Asset Prices in the Representative-Agent Economy with Background Risk*

Andrei Semenov†

York University

May 4, 2009

Abstract

We assume an economy with the representative agent that faces, in addition to the financial investment risk, an independent non-hedgeable adverse background risk. In this economy, the representative agent's consumption has the hedgeable and non-hedgeable (caused by non-hedgeable background risk factors) components. The associated pricing kernel is a function of the agent's optimal hedgeable consumption and the first two unconditional moments of the non-hedgeable consumption distribution. Empirical evidence is that this pricing kernel jointly explains the observed cross-section of the excess returns on risky assets and the risk-free rate with economically plausible values of the representative agent's preference parameters.

JEL Classification: G12
Keywords: Consumption CAPM, Equity Premium Puzzle, Incomplete Consumption Insurance, Risk-Free Rate Puzzle.

*The author thanks Melanie Cao, Pauline Shum, and Yisong Tian for helpful comments and suggestions and gratefully acknowledges financial support from the Social Sciences and Humanities Research Council of Canada (SSHRC), grant number 515199.

†Department of Economics, York University, Vari Hall 1028, 4700 Keele Street, Toronto, Ontario M3J 1P3, Canada, Phone: (416) 736-2100 ext.: 77025, Fax: (416) 736-5987, E-mail: asemenov@econ.yorku.ca

1 Introduction

The Lucas (1978) and Breeden (1979) consumption CAPM (standard consumption CAPM, hereafter) relates asset prices to the consumption and savings decisions of a single investor, interpreted as a representative for a large number of identical infinitely-lived investors. The power utility maximizing representative investor is assumed to freely trade in perfect capital markets without transaction costs, limitations on borrowing or short sales, and taxes. The stochastic discount factor (SDF), or pricing kernel, in this model is the discounted aggregate per capita consumption growth rate raised to the power of the negative utility curvature parameter (relative risk aversion (RRA) coefficient).

As argued in the literature, a problem with the standard consumption CAPM is that, when reasonably parameterized, this model yields the average equity premium, which is substantially lower than the observed excess return on the market portfolio over the risk-free rate and the representative agent must be assumed implausibly averse to risk for the model to fit the observed mean equity premium. This is the equity premium puzzle discussed by Mehra and Prescott (1985) and Hansen and Jagannathan (1991) among others. Another problem with the same model is that it yields the mean risk-free rate, which is much greater than the observed average return on the risk-free asset. The model explains the mean return on the risk-free asset only if the subjective time discount factor is greater than one (the representative investor has a negative rate of time preference). This is the risk-free rate puzzle as described in Weil (1989).

Since the Mehra and Prescott (1985) seminal paper, several ways to improve the standard consumption CAPM have been proposed in the literature. One response to the difficulties of this model is to generalize the utility function. Another way to improve the ability of the representative-agent consumption CAPM to fit empirical data on asset returns is to take into account the effects of psychological biases on asset prices. Some authors argue that various market frictions (such as transactions costs, limits on borrowing or short sales, etc.) may make aggregate consumption an inadequate proxy for the consumption of stock market investors and develop asset pricing models that do not rely so heavily on the aggregate consumption of the economy.

A possible explanation for the empirical failure of the standard representative-agent model is that this model abstracts from the lack of certain types of insurance such as insurance against labor income risk, loss of employment, inflation uncertainty, etc. The potential for the incomplete consumption insurance model to explain the equilibrium behavior of stock and bond returns, both in terms of the level of equilibrium rates and the discrepancy between equity and bond returns, was first suggested by Bewley (1982), Mehra and Prescott (1985), and Mankiw (1986).

The empirical evidence for the importance of the incomplete consumption insurance hypothesis for explaining asset returns is somewhat mixed. Aiyagari and Gertler (1991), Huggett (1993), Telmer (1993), Lucas (1994), Heaton and D. Lucas (1996, 1997), Jacobs (1999), and Cogley

(2002), for example, find no evidence that the assumption of incomplete consumption insurance can improve substantially the asset-pricing implications of the representative-agent consumption CAPM. However, Brav *et al.* (2002), Balduzzi and Yao (2007), and Kocherlakota and Pistaferri (2008) take the opposite view. Despite the difference in approaches, they find empirically that incomplete consumption insurance plays an important role in explaining the market premium. Brav *et al.* (2002), for example, obtain that, when incomplete consumption insurance is taken into account, the consumption CAPM can explain the mean equity premium with a RRA coefficient between three and four. Kocherlakota and Pistaferri (2008) show that the coefficient of RRA must be in the range from five to six for the incomplete consumption insurance model to explain the market excess return. The SDF proposed by Balduzzi and Yao (2007) enables to fit the equity premium with a value of the RRA coefficient, which is substantially lower than the estimate obtained for the standard representative-agent model, but still remains unrealistically high (greater than nine).

Although the hypothesis in incomplete consumption insurance allowed to make substantial progress in explaining the equity premium, there is still a problem with a joint explanation of the equity premium and the risk-free rate of return. For example, the pricing kernels proposed by Brav *et al.* (2002) and Kocherlakota and Pistaferri (2008) fit the observed excess return on the market portfolio over the risk-free rate at low values of RRA, but fare poorly when used to explain the risk-free rate. The both SDFs yield implausibly low estimates of the subjective time discount factor.

In this paper, we also investigate the role of the hypothesis of incomplete consumption insurance in explaining asset returns. However, our approach is different from the approaches in Brav *et al.* (2002), Balduzzi and Yao (2007), and Kocherlakota and Pistaferri (2008). Instead of considering heterogenous agents, we assume that there is a single representative investor that faces, in addition to the financial investment risk, a non-hedgeable background risk, which is adverse (i.e., has a non-positive expected value) and independent of the risk associated with the investment in a risky asset. The examples of such a risk are the losses from domestic political turmoil, the inflation risk, an uncertain income tax rate, the risk from natural disasters, etc.

The standard consumption CAPM is implicitly based on the assumptions of complete consumption insurance against the financial investment risk and the absence of non-hedgeable background risks. We refer the agent's RRA to uncertainty about the return on risky asset holdings in such an environment to as pure RRA. We argue that the utility in the standard model is, in fact, the indirect utility function, which may be different from the agent's original utility. In the absence of background risk, the two utility functions coincide. Gollier (2001) argues that risk vulnerability of preferences implies that in the presence of an independent non-hedgeable adverse background risk the indirect utility is more concave than the original utility. The curvature parameter of the original utility function characterizes the agent's pure RRA, while the indirect utility curvature parameter measures RRA in the presence of background risk (RRA "adjusted" for the background risk effect) or effective RRA. If preferences exhibit risk vulnerability, then the greater the background risk, the larger the effective RRA coefficient relative to the coefficient of pure RRA. Since the CRRA utility is risk vulnerable and the standard consumption CAPM is based on the assumption of the absence of background risk (and hence identifies the indirect and original utility functions), when erroneously interpreting the RRA coefficient in the standard model as the pure RRA coefficient, one can severely overestimate pure RRA if, in fact, the representative agent faces the independent non-hedgeable adverse background risk. In the presence of adverse background risk, the pure RRA coefficient may be in the acceptable range of values even if the curvature of the utility function in the standard consumption CAPM (the coefficient of effective RRA) is implausibly high and hence the so-called equity premium puzzle may simply be due to misinterpretation of the utility curvature parameter in the standard CAPM as the pure RRA coefficient.

By making the representative agent's utility function more concave, the independent non-hedgeable adverse background risk raises tomorrow's marginal utility of consumption in each non-hedgeable consumption state. This raises the expectation of the representative-agent's intertemporal marginal rate of substitution lowering thereby the equilibrium risk-free rate of return yielded by the model with background risk at given values of the agent's pure RRA and subjective time discount factor. It suggests that the consumption CAPM with background risk has the potential to jointly explain the observed equity premium and risk-free rate with plausible values of pure RRA and the subjective time discount factor.

Because under background risk the effective RRA coefficient differs from the curvature parameter of the original utility function, like in Campbell and Cochrane (1999) difference habit model, within our framework we are able to disentangle effective risk aversion and the utility curvature parameter. The attractive feature of our approach is that we can disentangle these two parameters under less restrictive assumptions and can avoid getting unrealistically volatile estimates of the effective RRA coefficient.

Our result is that the elasticity of intertemporal substitution (EIS) for the agent with the original utility function under background risk is identical to the EIS for the agent with indirect utility in the no background risk environment and is therefore equal to the inverse of the effective RRA coefficient. Since in the presence of background risk effective RRA differs from pure RRA, while they coincide in the standard consumption CAPM, this implies that the model with background risk enables us to disentangle pure risk aversion and the EIS in the expected utility framework. Under adverse background risk, the coefficient of effective RRA is greater than the pure RRA coefficient and hence, in contrast to the standard consumption CAPM, we can get a low value of the EIS even if the representative agent's aversion to financial risk is low.

We show that the effective RRA coefficient (and hence the EIS) is a function of the first two unconditional moments of the distribution of the non-hedgeable consumption and therefore may change over time even if the representative agent is assumed to have a time-invariant original utility function (i.e., the pure RRA coefficient does not change over time). This relates our approach to the strand of the literature that postulates that the attitudes towards financial risk are not fixed, but rather contingent upon the state of the world. The advantage of our approach is that the choice of the factors deemed to be relevant in explaining the investor's risk aversion and the functional form relating the coefficient of effective RRA to these factors obtain endogenously from the theoretical restrictions implied by a structural model.

The rest of the paper is organized as follows. In Section 2, we first derive the representative-agent consumption CAPM with the independent non-hedgeable background risk and analyze the potential of this model to solve both the equity premium and risk-free rate puzzles when the background risk has a non-positive mean. Then, we show that in the presence of an independent background risk the effective RRA coefficient is a function of the first two unconditional moments of the non-hedgeable consumption distribution and demonstrate that, when the background risk is adverse, as expected, the effective RRA coefficient is greater than the coefficient of pure RRA. The properties of the effective RRA are also investigated. We also show that under background risk the EIS is the reciprocal of the effective RRA coefficient. In Section 3, we describe data and investigate empirically the ability of the consumption CAPM with background risk to jointly explain the cross-section of excess returns on risky assets and the risk-free rate. Section 4 concludes.

2 A consumption-based asset pricing model with background risk

2.1 The optimal consumption choice problem

Consider the discrete-state intertemporal consumption choice problem of an infinitely living representative investor who maximizes the expected present value of discounted lifetime utility of consumption. Assume that uncertainty about the return on risky asset holdings is traded in a complete market. Suppose furthermore that the representative agent faces, in addition to the financial risk, an adverse background risk, which is non-hedgeable (i.e., the market for this risk is incomplete) and independent of the risk associated with the investment in a risky asset. The representative agent's consumption therefore depends not only on the return on risky asset holdings, but also on the non-hedgeable background risk, and is the sum of two components namely the hedgeable (the consumption that may be hedged using the financial market) and non-hedgeable consumption (the consumption caused by non-hedgeable background risk factors and hence uninsurable).

The agent is not able to self-insure against the background risk, but (knowing the distribution of the non-hedgeable consumption) can choose the consumption he is able to hedge in the financial market (the hedgeable consumption) that maximizes the expected present value of discounted lifetime utility of consumption[1]

$$U = \sum_{\tau=0}^{\infty} \delta^{\tau} E_t \left[E^{\Phi} \left[u \left(C_{t+\tau} + \Phi_{t+\tau} \right) \right] \right] \tag{1}$$

subject to the intertemporal budget constraint (which is the same as in the no background risk case)

$$W_{t+1} = \left(W_t - C_t \right) R_{i,t+1}, \tag{2}$$

where δ is the subjective time discount factor, $C_{t+\tau}$ is the representative agent's hedgeable consumption in period $t+\tau$, $\Phi_{t+\tau}$ is the non-hedgeable consumption, W_t is the representative agent's hedgeable wealth at time t, $R_{i,t+1}$ is the real gross return between time t and $t+1$ on asset i in which the agent holds a nonzero position, and $u(\cdot)$ is the representative-agent's period utility function.[2] The non-hedgeable consumption $\Phi_{t+\tau}$ is independent of both the hedgeable consumption $C_{t+\tau}$ and the risky payoff and is assumed to have a non-positive expected value. Note that the decision on the hedgeable consumption C_t is taken prior to the realization of Φ_t. E_t denotes the expectation over $C_{t+\tau}$ conditional on the information available to the agent at time t after the agent chooses his period t hedgeable consumption C_t and before he observes the realization of Φ_t. The notation E^{Φ} indicates the expectation over $\Phi_{t+\tau}$.

One of the first-order conditions, or Euler equations, describing the representative agent's optimal hedgeable consumption plan is

$$E^{\Phi} \left[u' \left(C_t + \Phi_t \right) \right] = \delta E_t \left[E^{\Phi} \left[u' \left(C_{t+1} + \Phi_{t+1} \right) \right] R_{i,t+1} \right], \tag{3}$$

where $u'(\cdot)$ denotes the first derivative of utility with respect to C_t.

The left-hand side of equation (3) is the expected (over the non-hedgeable consumption states) loss in utility if the representative investor buys another unit of asset i at time t and the right-hand side of this equation is the increase in discounted, expected utility the investor obtains from the extra payoff at time $t+1$ under background risk. In optimum, the investor equates the expected marginal loss and the expected marginal gain from holding asset i. Denote as $\Phi_{t,\lambda}$ the value of the non-hedgeable consumption Φ_t in state λ, $\lambda = 1, ..., N$. In order for marginal utility to be well-defined in any state λ, assume that the time t total consumption $C_{t,\lambda} = C_t + \Phi_{t,\lambda} > 0$.

[1] See also Franke *et al.* (1998) and Poon and Stapleton (2005).

[2] We assume that the first four derivatives of $u(\cdot)$ exist. As is conventional in the literature, we also assume that the representative agent is risk averse (i.e., $u'(\cdot) > 0$ and $u''(\cdot) < 0$) and prudent ($u'''(\cdot) > 0$). Kimball (1990) defines prudence as a measure of the sensitivity of the optimal choice of a decision variable to risk (of the intensity of the precautionary saving motive in the context of the consumption-saving decision under uncertainty). A precautionary saving motive is positive when $-u'(\cdot)$ is concave ($u'''(\cdot) > 0$) just as an individual is risk averse when $u(\cdot)$ is concave. Intuitively, the willingness to save is an increasing function of the expected marginal utility of future consumption. Since marginal utility is decreasing in consumption, the absolute level of precautionary savings must also be expected to decline as consumption rises. The condition $u''''(\cdot) < 0$ is necessary for decreasing absolute prudence.

Since $E^{\Phi}[u'(C_t+\Phi_t)]$ is known to the agent at time t after he chooses his period t hedgeable consumption C_t and before he observes the realization of the non-hedgeable consumption Φ_t and is therefore in the agent's set of information conditional on which he makes the decision on the optimal hedgeable consumption plan, we can divide both the left- and right-hand sides of equation (3) by $E^{\Phi}[u'(C_t+\Phi_t)]$ to get

$$E_t\left[\delta\frac{E^{\Phi}[u'(C_{t+1}+\Phi_{t+1})]}{E^{\Phi}[u'(C_t+\Phi_t)]}R_{i,t+1}\right]=1. \tag{4}$$

This is the consumption CAPM with background risk. In this model, the SDF is the discounted ratio of expectations of marginal utility over the non-hedgeable consumption states at two consecutive dates:

$$M_{t+1}=\delta\frac{E^{\Phi}[u'(C_{t+1}+\Phi_{t+1})]}{E^{\Phi}[u'(C_t+\Phi_t)]}. \tag{5}$$

In the absence of background risk,

$$E^{\Phi}\left[u'(C_t+\Phi_t)\right]=u'(C_t) \tag{6}$$

for any t and hence model (4) reduces to the conventional consumption CAPM with no background risk:

$$E_t\left[\delta\frac{u'(C_{t+1})}{u'(C_t)}R_{i,t+1}\right]=1, \tag{7}$$

in which the SDF is the discounted ratio of the marginal utility of consumption at time $t+1$ to the marginal utility of consumption at time t:

$$M_{t+1}=\delta\frac{u'(C_{t+1})}{u'(C_t)}. \tag{8}$$

2.2 The precautionary premium

In $E^{\Phi}[u'(C_t+\Phi_t)]$, C_t is a certain quantity and Φ_t is a random variable. Following Kimball (1990), we can hence write $E^{\Phi}[u'(C_t+\Phi_t)]$ as

$$E^{\Phi}\left[u'(C_t+\Phi_t)\right]=u'(C_t+E[\Phi_t]-\Psi_t)=u'\left(E^{\Phi}[C_t]-\Psi_t\right), \tag{9}$$

where $\Psi_t=\Psi(C_t,u(\cdot),\Phi_t)$ is an equivalent precautionary premium.

As follows from the first-order condition (3), the precautionary premium Ψ_t is the certain amount by which the representative agent is ready to reduce his consumption to escape the background risk while keeping his hedgeable consumption at the level as it would be in the presence of background risk. Franke *et al.* (1998) and Poon and Stapleton (2005) show that, in the case of HARA utility functions, the precautionary premium Ψ_t is positive, strictly increasing in the variability of Φ_t, and (except for exponential utility) strictly decreasing and convex in wealth (the level of hedgeable consumption, C_t, and the expected value of Φ_t, $E[\Phi_t]$, in our model).[3]

[3] The precautionary premium is a constant in the case of the exponential utility function.

In Section 2.1, we assumed that $C_{t,\lambda} = C_t + \Phi_{t,\lambda} > 0$ for any λ. Because this inequality holds in any state λ, it also holds in expectation, i.e.,

$$E^{\Phi}\left[C_t\right] = C_t + E\left[\Phi_t\right] > 0. \tag{10}$$

Equation (9) implies that for the marginal utility in the right-hand side of this equation to be well-defined, we now need $E^{\Phi}\left[C_t\right] - \Psi_t > 0$.

Using (9), we can write SDF (5) as

$$M_{t+1} = \delta \frac{u'\left(E^{\Phi}\left[C_{t+1}\right] - \Psi_{t+1}\right)}{u'\left(E^{\Phi}\left[C_t\right] - \Psi_t\right)}. \tag{11}$$

Notice that, as follows from condition (9), the precautionary premium Ψ_t is equivalent to the risk premium of Φ_t for the representative agent with utility function $-u'(\cdot)$, i.e.,

$$\Psi_t = \Pi\left(C_t, -u'(\cdot), \Phi_t\right). \tag{12}$$

Hence, by analogy with the risk premium,

$$\Psi_t \approx \frac{1}{2}\sigma^2_{\Phi,t}\left(-\frac{u'''\left(E^{\Phi}\left[C_t\right]\right)}{u''\left(E^{\Phi}\left[C_t\right]\right)}\right), \tag{13}$$

where

$$\sigma^2_{\Phi,t} = N^{-1}\sum_{\lambda=1}^{N}\left(\Phi_{t,\lambda} - E\left[\Phi_t\right]\right)^2 = N^{-1}\sum_{\lambda=1}^{N}\left(C_{t,\lambda} - E^{\Phi}\left[C_t\right]\right)^2. \tag{14}$$

Assume that the representative agent's utility function in (1) is CRRA:

$$u = \frac{\left(C_t + \Phi_t\right)^{1-\gamma} - 1}{1-\gamma}, \tag{15}$$

where the utility curvature parameter $\gamma > 0$, $\gamma \neq 1$.[4]

For this utility specification,

$$u'\left(E^{\Phi}\left[C_t\right]\right) = \left(E^{\Phi}\left[C_t\right]\right)^{-\gamma}, \tag{16}$$

$$u''\left(E^{\Phi}\left[C_t\right]\right) = -\gamma\left(E^{\Phi}\left[C_t\right]\right)^{-\gamma-1}, \tag{17}$$

$$u'''\left(E^{\Phi}\left[C_t\right]\right) = \gamma\left(\gamma+1\right)\left(E^{\Phi}\left[C_t\right]\right)^{-\gamma-2}, \tag{18}$$

$$u''''\left(E^{\Phi}\left[C_t\right]\right) = -\gamma\left(\gamma+1\right)\left(\gamma+2\right)\left(E^{\Phi}\left[C_t\right]\right)^{-\gamma-3}. \tag{19}$$

The precautionary premium Ψ_t for an agent with CRRA utility is hence

$$\Psi_t \approx \frac{1}{2}\sigma^2_{\Phi,t}\frac{\gamma+1}{E^{\Phi}\left[C_t\right]}. \tag{20}$$

[4] As γ approaches one, the power utility function (15) approaches the logarithmic utility $u\left(C_t + \Phi_t\right) = log\left(C_t + \Phi_t\right)$.

This implies that, with CRRA utility, for any t

$$u'\left(E^{\Phi}\left[C_t\right]-\Psi_t\right)=\left(E^{\Phi}\left[C_t\right]-\Psi_t\right)^{-\gamma}=\left(E^{\Phi}\left[C_t\right]\right)^{-\gamma}\left(1-\frac{\gamma+1}{2}\varsigma_{\Phi,t}^2\right)^{-\gamma}, \tag{21}$$

where $\varsigma_{\Phi,t}^2$ is the normalized variance of Φ_t:

$$\varsigma_{\Phi,t}^2=\frac{\sigma_{\Phi,t}^2}{\left(E^{\Phi}\left[C_t\right]\right)^2}=N^{-1}\sum_{\lambda=1}^{N}\left(\frac{C_{t,\lambda}-E^{\Phi}\left[C_t\right]}{E^{\Phi}\left[C_t\right]}\right)^2. \tag{22}$$

As follows from (21), we need to have $1-(\gamma+1)\varsigma_{\Phi,t}^2/2>0$ or, equivalently, $\gamma<2/\varsigma_{\Phi,t}^2-1$ to ensure that the marginal utility $u'\left(E^{\Phi}\left[C_t\right]-\Psi_t\right)$ is well-defined.[5]

Substituting (21) into (11) gives the following formula for the SDF in the consumption CAPM with background risk:

$$M_{t+1}=\delta\left(\frac{E^{\Phi}\left[C_{t+1}\right]}{E^{\Phi}\left[C_t\right]}\right)^{-\gamma}\left(\frac{1-\frac{\gamma+1}{2}\varsigma_{\Phi,t+1}^2}{1-\frac{\gamma+1}{2}\varsigma_{\Phi,t}^2}\right)^{-\gamma}. \tag{23}$$

Hence, in the presence of background risk the pricing kernel in the consumption CAPM is no longer a function of aggregate consumption per capita alone (as in the standard consumption CAPM), but is also a function of the first two unconditional moments of the distribution of the non-hedgeable consumption.

When the representative agent's utility function is (15), the consumption CAPM with background risk is

$$E_t\left[\delta\left(\frac{E^{\Phi}\left[C_{t+1}\right]}{E^{\Phi}\left[C_t\right]}\right)^{-\gamma}\left(\frac{1-\frac{\gamma+1}{2}\varsigma_{\Phi,t+1}^2}{1-\frac{\gamma+1}{2}\varsigma_{\Phi,t}^2}\right)^{-\gamma}R_{i,t+1}\right]=1. \tag{24}$$

If there is no background risk (i.e., $\Phi_{t,\lambda}=0$ for all λ and t and therefore $E^{\Phi}\left[C_t\right]=C_t$ and $\varsigma_{\Phi,t}^2=0$ for all t), then the marginal utility of consumption in (21) becomes

$$u'\left(E^{\Phi}\left[C_t\right]-\Psi_t\right)=u'\left(C_t\right)=C_t^{-\gamma} \tag{25}$$

and thus model (24) reduces to the standard consumption CAPM,

$$E_t\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}R_{i,t+1}\right]=1, \tag{26}$$

which is built based on the assumption of the absence of background risk.

[5] When the background risk is small (i.e., the value of $\varsigma_{\Phi,t}^2$ is small), the upper bound on the curvature parameter in utility γ is sufficiently high for the set of admissible values of γ to include all economically plausible values of this parameter. For example, with $\varsigma_{\Phi,t}^2=0.1$ the upper bound is 19. This is greater than 10, the highest plausible value for the curvature parameter γ as argued by Mehra and Prescott (1985).

2.3 Risk vulnerability and the asset pricing puzzles

The intuition is that the presence of a non-hedgeable adverse background risk increases the aversion of a decision maker to other independent risks. The preferences with such a property are said to exhibit risk vulnerability.[6]

As introduced by Kihlstrom *et al.* (1981) and Nachman (1982), define the following indirect utility function:

$$g(C_t) = E^{\Phi}\left[u(C_t + \Phi_t)\right]. \tag{27}$$

Gollier (2001) argues that, in the case of the background risk with a non-positive mean (i.e., $E[\Phi_t] \leqslant 0$) preferences exhibit risk vulnerability if and only if the indirect utility function $g(\cdot)$ is more concave than the original utility function $u(\cdot)$, i.e., for all C_t

$$-\frac{g''(C_t)}{g'(C_t)} = -\frac{E^{\Phi}\left[u''(C_t + \Phi_t)\right]}{E^{\Phi}\left[u'(C_t + \Phi_t)\right]} \geqslant -\frac{u''(C_t)}{u'(C_t)}. \tag{28}$$

As shown by Gollier (2001), this inequality holds if at least one of the following two conditions is satisfied: (i) absolute risk aversion is decreasing and convex and (ii) both absolute risk aversion and absolute prudence are positive and decreasing in wealth.[7] The latter condition is referred to as standard risk aversion, the concept introduced by Kimball (1993).[8]

To see how taking into account risk vulnerability can help solve the equity premium and risk-free rate puzzles, write the first-order conditions (4) in terms of the indirect utility function $g(\cdot)$:

$$E_t\left[\delta\frac{g'(C_{t+1})}{g'(C_t)}R_{i,t+1}\right] = 1. \tag{29}$$

As can be seen from conditions (4) and (29), the optimal hedgeable consumption plan of the representative agent with utility $u(\cdot)$ under background risk is identical to his optimal hedgeable consumption profile when he does not face background risk and has utility $g(\cdot)$.

Under the assumption that the representative agent has CRRA utility

$$g = \frac{C_t^{1-\beta} - 1}{1-\beta}, \tag{30}$$

we can write condition (29) as

$$E_t\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{-\beta}R_{i,t+1}\right] = 1. \tag{31}$$

[6] See Gollier (2001), for example.

[7] Under the assumption that the agent is risk averse (i.e., $u'(\cdot) > 0$ and $u''(\cdot) < 0$), the conditions $u'''(\cdot) > 0$ and $u''''(\cdot) < 0$ are necessary (but not sufficient) for respectively decreasing and convex absolute risk aversion. The condition $u'''(\cdot) > 0$ is also the necessary and sufficient condition for positive absolute prudence and the condition $u''''(\cdot) < 0$ is necessary (but not sufficient) for decreasing absolute prudence.

[8] Kimball (1993) shows that standard risk aversion implies risk vulnerability.

This is the standard consumption CAPM. In this model, there is no background risk and hence the indirect utility function coincides with the original utility. As a result, the utility curvature parameter β measures both effective and pure RRA to financial risk. The EIS in consumption is the reciprocal of risk aversion β. It has been shown empirically that, at economically realistic values of the utility curvature parameter β and the subjective time discount factor δ, this model substantially underestimates the average equity premium (this is the equity premium puzzle) and overestimates the average risk-free return (this is the risk-free rate puzzle). The consumption CAPM with no background risk (31) is able to fit the observed mean excess return on the market portfolio over the risk-free rate and the observed mean risk-free rate of return only if the utility function of the typical investor is implausibly concave (the representative agent is too averse to risk)[9] and the subjective time discount factor is greater than one (the agent has a negative rate of time preference).

Assume now that the representative agent faces background risk and his original utility function is (15). The CRRA utility function exhibits decreasing and convex absolute risk aversion and decreasing absolute prudence and is therefore risk vulnerable. Property (28) hence implies that, when $E[\Phi_t] \leqslant 0$, at the same hedgeable consumption level C_t, utility $u(\cdot)$ is less concave than utility $g(\cdot)$ and therefore the consumption CAPM with background risk must explain the observed excess equity return with a lower value of the utility curvature parameter (i.e., the pure RRA coefficient) compared with the standard consumption CAPM. Equivalently, the representative agent consumption CAPM with background risk should yield an equity premium, which is larger than the equity premium generated by model (31) with the same value of the coefficient of pure RRA, and therefore the model with background risk has the potential to solve the equity premium puzzle.

To see how the introduction of an independent non-hedgeable adverse background risk in the otherwise standard consumption CAPM affects the risk-free rate implied by the model, consider the unconditional equation that relates the risk-free interest rate $R_{f,t+1}$ to the pricing kernel M_{t+1}:

$$R_{f,t+1} = \left(E_t\left[M_{t+1}\right]\right)^{-1}. \tag{32}$$

In the presence of background risk, the SDF is given by (11) and hence we can write equation (32) as

$$R_{f,t+1} = \delta^{-1}\left(E_t\left[\frac{u'\left(E^{\Phi}\left[C_{t+1}\right]-\Psi_{t+1}\right)}{u'\left(E^{\Phi}\left[C_t\right]-\Psi_t\right)}\right]\right)^{-1}. \tag{33}$$

The value of the term $u'\left(E^{\Phi}\left[C_t\right]-\Psi_t\right)$ is known at time t. Suppose for simplicity that there is no background risk at time t, but there is background risk at time $t+1$. Since the background risk at time $t+1$ is assumed to be adverse (i.e., $E[\Phi_{t+1}] \leqslant 0$) and $\Psi_{t+1} > 0$, $E^{\Phi}[C_{t+1}] - \Psi_{t+1} < C_{t+1}$.

[9] In model (31), the RRA coefficient coincides with the utility curvature parameter and hence a high concavity of the utility function implies a high value of the coefficient of RRA.

Because the agent is risk averse (i.e., utility is concave), from this it immediately follows that in each state the ratio of marginal utilities at $t+1$ and t under background risk exceeds that in the no background risk case. As this is true in each state, this is true in expectation as well. Therefore, at a given value of the subjective time discount factor δ, the model with adverse background risk yields the interest rate, which is lower than the interest rate implied by the consumption CAPM with no background risk, or, equivalently, the consumption CAPM with adverse background risk enables to explain the observed risk-free rate of return with a lower value of the time discount factor compared with the standard model. This shows the potential of the model with background risk to solve the risk-free rate puzzle.

Economically, this may be explained as follows. Leland (1968), Sandmo (1970), and Drèze and Modigliani (1972), for example, argue that if the agent's absolute risk aversion is decreasing (i.e., the third derivative of the utility function is positive), then the presence of a non-hedgeable background risk leads the agent to save more in order to insure his future consumption against the additional variability caused by the non-hedgeable background risk.[10] The precautionary saving induced by incompleteness of the market for the background risk decreases the equilibrium rate of return on both the risky and risk-free assets. If the agent's preferences exhibit risk vulnerability, then the nonavailability of insurance against an additional independent non-hedgeable background risk makes the agent less willing to bear the financial risk and the equilibrium risky asset expected premium rises relative to the no background risk case. This suggests that the model, in which the representative agent faces in addition to aggregate dividend risk an independent non-hedgeable background risk, should predict a smaller bond return and a larger equity premium than would the representative agent CAPM with no background risk.

2.4 Risk aversion and the EIS under background risk

Suppose that at time t the representative agent faces the background risk and a lottery with an uncertain payoff Z_t. In Section 2.1, we assumed that the non-hedgeable consumption Φ_t is independent of both the hedgeable consumption C_t and the financial risk and hence Z_t and Φ_t are independent. For any distribution functions F_Z and G_Φ,

$$E^{Z,\Phi}\left[u\left(C_t+Z_t+\Phi_t\right)\right]=E^{\Phi}\left[u\left(C_t+E\left[Z_t\right]+\Phi_t-\Pi_t\right)\right], \tag{34}$$

where Π_t is the uncertain payoff's risk premium.

Assume further that the lottery is actuarially neutral (i.e., $E\left[Z_t\right]=0$) and the risk associated with the lottery is small. A Taylor approximation of the representative agent's utility

[10] Courbage and Rey (2007) stress that a positive third derivative of the utility function is still a necessary and sufficient condition for a positive precautionary saving motive when a non-financial background risk and the financial market risk are independent. They show that the set of sufficient conditions is more complex otherwise.

$u(C_t + Z_t + \Phi_t)$ around $C_t + \Phi_t$ yields

$$u(C_t + Z_t + \Phi_t) \approx u(C_t + \Phi_t) + u'(C_t + \Phi_t) Z_t + \frac{1}{2} u''(C_t + \Phi_t) Z_t^2 + o\left(Z_t^3\right), \tag{35}$$

where $o(x^n)$ means that $lim_{x \to 0}^n \; o(x^n)/x^n = 0$.

Since Z_t and Φ_t are independent, taking mathematical expectations of (35) with respect to Φ and Z, we obtain

$$E^{Z,\Phi}\left[u(C_t + Z_t + \Phi_t)\right] \approx E^{\Phi}\left[u(C_t + \Phi_t)\right] + \frac{1}{2}\sigma_{Z,t}^2 E^{\Phi}\left[u''(C_t + \Phi_t)\right], \tag{36}$$

where $\sigma_{Z,t}^2$ is the variance of Z_t.

A Taylor series expansion of $u(C_t + \Phi_t - \Pi_t)$ around $C_t + \Phi_t$ is

$$u(C_t + \Phi_t - \Pi_t) \approx u(C_t + \Phi_t) - \Pi_t u'(C_t + \Phi_t) + o\left(\Pi_t^2\right) \tag{37}$$

and hence

$$E^{\Phi}\left[u(C_t + \Phi_t - \Pi_t)\right] \approx E^{\Phi}\left[u(C_t + \Phi_t)\right] - \Pi_t E^{\Phi}\left[u'(C_t + \Phi_t)\right]. \tag{38}$$

It then follows from equation (34) that

$$\frac{1}{2}\sigma_{Z,t}^2 E^{\Phi}\left[u''(C_t + \Phi_t)\right] \approx -\Pi_t E^{\Phi}\left[u'(C_t + \Phi_t)\right] \tag{39}$$

implying

$$\Pi_t \approx \frac{1}{2}\sigma_{Z,t}^2 \left(-\frac{E^{\Phi}\left[u''(C_t + \Phi_t)\right]}{E^{\Phi}\left[u'(C_t + \Phi_t)\right]}\right), \tag{40}$$

where the term in parentheses is the coefficient of absolute risk aversion.

In the presence of an independent non-hedgeable background risk, the RRA coefficient of the agent with utility $u(\cdot)$ is then

$$\gamma_t^* = -\frac{E^{\Phi}\left[u''(C_t + \Phi_t)\right]}{E^{\Phi}\left[u'(C_t + \Phi_t)\right]} C_t. \tag{41}$$

This is the effective (or "adjusted" for the background risk effect) RRA coefficient.

Since Φ_t is independent of C_t,

$$g^{(n)}(C_t) = E^{\Phi}\left[u^{(n)}(C_t + \Phi_t)\right], \tag{42}$$

where $g^{(n)}$ and $u^{(n)}$ are the nth derivatives of $g(\cdot)$ and $u(\cdot)$, respectively, and hence

$$\gamma_t^* = -\frac{g''(C_t)}{g'(C_t)} C_t. \tag{43}$$

The above equation shows that the aversion to financial risk of the agent with the original utility $u(\cdot)$ under background risk is identical to the aversion to financial risk of the agent with the indirect utility $g(\cdot)$ when there is no background risk. This implies that under risk vulnerability there is a twofold effect of the background risk on the risk premium. First, unconcavifying the agent's utility function in the presence of an independent non-hedgeable background risk with a non-positive mean decreases the risk premium that the agent is ready to pay to escape the financial risk. However, the background risk makes the agent more risk averse to the financial risk and hence raises the risk premium, so that the total risk premium remains the same as if the agent has utility $g(\cdot)$ in the absence of background risk.

Denote as $\widetilde{\Pi}_t$ the risk premium we would observe if the agent's RRA were γ, where γ is the pure RRA coefficient. The proportion of the risk premium due to the background risk in the total risk premium the representative agent is ready to pay to avoid the financial risk is then

$$\frac{\Pi_t - \widetilde{\Pi}_t}{\Pi_t} \approx 1 - \frac{\gamma}{\gamma_t^*}. \tag{44}$$

Kimball (1992) defines the temperance premium Λ_t by the following condition:

$$E^\Phi \left[u''\left(C_t + \Phi_t\right)\right] = u''\left(E^\Phi\left[C_t\right] - \Lambda_t\right). \tag{45}$$

By analogy with the risk premium,

$$\Lambda_t \approx \frac{1}{2}\sigma_{\Phi,t}^2 \left(-\frac{u''''\left(E^\Phi\left[C_t\right]\right)}{u'''\left(E^\Phi\left[C_t\right]\right)}\right). \tag{46}$$

The conditions $u''''(\cdot) < 0$ and $u'''(\cdot) > 0$ (the necessary conditions for risk vulnerability) imply that Λ_t is positive.

Combining equation (41) with conditions (45) and (9), we have

$$\gamma_t^* = -\frac{u''\left(E^\Phi\left[C_t\right] - \Lambda_t\right)}{u'\left(E^\Phi\left[C_t\right] - \Psi_t\right)} C_t. \tag{47}$$

When utility has the power form (15),

$$\Lambda_t \approx \frac{1}{2}\sigma_{\Phi,t}^2 \frac{\gamma + 2}{E^\Phi\left[C_t\right]}, \tag{48}$$

$$u'\left(E^\Phi\left[C_t\right] - \Psi_t\right) = \left(E^\Phi\left[C_t\right] - \Psi_t\right)^{-\gamma} = \left(E^\Phi\left[C_t\right] - \frac{1}{2}\sigma_{\Phi,t}^2 \frac{\gamma+1}{E^\Phi\left[C_t\right]}\right)^{-\gamma}, \tag{49}$$

$$u''\left(E^\Phi\left[C_t\right] - \Lambda_t\right) = -\gamma\left(E^\Phi\left[C_t\right] - \frac{1}{2}\sigma_{\Phi,t}^2 \frac{\gamma+2}{E^\Phi\left[C_t\right]}\right)^{-\gamma-1}, \tag{50}$$

and therefore

$$\begin{aligned} \gamma_t^* &= C_t\gamma\left(E^\Phi\left[C_t\right] - \frac{1}{2}\sigma_{\Phi,t}^2\frac{\gamma+2}{E^\Phi\left[C_t\right]}\right)^{-\gamma-1}\left(E^\Phi\left[C_t\right] - \frac{1}{2}\sigma_{\Phi,t}^2\frac{\gamma+1}{E^\Phi\left[C_t\right]}\right)^{\gamma} \\ &= \gamma\frac{C_t}{E^\Phi\left[C_t\right]}\left(1 - \frac{\gamma+2}{2}\varsigma_{\Phi,t}^2\right)^{-\gamma-1}\left(1 - \frac{\gamma+1}{2}\varsigma_{\Phi,t}^2\right)^{\gamma}. \end{aligned} \tag{51}$$

Because

$$1 - \frac{\gamma+1}{2}\varsigma_{\Phi,t}^2 \geqslant 1 - \frac{\gamma+2}{2}\varsigma_{\Phi,t}^2, \tag{52}$$

the condition $1-(\gamma+2)\varsigma^2_{\Phi,t}/2>0$ or, equivalently, $\gamma<2\left(1/\varsigma^2_{\Phi,t}-1\right)$ is necessary and sufficient for γ^*_t to be well-defined.[11]

It can be seen from formula (51) that, as expected, the effective and pure RRA coefficients coincide (i.e., $\gamma^*_t=\gamma$) in the absence of background risk. The independent adverse (i.e., $E[\Phi_t]\leqslant 0$ implying $E^\Phi[C_t]\leqslant C_t$) background risk raises the agent's aversion to financial risk compared with the no background risk case, so that the effective RRA coefficient becomes greater than the coefficient of pure RRA ($\gamma^*_t>\gamma$).

If there is no background risk, then the effective RRA coefficient coincides with the pure RRA coefficient and is therefore constant over time. By contrast, when the agent is subject to an independent non-hedgeable adverse background risk, the coefficient of effective RRA depends not only on the original utility curvature parameter γ, but also on the agent's hedgeable consumption as well as on the first two unconditional moments of the distribution of the non-hedgeable consumption. Therefore, in the model with background risk, the measure of effective RRA may change over time even if the representative agent is assumed to have time-invariant utility (γ is constant over time). This relates our approach to the strand of the literature, which argues that the attitudes towards financial risk are not fixed, but rather contingent upon the state of the world. The intuition here is that an agent adjusts his aversion to financial risk given the problem that he faces. Within this approach, the RRA coefficient is usually restricted to be a function of some factors deemed to be relevant in explaining the investor's attitudes towards risk.[12] However, the choice of such factors and the particular functional form relating the coefficient of effective RRA to proxies for the state of the world are somewhat arbitrary. The attractive feature of our approach is that the set of factors and the form of the relationship between the effective RRA coefficient and these factors obtain endogenously from the theoretical restrictions implied by a structural model.

The values of the effective RRA coefficient γ^*_t for different levels of the agent's hedgeable consumption C_t, the first two unconditional moments of the distribution of the non-hedgeable consumption, $E[\Phi_t]$ and $\sigma^2_{\Phi,t}$, and the original utility curvature parameter γ are shown in Figure 1. Any adverse risk can be decomposed into a sure reduction in consumption and a pure (zero-mean) risk. Panel A of Figure 1 shows the values of the effective RRA coefficient in the case of a sure reduction in consumption ($\sigma^2_{\Phi,t}=0$, $E[\Phi_t]<0$) for C_t normalized to one for simplicity. The values of the effective RRA coefficient in the presence of a pure independent non-hedgeable background risk ($E[\Phi_t]=0$, $\sigma^2_{\Phi,t}>0$, and $C_t=1$) are plotted in Panel B. Panels C and D

[11] This restriction on the acceptable values for γ is only slightly stronger than the condition $\gamma<2/\varsigma^2_{\Phi,t}-1$ we need to be satisfied for marginal utility $u'\left(E^\Phi[C_t]-\Psi_t\right)$ to be well-defined. With $\varsigma^2_{\Phi,t}=0.1$, for example, the upper bound for the value of γ is now 18, instead of 19, the value implied by the restriction $\gamma<2/\varsigma^2_{\Phi,t}-1$. This new upper bound is still much greater than 10, the highest plausible value for γ suggested by Mehra and Prescott (1985).

[12] Bakshi and Chen (1996) and Gordon and St-Amour (2004), for example, suppose that the RRA coefficient is a decreasing function of the individual's wealth.

illustrate the influence of C_t on the effective RRA coefficient for $\sigma^2_{\Phi,t} = 0.1$ and $E[\Phi_t]$ set to -0.1 and 0, respectively. For each case, the values of the effective RRA coefficient γ^*_t are plotted for three different values of the curvature parameter in utility, i.e., $\gamma = 1$, $\gamma = 2$, and $\gamma = 5$.

As predicted by economic theory, an independent non-hedgeable adverse background risk raises the agent's aversion to financial risk. It is seen in Figure 1 that risk aversion of an agent with CRRA utility is an increasing and convex function of the sure reduction in consumption and the variance of the non-hedgeable consumption. An increase in the hedgeable consumption C_t lowers the agent's aversion to financial risk, so that the effective RRA coefficient approaches its value in the no background risk case (approaches the pure RRA coefficient γ) as both the expected value and variance of the non-hedgeable consumption become smaller relative to C_t. Thus, in the presence of an independent non-hedgeable adverse background risk, the agent's behavior towards financial risk exhibits decreasing RRA, while it is independent of the hedgeable consumption level in the no background risk environment. Finally, not surprisingly, the higher the agent's aversion to financial risk in the absence of background risk (i.e., the greater the value of γ), the more vulnerable the agent's aversion to financial risk to another independent risk.

Since the effective RRA increases in the sure reduction in consumption and the variance of the non-hedgeable consumption, the greater the variance of the non-hedgeable consumption, the larger the effective RRA coefficient relative to the coefficient of pure RRA and hence, as follows from equation (44), the greater the proportion of the risk premium attributed to the direct effect of the background risk (the smaller the fraction of the risk premium that may be explained by the curvature of the agent's utility function).

As we emphasized above, in the presence of an independent non-hedgeable adverse background risk, the effective RRA coefficient γ^*_t differs from the original utility curvature parameter γ (the coefficient of pure RRA). Campbell and Cochrane (1999) also disentangle attitudes towards financial risk from the utility curvature parameter. They assume that an agent's utility is a power function of the difference between the agent's total consumption and subsistence or habit requirements. When habit is external, the agent's aversion to financial risk in the Campbell and Cochrane (1999) model is given by the ratio of the utility curvature parameter γ to the surplus consumption ratio, which is defined as the fraction of total consumption that is surplus to habit. With this model, one can get a time-varying effective RRA coefficient as consumption rises or declines toward habit. However, Campbell and Cochrane (1999) need consumption to always be above habit requirements for marginal utility to be well-defined. This might be a problem in microeconomic models with exogenous consumption.[13] Another drawback of the Campbell and Cochrane (1999) difference model is that the effective RRA coefficient goes to infinity when consumption is close to habit even if γ is low. As a result, the model can produce implausibly volatile estimates of RRA.

In contrast to the Campbell and Cochrane (1999) model, as we argued above, in the model

[13] See Campbell *et al.* (1997).

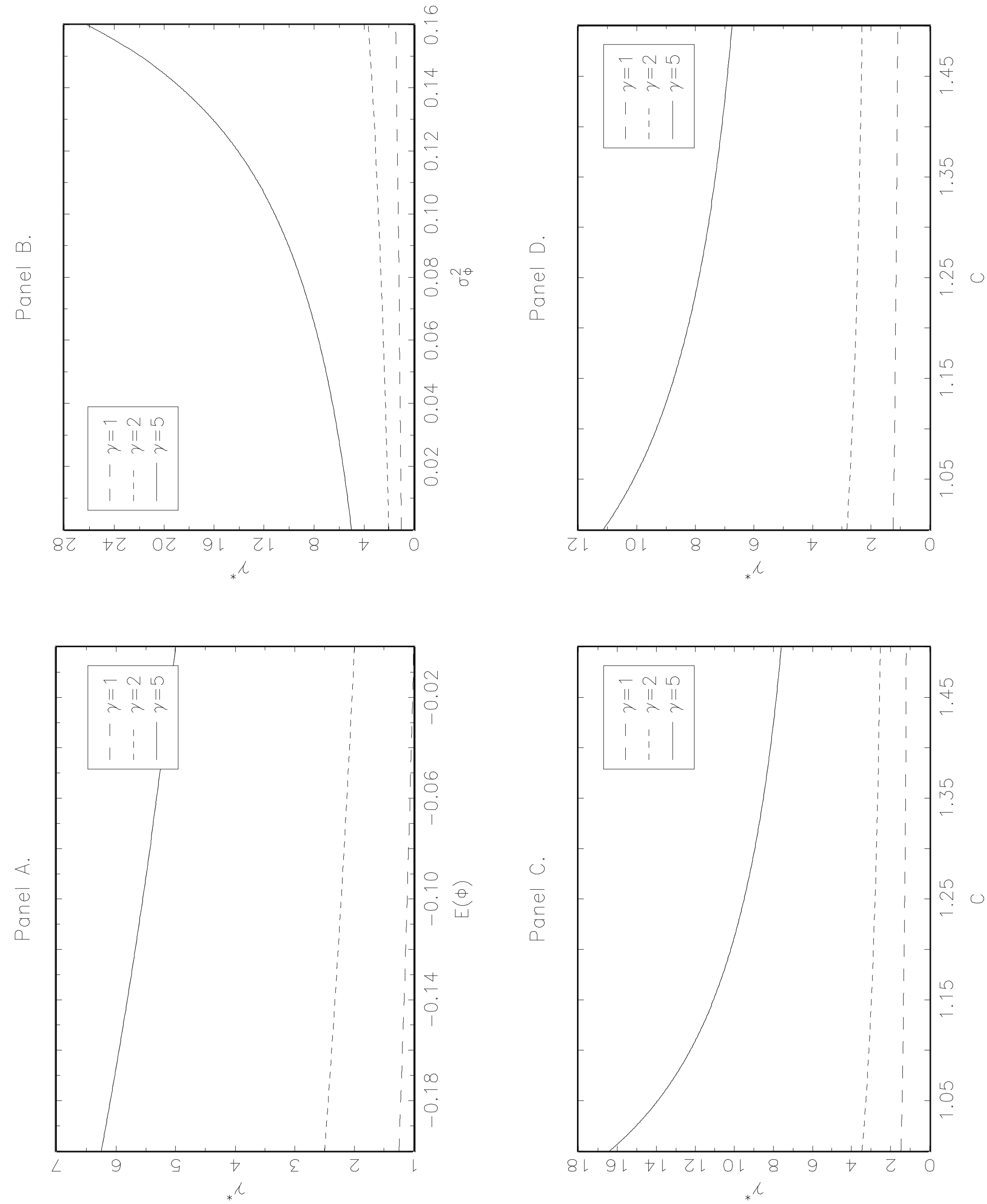

Figure 1: The coefficient of effective RRA in the case of a sure reduction in consumption ($\sigma^2_{\Phi,t} = 0$, $E[\Phi_t] < 0$, and $C_t = 1$) (Panel A) and in the case of a pure independent non-hedgeable background risk ($E[\Phi_t] = 0$, $\sigma^2_{\Phi,t} > 0$, and $C_t = 1$) (Panel B). The effect of the optimal hedgeable consumption C_t on the effective RRA coefficient for $\sigma^2_{\Phi,t} = 0.1$ and $E[\Phi_t]$ set to -0.1 (Panel C) and 0 (Panel D).

with background risk the marginal utility of consumption is always well-defined at economically plausible (less than 10) values of the original utility curvature parameter γ. As to the volatility of the estimate of the effective RRA coefficient, the results presented in Figure 1 suggest that the estimate of effective RRA in the model with background risk is unlike to by highly volatile over time unless the utility curvature parameter γ is quite high (say, greater than five). Although the Campbell and Cochrane (1999) habit formation model and the consumption CAPM with background risk are based on very different assumptions, in the both models the agent's attitudes towards financial risk are time varying, whereas RRA is constant over time in the standard consumption CAPM.

An important determinant of the investor's consumption and savings decisions is the EIS in consumption, which measures the sensitivity of changes in the investor's expected consumption growth rate between two periods to changes in interest rates. The EIS can be computed as the derivative of planned log consumption growth with respect to the log return on the risk-free asset. As argued above, the representative agent with utility $u(\cdot)$ facing background risk has the same optimal hedgeable consumption plan as the representative agent with utility $g(\cdot)$ in the no background risk environment. This suggests that the willingness of the investor with utility $u(\cdot)$ to move consumption between time periods in response to changes in the interest rate under background risk will be identical to that of the investor with utility $g(\cdot)$ in the absence of background risk, implying that the EIS in the model with an independent non-hedgeable background risk and utility $u(\cdot)$, which we write as $\phi_{u,t+1}$, is the same as in the case when there is no background risk and utility is $g(\cdot)$, i.e.,

$$\phi_{u,t+1} = \frac{\partial E_t\left[\Delta c_{t+1}\right]}{\partial r_{f,t+1}} = \phi_{g,t+1} = \frac{1}{\gamma^*_{t+1}} \leqslant \frac{1}{\gamma_{t+1}}, \tag{53}$$

where $\Delta c_{t+1} = ln\left(C_{t+1}/C_t\right)$ and $r_{f,t+1} = ln\left(R_{f,t+1}\right)$. The inequality is strict in the presence of background risk.

The above result implies that the model with background risk enables us to disentangle the coefficient of pure RRA and the EIS in the expected utility framework. Another implication is that, when the agent's preferences exhibit risk vulnerability, we can get a low value of the EIS even if the original utility function is not very concave, while in the standard consumption CAPM we need a large value of the utility curvature parameter to get a low value of elasticity. Since, as we showed above, the effective RRA coefficient may change over time depending on the first two unconditional moments of the non-hedgeable consumption distribution, so does the EIS.

3 Empirical investigation

3.1 The data

3.1.1 The consumption data

We use quarterly consumption data from the CEX, produced by the US Bureau of Labor Statistics (BLS). The CEX data available cover the period from 1980:Q1 to 2003:Q4. It is a collection of data on approximately 5000 households per quarter in the US. Each household in the sample is interviewed every three months over five consecutive quarters (the first interview is practice and is not included in the published data set). As households complete their participation, they are dropped and new households move into the sample. Thus, each quarter about 20% of the consumer units are new. The second through fifth interviews use uniform questionnaires to collect demographic and family characteristics as well as data on quarterly consumption expenditures for the previous three months made by households in the survey (demographic variables are based upon heads of households). Various income information and information on the employment of each household member is collected in the second and fifth interviews. As opposed to the Panel Study of Income Dynamics (PSID), which offers only food consumption data on an annual basis, the CEX contains highly detailed data on quarterly consumption expenditures.[14] The CEX attempts to account for an estimated 70% of total household consumption expenditures. Since the CEX is designed with the purpose of collecting consumption data, measurement error in consumption is likely to be smaller for the CEX consumption data compared with the PSID consumption data.

As suggested by Attanasio and Weber (1995), Brav *et al.* (2002), and Vissing-Jorgensen (2002), we drop all consumption observations for the years 1980 and 1981 because the quality of the CEX consumption data is questionable for this period. Thus, our sample covers the period from 1982:Q1 to 2003:Q4. Following Brav *et al.* (2002), in each quarter we drop households that do not report or report a zero value of consumption of food, consumption of nondurables and services, or total consumption. We also delete from the sample nonurban households, households residing in student housing, households with incomplete income responses, households that do not have a fifth interview, and households whose head is under 19 or over 75 years of age.

In the fifth (final) interview, the household is asked to report the end-of-period estimated market value of all stocks, bonds, mutual funds, and other such securities held by the consumer unit on the last day of the previous month as well as the difference in this estimated market value compared with the value of all securities held a year ago last month. Using these two values, we calculate each consumer unit's asset holdings at the beginning of a 12-month recall period in

[14] Food consumption is likely to be one of the most stable consumption components. Furthermore, as is pointed out by Carroll (1994), 95% of the measured food consumption in the PSID is noise due to the absence of interview training.

constant 2005 dollars. We consider four sets of households based on the reported amount of asset holdings at the beginning of a 12-month recall period. The first set consists of all households regardless of the reported amount of asset holdings. To take into account the limited participation of households in the capital market, we also consider households that report a positive amount of total asset holdings (the second set), households that report total assets equal to or exceeding $1000 (the third set), and, finally, households that report total assets equal to or exceeding $5000 (the fourth set).

As is conventional in the literature, the consumption measure used in this paper is consumption of nondurables and services. For each household, we calculate quarterly consumption expenditures for all the disaggregate consumption categories offered by the CEX. Then, we deflate obtained values in 2005 dollars with the CPI's (not seasonally adjusted, urban consumers) for appropriate consumption categories.[15] Aggregating the household's quarterly consumption across these categories is made according to the National Income and Product Account (NIPA) definition of consumption of nondurables and services. The household's per capita consumption growth between two quarters t and $t+1$ is defined as the ratio of the household's per capita consumption in quarters $t+1$ and t.[16] To mitigate observation error in individual consumption, we subject the households to a consumption growth filter and use the conventional z-score method to detect outliers. Following common practice for highly skewed data sets, in each time period we consider the consumption growth rates with z-scores greater than two in absolute value to be due to reporting or coding errors and remove them from the sample. The household's per capita consumption growth is deseasonalized using the multiplicative adjustments obtained from the per capita consumption growth rate, as explained below.

The consumption of the representative agent within each set of households is calculated as the average per capita consumption expenditures of the households in the set. For each set of households, the per capita consumption growth is seasonally adjusted by using multiplicative adjustments obtained from the X-12 procedure.

3.1.2 The returns data

The nominal quarterly value-weighted market capitalization-based decile index returns (capital gain plus all dividends) on all stocks listed on the NYSE, AMEX, and Nasdaq are from the Center for Research in Security Prices (CRSP) of the University of Chicago. Smallest stocks are placed in portfolio 1 and the largest in portfolio 10. The nominal quarterly value-weighted returns on the five NYSE, AMEX, and Nasdaq industry portfolios ((i) consumer durables, nondurables, wholesale, retail, and some services (laundries, repair shops), (ii) manufacturing, energy, and

[15] The CPI series are obtained from the BLS through CITIBASE.

[16] The quarterly consumption growth between dates t and $t+1$ is calculated if consumption is not equal to zero for both of the quarters (missing information is counted as zero consumption).

utilities, (iii) business equipment, telephone and television transmission, (iv) healthcare, medical equipment, and drugs, and (v) other) and the ten NYSE, AMEX, and Nasdaq industry portfolios ((i) consumer nondurables, (ii) consumer durables, (iii) manufacturing, (iv) oil, gas, and coal extraction and products, (v) business equipment, (vi) telephone and television transmission, (vii) wholesale, retail, and some services (laundries, repair shops), (viii) healthcare, medical equipment, and drugs, (ix) utilities, and (x) other) are from Kenneth R. French's web page.

The nominal quarterly risk-free rate is the 3-month US Treasury Bill secondary market rate on a per annum basis obtained from the Federal Reserve Bank of St. Louis. In order to convert from the annual rate to the quarterly rate, we raise the 3-month Treasury Bill return on a per annum basis to the power of 1/4.

The real quarterly returns are calculated as the quarterly nominal returns divided by the 3-month inflation rate based on the deflator defined for consumption of nondurables and services. We calculate the equity premium as the difference between the real equity return and the real risk-free rate.

Table I reports the descriptive statistics for the data set used in estimation.

3.2 The estimation procedure

We use Hansen and Jagannathan's (HJ, hereafter) (1991) volatility bounds to assess the empirical performance of the model with background risk. Our benchmark models are the standard representative-agent consumption CAPM and the consumption CAPM in which the SDF is the Taylor expansion of the equally weighted sum of the households' marginal rates of substitution up to cubic terms as proposed in Brav *et al.* (2002).

In the standard consumption CAPM, the SDF is

$$M_{t+1} = \delta \left(\frac{C_{t+1}}{C_t} \right)^{-\beta}. \tag{54}$$

The pricing kernel in Brav *et al.* (2002) is

$$M_{t+1} = \delta q_{t+1}^{-\alpha} \left[1 + \frac{\alpha(\alpha+1)}{2N} \sum_{i=1}^{N} \left(\frac{q_{i,t+1}}{q_{t+1}} - 1 \right)^2 - \frac{\alpha(\alpha+1)(\alpha+2)}{6N} \sum_{i=1}^{N} \left(\frac{q_{i,t+1}}{q_{t+1}} - 1 \right)^3 \right], \tag{55}$$

where $q_{i,t+1}$ is the household i's consumption growth rate between quarters t and $t+1$, q_{t+1} is the cross-sectional mean of the consumption growth rate, $q_{t+1} = N^{-1} \sum_{i=1}^{N} q_{i,t+1}$, and α is the utility curvature parameter. Since Brav *et al.* (2002) estimate the agent's RRA coefficient α under the assumption of background risk (which is, in contrast to our approach, is taken into account by considering the first three cross-sectional moments of the distribution of the household's consumption growth rate, while we consider a single representative agent facing an independent non-hedgeable adverse background risk), the parameter α in their framework is somewhat similar to the pure RRA coefficient.

As we showed in Section 2.2, the SDF in the consumption CAPM with background risk is

$$M_{t+1} = \delta \left(\frac{E^{\Phi}[C_{t+1}]}{E^{\Phi}[C_t]} \right)^{-\gamma} \left(\frac{1 - \frac{\gamma+1}{2} \varsigma^2_{\Phi,t+1}}{1 - \frac{\gamma+1}{2} \varsigma^2_{\Phi,t}} \right)^{-\gamma}. \tag{56}$$

The parameters of this pricing kernel cannot be estimated without knowledge of the first two unconditional moments of the hypothetical distribution of the representative agent's consumption, which are unobservable. We overcome this problem by using the estimated values of these variables instead of the true values. To estimate $E^{\Phi}[C_t]$ and $\varsigma^2_{\Phi,t}$ for all t, we adopt the following approach. As we mentioned in Section 2.1, the consumption CAPM with background risk described in this paper is based on the assumption of market completeness for uncertainty about the return on the risky asset and market incompleteness for the independent background risk. Hence, we may assume that in the absence of the independent non-hedgeable background risk the agents are able to equalize, state by state, their intertemporal marginal rates of substitution in consumption. This implies that CRRA utility maximizing investors are able to equalize, state by state, their hedgeable consumption growth rates and therefore variations in the observed consumption growth rate for individual investors may be attributed to different non-hedgeable background risks the agents faced in the time period under consideration.

We assume that the representative agent may be subject to the same independent non-hedgeable background risks as individual investors and hence the hypothetical distribution of the consumption growth rate for the representative agent may be well approximated by the observed distribution of the individual consumption growth rate. Under this assumption, we calculate $C_{t,\lambda}$, $\lambda = 1, ..., N$, as

$$C_{t,\lambda} = C_{t-1} q_{i,t}, \ \lambda = i = 1, ..., N, \tag{57}$$

where $q_{i,t}$ is the observed real consumption growth rate for household i between quarters $t-1$ and t.

We then estimate $E^{\Phi}[C_t]$ and $\varsigma^2_{\Phi,t}$ for all t as, respectively, the mean and the normalized variance of the generated in this manner hypothetical distribution of the representative agent's consumption:

$$\widehat{E^{\Phi}[C_t]} = N^{-1} \sum_{\lambda=1}^{N} C_{t,\lambda}, \tag{58}$$

$$\widehat{\varsigma}^2_{\Phi,t} = N^{-1} \sum_{\lambda=1}^{N} \left(\frac{C_{t,\lambda} - \widehat{E^{\Phi}[C_t]}}{\widehat{E^{\Phi}[C_t]}} \right)^2. \tag{59}$$

Substituting the $\widehat{E^{\Phi}[C_t]}$ and the $\widehat{\varsigma}^2_{\Phi,t}$ for the $E^{\Phi}[C_t]$ and the $\varsigma^2_{\Phi,t}$ in (56) yields the SDF

$$M_{t+1} = \delta \left(\frac{\widehat{E^{\Phi}[C_{t+1}]}}{\widehat{E^{\Phi}[C_t]}} \right)^{-\gamma} \left(\frac{1 - \frac{\gamma+1}{2} \widehat{\varsigma}^2_{\Phi,t+1}}{1 - \frac{\gamma+1}{2} \widehat{\varsigma}^2_{\Phi,t}} \right)^{-\gamma}. \tag{60}$$

Then, the SDF in the model with background risk becomes testable.

For each of the above SDFs ((54), (55), and (60)), we test the conditional Euler equations for the excess returns on risky assets

$$E_t\left[\mathbf{Z}_{t+1}M_{t+1}\right]=\mathbf{0} \tag{61}$$

and the risk-free rate

$$E_t\left[R_{f,t+1}M_{t+1}\right]=1, \tag{62}$$

where $\mathbf{Z}_{t+1}$ is the K-vector of time $t+1$ excess returns with elements $Z_{i,t+1}=R_{i,t+1}-R_{f,t+1}$, $i=1,...,K$, and $\mathbf{0}$ is the K-vector of zeroes.

Denote the vector of error terms in the Euler equation for the excess returns as $\boldsymbol{\epsilon}_{t+1}$, $\boldsymbol{\epsilon}_{t+1}=\mathbf{Z}_{t+1}M_{t+1}$, and the error term in the Euler equation for the risk-free rate as $\varepsilon_{f,t+1}$, $\varepsilon_{f,t+1}=R_{f,t+1}M_{t+1}-1$. Equations (61) and (62) imply that the error terms in the Euler equations are orthogonal to any variable x_t in the agent's time t information set and hence, at the true parameter vector,

$$E\left[\boldsymbol{\epsilon}_{t+1}x_t\right]=\mathbf{0}, \tag{63}$$

$$E\left[\varepsilon_{f,t+1}x_t\right]=0. \tag{64}$$

Since the true Euler equation error is contemporaneously uncorrelated with any lagged variable, we take as instruments x_t a constant, the aggregate per capita consumption growth rate, the returns on the CRSP equally and value-weighted indices, and the risk-free rate of return. All the variables are lagged one and two periods.[17]

A lower volatility bound for admissible SDFs $M^a_{t+1}(m)$, which have unconditional mean m and satisfy the orthogonality condition (63), can then be calculated as

$$\sigma\left(M^a_{t+1}(m)\right)=\left(m^2E\left[\mathbf{Z}_{t+1}x_t\right]'\boldsymbol{\Sigma}^{-1}E\left[\mathbf{Z}_{t+1}x_t\right]\right)^{1/2}, \tag{65}$$

where $\boldsymbol{\Sigma}$ is the unconditional variance-covariance matrix of $Z_{i,t+1}x_t$.

Following HJ (1991), we first treat m as an unknown parameter and look for the values of the SDF parameters, at which a considered SDF $M_{t+1}(m)$ satisfies the volatility bound (65), i.e.,

$$\frac{\sigma\left(M_{t+1}(m)\right)}{\sigma\left(M^a_{t+1}(m)\right)}>1 \tag{66}$$

or, equivalently,

$$\frac{\sigma\left(M_{t+1}(m)\right)}{\sigma\left(M^a_{t+1}(m)\right)}=\frac{\sigma\left(\widetilde{M}_{t+1}(\widetilde{m})\right)}{\left(\widetilde{m}^2E\left[\mathbf{Z}_{t+1}x_t\right]'\boldsymbol{\Sigma}^{-1}E\left[\mathbf{Z}_{t+1}x_t\right]\right)^{1/2}}>1, \tag{67}$$

[17] As emphasized by Hansen and Singleton (1982), time aggregation can make instruments correlated with the error term in the Euler equation. Hall (1988) argues that the use of variables lagged two periods helps to reduce this correlation as well as the effect of the mismatching of measurement time periods with planning time periods. As argued by Epstein and Zin (1991), the use as instruments variables lagged two periods require weaker assumptions on the information structure of the problem compared with the case of variables lagged one period. Ogaki (1988) demonstrates that the use of the second lag is consistent with the information structure of a monetary economy with cash-in-advance constraints.

where

$$\widetilde{M}_{t+1} = \frac{M_{t+1}}{\delta} \tag{68}$$

and $\widetilde{m} = T^{-1}\sum_{t=0}^{T-1}\widetilde{M}_{t+1}$.

Denote as θ the utility curvature parameter in the SDF $\widetilde{M}_{t+1}$.[18] For each instrument x_t, we look for the smallest value of the utility curvature parameter θ, say $\widehat{\theta}(x_t)$, at which inequality (67) holds.

Then, we find the optimal value of the estimate of the curvature parameter as the largest such value of $\widehat{\theta}(x_t)$:[19]

$$\widehat{\theta}^{opt} = \underset{\{x_t\}}{max}\,\widehat{\theta}(x_t). \tag{69}$$

This value of $\widehat{\theta}(x_t)$ is optimal in the sense that at $\theta = \widehat{\theta}^{opt}$ the conditional Euler equation (61) for the given set of risky assets holds for any instrument in the chosen instrument set.[20] Because $\theta = \beta$ in SDF (54), $\theta = \alpha$ in SDF (55), and $\theta = \gamma$ in SDF (60), $\widehat{\theta}^{opt}$ is the estimate of the pure RRA coefficient in the corresponding model. In the standard consumption CAPM, the effective RRA coefficient coincides with the coefficient of pure RRA and hence $\widehat{\theta}^{opt}$ is also the estimate of the effective RRA coefficient.

Denote as x_t^{opt} the instrument x_t, for which $\widehat{\theta}(x_t) = \widehat{\theta}^{opt}$. Since at the true parameter vector the orthogonality condition (64) holds for any x_t, it also holds for x_t^{opt} and therefore this orthogonality condition can be written as

$$E[(R_{f,t+1}M_{t+1} - 1)\,x_t^{opt}] = 0 \tag{70}$$

or, equivalently,

$$\delta E[R_{f,t+1}\widetilde{M}_{t+1}x_t^{opt}] = E[x_t^{opt}]. \tag{71}$$

Because we use a time series of the return on the risk-free asset, we estimate the subjective time discount factor δ as

$$\widehat{\delta} = \frac{\sum_{t=0}^{T-1} x_t^{opt}}{\sum_{t=0}^{T-1} R_{f,t+1}\widetilde{M}_{t+1}(\widehat{\theta}^{opt})x_t^{opt}}, \tag{72}$$

where $\widetilde{M}_{t+1}(\widehat{\theta}^{opt})$ is the value of $\widetilde{M}_{t+1}$ calculated at $\theta = \widehat{\theta}^{opt}$.

As a result, we obtain the estimates of the pricing kernel parameters ($\widehat{\theta}^{opt}$ and $\widehat{\delta}$), at which the conditional Euler equations (61) and (62) hold for the given set of instrumental variables and therefore the SDF is consistent with a given set of the excess returns on risky assets and the risk-free rate of return.

[18] $\theta = \beta$ in (54), $\theta = \alpha$ in (55), and $\theta = \gamma$ in SDF (60).

[19] This approach is somewhat similar to that in Bekaert and Liu (2004).

[20] Because $\widehat{\theta}(x_t) \leqslant \widehat{\theta}^{opt}$ for any instrument x_t and for any instrument the value of $\widehat{\theta}(x_t)$ is such that the pricing kernel satisfies the lower volatility bound at any $\theta \geqslant \widehat{\theta}(x_t)$, it also satisfies the lower volatility bound at $\theta = \widehat{\theta}^{opt}$ for any variable in the set of instruments.

As argued in Section 2.4, in the model with background risk the effective RRA coefficient differs from the utility curvature parameter γ (the coefficient of pure RRA) and is

$$\gamma_t^* = \gamma \frac{C_t}{E^{\Phi}\left[C_t\right]}\left(1-\frac{\gamma+2}{2}\varsigma_{\Phi,t}^2\right)^{-\gamma-1}\left(1-\frac{\gamma+1}{2}\varsigma_{\Phi,t}^2\right)^{\gamma}. \tag{73}$$

When estimating the coefficient of effective RRA in the consumption CAPM with background risk, we follow Kimball (1993) and Franke *et al.* (1998) and assume that the background risk is actuarially neutral, i.e., $E\left[\Phi_t\right] = 0$. Under this assumption, an estimator of the effective RRA coefficient is

$$\widehat{\gamma}_t^* = \widehat{\gamma}^{opt}\left(1-\frac{\widehat{\gamma}^{opt}+2}{2}\widehat{\varsigma}_{\Phi,t}^2\right)^{-\widehat{\gamma}^{opt}-1}\left(1-\frac{\widehat{\gamma}^{opt}+1}{2}\widehat{\varsigma}_{\Phi,t}^2\right)^{\widehat{\gamma}^{opt}}, \tag{74}$$

where $\widehat{\varsigma}_{\Phi,t}^2$ is from (59) and $\widehat{\gamma}^{opt}$ is $\widehat{\theta}^{opt}$ obtained for the pricing kernel (60) as explained above.

3.3 The results

We test the candidate SDFs using three different sets of risky assets: (i) the CRSP market capitalization-based decile portfolios (we also consider two subsets of this set of risky assets, namely the small stock portfolios (deciles 1-5) and the large stock portfolios (deciles 6-10)), (ii) the five NYSE, AMEX, and Nasdaq industry portfolios, and (iii) the ten NYSE, AMEX, and Nasdaq industry portfolios. For each set of risky assets, the estimation results are reported for the four sets of households defined in Section 3.1.1.

The estimation results for the unconditional and conditional versions of each model are quite similar with slightly higher point estimates of the pure RRA coefficient for the conditional version. This result is robust to the set of risky assets and the household layer. Since the conditional approach provides a more restrictive test of the model performance, in the following we focus on the results obtained for the conditional Euler equations. The results for the conditional versions of the standard model, the model with the SDF proposed in Brav *et al.* (2002), and the model with background risk are reported in Table II. The column labeled "SM" in this table shows the results for the standard model, the column labeled "BCG" presents the estimation results for the model with the pricing kernel proposed in Brav *et al.* (2002), and the column labeled "MBR" shows the estimates of the parameters for the model with background risk.

The results for the conditional standard consumption CAPM can be summarized as follows. When all households regardless of the reported amount of asset holdings are considered, the estimate of pure RRA is unrealistically high (from 16.6 to 26.0),[21] while the estimate of the subjective time discount factor is implausibly low (from 0.81 to 0.90).[22] When limited asset

[21] For this model, the effective and pure RRA coefficients coincide and hence the estimate of β in the standard model is the estimate of the both coefficients.

[22] Brav *et al.* (2002) also obtain low estimates of the subjective time discount factor in the standard consumption CAPM and argue that this result is due to error in the observed per capita consumption, which severely biases downward the estimated subjective time discount factor in the standard consumption CAPM.

market participation is taken into account, as expected, we obtain the estimates of pure RRA aversion that are quite lower than the estimates obtained for all households. These estimates range from 7.4 to 14.0 and hence are still too high to be recognized as economically realistic. The obtained estimates of the subjective time discount factor are greater and more realistic (they range from 0.90 to 0.99) compared with the estimates obtained for the whole set of households.

The model with the SDF expressed in terms of the cross-sectional mean, variance, and skewness of the household consumption growth rate, as proposed in Brav *et al.* (2002), explains the cross-sectional equity premium with the value of pure RRA that ranges from 5.6 to 8.9. These estimates are lower than the values of the pure RRA coefficient implied by the standard consumption CAPM, but are still unrealistically high. Another drawback of this pricing kernel is that it yields an estimate of the subjective time discount factor, which is implausibly low. This estimate is in the range from 0.55 to 0.68.[23]

In contrast to the standard model and the model proposed in Brav *et al.* (2002), the consumption CAPM with background risk explains the cross-section of asset returns and the risk-free rate at much lower values of the pure RRA coefficient and more realistic values of the subjective time discount factor. Under the assumption that the conditional Euler equations hold for all agents regardless of the reported amount of asset holdings, the estimate of the coefficient of pure RRA ranges from 4.8 to 5.7 and the estimate of the subjective time discount factor ranges from 0.92 to 0.96. Under limited asset market participation, the estimates of pure RRA are between 3.7 and 5.2 and the estimates of the subjective time discount factor are in the range from 0.89 to 0.96. The estimate of the pure RRA coefficient decreases as the threshold value in the definition of asset holders is raised.

To investigate whether the estimation results are sensitive to the size of stocks under consideration, we test the three models separately for the decile 1-5 and decile 6-10 portfolios. Our result is that the stock size does not affect significantly the performance of the models. As for the whole set of the market capitalization-based decile portfolios, for any subset of this set of risky assets both the standard model and the model proposed by Brav *et al.* (2002) yield too high estimates of pure RRA and too low estimates of the subjective discount factor, while for the model with background risk the estimates of the both parameters are within the acceptable range of values.

In the consumption CAPM with background risk, the estimate of the effective RRA differs from the estimate of the coefficient of pure RRA (the utility curvature parameter γ) and may change over time. We calculate the coefficient of effective RRA in the model with background risk for each quarter over the period from 1982:Q3 to 2003:Q4. As expected, the estimates of effective

[23] In Brav *et al.* (2002), the estimated subjective time discount factor in the pricing kernel that captures the mean, variance, and skewness of the cross-sectional distribution of the household consumption growth is also very low and below the estimate of the subjective time discount factor obtained for the standard model. The authors explain this by the fact that the observation error in household consumption is substantially higher than the observation error in per capita consumption.

RRA are greater than the estimates of the pure RRA coefficient and are not highly volatile.

When estimating the proportion of the risk premium due to the background risk, we use the time average estimate of the effective RRA coefficient under background risk as the value of γ_t^* in (44). The result is that for different sets of risky assets and sets of households the background risk accounts for from 34 to 44 percent of the risk premium.

Table II reports the estimates of the correlation between the effective RRA coefficient and the market premium, which is proxied by the real excess return on the value-weighted CRSP market portfolio over the risk-free rate, as well as the estimates of the correlation between the coefficient of effective RRA and the normalized variance of the non-hedgeable consumption, $\varsigma^2_{\Phi,t}$. Because the sampling distribution of Pearson's correlation coefficient r might be asymmetric, we use the Fisher transformation when testing the null hypothesis that the population correlation coefficient $\rho = 0$.[24] This transformation is applied to the sample correlation r and is defined by

$$z = 0.5 \ln \left(\frac{1+r}{1-r} \right). \tag{75}$$

The random variable z in (75) is normally distributed with standard deviation $1/\sqrt{T-3}$, where T is the sample size.

We find that in the model with background risk the correlation of the effective RRA coefficient with the market premium is very low and not statistically different from zero at any conventional level of significance. This result is not surprising because, as follows from equation (74), the agent's effective RRA is only a function of the utility curvature parameter γ (which is constant over time) and the normalized variance of the non-hedgeable consumption, $\varsigma^2_{\Phi,t}$ (which is assumed to be uncorrelated with the market premium). The correlation of the effective RRA coefficient with the normalized variance of the non-hedgeable consumption, $\varsigma^2_{\Phi,t}$, is close to one and significantly different from zero at the 5% level of significance.

The null hypothesis of serial correlation of the effective RRA coefficient is rejected statistically at any conventional level of significance for the entire set of households regardless of the reported amount of asset holdings. Under limited asset market participation, in most cases the first-order autocorrelation coefficient is positive and significantly different from zero at the 10% level (what might be due to serial correlation in the normalized variance of the non-hedgeable consumption), while the second-order autocorrelation coefficient is always low in absolute value and not significantly different from zero at any conventional significance level.[25]

Using the property that the EIS is equal to the reciprocal of the effective RRA coefficient, we calculate for each quarter the EIS in the consumption CAPM with background risk as the inverse

[24] See Fisher (1915, 1921).

[25] Fuller (1976) shows that under the IID null hypothesis the sample autocorrelation coefficients are asymptotically independent and normal with mean zero and standard deviation $1/\sqrt{T}$, i.e., $\widehat{\rho}_k \overset{a}{\sim} N(0, 1/T)$, where ρ_k is the kth-order autocorrelation coefficient.

of the coefficient of effective RRA obtained for this quarter under the assumption of background risk. The result is that the estimates of the EIS are only slightly volatile over time with the average value in the range from 0.10 to 0.19.

4 Concluding Remarks

In this paper, we assumed an economy in which a single representative investor faces, in addition to the insurable financial risk, an independent non-hedgeable adverse background risk. We presented empirical evidence that, in contrast to the previously proposed incomplete consumption insurance models, the asset-pricing model with the SDF calculated as the discounted ratio of expectations of marginal utilities over the non-hedgeable consumption states at two consecutive dates jointly explains the cross-section of risky asset excess returns and the risk-free rate with economically plausible values of the pure RRA coefficient and the subjective time discount factor. Consistently with earlier results, we reject empirically the model based on the assumption of no background risk (the standard consumption CAPM). The results are robust across different sets of stock-returns and threshold values in the definition of asset holders. Since the important components of the pricing kernel are the first two unconditional moments of the distribution of the non-hedgeable consumption, this supports the hypothesis that the independent non-hedgeable adverse background risk can account for the market premium and the return on the risk-free asset.

References

[1] Aiyagari, S. Rao, and Mark Gertler, 1991, Asset returns with transactions costs and uninsured individual risk, *Journal of Monetary Economics* 27, 311-331.

[2] Attanasio, Orazio P., and Guglielmo Weber, 1995, Is consumption growth consistent with intertemporal optimization? Evidence from the Consumer Expenditure Survey, *Journal of Political Economy* 103, 1121-1157.

[3] Bakshi, Gurdip S., and Zhiwu Chen, 1996, The spirit of capitalism and stock-market prices, *American Economic Review* 86, 133-157.

[4] Balduzzi, Pierluigi, and Tong Yao, 2007, Testing heterogeneous-agent models: An alternative aggregation approach, *Journal of Monetary Economics* 54, 369-412.

[5] Bekaert, Geert, and Jun Liu, 2004, Conditioning information and variance bounds on pricing kernels, *Review of Financial Studies* 17, 339-378.

[6] Bewley, Truman F., 1982, Thoughts on tests of the intertemporal asset pricing model, working paper, Evanston, Ill.: Northwestern University.

[7] Brav, Alon, George M. Constantinides, and Christopher C. Géczy, 2002, Asset pricing with heterogeneous consumers and limited participation: Empirical evidence, *Journal of Political Economy* 110, 793-824.

[8] Breeden, Douglas T., 1979, An intertemporal asset pricing model with stochastic consumption and investment opportunities, *Journal of Financial Economics* 7, 265–296.

[9] Campbell, John Y., and John Cochrane, 1999, By force of habit: a consumption-based explanation of aggregate stock market behavior, *Journal of Political Economy* 107, 205-251.

[10] Campbell, John Y., Andrew W. Lo, and A. Craig MacKinlay, 1997, *The Econometrics of Financial Markets* (Princeton University Press, Princeton).

[11] Carroll, Christopher D., 1994, How does future income affect current consumption, *Quarterly Journal of Economics* 109, 111-147.

[12] Cogley, Timothy, 2002, Idiosyncratic risk and the equity premium: Evidence from the Consumer Expenditure Survey, *Journal of Monetary Economics* 49, 309-334.

[13] Courbage, Christophe, and Béatrice Rey, 2007, Precautionary saving in the presence of other risks, *Economic Theory* 32, 417-424.

[14] Drèze, Jacques H., and Franco Modigliani, 1972, Consumption decision under uncertainty, *Journal of Economic Theory* 5, 308-335.

[15] Epstein, Larry G., and Stanley E. Zin, 1991, Substitution, risk aversion and the temporal behavior of consumption and asset returns: An Empirical analysis, *Journal of Political Economy* 99, 263-288.

[16] Fisher, Ronald Aylmer, 1915, Frequency distribution of the values of the correlation coefficient in samples of an indefinitely large population, *Biometrika* 10, 507-521.

[17] Fisher, Ronald Aylmer, 1921, On the 'probable error' of a coefficient of correlation deduced from a small sample, *Metron* 1, 3-32.

[18] Franke, Guenter, Richard C. Stapleton, and Marti G. Subrahmanyam, 1998, Who buys and who sells options: The role of options in an economy with background risk, *Journal of Economic Theory* 82, 89-109.

[19] Fuller, Wayne A., 1976, *Introduction to Statistical Time Series* (New York: Wiley).

[20] Gollier, Christian, 2001, *The economics of risk and time* (MIT Press, Cambridge, MA).

[21] Gordon, Stephen, and Pascal St-Amour, 2004, Asset returns and state-dependent risk preferences, *Journal of Business and Economic Statistics* 22, 241-252.

[22] Hall, Robert E., 1988, Intertemporal substitution in consumption, *Journal of Political Economy* 96, 339-357.

[23] Hansen, Lars Peter, and Ravi Jagannathan, 1991, Implications of security market data for models of dynamic economies, *Journal of Political Economy* 99, 225-262.

[24] Hansen, Lars Peter, and Kenneth J. Singleton, 1982, Generalized instrumental variables estimation of nonlinear rational expectations models, *Econometrica* 50, 1269-1286.

[25] Heaton, John, and Deborah Lucas, 1996, Evaluating the effects of incomplete markets on risk sharing and asset pricing, *Journal of Political Economy* 104, 443-487.

[26] Heaton, John, and Deborah Lucas, 1997, Market frictions, savings behavior, and portfolio choice, *Macroeconomic Dynamics* 1, 76-101.

[27] Huggett, Mark, 1993, The risk-free rate in heterogeneous-agent incomplete-insurance economies, *Journal of Economic Dynamics and Control* 17, 953-969.

[28] Jacobs, Kris, 1999, Incomplete markets and security prices: Do asset-pricing puzzles result from aggregation problems?, *Journal of Finance* 54, 123-163.

[29] Kihlstrom, Richard E., David Romer, and Steve Williams, 1981, Risk aversion with random initial wealth, *Econometrica* 49, 911-920.

[30] Kimball, Miles S., 1993, Standard risk aversion. *Econometrica* 61, 589-611.

[31] Kimball, Miles S., 1992, Precautionary motives for holding assets. In: Newmann, P., Falwell, J. (eds.) The new palgrave dictionary of money and finance, pp. 158–161. London: MacMillan.

[32] Kimball, Miles S., 1990, Precautionary savings in the small and in the large, *Econometrica* 58, 53-73.

[33] Kocherlakota, Narayana R., and Luigi Pistaferri, 2008, Asset pricing implications of Pareto optimality with private information, working paper.

[34] Leland Hayne E., 1968, Saving and uncertainty: The precautionary demand for saving, *Quarterly Journal of Economics* 82, 465-473.

[35] Lucas, Deborah, 1994, Asset pricing with undiversifiable income risk and short sales constraints: Deepening the equity premium puzzle, *Journal of Monetary Economics* 34, 325-341.

[36] Lucas Jr., Robert E., 1978, Asset prices in an exchange economy, *Econometrica* 46, 1429–1445.

[37] Mankiw, N. Gregory, 1986, The equity premium and the concentration of aggregate shocks, *Journal of Financial Economics* 17, 211-219.

[38] Mehra, Rajnish, and Edward C. Prescott, 1985, The equity premium: A puzzle, *Journal of Monetary Economics* 15, 145-162.

[39] Nachman, David C., 1982, Preservation of "more risk averse" under expectations, *Journal of Economic Theory* 28, 361-368.

[40] Ogaki, Masao, 1988, Learning about preferences from time trends, Ph.D. Dissertation, University of Chicago.

[41] Poon, Ser-Huang, and Richard C. Stapleton, 2005, *Asset Pricing in Discrete Time: A Complete Market Approach* (Oxford University Press, Oxford, New York).

[42] Sandmo, Agnar, 1970, The effect of uncertainty on saving decisions, *The Review of Economic Studies* 37, 353-360.

[43] Telmer, Chris, 1993, Asset-pricing puzzles and incomplete markets, *Journal of Finance* 48, 1803-1832.

[44] Vissing-Jorgensen, Annette, 2002, Limited asset market participation and the elasticity of intertemporal substitution, *Journal of Political Economy* 110, 825-853.

[45] Weil, Philippe, 1989, The equity premium puzzle and the risk-free rate puzzle, *Journal of Monetary Economics* 24, 401-421.

Table I. Descriptive Statistics

In this table, CGR_t is the aggregate per capita consumption growth rate, $\varsigma^2_{\Phi,t}$ is the normalized variance of the non-hedgeable consumption, $R_{VW,t}$ and $R_{EW,t}$ are the CRSP value- and equal-weighted indices, respectively, $R_{f,t}$ is the risk-free rate of return, $R_{Di,t}$ $(i = 1, ..., 10)$ are the returns on the CRSP market capitalization-based decile portfolios, $R_{5Ii,t}$ $(i = 1, ..., 5)$ and $R_{10Ii,t}$ $(i = 1, ..., 10)$ are the returns on the five and ten industry portfolios, respectively. The sample period is 1982:Q1 to 2003:Q4.

Variable	Minimum	Median	Maximum	Mean	St.Dev.	Skewness
			A. Consumption			
			A.1 All Households			
CGR_t	0.9039	0.9895	1.0664	0.9897	0.0339	0.0596
$\varsigma^2_{\Phi,t}$	0.0468	0.0721	0.1446	0.0733	0.0136	1.9611
			A.2 Households with Total Assets > \$0			
CGR_t	0.8835	1.0011	1.1348	0.9972	0.0500	0.0867
$\varsigma^2_{\Phi,t}$	0.0292	0.0731	0.1306	0.0732	0.0207	0.4479
			A.3 Households with Total Assets ⩾ \$1000			
CGR_t	0.8774	0.9973	1.1427	0.9965	0.0510	0.0901
$\varsigma^2_{\Phi,t}$	0.0292	0.0726	0.1306	0.0731	0.0200	0.3351
			A.4 Households with Total Assets ⩾ \$5000			
CGR_t	0.8776	0.9999	1.1618	0.9963	0.0552	0.2694
$\varsigma^2_{\Phi,t}$	0.0320	0.0765	0.1546	0.0764	0.0235	0.7064
			B. Asset Returns			
			B.1 Market Portfolio Returns and the Risk-Free Rate			
$R_{VW,t}$	0.7633	1.0346	1.2059	1.0269	0.0872	-0.4703
$R_{EW,t}$	0.7033	1.0257	1.3176	1.0321	0.1161	-0.0395
$R_{f,t}$	0.9933	1.0056	1.0178	1.0053	0.0048	0.2153
			B.2 Decile Portfolio Returns			
$R_{D1,t}$	0.7644	1.0434	1.2164	1.0363	0.0880	-0.4554
$R_{D2,t}$	0.7199	1.0262	1.2855	1.0302	0.1152	-0.3173
$R_{D3,t}$	0.7584	1.0364	1.2256	1.0297	0.0883	-0.5788
$R_{D4,t}$	0.7658	1.0242	1.2436	1.0236	0.0780	-0.6265
$R_{D5,t}$	0.6518	1.0381	1.3968	1.0303	0.1436	-0.1897
$R_{D6,t}$	0.7652	1.0398	1.2570	1.0266	0.1031	-0.2717
$R_{D7,t}$	0.7027	1.0321	1.3129	1.0338	0.1065	-0.1787
$R_{D8,t}$	0.7544	1.0320	1.2409	1.0341	0.0983	-0.2926
$R_{D9,t}$	0.8023	1.0352	1.2684	1.0243	0.0742	0.0048
$R_{D10,t}$	0.7606	1.0354	1.2076	1.0309	0.0958	-0.6319

Table I (continued)

Variable	Minimum	Median	Maximum	Mean	St.Dev.	Skewness
			B.3 5 Industry Portfolio Returns			
$R_{5I1,t}$	0.7319	1.0410	1.2417	1.0334	0.0917	-0.3746
$R_{5I2,t}$	0.7917	1.0337	1.1797	1.0259	0.0702	-0.8788
$R_{5I3,t}$	0.6793	1.0345	1.3409	1.0287	0.1228	-0.3808
$R_{5I4,t}$	0.7544	1.0320	1.2409	1.0341	0.0983	-0.2926
$R_{5I5,t}$	0.7606	1.0354	1.2076	1.0309	0.0958	-0.6319
			B.4 10 Industry Portfolio Returns			
$R_{10I1,t}$	0.7644	1.0434	1.2164	1.0363	0.0880	-0.4554
$R_{10I2,t}$	0.7199	1.0262	1.2855	1.0302	0.1152	-0.3173
$R_{10I3,t}$	0.7584	1.0364	1.2256	1.0297	0.0883	-0.5788
$R_{10I4,t}$	0.7658	1.0242	1.2436	1.0236	0.0780	-0.6265
$R_{10I5,t}$	0.6518	1.0381	1.3968	1.0303	0.1436	-0.1897
$R_{10I6,t}$	0.7652	1.0398	1.2570	1.0266	0.1031	-0.2717
$R_{10I7,t}$	0.7027	1.0321	1.3129	1.0338	0.1065	-0.1787
$R_{10I8,t}$	0.7544	1.0320	1.2409	1.0341	0.0983	-0.2926
$R_{10I9,t}$	0.8023	1.0352	1.2684	1.0243	0.0742	0.0048
$R_{10I10,t}$	0.7606	1.0354	1.2076	1.0309	0.0958	-0.6319

Table II

The Conditional Volatility Bounds Results

In this table, the column labeled "SM" shows the results for the standard consumption CAPM, the column labeled "BCG" presents the estimation results for the model with the pricing kernel proposed in Brav *et al.* (2002), and the column labeled "MBR" reports the estimates of the parameters for the model with background risk. *Pure RRA* is the pure RRA coefficient, δ is the subjective time discount factor, $\overline{\gamma}_t^*$ and $\sigma^2_{\gamma_t^*}$ are respectively the sample mean and variance of the effective RRA coefficient γ_t^*, μ is the proportion of the risk premium due to the background risk, $\overline{\phi}_t$ and $\sigma^2_{\phi_t}$ are respectively the sample mean and variance of the EIS. $z_i = 0.5ln\left(\left(1+r_i\right)/\left(1-r_i\right)\right)$, $i = 1, 2$, is the Fisher z-statistic with r_1 and r_2 being Pearson's coefficients of correlation between γ_t^* and the real excess return on the value-weighted CRSP market portfolio, $R_{VW,t} - R_{f,t}$, and between γ_t^* and $\varsigma^2_{\Phi,t}$, respectively. ρ_k is the kth-order autocorrelation coefficient of γ_t^*. Asymptotic standard errors are in parentheses.

Parameter	All			Assets > \$0			Assets ⩾ \$1000			Assets ⩾ \$5000		
	SM	BCG	MBR	SM	BCG	MBR	SM	BCG	MBR	SM	BCG	MBR
A. CRSP Market Capitalization-Based Decile Portfolios 1-10:												
Pure RRA	25.9816	8.213	5.6255	13.9201	8.468	5.1311	13.9919	8.807	5.1849	12.2080	7.879	4.9121
δ	0.8104	0.5549	0.9242	0.9382	0.5920	0.8890	0.9061	0.5551	0.8879	0.9033	0.5844	0.8867
$\overline{\gamma}_t^*$			10.1240			8.9163			8.9950			8.6133
$\sigma^2_{\gamma_t^*}$			3.8016			4.2795			3.8890			5.4466
μ			0.4443			0.4245			0.4236			0.4297
z_1			-0.0955			0.0336			-0.0190			0.0592
			(0.1098)			(0.1098)			(0.1098)			(0.1098)
z_2			1.6398			2.0262			2.0662			1.8556
			(0.1098)			(0.1098)			(0.1098)			(0.1098)
ρ_1			0.1009			0.1781			0.1236			0.1884
			(0.1091)			(0.1091)			(0.1091)			(0.1091)
ρ_2			-0.0397			-0.0809			0.0348			0.0595
			(0.1091)			(0.1091)			(0.1091)			(0.1091)
$\overline{\phi}_t$			0.1008			0.1169			0.1154			0.1220
$\sigma^2_{\phi_t}$			0.0001			0.0005			0.0004			0.0006

Table II (continued)

Parameter	All			Assets > \$0			Assets ⩾ \$1000			Assets ⩾ \$5000		
	SM	BCG	MBR	SM	BCG	MBR	SM	BCG	MBR	SM	BCG	MBR
						B. CRSP Market Capitalization-Based Decile Portfolios 1-5:						
Pure RRA	19.2395	6.756	5.0651	9.8380	6.754	4.3420	9.8170	7.023	4.3524	8.6606	6.259	4.0568
δ	0.8759	0.6037	0.9518	0.9817	0.6436	0.9403	0.9592	0.6109	0.9398	0.9550	0.6453	0.9397
$\overline{\gamma}_t^*$			8.5592			6.8930			6.8772			6.4060
$\sigma^2_{\gamma_t^*}$			1.7776			1.5752			1.3826			1.6558
μ			0.4082			0.3701			0.3671			0.3667
z_1			-0.1005			0.0282			-0.0220			0.0482
			(0.1098)			(0.1098)			(0.1098)			(0.1098)
z_2			1.7892			2.2392			2.2880			2.0956
			(0.1098)			(0.1098)			(0.1098)			(0.1098)
ρ_1			0.1171			0.1835			0.1395			0.2032
			(0.1091)			(0.1091)			(0.1091)			(0.1091)
ρ_2			-0.0356			-0.0765			0.0424			0.0613
			(0.1091)			(0.1091)			(0.1091)			(0.1091)
$\overline{\phi}_t$			0.1186			0.1490			0.1490			0.1609
$\sigma^2_{\phi_t}$			0.0002			0.0005			0.0005			0.0006

Table II (continued)

Parameter	All			Assets $>$ \$0			Assets $\geqslant$ \$1000			Assets $\geqslant$ \$5000		
	SM	BCG	MBR	SM	BCG	MBR	SM	BCG	MBR	SM	BCG	MBR
				C. CRSP Market Capitalization-Based Decile Portfolios 6-10:								
Pure RRA	16.5340	6.155	4.7779	8.3359	6.020	3.9608	8.2885	6.246	3.9547	7.3468	5.567	3.6550
δ	0.8992	0.6302	0.9619	0.9925	0.6746	0.9568	0.9736	0.6446	0.9560	0.9695	0.6794	0.9558
$\overline{\gamma}_t^*$			7.8283			6.0364			5.9934			5.5163
$\sigma^2_{\gamma_t^*}$			1.2048			0.9509			0.8225			0.9249
μ			0.3897			0.3438			0.3402			0.3374
z_1			-0.1028			0.0261			-0.0232			0.0436
			(0.1098)			(0.1098)			(0.1098)			(0.1098)
z_2			1.8684			2.3468			2.3992			2.2147
			(0.1098)			(0.1098)			(0.1098)			(0.1098)
ρ_1			0.1250			0.1857			0.1463			0.2087
			(0.1091)			(0.1091)			(0.1091)			(0.1091)
ρ_2			-0.0331			-0.0744			0.0454			0.0616
			(0.1091)			(0.1091)			(0.1091)			(0.1091)
$\overline{\phi}_t$			0.1294			0.1693			0.1701			0.1857
$\sigma^2_{\phi_t}$			0.0002			0.0006			0.0005			0.0007

Table II (continued)

Parameter	All			Assets > \$0			Assets ⩾ \$1000			Assets ⩾ \$5000		
	SM	BCG	MBR	SM	BCG	MBR	SM	BCG	MBR	SM	BCG	MBR
					D. 5 Industry Portfolios:							
Pure RRA	19.1530	6.737	5.0567	9.7889	6.731	4.3305	9.7670	6.999	4.3403	8.6177	6.238	4.0446
δ	0.8766	0.6017	0.9541	0.9809	0.6438	0.9404	0.9586	0.6113	0.9387	0.9555	0.6458	0.9384
$\overline{\gamma}_t^*$			8.5372			6.8661			6.8492			6.3778
$\sigma^2_{\gamma_t^*}$			1.7575			1.5518			1.3614			1.6273
μ			0.4077			0.3693			0.3663			0.3658
z_1			-0.1006			0.0282			-0.0220			0.0480
			(0.1098)			(0.1098)			(0.1098)			(0.1098)
z_2			1.7915			2.2424			2.2914			2.0991
			(0.1098)			(0.1098)			(0.1098)			(0.1098)
ρ_1			0.1174			0.1836			0.1397			0.2034
			(0.1091)			(0.1091)			(0.1091)			(0.1091)
ρ_2			-0.0356			-0.0764			0.0425			0.0613
			(0.1091)			(0.1091)			(0.1091)			(0.1091)
$\overline{\phi}_t$			0.1189			0.1496			0.1496			0.1616
$\sigma^2_{\phi_t}$			0.0002			0.0005			0.0005			0.0006

Table II (continued)

Parameter	All			Assets > \$0			Assets ⩾ \$1000			Assets ⩾ \$5000		
	SM	BCG	MBR	SM	BCG	MBR	SM	BCG	MBR	SM	BCG	MBR
						E. 10 Industry Portfolios:						
Pure RRA	20.9547	7.130	5.2252	10.8263	7.205	4.5620	10.8261	7.496	4.5832	9.5228	6.684	4.2917
δ	0.8658	0.5877	0.9488	0.9726	0.6270	0.9282	0.9481	0.5927	0.9283	0.9456	0.6270	0.9274
$\overline{\gamma}_t^*$			8.9866			7.4208			7.4255			6.9662
$\sigma^2_{\gamma_t^*}$			2.2073			2.0919			1.8527			2.3074
μ			0.4186			0.3852			0.3828			0.3839
z_1			-0.0992			0.0296			-0.0212			0.0511
			(0.1098)			(0.1098)			(0.1098)			(0.1098)
z_2			1.7459			2.1786			2.2253			2.0280
			(0.1098)			(0.1098)			(0.1098)			(0.1098)
ρ_1			0.1126			0.1821			0.1354			0.1997
			(0.1091)			(0.1091)			(0.1091)			(0.1091)
ρ_2			-0.0369			-0.0777			0.0404			0.0610
			(0.1091)			(0.1091)			(0.1091)			(0.1091)
$\overline{\phi}_t$			0.1131			0.1389			0.1384			0.1487
$\sigma^2_{\phi_t}$			0.0002			0.0005			0.0005			0.0006