Informed Trading, Predictable Noise Trading Activities and Market Manipulation

Jungsuk Han*

January, 2009

Abstract

Traditional models of informed trading typically assume the existence of noise trading activities which generate pure random noises in trading volumes. This paper studies a multi-period model of speculative trading in the presence of a systematic component of the noise trading activities which is privately observed by a monopolistic risk-averse informed trader. Because of the incentive to hide the magnitude of informed trading, the informed trader may comove with the mispricing caused by the systematic component of noise trading instead of engaging in arbitrage. The result implies that an arbitrager who has superior information on non-fundamentals such as investor sentiment may not always reduce the mispricing caused by them given private information on fundamentals. This paper demonstrates that market manipulation could easily occur in a standard Kyle model with relatively mild assumptions if private information has more than two dimensions: (i) fundamentals and (ii) non-fundamentals.

JEL Classification Codes: G12, G14, D82

Keywords: Market microstructure, informed trading, noise trading, market manipulation

*I am grateful to James Dow, Emeric Henry, Guillaume Plantin for many helpful discussions and comments. Any errors are mine alone. **Correspondence Information:** Jungsuk Han, London Business School, Regent's Park, London, NW1 4SA, tel: +44 (0)20 7262 5050 extension (8248) `mailto:jhan.phd2005@london.edu`

1 Introduction

Would a rational arbitrager correct the mispricing caused by noises in trading volume? The arbitrager might well trade against noise traders because he would benefit from engaging in arbitrage activities as long as there is no market frictions such as short-sale constraint, limited liability and limited investment horizon.[1] This paper attempts to answer the question in a slight different situation where a certain portion of noise trading activities are systematic, hence it is predictable to some degree. I assume that an arbitrager has superior information on both the liquidation value of a risky asset and the systematic component of noise trading activities. Since market makers attempt to predict the systematic portion of noise trading as well, market makers' forecasting error on the systematic component of noise trading causes mispricing. The results show that the arbitrager may not always correct the mispricing caused by noise trading even in the absence of any market friction as long as he has the incentive to camouflage his informed trading to further exploit the private information on fundamentals. While the arbitrager in limited arbitrage literature does not correct the mispricing due to the frictions while the arbitrager in this model intentionally amplifies the mispricing.

This paper develops a dynamic model of informed trading in the presence of an autoregressive component of noise trading and public information release. Kyle (1985) has shown that a monopolistic risk-neutral informed trader gradually reveals his private information through his trades over time when he holds private information on the risky asset. Holden and Subrahmanyam (1992) and Holden and Subrahmanyam (1994) extend this finding by incorporating competition among informed traders and risk aversion to the preference of informed traders. Their result shows that both competition among informed traders and risk aversion of informed traders make informed traders more aggressive in the initial stage, thus private information is revealed more quickly. This paper extends Kyle model by adding a systematic component of noise trading which follows an autoregressive process irrelevant to any fundamentals or infor-

[1]For example, Dow and Gorton (1994) show that an arbitrager who has limited investment horizon refrains from arbitrage because of the cost-of-carry associated with holding an arbitrage portfolio over an extended period of time.

mation. Furthermore, market makers in this model are allowed to collect public signals on the liquidation value of a risky asset beside the information from trading volumes.

The main difference between competitive models (e.g. Grossman and Stiglitz (1980), Wang (1993), Wang (1994) and He and Wang (1995)) and Kyle model (e.g. Kyle (1985), Holden and Subrahmanyam (1992), Holden and Subrahmanyam (1994), Back, Cao, and Willard (2000), Foster and Viswanathan (1996), and Bernhaedt and Miao (2004)) is that informed traders are not price-takers in Kyle model, and their impact on the equilibrium price is incorporated in informed traders' optimization problem. Unlike competitive models, the inclusion of traders' own impact on the equilibrium price leads to extra technical complication in solving equilibrium.

This paper makes difference from the previous literature in the strain of Kyle model on the following points: (i) This paper introduces a time-varying systematic component of noise trading as another dimension of private information to the informed trader. Therefore, the decision making of the informed trader is associated not only with his private information on fundamental factors but also with non-fundamental factors. (ii) It features public signals, which is observed by everyone in the economy. It enables us to explore the impact of public signals on informed trading in the presence of predictable noise trading activities. The result shows that it may force the informed trader to correct the mispricing instead of riding on it. (iii) To develop a more flexible version of Kyle model in discrete time, this paper adopts a more generalized approach similar to Wang (1994), and He and Wang (1995). This reformulation of Kyle model is simply a mathematical reinterpretation of the model for achieving extra tractablity. Thus, it does not alter any assumption in Kyle model, thereby keeping the properties of Kyle model unchanged. One of the benefit of this formulation is that it enables the model to feature dynamic learning of stochastic process using a linear filtering technique. This new reinterpretation also reveals that that the informed trader's demand for the risky asset at each period could be decomposed into two separate components according to their trading motives: (1) a corrective demand which is coming from a typical mean-variance demand for the excess return as a price-taker, and (2) a manipulative demand which is driven by price-controlling motives. This decomposition of the informed trader's demand plays a key role in revealing why the informed trader comoves with market makers' forecasting errors on the systematic component of noise trading instead of gaining immediate profit by engaging in arbitrage. Particularly, the decomposition of the informed trader's demand reveals that the comovement with the mispricing is caused only by manipulative demand, i.e., the informed trader suffers short-term losses to gain long-term profits by increasing noise in the market, which is market manipulation.

There are some empirical evidence supporting that informed traders could have superior information on noise trading activities. Brunnermeier and Nagel (2004) finds informed hedge funds prefer to ride bubbles because of predictable investor sentiment and limits to arbitrage. Chen, Hanson, Hong, and Stein (2008) finds that hedge funds engage in front-running strategies that exploit the predictable trades of mutual funds. Besides empirical facts, it is natural to assume that informed traders know more about noise trading activities since they are more able to distinguish uninformed order flows from informed order flows due to their private information. In case of pure random noises, however, even informed traders would not be able to have better estimation of such irrational trading volumes simply because there is no systematic patterns. As long as some systematic component persists in the uninformed order flows over time, informed traders would be able to form better estimation of such systematic components of uninformed order flows than other less informed traders. For the simplicity of analysis, I assume that the informed trader can observe the systematic part of noise trading directly at the start of every trading date while market makers attempt to learn it from the aggregate order flows.

One of the potential interpretations of the systematic component of noise trading featured in this paper is investor sentiment while other interpretations such as autocorrelated liquidity shocks are still valid. Investor sentiment refers to individual investors' irrational trading behavior which is uncorrelated to any fundamental factors. Empirical literature such as Lee, Shleifer, and Thaler (1991) suggests various phenomena unexplained by standard finance theory might be driven by investor sentiment. De Long, Shleifer, Summers, and Waldmann (1990a) studies a model where the unpredictability of investor sentiment deters rational arbitrageurs from correcting investor sentiment. Furthermore, De Long, Shleifer, Summers, and Waldmann (1990b) finds that rational speculation destabilizes the market when there exists noise trading in the form of positive feedback trading.

The informed trader in this paper takes a bit different stance from the ones in De Long, Shleifer, Summers, and Waldmann (1990a) or De Long, Shleifer, Summers, and Waldmann (1990b). The informed trader correctly observes a systematic part of noise trading (or investor sentiment) while he does not observe an unsystematic part (or pure noise). Therefore, the informed trader attempts to exploit the situation where he holds more information about noise trading compared to uninformed market makers. The informed trader in this economy does not completely correct the systematic part of noise trading not because investor sentiment poses extra risk to him but because it provides him with an extra camouflage in his trading activities.

The idea of trading against one's own private information has been studied by a large volume of literature on market manipulation, which finds informed traders may trade in the wrong direction to increase the noise in the trading volume. (e.g., Jarrow (1992), Allen and Gale (1992), Allen and Gorton (1992), Chakraborty and Yilmaz (2004a))[2] Most of papers in this line of literature adopts other models than Kyle model.[3] It is well known that an equilibrium with manipulative trading in Kyle model is ruled out under standard assumptions because of the monotonicity of the informed trader's equilibrium trading strategy. There exist a few exceptions which obtains manipulative trading with some variations of Kyle model such as Chakraborty and Yilmaz (2004b) which assumes that market makers are not certain about the existence of informed traders and possible trade sizes are finite, and Huddart, Hughes, and Levine (2001) which assumes that there exist mandatory disclosure laws. This paper shows that manipulative trading strategy could still easily happen in a variation of Kyle (1985) with standard assumptions if private information has more than two dimensions: (i) fundamentals and (ii) non-fundamentals. Therefore, this paper contributes to the literature of manipulative

[2]There are other types of market manipulation models such as Goldstein and Gumbel (2008), which studies the case of manipulating the prices without private information in the presence of feedback effect.

[3]For example, Chakraborty and Yilmaz (2004a) adopt variations of Glosten and Milgrom (1985) model.

informed trading by showing it with a standard Kyle model in the presence of predictable noise trading activities. That is, this paper proves that manipulative informed trading easily arises when private information includes non-fundamentals in a standard Kyle model, which originally rules out manipulative trading. Furthermore, this paper shows that such manipulative trading could be mitigated by releasing public signals while such stabilizing impact of public signals could deteriorate when public signals are correlated with the same systematic component of noise trading.

The paper is organized as follows: In Section 2, I describes the investment opportunities, participants in the trading, and information structure. In Section 3, I solve for the equilibrium order flow of the informed trader and show the existence of a linear equilibrium . In Section 4, I analyze the properties of the linear equilibrium using numerical analysis.

2 Model

Consider a multiperiod model of trading a risky asset where traders place market orders to competitive market makers. Trading occurs at trading dates $1, ..., T-1$, and the liquidation value of each share is paid to traders at the final date T. This could be considered as $T-1$ sequential auctions with unit time intervals in Kyle (1985)'s notation.

2.1 Investment Opportunities

There are two assets in the economy, which are traded at trading dates $1, ..., T-1$: a riskless asset yielding a return R with perfectly elastic supply, and a risky asset. Shares of assets are infinitely divisible. I normalize the gross return of the riskless asset R to one for simplicity, which makes holding each position of riskless asset equivalent to a cash position in the absence of inflation. The liquidation value of the risky asset at the final trade date T is given by V.

2.2 Traders in the Economy

There are three types of traders in the economy: market makers, a single informed trader, and uninformed noise traders. The informed trader and noise traders place market orders to market makers. That is, the informed trader and noise traders simultaneously choose the amount of shares they want to trade, then market makers set a price and trade the order flow to clear the market. Market makers observe the aggregate order flows submitted by the informed trader and noise traders, but do not observe the individual order flow submitted by each trader separately. Therefore, the existence of noise traders prevents the equilibrium order flow from fully revealing the informed trader's private information.

2.2.1 Informed Trader

The monopolistic informed trader can observe both private and public signals of the fundamental value of the risky asset. The informed trader has an initial wealth of W_0, and does not have any share of the risky asset initially. The informed trader has a constant absolute risk aversion (CARA) utility function, and maximizes his wealth at the final date T, i.e. $\mathcal{U}(W_T) = -e^{\gamma W_T}$ where γ is a risk aversion parameter. Let ΔX_t denote the informed trader's order flow for the risky asset at date t.

I keep the setting of a single informed trader for simplicity throughout the proof of the existence of a linear equilibrium and numerical analysis. In Section 3.6, I show that featuring extra informed traders would not change the nature of market manipulation problem. As it is shown in Holden and Subrahmanyam (1992) and Holden and Subrahmanyam (1994), a multiple informed trader assumption simply makes informed traders more aggressive in the initial stage due to competition. Therefore, increasing the number of informed trader does not change the prediction of this paper in other way.

2.2.2 Noise Trader

Noise traders are uninformed, and trade for other reasons than information such as liquidity reasons. Let ΔU_t denote noise traders' order flow for the risky asset at date t. The order flows from noise traders consist of two components: (1) demand driven by a certain systematic factor (or investor sentiment), (2) idiosyncratic shocks to noise traders' demand.

The process of the systematic factor S_t is given by a first-order autoregressive process:

$$S_{t+1} = a_S S_t + \epsilon_{S,t+1} \tag{1}$$

where $-1 < a_S < 1$ and $\epsilon_{S,t+1}$ is a shock to the systematic factor at trade date t, which follows normal distributions: $\epsilon_{S,t+1} \sim \mathcal{N}(0, \sigma^2_{S,t+1})$. Therefore, S_t is fluctuating around zero, and mean-reverting to zero in the steady state. Since S_t is irrational demand which is independent of fundamentals or information, it could be potentially interpreted as investor sentiment. Although another interpretation of S_t is still possible, I will refer to S_t as investor sentiment from now on for convenience.

Finally, the total demand of noise traders at trade date t could be written that

$$\Delta U_t = a_U S_t + \epsilon_{U,t+1} \tag{2}$$

where a_U is a non-negative scaling parameter and $\epsilon_{U,t+1}$ is an idiosyncratic shock to noise traders' demand at date t, which follows normal distributions: $\epsilon_{U,t+1} \sim \mathcal{N}(0, \sigma^2_{U,t+1})$. Note that I will normalize it to one throughout the numerical analysis in Section 4.

2.2.3 Market Maker

Market makers are risk neutral and competitive as in typical Kyle model. Since the competition among market makers drive their profit to zero, the price is set to market makers' expected liquidation value of the risky asset at the final date T. Although market makers observe public signals, they are not able to observe private signals of the informed trader. Thus, they attempt to infer private information of the informed trader using the aggregate order flow as well as public signals.

Let ΔZ_t denote the aggregate order flow by the informed traders and noise traders at date t. i.e.

$$\Delta Z_t \equiv \Delta X_t + \Delta U_t \ .$$

2.3 Information Structure

The prior information of market makers about the liquidation value of the risky asset V and investor sentiment S_t before the first trade date is common knowledge, and assume that the prior distributions are given by a certain distribution:

$$\begin{pmatrix} V \\ S_0 \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} v \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_V^2 & 0 \\ 0 & \frac{\sigma_S^2}{1-a_S^2} \end{pmatrix} \right),$$

where v is the mean of prior distribution of the liquidation value V. The prior of investor sentiment before the first trade date, S_0, is given as its steady state distribution, and is independent of the prior of the liquidation value of the risky asset.

The informed trader observes investor sentiment S_t privately at each trade date t. However, the informed trader is still not able to know investor sentiment in the future due its stochastic nature of the process. On the other hand, market makers are unable to observe the investor sentiment.[4] There are also public signals which both the informed trader and market makers receive at date t before they engage in any trading activities:

$$Y_t = V + a_Y S_t + \epsilon_{Y,t}, \tag{3}$$

[4]Even when the informed trader is not assumed to observe S_t directly, the informed trader would have superior information on S_t compared to market makers because he can infer past noise traders' order flows correctly from past aggregate order flows.

where a_Y is a non-negative constant, and $\epsilon_{Y,t}$ is a shock to public signal at trade date t. For example, the public signal Y_t is not distorted by investor sentiment S_t when $a_Y = 0$. On the other hand, the public signal Y_t is distorted by investor sentiment S_t when $a_Y > 0$.

Both the informed trader and market makers observe the past history of prices of the risky asset. Since market makers set the equilibrium price after observing the aggregate demand, the informed trader does not observe the equilibrium price until the next period. Therefore, the informed trader (I)'s information set at date t is given by:

$$\mathcal{F}_t^I = \{\mathcal{F}_0, V, P_{\tau-1}, S_t, Y_\tau : 1 \le \tau \le t\}, \tag{4}$$

where $\mathcal{F}_0$ denotes the common knowledge in the initial stage. On the other hand, a market maker (M)'s information set is given by:

$$\mathcal{F}_t^M = \{\mathcal{F}_0, P_{\tau-1}, \Delta Z_\tau, Y_\tau : 1 \le \tau \le t\}, \tag{5}$$

I will use the notation $\hat{x}_t^i \equiv E[x_t|\mathcal{F}_t^i]$ for any $i \in \{I, M\}$ (e.g. $\hat{V}_t^M \equiv E[x_t|\mathcal{F}_t^M]$). Now, I define state variables and shocks to the economy: (i) Denote $\Psi_t \equiv (V_t - P_{t-1}, S_t - a_S\hat{S}_{t-1}^M, Y_t - P_{t-1} - \hat{S}_{t-1}^M)^\top$ to be the vector of state variables. Since the informed trader can perfectly infer market makers' belief at each date t, one can easily observe that the informed trader knows Ψ_t correctly given his information set $\mathcal{F}_t^I$ at trade date t, i.e. $\hat{\Psi}_t^I \equiv E[\Psi_t|\mathcal{F}_t^I] = \Psi_t$ (ii) Denote $\varepsilon_t = (\epsilon_{S,t+1}, \epsilon_{U,t+1}, \epsilon_{Y,t+1})^\top$ to be the vector of shocks to the economy which has not yet arrived at trade date t. They are jointly normal, independent of each other, and independent over time. That is, the distribution of ε_t is given by $\varepsilon_{t+1} \sim \mathcal{N}(0, \Sigma_{t+1})$ where Σ_{t+1} is the covariance matrix of the shocks in which diagonal elements are $\sigma_{S,t+1}^2, \sigma_{U,t+1}^2, \sigma_{Y,t+1}^2$ respectively and other elements are all zero. Further assume that ε_t is independent of $E[V|\mathcal{F}_t^M]$.

3 Equilibrium

3.1 Equilibrium Order Flow

Consider a linear equilibrium in the economy.[5] There are three state factors which determines the equilibrium in this economy: fundamental factor $(V - P_{t-1})$, investor sentiment factor $(S_t - a_S \hat{S}^M_{t-1})$, and public announcement factor $(Y_t - P_{t-1} - a_Y a_S \hat{S}^M_{t-1})$. Each factor represents the difference between the true value and market makers' expectation of the liquidation value, investor sentiment, and error in public signal, respectively. Note that the first factor exactly matches the one in Kyle (1985) or Holden and Subrahmanyam (1994). This model requires two more factors than typical Kyle models since it features investor sentiment and public signals. The next theorem states that the equilibrium order flow of the informed trader at each trade date are given as a linear function of state variables.

Theorem 1 *In a linear equilibrium, the informed trader's order flows for the risky asset at trade date $1 \leq t < T$ are given by a linear function of state variables:*

$$\Delta X_t \quad = \quad a_{X,t}(V - P_{t-1}) + b_{X,t}(S_t - a_S \hat{S}^M_{t-1}) + c_{X,t}(Y_t - P_{t-1} - a_Y a_S \hat{S}^M_{t-1}) \ . \tag{6}$$

Equivalently,

$$\Delta X_t \quad = \quad \eta_t \Psi_t \ , \tag{7}$$

where $\eta_t \equiv (a_{X,t}, b_{X,t}, c_{X,t})$ and $\Psi_t \equiv (V - P_{t-1}, S_t - a_S \hat{S}^M_{t-1}, Y_t - P_{t-1} - a_Y a_S \hat{S}^M_{t-1})^\top$.

I will prove this theorem by assuming the above order flow and finding the informed trader's optimal order flow is indeed the same in a linear equilibrium. First, I show the learning problem of market makers, which determines the equilibrium price. Second, I solve the informed trader's optimization problem given the price function derived by the learning problem of market

[5] First of all, a linear equilibrium in this model makes more economic sense than potential nonlinear equilibria if any. Past literature has conjectured that there is no other equilibrium than a linear equilibrium in Kyle model, but has not been successful in showing it.

makers. Third, I show that the informed trader's optimal demand in equilibrium is indeed equal to the initial assumption, which proves the existence of the linear equilibrium.

3.2 Conditional Expectation

As I have mentioned earlier in the previous section, market makers observe the aggregate order flows, $\Delta Z_t \equiv \Delta X_t + \Delta U_t$. Define

$$\zeta_t \equiv \Delta X_t + \Delta U_t + a_{X,t} P_{t-1} + b_{X,t} a_S \hat{S}^M_{t-1} - c_{X,t}(Y_t - P_{t-1} - a_Y a_S \hat{S}^M_{t-1}) \tag{8}$$

Note that P_{t-1}, $\hat{S}^M_{t-1}$ and Y_t are all known to market makers at trade date t. Hence, observing the current order flow is equivalent to observing ζ_t given market makers' information set from the past period, $\mathcal{F}^M_{t-1}$, and the public signal, Y_t. That is, ζ_t is a sufficient statistic for the aggregate order flow ΔZ_t in equilibrium.[6] It is straight forward to show that ζ_t is equivalent to

$$\zeta_t = a_{X,t} V_t + (b_{X,t} + a_U) S_t + \epsilon_{U,t+1} \tag{9}$$

Market makers' updating belief on V at trade date t using ζ_t and Y_t could be solved by a simple Kalman filter:

Theorem 2 *Given the aggregate order flow for the risky asset and public news, $\hat{V}^M_t, \hat{S}^M_t$ is determined by the following linear filter:*

$$\begin{pmatrix} \hat{V}^M_t \\ \hat{S}^M_t \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & a_S \end{pmatrix} \begin{pmatrix} \hat{V}^M_{t-1} \\ \hat{S}^M_{t-1} \end{pmatrix} + \begin{pmatrix} k^\zeta_{V,t} & k^Y_{V,t} \\ k^\zeta_{S,t} & k^Y_{S,t} \end{pmatrix} \begin{pmatrix} \zeta_t - E[\zeta_t | \mathcal{F}^M_{t-1}] \\ Y_t - E[Y_t | \mathcal{F}^M_{t-1}] \end{pmatrix}$$

where $k^\zeta_{V,t}, k^Y_{V,t}, k^\zeta_{S,t}, k^Y_{S,t}$ are constants.

Proof See Appendix A. ∎

[6] In equilibrium, the informed trader's trading strategy $\eta_t \equiv (a_{X,t}, b_{X,t}, c_{X,t})$ is a common knowledge.

Since market makers are competitive and risk neutral, the equilibrium price is always equal to market makers' conditional expectation of the liquidation value at trade date t, i.e. $P_t = E[V|\mathcal{F}_t^M] = \hat{V}_t^M$. The informed trader's excess return at trade date t is given by $Q_{t+1} \equiv V - P_t$ assuming that he has a perfect knowledge on the risky asset, and does not have any borrowing constraints. The following lemma shows that the excess return from trading the risky asset at trade date t is determined by the informed trader's order flow, state variables, and shocks.

Lemma 3 *The equilibrium excess return at trade date t is represented by a linear function of ΔX_t, Ψ_t and ε_{t+1}.*

$$Q_{t+1} = a_{Q,t+1}\Delta X_t + b_{Q,t+1}\Psi_t + c_{Q,t+1}\varepsilon_{t+1} \tag{10}$$

where $a_{Q,t+1}$ is a constant, and $b_{Q,t+1}$, $c_{Q,t+1}$ are vectors of constants in proper order.

Proof See Appendix B. ∎

3.3 Informed Traders' Optimization problem

Since $Q_{t+1} \equiv V - P_t$, the informed trader's problem can be formulated as the following:

$$\begin{aligned} \max_{\Delta X_t} \quad & E\left[-e^{-\gamma W_T}\middle|\mathcal{F}_t^I\right] \\ \text{subject to} \quad & W_{t+1} = W_t + Q_{t+1}\Delta X_t \end{aligned} \tag{11}$$

This formulation of the informed trader's problem is a generalized version of Kyle model, which provides more flexibility in analyzing the informed trader's dynamic decision making problem. The main difference from a competitive market setting such as He and Wang (1995) is that the excess return Q_{t+1} is given as a function of the control variable ΔX_t,

In the following lemma, I show that the law of motion for the state vector Ψ_t is an autoregressive process with exogenous input ΔX_t. Unlike in competitive models such as Wang (1994) and He and Wang (1995), the state process is also affected by the control variable. Therefore, the informed trader is in fact able to affect the state process using the control variable for his own benefit.

Lemma 4 *The state vector Ψ_t is an autoregressive process with an exogenous input ΔX_t:*

$$\Psi_{t+1} = a_{\Psi,t+1}\Delta X_t + b_{\Psi,t+1}\Psi_t + c_{\Psi,t+1}\varepsilon_{t+1} \tag{12}$$

where $a_{\Psi,t+1}, b_{\Psi,t+1}, c_{\Psi,t+1}$ are matrix of constants in proper order.

Proof See Appendix C. ∎

Recall that the state vector at trade date t, Ψ_t, is perfectly observed by the informed trader, i.e. $\Psi_t = E[\Psi_t|\mathcal{F}_t^I]$. Therefore, we can solve the informed trader's optimization problem using the excess return from Lemma 3 and the state process from Lemma 4. The Bellman equation for the optimization problem (11) is given by

$$\begin{aligned} 0 = \underset{\Delta X_t}{Max} \quad & \{E[J(W_{t+1};\Psi_{t+1};t+1)|\mathcal{F}_t^I] - J(W_t;\Psi_t;t)\} \\ \text{subject to} \quad & W_{t+1} = W_t + Q_{t+1}\Delta X_t \\ & J(W_T;\Psi_T;T) = -e^{-\gamma W_T}. \end{aligned}$$

The solution for the optimization problem is derived according to the following lemma:

Lemma 5 *Suppose Q_{t+1} and Ψ_t are given by the following Gauss-Markov processes:*

$$\begin{aligned} Q_{t+1} &= a_{Q,t+1}\Delta X_t + b_{Q,t+1}\Psi_t + c_{Q,t+1}\varepsilon_{t+1} \\ \Psi_{t+1} &= a_{\Psi,t+1}\Delta X_t + b_{\Psi,t+1}\Psi_t + c_{\Psi,t+1}\varepsilon_{t+1} \end{aligned}$$

Then, the risk averse informed trader's optimal order flow for the risky asset at date t is given by a linear function of state vector at trade date t:

$$\Delta X_t = F_t\Psi_t \tag{13}$$

where F_t is a vector of proper order.

Proof See Appendix D. ∎

3.4 Solving the Equilibrium

Finally, the equilibrium is determined by solving the following equation system:

$$\eta_t = F_t \quad \text{for all } 0 < t < T \tag{14}$$

where η_t is a vector of unknowns at date t as is given in Theorem 1, and F_t is a solution derived using Lemma 3, Lemma 4, and Lemma 5 given η_t. Therefore, the existence of solution proves the existence of a linear equilibrium since it satisfies the assumption which I have made on the equilibrium order flow at the start of Section 3. Like other literature with Kyle model, an analytical solution of the equation system cannot be obtained in general. The numerical procedure of solving Equation (14) is described in Appendix F

3.5 Components of the Informed Trader's Order Flows

A further analysis on the informed trader's order flow shows that it could be decomposed into two separate components: (i) corrective demand which attempts to gain profits from informational rent as a price-taker, (ii) manipulative demand which attempts to gain profits from the price changes over time due to his own trading.

The result of of Lemma 5 in Appendix D reveals that the informed trader's optimal order flow at trade date t is given by

$$\Delta X_t = \frac{1}{\delta_1 + \delta_2}(\beta_1 + \beta_2)\Psi_t$$

where δ_1, δ_2 are constants, and β_1, β_2 are 3-vectors. By looking at the solution in Appendix D, one can observe that δ_1, β_1 are not related to any of the informed trader's impact on the excess return nor the state process. On the other hand, δ_2, β_2 are directly linked to the informed trader's impact on the excess return and the state process. Therefore, the following theorem is directly obtained from the result of Lemma 5.

Theorem 6 *The informed trader's demand consists of two separate parts: (i) corrective demand, (ii) manipulative demand:*

$$\Delta X_t = \underbrace{\left(\frac{1}{\delta_1}\beta_1 - \frac{\delta_2}{\delta_1(\delta_1+\delta_2)}\beta_1\right)\Psi_t}_{\textit{corrective demand}} + \underbrace{\left(\frac{1}{\delta_2}\beta_2 - \frac{\delta_1}{\delta_2(\delta_1+\delta_2)}\beta_2\right)\Psi_t}_{\textit{manipulative demand}} \tag{15}$$

where δ_1, δ_2 are constants, and β_1, β_2 are 3-vectors.

The first component is the informed trader's demand as a price taker, which consists of (i) a typical mean-variance utility maximizer as a price-taker: $\frac{1}{\delta_1}\beta_1$, and (ii) an adjustment term due to the risk regarding his own price impact: $-\frac{\delta_2}{\delta_1(\delta_1+\delta_2)}\beta_1$. The first term of corrective demand is exactly the same as competitive investors' demand in Wang (1994) or He and Wang (1995).[7] That is, corrective demand is defined as pure corrective demand ignoring his own price impact plus an adjustment term to the exposure to the risk by his own price impact.

Similarly, the second component is the informed trader's demand as a price manipulator, which consists of (i) a mean-variance utility maximizer as a price-manipulator: $\frac{1}{\delta_2}\beta_2$, and (ii) an adjustment term due to the risk regarding his own corrective demand: $-\frac{\delta_1}{\delta_2(\delta_1+\delta_2)}\beta_2$. One can also verify that the manipulative demand disappears at the final trade date $T-1$ since there is no more room for manipulating the state process.

3.6 Multiple Informed Traders

In this section, I will briefly show that the existence of multiple informed trader would not fundamentally change the findings of this model except for changing the degree of the informed

[7] Note that risk aversion parameter γ is included in δ_1 unlike Wang (1994), He and Wang (1995) for its notational difference.

trader's incentive of manipulative trading. More competition among the informed traders accelerates the revelation of private information through prices, thereby reducing the gains from market manipulation.

Instead of a single informed trader, suppose there are N informed traders who observe the liquidation value of the risky asset as well as the investor sentiment at every period. I further assume that they have identical preference, common knowledge and initial wealth. I denote ΔX_t^i to be ith informed trader's order flow at date t, and $\Delta \mathbb{X}_t \equiv \sum_{i=1}^N \Delta X_t^i$ to be the aggregate order flow of N informed traders.

Lemma 7 *The equilibrium excess return Q_t and state vector Ψ_t is given by an autoregressive process with an exogenous input $\Delta \mathbb{X}_t$:*

$$\begin{aligned} Q_{t+1} &= a_{Q,t+1}\Delta \mathbb{X}_t + b_{Q,t+1}\Psi_t + c_{Q,t+1}\varepsilon_{t+1} \\ \Psi_{t+1} &= a_{\Psi,t+1}\Delta \mathbb{X}_t + b_{\Psi,t+1}\Psi_t + c_{\Psi,t+1}\varepsilon_{t+1} \end{aligned}$$

where $a_{Q,t+1}, b_{Q,t+1}, c_{Q,t+1}, a_{\Psi,t+1}, b_{\Psi,t+1}, c_{\Psi,t+1}$ are matrices of constants in proper order. In equilibrium, i'th informed trader's order flow at period t is given by

$$\Delta X_t^i = \frac{1}{\delta_1 + \delta_2'}(\beta_1 + \beta_2)\Psi_t$$

where δ_1, δ_2' are constants, and β_1, β_2 are 3-vectors. δ_1, β_1 are related to corrective motives of trading while δ_2, β_2 are related to manipulative motives of trading. Furthermore, $\delta_1, \beta_1, \beta_2$ are not functions of the number of informed traders N while δ_2' is a function of N.

Proof See Appendix E. ∎

The result together with Theorem 6 shows that given all the things the same only the manipulative demand would be changed as the number of informed traders N while the corrective demand is unchanged.[8] Holden and Subrahmanyam (1992) and Holden and Subrahmanyam

[8] Although parameter values would change, the functional form of $\delta_1, \beta_1, \beta_2$ would remain the same regardless of the number of informed traders.

(1994) show that a multiple informed trader assumption makes informed traders more aggressive in the initial stage due to competition. That is, with reasonable parameters δ_2' would be an increasing function of N. With increasing δ_2' informed traders' incentive of manipulative trading would decrease, thereby increasing the share of corrective trading relatively. Therefore, featuring the competition among multiple informed traders would not change the nature of market manipulation found by this paper in the later section using numerical results while it would strictly decrease the informed traders' incentive of market manipulation.

4 Properties of Equilibrium

4.1 Informed Trading and Investor Sentiment

In this subsection, I assume that there is no public signal release in order to focus on studying the relationship between informed trading and investor sentiment.

[Insert Figure 1 here]

Figure 1 shows the impact of investor sentiment on price efficiency by comparing market makers' uncertainty on the liquidation value of the risky asset with and without investor sentiment. The information revelation through aggregate order flows is slower in the presence of investor sentiment. Since investor sentiment provides extra noise to the aggregate order flows, it naturally slows market makers' learning.

[Insert Table 1 here]

Table 1 compares the informed trader's equilibrium trading strategy between the case with and without investor sentiment. The informed trader becomes more aggressive when there exists investor sentiment. Even with the stronger intensity of informed trading, however, the informed trader is able to keep market makers less informed about the liquidation value using the camouflage of investor sentiment. By looking at the coefficients on investor sentiment factor, we can observe that the informed trader comove with market makers' forecasting errors on investor sentiment during earlier periods. Furthermore, such comovement with the mispricing due to investor sentiment gets more severe until trade date $t = 5$, then grows down afterwards. It can be also observed that the informed trader finally starts correcting the mispricing around the final trade date. Indeed, it could be easily shown that the informed trader always finds it optimal to correct investor sentiment at trade date $T - 1$:

Corollary 8 *When the informed trader's optimization problem is a static problem instead of a dynamic problem (i.e. $t = T - 1$), the informed trader's optimal order flow is given by the following:*

$$\Delta X_t = \frac{V - P_{t-1}}{\Gamma_t} - \frac{k^{\zeta}_{V,t}(S_t - a_S \hat{S}^M_{t-1})}{\Gamma_t},$$

where $\Gamma_t \equiv k^{\zeta}_{V,t}(2 + \gamma k^{\zeta}_{V,t}\sigma^2_{U,t+1})$.

Since $\Gamma_t, k^{\zeta}_{V,t}$ are positive constants[9], Corollary 8 implies that the informed trader always bets against market makers' error of forecasting investor sentiment, $S_t - a_S \hat{S}^M_{t-1}$ at the final trade date $T - 1$. Thus, if the informed trader ever bets on investor sentiment, it is because of the dynamic property of his optimization problem. That is, the informed trader might find it profitable to comove with investor sentiment because the expected profit which he will achieves in the future by manipulating prices dominates the profit which he achieves by correcting investor sentiment at the current period. The next result in fact reveals that the informed trader comoves with investor sentiment out of price-controlling motives.

[Insert Table 2 here]

Table 2 reports the decomposition of the informed traders' order flow into two separate components defined in Section 3.5: (i) corrective demand, and (ii) manipulative demand. It

[9] Γ_t is positive due to the second order condition of the informed trader's optimization problem. $k^{\zeta}_{V,t}$ could also be shown to be positive at trade date $T - 1$ in the linear equilibrium.

reveals that the comovement of the informed trader with investor sentiment is driven by his manipulative demand rather than his corrective demand. We can clearly observe that the corrective demand corrects the error in market makers' forecasting investor sentiment in every period, however, the manipulative demand which comoves with investor sentiment dominates the corrective demand except for a few trade dates near the liquidation date T. That is, the manipulative demand overwhelms the corrective demand during early trading dates. While correcting the mispricing due to investor sentiment could give short-term profits, comoving with the mispricing give better long-term profits. The informed trader finds that the long-term profits from comovement with investor sentiment factor overwhelms the short-term protifs from correcting the mispricing during early trading dates. As a result, the informed trader trades in the wrong direction regarding investor sentiment to increase the noise in the market during early trading dates. The result is in line with recent empirical observations such as Brunnermeier and Nagel (2004): the informed trader magnifies irrational demands of noise traders during early periods of trading, and corrects it near the liquidation of the risky asset.

[Insert Figure 2 here]

Figure 2 shows market makers' uncertainty about investor sentiment. Since investor sentiment evolves over time, the uncertainty goes back to the steady state level unless market makers keep learning new information on it. The figure shows an interesting comparison with Figure 1, which shows monotone-decreasing market makers' uncertainty about the liquidation value. While the uncertainty about the liquidation value is rather gradually decreasing, the uncertainty about investor sentiment decreases rapidly at first, but picks up slowly afterwards. When the final trade date is far away, the informed trader deliberately chooses to reveal less about the liquidation value of the risky asset at the cost of revealing more about investor sentiment by comoving with it. This shows a case where informed traders may ride a bubble driven by investor sentiment for informational reason because informed traders are able to less reveal their private information by leaning toward investor sentiment.

[Insert Figure 3 here]

Figure 3 also shows that the price sensitivity to the aggregate order flow which could be interpreted as the reverse of liquidity in the market. It reveals that investor sentiment provides higher liquidity to the informed trader over all. However, the liquidity becomes suddenly lower near the final trade date when there exists investor sentiment. It is because the aggregate order flow becomes suddenly very informative near the final trade date because the informed trader suddenly starts correcting investor sentiment. Since investor sentiment prevents private information from being revealed, private information which would have been revealed without investor sentiment gets accumulated over time without being revealed. Moreover, the informed trader starts correcting aggressively the mispricing due to investor sentiment near the liquidation of the risky asset. As a result, the revelation of these accumulated signals near the final trade date with dampened noises from investor sentiment makes market makers more sensitive to the aggregate order flows. Therefore, market depth would increase gradually with sharp decrease just near announcements if informed traders have been manipulating the market.

4.2 Informed Trading and Public News

In this section, I study the impact of public singals on informed trading. The release of public information weakens the informed trader's incentive to manipulate the market, however, such stabilizing effect of public information would deteriorate if public information is also distorted by investor sentiment. The result is robust whether information release is given as a single shock or sequential shocks.

4.2.1 Single Public Information Release

Consider the arrival of a single public signal at one specific date $t = 3$. The variance of the signal is given by $\sigma_{Y,3} = 1$, and $\sigma_{Y,t} = 10^6$ for all $t \neq 3$. i.e., the accuracy would be considered as $1/\sigma_{Y,3} = 1$, and $1/\sigma_{Y,t} = 1/10^6 \approx 0$ for all $t \neq 3$.

[Insert Figure 4 Here]

Figure 4 reports market makers' uncertainty about the liquidation value when there is a public announcement at $t = 3$. It shows that the uncertainty reduction due to the pubic announcement is bigger when the public signal is not affected by investor sentiment.

[Insert Table 3 and Table 4 Here]

Table 3 and 4 show the informed trader's trading strategy when there is a relatively accurate public announcement at date $t = 3$. We can observe that the informed trader corrects the mispricing due to investor sentiment before the announcement date $t = 3$ unlike the case without any pubic announcement. Therefore, the release of public information stabilizes the market by mitigating the informed trader's incentive to manipulate the market. We can also observe that correction of the mispricing due to investor sentiment when the announcement is affected by investor sentiment (Table 4) is weaker than the correction of the mispricing when the announcement is not affected by investor sentiment (Table 3). Therefore, the price-stabilizing effect of the public signal is reduced when the signal is distorted by investor sentiment.

4.2.2 Sequential Public Information Release

Consider sequential arrivals of public signal at each date. Note that the signal is chosen to be relatively noisier compared to the single information shock case: $\sigma_{Y,t} = 5$ for all $0 < t < T$. i.e, the accuracy of signals could be considered as $1/\sigma_{Y,t} = 0.2$ for all $0 < t < T$.

[Insert Figure 5 Here]

Figure 5 reports a similar result as the single arrival of public signal about the price efficiency. It shows that the uncertainty reduction due to the pubic announcement is bigger when the public signal is not affected by investor sentiment.

[Insert Table 5 and Table 6 Here]

Table 5 and 6 show the informed trader's trading strategy with public signal arriving at each trade date. As the case with single information shock, we can observe that the intensity of riding the mispricing due to investor sentiment becomes weaker compared to the case without any public signal. We can also observe that the correction of investor sentiment when public signal is affected by investor sentiment (Table 6) is weaker than the one when public signal is not affected by investor sentiment (Table 5). It also confirms that the price-stabilizing effect of public signal is reduced when the signal is distorted by the same source of noise.

Therefore, this section develops an implication for the policy of market stabilization. Not surprisingly, pubic information needs to be revealed as much as possible to mitigate the destabilizing impact of market manipulation. Furthermore, such public signal needs to be free from the source of noise in the market. For example, the release of news which are potentially affected market prevalent investor sentiment would not be very helpful for stabilizing the market according to the prediction of this paper.

5 Conclusion

This paper attempts to answer the question whether a rational arbitrager who has superior information about non-fundamental factor such as noise trading activities would reduce the mispricing caused by such non-fundamentals in the market. I analyze a dynamic model of informed trading in the presence of an autoregressive component of noise trading which is privately observed by a monopolistic risk-averse informed trader. To develop a more flexible version of Kyle model in discrete time, this paper adopts a more generalized approach similar to Wang (1994), and He and Wang (1995). Using the reinterpreted version of Kyle model, this paper shows that the informed trader's demand for the risky asset at each period could be decomposed into two separate components according to their trading motives: (i) a corrective demand which is a typical price-taking mean-variance demand for the excess return, and (ii) a manipulative demand which is driven by price-controlling motives. The result shows that the informed trader may ride on the mispricing caused by the systematic component of noise trading during early periods of trading, and only start correcting it near the liquidation of the risky asset. The decomposition of the informed trader's demand reveals that such comovement is caused by manipulative demand, i.e., the informed trader suffers short-term losses to gain long-term profits by increasing noise in the market, which is market manipulation. The informed trader chooses to comove with irrational demands because such comovement allows him to reveal less information through his trading volume, which leads to more profit in later periods. Furthermore, I show that the release of public signals help stabilize prices to some degree since it reduces the informed traders' incentive to manipulate the market. However, such price-stabilizing effect of public information is severely weakened when public information is also distorted by the same irrationality of noise traders. The result implies that an arbitrager who has superior information on non-fundamentals such as investor sentiment may not always reduce the mispricing caused by non-fundamentals given private information on fundamentals. This paper demonstrate that manipulative trading could occur under standard Kyle model setting if private information includes both fundamentals and non-fundamentals.

Appendices

Appendix A

Linear filtering problem could be solved by a standard algorithm called Kalman filter. (For example, see Hamilton (1994) or Wang (1994))

Lemma A.1 *Let ξ_t denote a n-vector of state variables, y_t denote a m-vector of observed signals. Suppose the dynamics of y_t is given by the following system of equations:*

$$\begin{aligned} \xi_t &= A_t \xi_{t-1} + B_t \epsilon_{\xi,t} \\ y_t &= H_t \xi_t + \epsilon_{y,t} \end{aligned}$$

where A_t, B_t and H_t are matrices of parameters of dimension $n \times n, n \times k, m \times n,$ respectively. $\epsilon_{\xi,t}$ and $\epsilon_{y,t}$ are k-vector and m-vector of innovations, respectively. $\epsilon_{\xi,t}$ and $\epsilon_{y,t}$ are independent, and their distributions are given by $\epsilon_{\xi,t} \sim \mathcal{N}(0, \mathcal{Q}_t)$ and $\epsilon_{y,t} \sim \mathcal{N}(0, R_t)$. Then, the conditional expectation and variance of ξ_t is given by the following recursive filters:

$$\hat{\xi}_t = A_t\hat{\xi}_{t-1} + K_t(y_t - H_tA_t\hat{\xi}_{t-1}) \tag{A.1}$$

$$O_t = (I_n - K_tH_t)(A_tO_{t-1}A_t^\top + B_t\mathcal{Q}_tB_t^\top) \tag{A.2}$$

where $K_t = (A_tO_{t-1}A_t^\top + B_t\mathcal{Q}_tB_t^\top)H_t^\top[H_t(A_tO_{t-1}A_t^\top + B_t\mathcal{Q}_tB_t^\top)H_t^\top + R_t]^{-1}$, and I_n is a $(n \times n)$ identity matrix.

Proof of Theorem 2: In case of market makers' filtering problem, the state variables are $\xi_t \equiv (V, S_t)^\top$ and the observed variables are $y_t \equiv (\zeta_t, Y_t)^\top$. Also, innovations are given by $\epsilon_{\xi,t} \equiv \epsilon_{S,t+1}$, $\epsilon_{y,t} \equiv [\epsilon_{U,t+1}, \epsilon_{Y,t}]^\top$, and coefficients are given by

$$\begin{aligned}
A_t &= \begin{pmatrix} 1 & 0 \\ 0 & a_S \end{pmatrix}, \\
B_t &= \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \\
H_t &= \begin{pmatrix} a_{X,t} & a_U + b_{X,t} \\ 1 & a_Y \end{pmatrix}, \\
\mathcal{Q}_t &= \sigma_{S,t+1}^2 , \\
R_t &= \begin{pmatrix} \sigma_{U,t+1}^2 & 0 \\ 0 & \sigma_{Y,t}^2 \end{pmatrix}.
\end{aligned}$$

Using Lemma A.1, we could derive the following Kalman filter of market makers:

$$\begin{pmatrix} \hat{V}_t^M \\ \hat{S}_t^M \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & a_S \end{pmatrix} \begin{pmatrix} \hat{V}_{t-1}^M \\ \hat{S}_{t-1}^M \end{pmatrix} + K_t \begin{pmatrix} \zeta_t - E[\zeta_t|\mathcal{F}_{t-1}^M] \\ Y_t - E[Y_t|\mathcal{F}_{t-1}^M] \end{pmatrix}$$

where $K_t = (A_t O_{t-1} A_t^\top + B_t \mathcal{Q}_t B_t^\top) H_t^\top [H_t (A_t O_{t-1} A_t^\top + B_t \mathcal{Q}_t B_t^\top) H_t^\top + R_t]^{-1}$. Also, the mean-square error of forecasting is given by

$$O_t \quad = \quad (I_n - K_t H_t)(A_t O_{t-1} A_t^\top + B_t \mathcal{Q}_t B_t^\top).$$

▮

Appendix B

Proof of Lemma 3:

Since $E[\zeta_t|\mathcal{F}_{t-1}^M] = b_{X,t} V_t$ and $E[Y_t|\mathcal{F}_{t-1}^M] = V_t$, it could be shown that

$$\begin{aligned} \hat{V}_t^M &= \hat{V}_{t-1}^M + k_{V,t}^\zeta [\zeta_t - (a_{X,t} \hat{V}_{t-1}^M + (b_{X,t} + a_U) a_S \hat{S}_{t-1}^M)] + k_{V,t}^Y [Y_t - (\hat{V}_{t-1}^M + a_Y a_S \hat{S}_{t-1}^M)] \\ &= \hat{V}_{t-1}^M + k_{V,t}^\zeta [\Delta X_t + \Delta U_t + a_{X,t} P_{t-1} + b_{X,t} a_S \hat{S}_{t-1}^M - c_{X,t}(Y_t - P_{t-1} - a_Y a_S \hat{S}_{t-1}^M) \\ &\quad -(a_{X,t} \hat{V}_{t-1}^M + (b_{X,t} + a_U) a_S \hat{S}_{t-1}^M)] + k_{V,t}^Y (V - P_{t-1} - a_Y a_S \hat{S}_{t-1}^M) \\ &= P_{t-1} + k_{V,t}^\zeta \Delta X_t + a_U k_{V,t}^\zeta (S_t - a_S \hat{S}_{t-1}^M) + (-c_{X,t} k_{V,t}^\zeta + k_{V,t}^Y)(Y_t - P_{t-1} - a_Y a_S \hat{S}_{t-1}^M) + k_{V,t}^\zeta \epsilon_{U,t+1} \ . \end{aligned}$$

Since $Q_{t+1} \equiv V - P_t$, the excess return is given by

$$\begin{aligned} Q_{t+1} &= -k_{V,t}^\zeta \Delta X_t + (V - P_{t-1}) - a_U k_{V,t}^\zeta (S_t - a_S \hat{S}_{t-1}^M) + (c_{X,t} k_{V,t}^\zeta - k_{V,t}^Y)(Y_t - P_{t-1} - a_Y a_S \hat{S}_{t-1}^M) \\ &\quad -k_{V,t}^\zeta \epsilon_{U,t+1} \ . \end{aligned}$$

Equivalently, the equilibrium excess return at date t could be represented as

$$Q_{t+1} = a_{Q,t+1}\Delta X_t + b_{Q,t+1}\Psi_t + c_{Q,t+1}\varepsilon_{t+1}$$

where

$$\begin{aligned}
a_{Q,t+1} &\equiv -k^{\zeta}_{V,t}, \\
b_{Q,t+1} &\equiv \left(1, -a_U k^{\zeta}_{V,t}, c_{X,t}k^{\zeta}_{V,t} - k^{Y}_{V,t}\right), \\
c_{Q,t+1} &\equiv \left(0,\ -k^{\zeta}_{V,t},\ 0\right).
\end{aligned}$$

$\blacksquare$

Appendix C

Proof of Lemma 4:

Note that

$$\begin{aligned}
V - P_t &= -k^{\zeta}_{V,t}\Delta X_t + (V - P_{t-1}) - a_U k^{\zeta}_{V,t}(S_t - a_S\hat{S}^M_{t-1}) \\
&\quad +(c_{X,t}k^{\zeta}_{V,t} - k^{Y}_{V,t})(Y_t - P_{t-1} - a_Y a_S\hat{S}^M_t) - k^{\zeta}_{V,t}\epsilon_{U,t+1}, \\
S_{t+1} - a_S\hat{S}^M_t &= -a_S k^{\zeta}_{S,t}\Delta X_t + a_S(1 - a_U k^{\zeta}_{S,t})(S_t - a_S\hat{S}^M_{t-1}) \\
&\quad +a_S(c_{X,t}k^{\zeta}_{S,t} - k^{Y}_{S,t})(Y_t - P_{t-1} - a_Y a_S\hat{S}^M_t) - a_S k^{\zeta}_{S,t}\epsilon_{U,t+1} + \epsilon_{S,t+1}, \\
Y_{t+1} - P_t - a_Y a_S\hat{S}^M_t &= (V - P_t) + a_Y(S_{t+1} - a_S\hat{S}^M_t) + \epsilon_{Y,t+1}.
\end{aligned}$$

Therefore, it is straight forward to show that

$$\Psi_{t+1} = a_{\Psi,t+1}\Delta X_t + b_{\Psi,t+1}\Psi_t + c_{P,t+1}\varepsilon_{t+1}$$

where

$$
\begin{aligned}
a_{\Psi,t+1} &= (-k^{\zeta}_{V,t}, -a_S k^{\zeta}_{S,t}, -k^{\zeta}_{V,t} - a_Y a_S k^{\zeta}_{S,t})^{\top}, \\
b_{\Psi,t+1} &= \begin{pmatrix} 1 & -a_U k^{\zeta}_{V,t} & c_{X,t}k^{\zeta}_{V,t} - k^{Y}_{V,t} \\ 0 & a_S(1 - a_U k^{\zeta}_{S,t}) & a_S(c_{X,t}k^{\zeta}_{S,t} - k^{Y}_{S,t}) \\ 1 & -a_U k^{\zeta}_{V,t} + a_Y a_S(1 - a_U k^{\zeta}_{S,t}) & c_{X,t}k^{\zeta}_{V,t} - k^{Y}_{V,t} + a_Y a_S(c_{X,t}k^{\zeta}_{S,t} - k^{Y}_{S,t}) \end{pmatrix}, \\
c_{\Psi,t+1} &= \begin{pmatrix} 0 & -k^{\zeta}_{V,t} & 0 \\ 1 & -a_S k^{\zeta}_{S,t} & 0 \\ a_Y & -k^{\zeta}_{V,t} - a_Y a_S k^{\zeta}_{S,t} & 1 \end{pmatrix}.
\end{aligned}
$$

∎

Appendix D

There is a standard formula which computes the certainty equivalence of expected utilities in case of CARA utilities. (For example, see Dow and Rahi (2003))

Lemma D.1 *Suppose A is a symmetric $m \times m$ matrix, b is an m-vector, d is a scalar, and w is an m-dimensional normal variate: $w \sim N(0, \Sigma)$, Σ positive definite. Then, we can find the following certainty equivalence of expected utilities if $(I - 2\Sigma A)$ is positive definite*

$$
E\Big[\exp(w^{\top}Aw + b^{\top}w + d)\Big] = |I - 2\Sigma A|^{-\frac{1}{2}} \exp\Big[\frac{1}{2}b^{\top}(I - 2\Sigma A)^{-1}\Sigma b + d\Big]. \tag{D.1}
$$

Proof of Lemma 5:

Conjecture that the value function has the form as the following:

$$
J(W_t; \Psi_t; t) = -\exp\Big(-\gamma_t W_t - \frac{1}{2}{\Psi_t}^{\top}\Omega_t \Psi_t + \kappa_t\Big) \tag{D.2}
$$

Then, it leads to

$$
\begin{aligned}
&E[J(W_{t+1};\Psi_{t+1};t+1)|\mathcal{F}_t^I] \\
&= E\Big[-\exp\Big(-\gamma_{t+1}W_{t+1} - \frac{1}{2}(\Psi_{t+1}{}^\top\Omega_{t+1}\Psi_{t+1}) + \kappa_{t+1}\Big)\Big|\mathcal{F}_t^I\Big] \\
&= E\Big[-\exp\Big(-\gamma_{t+1}\Big\{W_t + \Delta X_t(a_{Q,t+1}\Delta X_t + b_{Q,t+1}\Psi_t + c_{Q,t+1}\varepsilon_{t+1})\Big\} \\
&\quad - \frac{1}{2}(a_{\Psi,t+1}\Delta X_t + b_{\Psi,t+1}\Psi_t + c_{\Psi,t+1}\varepsilon_{t+1})^\top\Omega_{t+1}(a_{\Psi,t+1}\Delta X_t + b_{\Psi,t+1}\Psi_t + c_{\Psi,t+1}\varepsilon_{t+1}) + \kappa_{t+1}\Big)\Big|\mathcal{F}_t^I\Big] \\
&= E\Big[-\exp\Big(-\gamma_{t+1}\Big\{W_t + \Delta X_t(a_{Q,t+1}\Delta X_t + b_{Q,t+1}\Psi_t)\Big\} \\
&\quad - \frac{1}{2}(a_{\Psi,t+1}\Delta X_t + b_{\Psi,t+1}\Psi_t)^\top\Omega_{t+1}(a_{\Psi,t+1}\Delta X_t + b_{\Psi,t+1}\Psi_t) \\
&\quad - \Big\{\gamma_{t+1}c_{Q,t+1}^\top\Delta X_t + c_{\Psi,t+1}^\top\Omega_{t+1}(a_{\Psi,t+1}\Delta X_t + b_{\Psi,t+1}\Psi_t)\Big\}^\top\varepsilon_{t+1} \\
&\quad - \frac{1}{2}\varepsilon_{t+1}{}^\top c_{\Psi,t+1}^\top\Omega_{t+1}c_{\Psi,t+1}\varepsilon_{t+1} + \kappa_{t+1}\Big)\Big|\mathcal{F}_t^I\Big]
\end{aligned}
$$

Using Lemma D.1, it can be shown that

$$
\begin{aligned}
&E[J(W_{t+1};\Psi_{t+1};t+1)|\mathcal{F}_t^I] \\
&= -\rho_{t+1}\exp\Big(-\gamma_{t+1}\Big\{W_t + \Delta X_t(a_{Q,t+1}\Delta X_t + b_{Q,t+1}\Psi_t)\Big\} \\
&\quad - \frac{1}{2}(a_{\Psi,t+1}\Delta X_t + b_{\Psi,t+1}\Psi_t)^\top\Omega_{t+1}(a_{\Psi,t+1}\Delta X_t + b_{\Psi,t+1}\Psi_t) \\
&\quad + \frac{1}{2}\Big\{\gamma_{t+1}c_{Q,t+1}^\top\Delta X_t + c_{\Psi,t+1}^\top\Omega_{t+1}(a_{\Psi,t+1}\Delta X_t + b_{\Psi,t+1}\Psi_t)\Big\}^\top\Xi_{t+1} \\
&\quad \times \Big\{\gamma_{t+1}c_{Q,t+1}^\top\Delta X_t + c_{\Psi,t+1}^\top\Omega_{t+1}(a_{\Psi,t+1}\Delta X_t + b_{\Psi,t+1}\Psi_t)\Big\} + \kappa_{t+1}\Big)
\end{aligned}
$$

where $\Xi_{t+1} \equiv (\Sigma_{t+1}^{-1} + c_{\Psi,t+1}^\top\Omega_{t+1}c_{\Psi,t+1})^{-1}$ and $\rho_{t+1} = \sqrt{|\Xi_{t+1}|/|\Sigma_{t+1}|}$.

The first-order condition yields

$$(\delta_1 + \delta_2)\Delta X_t = (\beta_1 + \beta_2)\Psi_t$$

where

$$
\begin{aligned}
\delta_1 &= \gamma_{t+1}^2 c_{Q,t+1} \Xi_{t+1} c_{Q,t+1}^\top \\
\delta_2 &= -2\gamma_{t+1} a_{Q,t+1} - a_{\Psi,t+1}^\top \Omega_{t+1} a_{\Psi,t+1} + (c_{\Psi,t+1}^\top \Omega_{t+1} a_{\Psi,t+1})^\top \Xi_{t+1} c_{\Psi,t+1}^\top \Omega_{t+1} a_{\Psi,t+1} \\
&\quad + 2\gamma_{t+1} c_{Q,t+1} \Xi_{t+1} c_{\Psi,t+1}^\top \Omega_{t+1} a_{\Psi,t+1} \\
\beta_1 &= \gamma_{t+1}(b_{Q,t+1} - c_{Q,t+1} \Xi_{t+1} c_{\Psi,t+1}^\top \Omega_{t+1} b_{\Psi,t+1}) \\
\beta_2 &= a_{\Psi,t+1}^\top \Omega_{t+1} b_{\Psi,t+1} - (c_{\Psi,t+1}^\top \Omega_{t+1}^\top a_{\Psi,t+1})^\top \Xi_{t+1} c_{\Psi,t+1}^\top \Omega_{t+1} b_{\Psi,t+1}
\end{aligned}
$$

Therefore, the solution for the optimization problem is given by

$$
\Delta X_t = F_t \Psi_t, \quad \text{for all } 1 \leq t < T \tag{D.3}
$$

where $F_t \equiv (\delta_1 + \delta_2)^{-1}(\beta_1 + \beta_2)$.

The second-order condition yields the condition for optimality:

$$
\delta_1 + \delta_2 > 0 \tag{D.4}
$$

Define

$$
\begin{aligned}
M_t \equiv\ & F_t^\top (\delta_1 + \delta_2) F_t \\
& - (c_{\Psi,t+1}^\top \Omega_{t+1} b_{\Psi,t+1})^\top \Xi_{t+1} (c_{\Psi,t+1}^\top \Omega_{t+1} b_{\Psi,t+1}) \\
& + b_{\Psi,t+1}^\top \Omega_{t+1} b_{\Psi,t+1}
\end{aligned}
$$

Then, we derive

$$
E[J(W_{t+1}; \Psi_{t+1}; t+1) | \mathcal{F}_t^I] = -\rho_{t+1} \exp\left(-\gamma_{t+1} W_t - \frac{1}{2} \Psi_t{}^\top M_t \Psi_t + \kappa_{t+1}\right) \tag{D.5}
$$

From (D.3)and(D.5), we obtain the following for $t < T$:

$$\gamma_t = \gamma_{t+1},\ \Omega_t = M_t,\ \text{and}\ \kappa_t = \kappa_{t+1} - 2\log\rho_{t+1}, \tag{D.6}$$

and for $t = T$:

$$\gamma_T = \gamma,\ \Omega_T = \Theta_{3,3},\ \text{and}\ \kappa_T = 0. \tag{D.7}$$

where $\Theta_{3,3}$ is a 3×3 matrix of zeros. Then, we can recursively solve for λ_t, Ω_t and κ_t, which proves the conjecture on the value function is indeed correct. ∎

Appendix E

Proof of Lemma 7: Assume the same order flow of each informed trader's order flow in Theorem 1. Then, the sufficient statistic at date t which market makers observe in case of N informed traders is defined as

$$\begin{aligned}
\zeta_t &\equiv \Delta\mathbb{X}_t + \Delta U_t + Na_{X,t}P_{t-1} + Nb_{X,t}a_S\hat{S}^M_{t-1} - Nc_{X,t}(Y_t - P_{t-1} - a_Y a_S \hat{S}^M_{t-1}) && \text{(E.1)}\\
&= Na_{X,t}V + (Nb_{X,t} + a_U)S_t + \epsilon_{U,t+1}. && \text{(E.2)}
\end{aligned}$$

Define $a'_{X,t} = Na_{X,t}, b'_{X,t} = Nb_{X,t}$. By simply substituting $a_{X,t}, b_{X,t}$ and ΔX_t with $a'_{X,t}, b'_{X,t}$ and $\Delta\mathbb{X}_t$ in the proof, we could obtain the same result for Theorem 2, Lemma 3, Lemma 4. Now, I replicate the proof of Lemma 5. Trader i's expected utility given the same conjecture as in (D.2) is given by

$$\begin{aligned}
&E[J(W^i_{t+1};\Psi_{t+1};t+1)|\mathcal{F}^I_t]\\
&= E\Big[-\exp\Big(-\gamma_{t+1}\Big\{W^i_t + \Delta X^i_t(a_{Q,t+1}\Delta\mathbb{X}_t + b_{Q,t+1}\Psi_t + c_{Q,t+1}\varepsilon_{t+1})\Big\}\\
&\quad - \frac{1}{2}(a_{\Psi,t+1}\Delta\mathbb{X}_t + b_{\Psi,t+1}\Psi_t + c_{\Psi,t+1}\varepsilon_{t+1})^\top\Omega_{t+1}(a_{\Psi,t+1}\Delta\mathbb{X}_t + b_{\Psi,t+1}\Psi_t + c_{\Psi,t+1}\varepsilon_{t+1}) + \kappa_{t+1}\Big)\Big|\mathcal{F}^I_t\Big].
\end{aligned}$$

Define $\Delta \bar{X}_t$ to be the average of the other informed traders' order flows. Therefore, the aggregate informed order flow is given by $\Delta \mathbb{X}_t = \Delta X_t^i + (N-1)\Delta \bar{X}_t$. Substituting this into ith trader's expected utility and evaluating the first-order condition yields[10]

$$(\delta_1 + \delta_2)\Delta X_t^i + (N-1)(\delta_2 - \gamma_{t+1} c_{Q,t+1} \Xi_{t+1} c_{\Psi,t+1}^\top \Omega_{t+1} a_{\Psi,t+1})\Delta \bar{X}_t = (\beta_1 + \beta_2)\Psi_t$$

where $\delta_1, \delta_2, \beta_1, \beta_2$ are identical to the solution in Appendix D. As in Holden and Subrahmanyam (1992), $\Delta X_t^i = \Delta \bar{X}_t$ in equilibrium. Substituting this yields the equilibrium order flow of ith informed trader

$$\Delta X_t^i = (\delta_1 + \delta_2')^{-1}(\beta_1 + \beta_2)\Psi_t$$

where

$$\delta_2' \quad = \quad N(\delta_2 - \gamma_{t+1} c_{Q,t+1} \Xi_{t+1} c_{\Psi,t+1}^\top \Omega_{t+1} a_{\Psi,t+1}).$$

The rest of the proof stays the same, and it proves the initial conjecture of equilibrium order flow is indeed correct. Thus, it finishes the proof. ∎

Appendix F

Numerical Procedure for Section 3.4: I follow a similar scheme as in Holden and Subrahmanyam (1994) to obtain numerical solutions for the equation system (14). First, I assume an arbitrary value for market makers' mean-squared error of forecasting at the final date T, which is denoted by $\tilde{O}_T$. By the definition of market makers' mean-square of forecasting at trade date t, the following should be true:

$$\tilde{O}_{t+1} = \begin{pmatrix} o_{t+1}^V & o_{t+1}^{VS} \\ o_{t+1}^{VS} & o_{t+1}^S \end{pmatrix}$$

[10] One is advised to follow the same step in Appendix D.

where

$$
\begin{aligned}
o_{t+1}^{V} &= E[(V_t - \hat{V}_t^M)(V_t - \hat{V}_t^M)|\mathcal{F}_t^M], \\
o_{t+1}^{VS} &= E[(V_t - \hat{V}_t^M)(S_t - \hat{S}_t^M)|\mathcal{F}_t^M], \\
o_{t+1}^{S} &= E[(S_t - \hat{S}_t^M)(S_t - \hat{S}_t^M)|\mathcal{F}_t^M].
\end{aligned}
$$

Note that

$$
\zeta_t = a_{X,t}V + (b_{X,t} + a_U)S_t + \epsilon_{U,t+1}, \tag{F.1}
$$

$$
Y_t = V + a_Y S_t + \epsilon_{Y,t}. \tag{F.2}
$$

Using (F.1) and (F.2), Kalman gain matrix K_t can be easily derived using $\tilde{O}_{t+1}$ like the following:

$$
k_{V,t}^{\zeta} = \frac{1}{\sigma_{U,t+1}^2}[a_{X,t}o_{t+1}^{V} + (b_{X,t} + a_U)o_{t+1}^{VS}], \tag{F.3}
$$

$$
k_{V,t}^{Y} = \frac{1}{\sigma_{Y,t}^2}[o_{t+1}^{V} + a_Y o_{t+1}^{VS}] \tag{F.4}
$$

$$
k_{S,t}^{\zeta} = \frac{1}{\sigma_{U,t+1}^2}[a_{X,t}o_{t+1}^{VS} + (b_{X,t} + a_U)o_{t+1}^{S}], \tag{F.5}
$$

$$
k_{S,t}^{Y} = \frac{1}{\sigma_{Y,t}^2}[o_{t+1}^{VS} + a_Y o_{t+1}^{S}]. \tag{F.6}
$$

Therefore, it allows us to solve (14) for η_{T-1} at date $T-1$. Then, $\tilde{O}_{T-1}$ could be solved like the following:

$$
\begin{aligned}
o_t^V &= \frac{1}{D_t}\left[(1 - (b_{X,t} + a_U)k_{S,t}^{\zeta} - a_Y k_{S,t}^{Y})o_{t+1}^{V} + [(b_{X,t} + a_U)k_{V,t}^{\zeta} + a_Y k_{V,t}^{Y}]o_{t+1}^{VS}\right], \\
o_t^{VS} &= \frac{1}{a_S D_t}\left[(1 - (b_{X,t} + a_U)k_{S,t}^{\zeta} - a_Y k_{S,t}^{Y})o_{t+1}^{VS} + [(b_{X,t} + a_U)k_{V,t}^{\zeta} + a_Y k_{V,t}^{Y}]o_{t+1}^{S}\right], \\
o_t^{S} &= \frac{1}{a_S^2 D_t}\left[(a_{X,t}k_{S,t}^{\zeta} + k_{S,t}^{Y})o_{t+1}^{VS} + [1 - a_{X,t}k_{V,t}^{\zeta} - k_{V,t}^{Y}]o_{t+1}^{S}\right] - \frac{\sigma_{S,t}^2}{a_S^2}.
\end{aligned}
$$

where

$$D_t \equiv \left|1 + [a_{X,t}a_Y - (b_{X,t} + a_U)](k^{\zeta}_{V,t}k^{Y}_{S,t} - k^{\zeta}_{S,t}k^{Y}_{V,t}) - a_{X,t}k^{\zeta}_{V,t} - k^{Y}_{V,t} - (b_{X,t} + a_U)k^{\zeta}_{S,t} - a_Y k^{Y}_{S,t}\right|.$$

I continue to solve for η_{T-2}, and so on until I solve for $\tilde{O}_0$. Using trial and error method, find $\tilde{O}_T$ which leads to $\tilde{O}_0 = O_0$, which completes solving the equation system numerically.

References

Allen, F., and D. Gale, 1992, "Differential Information and Dynamic Behavior of Stock Trading Volume," *Review of Financial Studies*, 5(3), 503–529.

Allen, F., and G. Gorton, 1992, "Stock price manipulation, market microstructure and asymmetric information," *European Economic Review*, 36(2-3), 624–630.

Back, K., C. H. Cao, and G. A. Willard, 2000, "Imperfect Competition among Informed Traders," *Journal of Finance*, 55(5), 2117–2155.

Bernhaedt, D., and J. Miao, 2004, "Informed Trading When Information Becomes Stale," *Journal of Finance*, 59(1), 339–390.

Brunnermeier, M. K., and S. Nagel, 2004, "Hedge Funds and the Technology Bubble," *Journal of Finance*, 59(5), 2013–2040.

Chakraborty, A., and B. Yilmaz, 2004a, "Informed manipulation," *Journal of Economic Theory*, 114(1), 132–152.

——— , 2004b, "Manipulation in market order models," *Journal of Financial Markets*, 7(2), 187–206.

Chen, J., S. Hanson, H. Hong, and J. C. Stein, 2008, "Do Hedge Funds Profit from Mutual-Fund Distress?," National Bureau of Economic Research Working Paper No.

De Long, J. B., A. Shleifer, L. H. Summers, and R. J. Waldmann, 1990a, "Noise Trader Risk in Financial Markets," *Journal of Political Economy*, 98(4), 703–738.

——— , 1990b, "Positive Feedback Investment Strategies and Destabilizing Rational Speculation," *Journal of Finance*, 45(2), 374–397.

Dow, J., and G. Gorton, 1994, "Arbitrage Chains," *Journal of Finance*, 49(3), 819–849.

Dow, J., and R. Rahi, 2003, "Informed Trading, Investment and Welfare," *Journal of Business*, 76(3), 439–454.

Foster, F. D., and S. Viswanathan, 1996, "Strategic Trading When Agents Forecast the Forecasts of Others," *Journal of Finance*, 51(4), 1437–1478.

Glosten, L., and P. Milgrom, 1985, "Bid, Ask, and Transaction Prices in a Specialist Market with Heterogeneously Informed Traders," *Journal of Financial Economics*, 14(1), 71–100.

Goldstein, I., and A. Gumbel, 2008, "Manipulation and the Allocational Role of Prices," *Review of Economic Studies*, 75(1), 133–164.

Grossman, S. J., and J. E. Stiglitz, 1980, "On the Impossiblity of Informationally Efficient Markets," *American Economic Review*, 70(3), 393–408.

Hamilton, J. D., 1994, *Time Series Analysis*. Princeton University Press.

He, H., and J. Wang, 1995, "Differential Information and Dynamic Behavior of Stock Trading Volume," *Review of Financial Studies*, 8(4), 919–972.

Holden, C. W., and A. Subrahmanyam, 1992, "Long-lived Private Information and Imperfect Competition," *Journal of Finance*, 47(1), 247–270.

——— , 1994, "Risk Aversion, Imperfect Competition and Long-lived Information," *Economics Letters*, 44, 181–190.

Huddart, S., J. S. Hughes, and C. B. Levine, 2001, "Public Disclosure and Dissimulation of Insider Trades," *Econometrica*, 69(3), 665–681.

Jarrow, R. A., 1992, "Market manipulation, bubbles, corners, and short squeezes," *Journal of Financial and Quantitative Analysis*, 27(3), 311–336.

Kyle, A. S., 1985, "Continuous Auctions and Insider Trading," *Econometrica*, 53(6), 1315–1335.

Lee, C. M. C., A. Shleifer, and R. H. Thaler, 1991, "Investor Sentiment and the Closed-End Fund Puzzle," *Journal of Finance*, 46(1), 75–109.

Wang, J., 1993, "A Model of Intertemporal Asset Prices Under Asymmetric Information," *Review of Financial Studies*, 60, 249–282.

——— , 1994, "A Model of Competitive Stock Trading," *Journal of Political Economy*, 102(1), 127–168.

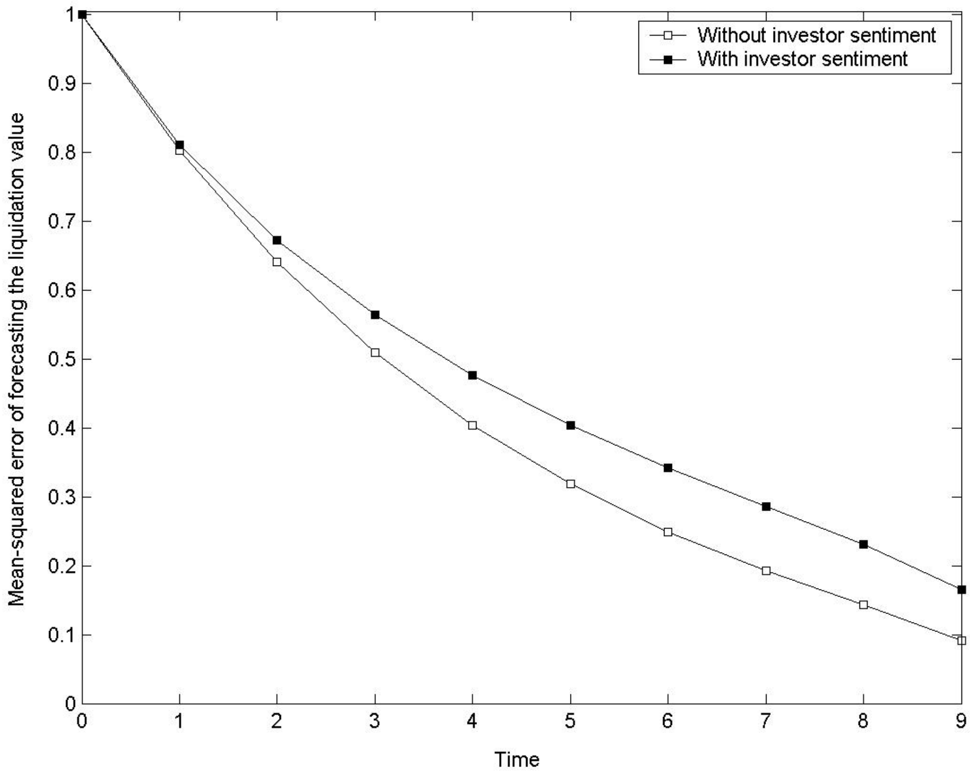

Figure 1. Mean-squared error of forecasting the liquidation value (o_t^V) over time with or without investor sentiment

Market makers' mean-squared error of forecasting the liquidation value at trade date t is given by $o_t^V = E[(V - \hat{V}_t^M)(V - \hat{V}_t^M)|\mathcal{F}_t^M]$. The line with black squares denotes o_t^V without investor sentiment, which corresponds to Holden and Subrahmanyam (1994). The line with white squares denotes o_t^V with investor sentiment ($\sigma_S = 0.2$). Public signals are not given. Parameter values are given by $T = 10$, $\gamma = 4$, $\sigma_V = 1$, $\sigma_U = 1$, $a_U = 1$.

	Without investor sentiment	With investor sentiment	
t	$a_{X,t}$	$a_{X,t}$	$b_{X,t}$
1	0.4958	0.5329	0.0308
2	0.5605	0.6439	0.0635
3	0.6341	0.7528	0.0913
4	0.7182	0.8622	0.1125
5	0.8153	0.9752	0.1238
6	0.9322	1.0948	0.1171
7	1.0877	1.2249	0.0742
8	1.3431	1.3765	-0.0485
9	2.0122	1.6678	-0.3517

Table 1. Coefficients of informed trading on fundamental factor $V - P_{t-1}$ and investor sentiment factor $S_t - a_S \hat{S}^M_{t-1}$ with or without investor sentiment

$a_{X,t}$ denotes the coefficient of the informed trader's order flow at trade date t on fundamental factor $V - P_{t-1}$. $b_{X,t}$ denotes the coefficient of the informed trader's order flow at trade date t on investor sentiment factor $S_t - a_S \hat{S}^M_{t-1}$. The first column reports the coefficients on order flow without investor sentiment, the next two columns report the coefficients on order flow with investor sentiment ($\sigma_S = 0.2$). Public signals are not given. Parameter values are given by $T = 10$, $\gamma = 4$, $\sigma_V = 1$, $\sigma_U = 1$, $a_U = 1$.

	Trading strategy		Corrective demand		Manipulative demand	
t	$a_{X,t}$	$b_{X,t}$	$a_{X,t}$	$b_{X,t}$	$a_{X,t}$	$b_{X,t}$
1	0.5329	0.0308	0.8469	**-0.1710**	-0.3140	**0.2018**
2	0.6439	0.0635	1.0563	**-0.1815**	-0.4124	**0.2451**
3	0.7528	0.0913	1.2680	**-0.1932**	-0.5153	**0.2844**
4	0.8622	0.1125	1.4815	**-0.2062**	-0.6193	**0.3187**
5	0.9752	0.1238	1.6912	**-0.2211**	-0.7160	**0.3448**
6	1.0948	0.1171	1.8829	**-0.2393**	-0.7881	**0.3564**
7	1.2249	0.0742	2.0249	**-0.2647**	-0.8000	**0.3389**
8	1.3765	-0.0485	2.0399	**-0.3051**	-0.6633	**0.2566**
9	1.6678	-0.3517	1.6678	**-0.3517**	0	**0**

Table 2. Informed trader's trading strategy and the decomposition

$a_{X,t}$ denotes the coefficient of the informed trader's order flow at trade date t on fundamental factor $V - P_{t-1}$. $b_{X,t}$ denotes the coefficient of the informed trader's order flow at trade date t on investor sentiment factor $S_t - a_S \hat{S}^M_{t-1}$. The first two columns report the informed trader's strategy which are the sums of competitive and manipulative demands, and the next four columns report the decomposition of it. Public signals are not given. Parameter values are given by $T = 10$, $\gamma = 4$, $\sigma_V = 1$, $\sigma_S = 0.2$, $\sigma_U = 1$, $a_U = 1$.

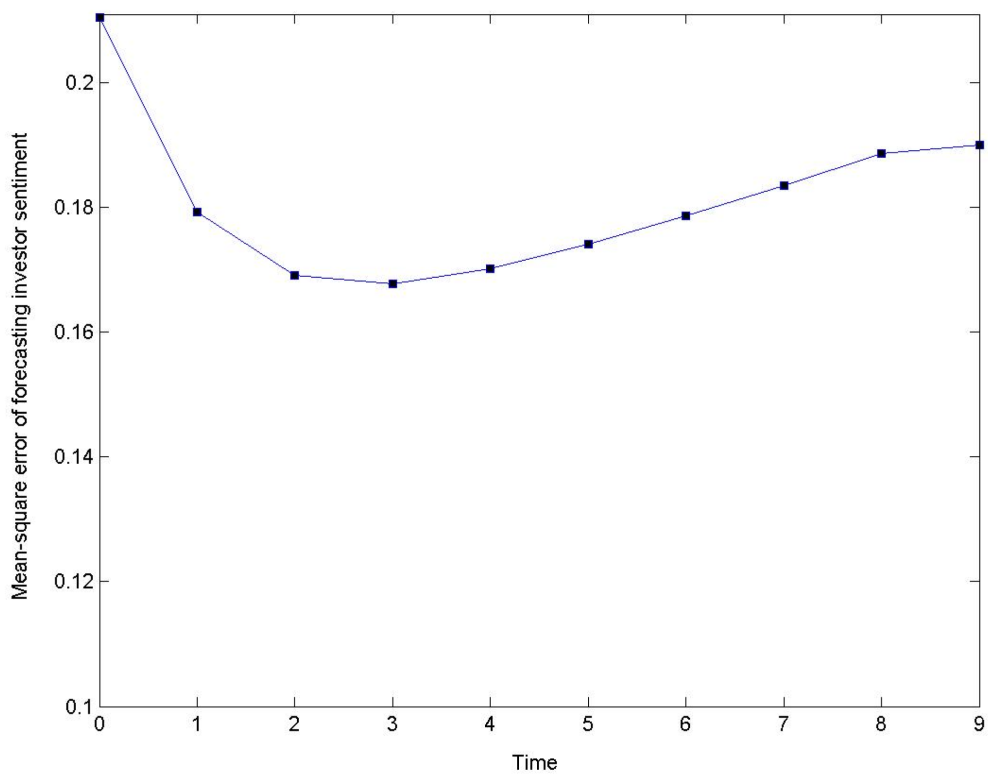

Figure 2. Mean-squared error of forecasting investor sentiment (o_t^S) over time

Market makers' mean-squared error of forecasting investor sentiment at trade date t is given by $o_t^S = E[(S_t - \hat{S}_t^M)(S - \hat{S}_t^M)|\mathcal{F}_t^M]$. The line with white squares denotes o_t^V with investor sentiment ($\sigma_S = 0.2$). Public signals are not given. Parameter values are given by $T = 10$, $\gamma = 4$, $\sigma_V = 1$, $\sigma_U = 1$, $\sigma_S = 0.2$, $a_U = 1$.

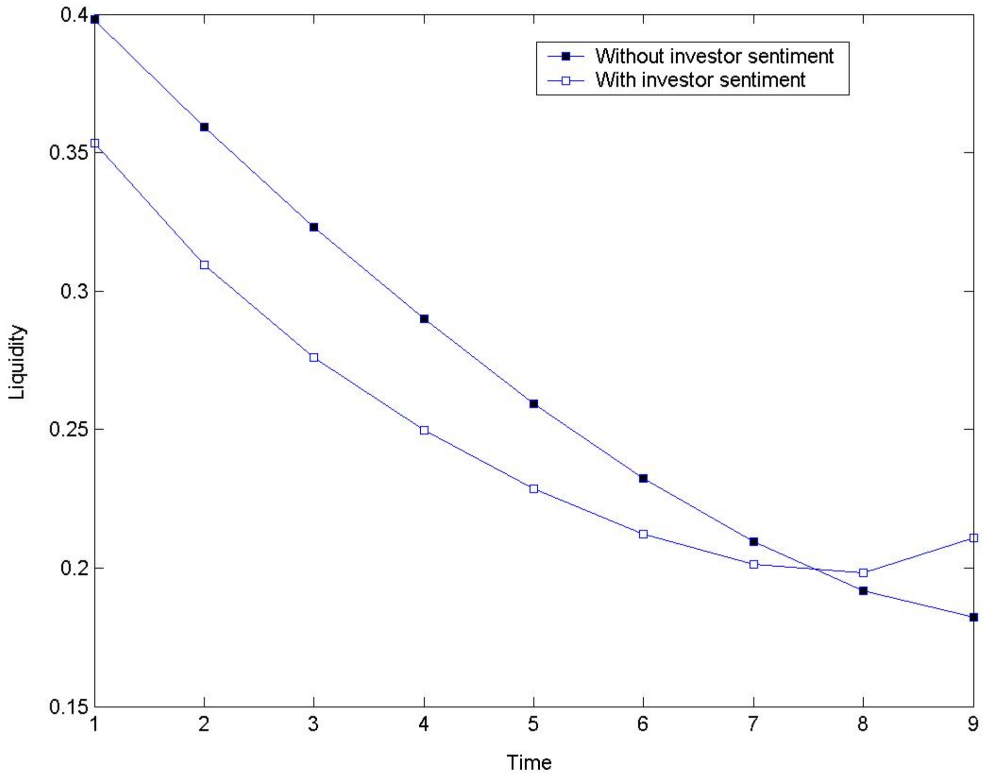

Figure 3. Liquidity over time with or without investor sentiment

Liquidity parameter is measured by the price sensitivity to the aggregate order flow ($k^{\zeta}_{V,t}$). The line with black squares denotes liquidity with investor sentiment. The line without white squares denotes liquidity without investor sentiment ($\sigma_S = 0.2$). Parameter values are given by $T = 10$, $\gamma = 4$, $\sigma_V = 1$, $\sigma_U = 1$, $a_U = 1$.

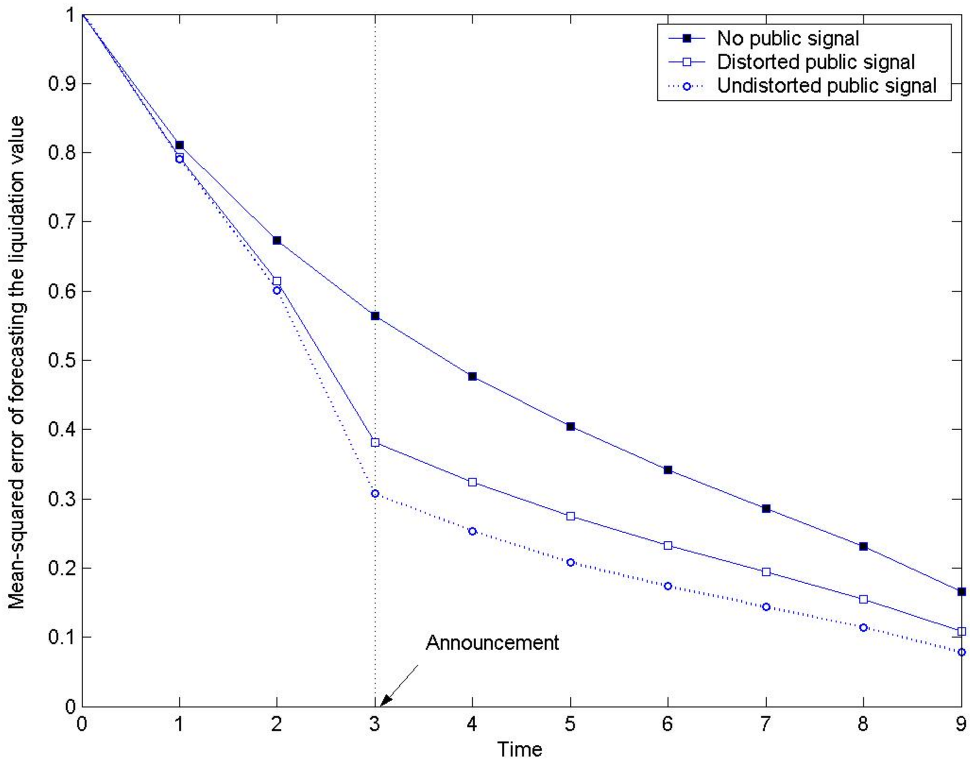

Figure 4. Price efficiency parameter o_t^V over time in the presence of public signals

Price efficiency parameter is measured by market makers' mean-squared error of forecasting at trade date t, i.e. $o_t^V = E[(V - \hat{V}_t^M)(V - \hat{V}_t^M)]$. Solid line denotes o_t^V when the public signal is not affected by investor sentiment, i.e. $a_Y = 0$. Dotted line denotes o_t^V when the public signal is affected by investor sentiment, i.e. $a_Y = 1$. Parameter values are given by $T = 10, \gamma = 4,\ \sigma_V = 1,\ \sigma_S = 0.2, \sigma_U = 1, \sigma_{Y,t} = 10^6$ for all $t \neq 3, \sigma_{Y,t} = 1$ for $t = 3, a_U = 1$.

	η_t			Corrective demand			Manipulative demand		
t	$a_{X,t}$	$b_{X,t}$	$c_{X,t}$	$a_{X,t}$	$b_{X,t}$	$c_{X,t}$	$a_{X,t}$	$b_{X,t}$	$c_{X,t}$
1	0.5594	**-0.0685**	0	0.7786	-0.2078	0	-0.2192	0.1393	0
2	0.7341	**-0.2025**	0	0.8244	-0.2603	0	-0.0902	0.0578	0
3	1.0107	0.0942	-0.3738	1.8068	-0.2442	-0.6917	-0.7961	0.3384	0.3179
4	1.1893	0.1052	0	2.1247	-0.2641	0	-0.9354	0.3692	0
5	1.3876	0.0813	0	2.3517	-0.2835	0	-0.9641	0.3648	0
6	1.5590	0.1532	0	2.9144	-0.2995	0	-1.3554	0.4528	0
7	1.7627	0.0984	0	3.1210	-0.3253	0	-1.3582	0.4237	0
8	2.0012	-0.0483	0	3.1116	-0.3615	0	-1.1105	0.3131	0
9	2.4984	-0.3827	0	2.4984	-0.3827	0	0	0	0

Table 3. Informed trader's trading strategy and the decomposition given public announcement at $t = 3$ without distortions by investor sentiment

$a_{X,t}$ denotes the coefficient of the informed trader's order flow at trade date t on fundamental factor $V - P_{t-1}$. $b_{X,t}$ denotes the coefficient on investor sentiment factor $S_t - a_S\hat{S}^M_{t-1}$. $c_{X,t}$ denotes the coefficient on public signal factor $Y_t - P_{t-1} - a_Y a_S \hat{S}^M_{t-1}$. The first three columns report the informed trader's strategy which are the sums of competitive and manipulative demands, and the next six columns report the decomposition of it. Parameter values are given by $T = 10$, $\gamma = 4$, $\sigma_V = 1$, $\sigma_S = 0.2$, $\sigma_U = 1$, $\sigma_{Y,t} = 10^6$ for all $t \neq 3, \sigma_{Y,t} = 1$ for $t = 3$, $a_Y = 0$, $a_U = 1$.

	Trading strategy			Corrective demand			Manipulative demand		
t	$a_{X,t}$	$b_{X,t}$	$c_{X,t}$	$a_{X,t}$	$b_{X,t}$	$c_{X,t}$	$a_{X,t}$	$b_{X,t}$	$c_{X,t}$
1	0.5590	**-0.0525**	0	0.7867	-0.2000	0	-0.2276	0.1475	0
2	0.7335	**-0.0556**	0	0.8558	-0.1807	0	-0.1223	0.1251	0
3	1.0069	0.1309	-0.3305	1.7814	-0.2275	-0.5606	-0.7745	0.3584	0.2301
4	1.1295	0.1365	0	1.9997	-0.2428	0	-0.8701	0.3793	0
5	1.2558	0.1502	0	2.2626	-0.2581	0	-1.0067	0.4083	0
6	1.3943	0.1313	0	2.4627	-0.2772	0	-1.0684	0.4085	0
7	1.5455	0.0846	0	2.6297	-0.3016	0	-1.0842	0.3863	0
8	1.7297	-0.0646	0	2.5726	-0.3391	0	-0.8429	0.2745	0
9	2.1041	-0.3699	0	2.1041	-0.3699	0	0	0	0

Table 4. Informed trader's trading strategy and the decomposition given public announcement at $t = 3$ with distortions by investor sentiment

$a_{X,t}$ denotes the coefficient of the informed trader's order flow at trade date t on fundamental factor $V - P_{t-1}$. $b_{X,t}$ denotes the coefficient on investor sentiment factor $S_t - a_S \hat{S}^M_{t-1}$. $c_{X,t}$ denotes the coefficient on public signal factor $Y_t - P_{t-1} - a_Y a_S \hat{S}^M_{t-1}$. The first three columns report the informed trader's strategy which are the sums of competitive and manipulative demands, and the next six columns report the decomposition of it. Parameter values are given by $T = 10$, $\gamma = 4$, $\sigma_V = 1$, $\sigma_S = 0.2$, $\sigma_U = 1$, $\sigma_{Y,t} = 10^6$ for all $t \neq 3, \sigma_{Y,t} = 1$ for $t = 3$, $a_Y = 1$, $a_U = 1$.

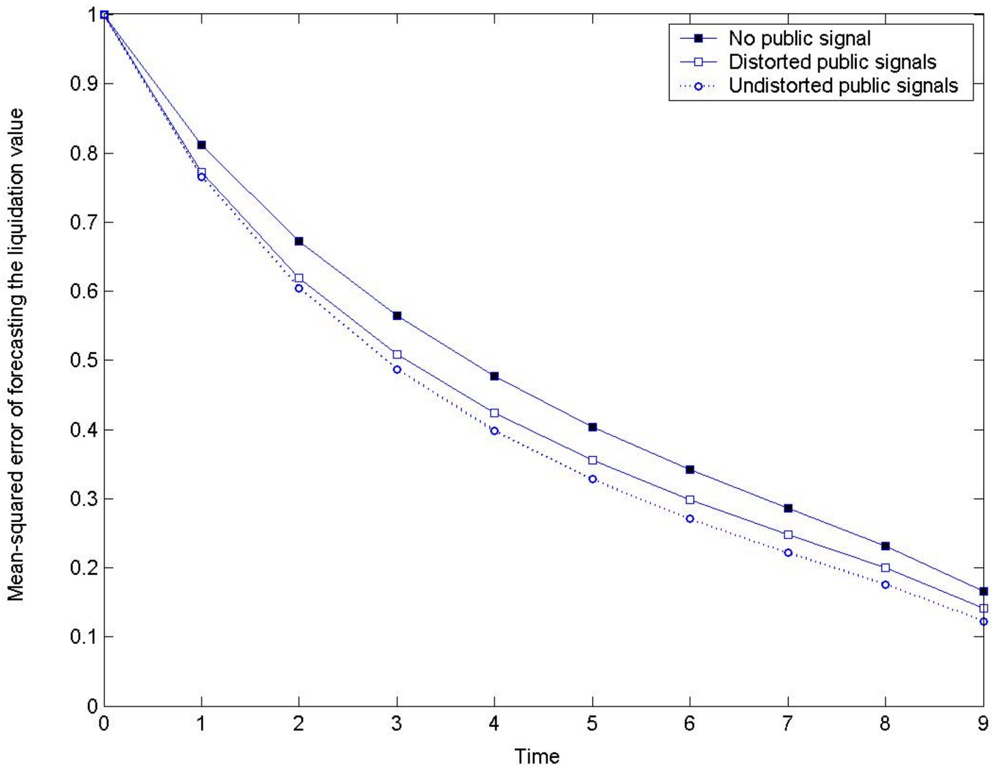

Figure 5. Mean-squared error of forecasting the liquidation value (o_t^V) over time with or without distortions in public signals

Market makers' mean-squared error of forecasting the liquidation value at trade date t is given by $o_t^V = E[(V - \hat{V}_t^M)(V - \hat{V}_t^M)|\mathcal{F}_t^M]$. The line with black squares denotes o_t^V without public signals. The line with white squares denotes o_t^V with distorted public signals ($a_Y = 0$). The dotted line with white circles denotes o_t^V with undistorted public signals ($a_Y = 1$). Parameter values are given by $T = 10$, $\gamma = 4$, $\sigma_V = 1$, $\sigma_S = 0.2$, $\sigma_U = 1$, $\sigma_Y = 5$, $a_U = 1$.

	Trading strategy			Corrective demand			Manipulative demand		
t	$a_{X,t}$	$b_{X,t}$	$c_{X,t}$	$a_{X,t}$	$b_{X,t}$	$c_{X,t}$	$a_{X,t}$	$b_{X,t}$	$c_{X,t}$
1	0.5654	-0.0251	-0.0217	0.8256	-0.1922	-0.0318	-0.2602	0.1670	0.0100
2	0.6929	0.0116	-0.0206	1.0627	-0.2032	-0.0321	-0.3698	0.2148	0.0115
3	0.8214	0.0470	-0.0192	1.3165	-0.2149	-0.0319	-0.4950	0.2620	0.0127
4	0.9535	0.0782	-0.0179	1.5851	-0.2281	-0.0313	-0.6317	0.3063	0.0134
5	1.0917	0.1002	-0.0166	1.8604	-0.2434	-0.0303	-0.7687	0.3435	0.0136
6	1.2392	0.1034	-0.0156	2.1221	-0.2619	-0.0287	-0.8829	0.3654	0.0131
7	1.4005	0.0674	-0.0147	2.3277	-0.2874	-0.0262	-0.9272	0.3548	0.0115
8	1.5891	-0.0546	-0.0142	2.3780	-0.3265	-0.0223	-0.7889	0.2719	0.0081
9	1.9571	-0.3643	-0.0151	1.9571	-0.3643	-0.0151	0	0	0

Table 5. Informed trader's strategy and the decomposition given sequential public signals without distortions by investor sentiment

$a_{X,t}$ denotes the coefficient of the informed trader's order flow at trade date t on fundamental factor $V - P_{t-1}$. $b_{X,t}$ denotes the coefficient on investor sentiment factor $S_t - a_S \hat{S}^M_{t-1}$. $c_{X,t}$ denotes the coefficient on public signal factor $Y_t - P_{t-1} - a_Y a_S \hat{S}^M_{t-1}$. The first three columns report the informed trader's strategy which are the sums of competitive and manipulative demands, and the next six columns report the decomposition of it. Parameter values are given by $T = 10$, $\gamma = 4$, $\sigma_V = 1$, $\sigma_S = 0.2$, $\sigma_U = 1$, $\sigma_{Y,t} = 5$ for all $0 < t < 10$, $a_Y = 0$, $a_U = 1$.

	Trading Strategy			Corrective demand			Manipulative demand		
t	$a_{X,t}$	$b_{X,t}$	$c_{X,t}$	$a_{X,t}$	$b_{X,t}$	$c_{X,t}$	$a_{X,t}$	$b_{X,t}$	$c_{X,t}$
1	0.5644	0.0051	-0.0216	0.8416	-0.1781	-0.0307	-0.2772	0.1831	0.0091
2	0.6909	0.0488	-0.0189	1.0859	-0.1877	-0.0286	-0.3950	0.2365	0.0097
3	0.8147	0.0866	-0.0166	1.3369	-0.1991	-0.0262	-0.5222	0.2857	0.0097
4	0.9383	0.1159	-0.0145	1.5911	-0.2127	-0.0237	-0.6528	0.3286	0.0092
5	1.0645	0.1326	-0.0126	1.8393	-0.2286	-0.0209	-0.7748	0.3612	0.0083
6	1.1962	0.1281	-0.0110	2.0629	-0.2482	-0.0179	-0.8667	0.3762	0.0069
7	1.3373	0.0834	-0.0093	2.2245	-0.2748	-0.0145	-0.8872	0.3582	0.0052
8	1.4992	-0.0458	-0.0075	2.2369	-0.3157	-0.0105	-0.7377	0.2699	0.0030
9	1.8161	-0.3585	-0.0054	1.8161	-0.3585	-0.0054	0	0	0

Table 6. Informed trader's strategy and the decomposition given sequential public signals with distortions by investor sentiment

$a_{X,t}$ denotes the coefficient of the informed trader's order flow at trade date t on fundamental factor $V - P_{t-1}$. $b_{X,t}$ denotes the coefficient on investor sentiment factor $S_t - a_S \hat{S}^M_{t-1}$. $c_{X,t}$ denotes the coefficient on public signal factor $Y_t - P_{t-1} - a_Y a_S \hat{S}^M_{t-1}$. The first three columns report the informed trader's strategy which are the sums of competitive and manipulative demands, and the next six columns report the decomposition of it. Parameter values are given by $T = 10$, $\gamma = 4$, $\sigma_V = 1$, $\sigma_S = 0.2$, $\sigma_U = 1$, $\sigma_{Y,t} = 5$ for all $0 < t < 10$, $a_Y = 1$, $a_U = 1$.