Can static models predict implied volatility surfaces? Evidence from OTC currency options*

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This version: January, 2009

Abstract

Despite advances in describing the characteristics and dynamics of non–flat implied volatility surfaces, relatively little has been said regarding the practical problem of implied volatility surface forecasting. Taking an explicitly out–of–sample forecasting approach, we propose a simple–to–estimate parametric decomposition of the implied volatility surface that combines and extends previous research in several respects. Using daily data from OTC options on 22 different currencies quoted against the U.S. $, we demonstrate that the approach yields intuitive and easy to communicate factors that achieve excellent in–sample fit, and whose time–variation capture the dynamics of the surface. Static econometric models for the factors are estimated and used for making short and long–term prediction of implied volatility surfaces. Results indicate that in comparison to leading benchmarks, the forecasts of 5 to 20–weeks–ahead are much superior across all surfaces.

JEL classification: C32; C53; G13; F37

Keywords: Implied volatility surfaces; Factor model; Forecasting.

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*We thank participants and discussants at the Asian FA/NFA conference in Yokohama, Japan, the 2nd Risk Management conference in Singapore, the 15th GFA meeting in Hangzhou, P. R. China, the 6th HFAA meeting in Patras, Greece and seminar participants at the Athens University of Economics & Business and the University of Piraeus for comments and suggestions. Any errors are our own responsibility.

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1 Introduction

Observed option prices implicitly contain information about the volatility expectations of market participants. Using an option pricing model, these volatility expectations can be extracted, and if market participants are rational, then these implied volatilities should contain all the information that is relevant for the pricing, hedging and management of option contracts and portfolios.

Contrary to the Black–Scholes–Merton assumption of constant (or deterministically time–dependent) volatility, the empirical pattern of option–implied volatilities has two features that have attracted the interest of researchers and practitioners in financial modeling: First, the volatilities implied from observed contracts systematically vary with the options strike price K and date to expiration T, giving rise to an instantaneously non–flat *implied volatility surface* (hereafter IVS). Canina and Figlewski (1993) and Rubinstein (1994) provide evidence that when plotted against moneyness, $m = K/S$ (the ratio of the strike price to the underlying spot price), implied volatilities exhibit either a 'smile' or a 'skew', while Heynen et. al. (1994), Xu and Taylor (1994) and Campa and Chang (1995) show that implied volatilities are a function of time to expiration and thus exhibit a 'term structure'. The second feature is that the IVS changes dynamically over time as prices in the options market respond to new information that affects investors' beliefs and expectations.

Three popular approaches to modeling this empirically observed profile of the IVS can be identified in the literature. The *no–arbitrage approach*, inspired by the stochastic interest rate literature, where stochastic volatility models are calibrated to today's IVS so as to preclude arbitrage, with prominent examples offered by Dupire (1993), Derman and Kani (1998) and Ledoit and Santa–Clara (1998) among others. Secondly, *linear parametric specifications* linking implied volatility to time to maturity and moneyness (Dumas, Fleming and Whaley (1998), Peña, Rubio and Serna (1999), etc.), and finally (statistical) *latent factors* that are identified in the dynamics of the IVS, as in Skiadopoulos et. al. (1999), Cont and da Fonseca (2002) and Mixon (2002).

Despite the relative success of the approaches for pricing and hedging purposes, surprisingly little has been said regarding the practical problem of implied volatility surface forecasting. The arbitrage–free stochastic volatility literature has little to say about forecasting, as it is concerned primarily with fitting the IVS at a point in time. Moreover, most contributions offered by latent factor decompositions of the IVS, although concerned with dynamics and thus potentially useful for forecasting, focus mainly on in–sample fit as opposed to out–of–sample predictions.[1]

In this paper, we take an explicitly out–of–sample forecasting approach that originates from the literature that employs parametric $\sigma_{IV}(m, T)$ specifications which have proved very successful in explaining deviations from a flat IVS in–sample.

Using daily implied volatility surfaces from a cross–section of options on 22 different currencies quoted against the U.S. $ from the OTC market, we decompose, period–by–period, each surface into seven linear parameters that evolve dynamically, and can be interpreted as factors. Unlike existing factor decompositions of the IVS, in which both the latent factors and the factor loadings are estimated (e.g. Mixon (2002)), our specification imposes structure on the factor loadings. This facilitates highly precise estimation of the factors and their intuitive interpretation.

We propose simple econometric models for the factors that are used for predicting the future IVS, by forecasting the factors forward. Results are encouraging; the factors achieve excellent in–sample fit across different FX options IVSs, and can be used to produce medium to long–term predictions (from 5 to 20 weeks ahead) that are noticeably more accurate than standard benchmarks and competing models.

A few existing papers are related to our approach, with Diebold and Li (2006) and Gonçalves and Guidolin (2006) closest in spirit. Both papers first apply parametric specifications at the cross–sectional level, and then fit time series models on the factors extracted from the first step. Moreover, both papers are concerned with forecasting: the yield curve and the IVS of S&P 500 index options respectively.

Diebold and Li (2006) (hereafter DL06) apply a Nelson and Siegel (1987) type of specification to the yield curve derived from the cross–section of U.S. government bond prices, and the estimated coefficients of the specification are fitted to an autoregressive model. Gonçalves and Guidolin (2006) (hereafter GG06) first estimate a five–parameter version of the ad hoc implied volatility model of Dumas et. al. (1998) on the volatility surface implied by S&P 500 index options. They then model the time evolution of estimated parameters with a vector autoregressive model.

The decomposition of the IVS we propose here can be though of as an extension of the GG06 framework in three respects. First, we allow potential

[1] See however the recent results in Chalamandaris and Tsekrekos (2008).

asymmetries in the shape of both the implied volatility smile and the decay of the smile with respect to the options' maturity. Secondly, we employ the DL06 factorisation of the Nelson and Siegel (1987) parsimonious model to extract factors that govern the dynamics of the IVS term–structure. Finally, we investigate a number of alternative modeling specifications for the IVS–extracted factors, with various degrees of in–sample fit and out–of–sample predicting accuracy. A simple vector autoregressive model of the factors (with Bayesian updating) appears superior amongst competing model specifications.

The rest of the paper is organised as follows: Section 2 describes the data, presents our methodology for decomposing the implied volatility surface into intuitive dynamic factors and discusses the estimation results of this methodology. In section 3 we estimate a series of static econometric models that can capture the time–series dynamics of the factors. Section 4 is devoted to the assessment of the out–of–sample forecasting performance of our approach, while Section 5 concludes the paper.

2 The implied volatility surface

2.1 The data

The data used in this study consist of daily time–series of implied volatilities for a cross–section of OTC currency options on 24 different currencies quoted against the U.S. dollar, kindly supplied by a major market participant. The time series are from 1/1/1999 to 21/5/2007, a total of 2,184 weekdays. The currencies examined and some exchange rate statistics are reported in Table 1.

In comparison to exchange–traded currency options, the OTC market is far more liquid. According to a Bank of International Settlements survey (2007), the outstanding notional amount of OTC currency options on the U.S. $ in June 2008 was approximately 11.9 trillion US$. The corresponding amount of exchange–traded currency options was 190.62 billion US$, just over 1.5% of the notional amount outstanding in the OTC market.

As is typical in such markets, dealers do not quote option prices denominated in currency units but rather implied volatilities, which are then conventionally converted into prices using the Garman and Kohlhagen (1983)

Code	Currency	Average	Min-Max
AUD	Australian \$	1.559	1.195-2.071
BRL	Brazilian Real	2.426	1.207-3.945
CAD	Canadian \$	1.381	1.085-1.613
CHF	Swiss Franc	1.431	1.134-1.825
CLP	Chilean Peso	592.7	468.0-758.2
CZK	Czech Koruny	30.12	20.59-42.04
DKK	Danish Krone	6.912	5.455-9.005
EUR	Euro	0.929	0.732-1.209
GBP	British £	0.607	0.498-0.728
HUF	Hungarian Forint	235.4	179.9-317.1
ILS	Israeli (New) Sheqel	4.381	3.938-5.010
JPY	Japanese ¥	115.1	101.5-134.8
KRW	South Korean Won	1137.6	913.7-1369.0
NOK	Norwegian Kroner	7.469	5.944-9.589
NZD	New Zealand \$	1.833	1.340-2.551
PLN	Polish Zlotych	3.755	2.751-4.715
SEK	Swedish Kronor	8.402	6.594-11.027
SGD	Singapore \$	1.703	1.511-1.854
THB	Thailand Baht	40.37	31.80-45.82
TWD	Taiwanese (New) \$	33.05	30.35-35.16
VEB	Venezuelan Bolivar Fuerte	1.401	0.560-2.140
ZAR	South African Rand	7.405	5.615-13.60

Table 1: Average, minimum and maximum middle exchange rates of 22 different currencies against the Euro from January 1999 to May 2007. Source: Federal Reserve Bank of New York & Reuters.

version of the Black and Scholes (1973) option pricing formula:

$$c = Se^{-r_f T}\mathcal{N}(d) - Ke^{-r_d T}\mathcal{N}\left(d - \sigma\sqrt{T}\right) \quad (1)$$

$$p = Ke^{-r_d T}\mathcal{N}\left(-d + \sigma\sqrt{T}\right) - Se^{-r_f T}\mathcal{N}(-d) \quad (2)$$

where

$$d = \frac{\ln(S/K) + (r_d - r_f + \sigma^2/2)\,T}{\sigma\sqrt{T}} \quad (3)$$

with r_d, r_f the risk–free interest rate in the domestic and the foreign country respectively, S the spot exchange rate, K the strike price of the option, T the time to option maturity in years, σ the exchange rate's volatility and $\mathcal{N}(.)$ the standard cumulative normal distribution.

The moneyness of the option is measured by its (Black–Scholes) delta:

$$\Delta^{BS} = \frac{\partial O}{\partial S} = \begin{cases} e^{-r_f T} \mathcal{N}(d), & \text{if the option } O \text{ is a call} \\ \\ e^{-r_f T} \left[\mathcal{N}(d) - 1\right], & \text{if the option } O \text{ is a put} \end{cases} \tag{4}$$

The industry convention is to quote, for each maturity, implied volatilities for portfolios of options such as delta–neutral straddles and risk reversals or butterfly spreads of a certain Δ^{BS}. From these, the implied volatility for at–the–money (ATM) options and for out–of–the–money (OTM) calls and puts can be inferred.[2]

Our data–set consists of implied volatilities for the following fourteen expirations: 1 week, 1 month, 2 months, 3 months, 6 months, 9 months, 12 months, 18 months, 2–5 years, 7 years and 10 years. For each of these maturities, the implied volatility is observed for options with five different Black–Scholes deltas: OTM puts with $\Delta^{BS} = -0.10$ and $\Delta^{BS} = -0.25$, ATM calls/puts and OTM calls with $\Delta^{BS} = 0.10$ and $\Delta^{BS} = 0.25$. Hence, for each exchange rate and on each observation date, a vector of 70×1 implied volatilities is observed.

Of course, not all currencies in our sample and not all option expirations are of equal trading intensity and variation. To eliminate the possibility that thinly–traded segments of an IVS influence our results, we exclude option maturities whose implied volatility is missing or remains unchanged for more than 70% of weekdays in our sample period. Then, to ensure that each IVS is continuous in the time domain, we discard parts of the sample that cause gaps of missing values longer than 4 weekdays. Applying the above two criteria ensures that in our reduced (both in maturities and in eligible weekdays) sample the entire surface under consideration is active. Table 2 reports the starting date and the number of days remaining in our sample after the above criteria have been applied.

Several different profiles of implied volatility surfaces are observed in our sample period. As an indication, in Figures 1–4 the average IVS profile and the daily standard deviation of the IVS from EUR/USD and NZD/USD options are plotted. In the EUR/USD case, the implied volatility surface exhibits a clear symmetric "smile" with an increasing term structure on average, and a fair amount of variability around this average profile (ranging from a fourth to a tenth of its typical value). In contrast, the NZD/USD

[2] Carr and Wu (2007a) and Malz (1996) demonstrate this in detail, in their excellent discussions on OTC currency option quoting and trading conventions.

Currency	Start Date	# of days	Currency	Start Date	# of days
AUD	05-Sep-2000	1744	JPY	04-Jul-2000	1806
BRL	27-Apr-1999	2178	KRW	27-Apr-1999	2178
CAD	27-Apr-1999	2178	NOK	27-Apr-1999	2178
CHF	04-Sep-2000	1745	NZD	27-Apr-1999	2178
CLP	29-Aug-2000	1739	PLN	27-Apr-1999	2178
CZK	27-Apr-1999	2178	SEK	27-Apr-1999	2178
DKK	27-Apr-1999	2178	SGD	xx05-Dec-2005	1610
EUR	04-Sep-2000	1745	THB	27-Apr-1999	2178
GBP	05-Sep-2000	1744	TWD	05-Sep-2000	1744
HUF	15-Jun-2000	1506	VEB	14-May-2000	1645
ILS	28-Apr-1999	2177	ZAR	08-Jul-2000	1803

Table 2: For each of the twenty two different currency options in our sample, the table reports the starting date and the number of trading days in the time series. The end date in all time series is 21/5/2007.

implied volatility surface exhibits a "skew", with either an increasing or a humped–shaped term structure, and a significantly asymmetric variability for short maturities. Similar patterns emerge in all currencies examined; to conserve space the corresponding figures for the remaining 22 currencies are relegated to Appendix D (available from the authors upon request).

Given the origin of the data, one possible criticism is that idiosyncratic effects, specific to the market participant supplying the quotes, could influence the analysis. There are however reasons to believe that such effects (if any) are not strongly affecting our analysis. First, our focus here is on systematic factors in the volatility surface, not on specific events or outliers of the surface. Secondly, given the liquidity of the market and the size of the market participant supplying the data, it should be fairly unlikely that our data are substantially away from typical values. Cross–checking a randomly

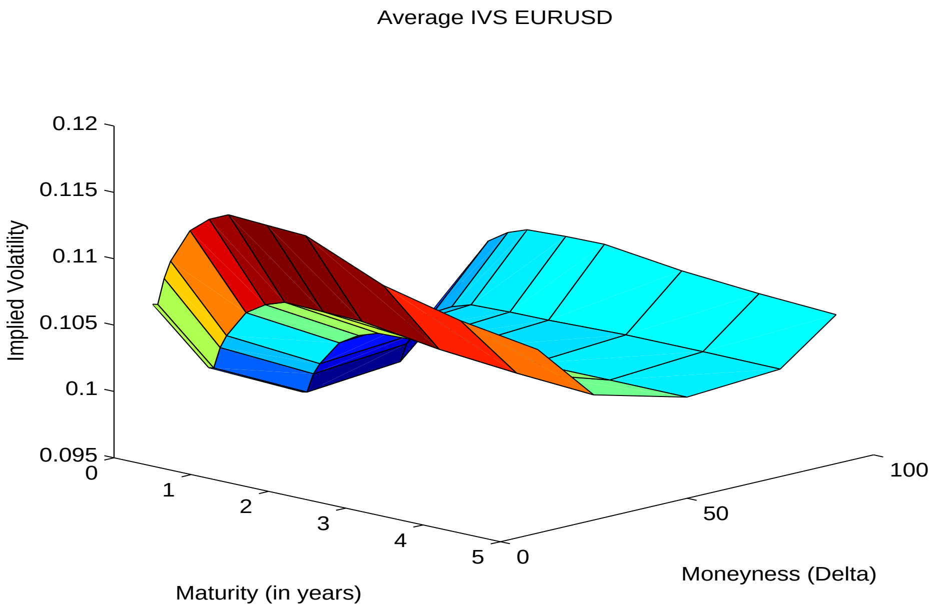

Figure 1: Average implied volatility surface from EUR/USD options, for the period 4/9/2000–21/5/2007.

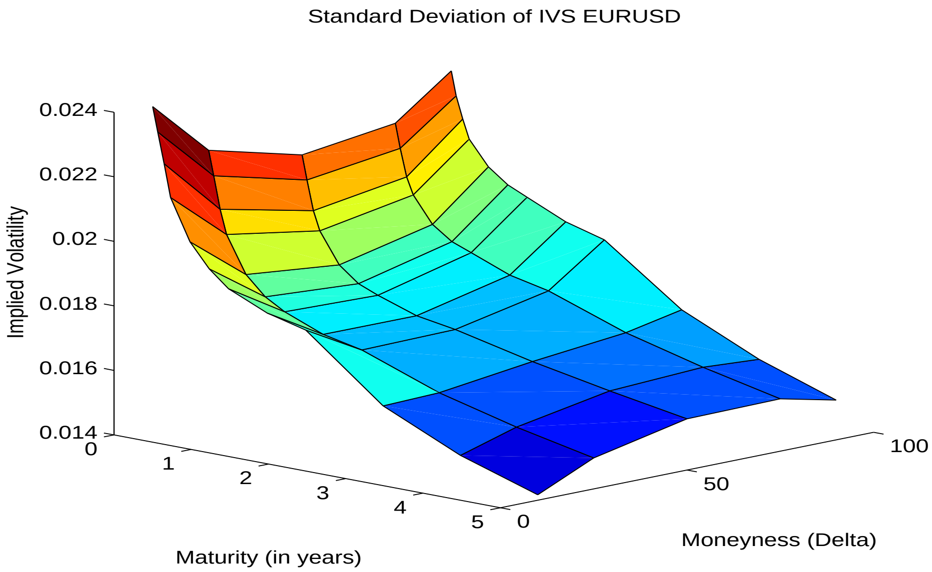

Figure 2: Daily standard deviation of EUR/USD implied volatilities as a function of moneyness and time to maturity for the period 4/9/2000–21/5/2007.

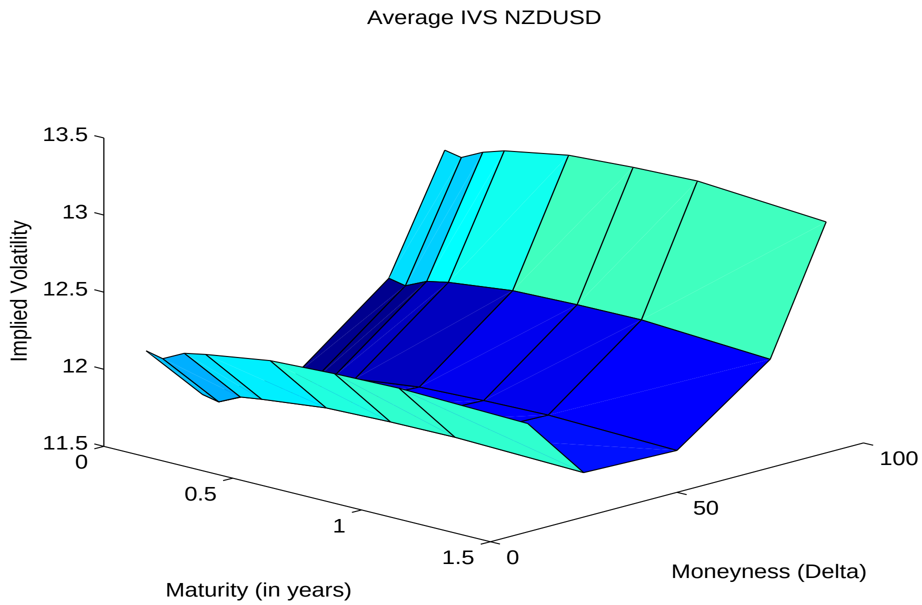

Figure 3: Average implied volatility surface from NZD/USD options, for the period XX/12/2003–21/5/2007.

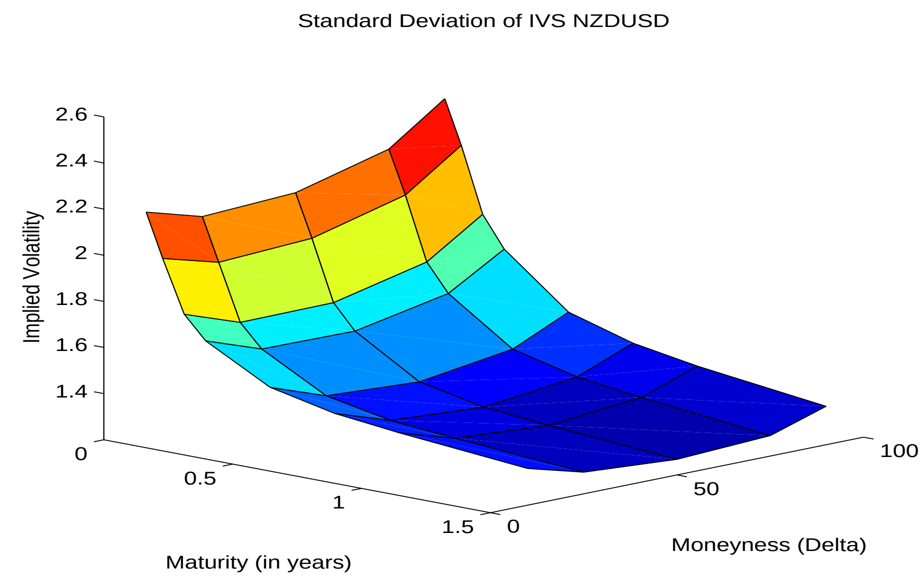

Figure 4: Daily standard deviation of NZD/USD implied volatilities as a function of moneyness and time to maturity for the period XX/12/2005–21/5/2007.

selected subsample of our data set with the implied volatility quotes from another data vendor (Bloomberg) reveals that this is indeed the case.

Of course using OTC data has many advantages in comparison to exchange–traded data. Besides superior liquidity, OTC currency options are available for longer maturities than the currency options traded in exchanges. Moreover, OTC options have a constant time–to–maturity, unlike exchange–traded options whose maturity varies from day to day. In practical terms, this alleviates the need for grouping options into maturity bins (see for example Skiadopoulos et. al. (1999)) or for creating synthetic *fixed–maturity* series via interpolation (as in Alexander (2001)). This should translate to less noisy IVSs and more precision in the identification of factors affecting their dynamics. Similar OTC currency options data have been used in previous studies by Campa and Chang (1995), (1998), Carr and Wu (2007a), (2007b) and Christoffersen and Mazzotta (2005); the latter study actually concludes that OTC currency options data are of superior quality for volatility forecasting purposes.

2.2 Decomposition of the implied volatility surface

A common practice in describing the implied volatility surface on any given day, is to fit cross–sectionally a parametric specification of some "moneyness" metric and time–to–maturity, (Dumas et. al. (1998) for example). Similarly, in this paper we propose decomposing the daily IVS into seven parametric indicators, each one with a natural interpretation regarding deviations from a theoretically flat surface.

Each day, we estimate the following cross–sectional model

$$\sigma_{IV,i} = \sum_{j=1}^{7} \beta_j I_{i,j} + \epsilon_i \tag{5}$$

where ϵ_i the random error term, $i = 1, \ldots, N$ with N the number of implied volatilities in each daily cross section (a maximum of 70), and

$I_{i,1} = 1$	Flat level
$I_{i,2} = \mathbf{1}_{\{\Delta_i > 0.5\}} \Delta_i^2$	Right "smile"
$I_{i,3} = \mathbf{1}_{\{\Delta_i < 0.5\}} \Delta_i^2$	Left "smile"
$I_{i,4} = \frac{1 - e^{-\lambda T_i}}{\lambda T_i}$	Short–term

$$I_{i,5} = \frac{1-e^{-\lambda T_i}}{\lambda T_i} - e^{-\lambda T_i} \quad \text{Medium–term}$$

$$I_{i,6} = \mathbf{1}_{\{\Delta_i > 0.5\}} \Delta_i T_i \quad \text{Right "smile" attenuation}$$

$$I_{i,7} = \mathbf{1}_{\{\Delta_i < 0.5\}} \Delta_i T_i \quad \text{Left "smile" attenuation}$$

In the definitions above, $\mathbf{1}_x$ denotes an indicator function that takes the value of one if statement x is true, and zero otherwise, T_i is the time–to–maturity of the contract with implied volatility i, while

$$\Delta_i = \left| \left| \Delta_i^{BS} \right| - \mathbf{1}_{\Delta_i^{BS} < 0} \right| \tag{6}$$

is a "moneyness" metric, that is one–to–one with the Black–Scholes delta of contract i in the cross section.[3]

The interpretation of the seven indicators should be apparent: $I_{i,1}$ is flat, thus β_1 captures the (mean) level of the IVS on any given day. Indicators $I_{i,2}$ and $I_{i,3}$ are quadratic in Δ, describing the "smile" of the surface. By dividing the implied volatility smile into a left (from OTM calls) and a right (from OTM puts) component, we can capture through β_2, β_3 any asymmetries due to differential investor risk aversion towards the two currencies of the exchange rate.

Indicators $I_{i,4}$ and $I_{i,5}$ are functions of time–to–maturity, and collectively account for the term–structure of the IVS. They are adopted from the recent Diebold and Li (2006) factorisation of the Nelson and Siegel (1987) parsimonious term structure model that has been proved quite successful in forecasting the yield curve. In this parametrisation, λ governs the exponential decay rate: small values produce slow decay and thus better fit of the term structure at long maturities, whereas large values produce fast decay and a better fit at short maturities. The coefficients β_4 and β_5 capture the (average) short–end and medium part of the term structure of the IVS on any given day.

The last two indicators account for a common feature of the IVS, the "flattening" of the smile as the time to maturity increases.[4] Thus, it seems important to allow the intensity of the smile to vary with the maturity, independently for each side of the surface. The coefficients β_6 and β_7 capture

[3] It should be clear that (6) is a simple transformation of $\Delta^{BS} \in [-1, 1]$ into $\Delta \in [0, 1]$.

[4] An interpretation of this property is usually the increased uncertainty about the direction of future paths: the current exchange rate seems almost equally probable with a reasonably OTM strike, as long as the time to expiry is long enough.

the attenuation of the smile with time–to–maturity, for OTM puts and calls respectively.

To our knowledge, the parametric decomposition of the IVS we propose in (5) is novel, in that it combines the possibility of asymmetric and attenuating smiles with the parsimonious modeling of the term–structure of the IVS. Peña, Rubio and Serna (1999), in their investigation of volatility implied from index options in the Spanish market, have shown that allowing for an asymmetric smile achieves good in–sample fit. However, their work, which focuses to the closest to maturity options and ignores the term–structure dimension of the surface, is not concerned with whether such a decomposition can be employed for forecasting purposes.

Gonçalves and Guidolin (2006) have used a similar decomposition of the IVS of S&P500 index options that has proved successful in predictions. In comparison to (5), the model they fit in the cross–section is symmetric (i.e. in our notation, $\beta_2 + \beta_3$ and $\beta_6 + \beta_7$ are their smile and attenuation coefficients) and linear in the time–to–maturity T_i. Although, equation (5) requires estimation of two extra parameters in comparison to their model, our results indicate that this increased modeling flexibility is crucial both for in–sample fitting performance and for out–of–sample forecasting accuracy.

Before turning to the in–sample fitting results, a notes is in order: Equation (5) is not linear in λ and can not be estimated with OLS. Instead of resorting to nonlinear least squares, we use a grid–search procedure for each surface, where λ is set equal to its median estimated value, by minimising the sum of squared errors each day. Once $\widehat{\lambda}$ is determined, equation (5) is estimated with ordinary least–squares.

Table 3 summarises the goodness of fit of our parametric specification in (5) to the time series of IVSs. For comparison purposes, we also fit the simpler specification in Gonçalves & Guidolin (2006, eq. (1)). Naturally, the increased flexibility of our specification results in better in–sample fitting, as the average adjusted R^2's indicate. In none of the surfaces is the average adjusted R^2 less than 95%, a distinct improvement over the corresponding measures of fit of the Gonçalves & Guidolin (2006) specifications. With the exception of the CZK/USD and THB/USD surfaces, our specification achieves a minimum goodness of fit of 60% and above; in contrast, the minimum adjusted R^2 of the simpler specification is less than 60% in 14 out of the 24 surfaces.

Table 4 reports the percentage of sample days in which the estimated coefficients $\widehat{\beta}' = \left\{\widehat{\beta}_1, \ldots, \widehat{\beta}_7\right\}$ are statistically significant at $\alpha = 5\%$. This

Currency code	Chalamandaris & Tsekrekos, equation (5)					Gonçalves & Guidolin (2006, eq. (1))		
	Average $R^2_{\text{adj.}}$	Max $R^2_{\text{adj.}}$	Min $R^2_{\text{adj.}}$	Median $R^2_{\text{adj.}}$	Median $\widehat{\lambda}$	Average $R^2_{\text{adj.}}$	Max $R^2_{\text{adj.}}$	Min $R^2_{\text{adj.}}$
AUD	96.87	99.26	79.18	97.32	4.34	81.97	99.15	33.23
BRL	97.69	99.45	62.52	98.35	3.81	95.45	99.76	59.86
CAD	97.38	99.75	87.33	97.68	9.31	92.07	99.75	60.20
CHF	95.32	99.13	78.58	95.68	5.11	77.29	98.34	28.84
CLP	97.19	99.44	66.76	97.44	3.86	94.94	99.88	66.50
CZK	94.77	99.69	14.65	98.24	7.83	87.71	99.43	3.76
DKK	97.28	99.68	73.82	97.64	9.24	89.88	99.33	56.88
EUR	96.08	99.28	85.39	96.42	4.92	77.51	98.53	24.90
GBP	94.60	99.17	67.15	95.31	3.49	80.16	98.38	33.79
HUF	96.84	99.62	62.32	98.47	5.63	94.54	99.81	57.89
ILS	98.44	99.53	62.78	98.82	9.29	96.82	99.54	72.81
JPY	94.40	98.91	66.26	96.27	4.80	89.89	98.02	61.04
KRW	95.97	99.32	64.01	97.09	3.92	92.42	99.30	25.95
NOK	96.99	99.80	73.87	97.31	8.10	89.81	99.27	56.71
NZD	97.84	99.72	80.87	98.30	8.08	90.28	99.57	49.10
PLN	98.38	99.87	94.76	98.50	8.18	96.57	99.76	78.36
SEK	97.07	99.80	74.42	97.46	7.91	89.82	99.37	48.47
SGD	96.96	99.50	77.86	97.51	2.73	92.52	99.42	58.77
THB	96.47	99.65	59.04	96.92	2.77	91.77	99.02	19.11
TWD	96.08	99.65	76.51	96.94	0.84	89.41	99.21	48.90
VEB	97.88	99.51	88.89	98.18	6.86	86.96	98.87	64.75
ZAR	96.42	99.85	83.29	97.78	4.31	91.72	99.28	48.61

Table 3: The table reports the average, maximum, minimum and median adjusted R^2 of fitting equation (5) by ordinary least squares every day, in each of the twenty two exchange rate implied volatility surfaces in our sample, once λ is fixed at the reported values. The selected values of λ are medians of daily estimates that minimise the sum of squared errors in the cross section. The length of each time series is reported in Table 2. For comparison purposes, the average, maximum and minimum adjusted R^2 of fitting equation (1) in Gonçalves & Guidolin (2006) are also reported.

Currency code	Percentage of statistically significant (at $\alpha = 5\%$) daily estimates													
	Chalamandaris & Tsekrekos, equation (5)								Gonçalves & Guidolin (2006, eq. (1))					
	$\widehat{\beta_1}$	$\widehat{\beta_2}$	$\widehat{\beta_3}$	$\widehat{\beta_4}$	$\widehat{\beta_5}$	$\widehat{\beta_6}$	$\widehat{\beta_7}$	$\overline{DW}$	$\widehat{\beta_0}$	$\widehat{\beta_1}$	$\widehat{\beta_2}$	$\widehat{\beta_3}$	$\widehat{\beta_4}$	$\overline{DW}$
AUD	100	82.6	100	95.1	73.3	62.4	86.9	1.74	100	89.5	100	90.0	50.1	0.70
BRL	100	84.3	100	88.8	47.4	24.1	85.0	2.41	100	100	92.1	94.7	76.4	1.35
CAD	100	93.5	98.6	83.6	43.1	30.7	39.7	1.73	100	79.8	100	88.6	31.5	0.93
CHF	100	99.9	89.3	90.0	52.1	75.2	61.0	1.99	100	80.9	100	77.2	37.0	0.79
CLP	100	70.6	99.9	78.3	29.8	4.9	98.4	2.56	100	98.0	100	87.5	96.7	1.50
CZK	100	90.6	84.7	85.4	51.6	30.6	61.3	1.74	100	66.7	96.4	90.1	45.4	0.96
DKK	100	99.0	92.5	87.1	51.3	38.8	25.0	1.93	100	73.0	100	91.9	34.6	0.79
EUR	100	99.9	91.6	93.6	55.9	80.3	54.2	2.10	100	78.9	100	83.4	34.2	0.74
GBP	100	99.5	95.5	94.6	67.8	61.8	42.3	1.73	100	72.3	100	92.2	38.9	0.78
HUF	100	81.7	87.5	84.0	27.1	39.5	46.1	2.00	100	97.3	100	88.8	46.6	1.13
ILS	100	79.0	99.9	81.6	51.4	14.9	37.5	1.81	100	99.1	100	91.7	29.9	1.28
JPY	100	99.3	64.0	77.6	24.4	73.4	83.6	1.96	100	93.0	100	82.8	89.5	1.39
KRW	100	82.0	95.1	84.6	53.5	40.6	49.9	1.77	100	76.9	100	93.0	55.9	1.12
NOK	100	99.2	92.2	87.6	45.3	36.8	25.6	1.87	100	74.3	100	90.2	37.3	0.80
NZD	100	79.2	100	91.2	51.9	42.7	41.0	1.69	100	85.4	100	95.1	26.2	0.68
PLN	100	60.2	100	78.5	22.8	51.1	57.9	2.37	100	100	100	87.0	64.3	1.44
SEK	100	99.2	92.3	87.2	49.3	34.1	25.6	1.90	100	73.7	100	90.1	36.8	0.81
SGD	100	96.3	89.8	78.8	64.7	64.7	63.8	1.98	100	79.3	100	92.2	63.0	1.17
THB	100	79.5	96.8	82.0	63.4	44.5	54.2	1.78	100	72.1	97.4	92.6	56.1	1.20
TWD	100	88.4	80.7	92.5	76.3	51.4	41.8	1.80	100	90.8	99.9	99.9	49.5	0.86
VEB	100	91.9	99.3	95.3	80.9	2.1	22.1	1.89	100	91.6	11.5	97.1	13.7	0.38
ZAR	100	65.9	99.9	82.4	44.8	42.0	69.1	2.37	100	100	100	84.5	58.6	1.54

Table 4: For each of the twenty two exchange rate implied volatility surfaces the table reports the percentage of in–sample days in which the coefficients of the cross sectional estimation, equation (5), were statistically significant at 5%. $\overline{DW}$ is the average, over all sample days, Durbin–Watson statistics of the cross sectional estimation residuals. The sample size for each implied volatility surface is reported in Table 2. For comparison purposes, the corresponding percentages and statistics from fitting equation (1) in Gonçalves & Guidolin (2006) are also reported.

should give readers an indication as to the source of the fitting improved performance of our specification in (5). Obviously, the level and "smile" indicators are extremely important in explaining the surfaces in our sample. In all but three cases ($\widehat{\beta}_2$ in PLN, ZAR and $\widehat{\beta}_3$ in JPY) these coefficients add to the explanatory power of the specification in more than 70% of sample days.

Similarly, the term–structure indicators appear important in explaining the daily implied volatility surface, with the short–term one (I_4) contributing slightly more than the medium–term, I_5. The two "smile attenuation" indicators seem the least important in explaining most surfaces cross sectionally; however, allowing the slope of the smile to be different for OTM puts and calls seems to add to the fitting performance of (5) (compared to the "symmetric treatment in Gonçalves & Guidolin (2006)).

Table 4 also reports $\overline{DW}$, the average Durbin–Watson statistic for the residuals of the daily cross–sectional regressions. Comparing the average DW statistics brings forward an important improvement aspect of our specification over the simpler, symmetric one: unexplained residuals appear uncorrelated on average, much more than the residuals of the alternative specification. The improvement is universal, across all currency IVSs, and makes one far more confident regarding the appropriateness of the proposed surface decomposition.

Having established the appropriateness of the decomposition in (5), we follow the work of Diebold and Li (2006) and Gonçalves & Guidolin (2006) that interpret the time–varying parameters $\widehat{\beta}' = \left\{\widehat{\beta}_1, \widehat{\beta}_2, \widehat{\beta}_3, \widehat{\beta}_4, \widehat{\beta}_5, \widehat{\beta}_6, \widehat{\beta}_6\right\}$ as factors, whose dynamic evolution drive the implied volatility surface. Doing so facilitates the precise estimation of the factors (from their loadings, I_i) and their interpretation as deviations from a Black–Scholes flat surface.

In what follows, we model the dynamics of the factors (for each surface separately) using simple econometrics specifications. These specifications are then used to forecast the factors and subsequently the whole surface forward, and their predicting ability is compared with numerous benchmarks that have been proposed in the literature.

3 Modeling the time–variation of implied volatility surfaces

As a means of determining the appropriate modeling approach for the factors extracted from the implied volatility surfaces, a number of unit–root tests are performed on the time series of $\widehat{\beta}$'s, with the results reported in Tables 5 and 6.

The p–values of augmented Dickey–Fuller (ADF) and Elliot, Rothenberg and Stock (1996, ERS) tests are reported, along with LM statistics from the Kwiatkowski, Phillips, Schmidt, and Shin (1992, KPSS) unit–root test.[5]

The results suggest that in most of the 24 exchange rate surfaces, $\widehat{\beta}_1$ may have unit roots, while $\widehat{\beta}_2, \widehat{\beta}_3, \widehat{\beta}_4, \widehat{\beta}_5, \widehat{\beta}_6, \widehat{\beta}_7$ not so. Although our approach in this paper is purely a forecasting one, to account for the mixed stationarity results we replicate the analysis by fitting the indicators $I_1, \dots, I_7$ to the daily changes of implied volatilities in each surface:

$$\Delta\sigma_{IV,i} = \sum_{j=1}^{7} \gamma_j I_{i,j} + \varepsilon_i \tag{7}$$

and also use the dynamics of factors $\widehat{\gamma}$ for forecasting purposes.

We investigate a number of alternative modeling specifications, with regards to the in–sample dynamics of the factors $\widehat{\beta}$ and $\widehat{\beta}$. More specifically, the following are estimated and used in forecasting:

[a] Univariate AR(1) model of factors extracted from the daily IVS

$$\widehat{\beta}_{i,t} = \widehat{c}_i + \widehat{\phi}_i \widehat{\beta}_{i,t-1} + u_{i,t} \tag{8}$$

for $i = 1, \dots, 7$

[b] Univariate AR(1) model of factors extracted from the daily change of the IVS

$$\widehat{\gamma}_{i,t} = \widehat{c}_i + \widehat{\psi}_i \widehat{\gamma}_{i,t-1} + e_{i,t} \tag{9}$$

with $\widehat{\gamma}_{i,t}$, $i = 1, \dots, 7$ from (7)

[c] VAR(1) model of factors extracted from the daily IVS

$$\widehat{\beta}_t = \widehat{\mathbf{c}} + \widehat{\mathbf{\Phi}} \widehat{\beta}_{t-1} + \mathbf{u}_t \tag{10}$$

[5] The Bayesian Information Criterion of Schwarz (1978) is used to choose the lags in the ADF and ERS tests.

Currency Code	$\widehat{\beta}_1$ ERS	$\widehat{\beta}_1$ ADF	$\widehat{\beta}_1$ KPSS	$\widehat{\beta}_1$ $k^\star$	$\widehat{\beta}_2$ ERS	$\widehat{\beta}_2$ ADF	$\widehat{\beta}_2$ KPSS	$\widehat{\beta}_2$ $k^\star$	$\widehat{\beta}_3$ ERS	$\widehat{\beta}_3$ ADF	$\widehat{\beta}_3$ KPSS	$\widehat{\beta}_3$ $k^\star$	$\widehat{\beta}_4$ ERS	$\widehat{\beta}_4$ ADF	$\widehat{\beta}_4$ KPSS	$\widehat{\beta}_4$ $k^\star$
AUD	0.860	0.940	10.78	12	0.067	0.111	13.16	11	0.513	0.069	10.18	8	0.570	0.018	3.66	11
BRL	0.948	0.165	5.03	12	0.883	0.000	18.45	4	0.880	0.000	15.50	2	0.691	0.000	4.28	2
CAD	0.759	0.675	29.93	5	0.003	0.004	10.50	5	0.866	0.015	13.53	12	0.000	0.000	1.16	12
CHF	0.998	0.618	11.21	12	0.226	0.259	13.47	10	0.692	0.017	4.95	11	0.013	0.000	1.02	12
CLP	0.080	0.007	8.85	6	0.449	0.099	30.24	4	0.623	0.009	7.71	7	0.034	0.001	2.20	10
CZK	0.979	0.033	69.23	2	0.931	0.025	8.31	12	0.935	0.109	12.98	6	0.092	0.014	0.00	12
DKK	0.899	0.988	11.03	11	0.897	0.000	22.90	5	0.158	0.000	8.30	10	0.002	0.000	2.41	12
EUR	0.998	0.502	14.00	12	0.200	0.186	10.68	12	0.734	0.028	6.01	11	0.000	0.000	0.00	12
GBP	0.912	0.717	9.55	10	0.243	0.012	20.77	2	0.812	0.005	77.86	1	0.082	0.002	1.03	12
HUF	0.267	0.081	8.17	4	0.591	0.167	19.93	6	0.827	0.543	11.87	6	0.001	0.001	2.18	7
ILS	0.870	0.678	17.58	8	0.847	0.409	25.50	9	0.604	0.731	19.12	8	0.000	0.000	0.00	12
JPY	0.996	0.147	21.22	9	0.611	0.006	8.44	1	0.048	0.030	42.54	2	0.071	0.006	8.84	8
KRW	0.971	0.030	95.06	2	0.314	0.000	28.52	2	0.133	0.000	9.86	7	0.627	0.000	7.43	8
MYR	0.982	0.824	173.80	0	0.785	0.570	22.24	2	0.966	0.367	47.25	4	0.672	0.635	33.47	2
NOK	0.501	0.644	3.87	10	0.876	0.000	56.98	2	0.185	0.003	7.90	11	0.026	0.000	1.74	12
NZD	0.671	0.685	2.52	12	0.011	0.019	12.05	11	0.469	0.005	4.08	11	0.043	0.002	4.90	10
PLN	0.982	0.055	18.62	5	0.524	0.004	6.41	11	0.059	0.044	25.04	7	0.000	0.000	4.38	8
SEK	0.604	0.777	5.25	11	0.924	0.000	12.00	10	0.189	0.002	7.66	11	0.019	0.000	3.22	12
SGD	0.043	0.016	4.23	8	0.139	0.076	14.29	6	0.115	0.017	9.95	12	0.004	0.000	0.00	3
THB	0.793	0.013	26.71	6	0.239	0.002	19.16	3	0.067	0.066	12.70	11	0.542	0.006	10.11	12
TWD	0.113	0.001	24.25	2	0.553	0.444	18.05	6	0.129	0.200	21.79	6	0.002	0.005	10.21	4
VEB	0.785	0.489	32.64	2	0.946	0.432	8.85	11	0.964	0.729	39.77	5	0.110	0.027	17.76	3
ZAR	0.373	0.078	13.38	5	0.475	0.016	4.57	12	0.016	0.025	3.37	12	0.000	0.000	8.02	3

Table 5: The table reports the results of unit–root tests performed on the time series of estimates $\widehat{\beta}_1, \widehat{\beta}_2, \widehat{\beta}_3, \widehat{\beta}_4$ from equation (5). Under ERS, p–values of the Elliot, Rothenberg and Stock (1996) test are reported; under ADF, p–values of the augmented Dickey–Fuller test, and under KPSS, LM statistics of the Kwiatkowski, Phillips, Schmidt, and Shin (1992) test. The critical value of the KPSS test at $\alpha = 1\%$ is 0.739. $k^\star$ stands for the estimated residual spectrum at frequency zero that is necessary for the KPSS tests.

Currency	$\widehat{\beta}_5$				$\widehat{\beta}_6$				$\widehat{\beta}_7$			
Code	ERS	ADF	KPSS	$k^\star$	ERS	ADF	KPSS	$k^\star$	ERS	ADF	KPSS	$k^\star$
AUD	0.000	0.000	1.81	12	0.134	0.061	28.25	6	0.913	0.496	20.44	8
BRL	0.000	0.000	2.69	12	0.029	0.000	3.25	5	0.561	0.102	13.01	8
CAD	0.068	0.000	1.64	12	0.848	0.000	13.03	4	0.521	0.004	8.31	12
CHF	0.030	0.000	1.78	12	0.324	0.406	39.54	5	0.529	0.003	39.24	3
CLP	0.004	0.000	4.00	12	0.005	0.000	18.84	5	0.786	0.066	29.55	5
CZK	0.000	0.000	9.65	3	0.000	0.000	17.58	6	0.070	0.004	8.18	4
DKK	0.000	0.000	5.74	9	0.832	0.000	6.22	9	0.229	0.000	1.32	12
EUR	0.021	0.000	1.86	12	0.663	0.721	23.26	9	0.839	0.061	12.04	11
GBP	0.000	0.000	1.26	10	0.012	0.016	44.26	3	0.846	0.005	14.95	7
HUF	0.000	0.000	2.58	8	0.125	0.096	10.23	7	0.059	0.073	13.78	8
ILS	0.468	0.008	9.32	12	0.049	0.008	10.67	9	0.173	0.004	9.47	8
JPY	0.007	0.000	4.65	12	0.811	0.135	77.06	2	0.635	0.004	3.13	3
KRW	0.385	0.000	2.10	9	0.892	0.000	11.19	3	0.820	0.002	54.75	2
MYR	0.136	0.135	22.11	3	0.358	0.083	26.61	3	0.932	0.310	63.12	2
NOK	0.123	0.000	4.50	11	0.787	0.000	6.34	9	0.197	0.000	1.37	12
NZD	0.108	0.000	2.56	12	0.000	0.000	6.97	12	0.137	0.003	7.12	12
PLN	0.315	0.000	2.02	12	0.496	0.236	24.56	8	0.562	0.092	32.24	6
SEK	0.057	0.000	2.44	12	0.808	0.000	4.63	11	0.238	0.000	1.43	9
SGD	0.191	0.002	15.52	4	0.086	0.050	9.92	7	0.311	0.533	22.46	7
THB	0.069	0.000	1.19	12	0.787	0.008	27.88	4	0.910	0.033	14.12	10
TWD	0.647	0.140	8.01	11	0.293	0.157	13.81	7	0.006	0.011	8.42	6
VEB	0.026	0.039	7.63	12	0.057	0.084	9.79	11	0.757	0.383	17.32	11
ZAR	0.000	0.000	5.93	3	0.000	0.000	6.10	3	0.063	0.041	11.26	12

Table 6: The table reports the results of unit–root tests performed on the time series of estimates $\widehat{\beta}_5, \widehat{\beta}_6, \widehat{\beta}_7$ from equation (5). Under ERS, p–values of the Elliot, Rothenberg and Stock (1996) test are reported; under ADF, p–values of the augmented Dickey–Fuller test, and under KPSS, LM statistics of the Kwiatkowski, Phillips, Schmidt, and Shin (1992) test. The critical value of the KPSS test at $\alpha = 1\%$ is 0.739. $k^\star$ stands for the estimated residual spectrum at frequency zero that is necessary for the KPSS tests.

with $\widehat{\beta}'_t = \left\{\widehat{\beta}_1, \widehat{\beta}_2, \widehat{\beta}_3, \widehat{\beta}_4, \widehat{\beta}_5, \widehat{\beta}_6, \widehat{\beta}_7\right\}$
[d] VAR(1) model of factor extracted from the daily change of the IVS

$$\widehat{\gamma}_t = \widehat{\mathbf{c}} + \widehat{\boldsymbol{\Psi}}\widehat{\gamma}_{t-1} + \mathbf{e}_t \tag{11}$$

with $\widehat{\gamma}'_t = \{\widehat{\gamma}_1, \widehat{\gamma}_2, \widehat{\gamma}_3, \widehat{\gamma}_4, \widehat{\gamma}_5, \widehat{\gamma}_6, \widehat{\gamma}_7\}$
[e] ECM(1) with r common trends

$$\widehat{\gamma}_t = \widehat{\mathbf{c}} + \widehat{\boldsymbol{\Gamma}}\widehat{\gamma}_{t-1} + \widehat{\boldsymbol{\Theta}}\widehat{\beta}_{t-1} + \mathbf{v}_t \tag{12}$$

with $\widehat{\beta}_t, \widehat{\gamma}_t, d$ as before and $\widehat{\boldsymbol{\Theta}}$ a matrix of coefficients for the cointegrating vectors (that is of rank r).[6]
[f] A Bayesian variant of the VAR(1) specification of factors in [c], with the prior means and variances determined by the Doan, Litterman and Sims (1984) procedure.[7]
[g] An ARFI$(1, d)$ model of factors extracted from the daily IVS, with d the order of fractional integration (see Baillie (**?**)).

To avoid cluttering the reader, we have decided to suppress the estimation results of the 7 aforementioned econometric specifications. These are of course included in Appendix D, and are available upon request. However, the focus of this work is the forecasting ability of such static econometric models of factor extracted from the daily dynamics of IVSs from the OTC FX options market, and this is what we turn to in the section that follows.

4 Out–of–sample forecasting performance

An intuitive decomposition of the implied volatility surface like equation (5), and a parsimonious model of its factor dynamics like the 7 specifications estimated in the previous section should not only fit well in–sample, but also forecast well out–of–sample. Because the IVS depends only on $\widehat{\beta}_t$ in (5), forecasting the future surface is equivalent to producing good forecasts of $\widehat{\beta}_{t+h}$ via any of the econometric specifications considered.

In this section we undertake just such a forecasting exercise. For each IVS in our sample, we first fit equation (5) on the first 100 weeks of our sample period (approximately the first 500 trading days), and then estimate

[6] The procedure by MacKinnon (1996) is used to generate critical values for both the trace and maximal eigenvalue statistics for the ECM(1).

[7] Details regarding the overall tightness and lag–decay hyperparameters of the procedure are relagated in Appendix D (available from the authors upon request).

the 7 econometric specifications introduced earlier, on the time series of the $\widehat{\beta}$ factors that are produced in the first step.

Each day, we estimate and forecast recursively for $h = 1, 2, 5, 10, 15$ and 20 weeks ahead, and the accuracy of the forecasts is assessed through a number of simple and more sophisticated benchmarks.

The benchmarks we employ for comparisons are:

(1) Random walk:

$$\sigma_{IV,i,t+h} = \sigma_{IV,i,t} \tag{13}$$

for $i = 1, \ldots, N$ where N the number of options available in each day (at most 70)

(2) Univariate AR(1) on implied volatility levels:

$$\widehat{\sigma}_{IV,i,t+h} = \widehat{c}_i + \widehat{\varphi}_i \sigma_{IV,i,t} \tag{14}$$

(3) Univariate AR(1) on implied volatility first differences:

$$\widehat{\Delta\sigma}_{IV,i,t+h} = \widehat{c}_i + \widehat{\varphi}_i \Delta\sigma_{IV,i,t} \tag{15}$$

(4) VAR(1) model of IVS principal components:
We first perform a principal components analysis on the full set of (at most) seventy impled volatilities $\sigma'_{IV,t} = \{\sigma_{IV,1,t}, \ldots, \sigma_{IV,N,t}\}$ of each surface for the first 100 weeks, effectively decomposing the implied volatility covariance matrix as VLV^T, where the diagonal elements of L are the eigenvalues and the columns of V are the associated eigenvectors. Denote the largest three eigenvalues by λ_1, λ_2 and λ_3, and denote the associated eigenvectors by v_1, v_2 and v_3. The first three principal components $\mathbf{f}'_t = \{f_{1,t}, f_{2,t}, f_{3,t}\}$ are then defined by $f_{j,t} = v'_j \sigma_{IV,t}$. We then use a VAR(1) model to produce forecasts of the principal components:

$$\widehat{\mathbf{f}}_{t+1} = \widehat{\mathbf{c}} + \widehat{\mathbf{\Omega}} \mathbf{f}_{t-1} \tag{16}$$

and produce forecasts of implied volatilities as

$$\widehat{\sigma}_{IV,t+h} = v_1 \widehat{f}_{1,t+h} + v_2 \widehat{f}_{2,t+h} + v_3 \widehat{f}_{3,t+h} \tag{17}$$

(5) VAR(1) model of first differences IVS principal components:
Exactly the same as (4) above, only principal components analysis is performed on the full set of (at most) seventy impled volatilities first differences $\Delta\sigma'_{IV,t} = \{\sigma_{IV,1,t} - \sigma_{IV,1,t-1}, \ldots, \sigma_{IV,N,t} - \sigma_{IV,N,t-1}\}$ of each surface for the first 100 weeks.

(6) The forecasting procedure outlined in Gonçalves & Guidolin (2006), that is based on a vector autoregressive representation of an IVS cross–sectional decomposition that is similar to our (5), but symmetric in "smile" and "smile attenuation", and linear in time–to–maturity.

To assess the predicting ability of competing models and benchmarks across different forecast horizons, mean absolute errors (MAE) and root mean squared errors (RMSE) are computed and reported; the latter in Table 7, while the former in Appendix 6. To facilitate the comparison across models, all MAE and RMSE of h–week–ahead forecasts are divided by the corresponding MAE and RMSE of the random walk model in equation (13).

Naturally, for IVS predictions 1 and 2 weeks into the future, none of the models (or benchmarks) can improve over the simple random walk considerably and/or consistently (across all exchange rate surfaces). However, for predictions 5 weeks into the future or longer, most of the simple factor models we consider improve over the "naive" random walk considerably: increased forecast accuracy of 30-50% over the random walk seems to be the usual case in most of the IVSs.

Obviously, not all factor models examined appear equally successful in making accurate implied volatility forecasts. Comparing across models, it can be seen that the specifications that are based on the dynamics of the $\widehat{\beta}$ factors dominate almost universally those based on the dynamics of $\widehat{\gamma}$. This is in line with the findings of Diebold and Li (2006) in yield–curve forecasting, and might help in explaining the inability of recent studies to predict the changes of volatility indices (see e.g. Konstantinidi et. al. (**?**)).

In only 5 of the currency options IVSs considered, do the $\widehat{\gamma}$–models consistently beat the random walk benchmark. In contrast, this is true only in 4 surfaces for the models based on the dynamics of $\widehat{\gamma}$'s from equation (5). Between these models, ARFI$(1, d)$ seems to be performing the worst, while the Bayesian VAR(1) slight better than the others.

Comparing with the benchmarks, the factor models considered appear to be performing better or equally well in all surfaces. The models based on the dynamics of $\widehat{\beta}$'s strictly dominate simple AR(1) (in σ_{IV}'s and in their first differences), perform better than principal components in the majority of surfaces, and achieve comparable forecasting accuracy with the methodology of Gonçalves and Guidolin (2006).

Similar results emerge from the comparison of MAE of forecasts in Table 6.1. We can conclude that the simple decomposition of the IVS into seven intuitive factors can achieve excellent in–sample fit, whose dynamics can

Forecast accuracy measure: RMSE												
AUD												
Weeks	$\widehat{\beta}$ factors				$\widehat{\gamma}$ factors			Benchmarks				
Ahead	AR(1)	VAR(1)	BVAR(1)	ARFI$(1,d)$	AR(1)	VAR(1)	ECM	σ,AR(1)	$\Delta\sigma$,AR(1)	PC	PC,Δ	GG06
1	0.974	0.973	0.972	1.021	1.010	1.021	1.081	9.088	9.218	0.972	1.018	1.054
2	0.933	0.924	0.927	0.995	0.988	1.016	1.098	6.778	6.963	0.935	1.003	0.964
5	0.790	0.891	0.887	0.964	0.958	0.979	1.093	4.684	4.959	0.871	0.980	0.894
10	0.721	0.828	0.821	0.939	0.926	0.942	1.098	3.434	3.756	0.792	0.938	0.825
15	0.638	0.679	0.677	0.899	0.894	0.905	1.045	2.912	3.256	0.664	0.901	0.659
20	0.574	0.545	0.549	0.886	0.871	0.885	0.993	2.598	2.958	0.559	0.882	0.530
BRL												
1	0.991	0.987	0.985	1.131	1.048	1.026	1.074	4.432	4.410	0.972	0.994	1.011
2	0.928	0.862	0.861	1.086	1.031	1.007	1.039	3.229	3.189	0.943	0.984	0.868
5	0.755	0.790	0.784	1.066	1.012	0.989	1.000	2.475	2.376	0.909	0.972	0.777
10	0.586	0.712	0.705	1.034	1.015	0.987	0.999	1.935	1.773	0.835	0.974	0.699
15	0.501	0.657	0.651	1.016	1.015	0.982	0.988	1.858	1.648	0.774	0.972	0.628
20	0.442	0.597	0.593	1.020	1.021	0.990	0.981	1.808	1.556	0.755	0.986	0.560
CAD												
1	1.087	1.035	1.033	1.058	1.068	1.034	1.041	5.382	5.418	1.002	1.016	1.042
2	1.098	1.036	1.036	1.046	1.054	1.025	1.027	3.878	3.930	1.001	1.016	1.013
5	1.074	0.997	0.998	1.004	1.023	1.033	1.033	2.418	2.501	0.973	1.028	0.956
10	1.066	0.961	0.964	1.037	1.045	1.052	1.055	1.699	1.809	0.940	1.050	0.926
15	1.117	1.043	1.046	1.101	1.079	1.074	1.078	1.611	1.771	1.030	1.074	1.023
20	1.116	1.076	1.078	1.136	1.111	1.085	1.091	1.505	1.705	1.067	1.086	1.061
CHF												
1	1.008	0.976	0.973	0.984	0.979	1.022	1.040	8.779	8.660	0.972	1.044	1.156
2	1.002	0.949	0.946	0.957	0.971	1.011	0.988	6.868	6.684	0.908	1.017	1.045
5	1.004	0.833	0.834	0.923	0.944	0.951	0.929	4.931	4.651	0.822	0.949	0.916
10	0.887	0.750	0.752	0.840	0.869	0.875	0.854	4.101	3.732	0.743	0.880	0.793
15	0.747	0.704	0.705	0.785	0.791	0.800	0.749	3.379	3.030	0.706	0.803	0.723
20	0.672	0.674	0.675	0.763	0.774	0.787	0.719	2.800	2.493	0.675	0.789	0.682

continued

continued

Weeks Ahead	$\widehat{\beta}$ factors AR(1)	VAR(1)	BVAR(1)	ARFI$(1,d)$	$\widehat{\gamma}$ factors AR(1)	VAR(1)	ECM	Benchmarks σ,AR(1)	$\Delta\sigma$,AR(1)	PC	PC,Δ	GG06
CLP												
1	1.124	0.971	0.970	1.154	1.157	1.019	1.018	3.331	3.361	0.912	0.974	0.981
2	1.025	0.965	0.963	1.089	1.106	0.996	0.997	2.377	2.421	0.866	0.978	0.983
5	0.847	0.960	0.954	1.016	1.038	1.020	1.021	1.765	1.843	0.779	1.003	0.963
10	0.743	0.898	0.893	0.987	1.007	1.007	1.005	1.311	1.409	0.780	0.988	0.886
15	0.698	0.890	0.888	1.022	1.017	1.026	1.020	1.164	1.264	0.744	1.005	0.888
20	0.726	0.829	0.826	1.007	1.023	1.009	1.000	1.047	1.110	0.738	1.000	0.819
CZK												
1	1.305	1.006	0.993	1.345	1.354	1.029	1.037	4.844	4.837	1.022	1.049	0.992
2	1.210	0.989	0.956	1.343	1.313	0.966	0.985	4.563	4.549	1.001	0.968	0.976
5	1.110	1.086	1.061	1.096	1.078	1.050	1.043	4.383	4.352	1.041	1.053	1.057
10	1.145	1.053	1.071	1.244	1.082	1.136	1.114	3.657	3.610	1.029	1.132	1.108
15	1.117	0.971	0.999	1.268	1.155	1.196	1.174	3.159	3.103	0.974	1.180	1.000
20	1.086	0.900	0.925	1.320	1.086	1.184	1.159	2.573	2.515	0.881	1.168	0.908
DKK												
1	0.950	1.000	0.992	0.967	0.988	1.075	1.068	3.406	3.468	0.952	0.979	0.979
2	0.945	0.973	0.970	0.966	0.994	1.019	1.010	2.836	2.938	0.952	0.975	0.967
5	0.895	1.006	1.001	0.970	0.989	1.011	1.011	2.233	2.423	0.976	0.991	0.993
10	0.854	0.954	0.953	0.975	0.993	1.015	1.012	1.790	2.052	0.920	0.996	0.949
15	0.854	0.926	0.925	1.007	1.011	1.007	1.001	1.616	1.925	0.891	0.995	0.918
20	0.815	0.862	0.861	1.021	1.027	0.996	0.987	1.402	1.715	0.841	0.992	0.855
EUR												
1	0.973	1.071	1.056	0.965	0.961	1.042	1.052	14.842	14.696	0.977	1.025	1.241
2	0.964	1.036	1.025	0.947	0.961	1.030	1.039	11.458	11.239	0.926	1.010	1.108
5	0.940	0.834	0.833	0.906	0.915	0.937	0.944	7.815	7.495	0.778	0.927	0.903
10	0.811	0.697	0.698	0.818	0.827	0.842	0.851	6.007	5.617	0.640	0.842	0.725
15	0.671	0.624	0.624	0.731	0.737	0.748	0.765	4.759	4.398	0.596	0.750	0.636
20	0.596	0.590	0.590	0.711	0.720	0.741	0.760	3.845	3.528	0.577	0.738	0.598
GBP												
1	0.975	0.980	0.979	1.031	1.025	1.082	1.088	5.364	5.242	1.016	1.043	1.113
2	0.920	0.957	0.958	0.992	0.986	1.027	1.038	4.036	3.867	0.997	1.013	1.029
5	0.845	0.962	0.957	0.980	0.967	0.974	0.984	3.067	2.810	0.948	0.979	0.978
10	0.801	0.947	0.938	0.938	0.917	0.937	0.959	3.096	2.717	0.891	0.947	0.953
15	0.664	0.727	0.725	0.844	0.838	0.893	0.924	2.853	2.457	0.701	0.897	0.739
20	0.641	0.661	0.663	0.815	0.814	0.870	0.905	2.506	2.143	0.657	0.875	0.676

continued

continued

Weeks Ahead	$\widehat{\beta}$ factors AR(1)	VAR(1)	BVAR(1)	ARFI$(1,d)$	$\widehat{\gamma}$ factors AR(1)	VAR(1)	ECM	Benchmarks σ,AR(1)	$\Delta\sigma$,AR(1)	PC	PC,Δ	GG06
HUF												
1	1.043	1.076	1.068	1.060	1.082	1.035	1.086	4.722	4.804	1.050	1.087	1.066
2	1.019	1.080	1.075	1.032	1.053	1.010	1.056	3.309	3.418	1.024	1.026	1.069
5	0.950	0.994	0.990	1.010	1.046	1.031	0.997	2.339	2.516	0.933	1.018	0.984
10	0.822	0.791	0.789	0.994	1.072	1.055	0.800	1.538	1.758	0.796	1.035	0.777
15	0.760	0.745	0.745	0.981	1.085	1.062	0.815	1.305	1.568	0.769	1.047	0.737
20	0.767	0.769	0.769	0.983	1.096	1.082	0.811	1.105	1.360	0.799	1.066	0.767
ILS												
1	1.083	1.051	1.048	1.095	1.094	1.154	1.147	NaN	NaN	1.011	1.017	1.093
2	1.040	1.046	1.043	1.069	1.070	1.012	1.011	NaN	NaN	1.001	1.006	1.100
5	1.000	1.001	0.999	1.052	1.051	1.055	1.048	NaN	NaN	0.931	1.012	1.020
10	0.969	0.943	0.940	1.079	1.069	1.079	1.066	NaN	NaN	0.857	1.047	0.918
15	0.967	0.912	0.907	1.141	1.129	1.111	1.094	NaN	NaN	0.832	1.085	0.908
20	0.910	0.871	0.863	1.140	1.156	1.141	1.121	NaN	NaN	0.763	1.117	0.847
JPY												
1	1.004	1.036	1.031	1.017	1.012	1.015	1.085	6.931	6.924	0.950	0.950	1.081
2	0.994	1.028	1.022	1.033	1.021	1.022	1.118	5.312	5.299	0.949	0.984	1.043
5	0.898	0.914	0.911	1.034	0.998	1.002	1.030	4.010	3.971	0.846	0.985	0.925
10	0.761	0.765	0.764	1.001	0.954	0.982	0.880	3.264	3.185	0.726	0.968	0.791
15	0.727	0.729	0.728	0.953	0.939	0.967	0.831	3.056	2.934	0.700	0.949	0.735
20	0.647	0.645	0.645	0.928	0.897	0.924	0.824	3.016	2.835	0.624	0.903	0.642
KRW												
1	1.019	0.979	0.978	1.016	1.016	1.009	1.002	4.040	4.020	0.987	1.023	1.030
2	1.007	0.957	0.956	0.995	0.999	1.011	1.007	3.092	3.061	0.947	1.007	1.009
5	0.960	0.933	0.931	0.972	0.971	1.021	1.020	2.294	2.235	0.874	1.008	0.972
10	0.897	0.890	0.888	0.962	0.963	1.032	1.037	2.022	1.912	0.811	1.022	0.903
15	0.865	0.868	0.867	0.936	0.952	1.004	1.013	2.173	1.994	0.813	0.983	0.849
20	0.775	0.773	0.772	0.913	0.926	0.988	1.001	1.962	1.761	0.744	0.965	0.758
NOK												
1	0.992	1.015	1.012	1.014	1.007	1.020	1.036	2.984	3.015	0.982	1.007	1.006
2	0.961	1.023	1.021	0.998	0.987	1.016	1.065	2.402	2.453	0.982	0.994	0.986
5	0.915	1.022	1.020	1.009	0.999	1.037	1.084	1.878	1.981	0.995	1.019	0.991
10	0.883	0.962	0.962	1.002	1.006	1.034	1.017	1.586	1.753	0.960	1.021	0.943
15	0.906	0.941	0.943	1.048	1.024	1.030	0.960	1.488	1.716	0.949	1.012	0.938
20	0.861	0.889	0.891	1.059	1.048	1.012	0.914	1.292	1.539	0.903	1.004	0.888

continued

continued

Weeks Ahead	$\widehat{\beta}$ factors AR(1)	$\widehat{\beta}$ factors VAR(1)	$\widehat{\beta}$ factors BVAR(1)	$\widehat{\beta}$ factors ARFI$(1,d)$	$\widehat{\gamma}$ factors AR(1)	$\widehat{\gamma}$ factors VAR(1)	$\widehat{\gamma}$ factors ECM	Benchmarks σ,AR(1)	Benchmarks $\Delta\sigma$,AR(1)	Benchmarks PC	Benchmarks PC,Δ	Benchmarks GG06
NZD												
1	1.023	0.999	0.998	1.058	1.051	1.058	1.056	3.692	3.721	0.994	1.045	1.048
2	1.027	0.986	0.984	1.094	1.080	1.041	1.043	2.932	2.980	0.981	1.029	1.003
5	0.945	0.936	0.934	1.077	1.049	1.036	1.039	1.897	1.972	0.935	1.026	0.937
10	0.887	0.865	0.864	1.086	1.043	1.044	1.045	1.328	1.427	0.869	1.039	0.869
15	0.844	0.803	0.803	1.103	1.053	1.060	1.061	1.126	1.247	0.811	1.055	0.806
20	0.824	0.760	0.760	1.156	1.078	1.081	1.082	1.038	1.177	0.773	1.075	0.756
PLN												
1	1.058	1.004	1.002	1.056	1.056	1.058	1.066	4.335	4.357	0.966	1.007	1.001
2	1.101	0.935	0.934	1.073	1.071	1.047	1.046	3.467	3.500	0.918	1.022	0.927
5	1.130	0.850	0.850	1.099	1.100	1.108	1.104	2.501	2.561	0.863	1.087	0.842
10	0.978	0.789	0.789	1.147	1.139	1.122	1.117	1.876	1.962	0.821	1.115	0.787
15	0.885	0.764	0.765	1.195	1.182	1.165	1.157	1.750	1.865	0.804	1.161	0.773
20	0.840	0.750	0.751	1.210	1.188	1.164	1.157	1.656	1.788	0.798	1.161	0.758
SEK												
1	1.015	0.945	0.942	1.019	1.013	0.998	1.008	2.735	2.766	0.941	0.968	0.973
2	1.059	0.986	0.984	1.062	1.051	1.015	1.145	2.380	2.432	0.963	0.988	0.996
5	0.951	0.979	0.977	1.023	1.024	1.010	1.143	1.747	1.842	0.926	1.003	0.982
10	0.865	0.955	0.954	1.010	1.018	1.014	1.069	1.450	1.597	0.894	1.005	0.946
15	0.863	0.916	0.917	1.064	1.043	1.018	0.985	1.357	1.554	0.883	1.012	0.921
20	0.806	0.855	0.855	1.067	1.064	1.010	0.922	1.215	1.436	0.833	1.010	0.859
SGD												
1	1.071	0.993	0.991	1.071	1.095	1.023	1.021	5.299	5.331	0.973	0.983	1.013
2	1.037	0.947	0.945	1.042	1.068	1.004	1.001	3.789	3.828	0.932	0.981	0.942
5	0.935	0.784	0.787	0.961	1.024	1.017	1.018	2.476	2.526	0.766	1.003	0.776
10	0.731	0.633	0.634	0.907	0.997	1.008	1.015	1.930	2.012	0.628	1.004	0.626
15	0.639	0.591	0.592	0.917	1.003	0.997	1.012	1.799	1.902	0.589	0.996	0.588
20	0.607	0.598	0.598	0.930	0.983	0.980	1.001	1.790	1.895	0.593	0.978	0.596
THB												
1	1.070	1.072	1.069	1.078	1.083	1.082	1.089	8.048	8.014	0.985	0.980	1.088
2	1.085	1.090	1.087	1.104	1.116	1.093	1.092	5.942	5.888	0.974	1.011	1.093
5	0.954	1.052	1.051	1.004	1.033	1.073	1.076	4.060	3.968	0.936	1.042	1.056
10	0.990	1.095	1.095	1.084	1.117	1.154	1.158	3.536	3.378	0.982	1.118	1.112
15	1.200	1.270	1.270	1.279	1.244	1.272	1.278	3.046	2.824	1.187	1.237	1.288
20	1.300	1.329	1.329	1.407	1.354	1.360	1.367	2.450	2.177	1.285	1.322	1.337

continued

continued

Weeks Ahead	$\widehat{\beta}$ factors AR(1)	VAR(1)	BVAR(1)	ARFI$(1,d)$	$\widehat{\gamma}$ factors AR(1)	VAR(1)	ECM	Benchmarks σ,AR(1)	$\Delta\sigma$,AR(1)	PC	PC,Δ	GG06
TWD												
1	1.046	1.057	1.049	1.026	1.028	1.068	1.145	4.191	4.295	0.976	1.002	1.064
2	1.054	1.056	1.044	1.028	1.027	1.029	1.102	2.967	3.103	0.947	0.991	1.031
5	0.987	0.917	0.911	1.002	0.970	1.017	0.997	1.944	2.130	0.849	0.999	0.885
10	0.988	0.862	0.858	1.063	1.009	1.030	0.921	1.693	1.925	0.812	1.019	0.823
15	0.837	0.737	0.733	1.091	1.016	1.068	0.824	1.625	1.870	0.703	1.054	0.709
20	0.800	0.735	0.731	1.112	1.030	1.089	0.834	1.634	1.868	0.711	1.082	0.712
VEB												
1	1.050	1.144	1.102	1.018	1.024	1.029	1.246	14.429	14.373	0.433	0.364	2.605
2	1.126	1.278	1.207	1.055	1.070	1.051	1.679	14.399	14.277	0.581	0.433	2.647
5	1.405	1.609	1.540	1.266	1.327	1.225	2.284	14.430	14.140	0.850	0.764	2.777
10	1.791	1.990	1.989	1.718	1.835	1.697	2.628	14.479	13.947	0.996	1.396	2.958
15	2.063	2.181	2.185	2.246	2.437	2.260	2.984	14.485	13.752	1.146	2.039	3.062
20	1.170	1.188	1.181	1.200	1.250	1.226	1.341	2.482	2.327	1.128	1.213	1.231
ZAR												
1	1.021	1.039	1.036	1.017	1.025	0.999	0.998	5.815	5.776	1.032	1.008	1.104
2	1.036	1.017	1.013	1.035	1.051	1.010	1.009	3.840	3.790	0.991	1.009	1.018
5	0.957	0.965	0.962	1.010	1.047	1.045	1.038	2.545	2.460	0.920	1.049	0.945
10	0.930	0.896	0.895	1.102	1.146	1.107	1.091	1.899	1.798	0.854	1.100	0.873
15	0.894	0.862	0.861	1.129	1.172	1.163	1.140	1.595	1.492	0.828	1.158	0.846
20	0.972	0.931	0.930	1.224	1.258	1.232	1.197	1.539	1.445	0.908	1.237	0.936

Table 7: For the volatility surfaces implied by the twenty two different currency options in our sample, the table presents the root mean squared errors (RMSE) of out–of–sample h–week ahead forecasts, of seven models and five benchmarks, all divided by the root mean squared errors of h–week ahead forecasts produced by the random walk model, equation (13). Models of the $\widehat{\beta}$ and $\widehat{\gamma}$ factors refer to cases [a]–[h] on pp. 16–19 and the are estimated over the first 100 weeks of our sample period; then forecasts and estimates are made recursively until 21/5/2007. Benchmarks refer to cases (1)–(6) on pp. 20–21.

assist in making accurate medium to long–term predictions of the future IVS.

5 Conclusions

No single empirically observed deviation from the Black–Scholes–Merton option pricing framework has attracted more research effort than the nonconstant pattern of implied volatility versus the moneyness and time to maturity dimensions.

Despite advances in describing the characteristics and dynamics of non–flat implied volatility surfaces and recent general equilibrium structural models that have proposed economic justifications for the existence of the phenomenon, relatively little has been said regarding the practical problem of implied volatility surface forecasting.

In this paper, we take an explicitly out–of–sample forecasting approach. We propose a simple–to–estimate parametric decomposition of the implied volatility surface that combines and extends previous research in several respects. Using daily data from a cross–section of options on 22 different currencies quoted against the U.S. $ from the OTC market, we demonstrate that the approach yields intuitive and easy to communicate factors that achieve excellent in–sample fit, and whose time–variation capture the dynamics of the surface.

Simple econometric models for the factors are estimated and used for making short and long–term prediction of implied volatility surfaces. Although the 1–month–ahead (5 weeks) forecasting results are no better than those of random walk and other leading benchmarks, the forecasts of 5 to 20–weeks–ahead are much superior.

In concluding the paper we would like to stress that although the factor models we consider are not arbitrage–free, they are based on sound theoretical justifications and established empirical practices that can explain their forecasting success. On the theoretical front, the factor models we examine can be considered reduced–form analogs of more structural models, such as that proposed by Garcia, Luger and Renault (2003). There, predictability in the IVS dynamics arises as a consequence of investors' learning (from option prices) about the processes of fundamentals that are driven by persistent factors. Our simple econometric models seem to pick up this theoretically justified predictability. Moreover, empirical intuition and experience has established that the "shrinkage perspective, which tends to produce seemingly naive but truly sophisticatedly simple models (of which ours is one example), may be very appealing when the goal is forecasting", as Diebold and Li (2006, p. 362) argue. In our setting, by imposing strict structure on the factors extracted from the implied volatility surfaces, seems to help in improving medium to long–term forecasts.

6 Appendix: Additional forecasting results

In the lengthy table that follows, we report forecasting mean absolute errors (MAE) of competing models and benchmarks. Again, to facilitate comparisons across models all h–week–ahead MAE are standardised (divided by) the corresponding MAE of the random walk model in equation (13).

Forecast accuracy measure: MAE												
AUD												
Weeks	$\widehat{\beta}$ factors				$\widehat{\gamma}$ factors			Benchmarks				
Ahead	AR(1)	VAR(1)	BVAR(1)	ARFI$(1,d)$	AR(1)	VAR(1)	ECM	σ,AR(1)	$\Delta\sigma$,AR(1)	PC	PC,Δ	GG06
1	0.980	0.981	0.980	1.036	1.026	1.006	1.076	10.201	10.359	0.967	1.013	1.065
2	0.915	0.934	0.936	0.992	0.985	1.012	1.107	8.061	8.299	0.941	0.987	1.015
5	0.767	0.807	0.802	0.959	0.962	0.969	1.060	5.166	5.501	0.790	0.962	0.850
10	0.707	0.774	0.769	0.930	0.927	0.938	1.067	3.833	4.244	0.749	0.928	0.797
15	0.592	0.609	0.608	0.875	0.892	0.885	0.979	3.077	3.487	0.598	0.875	0.606
20	0.525	0.478	0.479	0.862	0.866	0.868	0.916	2.688	3.096	0.483	0.861	0.467
BRL												
1	1.011	1.050	1.048	1.103	1.055	1.012	1.097	5.672	5.623	0.999	0.997	1.076
2	0.957	0.880	0.879	1.057	1.035	0.992	1.025	4.003	3.925	0.970	0.976	0.893
5	0.786	0.858	0.851	1.021	1.008	1.005	1.011	3.038	2.868	0.934	0.995	0.840
10	0.643	0.838	0.830	1.030	1.036	1.069	1.050	2.508	2.211	0.864	1.067	0.813
15	0.544	0.788	0.781	1.021	1.042	1.088	1.058	2.379	2.009	0.787	1.086	0.729
20	0.511	0.747	0.742	1.075	1.065	1.168	1.116	2.440	1.983	0.761	1.172	0.684
CAD												
1	1.056	1.029	1.027	1.057	1.070	1.037	1.044	6.116	6.155	0.988	1.020	1.023
2	1.101	1.036	1.036	1.059	1.072	1.027	1.029	4.364	4.421	1.010	1.023	1.016
5	1.041	0.962	0.962	1.009	1.031	1.036	1.037	2.587	2.680	0.924	1.033	0.918
10	1.007	0.911	0.912	1.043	1.050	1.055	1.058	1.785	1.907	0.892	1.054	0.876
15	1.092	1.024	1.027	1.104	1.084	1.060	1.066	1.728	1.897	1.003	1.058	1.018
20	1.112	1.081	1.084	1.112	1.104	1.046	1.056	1.644	1.857	1.075	1.045	1.094
CHF												
1	1.036	0.974	0.972	0.990	0.976	1.022	1.026	12.008	11.814	1.024	1.060	1.230
2	0.997	0.930	0.927	0.951	0.962	0.999	0.955	8.896	8.626	0.914	1.016	1.046
5	1.003	0.809	0.811	0.932	0.950	0.957	0.902	6.043	5.657	0.801	0.951	0.884
10	0.992	0.752	0.755	0.824	0.843	0.851	0.881	5.336	4.806	0.752	0.852	0.817
15	0.811	0.687	0.689	0.731	0.743	0.753	0.769	4.307	3.805	0.691	0.758	0.730
20	0.687	0.645	0.645	0.702	0.713	0.729	0.704	3.398	2.968	0.648	0.727	0.669

continued

continued

Weeks Ahead	$\widehat{\beta}$ factors AR(1)	VAR(1)	BVAR(1)	ARFI$(1,d)$	$\widehat{\gamma}$ factors AR(1)	VAR(1)	ECM	Benchmarks σ,AR(1)	$\Delta\sigma$,AR(1)	PC	PC,Δ	GG06
CLP												
1	1.194	1.040	1.038	1.205	1.217	1.050	1.050	4.188	4.233	0.961	1.015	1.056
2	1.020	1.027	1.024	1.045	1.097	1.008	1.009	2.883	2.944	0.898	1.000	1.027
5	0.847	0.984	0.976	0.987	1.020	1.008	1.009	1.992	2.089	0.793	0.995	0.960
10	0.776	0.964	0.957	1.000	1.018	1.009	1.009	1.441	1.555	0.810	0.988	0.926
15	0.741	0.913	0.909	1.005	1.008	1.011	1.007	1.255	1.379	0.789	0.981	0.886
20	0.708	0.838	0.834	0.990	1.019	0.987	0.977	1.043	1.127	0.726	0.982	0.812
CZK												
1	1.372	1.080	1.060	1.367	1.365	1.085	1.095	8.015	8.008	1.109	1.118	1.089
2	1.257	1.086	1.060	1.277	1.253	0.968	0.988	6.169	6.157	1.089	0.989	1.092
5	1.220	1.171	1.146	1.077	1.068	1.045	1.040	5.060	5.039	1.139	1.038	1.170
10	1.239	1.132	1.149	1.103	1.056	1.098	1.075	4.007	3.981	1.136	1.090	1.176
15	1.155	1.011	1.037	1.118	1.083	1.114	1.094	3.383	3.357	1.047	1.103	1.044
20	1.032	0.899	0.923	1.087	0.997	1.082	1.058	2.696	2.676	0.916	1.071	0.923
DKK												
1	0.948	1.037	1.028	0.980	1.024	1.050	1.043	4.119	4.191	0.981	0.991	1.032
2	0.942	1.024	1.018	0.971	1.019	1.028	1.019	3.185	3.298	0.967	0.992	1.007
5	0.878	1.022	1.014	0.959	0.992	1.016	1.013	2.352	2.561	0.962	1.001	0.997
10	0.875	1.003	1.000	1.003	1.003	1.018	1.016	1.902	2.205	0.954	0.995	0.995
15	0.808	0.904	0.903	1.023	1.027	1.012	1.004	1.565	1.904	0.863	1.001	0.887
20	0.760	0.820	0.819	1.032	1.029	1.002	0.994	1.326	1.672	0.805	0.999	0.810
EUR												
1	0.996	1.044	1.028	0.963	0.960	1.046	1.053	20.140	19.929	1.018	1.043	1.291
2	0.945	1.016	1.004	0.938	0.953	1.023	1.032	14.706	14.415	0.929	1.016	1.102
5	0.949	0.827	0.827	0.920	0.928	0.944	0.951	9.953	9.534	0.773	0.935	0.884
10	0.923	0.727	0.730	0.855	0.851	0.860	0.861	8.002	7.469	0.668	0.855	0.759
15	0.706	0.595	0.596	0.709	0.719	0.727	0.749	5.809	5.343	0.559	0.734	0.616
20	0.636	0.586	0.586	0.704	0.711	0.732	0.748	4.696	4.280	0.570	0.728	0.602
GBP												
1	0.986	1.025	1.018	1.025	1.020	1.120	1.128	6.993	6.819	1.030	1.055	1.167
2	0.942	0.986	0.984	0.997	0.991	1.044	1.051	5.186	4.953	1.019	1.020	1.060
5	0.836	0.931	0.924	0.971	0.971	0.985	0.995	3.860	3.514	0.911	0.982	0.965
10	0.780	0.916	0.903	0.939	0.924	0.946	0.966	3.825	3.333	0.870	0.946	0.907
15	0.640	0.679	0.678	0.828	0.826	0.882	0.909	3.448	2.964	0.688	0.883	0.698
20	0.598	0.595	0.597	0.793	0.797	0.862	0.897	2.942	2.515	0.609	0.861	0.618

continued

continued

Weeks Ahead	$\widehat{\beta}$ factors AR(1)	VAR(1)	BVAR(1)	ARFI(1, d)	$\widehat{\gamma}$ factors AR(1)	VAR(1)	ECM	Benchmarks σ,AR(1)	$\Delta\sigma$,AR(1)	PC	PC,Δ	GG06
HUF												
1	1.085	1.066	1.058	1.100	1.122	1.065	1.092	5.257	5.338	1.045	1.090	1.069
2	1.022	1.046	1.042	1.017	1.038	1.007	1.028	3.481	3.585	1.004	1.007	1.038
5	0.916	0.980	0.976	1.001	1.047	1.045	0.979	2.461	2.638	0.917	1.009	0.973
10	0.849	0.808	0.805	1.026	1.104	1.078	0.779	1.607	1.830	0.804	1.065	0.794
15	0.781	0.772	0.770	1.010	1.106	1.097	0.819	1.284	1.519	0.788	1.094	0.759
20	0.792	0.787	0.787	0.988	1.074	1.075	0.815	1.098	1.307	0.815	1.057	0.789
ILS												
1	1.138	1.056	1.053	1.122	1.104	1.143	1.136	NaN	NaN	1.049	1.014	1.143
2	1.093	1.070	1.069	1.121	1.094	1.011	1.008	NaN	NaN	1.041	1.002	1.156
5	1.023	1.035	1.032	1.083	1.070	1.054	1.046	NaN	NaN	0.966	1.006	1.059
10	0.952	0.917	0.914	1.070	1.071	1.068	1.058	NaN	NaN	0.844	1.046	0.901
15	0.982	0.887	0.883	1.135	1.118	1.100	1.087	NaN	NaN	0.828	1.087	0.900
20	0.932	0.848	0.843	1.137	1.138	1.120	1.104	NaN	NaN	0.772	1.107	0.836
JPY												
1	1.010	1.029	1.023	1.024	1.021	1.010	1.081	8.798	8.782	0.925	0.918	1.072
2	1.028	1.043	1.037	1.069	1.058	1.022	1.125	6.908	6.879	0.945	0.958	1.059
5	0.892	0.910	0.908	1.067	1.037	1.009	1.039	4.893	4.816	0.824	0.972	0.921
10	0.782	0.783	0.782	1.035	0.987	1.018	0.904	3.955	3.796	0.731	0.996	0.806
15	0.744	0.730	0.729	0.995	0.959	0.991	0.809	3.660	3.441	0.708	0.960	0.730
20	0.607	0.599	0.599	0.896	0.869	0.896	0.781	3.430	3.133	0.578	0.864	0.586
KRW												
1	1.008	0.986	0.985	1.013	1.012	1.018	1.003	4.917	4.891	0.998	1.025	1.059
2	1.003	0.963	0.961	1.011	1.007	0.998	0.987	3.510	3.473	0.945	0.993	1.023
5	0.911	0.893	0.891	0.939	0.925	0.970	0.964	2.424	2.360	0.827	0.958	0.942
10	0.882	0.865	0.864	0.978	0.984	1.039	1.048	2.101	1.969	0.806	1.043	0.885
15	0.839	0.840	0.839	0.937	0.965	0.975	0.992	2.229	2.014	0.784	0.975	0.819
20	0.734	0.735	0.734	0.895	0.921	0.970	0.985	1.979	1.751	0.702	0.954	0.715
NOK												
1	0.976	1.037	1.033	1.009	1.004	0.994	1.056	3.488	3.527	1.008	1.006	1.046
2	0.974	1.035	1.033	1.014	1.000	1.020	1.089	2.622	2.680	0.983	0.989	1.017
5	0.902	1.032	1.029	1.009	1.006	1.027	1.089	1.970	2.082	0.976	1.014	1.016
10	0.886	0.993	0.992	1.014	1.011	1.038	1.039	1.666	1.839	0.980	1.023	0.975
15	0.861	0.907	0.909	1.055	1.031	1.016	0.903	1.503	1.734	0.917	1.001	0.903
20	0.818	0.858	0.861	1.067	1.057	1.013	0.838	1.306	1.547	0.878	1.001	0.852

continued

continued

Weeks Ahead	$\widehat{\beta}$ factors AR(1)	$\widehat{\beta}$ factors VAR(1)	$\widehat{\beta}$ factors BVAR(1)	$\widehat{\beta}$ factors ARFI$(1,d)$	$\widehat{\gamma}$ factors AR(1)	$\widehat{\gamma}$ factors VAR(1)	$\widehat{\gamma}$ factors ECM	Benchmarks σ,AR(1)	Benchmarks $\Delta\sigma$,AR(1)	Benchmarks PC	Benchmarks PC,Δ	Benchmarks GG06
NZD												
1	1.048	0.997	0.998	1.067	1.059	1.046	1.046	4.024	4.057	1.001	1.048	1.063
2	1.047	0.992	0.990	1.080	1.078	1.044	1.048	3.211	3.265	0.991	1.035	1.024
5	0.954	0.941	0.939	1.057	1.043	1.038	1.040	2.065	2.155	0.934	1.026	0.944
10	0.917	0.880	0.877	1.085	1.053	1.054	1.053	1.443	1.568	0.869	1.049	0.882
15	0.860	0.800	0.799	1.119	1.068	1.069	1.066	1.146	1.296	0.800	1.065	0.797
20	0.835	0.753	0.753	1.160	1.078	1.076	1.075	1.015	1.182	0.768	1.071	0.747
PLN												
1	1.058	1.023	1.020	1.080	1.077	1.056	1.060	5.060	5.088	0.983	1.000	0.999
2	1.139	0.982	0.981	1.116	1.108	1.050	1.051	4.113	4.156	0.949	1.020	0.947
5	1.178	0.857	0.857	1.123	1.122	1.090	1.088	2.817	2.890	0.875	1.068	0.853
10	1.060	0.831	0.831	1.139	1.142	1.130	1.124	2.058	2.162	0.853	1.125	0.834
15	0.981	0.823	0.823	1.154	1.186	1.165	1.157	1.909	2.046	0.858	1.160	0.833
20	0.939	0.815	0.816	1.162	1.179	1.160	1.151	1.830	1.993	0.876	1.158	0.822
SEK												
1	1.057	1.010	1.006	1.071	1.081	1.010	1.073	3.377	3.417	0.981	1.016	1.038
2	1.011	0.986	0.983	1.039	1.063	1.010	1.128	2.627	2.687	0.952	0.997	1.008
5	0.891	0.993	0.990	0.985	1.013	1.001	1.158	1.861	1.966	0.912	0.998	0.982
10	0.873	0.985	0.983	1.003	1.008	1.024	1.082	1.532	1.689	0.912	1.015	0.975
15	0.836	0.912	0.912	1.067	1.048	1.006	0.955	1.396	1.598	0.863	1.001	0.914
20	0.746	0.813	0.813	1.069	1.072	1.016	0.849	1.198	1.405	0.785	1.015	0.814
SGD												
1	1.062	0.971	0.970	1.082	1.102	1.002	1.003	6.228	6.254	0.935	0.961	1.018
2	0.980	0.937	0.932	1.026	1.044	0.993	0.992	4.238	4.270	0.938	0.972	0.943
5	0.869	0.765	0.765	0.966	1.025	1.012	1.012	2.692	2.742	0.750	0.999	0.754
10	0.706	0.607	0.608	0.893	1.000	1.010	1.012	2.049	2.120	0.599	1.009	0.600
15	0.616	0.567	0.567	0.893	0.984	0.959	0.981	1.840	1.927	0.562	0.962	0.560
20	0.620	0.608	0.608	0.905	0.965	0.969	0.983	1.900	1.999	0.602	0.972	0.605
THB												
1	1.161	1.133	1.129	1.123	1.124	1.092	1.104	10.230	10.148	1.022	0.984	1.156
2	1.208	1.187	1.184	1.131	1.133	1.096	1.096	7.517	7.396	1.050	1.005	1.197
5	1.055	1.159	1.159	1.004	1.019	1.067	1.070	4.830	4.639	1.025	1.024	1.167
10	1.042	1.149	1.149	1.073	1.102	1.131	1.139	3.869	3.573	1.015	1.088	1.161
15	1.195	1.284	1.284	1.286	1.276	1.294	1.309	3.458	3.068	1.170	1.247	1.299
20	1.265	1.314	1.314	1.498	1.487	1.476	1.492	2.823	2.400	1.250	1.430	1.319

continued

continued

Weeks Ahead	$\widehat{\beta}$ factors AR(1)	VAR(1)	BVAR(1)	ARFI$(1,d)$	$\widehat{\gamma}$ factors AR(1)	VAR(1)	ECM	Benchmarks σ,AR(1)	$\Delta\sigma$,AR(1)	PC	PC,Δ	GG06
TWD												
1	1.021	1.062	1.050	1.011	1.019	1.056	1.145	4.650	4.763	0.976	0.992	1.096
2	1.037	1.090	1.075	0.988	1.015	1.021	1.141	3.406	3.562	0.955	0.986	1.062
5	0.956	0.938	0.934	0.954	0.948	1.028	1.007	2.107	2.308	0.853	1.005	0.899
10	1.050	0.927	0.924	1.057	1.007	1.042	0.971	1.859	2.099	0.872	1.035	0.891
15	0.846	0.748	0.745	1.037	0.989	1.072	0.832	1.608	1.816	0.711	1.057	0.728
20	0.840	0.768	0.765	1.079	1.020	1.091	0.883	1.640	1.815	0.732	1.088	0.751
VEB												
1	1.063	1.102	1.090	1.008	1.026	1.033	1.161	16.569	16.508	0.412	0.355	2.851
2	1.158	1.240	1.191	1.054	1.090	1.065	1.493	16.574	16.437	0.541	0.404	2.868
5	1.468	1.620	1.543	1.310	1.399	1.272	2.093	16.627	16.299	0.822	0.803	2.902
10	1.897	2.038	2.053	1.804	1.972	1.814	2.544	16.674	16.072	1.008	1.570	2.939
15	2.210	2.273	2.320	2.399	2.666	2.459	2.961	16.685	15.852	1.125	2.335	2.939
20	1.526	1.504	1.506	1.717	1.852	1.770	1.914	5.227	4.925	1.154	1.751	1.677
ZAR												
1	1.092	1.116	1.113	1.037	1.041	1.008	1.001	7.831	7.758	1.097	1.031	1.151
2	1.183	1.153	1.148	1.126	1.133	1.030	1.024	5.373	5.274	1.094	1.019	1.142
5	1.099	1.116	1.111	1.108	1.130	1.097	1.080	3.492	3.329	1.039	1.080	1.106
10	1.003	0.955	0.953	1.189	1.234	1.156	1.131	2.278	2.109	0.885	1.144	0.932
15	0.919	0.878	0.876	1.197	1.248	1.204	1.175	1.801	1.639	0.828	1.200	0.868
20	1.010	0.959	0.958	1.252	1.308	1.259	1.224	1.677	1.519	0.929	1.260	0.963

Table 6.1: For the volatility surfaces implied by the twenty two different currency options in our sample, the table presents the mean absolute errors (MAE) of out–of–sample h–week ahead forecasts, of seven models and five benchmarks, all divided by the mean absolute errors of h–week ahead forecasts produced by the random walk model, equation (13). Models of the $\widehat{\beta}$ and $\widehat{\gamma}$ factors refer to cases [a]–[h] on pp. 16–19 and the are estimated over the first 100 weeks of our sample period; then forecasts and estimates are made recursively until 21/5/2007. Benchmarks refer to cases (1)–(6) on pp. 20–21.

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