) consider a bank whose equity grows through retained earnings over time. The model is calibrated to U.S. banking data from 1984 to 1993 and finds a U-shaped relationship between equity capital and risk. Undercapitalized banks take large risks because of the deposit insurance's coverage of bankruptcy costs. Risk taking is decreasing in capital up to a critical level of capitalization at which additional capital increases risk taking because of the increasing marginal benefit of gambling.

In a seminal piece, Blum (1999) shows how the core tradeoffs governing banks' equity growth are affected by changes in capital regulation in a two period framework. Investing in risk-free and/or risky assets the bank chooses the level of risk and the deposit amount at the beginning of each period. In a key finding, Blum demonstrates the potential for capital requirements to *increase* risk taking due to an intertemporal tradeoff between the bank's desire for profit maximization and survival in the presence of regulatory leverage constraints.

This paper extends the critical intertemporal tradeoffs uncovered by Blum. While the issues of asymmetric information are also important, Blum's basic framework allows for a clear analytical characterization of the decision problem based on the bank's incentives and its determinants. Five key extensions are presented: (1) The framework is generalized by accounting for per period dividends through a retention rate applied to the bank's per period net income. The realistic introduction of dividend payments reveals important regulatory implications because of the effect of cash outflows on future profits from retained earnings. (2) A discount factor that is a function of the bank shareholder's required rate of return is introduced. This allows an incorporation of shareholders' preferences over time and provides results with respect to different types of shareholder (patient versus impatient). (3) A rigorous investigation of the intertemporal effects of equity growth and deposit level choices on the bank's risk taking through the influence of the retention rate, discount factor, and return on the risky asset. (4) A proof of the stationarity property of the socially optimal risk outcome. (5) The provision of regulatory guidance for two key scenarios and the derivation of the optimal level of regulation for the realistic case of an "old bank" that faces regulation introduction.

3. The Model without Regulation

We build on a standard financial intermediation model in which banks use funds to produce loans (Klein 1971). For brevity all derivations below have been omitted but are available from the authors by request.

3.1. Decision making over two periods

To examine the intertemporal effects of capital regulation, we focus on the analysis of the decision problem of one bank over two periods as in Blum (1999). The model has four types of agents in a risk neutral economy: one bank, depositors, loan seeking entrepreneurs, and the government (providing deposit insurance, offering risk free assets, and exercising regulatory capital adequacy rules).

The bank's goal is to maximize the present value of future cash flows over two periods. At the beginning of each period, the bank faces deposit collection and asset allocation. At $t=0$ (beginning of period 1), provided with an exogenous positive amount of capital[1] $E_0 \geq 1$ and accounting for constraints (if applicable), the bank chooses (and instantly collects) deposits $D_0>0$ for its investment in risky loans to entrepreneurs and/or risk free assets (if desired). The deposit rate is an increasing function of deposits. It is assumed that the deposit cost function is strictly convex[2] and that the government provides deposit insurance[3] so that depositors are paid if the bank fails.

At $t=1$ (the end of period 1 and beginning of period 2), the bank collects the principal and interest of its assets for the first period to recover its equity and repay depositors, where the sum

[1]This realistic assumption is based on seminal research on a firm's choice of capital structure (Modigliani & Miller 1959; Modigliani & Miller 1963; Baxter 1967). In the presence of bankruptcy costs, a firm chooses a positive amount of equity. In the presence of a tax shield for debt capital, the leverage tradeoff between tax advantages and increasing bankruptcy cost yields an interior solution for the optimal capital structure (balance theorem).

[2]Gennotte & Pyle (1991) make a similar assumption using a convex cost function of loan initiation and argue that the cost of loan initiation increases as the bank's size increases. The growth rate of these costs is increasing, for example, because the bank engages in loan markets outside of its core expertise as it grows its business.

[3]The deposit insurance assumption is known to "best reflect reality" and makes the supply of deposits by depositors independent of the bank's risk level choice (Hellmann *et al.* 2000). For simplicity, we ignore any deposit insurance fees as well as intermediation cost for the bank.

of period 1 deposit principal and interest payments are represented by $C(D_0)$. The first period net income NI_1 equals the difference between the sum of assets and returns and $C(D_0)$. If $NI_1<0$ the bank incurs a partial (if $|NI_1|<E_0$) or total loss (if $|NI_1|\geq E_0$), where the bank fails in the latter case. If $NI_1>0$ the bank applies the retention rate[4] $0\leq b\leq 1$ and pays dividends $DIV_1=(1-b)NI_1$ retains earnings $RE_1=bNI_1$. Accordingly, the bank shareholder's period 2 equity to invest equals $E_1=E_0+RE_1$.[5]

At the beginning of the second period ($t=1$), the bank chooses deposits D_1 to invest together with E_1. Following Blum (1999), we ignore uncertainty in the second period and assume that the bank will receive the expected value of the random variable R, denoted $\overline{X}>R_f$.[6] At $t=2$ (end of period 2), the bank collects returns, pays deposit principal and interest, and determines its final net worth E_2. Accordingly, the cash flow following the final period 2 is DIV_2+E_2.

3.2. Asset risk and return

Risk free assets pay a positive per period interest of the amount r_f (in percent) so that the gross return (principal and interest) is $R_f=(1+r_f)$. If the bank invests all funds in the risk free portfolio, it receives $R_f(E_t+D_t)$ at the end of period $t+1$. Risky loans ("gambling assets") offer state dependent returns. Assuming two states, the risky asset's per period net gross return R has two mutually exclusive states:

$$R=\begin{cases} X & \text{with probability} \quad P(X), \\ L & \text{with probability} \quad 1-P(X). \end{cases} \tag{1}$$

The degree of risk taken is endogenous through the bank's choice of the gross return X and it is assumed that $X>R_f$ and $\overline{X}>R_f$. Then the probability of gambling success $P(X)$ has a maximum at $X=R_f$[7], which implies $P'(X)<0$ for higher returns. We further assume $P''(X)<0$ (see Fig. 1), which seems reasonable and is also consistent with equilibrium loan rationing at higher levels of risk (see e.g. Stiglitz and Weiss (1981)). In this setting the expected per period gross return of the risky asset is:

$$E(R)=P(X)X+(1-P(X))L \tag{2}$$

$1-P(X)$ is the probability of an unsuccessful gamble in which $L=0$, expressing the case of negative net income depleting the period's beginning equity. In this case, the bank is insolvent and defaults. This reduces the per period expected return of equation (2) to:

$$E(R)=P(X)X \tag{3}$$

When investing in risky assets, the bank chooses an optimal risk level X^* yielding an expected per period return of $E(R)=P(X^*)X^*$. X is continuous over the interval $R_f\leq X<X^{max}$ and $P(X)>0$. A rational bank's lowest return choice is the risk free return R_f. The incentive to choose a positive level of risk $X>R_f$ stems from a private return X exceeding the risk free return R_f so that $E'(R_f)>0$, which assumes $P'(R_f)>-1/R_f$. To prevent infinite risk taking, we assume that the

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[4]It is assumed that the bank takes the retention rate as given by, for example, following an industry or historical average, or in response to shareholders' preferences. Also, the retention rate applies to both periods and does not change. Blum's model (1999) assumes the special case of $b=1$ which we generalize by allowing $0\leq b\leq 1$.

[5]For $b=0$, $E_1=E_0$; and for $b=1$, $E_1=E_0+NI_1$.

[6]Blum (1999) points out that a true replication of the period 1 structure for period 2 renders the model intractable. Because the analysis focuses on the investigation of the bank's behavior at the beginning of period 1, the use of the reduced form of the risky asset in period 2 does not change the general results.

[7]For the risk free asset, $P(R_f)=1$.

expected return decreases above an optimal X^* (see Fig. 2).[8] Following the characteristics of the probability function, the expected return of equation (3) is strictly concave[9] with a maximum at X^*.

Given this framework, we next solve for the level of risk that would be chosen by a social planner (a first-best benchmark) and by an unregulated bank.

3.3. Socially optimal risk choice

Equation (3) provides the derivation of first best risk choices that serves as a comparative benchmark for different regulation scenarios. Ignoring any bankruptcy cost and over a single period, a risk neutral social planner (*SP*) would choose the bank's optimal risk level X^* to maximize the single period expected returns from equation (3), implying:

$$E'(R) = P'(X^*)X^* + P(X^*) = 0 \tag{4}$$

Over two periods, the *SP* optimization problem includes dividends, retained earnings and discounting. At the end of the first period, expected (gross) returns per dollar initially put in the risky asset (at $t=0$) are $E(R)_1=P(X)X$. Cash flow for the *SP* after period 1 is the dividend payment $(1-b)(E(R)_1-1)$ where $0\leq b\leq 1$. The remainder, consisting of the retained expected return as well as the initial investment amount $b(E(R)_1-1+1)$, is reinvested in period 2 and again earns $P(X)X$ during that period. The cash flow after period 2 at $t=2$ is $P(X)X\, b(E(R)_1-1+1)$.

The *SP* applies the discount factor $0<\theta\leq 1$ with $\theta=1/(1+r_{SP})$ ($r_{SP}>0$ represents the social discount rate) and chooses X to maximize the present value of expected future returns:

$$F_{SP} = \max_X \left\{ \theta(1-b)\left(P(X)X-1\right) + \theta^2 P(X)X\left(b\left(P(X)X-1\right)+1\right) \right\} \tag{5}$$

The *SP* optimal choice for X requires:

$$\frac{dSP}{dX} = \left(P'(X)X + P(X)\right)\theta\left((1-b)(1+\theta) + 2b\theta P(X)X\right) = 0 \tag{6}$$

The necessary condition requires a level of risk to be such that the additional returns be exactly balanced by the value of the reduction in the probability of success. Since $\theta((1-b)(1+\theta)+2b\theta P(X)X)>0$ equation (6) will be satisfied only if $P'(X)X+P(X)=0$. This is exactly the single period optimality condition of equation (4). It provides the relevant comparative benchmark, which is independent of the social discount rate θ and the bank's retention rate b.

Proposition 1. *The optimality condition of the social planner's risk choice is stationary and is independent of discounting and dividend parameters.*

This first best solution implies that a bank should choose some positive level of risk $X^*>R_f$ despite the positive probability of default. This idea is consistent with the normal role of banks as financial intermediaries. In the relevant range $R_f<X<X$, the risky asset offers a higher expected return than the risk free asset (see Fig. 2). The level X^M of risk provides the upper bound because $E(R_f)=E(X^M)$ given strict concavity of expected returns. Therefore, an unregulated bank chooses to invest all funds in the risky asset choosing from a risk level $R_f<X<X^M$.

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[8] We can think of each risky project as being funded by a mix of bank loans and owner's equity. Given positive costs of bankruptcy, this implies a unique interior optimal level of leverage.

[9] Because $P'(X)<0$ and $P''(X)<0$ we get $E''(R)=P''(X)X+2P'(X)<0$.

3.4. An unregulated bank (case UU)

For unregulated periods, the bank's deposit choice D_t refers to deposits for the risky asset investment. Furthermore it needs to be determined what risk level the bank is choosing. To begin the model setup, consider the case of a certain successful gamble. By investing all equity and deposits, the period 1 net income is:

$$NI_1 = X(E_0 + D_0) - C(D_0) - E_0 \tag{7}$$

Net income equals the gross return on all invested funds $X(E_0+D_0)$ less the principal and interest payments $C(D_0)$ and initial equity capital E_0. The bank divides the net income into dividends DIV_1 and retained earnings RE_1 by applying the retention rate $bDIV_1$. Therefore, the dividend, retained earnings RE_1, equity capital E_1, and cash flow CF_1 after period 1 (at $t=1$) are:

$$DIV_1 = (1-b)NI_1 = (1-b)[X(E_0 + D_0) - C(D_0) - E_0] \tag{8}$$

$$RE_1 = bNI_1 = b[X(E_0 + D_0) - C(D_0) - E_0] \tag{9}$$

$$E_1 = E_0 + RE_1 = E_0 + bNI_1 = E_0 + b[X(E_0 + D_0) - C(D_0) - E_0] \tag{10}$$

$$CF_1 = DIV_1 = (1-b)NI_1 = (1-b)[X(E_0 + D_0) - C(D_0) - E_0] \tag{11}$$

The bank receives a total of two cash flows over two periods: dividends after period 1 DIV_1 and the sum of dividends DIV_2 and net worth (equity value E_2) after period 2 ($\overline{X}$ is the second period return). To maximize the expected present value of these future cash flows, they are discounted at the required rate of return r_e (in percent) using the discount factor $\delta=1/(1+r_e)$:

$$F_{UU} = \max_{D_0, D_1, X} \{P(X)[\delta(1-b)\left(X(E_0 + D_0) - C(D_0) - E_0\right) + \delta^2(\overline{X}\left(E_1 + D_1\right) - C(D_1))]\} \tag{12}$$

The first order conditions for the unregulated bank are:

$$\partial F_{UU} / \partial D_0 = P(X)\delta(1 - b + b\delta\overline{X})(X - C'(D_0)) = 0 \tag{13}$$

$$\partial F_{UU} / \partial D_1 = P(X)\delta^2(\overline{X} - C'(D_1)) = 0 \tag{14}$$

$$\partial F_{UU} / \partial X = P'(X)\delta[(1-b)NI_1 + \delta(\overline{X}\left(E_1 + D_1\right) - C(D_1))] + P(X)\delta(E_0 + D_0)(1 - b + \delta\overline{X}b) = 0 \tag{15}$$

Necessary conditions (13) and (14) require the marginal cost of the last deposit unit in each period to equal its marginal benefit, or $X=C'(D_0)$ and $\overline{X}=C'(D_1)$. Fig. 3 displays the necessary conditions with marginal costs as a function of deposits and the bank's optimal choices $D_0=A$ and $D_1=B$.

The first term on the right side of equation (15) is the expected marginal cost of taking on more risk (as $P'(X)<0$). The second term is the expected marginal return of increasing risk, where returns are sum of the discounted gross contribution of total period 1 funds to period 1 dividends and the discounted gross contribution of total period 1 funds to period 2 net income through retained earnings. Equation (15) requires the optimal level of risk to balance expected marginal costs and returns:

$$P(X) + P'(X)X - P'(X)\left(\frac{(1 - b + b\delta\overline{X})(E_0 + C(D_0)) - E_0\delta\overline{X} - \delta(\overline{X}D_1 - C(D_1))}{(E_0 + D_0)(1 - b + \delta\overline{X}b)}\right) = 0 \tag{16}$$

Equation (16) differs from the first best case in equation (4) through the third term. The numerator in parentheses determines the sign of the term. The last term $\delta(\overline{X}D_1\text{-}C(D_1))$ represents discounted period 2 profits from deposits. If these are high, the bank has an incentive to avoid risk so as to increase the probability of success, perhaps lower than X^*. In these circumstances capital regulation would not be expected to improve the outcome. We therefore focus on the more interesting and empirically relevant case in which discounted period 2 profits are small enough that the bank has an incentive to take more risk:

$$(1-b+b\delta\overline{X})(E_0+C(D_0))-E_0\delta\overline{X}>\delta(\overline{X}D_1-C(D_1)) \quad (17)$$

Using the threshold (maximum) level of discounted period 2 profits from deposits $\delta(\overline{X}D_1\text{-}C(D_1))$ condition (17) restricts the third term of equation (16) to be negative so that $P'(X)X+P(X)<0$. As a result, the unregulated bank takes on too much risk ($X_{UU}>X^*$).

To clarify the interrelationship between retention rate b, required rate of return r_e and period 2 risky asset return $\overline{X}$, we first discuss the special case in which the a dividend payout rate of 100% ($b=0$). Then the period 2 deposit profit threshold in (17) is becomes $E_0(1\text{-}\delta\overline{X})+C(D_0)$. The absolute threshold value is smaller (larger) than $C(D_0)$ for $\overline{X}>(<)1+r_e$, making it less (more) likely for condition (17) to hold. This relationship between $\overline{X}$ and r_e can be referred to as the "patient (impatient) shareholder" condition, because if $\overline{X}>(<)1+r_e$, the shareholder's required rate of return is relatively low (high) which indicates that future cash flows are valued relatively high (low). Thus, when $b=0$, condition (17) is more likely to be true when the bank shareholder is more impatient. More generally, condition (17) is more likely to hold as b increases.

Equation (17) is based on the intertemporal inverse relationship between the profits from period 2 deposits and the level of risk over both periods. The higher (lower) the discounted period 2 profits on deposits $\delta(\overline{X}D_1\text{-}C(D_1))$, the lower (higher) the absolute value of the third term in equation (16) and the lower (higher) the unregulated bank's chosen risk level $X_{UU}>X^*$. An increase in risk increases the capital base after period 1 in case of a successful gamble, but it also increases the chance of bankruptcy. Taking on a lower level of risk involves the intertemporal tradeoff between lower expected equity value E_1 after period 1 and an increased chance not to fail in order to obtain period 2 profits.

Condition (17) can be numerically assessed using historical parameter values. To simulate an unregulated bank, let period 1 total liabilities plus equity be 15 units, composed of 1 unit of E_0 and 14 units of D_0. This translates into a leverage ratio of 6.7% which is representative of pre-Basel regulation leverage ratios and a lower leverage ratio than the Basel I requirement level of 8%. For simplicity, we ignore the effect of effect of retained earnings and assume the same capital amount and structure for the second period ($E_0=E_1=1$ and $D_0=D_1=14$). Based on FDIC per institution historical averages from 1966 to 2006, let the dividend payout rate be 60% which is equal to a retention rate of 40% (Federal Deposit Insurance Corporation 2008). The cost of funding is estimated to be 4% on the basis of the 6 month London Eurodollar deposit rates as reported in the Federal Reserve's H.15 statistical interest rate releases from December 27th, 2005 and August 4th, 2008 (The Federal Reserve Board 2008). Therefore, the per period gross costs of funding are equal to $C(D_0)=C(D_1)=14*1.04=14.56$. The same data source provides the risk free rate to be equal to 2% based on the 6 month Treasury yield. The expected risky asset return $\overline{X}$ estimate is based on the average net interest rate margin level reported by the FDIC (Federal Deposit Insurance Corporation 2005). We suggest a margin of 4% which results in an expected gross return of $\overline{X}=1.08$ on the basis of the funding costs mentioned above. The discount factor δ is assumed to be based on a required rate of return of 13% which results in a value of 0.89. Under these values, condition (17) is satisfied.[10]

Proposition 2. *Under the realistic assumption that the bank's period 2 deposit profits are not excessive, the unregulated bank chooses to invest in the risky asset only. The chosen risk level X_{UU} is suboptimal and exceeds the socially optimal level X^*.*

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[10]The resulting values for equation (17) are 14.3>0.5 for $b=0.4$, 14.6>0.5 for $b=0$, and 15.7>0.6 for $b=1$.

The analysis of the influence of the retention rate on the optimal risk choice yields:

$$\partial X_{UU} / \partial b = \frac{\delta^4 (1-b+b\delta\overline{X})P(X)^2 C''(D_1)C''(D_0)(1-\delta\overline{X})(P(X)(D_0+E_0)+P'(X)NI_1)}{|H_3|} \tag{18}$$

Where $|H_3|$ is the determinant of the Hessian matrix. The sign of this expression depends on the numerator, since $|H_3|$ must be negative for X_{UU} to be an optimal choice for the bank. The numerator depends critically on two relationships. The first relates to the "impatient shareholder" condition $\overline{X}<1/\delta$ (equivalently $\overline{X}<1+r_e$) which makes $1-\delta\overline{X}$ positive (and negative in case the "patient shareholder" condition holds) The second important relationship is between the marginal return from taking on risk $P(X)(E_0+D_0)$ and the marginal cost of that risk taking $P'(X)NI_1$. If the effect of the retention rate b on marginal returns exceeds that of marginal costs $P(X)(E_0+D_0)+P'(X)NI_1>0$ and vice versa.

The combination of these two relationships determines the effect of the retention rate. For an impatient shareholder, if the effect on marginal returns exceeds that on marginal costs then $\partial X/\partial b<0$ and an increase in the retention rate decreases the amount of risk the unregulated bank takes on. However, if the effect on marginal costs exceeds that on marginal returns, $\partial X/\partial b>0$ and increasing the retention rate would lead to an increase in risk taking by the unregulated bank. The implications flip if the shareholder is patient. If the effect on marginal returns exceeds (is less than) that on marginal costs then $\partial X/\partial b>0$ (<0) and increasing the retention rate would lead to an increase (decrease) in risk taking by the unregulated bank.

Proposition 3. *Considering a patient (impatient) bank shareholder, whose required gross return r_e+1 is less (greater) than the expected return from the risky asset in period 2 $\overline{X}$, an increase in the retention rate decreases (increases) risk taking by the bank.*

In sum, unregulated banks choose a risk level that is suboptimal, exceeding the level of the first best case. The assumption of a dominating return effect yields a positive (negative) relationship between the level of risk taking and the retention rate for the impatient (patient) shareholder. However, these findings depend on the firm and reliable commitment that the regulator will not introduce regulation in the future. As it is hard to conceive of a mechanism by which a regulator could credibly commit to never regulate in the future, the implications of this being relaxed and the consequences of temporally distinct implementation of regulation is the focus of what follows.

4. Bank Behavior under Regulation

In this section, we solve for the risk levels selected by banks subject to capital regulations. Two scenarios are considered. First we analyze an "old" bank for which regulation is implemented only in the second period, corresponding to the situation faced by banks before the introduction of the Basel I requirements in 1988.[11] Then we analyze a "new" bank facing capital

[11]The case of a bank regulated only in period 1 is empirically less relevant. That analysis is available from the authors and shows that the bank's chosen risk level is always socially nonoptimal ($X_{RU}>X^*$), exhibiting a negative relationship with the capital adequacy ratio. The second best solution is attained at the tightest regulation level.

regulation in both periods. We model the regulation as a fraction of risk-weighted assets, as in the Basel accord's "Cooke ratio."[12]

4.1. An old bank faces new regulation (case UR)

In this case the old bank correctly anticipates the introduction of a capital adequacy requirement in the second period. With a temporal separation in regulation, the bank's investment in the risky asset in period 2 is constrained while the period 1 investment remains unregulated. This case is of special interest for two reasons. (1) If it is the case that the introduction of capital requirements induces a bank to take a higher risk before regulation introduction, this result could help explain the high number of bank failures at the time of the Basel I introduction. (2) This analysis can provide guidance in forecasting risk taking behavior of banks in those countries that consider accepting the Basel rules in the future.

In our stylized model, the minimum Cooke ratio C_t equals equity capital as a percentage of total risky loan assets.[13] Therefore, a bank can acquire a maximum of risky loan assets of $E_t/C_t=M_tE_t$ in a regulated period t, where equity multiplier M_t ranges from 1 (for C_t=1) to infinity (for a very small value of C_t). The capital ratio's potential range is $0<C_t\leq 1$, with lower (higher) values representing looser (tighter) regulation. The tightest regulation C_t=1 effectively forbids any leverage in funding loans.

We analyze the relevant case in which the regulation is binding. As before, the bank's deposit choice D_0 refers to deposits for the risky asset investment. By regulation, the maximum amount invested in the risky asset in period 2 (at a gross return of $\overline{X}$) is limited to M_1E_1. To fund these risky assets, a matching deposit amount of D_1^g ("period 2 gambling deposits") is acquired at $t=1$ as governed by $E_1+D_1^g=M_1E_1$. Because investments in the risky asset are capped at M_1E_1, the bank may wish to make additional investments and so needs to hire additional funds. Therefore, the bank's regulated period deposit choice D_1 ("period 2 risk free deposits") refers to deposits invested in the risk free asset, as risky deposits are dictated by the regulatory constraint. Therefore, total deposits and associated costs are:

$$D_1^t = D_1 + D_1^g = D_1 + (M_1 - 1)E_1 = D_1 + (M_1 - 1)(E_0 + b(X(E_0 + D_0) - C(D_0) - E_0)) \qquad (19)$$

$$C(D_1^t) = C(D_1^g + D_1) = C(E_1(M_1 - 1) + D_1) \qquad (20)$$

At $t=1$ NI_1, DIV_1, RE_1, E_1, and CF_1 are identical to those of case UU. The period 2 net income at $t=2$, NI_2, equals the gross return on all invested funds which include earnings from the risky asset $\overline{X}M_1E_1$ and the risk free asset R_fD_1 less the sum of the principal and interest payments for all deposits $C(D_1^t)$ and the initial equity capital E_1. The bank's problem to maximize its expected cash flows over the two periods becomes, using the discount factor δ, becomes:

$$\begin{aligned} F_{UR} = \max_{D_0, D_1, X} \{ & P(X)[\delta(1-b)(X(E_0 + D_0) - C(D_0) - E_0) \\ & + \delta^2 (M_1 \overline{X}\left(E_0 + b(X(E_0 + D_0) - C(D_0) - E_0\right) + D_1 R_f - C(D_1^t))]\} \end{aligned} \qquad (21)$$

subject to $D_1 \geq 0$ and $E_1+D_1^g \leq M_1E_1$.

The first order conditions for the in period 2 only regulated bank are:

$$\partial F_{UR} / \partial D_0 = P(X)\delta(X - C'(D_0))(1 - b + \delta b(M_1 \overline{X} - \left(M_1 - 1\right) C'(D_1^t))) = 0 \qquad (22)$$

[12] The acceptance of the Basel framework in 1988 and the passage of the FDIC Improvement Act in 1991 have effectively led to a system of parallel leverage requirements in the U.S. We analyze only the former requirement.

[13] Because our model assumes homogeneous loans, risk weighted assets equal the total amount of risky assets. This corresponds to weights of 100% and 0% for risky loans and risk free assets respectively.

$$\partial F_{UR} / \partial D_1 = P(X)\delta^2(R_f - C'(D_1^t)) \le 0; D_1(P(X)\delta^2(R_f - C'(D_1^t))) = 0; D_1 \ge 0 \tag{23}$$

$$\begin{aligned}\partial F_{UR} / \partial X = {} & P'(X)\delta((1-b)NI_1 + \delta\overline{X}M_1E_1 + D_1R_f - C(D_1^t)) \\ & + P(X)\delta(E_0 + D_0)(1 - b + b\delta(M_1\overline{X} - (M_1 - 1)C'(D_1^t))) = 0\end{aligned} \tag{24}$$

As in case *UU*, the bank's deposit choice for the unregulated period refers to deposits (here D_0) for the risky asset investment. The necessary condition (22) requires the marginal cost of the last unit of deposits in period 1 to equal its marginal benefit X (point *A* in Fig. 3).

The Kuhn-Tucker optimality condition (23) governs the bank's regulated period deposit choice D_1 choice for risk free deposits (as risky deposits D_1^g are dictated by the regulatory constraint). To illustrate, consider Fig. 3 and a threshold $D_1^g=B$ where regulation becomes just binding. At that point, $C'(D_1^t)=\overline{X}$ so that $C'(D_1^t)>R_f$ and the bank chooses not to hire additional period 2 risk free deposits ($D_1=0$). A tightening of regulation is shown by point $D_1^g=C$. Any further tightening of regulation results in risk free deposits being hired until the last unit of total deposits collected $C'(D_1^t)$ equals the risk free rate R_f.

Similar to equation (15), the optimality condition for the risk level X in equation (24) has two components on the right side. In the marginal cost term (first term), an incremental increase in risk lowers $P(X)$ and reduces the expected present value of both periods' cash flows. Because the investment in the risky asset is capped, the benefit of an increment of risk taking only applies up to the maximum amount of M_1E_1. If the bank invests in risk free assets, additional risk only boosts the funds used for risky assets so that risk taking results in lost leverage on profits, due to a lower profit margin per dollar of total funds invested compared to the unregulated case. Hence, the marginal cost of risk taking is reduced, with a stronger effect for more patient bank shareholder. We refer to this as the "*profit effect*."

The marginal return of increasing risk (second term in (24)) depends on the amount of total funds invested in the risky asset at $t=0$. There are two components to this return effect, the first through period 1 dividends, and the second through the effect of period 1 retained earnings on period 2 net income. A successful gamble results in a higher private return earned from the risky investment in period 1 resulting in higher dividends and retained earnings at $t=1$. However, the equity multiplier caps the period 2 risky asset investment and therefore restricts the return effect of period 1 retained earnings on period 2 profits in comparison to the unregulated case. This "*multiplier effect*" explains a decrease in the marginal return of risk taking, because tighter regulation (lower equity multiplier) proportionally decreases gains from risk taking.

In addition, the increased marginal value of equity under regulation ($M_1\overline{X}-(M_1-1)C'(D_1^t)>\overline{X}$) provides an incentive for the bank to increase its risk taking at the beginning of period 2. This "*leverage effect*" increases the marginal return of risk taking as an opposing force to the multiplier effect and expresses the additional value of equity in the future period to the regulated bank.

Given the countervailing influences of the profit, multiplier and leverage effects the overall effect of the equity multiplier M_1 on the bank risk's choice is ambiguous. The comparative static effect of X with respect to a binding M_1 is:

$$\partial X / \partial M_1 = \frac{A \times B(G+I)}{|H_3|} \tag{25}$$

where

$$A = -\delta^5 P(X)^2 C''(D_0)C''(D_1^t) \tag{26}$$

$$B = C''(D_0)(1 - b + b\delta(M_1\overline{X} - (M_1 - 1)C'(D_1^t))) \tag{27}$$

$$G = P(X)(\overline{X} - C'(D_1^t))b(D_0 + E_0) \quad (28)$$

$$I = P'(X)(\overline{X} - R_f)E_1 \quad (29)$$

The sign of equation (25) depends on the numerator, since the denominator must be negative for an optimal choice. Within the numerator, $A \times B$ and I are negative. G is positive for $\overline{X}>C'(D_1^t)$ and zero for $\overline{X}=C'(D_1^t)$. Consider period 2 regulation to be just binding at an upper bound $\overline{M}_1$ which equals a lower bound $\underline{C}_1$. At that point, the amounts of total period 2 deposits and period 2 gambling deposits are equal ($D_1^t=D_1^g$), which is shown as point B in Fig. 3; and $\overline{X}=C'(D_1^t)$, which results in $G=0$. In this case, the numerator is positive and $\partial X/\partial M_1<0$. This finding (equivalently $\partial X/\partial C_1>0$) demonstrates that tighter regulation induces higher risk. When period 2 regulation becomes binding, profit and leverage effects (due to decreased marginal cost of risk taking and increased marginal value of equity in period 2, respectively) dominate the multiplier effect of reduced marginal returns. As regulation tightens, it forces the bank to invest less in the risky asset, which translates into less hiring of period 2 gambling deposits. This is represented by moving from point B in Fig. 3 to the left towards point C. The tightening of the Cooke ratio makes G increase because $\overline{X}-C'(D_1^t)$ increases.

Whenever G+I<0 and regulation is tightened from the upper bound $\overline{M}_1$ (lower bound $\underline{C}_1$) by decreasing (increasing) the equity multiplier (Cooke ratio), risk taking is increased. Increasing regulation makes the bank increase risk as regulation tightens because of a prevailing leverage effect. This effect is strongest at the point where regulation just becomes binding, because at that point, the marginal value of period 2 equity is highest. Also, the risk increase effect is less pronounced when the retention rate b is higher, because of a less dominant leverage effect when retained earnings and future equity are larger, or "less scarce".

Eventually $G+I=0$ and a maximum risk level is obtained for the Cooke ratio C_1^{max}, where this point is attained faster for higher a retention rate b. Further increases in C_1 make the negative multiplier effect dominate both the positive leverage affect on the marginal return and the negative profit effect on marginal cost, so that $G+I>0$. Therefore, high levels of regulation decrease the marginal return (which is proportional to the equity multiplier) faster than the marginal cost of risk taking decreases, causing risk to decrease as regulation tightens. At the tightest regulatory level M_1=1 the marginal value of equity has decreased to the level of the unregulated bank $\overline{X}$ and the leverage effect has completely vanished. The result is that the bank decreases risk for $C_1^{max}<C_1\leq 1$.

This is an important result because it (1) states that capital regulation may produce the opposite behavior of its intended purpose for a certain range of regulation level $\underline{C}_1\leq C1<C_1^{max}$, (2) may provide support to explain the increased number of bankruptcies following the introduction of Basel 1 capital adequacy rules, and (3) may provide guidance for countries that are discussing to accept the Basel framework in the future. It is worth noting that the above described relationship between risk taking and regulation level even holds if the bank is only expecting that regulation is introduced in period 2.

Proposition 4. *The "old" bank regulated only in period 2 faces an incentive to take additional risk in both periods due to the initially dominating leverage effect on marginal returns. Eventually, tightening regulation decreases risk based on the profit effect on marginal costs.*

As discussed above, the tightening of the equity multiplier towards $M_1=1$ eventually leads to a reduction of risk. However, even with the most stringent regulation of $M_1=1$, the bank will choose to exceed the optimal risk level of the unregulated bank. This result follows from comparing the unregulated equation (16) with a rearrangement of equation (24):

$$\partial F_{UR}/\partial X = P(X) + P'(X)X - P'(X)\left(\frac{(1-b+\delta b\overline{X})(E_0 + C(D_0)) - E_0\delta\overline{X} - \delta(R_f D_1 - C(D_1^t))}{(E_0 + D_0)(1-b+\delta\overline{X}b)}\right) \tag{30}$$

Equations (16) and (30) are identical apart from period 2 deposit profits. With a positive denominator the sign of (30) depends on the numerator. Under the period 2 deposit profit condition (17) we found the unregulated level of risk to exceed the socially optimal outcome. With regulation, if $M_1=1$ the bank cannot lever its equity for the risky investment. Therefore, risky deposits $D_1^g=0$ are dictated by the regulatory constraint. The amount of the bank's regulated period deposit choice D_1 for risk free deposits follows from equation (23) and is represented by point C in Fig. 3, at which $R_f=C'(D_1^t)$. Since $D_1^t=D_1$, period 2 deposit profits are $R_fD_1-C(D_1)$. Comparing these period 2 deposit profits to those of the unregulated bank, which are $\overline{X}D_1-C(D_1)$ as shown in equation (16) demonstrates that regulated profits from deposits in period 2 are smaller because $\overline{X}>R_f$, making the parenthesized term of equation (30) larger than that of equation (16). Therefore, the bank's chosen risk level for $M_1=1$ in the case UR is higher than in the unregulated case UU, or $X_{UR}>X_{UU}$. This means that even the tightest period 2 regulation cannot reduce the bank's risk choice at $t=0$ risk as low as that of an unregulated bank X_{UU}, which in turn is higher than the first best outcome X^*.

This startling finding demonstrates that the Cooke ratio is inherently the wrong regulatory tool to achieve the first best outcome when imposed in period 2.[14] Moreover, it cannot reduce risk below the unregulated level.

Proposition 5. *The prohibition of leverage in period 2 results in a risk level choice at the beginning of period 1 of X_{UR} that exceeds the risk level X_{UU} of the unregulated case.*

In sum, for capital requirements $M_1\geq\overline{M}_1$ ($C_1\leq\underline{C}_1$) that are not binding or just binding, banks choose the same risk in cases UU and UR ($X_{UR}=X_{UU}$). If $M_1<\overline{M}_1$ ($C_1>\underline{C}_1$), the bank initially increases risk and begins to hire risk free deposits for a level of M resulting in total period 2 gambling deposits of $D_1^g<B$. Risk eventually decreases to a level that still exceeds the risk level of the unregulated bank ($X_{UR}>X_{UU}$) at the tightest regulation $M_1=1$. Therefore, neither the first nor the second best solution can be attained.

Given the above analysis, the question for the optimal level of regulation can be answered. The lowest attainable risk level is the risk level X_{UU} chosen by the unregulated bank. Any period 2 regulation starting at the point where it becomes binding $\underline{C}_1<C_1<1$ results in a risk level $X_{UR}>X_{UU}$. If the regulator targets the lowest attainable risk level, the suggested action is not regulating at all. This important result states that the regulator's attempt to make banking safer in period 2 yields an increased level of risk in both periods compared to the unregulated case. Because this result holds even if the bank is simply expecting the introduction of regulation in the second period, the outcome suggests that the regulator should not announce future capital regulation to the bank. Instead, an unexpected introduction at the beginning of period 2 can decrease risk. However, given the public discussion about national and international regulation,

[14]This means that, while the Cooke Ratio might be able to attain the desired policy goals for banks that were chartered after its implementation (a separate question addressed in the *RR* section below), it cannot bring pre-existing banks into conformity with the socially optimal level of risk.

a surprise introduction of capital adequacy rules does not appear to be realistic, and would also entail substantial transitional costs of compliance. Moreover, an unexpected regulation would require both a prior credible commitment by the regulator not to regulate (which is unlikely, as noted above) and then a violation of that commitment. Therefore, this case realistically portrays the risk choice of an unregulated bank that expects regulation in period 2. Fig. 4 summarizes the findings for this case.

Proposition 6. *Even if the regulator targets the lowest attainable risk level for an "old" bank regulated only in period 2, the regulation does not reduce risk at all.*

4.2. Regulation in both periods (case RR)

This scenario describes the other common situation of a new bank that faces a binding capital constraint of $C_{RR}=1/M_{RR}$ in both periods. In each period, the bank can acquire risky assets $E_t/C_{RR}=M_{RR}E_t$ funded with equity E_t and gambling deposits $D_t^g=(M_{RR}-1)E_t$ as dictated by the regulatory constraint. The bank's regulated per period deposit choice $D_t\geq 0$ refers to deposits invested in the risk free asset

Given regulation in both periods net income, dividends, retained earnings, equity, and cash flows in both periods use the two-period equity multiplier M_{RR}. Total deposits in each period are:

$$D_0^t = D_0^g + D_0 = (M_{RR}-1)E_0 + D_0 \tag{31}$$

$$D_1^t = D_1 + D_1^g = D_1 + (M_{RR}-1)(E_0 + b(XM_{RR}E_0 + R_f D_0 - C(D_0^t) - E_0)) \tag{32}$$

Using the discount factor δ, the bank solves the following maximization problem:

$$\begin{aligned} F_{RR} = \max_{D_0, D_1, X} \{ & P(X)[\delta(1-b)(XM_{RR}E_0 + R_f D_0 - C(D_0^t) - E_0) \\ & + \delta^2 (M_{RR}\overline{X}(E_0 + b(XM_{RR}E_0 + R_f D_0 - C(D_0^t) - E_0)) + D_1 R_f - C(D_1^t))]\} \end{aligned} \tag{33}$$

subject to $D_t\geq 0$ and $E_t+D_t^g\leq M_{RR}E_t$. The first order conditions are:

$$\begin{aligned} &\partial F_{RR}/\partial D_0 = P(X)\delta(R_f - C'(D_0^t))(1-b+\delta b(M_{RR}\overline{X} - (M_{RR}-1)C'(D_1^t))) \leq 0; \\ &D_0[P(X)\delta(R_f - C'(D_0^t))(1-b+\delta b(M_{RR}\overline{X} - (M_{RR}-1)C'(D_1^t)))] = 0; D_0 \geq 0 \end{aligned} \tag{34}$$

$$\partial F_{RR}/\partial D_1 = P(X)\delta^2(R_f - C'(D_1^t)) \leq 0; D_1(P(X)\delta^2(R_f - C'(D_1^t))) = 0; D_1 \geq 0 \tag{35}$$

$$\begin{aligned} &\partial F_{RR}/\partial X = P'(X)\delta((1-b)NI_1 + \delta M_{RR}\overline{X}E_1 + D_1 R_f - C(D_1^t)) \\ &+ P(X)\delta E_0 M_{RR}(1-b+b\delta(M_{RR}\overline{X} - (M_{RR}-1)C'(D_1^t))) = 0 \end{aligned} \tag{36}$$

The bank's choice to hire risk free deposits D_t in both, either, or no periods is expressed in equations (34) and (35) and illustrated in Fig. 3. The point at which regulation level M_{RR} binds in period 1 and becomes binding in period 2 is characterized by $D_t^g=B$ where the marginal cost of the last unit of deposits hired equals the expected risky asset return $C'(D_t^t)=\overline{X}$. At this point, $D_t=0$ and $R_f<C'(D_t^t)$ (from conditions (34) and (35)). If regulation tightens so that $(M_{RR}-1)E_t<C$, the bank chooses risk free deposits $D_t>0$ until $C'(D_t^t)=R_f$.

As in the previous cases, the risk level optimality condition (36) consists of two terms on the right side. $P'(X)$ can again be interpreted as the marginal cost of risk taking that explains the bank's desire to take additional risk based on the profit effect. Again, the second term represents the marginal return of risk taking that is subject to risk reducing multiplier and risk increasing leverage effect as regulation tightens (see the discussion for equation (24)).

The analysis of the overall effect of profit, multiplier, and leverage effects on the bank's chosen risk level depends on the parameters, as well as on the cost and the probability function. Given the previous outcomes, the three expected results for a bank that faces a capital constraint

C_{RR} applicable to each period are (see Fig. 5): (1) A not binding Cooke ratio C_{RR} in period 1 results in a bank risk level X_{RR}=X_{UU}. (2) For capital requirements binding only in period 1, the bank initially decreases risk (X_{RR}<X_{UU}) because of the dominating multiplier effect on marginal returns.[15] (3) As regulation tightens so that it also becomes binding in the second period, the bank behaves as described for case *UR:* It initially increases risk (driven by the leverage and profit effect), and eventually decreases risk again (due to the increasing multiplier effect).

The sensitivity analysis reflects this relationship between regulation level M_{RR} and bank risk level X:

$$\partial X / \partial M_{RR} = \frac{S}{|H_3|}(T(U+V)+W) \tag{37}$$

where

$$S = -\delta^4 P(X)^2 C''(D_1^t) \tag{38}$$

$$T = 1-b+b\delta(M_{RR}\overline{X} - C'(D_1^t)(M_{RR}-1)) \tag{39}$$

$$U = P(X)C''(D_0^t)E_0(T + b\delta M_{RR}(\overline{X} - C'(D_1^t))) \tag{40}$$

$$V = P'(X)C''(D_0^t)(\delta(\overline{X} - R_f)(E_1 + E_0 bM_{RR}(X - R_f)) + E_0(X - R_f)(1-b+b\delta R_f)) \tag{41}$$

$$W = P'(X)b\delta(1-b+b\delta(M_{RR}\overline{X} - R_f(M_{RR}-1)))(R_f - C'(D_0^t))^2(\overline{X} - C'(D_1^t)) \tag{42}$$

Because of S and $|H_3|$ being negative the sign of expression (37) depends on $T(U+V)+W$. For a regulatory equal to the period 2 binding level $\overline{C}_{RR}$ the bank optimizes using $\overline{X}=C'(D_t^t)$ yielding $W=0$. Assuming a combination of parameters, cost, and probability functions so that $U+V>0$, we get $\partial X/\partial M_{RR}>0$ describing an inverse relationship between risk and regulation level (see footnotes 11 and 15).

Further tightening of regulation results in $\overline{X}>C(D_t^t)$ and W being negative, outweighing the positive net effect of $U+V$. Therefore the bank increases risk due to the dominant leverage effect as described for case *UR*. This effect is stronger the higher the retention rate and the lower the bank shareholder's required rate of return. For a given parameter set, the effect is the strongest at $\overline{C}_{RR}$ and gets weaker as regulation increases further because of the decreasing difference between R_f-C'(D_0^t) describing the decreasing leverage effect. Also, if the cost function is more convex, the desire to take additional risk is stronger. This can be explained by a more pronounced profit effect, and by the fact that a more convex cost function makes it harder to build future equity, therefore increasing the marginal value of equity (leverage effect).

$\partial X/\partial M_{RR}$ remains negative until regulation tightens to a level so that $(M_{RR}-1)E_t$ is equal to point C in Fig. 3 at which $R_f = C'(D_0^t)$ so that W=0. Because of $\overline{X}-C(D_1^t)>0$ which increases in value as regulation tightens, U increases so that $U+V$>0. Because of the dominating multiplier effect risk decreases up to the tightest point of regulation ($C_{RR}=1$) as shown in Fig. 3.

This effect of a risk decrease can be proven by analyzing the first order condition at the tightest level of regulation $M_{RR}=1$. Rearranging the first order condition (36) gives:

$$\begin{aligned}\partial F_{RR} / \partial X &= P(X) + P'(X)X \\ &+ P'(X)\left(\frac{(1-b+\delta b\overline{X})(R_f D_0 - C(D_0)) + \delta(R_f D_1 - C(D_1)) - E_0(1-b)(1-\delta\overline{X})}{E_0(1-b+\delta b\overline{X})}\right)\end{aligned} \tag{43}$$

[15]For case *RU* it can be shown that the multiplier effect always dominates the profit effect so that tighter regulation always decreases risk (see footnote 11).

The first two components in the numerator of the term in parentheses and the denominator are positive. If the last term $E_0(1\text{-}b)(1\text{-}\delta\overline{X})$ is zero, the term is parentheses term of equation (43) is positive. This means that $P(X)+P'(X)$ in equation (43) is positive which expresses that the bank's chosen risk level for $M_1=1$ is below the first best level. Such a low risk level can be achieved when (1) no dividends are paid $(b=1)$, or (2) the bank shareholder is patient or indifferent $(\overline{X}\geq 1/\delta)$ in which case $E_0(1\text{-}b)(1\text{-}\delta\overline{X})<0$, or (3) the bank shareholder is impatient ($\overline{X}<1/\delta$) in combination with a very low retention rate b. In all these cases the bank's chosen risk level is less than the first best outcome because the following condition holds:

$$(1-b+\delta b\overline{X})(R_f D_0 - C(D_0)) + \delta(R_f D_1 - C(D_1)) > E_0(1-b)(1-\delta\overline{X}) \tag{44}$$

This is a significant result as it expresses that the bank regulated in both periods can be influenced to decrease risk to the level of the first best outcome and even below. An important observation is that expression (44) is more likely to be true for smaller banks as measured by E_0.

Proposition 7. *Binding period 1 regulation causes bank risk to decrease due to the dominant multiplier effect on marginal returns. From the point of binding period 2 regulation the bank initially increases risk due to the prevailing leverage effect, and eventually decreases risk. Under some conditions the regulator can reach the first best risk level.*

To derive the socially optimal regulation, the general form ($M_{RR}>1$) of the equation (43) is used:

$$\begin{aligned}\partial F_{UR}/\partial X = {} & P(X) + P'(X)X \\ & + \left[\frac{P'(X)((1-b+\delta b M_{RR}\overline{X})(R_f D_0 - C(D_0^t)) + \delta(R_f D_1 - C(D_1^t)) - E_0(1-b)(1-\delta\overline{X}M_{RR}))}{M_{RR}E_0(1-b+\delta b M_{RR}\overline{X})}\right. \\ & \left. - \frac{P(X)b\delta M_{RR}E_0 C'(D_1^t)(M_1-1)}{M_{RR}E_0(1-b+\delta b M_{RR}\overline{X})}\right]\end{aligned} \tag{45}$$

Setting the last term equal to zero and solving for M_{RR}^* gives:

$$M_{RR}^* = (Y \pm \sqrt{Y^2 - 4QZ})/2Q \tag{46}$$

where

$$Y = P(X)b\delta E_0 C'(D_1^t) \tag{47}$$

$$Q = P'(X)(\delta b\overline{X}(R_f D_0 - C(D_0^t)) + E_0\delta\overline{X}(1-b)) + Y \tag{48}$$

$$Z = P'(X)((1-b)(R_f D_0 - C(D_0^t) - E_0) + \delta(R_f D_1 - C(D_1^t))) \tag{49}$$

The corresponding Cooke ratio is equal to:

$$C_{RR}^* = 2Q/(Y \pm \sqrt{Y^2 - 4QZ}) \tag{50}$$

An important implication of the optimal Cooke ratio C_{RR}^* is that it is a function of the initial shareholder value E_0. This is remarkable since the current Basel framework of risk based capital regulation does not discriminate regarding the size of banks. In addition, the optimal Cooke ration is also a function of the discount factor δ, retention rate b, risk free period 1 deposits D_0, total period 1 cost of debt $C(D_0^t)$, total period 2 cost of debt $C(D_1^t)$, marginal cost of period 2 debt $C'(D_1^t)$, the risk free gross return R_f, and period 2 return rate $\overline{X}$.

Equation (50) has two solutions in general. For an optimal policy to exist, the discriminant Y^2-$4QZ$ must be nonnegative.[16] Substituting equations (47) to (49) into $Y^2-4QZ\geq 0$, and focusing on the simplified case of no dividends ($b=1$) and $\delta=1$, we obtain the following requirement for a real value of the optimal Cooke ratio:

$$(P(X)E_0C'(D_1^t))^2 \geq 4P'(X)(R_fD_1 - C(D_1^t))(P'(X)(\overline{X}(R_fD_0 - C(D_0^t))) + P(X)E_0C'(D_1^t)) \tag{51}$$

Consider the case of a optimal regulation that requires a somewhat high regulatory level as portrayed in Fig. 3. In such case the per period gambling deposits D_t^g are to the left of point C. For some sufficiently small $D_t^g<C$ it is the case that the per period gross return from risk free deposits $R_fD_t=R_f(D_t^t-D_t^g)$ exceeds the total per period deposit costs $C(D_t^t)$ (area under the cost curve up to point C). This is true for $M_{RR}=1$ and holds for $M_{RR}<1$ until $R_fD_t=C(D_t^t)$. Consequently, for such high regulatory levels it is the case that $R_fD_t>C(D_t^t)$, where a more convex cost function is resulting in a wider the range of regulatory levels guaranteeing this condition. Since $(P(X)E_0C'(D_1^t))^2$ is positive condition (51) holds for $R_fD_t=C(D_t^t)$, as well as for $R_fD_t>C(D_t^t)$ if we assume a combination of parameters, cost, and probability functions so that $P(X)E_0C'(D_1^t)>|P'(X)(\overline{X}(R_fD_0-C(D_0^t)))|$. However, because it has to be the case that $M_{RR}\geq 1$, the additional requirement is that $(-Y\pm\sqrt{(Y^2-4QZ)})/2Q)\geq 1$ or $\pm\sqrt{(Y^2-4QZ)}\geq Y+2Q$.

For the rare case where $Y^2-4QZ=0$ for $0<b\leq 1$, we know that Y is negative. The previously used condition $R_fD_t>C(D_t^t)$ for high regulatory levels assures that Q is negative which leads to $M_{RR}^*=Y/2Q<0$. Therefore, this case is never policy relevant (given the simplifying assumptions made above).

Another simplified case occurs for no retained earnings ($b=0$) and $\delta=1$, for which $Y=0$, $Q=P'(X)E_0\overline{X}$, and $Z=P'(X)(R_fD_0+R_fD_1-C(D_0^t)-C(D_1^t)-E_0)$. Q is negative and Z depends on the level of regulation determining the sign of $(R_fD_0+R_fD_1-C(D_0^t)-C(D_1^t)-E_0)$. For lower levels of regulation and/or large banks (high E_0) this expression can be negative resulting in a positive Z. Given this scenario the discriminant is nonnegative, similarly offering two solutions.

The result of two suggested solutions is not surprising as it was discussed earlier that risk increases starting at $\overline{C}_{RR}$ before decreasing again towards $C_{RR}=1$. With reference to Fig. 5, only the lower value of M_{RR}^* (representing a higher C_{RR}^*) of the two solutions qualifies as the optimal regulatory rule, because the higher M_{RR}^* value is not policy relevant since the bank chooses a risk level above the first best outcome for a regulatory level $\underline{C}_{RR}\leq C_{RR}\leq\overline{C}_{RR}$ (see Fig. 5). Provided the conditions discussed above, the suggested action for the regulator is to regulate using the socially optimal equity multiplier is:

$$C_{RR}^* = 2Q/(Y - \sqrt{Y^2 - 4QZ}) \tag{52}$$

Proposition 8. *For the new bank regulated in both periods the regulator can regulate using the optimal Cooke ratio C_{RR}^* given by equation (52) to attain the first best outcome.*

In sum, the important outcome of this case *RR* is that, given certain previously described conditions and in contrast to the case of an old bank *UR*, the regulator can indeed decrease risk to or even below the level of the first best outcome.

[16] The special case $Y^2-4QZ=0$ yields one distinct root $M_{RR}^*=-Y/2Q$.

5. Conclusions and Extensions

This paper has shown that regulatory capital requirements can have different effects on banks' risk taking behavior depending on whether a bank was chartered before or after the introduction of the regulation. In the former case, the optimal regulatory standard ("Cooke ratio") was shown to be a function of bank size and other factors, and can achieve the first-best outcome under particular conditions. In the latter case, capital regulation cannot ever decrease risk under the conditions of the analysis, unless banks fail to anticipate the introduction of the regulation. The dependence of optimal regulation on bank size and on the timing of the regulation has not been recognized in previous studies.

Our intertemporal model identifies three effects that either increase or decrease risk taking as a function of regulatory stringency. As regulation tightens, the multiplier effect on the marginal return of risk taking decreases banks' risk taking because the gain from gambling is decreased proportionally with the equity multiplier. In contrast, banks will tend to choose higher risk due to the profit effect, which decreases the marginal cost of risk taking (the bank has less to lose); and the leverage effect, which increases the marginal return of risk taking due to an increased marginal value of equity.

Our framework also indicated that an unregulated bank will choose a level of risk exceeding the socially optimal level, providing a motive for regulatory intervention. The optimization provides another important observation: A shareholder who is patient and values future cash flows highly will choose lower risk as the bank's retention rate increases. Therefore, regulation is less urgent when banks pay low dividends, because the patient shareholder decreases risk to obtain higher future cash flows. This effect of dividend policy has not been previously recognized.

Our model provides multiple possibilities for further extension. First, for convenience and tractability, we abstracted from uncertainty in period 2 by considering the reduced form of the gambling asset in that period. This assumption can be relaxed and may require the derivation of numerical solutions.

Because this model has provided a comprehensive analysis of the Cooke ratio, future work can derive and compare parallel results for a simple leverage ratio requirement $E_0/(E_0+D_0)$. This analysis may lead to a potentially simpler expression and may answer which ratio provides broader parameter value regions of feasibility or optimality.

The bank's risk optimality condition suggests the possibility of alternative regulatory tools. For example, a rate of return regulation may be able to achieve a socially optimal level of bank risk. Such a tool can also be configured as a subordinated debt requirement – thus lending further support to some previous proposals – or perhaps as a regulatory tax. Alternatively, it might be possible to regulate banks' retention rate as a way of favorably influencing banks' choice of risk.

The analysis of banks subject to ongoing regulation (case *RR*) identified specific conditions (patient shareholder, low retention rate, and low degree of convexity of the cost function) under which the first best outcome is attainable. Future work can empirically evaluate whether and when these conditions hold and can provide numerical solutions on the basis of historical parameters. Moreover, the sensitivity of optimal capitalization to bank size can usefully be explored in more detail, based on a variety of historically grounded parameter values. A more thorough analysis of this feature would also take into account the nonlinearity of the key relationship, to propose improved forms of regulatory capital requirements.

Since our model predicts that "older" banks would have chosen a higher level of risk than "newer" banks, this hypothesis can be tested empirically in the future. Related to this idea, the analysis suggests that mergers between an older bank and a newer bank might be expected to alter the risk profile of the target bank to mimic that of the acquiring bank, a possibility that has important policy implications.

Future work may also seek to address the question of parallel regulation of national and international binding constraints. Also, incorporating taxes into the existing model may provide improved policy advice for international regulation frameworks (such as Basel II) that are applicable in different national and international tax environments. Additionally, the important question of the effects of amendments of an existing regulatory framework on "old" and "new" banks could be analyzed and empirically tested.

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Figures

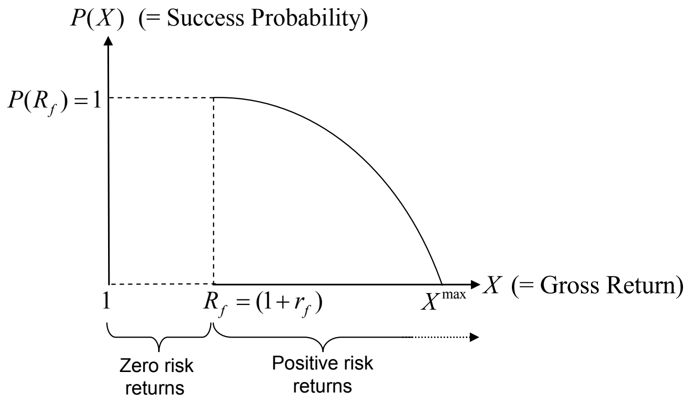

Fig. 1. Per period probability of success.

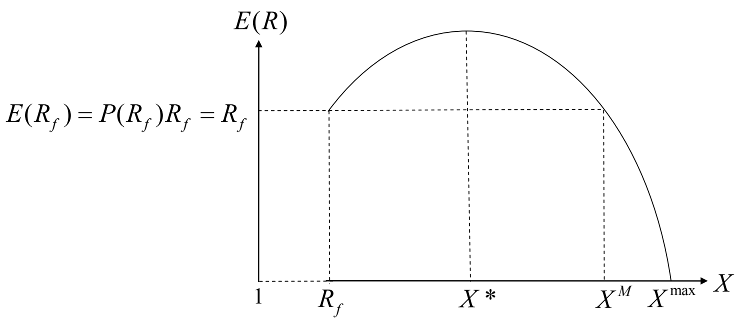

Fig. 2. Expected per period return of the risky asset.

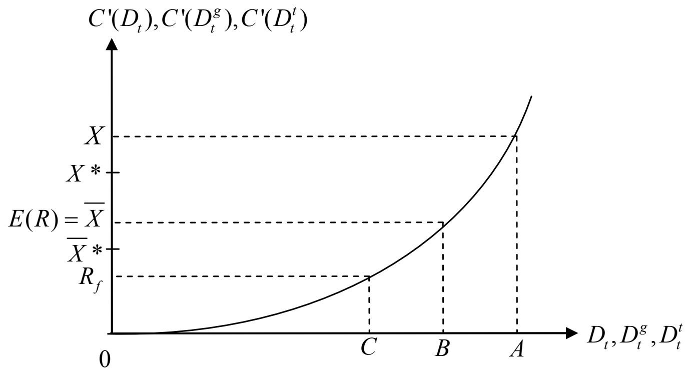

Fig. 3. Marginal cost of deposits as a function of deposits.

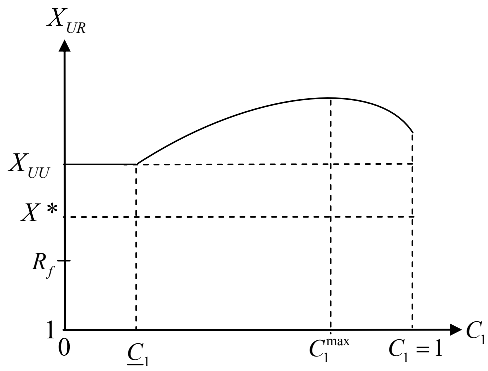

Fig. 4. Case *UR*: Risk as a function of the period 2 Cooke ratio.

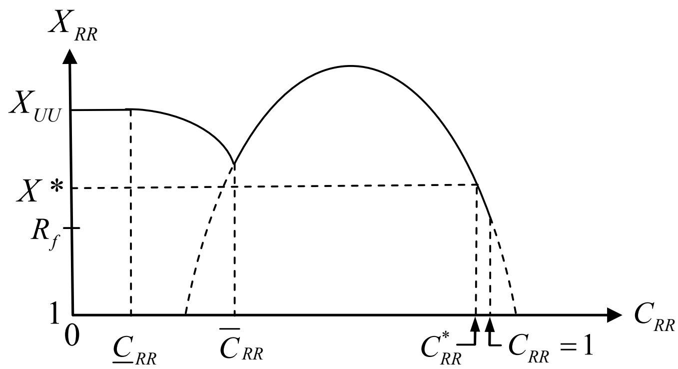

Fig. 5. Case *RR*: Risk as a function of the two period Cooke ratio