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Why and How to Integrate Liquidity Risk into a VaR-Framework

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Why and How to Integrate Liquidity Risk into a VaR-Framework

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We integrate liquidity risk measured by the weighted spread into a Value-at-Risk (VaR) framework. The weighted spread measure extracts liquidity costs by order size from the limit order book. We show that it is precise from a risk perspective in a wide range of clearly defined situations.

Using a unique, representative data set provided by Deutsche Börse AG, we find liquidity risk to increase traditionally-measured price risk by over 25%, even at standard 10-day horizons and for liquid DAX stocks. We also show that the common approach of simply adding liquidity risk to price risk substantially overestimates total risk because correlation between liquidity and price is neglected. Our results are robust with respect to changes in risk measure, to sample periods and to effects of portfolio diversification.

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1 Introduction

Liquidity as the ease of trading an asset has lately received much attention in the academic world and in practice. Still, many risk management systems assume, that a position can be bought or sold without cost if the liquidation horizon is long enough. While this is a traditional assumption in theoretical, perfect markets, in real financial markets liquidity costs can get quite substantial. Even for liquid stocks like those in the DAX index, consisting of the 30 largest German companies, liquidity costs rise to over 100 basis points when trading larger positions.[1]

From a risk management perspective, liquidity risk is the potential loss due to the time-varying cost of trading. Despite high current interest, liquidity risk measurement is still under development.[2] A range of approaches has been suggested in the academic literature.

Bangia, Diebold, Schuermann and Stroughair (1999) use the quoted bid-ask-spread as liquidity measure. In a parametric Value-at-Risk (VaR) approach, they add the mean-variance-estimated worst spread to the price risk of an asset.[3] Their approach is quickly implementable with easily available data but neglects, that only small positions can be traded at the quoted spread.[4] Liquidity cost for larger positions can thus be underestimated. Their add-on approach also implicitly assumes perfect correlation between prices and liquidity cost, i.e. that worst price loss and highest costs will occur simultaneously in crises. While this greatly simplifies calculations, it will overestimate risk if correlations are less than perfect.

In a VaR-approach Berkowitz (2000) has included the fact, that liquidity costs increase with the size of the position beyond the quoted spread. This so-called price impact is estimated as a linear function from transaction prices. However, price impact functions are generally not linear[5] and precise estimation of individual-stock liquidity from transaction data is difficult at best[6]. In addition, Berkowitz assumes that liquidity costs and returns are independent, i.e. zero return-liquidity correlation, which can also be doubted.

Other empirical frameworks have been suggested by Francois-Heude and Van Wynendaele (2001) and Angelidis and Benos (2006), but also suffer from im-

[1] Liquidity cost for positions above € 1 million, cp. Stange and Kaserer (2008).
[2] Cp. Basel committee (2005), p.10.
[3] This is similar to some practical risk management systems who value positions at bid prices, but also accounts for the time-variation of liquidity cost.
[4] Market makers are only required to trade positions up to a certain size, the 'spread depth' or 'normal market size', at the quoted spread.
[5] Cp. non-linearity of price impact function found by Hasbrouck (1991); Hausman et al. (1992); Stange and Kaserer (2008).
[6] Cp. discussions and approaches in Amihud (2002); Pastor and Stambaugh (2003).

precise liquidity approximations and specific but untested assumptions concerning liquidity-price correlations. In a different stream of literature on optimal trading strategies, empirical estimation procedures have yet to be developed before they can be useful in practical risk management.[7]

Giot and Grammig (2005) is closest to our approach, using weighted spread in an intraday VaR-framework. Weighted spread measures the liquidity cost of a specific order size as the average spread in the limit order book weighted by individual limit-order sizes.[8] It generalizes the approach of Bangia et al. beyond the quoted spread and is a precise price-impact measure when immediately transacting against the limit order book. Giot and Grammig circumvent the problem of liquidity-price correlation by modeling t-distributed net-returns, i.e. returns net of liquidity costs. The specific distributional assumption is, however, not empirically tested. While providing insight on the intraday structure of liquidity risk, results for daily and longer horizons are naturally outside their scope.

In this paper, we also use the weighted-spread liquidity measure and address three open issues. First, we clearly identify the situations, in which weighted spread can be validly employed in the risk management context. Second, we analyze the magnitude of liquidity impact at standard, larger-than-intraday horizons in a representative sample of stocks. While it is plausible and empirically proven that liquidity risk is economically significant at intraday horizons,[9] it is unclear if rendered negligible at standard daily or even 10-day horizons. General price risk increases the longer the forecasting horizon, liquidity is a one-time cost much less dependent on the horizon. Thus, the liquidity risk component in total risk will be smaller for longer horizons. As a consequence, liquidity risk might be negligible in liquid stock markets at standard horizons and neglect by risk frameworks could be justified. Existing estimates of the liquidity component are based on very small samples and on imprecise liquidity risk measures.[10] Third, we empirically clarify, whether tail correlation between price

[7] Cp. Almgren and Chriss (2000); Hisata and Yamai (2000); Almgren (2003); Dubil (2003); Jarrow and Protter (2005); Engle and Ferstenberg (2007), who devise optimal trading strategies to minimize liquidity risk, but are yet empirically untraceable.

[8] This measure corresponds to the cost of a round-trip (CRT) by Irvine et al. (2000). Similar measures are used in other contexts by Coppejans et al. (2001); Gomber and Schweickert (2002); Gomber et al. (2004); Domowitz et al. (2005).

[9] Francois-Heude and Van Wynendaele (2001) find a 2-21% contribution of intraday price impact in one stock over four months. Giot and Grammig (2005) show that 30-minute, intraday liquidity-adjusted VaR is 11-30% for three large stocks over three months. In a seven month sample of 60 stocks, Angelidis and Benos (2006) estimate that liquidity risk constitutes 11% of total intraday VaR in low capitalization stocks. Lei and Lai (2007) reveal a 30% total intraday risk contribution by liquidity in 41 small price stocks over 12 months.

[10] At daily horizon, Bangia et al. (1999) find underestimation of total VaR by 25-30% in emerging market currencies when looking at bid-ask-spread liquidity. Le Saout (2002) estimates for 41

and liquidity costs is perfect or not. This assumption simplifies risk calculation in parametric frameworks and was criticized, but so far untested.[11]

For our empirical analysis, we use - as far as we know - the most representative sample of daily weighted spread available to academia.[12] It contains a data set of the Xetra Liquidity Measure (XLM) for 160 stocks over 5.5 years, which is readily available from Deutsche Börse.

The remainder of the paper is organized as follows. Section 2 defines liquidity, the liquidity measure and liquidity risk and discusses the situations, when our approach is valid. In section 3 we describe our empirical data set and the empirical results. Section 4 summarizes and concludes.

2 Theoretical framework and assumptions

In section 2.1 we first define liquidity from a cost perspective, characterize the situational assumptions in which our framework can be applied and describe our empirical liquidity measure. In 2.2 we introduce our risk estimation approach

We will first define liquidity from a cost perspective, describe our empirical liquidity cost measure and characterize the situations, in which the measure can be applied in section . In 2.2 we introduce the risk estimation approach and definitions as well as a risk decomposition to uncover structural insights.

2.1 Liquidity cost framework

2.1.1 Definition of liquidity

We define illiquidity as the cost of trading an asset relative to fair value.[13] Fair value is assumed to be the mid-point of the bid-ask-spread. We distinguish three components of the relative liquidity cost $L_t(q)$ in percent of the mid-price[14] for an order quantity q at time t

$$L_t(q) := T(q) + PI_t(q) + D_t(q) \tag{1}$$

stocks over 28 months, that the bid-ask-spread liquidity component can represent 50% of the total daily risk for illiquid stocks.

[11] Critique brought forward by Francois-Heude and Van Wynendaele (2001), Angelidis and Benos (2006), Loebnitz (2006), Lei and Lai (2007) and Jorion (2007).

[12] Usual samples are restricted to few stocks over few months, because weighted spread has to be manually calculated from the whole intraday order book, which is highly computational extensive.

[13] Cp. Dowd (2001), p. 187 ff. and Buhl (2004); Amihud and Mendelson (2006).

[14] Mid-price is the mid-point of the bid-ask-spread.

where $T(q)$ are direct trading costs, $PI_t(q)$ is the price impact vs. mid-price due to the size of the position, $D_t(q)$ are delay costs if a position cannot be traded immediately.[15]

Direct trading costs, also called explicit transaction costs, include exchange fees, brokerage commissions and transaction taxes. Their main characteristic is that they are deterministic.[16] The *price impact* is the difference between the transaction price and the mid-price. They result from imperfectly elastic demand and supply curve for stocks at a specific point in time. For small volumes this is the bid-ask-spread, but for larger volumes price impact is larger. *Delay costs* comprise costs for searching a counter-party and the cost imposed on the investor due to bearing price risk and price impact risk during the execution delay.[17] For many assets like most stocks and bonds on an exchange search costs are negligibly small, but costs of additional risk during delay can remain large. This cost definition takes a practical, concrete investor's perspective and can integrate other definitions in the literature.

2.1.2 Situational assumptions

To simplify the concrete approach, we will look at well-defined situations with specific types of assets. This section develops the four characterizing assumptions, under which our framework can be validly applied.

First, we assume that direct trading costs are zero, $T(q) = 0$. For very large or institutional traders in developed markets, T(q) can generally be considered negligible. On the Xetra system of the Deutsche Börse, for example, institutional traders pay only around 0.5 bp as transaction fee.[18] Transaction cost T(q) can also be neglected if time variation of liquidity is of major interest.

The second characteristic concerns data availability. Because we focus on the price impact of a specific position size, this type of price impact data needs to be available. This is most probably true in markets with an electronic limit order book, where limit order book data is made available, such as the London Stock Exchange, the NASDAQ, the Frankfurt Xetra or the Euronext. We provide an exact description of our liquidity measure and its calculation from the limit order book in the next subsection 2.1.3.

Third, we look at assets positions, which are continuously tradable during crises. This means, that no (or very few) zero trading days occur and the position size is not

[15] This closely follows Amihud and Mendelson (2006), but additionally differentiates by the size of the position.
[16] Cp. Loebnitz (2006), p.18 f.
[17] Almgren (2003) calls price impact risk "trading enhanced risk".
[18] Cp. Deutsche Boerse (2008), p.6 ff.

larger than market depth. This is a close approximation for most stocks, which have no or very few zero trading days. Therefore, investors are not forced to delay the execution of a transaction and costs from forced delay are zero. Scanning our data of 160 German stocks over 5.5 years shows that this assumption is less restrictive than it first seems. Even for less continuously traded stocks in our sample, trading gets continuous during market turmoils. Zero trading days seem to occur mainly in calmer market periods. We hypothesize that tumbling market prices attract traders, who want to liquidate positions or to stop loss via limit orders, which ensures continuous trading. However, we leave a rigorous analysis of this aspect to future research.

Fourth, we assume that deliberate, strategic delay has no significant benefit, i.e. we assume that positions can be equally good instantly liquidated against the limit order book.[19] So, we neglect any (potential) effect of optimal trading strategies, which balance the increased price risk of delay against reduced liquidity cost by trading smaller quantities.[20] In our view, this is a reasonable assumption in four cases. When we take the worst case perspective of impatient traders, a common risk assumption, potential benefits are consciously neglected. Benefits are also non-existent, if informational content of our trade is too high. The trader wants to trade immediately on an informational advantage, which would be revealed by trading more slowly or dissolve over time. Adverse informational effects are also possible, i.e. trading more slowly could have price effects because the market assumes informational advantage, which is not present in reality.[21] Immediate liquidation is fair, too, if liquidity prices are efficient and a traders risk aversion is greater or equal to that of the market.[22] In this case, marginal gain from lower liquidity costs by delaying a transaction balances the marginal loss due to higher price risk. Finally, optimal trading strategies might not be feasible in times of market stress,[23] because the optimization parameters are not stable or strategic trading is not always possible.

[19] This also neglects liquidation via limit instead of market orders as well as up-floor or over-the-counter trading.

[20] Cp. for example Almgren and Chriss (1999, 2000); Almgren (2003); Bertsimas and Lo (1998) and others.

[21] Technically expressed as high permanent price impact rendering optimal trading strategies useless.

[22] If liquidity costs are too high, liquidity providers will enter with limit orders, because liquidity costs, i.e. their profits, will compensate for the additional risk during the delay until the limit order is executed. If liquidity costs are too low, market orders and withdrawn limit orders will deplete the order book, because nobody is willing to take price risk during delay.

[23] A point raised in Jarrow and Protter (2005), p.9.

If there is no forced or deliberate delay, delay cost are zero ($D(q) = 0$) and as a consequence, our total liquidity cost can be fairly measured with the price impact from immediate execution.

$$L(q) = PI(q)$$

The first two assumptions are generally less critical. Although the latter two assumptions place restrictions on the range of applications, the discussion shows, that our approach is still valid in a large variety of situations, especially if markets are fairly liquid, positions are not too large and we take a worst case perspective.

2.1.3 The weighted spread liquidity measure

We have obtained our liquidity data from the Xetra system of the Frankfurt Stock Exchange covering the bulk of stock transactions in Germany.[24] Xetra is an electronic trading platform by Deutsche Börse, which is among the top 10 largest stock exchanges in the world. Trading starts with an opening auction at 9 a.m., is interrupted by an intraday auction around 1 p.m. and ends with a closing auction finished at 5.30 p.m.. In between, trading is continuous. An electronic order book collects all limit and market orders from market participants and matches them on price, followed by time priority. The order book is anonymous, but visible to all market participants. However, traders can also submit invisible, "iceberg" orders to trade large volumina, where traded volume is only revealed up to a certain size and a similar order of equal size will be initiated once the first limit order is transacted. For illiquid stocks, market makers post bid- and ask quotes up to a prespecified minimum quotation volume.[25]

We measure price impact with the Xetra Liquidity Measure (XLM). XLM is a weighted spread measure, which provides the liquidity cost of a round trip of size q compared to its fair value.[26] The Xetra system automatically calculates XLM from the visible and invisible part of the limit order book. Mathematically, XLM is defined as follows. The weighted bid-price $b_t(v)$ for selling v number of shares is calculated as

$$b_t(v) = \frac{\sum_i b_{i,t} v_{i,t}}{v} \tag{2}$$

where $b_{i,t}$ and $v_{i,t}$ are the bid-prices in € and bid-volumes of individual limit orders at time t sorted by price priority. Individual limit order volume add up to v shares,

[24]Cp. Deutsche Boerse (2005).
[25]Cp. Deutsche Boerse (2004).
[26]Fair value is set at the mid-price of the bid-ask-spread P_{mid}.

$\sum_i v_i = v$. The weighted ask-price is calculated analogously. XLM is then calculated as the weighted spread in basis points (bp) for predefined order sizes q

$$XLM(q) = \frac{a_t(v) - b_t(v)}{P_{mid}} \times 100 \tag{3}$$

where P_{mid} is the mid-price of the quoted (minimum) spread and $q = v \times P_{mid}$ is the size of the position measured in Euro-mid-price value.

Graphically, XLM is the area between the bid- and the ask-curve in the limit order book up to the order size q divided by the mid-price value (see figure 1 on page 30). XLM calculates the price impact of an order of size q in basis points. It can also be seen as the relative liquidity discount for a round-trip of an order of size q.[27] XLM is an ex-ante measure, because it calculates the cost from committed liquidity in the order book - including hidden 'iceberg orders' - and neglects any hidden liquidity.[28]

Liquidity cost L(q) is then estimated from a transaction perspective. As a per-transaction figure has much more practical meaning than a per-round-trip figure, we assume that the order book is symmetrical on average.[29] Therefore, we can calculate the price impact per transaction under the situational assumptions outlined in section 2.1.2 as

$$L(q) = PI(q) = \frac{XLM(q)}{2} \tag{4}$$

In contrast to other price impact proxies, measure (4) is a precise measure of the ex-ante, order-size differentiated liquidity cost at and beyond the bid-ask-spread depth.[30] However, it is important to notice, that this liquidity cost measure increases computational complexity, because the price impact curve must be estimated, at least with a liquidity cost vector. In addition, concrete position sizes must be interpolated between vector entries. Nevertheless, additional computations are limited as long as weighted spread is provided by the exchange, like in the case of XLM, and not manually calculated from the intraday order book.

There are important similarities and differences between XLM and the quoted bid-ask-spread. The quoted spread is the simplest version of an ex-ante liquidity measure, but is valid only up to quoted depth. XLM is its natural generalization, because it extends beyond best bid-ask-prices to the rest of the order book. The bid-ask-spread is the minimum weighted spread (for small order sizes). However,

[27] Gomber and Schweickert (2002) provide further theoretical background.

[28] Cp. Irvine et al. (2000), p.4.

[29] Liquidity cost estimation could gain further precision, if exchanges would provide buy-side and sell-side weighted half-spread data.

[30] Up to now, it been empirically impossible to distill precise price impact measures for single assets from ex-post transaction data (cp. Amihud (2002); Pastor and Stambaugh (2003)).

spread is usually not measured for constant order sizes q, because quoted depth differs between stocks. Spread also has upper bounds regulated by the exchange protocol, if there is market maker coverage.[31] This is a first indicator that liquidity-cost dynamics will be different when moving beyond the minimum bid-ask-spread.

2.2 Liquidity risk framework

2.2.1 Measurement approach

We want to calculate liquidity risk estimates as precise as possible. Therefore, we use the historical, empirical distribution instead of a parametric approach to estimate percentiles. This approach is possible due to our large sample and has the advantage that we do not have to make any assumption regarding the distribution of liquidity. This is important, because liquidity distributions are often far from normal.[32] The development of a correct parametrization is left to future research.

To use percentiles of the historical distribution, we have to rely on the full sample period, because short samples have not enough observations to finely estimate percentiles. We also deliberately accept that risk might be different at different estimation periods. As robustness test, we later look into time variation in a parametric framework to test if those drawbacks have any significant impact (see section 3.4.2).

Similarly, we measure risk ex-post and not ex-ante. This avoids any distortion through a specific forecasting method, which is similarly a point left for future development.

2.2.2 Definition of risk measures

Before we turn to defining liquidity risk, we start with the definition of price risk. We use standard risk statistics, against which we will measure the impact of liquidity risk.

Price and return are described in the usual framework of

$$P_{mid,t} = P_{mid,t-\Delta t} \times exp(r_{t,\Delta t})$$

where P_{mid} is defined as the mid-price $P_{mid,t} = \frac{a_t+b_t}{2}$ with a_t and b_t being the (best) ask- and bid-price at time t respectively. $r_{t,\Delta t}$ is the Δt-period continuous mid-price return at time t, i.e., $r_{t,\Delta t} = ln(P_{mid,t}/P_{mid,t-\Delta t})$. We take a traditional approach

[31] On Xetra illiquid stocks, defined by XLM and past volume, are covered by market makers. If the stock is not covered, the bid-ask-spread corresponds to the minimum spread in the order book.

[32] Cp. Stange and Kaserer (2008) for a detailed discussion of the properties of XLM.

from a value-at-risk (VaR) perspective and define price risk as the relative VaR at the $(1-\alpha)$-percent confidence level over the horizon Δt

$$VaR^{\alpha,\Delta t}_{price} = 1 - exp(r^{\alpha}_{t,\Delta t}) \tag{5}$$

where $r^{\alpha}_{t,\Delta t}$ is the α-percentile of Δt-period return distribution. Consequently, VaR_{price} measures the maximum percentage loss over the period Δt with a confidence of $(1-\alpha)$-percent.

Analogously, we measure total risk including liquidity risk. To calculate the impact of liquidity, we define the Δt-period *net return* in t as the sum of the continuous mid-price return and the liquidity discount converted to a continuous value, $l_t(q) = ln\,(1 - L_t(q))$.

$$rnet_{t,\Delta t}(q) = r_{t,\Delta t} + l_t(q) \tag{6}$$

Please note the difference of (6) to net-price returns.[33] Using net returns instead of net-price returns, we implicitly assume that the liquidity cost of entering a position has already been properly accounted for. If we used net-price returns, the implicit assumption would be that not only the liquidity cost of entering a position, but also the expected liquidity cost of the liquidation is properly accounted for already when entering it. We believe that our assumption is more realistic in practice.

Price is then calculated as

$$P_{net,t}(q) = P_{mid,t-\Delta t} \times exp(r_{t,\Delta t} + l_t(q)) \tag{7}$$

where $P_{net,t}(q)$ is the achievable transaction price.

The Δt-period *liquidity-adjusted total risk* is then defined in a VaR-framework as the empirical α-percentile of the net-return distribution.

$$VaR^{\alpha,\Delta t}_{total}(q) = 1 - exp(rnet^{\alpha}_{t,\Delta t}(q)) \tag{8}$$

VaR_{total} is the maximum percentage loss due to mid-price risk and liquidation cost over the period Δt with a confidence of $(1-\alpha)$-percent. This specification covers the real dynamics of the net return on a certain stock position. It is practical but also more general than existing approaches in the following ways:

1. We use a more precise liquidity measure than most papers by covering more aspects of liquidity. Specifically, we account for the impact of order-size on liquidity. This extends the approach of Bangia et al. (1998, 1999), where

[33] I.e. $ln\,([P_{mid,t} \times (1 - L_t(q))]\,/\,[P_{mid,t-1} \times (1 - L_{t-1}(q))])$.

liquidity costs of any order size is proxied for with the bid-ask-spread. The XLM measure is also more precise than the one used in Berkowitz (2000), Francois-Heude and Van Wynendaele (2001) or Angelidis and Benos (2006).

2. As we take empirical percentiles instead of a parametric method, we avoid any distributional assumption, especially on liquidity cost, such as in Giot and Grammig (2005). Our approach will capture non-normality of the distribution as well, which is made possible by our large sample size.

3. Our approach takes percentiles of the net return distribution and does not treat price risk and liquidity separately. We look at the dynamics of net returns which combines the mid-price-return dynamics and liquidity cost dynamics. Instead of adding distribution percentiles of liquidity and price risk separately, we acknowledge that liquidity cost and mid-price might not be perfectly correlated. While it is possible that large liquidity discounts and low prices coincide, this must not be the case.

2.2.3 Risk decomposition

To uncover the structure of the liquidity impact, we decompose total risk into its components. We define relative liquidity impact $\lambda(q)$ as

$$\lambda(q) = \frac{VaR_{total}(q) - VaR_{price}}{VaR_{price}} \tag{9}$$

$\lambda(q)$ is the maximum percentage loss due to the liquidity in relation to price risk. It can be interpreted as the error made when ignoring liquidity. It is therefore a measure of the relative significance of liquidity in the risk management context. In addition, it can be used as a scaling factor with which price risk would need to be adjusted in order to correctly account for liquidity. We measure it relative to price risk, because absolute liquidity impact has little meaning by itself for our type of analysis.

In order to uncover the effect of tail correlation between liquidity and price, we define liquidity cost risk as the relative worst liquidity cost

$$VaR^{\alpha}_{liquidity}(q) = 1 - exp(l^{\alpha}_{t,\Delta t}(q)) \tag{10}$$

with $l^{\alpha}_{t,\Delta t}$ being the empirical percentile of the continuous liquidity discount. This is the maximum percentage loss due to liquidity cost at an $(1-\alpha)$-percent confidence level.

We can now apply a further decomposition of total risk and define the correlation factor $\kappa(q)$ as residual of

$$VaR_{total}(q) = VaR_{price} + VaR_{liquidity}(q) + \kappa(q) \times VaR_{liquidity}(q) \tag{11}$$

Naturally, this is just a further decomposition of the liquidity impact

$$\lambda(q) = \frac{VaR_{liquidity}}{VaR_{price}}(1 + \kappa(q)) \tag{12}$$

$\kappa(q)$ measures the tail correlation factor between mid-price return and liquidity cost, the proportion of liquidity risk, that is diversified away due to tail correlation. If tail correlation is perfect, $\kappa(q)$ is zero and worst mid-prices and worst liquidity costs can be added to get total risk.[34] If there is some diversification between cost and price, $\kappa(q)$ will become negative.

The liquidity impact $\lambda(q)$ contains the following conceptual components. First, it contains the mean liquidity discount for the position of size q - in contrast to other approaches. This is suitable as positions are usually valued at mid-prices already neglecting mean liquidity costs. Second, it includes negative deviations from the mean cost as measured by volatility and higher moments. Third, possible diversification effects between price and liquidity are included and reduce liquidity risk. If liquidity cost and mid-prices have a less than perfect, negative tail correlation ($\kappa(q) < 0$), a liquidity risk estimate based on the α-percentile of the liquidity cost distribution as in (10) will be incorrectly higher than based on the net-return distribution as in (9).

2.2.4 Interpretation of time horizon

The time horizon in the VaR framework is usually the time required to orderly liquidate an asset. It is differentiated between asset classes but usually assumed constant within one asset class such as stocks.[35]

We would like to stress that in the framework presented above, the time horizon Δt gets a more specific interpretation than usual. If we assume, for example, a standard 10-day period ($\Delta t = 10$), total risk measure (8) calculates a 10-day risk forecast, which is the time for management to decide and react. At day 10 the stock position will be instantly liquidated.

This interpretation is consistent with a general view on "orderly liquidation", where the time required comprises management reaction time as well as the liquidation time. It stands, however, in slight contrast to a more narrow view of "orderly

[34] This corresponds, for example, to the assumption and approach of Bangia et al. (1999).
[35] Cp. for example Jorion (2001), p. 24.

liquidation" that the time horizon of 10 days represents the period, during which a position is continuously liquidated.

Both interpretations are, however, valid in certain situations. In a situation, where very large positions can be liquidated without much time pressure, a continuous liquidation over a certain time period is valid. This is also the situation, where optimal trading strategies can be applied to maximize the net sales proceeds. In our framework, we are looking at a situation characterized in 2.1.2, which justifies instant liquidation. If we look at impatient traders or equivalently at the worst case, we do not allow for mitigation of some of the liquidity cost by allowing continuous liquidation. In such a case, "orderly liquidation" needs to be more generally defined and our approach is suitable.

3 Empirical results

In the empirical part, section 3.1 describes our data set, 3.2 provides some market background to our analysis. Section 3.3 presents our empirical results and 3.4 contains our robustness tests.

3.1 Description of data

Our sample consists of 5.5 years of daily XLM data (July 2002 to January 2008) for all 160 stocks in the four major German stock indices (DAX, MDAX, SDAX, TecDAX).[36] In total, we therefore cover a market capitalization of approximately € 1.2 trillion, which represents the largest part of the market capitalization in Germany.[37] As far as we know, this is the most representative sample on weighted spread available to academia.

We received XLM data for all days, where a stock was included in one of the four indices.[38] Daily values are calculated by Xetra as the equal-weighted average of all available by-minute data points.[39] XLM(q) comprises for each day the weighted spread for 10 standardized order sizes q. Standardized order size reach from € 25.000 to € 5 million in the DAX and from € 10.000 to € 1 million in all other indices. In

[36]The DAX contains the 30 largest publicly listed companies in Germany (by free-float market volume), the MDAX the subsequent 70 largest before 24.03.2003 and 50 largest thereafter and the SDAX the following 50 largest. The TecDAX, introduced during the sample period on 24.03.2003, comprises the 30 largest technology stocks.

[37]As of 1/2008.

[38]Therefore, our sample is non-constant containing 275 different stocks, but only 160 stocks at one point in time.

[39]This comprises a maximum of 1,060 measurements during continuous trading.

addition to XLM data, we obtained the day-closing bid-ask-spread s at the Xetra trading system from Datastream.

Three stocks were excluded from the analysis due to missing XLM or Datastream data.[40] We also had to eliminate 408 XLM observations, where liquidity data were available outside the standardized volume class structure described above, to ensure that our estimates remain representative in each volume class.[41] These exclusions left 99.9% of the stock-days in the sample.[42]

In total, our remaining sample contains 1.8 million observations for the 1424 trading days. We break our total sample into four sub-samples, each containing the stocks of one index.

3.2 Market background

As background to our analysis, table 6 on page 31 summarizes market conditions during the sample period. Markets were bullish in the largest part of the sample period. We also captured the downturns in the second half of 2002 and the first month of 2008. Due to beginning and end-of-period declines, overall return was rather average at 8% p.a.. Naturally, market capitalization increased similar to returns. Market capitalization is several times larger in the DAX than in all other indices. MDAX contained the second largest average market capitalization stocks, followed by TecDAX and SDAX. Volatility exhibited a similar, but reversed pattern than returns. Due to the bullish period, our sample is probably rather positively biased.

Daily transaction volume strongly increased during the sample period, which is already a plausible indicator for improving liquidity. Transaction volume was largest in the DAX, in the other indices it was several magnitudes smaller. Contrary to the general positive trend, transaction volume in the TecDAX remained steady after its initiation in 2003 and exhibits a level slightly lower than the MDAX. SDAX transaction volume was again several times smaller than in MDAX or TecDAX. The high diversity in transaction volumes underlines the representativeness of our sample.

[40] Procon Multimedia (in SDAX between 10/2002 and 03/2003) and Medisana (in SDAX between 12/2002 and 03/2003). Data could not be obtained for Sparks Networks (in SDAX between 06/2004 and 12/2005), because it was not available in Datastream anymore.

[41] Less than 0.01% of all observations were available for connected periods of less than seven days. We assume that the automatic calculation routine of the Xetra computer was extended to non-standard order sizes during trial periods.

[42] 323.670 of the total of 323.953 stock-days.

3.3 Liquidity impact and its components

In this section, we analyze the significance of liquidity in standard risk measures and its components. We will not discuss absolute risk levels in detail. The interested reader will find estimates of absolute price risk and absolute total risk in the appendix.

3.3.1 Magnitude of liquidity impact

As a starting point, we look at the total impact of liquidity $\lambda(q)$ on risk in a standard 10-day, 99% confidence-level VaR-setting according to equation (9). These parameters are typically used in a Basel II framework.[43] Table 7 on page 32 presents statistics on the overall liquidity impact $\lambda(q)$ by order size and index at a horizon often used in risk management systems.

On average over all stocks and across all order sizes, total risk - including liquidity risk - is 10% higher than price risk alone. DAX is generally the index with the lowest liquidity risk, while MDAX and TecDAX are second. SDAX consistently shows the highest liquidity impact levels across all order sizes. This finding is consistent with trading volumes and market values discussed in section 3.2.

There is strong variation in liquidity impact between indices and within indices as indicated by standard deviations. Variation is of the same order of magnitude than the level. Impact is practically zero ($\leq$1%) in small order sizes of the DAX ($<$€ 250 thsd.). Liquidity impact can easily rise above 20% in large stock positions of the DAX or medium stock positions in small stocks. In an average € 1 million SDAX-positions, liquidity impact on risk rises to 30% of price risk at a 10-day horizon.

Especially interesting is the liquidity impact calculated with spread as revealed in the min-column.[44] Impact remains rather small across all stocks and comparable to the liquidity impact measured with XLM(10) and XLM(25) respectively. In SDAX and TecDAX it is slightly higher than in the smallest XLM bracket. Since median risk levels are comparable, this effect is probably due to few outliers as XLM and spread data come from two different databases.

Liquidity impact generally increases with order size.[45] To more systematically analyze this size effect, we separately estimated the impact of doubling order size

[43]Cp. Dowd (2001), p.51.

[44]This corresponds to the risk measurement approach suggested by Bangia et al. (1999) applied to stocks.

[45]The decrease in the average SDAX position between € 250 thsd. and € 500 thsd. results from a non-constant sample effect. Large SDAX positions were continuously tradable only in later years. Therefore, risk estimates for large SDAX positions are calculated on a more liquid period depressing liquidity impacts compared to more continuously traded small positions.

on $\lambda(q)$ in percent in the last column. To do so, we regress the log row statistics on log order size including a constant intercept.[46] Size impact is the coefficient on log-size and indicates the curvature of the price impact function. It specifically investigates into the importance of price impact data in contrast to spread data only and abstracts from the different levels in liquidity risk between indices. Generally, the estimated price impact statistic is positive but smaller than one, which shows, that the liquidity impact (risk) function is concave.[47] The price impact is larger in the DAX, than in the other indices. Here, the difference between small, liquid and larger, less-liquid positions is especially pronounced. With size impact of 0.78, liquidity impact almost doubles in the DAX when doubling order size. In the other indices, liquidity impact is already large at small positions - hence the lower curvature. All size impacts are statistically significant at the 1%-level. The economically large size-impact statistic underlines the importance of using order book information beyond the spread for risk estimation - even in the DAX.

These results have important consequences for risk estimation techniques. First, we find that liquidity is an important component in total risk, especially in larger order sizes, where the price impact estimation error relative to price risk rises up to 30% at 10-day horizons. Second, estimating liquidity risk with spread data is no valid alternative, as liquidity risk impact in this size class is very small and strongly increases with size. Third, large variations indicate that constant scaling of price risk across all stocks, "hair cuts", are probably insufficient and liquidity has to be accounted for specifically for each stock.

3.3.2 Correlation effect

Next, we would like to specifically look into the tail correlation between mid-price return and liquidity cost. A correlation factor $\kappa(q)$ of zero corresponds to perfect tail correlation between liquidity and mid-price return. It mirrors the case that liquidity costs are highest when prices are lowest. Table 8 on page 33 shows the results based on 10-day, 99% VaR according to (11). Mean correlation factors ranges between 40% and 60% of liquidity risk. On average, 60% of the liquidity risk is diversified away. The negative correlation factor reveals that large, illiquid positions get more liquid in crises. Stock market crashes seem to attract liquidity, which allows to liquidate less-liquid positions more cost-efficiently, however at lower prices. Since over half of

[46] Ordinary least-squared regression equation is $log(Stat(q)) = c + log(q) + \epsilon$, with stat being the row statistic and c a constant intercept.

[47] This is consistent as already the price impact cost function is empirically found to be concave; cp. Hasbrouck (1991); Hausman et al. (1992).

the liquidity risk is diversified away, liquidity risk would be overestimated by about 100% at larger sizes when neglecting correlation (cp. equation (12)).

Correlation factors are quite uniform across order sizes and indices at around 55-65%. Only in the DAX it is slightly lower at about 40%. Correlation plays an even larger role at the spread level, where it is consistently higher than in larger order sizes. This underlines the different dynamics between the spread, quoted by market makers, and weighted spread, which emerges from free market competition. Cross-sectional standard deviation is also quite constant. The size-independent nature is underlined by the statistically and economically insignificant price impact statistic.[48]

The $\kappa(q)$-statistic should be treated with care. The effect of correlation on total risk is substantial only if the liquidity risk is also substantial (cp. equation (12)). As liquidity risk is quite low at small positions the overall error remains small and the violation is less critical.

Overall, these empirical results refute the common assumption of perfect tail correlation, i.e. that it is reasonable to simple add up price and liquidity risk. Doing so would overestimate total risk, especially in large, more illiquid order sizes. These results resolve the discussion, whether the perfect tail correlation assumption is valid or not. Our representative, empirical results are in line with the argument of Francois-Heude and Van Wynendaele (2001), who criticize the perfect correlation assumption of Bangia et al. (1999). However, the overall effect of this assumption remains small if the liquidity impact is small in total. It might also be different in other assets like currencies, which were analyzed by Bangia et al. (1999), but we see no a priori reason why this should be the case. We also hypothesize that correlation effects should be similar for other liquidity cost measures, because they proxy for the same phenomenon. Overall, our results indicate, that tail correlation is important and should be taken into account in illiquid stock positions.

3.3.3 Liquidity impact at shorter horizons

Risk on a 10-day horizon calculated above, provides a comparable reference to the standard statistics usually requested by financial regulators. However, as noted already in section 2.2.2, when correctly and directly accounting for liquidity risk, the 10-day horizon gets the notion of management reaction time instead of liquidation time. In order to stick to the original intention behind VaR, what a portfolio is worth in the worst case, we also calculate VaR at a 1-day horizon. This statistic is also more comparable to the intraday results available so far.

[48]Estimated in a linear regression of the distribution statistic on size.

Table 9 shows the liquidity impact $\lambda(q)$ for a 1-day, 99% VaR according to (9). As expected, the relative liquidity impact magnifies when shortening horizons, because price risk is reduced while absolute liquidity risk remains unchanged. The structure between indices remains unchanged. While still being negligible in small DAX positions, total risk including liquidity is almost double the price risk for large positions. Average € 1 million SDAX positions have a >90% liquidity risk impact. Even in some small positions, liquidity plays a substantial role with liquidity impact surpassing 10% in the SDAX for small position sizes.

The size-impact statistic reveals a very similar curvature in magnitude in the daily compared with the 10-day case. All size impacts are statistically significant at the 1% level. Correlation effects are similar in structure but larger in magnitude when compared to the 10-day horizon.[49] Our results are comparable to the 2-30% range found in other studies.[50]

3.4 Robustness tests

3.4.1 Effect of using the expected short-fall measure

Recently, literature has discussed coherent risk measures as alternative to Value-at-Risk to overcome the shortfalls of VaR like non sub-additivity.[51] This raises the question, if our results would change significantly when switching to a different risk measure. To test if our results are robust or specific to the VaR, we calculate expected shortfall,[52] which is the expected loss in the worst α-percent of the cases. We continue to use our basic approach detailed in 2.2.2 on page 8, but we replace VaR with expected shortfall (ES) defined as follows.

$$ES^{\alpha,\Delta t} = E(r|r < r^{\alpha}) \tag{13}$$

When we calculate risk based on expected shortfall instead of value-at-risk as displayed in table 10 effects of order size get accentuated. Generally speaking, results are structurally similar when measuring risk as ES compared to VaR. While total risk estimates increase, the impact of liquidity is comparable even in the tail of the distribution. Our methodology and results are therefore quite robust to a change to the expected shortfall risk measure.

[49] Results available on request.
[50] Cp. Francois-Heude and Van Wynendaele (2001); Giot and Grammig (2005); Angelidis and Benos (2006).
[51] Cp. Artzner et al. (1997); Acerbi and Scandolo (2007).
[52] Also called 'conditional value-at-risk' or 'expected tail loss'.

3.4.2 Effects of time variation

As further robustness test, we calculate monthly, rolling estimates of lambda to counter concerns that our results are due to the long estimation period.[53] This test also addresses any concerns for non-constant-sample bias, because we calculate risk estimates only on stocks included in the index due to data availability. Because empirical percentiles cannot be calculated on monthly samples of daily data, we chose a straight-forward mean-variance estimation procedure. For each date, we calculate the 20-day backward variance σ_r of continuous price return and assume that daily expected return is zero. Relative price risk on a 99% confidence level is then defined as

$$VaR^{1\%}_{price} = 1 - exp\left(-2.33 \times \sigma_r\right) \tag{14}$$

Similarly, we calculated liquidity-adjusted total risk with the mean μ_{rnet} and variance σ_{rnet} of 20-day backward net-return distribution

$$VaR^{1\%}_{total}(q) = 1 - exp\left(\mu_{rnet}(q) - 2.33 \times \sigma_{rnet}(q)\right) \tag{15}$$

with net returns calculated according to equation (6). We then calculate the liquidity impact $\lambda(q)$ according to equation (9). Neglect of negative skewness and high kurtosis (fat tails) makes this procedure simple, but might underestimate risk. Due to the underestimation, absolute values need to be treated with care, but are still - as lower bound - a suitable indicator for the time variation of the liquidity impact on risk, especially if higher moments are fairly constant.

Results for $\lambda(q)$ on the basis of a 10-day, 99% VaR according to (9) and (15) are displayed in table 11. The impact of liquidity on risk has generally declined over time across all indices. In all years, the liquidity impact strongly increased with order size as the size-impact statistic reveals. Our prior finding of the index rank (DAX, MDAX / TecDAX, SDAX) is confirmed and stable over time. TecDAX, however, was shortly more liquid after its initiation in 2003 until 2004. Although to be interpreted with care, the liquidity impact probably remained non-negligible during the low-risk period from 2006-2007. The impact of liquidity on total risk was certainly economically significant in the crises periods of 2002-2003 and in 2008.

Results for the whole panel ('all') have to be treated with care, because they are distorted by the non-constant sample effect. Over the years, the liquidity of less-liquid stocks strongly improved, which made their liquidity cost data increasingly available. As consequence, less-liquid, high-cost stocks are increasingly included in the sample, which increases the average risk estimate. However, individual year

[53] Rolling total risk estimates are shown in the appendix.

estimates have almost no sample bias and underline, that liquidity impact is economically significant.

If skewness and kurtosis would be included, these findings are also likely to get confirmed, as the one-time liquidity cost deduction will probably introduce additional skewness, which keeps the relation between price and liquidity risk valid. Overall, this confirms that liquidity price impact is economically significant enough to encourage integration into risk measurement systems.

3.4.3 Effects of portfolio diversification

We showed, that liquidity risk is economically significant when looking at individual stocks in the different indices. But does this result persist when looking at portfolios of stocks? If diversification between mid-prices of different stocks is larger than between liquidity of different stocks, liquidity impact might be substantially reduced.

To test the robustness of our results against effects of portfolio diversification, we calculated daily value-weighted index returns and determined liquidity impact $\lambda(q)$ based on a 10-day, 99% VaR according to (9). While our methodology does not use optimized position weights, a value weighted portfolio should show effects of diversification if there are any. Results are displayed in table (12). Estimates demonstrate, that liquidity impact on the portfolio level is of similar magnitude than on the average individual stock level (cp. table 7 on page 32). Especially in larger sizes, liquidity impact is increased at the portfolio level, e.g. it rises to 54% for the € 1 million position in the SDAX portfolio compared to 30% for the average individual stock position. This must be driven by larger liquidity commonality in larger sizes, i.e. diversification in liquidity between stocks decreases with larger sizes. Even for the all-stock portfolio liquidity impact levels are higher than for the average stock. Overall, our results are robust to diversification effects in stock portfolios.

4 Conclusion and outlook

In this paper, we modeled liquidity risk based on the weighted spread liquidity measure in a Value-at-Risk framework. The main advantage over existing approaches is the higher precision of the weighted spread, which calculates liquidity cost differentiated by order size, i.e. the price impact, from the limit order book.

We argued that weighted spread is a valid liquidity measure from a risk perspective in a wide range of situations, which we defined clearly. If we look at limit order book markets, where this type of data is available and from the perspective of institutional investors, for whom other direct trading costs are negligible, two situational assumptions are critical.

First, our approach works most precise for continuously tradable asset positions, for example for small to medium sized positions in developed stock markets. Positions cannot be too large, like block holdings, and markets have to be fairly liquid with few zero trading days. If this is not the case, forced execution delay can incur costs which we have neglected.[54]

Second, we assume that deliberate, strategic delay has no significant benefit, which renders optimal trading strategies useless.[55] This is a fair assumption in four possible cases. We can take a worst case perspective, e.g. because external restrictions require to close whole positions immediately. Any possible benefit from delay is then consciously ignored. From a risk perspective, strategic delay also remains with unrealizable benefit, if the optimal trading strategy is non-stable in crises situations and can therefore not provide any benefit on an expected basis. Further, if liquidity prices are efficient in fairly liquid markets, strategic delay has per definition no marginal benefit. Finally, optimal trading strategies are also useless, if the real or perceived (i.e. adverse) informational content of the trade is high and delay only increases the probability of adverse price movements.

This discussion shows that these cases cover a variety of situations. Overall, our approach is most valid for up to medium sized positions in generally continuously trading markets.

We then defined liquidity-adjusted Value-at-Risk of a specific position in a straight-forward manner based on net return, i.e. mid-price return less the weighted spread of the position. This definition avoids any distortion through correlation assumptions between liquidity and price return, which we analyzed separately.

Empirically, we find that impact of liquidity relative to price risk is small at small order sizes, especially at the spread level ($<$10% for 10-day, 99% VaR). However, it increases to 20-30% of price risk in larger sizes in illiquid indices as well as in the DAX. Results aggravate if we switch to daily VaR-horizons.

We also took a detailed look at tail correlation between liquidity and mid-price returns and showed that it is non-negligible. Liquidity risk would be overestimated by 100% if correlations are ignored. In the cases we identified above, where liquidity risk is an economically significant component of total risk, total risk will be

[54]We argue that this covers quite a range of assets, because even assets with low trading seem to get more continuously traded in crises situations, which makes our approach applicable.

[55]Optimal trading strategies try to minimize total liquidation costs by delaying parts of a transaction. The gain from lower liquidity cost of smaller order sizes is balanced against the additional price risk for the delayed part of the position in an optimal way.

severely overestimated if liquidity cost risk is simply added to existing risk measures. Therefore, several common approaches should be adapted to avoid this distortion.

We find that results are structurally similar when using expected shortfall instead of VaR risk measures. Our results are therefore transferable. To check the time robustness of these findings, we employ a monthly, rolling mean-variance estimation method. Results are confirmed. Results are also similar for portfolios of stocks, when portfolio diversification is accounted for.

Overall, we strongly advocate the use of weighted spread data like XLM to improve risk estimates. Liquidity constitutes a large part of total risk, especially in larger positions and at short horizons - even in more liquid market segments.

Several venues are still open for future research. Because we have used empirical ex-post risk measurement to avoid any distortion by a specific choice of parametrization or forecasting, appropriate techniques will need to be selected. It will also be helpful to test the prceision of our estimates against real transaction data. Future research can also address two assumptions to extend this approach to a larger realm of situations and assets. The empirical integration of delay risk is still unsolved as is the empirical questions when liquidity prices are efficient. Further insights into when and under which circumstances delay occurs will also help to advance this line of thinking. Another simplifying advance would be a method which directly integrates liquidity-price-correlation in a parametric approach when adding liquidity to price risk. Tackling these research areas will help to further advance the integration of liquidity into risk measurement.

5 Appendix

Index	Mean	Median	Std. Dev.	Obs.
DAX	16,3%	14,7%	4,4%	43.767
MDAX	17,2%	15,8%	8,1%	76.750
SDAX	19,5%	17,7%	7,3%	72.373
TECDAX	24,3%	22,6%	9,2%	38.070
All	18,9%	17,4%	7,9%	230.960

Table 1: Price risk (VaR, 10 day, 99%)

This table contains distribution statistics on price risk calculated as 10-day, 99% VaR according to equation (5).

Index	Mean	Median	Std. Dev.	Obs.
DAX	5,6%	5,5%	1,1%	43.710
MDAX	6,1%	5,7%	2,2%	71.458
SDAX	7,2%	6,5%	3,0%	72.313
TECDAX	8,2%	7,9%	1,8%	36.801
All	6,7%	6,0%	2,5%	224.282

Table 2: Price risk (VaR, 1 day, 99%)

This table contains distribution statistics on price risk calculated as 1-day, 99% VaR according to equation (5).

Total risk, VaR(10 day, 99%)		Order size (in thsd. Euro)																Size
abs., liquidity-adjusted in %		Min	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All	impact
DAX	Mean	17%	n/a	17%	17%	n/a	17%	n/a	17%	17%	n/a	18%	20%	20%	21%	21%	18%	0.82 ***
	Median	15%	n/a	15%	15%	n/a	15%	n/a	15%	16%	n/a	17%	18%	18%	17%	18%	17%	0.65 ***
	Std. Dev.	4%	n/a	5%	5%	n/a	5%	n/a	5%	5%	n/a	5%	8%	7%	7%	8%	6%	0.56 ***
	Obs.	42,129	n/a	42,710	42,710	n/a	42,710	n/a	42,710	42,706	n/a	42,663	41,716	39,970	38,225	36,343	412,463	
MDAX	Mean	18%	19%	20%	20%	20%	20%	20%	22%	23%	22%	24%	n/a	n/a	n/a	n/a	21%	0.98 ***
	Median	17%	17%	17%	17%	17%	17%	18%	19%	21%	20%	20%	n/a	n/a	n/a	n/a	18%	0.91 ***
	Std. Dev.	8%	9%	9%	9%	9%	9%	9%	9%	10%	10%	11%	n/a	n/a	n/a	n/a	9%	0.36 ***
	Obs.	73,234	74,779	74,574	73,930	73,291	72,671	71,357	68,520	60,784	53,461	46,741	n/a	n/a	n/a	n/a	670,108	
SDAX	Mean	21%	20%	21%	22%	24%	24%	25%	27%	28%	30%	35%	n/a	n/a	n/a	n/a	24%	2.87 ***
	Median	19%	18%	18%	20%	21%	21%	22%	23%	26%	29%	30%	n/a	n/a	n/a	n/a	21%	2.70 ***
	Std. Dev.	8%	7%	7%	8%	9%	9%	9%	10%	9%	10%	16%	n/a	n/a	n/a	n/a	9%	1.30 **
	Obs.	70,048	69,081	64,254	60,824	57,798	54,871	49,291	39,780	23,114	13,985	8,363	n/a	n/a	n/a	n/a	441,361	
TECDAX	Mean	25%	21%	22%	22%	22%	22%	23%	24%	26%	26%	30%	n/a	n/a	n/a	n/a	23%	1.63 ***
	Median	23%	19%	20%	20%	20%	20%	21%	22%	24%	24%	28%	n/a	n/a	n/a	n/a	20%	1.63 ***
	Std. Dev.	9%	8%	8%	9%	8%	9%	9%	9%	10%	9%	10%	n/a	n/a	n/a	n/a	9%	0.41 ***
	Obs.	36,980	37,133	37,133	37,126	37,075	36,949	36,299	33,958	26,995	20,641	16,031	n/a	n/a	n/a	n/a	319,340	
All	Mean	20%	n/a	20%	20%	n/a	21%	n/a	22%	23%	n/a	23%	n/a	n/a	n/a	n/a	21%	0.96 ***
	Median	18%	n/a	18%	18%	n/a	19%	n/a	20%	21%	n/a	21%	n/a	n/a	n/a	n/a	19%	0.96 ***
	Std. Dev.	8%	n/a	8%	8%	n/a	9%	n/a	9%	9%	n/a	11%	n/a	n/a	n/a	n/a	9%	0.68 **
	Obs.	222,391	n/a	218,671	214,590	n/a	207,201	n/a	184,968	153,599	n/a	113,798	n/a	n/a	n/a	n/a	1,843,272	

Table 3: Absolute liquidity-adjusted total risk (VaR, 10 day, 99%)

This tables shows cross-sectional statistics on empirical, absolute total risk including a liquidity adjustment according to equation (8); min-column measures risk at minimum spread level; all-column is average over all standardized order sizes, i.e. without minimum; size impact is the increase in risk in percentage points when doubling order size, measured as coefficient in 10^{-2} of log-size in a regression of the distribution statistic on log-size including an intercept; * indicates 10%, ** 5% and *** 1% confidence level of being different from zero based on a two-tailed test.

Total risk, VaR(1 day, 99%) abs., liquidity-adjusted in %		Order size (in thsd. Euro)																Size impact
		Min	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All	
DAX	**Mean**	6%	n/a	6%	6%	n/a	6%	n/a	6%	6%	n/a	7%	9%	10%	10%	10%	7%	0.88 ***
	Median	6%	n/a	6%	6%	n/a	6%	n/a	6%	6%	n/a	6%	8%	8%	9%	9%	6%	0.58 ***
	Std. Dev.	1%	n/a	1%	1%	n/a	1%	n/a	1%	1%	n/a	2%	5%	4%	4%	4%	3%	0.73 ***
	Obs.	42,129	n/a	42,710	42,710	n/a	42,710	n/a	42,710	42,706	n/a	42,663	41,716	39,970	38,225	36,343	412,463	
MDAX	**Mean**	6%	6%	6%	6%	7%	7%	7%	8%	10%	10%	11%	n/a	n/a	n/a	n/a	8%	1.16 ***
	Median	6%	6%	6%	6%	6%	7%	7%	8%	10%	9%	9%	n/a	n/a	n/a	n/a	7%	0.90 ***
	Std. Dev.	2%	2%	3%	2%	2%	2%	3%	3%	4%	4%	5%	n/a	n/a	n/a	n/a	3%	0.57 ***
	Obs.	73,234	74,779	74,574	73,930	73,291	72,671	71,357	68,520	60,784	53,461	46,741	n/a	n/a	n/a	n/a	670,108	
SDAX	**Mean**	9%	8%	8%	9%	11%	12%	12%	14%	15%	17%	22%	n/a	n/a	n/a	n/a	11%	2.73 ***
	Median	7%	7%	8%	9%	9%	10%	11%	11%	13%	15%	16%	n/a	n/a	n/a	n/a	9%	1.91 ***
	Std. Dev.	5%	4%	3%	5%	7%	7%	7%	10%	7%	9%	16%	n/a	n/a	n/a	n/a	7%	1.99 ***
	Obs.	70,048	69,081	64,254	60,824	57,798	54,871	49,291	39,780	23,114	13,985	8,363	n/a	n/a	n/a	n/a	441,361	
TECDAX	**Mean**	9%	7%	7%	8%	8%	9%	9%	10%	12%	13%	15%	n/a	n/a	n/a	n/a	9%	1.69 ***
	Median	8%	7%	7%	7%	8%	8%	9%	10%	12%	11%	13%	n/a	n/a	n/a	n/a	8%	1.37 ***
	Std. Dev.	3%	2%	2%	2%	2%	3%	4%	4%	5%	5%	6%	n/a	n/a	n/a	n/a	4%	0.94 ***
	Obs.	36,980	37,133	37,133	37,126	37,075	36,949	36,299	33,958	26,995	20,641	16,031	n/a	n/a	n/a	n/a	319,340	
All	**Mean**	7%	n/a	7%	7%	n/a	8%	n/a	9%	10%	n/a	11%	n/a	n/a	n/a	n/a	9%	1.14 ***
	Median	7%	n/a	7%	7%	n/a	7%	n/a	8%	9%	n/a	9%	n/a	n/a	n/a	n/a	7%	0.65 ***
	Std. Dev.	4%	n/a	3%	3%	n/a	5%	n/a	6%	5%	n/a	7%	n/a	n/a	n/a	n/a	5%	1.03 **
	Obs.	222,391	n/a	218,671	214,590	n/a	207,201	n/a	184,968	153,599	n/a	113,798	n/a	n/a	n/a	n/a	1,843,272	

Table 4: Absolute liquidity-adjusted total risk (VaR, 1 day, 99%)

This tables shows cross-sectional statistics on empirical, absolute total risk including a liquidity adjustment according to equation (8); min-column measures risk at minimum spread level; all-column is average over all standardized order sizes, i.e. without minimum; size impact is the increase in risk in percentage points when doubling order size, measured as coefficient in 10^{-2} of log-size in a regression of the distribution statistic on log-size including an intercept; * indicates 10%, ** 5% and *** 1% confidence level of being different from zero based on a two-tailed test.

L-adj. VaR(10d, 99%) in %		Order size (in thsd. Euro)																Size impact
		Min	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All	
DAX	2002	24%	n/a	24%	24%	n/a	24%	n/a	24%	25%	n/a	26%	28%	29%	30%	31%	26%	1.32 ***
	2003	16%	n/a	17%	17%	n/a	17%	n/a	17%	17%	n/a	18%	20%	21%	21%	22%	18%	0.94 ***
	2004	9%	n/a	9%	9%	n/a	9%	n/a	10%	10%	n/a	10%	11%	12%	12%	13%	10%	0.70 ***
	2005	8%	n/a	8%	8%	n/a	8%	n/a	8%	8%	n/a	8%	9%	9%	9%	10%	9%	0.29 ***
	2006	9%	n/a	9%	9%	n/a	9%	n/a	10%	10%	n/a	10%	10%	10%	10%	11%	10%	0.29 ***
	2007	10%	n/a	10%	10%	n/a	10%	n/a	10%	10%	n/a	10%	11%	11%	11%	11%	11%	0.22 ***
	2008	16%	n/a	15%	15%	n/a	15%	n/a	15%	15%	n/a	15%	15%	16%	16%	16%	15%	0.18 **
	All	12%	n/a	12%	12%	n/a	12%	n/a	12%	12%	n/a	13%	13%	13%	14%	14%	13%	0.37 ***
	Δ2002-2008 [a.]	-12%	n/a	-12%	-12%	n/a	-12%	n/a	-12%	-13%	n/a	-14%	-15%	-16%	-16%	-17%	-14%	-0.96
MDAX	2002	21%	21%	21%	22%	22%	23%	24%	26%	28%	27%	27%	n/a	n/a	n/a	n/a	23%	1.69 ***
	2003	16%	16%	16%	16%	16%	16%	17%	19%	21%	23%	24%	n/a	n/a	n/a	n/a	17%	1.87 ***
	2004	11%	12%	12%	12%	12%	12%	13%	13%	16%	17%	19%	n/a	n/a	n/a	n/a	13%	1.46 ***
	2005	11%	11%	11%	11%	11%	11%	11%	11%	13%	14%	15%	n/a	n/a	n/a	n/a	12%	0.80 ***
	2006	12%	12%	12%	12%	12%	12%	13%	13%	13%	14%	15%	n/a	n/a	n/a	n/a	13%	0.59 ***
	2007	13%	13%	13%	13%	13%	13%	14%	14%	14%	15%	15%	n/a	n/a	n/a	n/a	14%	0.49 ***
	2008	20%	19%	19%	20%	20%	20%	20%	20%	20%	21%	22%	n/a	n/a	n/a	n/a	20%	0.50 ***
	All	14%	14%	14%	14%	14%	14%	14%	15%	16%	16%	17%	n/a	n/a	n/a	n/a	15%	0.65 ***
	Δ2002-2008 [a.]	-7%	-7%	-7%	-8%	-8%	-8%	-9%	-11%	-13%	-11%	-10%	n/a	n/a	n/a	n/a	-8%	-1.12
SDAX	2002	24%	23%	27%	35%	41%	47%	47%	32%	4%	n/a	n/a	n/a	n/a	n/a	n/a	28%	-1.45
	2003	19%	19%	20%	21%	23%	25%	28%	31%	35%	28%	29%	n/a	n/a	n/a	n/a	22%	2.98 ***
	2004	14%	15%	15%	16%	18%	20%	21%	25%	31%	31%	33%	n/a	n/a	n/a	n/a	18%	4.49 ***
	2005	13%	13%	13%	13%	14%	15%	16%	18%	24%	27%	30%	n/a	n/a	n/a	n/a	15%	3.83 ***
	2006	14%	14%	14%	15%	15%	15%	16%	18%	23%	27%	31%	n/a	n/a	n/a	n/a	17%	3.53 ***
	2007	15%	15%	15%	15%	15%	16%	16%	17%	20%	23%	26%	n/a	n/a	n/a	n/a	17%	2.22 ***
	2008	20%	22%	22%	22%	21%	21%	22%	24%	28%	33%	35%	n/a	n/a	n/a	n/a	23%	2.80 ***
	All	16%	16%	16%	16%	17%	17%	18%	20%	23%	25%	28%	n/a	n/a	n/a	n/a	18%	2.59 ***
	Δ2002-2008 [a.]	-8%	-7%	-11%	-18%	-24%	-29%	-28%	-13%	19%	-3%	-1%	n/a	n/a	n/a	n/a	-11%	3.95
TecDAX	2002	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
	2003	21%	21%	21%	22%	22%	23%	24%	25%	27%	29%	28%	n/a	n/a	n/a	n/a	23%	1.87 ***
	2004	17%	17%	17%	17%	17%	18%	19%	20%	22%	24%	25%	n/a	n/a	n/a	n/a	18%	1.86 ***
	2005	13%	13%	13%	13%	13%	14%	14%	15%	17%	20%	21%	n/a	n/a	n/a	n/a	15%	1.76 ***
	2006	15%	15%	15%	15%	15%	15%	16%	16%	18%	20%	23%	n/a	n/a	n/a	n/a	16%	1.49 ***
	2007	16%	16%	16%	16%	16%	16%	17%	17%	18%	20%	21%	n/a	n/a	n/a	n/a	17%	1.03 ***
	2008	24%	23%	23%	23%	23%	23%	23%	24%	26%	26%	29%	n/a	n/a	n/a	n/a	24%	1.11 ***
	All	16%	16%	16%	16%	17%	17%	17%	18%	20%	21%	23%	n/a	n/a	n/a	n/a	18%	1.45 ***
	Δ2002-2008 [a.]	-5%	-5%	-5%	-5%	-5%	-6%	-6%	-7%	-8%	-7%	-6%	n/a	n/a	n/a	n/a	-5%	-0.54
All	2002	22%	n/a	23%	23%	n/a	24%	n/a	25%	26%	n/a	26%	n/a	n/a	n/a	n/a	24%	0.96 ***
	2003	18%	n/a	18%	18%	n/a	19%	n/a	20%	21%	n/a	21%	n/a	n/a	n/a	n/a	20%	0.96 ***
	2004	13%	n/a	13%	14%	n/a	15%	n/a	15%	16%	n/a	15%	n/a	n/a	n/a	n/a	15%	0.61 **
	2005	11%	n/a	11%	11%	n/a	12%	n/a	13%	14%	n/a	14%	n/a	n/a	n/a	n/a	12%	0.96 ***
	2006	13%	n/a	13%	13%	n/a	13%	n/a	14%	15%	n/a	16%	n/a	n/a	n/a	n/a	14%	0.84 ***
	2007	14%	n/a	14%	14%	n/a	14%	n/a	15%	16%	n/a	17%	n/a	n/a	n/a	n/a	15%	0.84 ***
	2008	20%	n/a	20%	20%	n/a	20%	n/a	21%	22%	n/a	23%	n/a	n/a	n/a	n/a	21%	0.84 ***
	All	14%	n/a	14%	15%	n/a	15%	n/a	16%	16%	n/a	17%	n/a	n/a	n/a	n/a	15%	0.72 ***
	Δ2002-2008 [a.]	-8%	n/a	-8%	-9%	n/a	-9%	n/a	-9%	-10%	n/a	-9%	n/a	n/a	n/a	n/a	-9%	-0.30

Table 5: Liquidity-adjusted total risk on 10 day horizon (rolling estimate, VaR, 10-day, 99%)

Table shows liquidity-adjusted total risk by sub-sample according to equation (9) calculated with a rolling mean-variance estimation; a. statistic shows absolute change between 2003 and 2008 when 2002 number not available; min-column measures risk at minimum spread level; all-column is average over all standardized order sizes, i.e. without minimum; size impact is the coefficient in 10^{-2} of log-size in a regression of the distribution statistic on log-size including an intercept; * indicates 10%, ** 5% and *** 1% confidence level of being different from zero based on a two-tailed test.

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Zusammenfassung (deutsch)

In diesem Artikel zeigen wir auf, wie Liquiditätsrisiko mit Hilfe der gewichteten Geld-Brief-Spanne in einem Standardrisikoansatz (Value-at-Risk) gemessen werden kann. Die gewichtete Geld-Brief-Spanne misst Liquiditätskosten gestaffelt nach Ordergröße und wird aus dem Limit-Order-Buch berechnet. Wir zeigen auf, dass dieses Liquiditätsmaß unter Risikoaspekten in einer Vielzahl von Situationen Kosten korrekt abbildet. Insbesondere wird der Kostenanstieg mit der Größe der Position, der sogenannte Preiseinfluss, präzise einbezogen.

Für unsere empirische Analyse verwenden wir einen bislang einzigartigen, repräsentativen Datensatz des Xetra Liquiditätsmasses (XLM). Er enthält die tägliche gewichtete Geld-Brief-Spanne für 160 Aktien über die letzten 5,5 Jahre und wurde uns freundlicherweise von der Deutschen Börse AG zur Verfügung gestellt.

Wir können zeigen, dass Liquiditätsrisiko traditionelle Preisrisikomaße um über 25% erhöht, gemessen auf einen langen 10-Tages Vorhersagehorizont. Selbst im liquiden DAX kann dieses Ausmaß für große Positionen beobachtet werden. Wechselt man zu täglichen Vorhersagehorizonten, so verdoppelt sich der Einfluss von Liquidität noch einmal. Die Resultate verdeutlichen, dass Liquiditätsrisiko so substantiell ist, dass es nicht vernachlässigt werden kann.

In einer weiteren Analyse legen wir dar, dass Liquiditätsrisiko nicht einfach zu Preisrisiko hinzuaddiert werden darf, wie dies von einigen Methoden vorgeschlagen wird. Da hohe Liquiditätskosten und niedrige Preise in Krisensituationen nicht gleichzeitig auftreten, wird in diesen Methoden das Risiko substantiell falsch gemessen. Liquiditätsrisiko wird um 100% überschätzt, wenn man die Korrelation zwischen Liquidität und Preis nicht beachtet. Mit unserer Messmethode berücksichtigen wir diese Problematik. Unsere Resultate sind robust, auch wenn man andere Risikomasse verwendet, verschiedene Zeiträume betrachtet oder Diversifikation in Portfolios einbezieht.

Insgesamt plädieren wir dafür, das Liquiditätsmaß 'gewichtete Geld-Brief-Spanne', wie z.B. XLM, wenn möglich in Risikomessungen zu verwenden. Es lässt sich einfach in bestehende Risikoansätze einbauen und besitzt zahlreiche Vorteile gegenüber bisherigen Risikomessmethoden.

Practicioner abstract

In this paper, we analyze a method to measure liquidity risk in a standard Value-at-Risk (VaR) framework. This method uses the weighted spread liquidity measure, which calculates the liquidity cost of transacting a position of a specific size.

Weighted spread is the average spread weighted by individual limit order sizes in the limit-order-book. It is superior to many liquidity measures, because it respects that liquidity cost rise sharply with the size of a position. Although weighted spread is calculated under the assumption, that a position is immediately executed as market order against the limit order book, it is shown to be precise in a large range of situations. Valid applications include small- to medium-sized positions in more liquid limit-order book markets, e.g. many developed stock markets.

Weighted spread can be manually calculated from intraday data or is readily available at daily frequencies from exchanges like Deutsche Börse AG. For our empirical analysis, we use a unique, representative data set of the Xetra Liquidity Measure (XLM) by Deutsche Börse, a daily sample of weighted spread for 160 stocks over the last 5.5 years.

We find liquidity risk to increase traditionally-measured price risk by over 25%, even at standard 10-day horizons. Also for liquid DAX stocks, the magnitude is similarly high when trading large positions. When switching to daily horizons, liquidity impact more than doubles. This economically highly significant magnitude of liquidity risk cannot be neglected. Therefore, we argue that liquidity risk needs to be taken into account, even for more liquid stocks and at larger forecast horizons.

We also show that liquidity risk cannot be simply added to existing price risk measures, a method proposed in several papers. Because high liquidity cost and low market prices do not occur simultaneously, adding both worst cases substantially overestimates liquidity risk, in our estimates by over 100%. Instead, these liquidity - price - correlations need to be integrated into risk measures, but it remains unclear up to now, how this could be done. Alternatively, risk can be calculated based on net return, mid-price return less liquidity cost, which is the approach we chose for this paper.

We also show, that our results are quite robust and do not structurally change when using the 'expected shortfall' risk measure or when looking at different sample periods. Results remain similar, too, when taking into account diversification effects within portfolios of stocks.

Overall, we strongly advocate the use of weighted spread measures like XLM in risk measurement - where applicable. Its integration in existing, standard frameworks is easy and provides substantial improvements over existing mark-to-market and other liquidity-adjusted risk measures.

Tables and figures

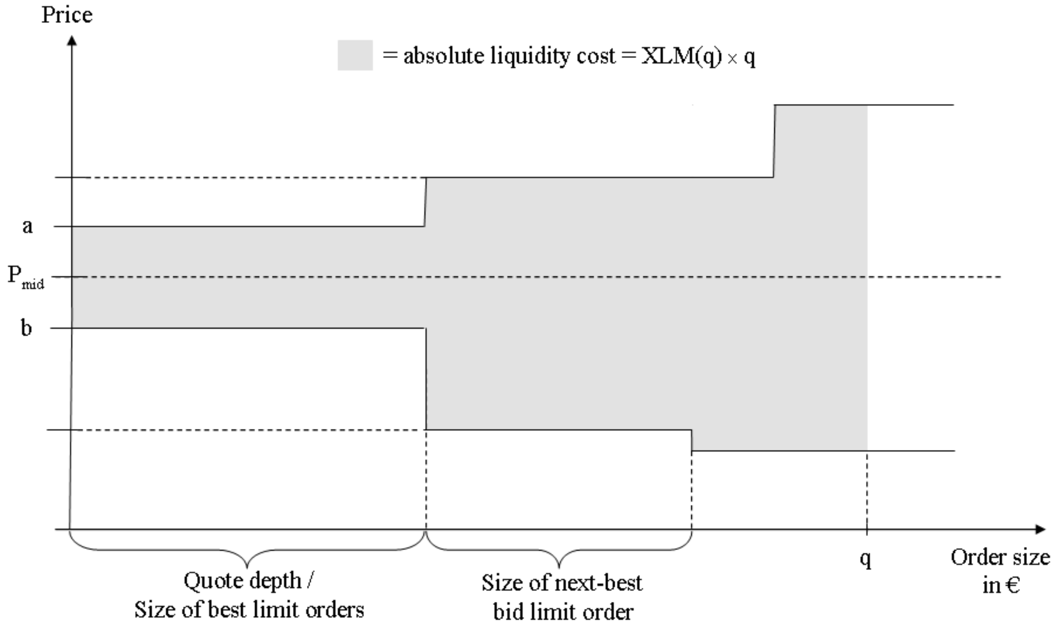

Figure 1: XLM as the area between the limit order curves

Figure 1 shows a graphical representation of the order book; P_{mid} is the mid-price of the bid-ask-spread, a is the ask price, b is the bid price, q is the size of the position in € mid-price value, XLM(q) is the weighted spread measuring ex-ante liquidity cost for a round-trip of size q.

Market segment overview	II/2002	2003	2004	2005	2006	2007	1/2008	Total period[a]
Average continuous period return [b]								
DAX	-52%	24%	6%	27%	20%	22%	-15%	6%
MDAX	-23%	39%	15%	25%	25%	-1%	-12%	12%
SDAX	-36%	35%	11%	28%	29%	4%	-14%	10%
TecDAX	n/a	52%	3%	26%	24%	32%	-25%	23%
Total	-35%	24%	10%	26%	24%	11%	-15%	8%
Average period return volatility (annualized) [c]								
DAX	64%	41%	22%	19%	23%	25%	51%	30%
MDAX	54%	39%	28%	26%	30%	35%	59%	35%
SDAX	65%	47%	35%	31%	36%	38%	58%	40%
TecDAX	n/a	54%	42%	31%	38%	44%	71%	42%
Total	60%	44%	32%	27%	32%	36%	59%	37%
Average free-float market capitalization in million Euro								
DAX	15.217	14.615	17.983	20.350	24.357	29.949	29.325	21.008
MDAX	1.043	1.330	1.940	2.537	3.734	3.797	3.121	2.453
SDAX	106	235	320	393	500	775	640	418
TecDAX	n/a	725	863	898	995	1.221	1.204	955
Total	3.639	3.483	4.319	4.998	6.154	7.379	7.009	5.160
Average daily transaction volume in thsd. Euro								
DAX	93.500	94.399	98.037	119.563	165.833	250.835	351.793	144.040
MDAX	1.384	2.297	4.035	6.242	11.034	18.243	22.351	7.557
SDAX	36	160	237	514	958	2.129	2.081	780
TecDAX	n/a	1.813	2.345	2.308	4.769	7.946	11.430	4.052
Total	20.431	19.543	20.268	25.206	35.797	54.891	75.739	31.020

Table 6: Market conditions during sample period

Table shows per-stock averages; a. annualized; b. Includes dividend returns, because price series are adjusted for corporate capital actions; c. volatility has been annualized with $\sqrt{250}$; All values equal-weighted.

λ(q), VaR(10 day, 99%) in % of price risk		Order size (in thsd. Euro)																Size
		Min	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All	impact
DAX	Mean	1%	n/a	1%	1%	n/a	1%	n/a	2%	4%	n/a	9%	16%	21%	24%	26%	10%	0.78 ***
	Median	1%	n/a	1%	1%	n/a	1%	n/a	1%	3%	n/a	6%	11%	19%	20%	25%	3%	0.78 ***
	Std. Dev.	1%	n/a	0%	0%	n/a	1%	n/a	2%	4%	n/a	8%	14%	16%	16%	18%	14%	0.84 ***
	Obs.	42,129	n/a	42,710	42,710	n/a	42,710	n/a	42,710	42,706	n/a	42,663	41,716	39,970	38,225	36,343	412,463	
MDAX	Mean	2%	2%	2%	3%	4%	5%	7%	11%	15%	19%	21%	n/a	n/a	n/a	n/a	8%	0.58 ***
	Median	2%	1%	2%	2%	3%	4%	5%	9%	11%	14%	17%	n/a	n/a	n/a	n/a	4%	0.62 ***
	Std. Dev.	2%	3%	3%	5%	4%	5%	6%	9%	18%	35%	63%	n/a	n/a	n/a	n/a	22%	0.62 ***
	Obs.	69,578	74,779	74,574	73,930	73,291	72,671	71,357	68,520	60,784	53,461	46,741	n/a	n/a	n/a	n/a	670,108	
SDAX	Mean	9%	6%	7%	10%	13%	16%	20%	23%	22%	22%	30%	n/a	n/a	n/a	n/a	14%	0.35 ***
	Median	3%	3%	4%	7%	8%	9%	11%	14%	19%	20%	23%	n/a	n/a	n/a	n/a	8%	0.44 ***
	Std. Dev.	52%	8%	11%	16%	20%	29%	44%	48%	17%	15%	34%	n/a	n/a	n/a	n/a	27%	0.23 *
	Obs.	69,988	69,081	64,254	60,824	57,798	54,871	49,291	39,780	23,114	13,985	8,363	n/a	n/a	n/a	n/a	441,361	
TECDAX	Mean	3%	1%	2%	3%	4%	5%	8%	11%	16%	18%	18%	n/a	n/a	n/a	n/a	7%	0.64 ***
	Median	1%	1%	1%	2%	3%	4%	6%	8%	13%	18%	16%	n/a	n/a	n/a	n/a	3%	0.66 ***
	Std. Dev.	5%	1%	1%	2%	4%	8%	7%	10%	18%	12%	13%	n/a	n/a	n/a	n/a	10%	0.66 ***
	Obs.	35,741	37,133	37,133	37,126	37,075	36,949	36,299	33,958	26,995	20,641	16,031	n/a	n/a	n/a	n/a	319,340	
All	Mean	4%	n/a	3%	5%	n/a	7%	n/a	12%	13%	n/a	17%	n/a	n/a	n/a	n/a	10%	0.44 ***
	Median	2%	n/a	2%	2%	n/a	3%	n/a	7%	10%	n/a	13%	n/a	n/a	n/a	n/a	4%	0.62 ***
	Std. Dev.	30%	n/a	7%	10%	n/a	16%	n/a	25%	16%	n/a	42%	n/a	n/a	n/a	n/a	20%	0.42 **
	Obs.	217,436	n/a	218,671	214,590	n/a	207,201	n/a	184,968	153,599	n/a	113,798	n/a	n/a	n/a	n/a	1,843,272	

Table 7: Liquidity impact on risk (VaR, 10 day, 99%)

Table shows cross-sectional statistics of lambda, which is the impact of liquidity in percent of price risk according to (9); min-column measures risk at minimum spread level; all-column is average over all standardized order sizes, i.e. without minimum; size impact is the coefficient of log-size regressed on the log distribution statistic including an intercept; * indicates 10%, ** 5% and *** 1% confidence level of being different from zero based on a two-tailed test.

κ(q) VaR(10 day, 99%)		Order size (in thsd. Euro)																Size
in % of liquidity risk		Min	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All	impact
DAX	**Mean**	-64%	n/a	-40%	-41%	n/a	-43%	n/a	-45%	-49%	n/a	-34%	-41%	-40%	-40%	-41%	-41%	0.00
	Median	-68%	n/a	-41%	-41%	n/a	-41%	n/a	-44%	-48%	n/a	-49%	-47%	-42%	-41%	-41%	-44%	0.00
	Std. Dev.	21%	n/a	15%	15%	n/a	16%	n/a	17%	18%	n/a	87%	47%	30%	24%	19%	36%	0.00
	Obs.	42,129	n/a	42,710	42,710	n/a	42,710	n/a	42,710	42,706	n/a	42,663	41,716	39,970	38,225	36,343	412,463	
MDAX	**Mean**	-70%	-62%	-62%	-61%	-63%	-64%	-63%	-63%	-64%	-60%	-59%	n/a	n/a	n/a	n/a	-62%	0.00 **
	Median	-72%	-66%	-66%	-62%	-63%	-67%	-64%	-61%	-62%	-63%	-60%	n/a	n/a	n/a	n/a	-63%	0.00 **
	Std. Dev.	15%	17%	17%	18%	16%	16%	17%	16%	18%	24%	17%	n/a	n/a	n/a	n/a	18%	0.00
	Obs.	69,578	74,779	74,574	73,930	73,291	72,671	71,357	68,520	60,784	53,461	46,741	n/a	n/a	n/a	n/a	670,108	
SDAX	**Mean**	-67%	-61%	-65%	-65%	-66%	-66%	-64%	-60%	-57%	-59%	-54%	n/a	n/a	n/a	n/a	-63%	0.00 ***
	Median	-68%	-61%	-65%	-65%	-65%	-66%	-64%	-59%	-56%	-61%	-57%	n/a	n/a	n/a	n/a	-64%	0.00 **
	Std. Dev.	19%	19%	16%	18%	16%	16%	18%	15%	17%	18%	16%	n/a	n/a	n/a	n/a	17%	0.00
	Obs.	69,988	69,081	64,254	60,824	57,798	54,871	49,291	39,780	23,114	13,985	8,363	n/a	n/a	n/a	n/a	441,361	
TECDAX	**Mean**	-72%	-66%	-66%	-64%	-66%	-66%	-67%	-66%	-63%	-65%	-62%	n/a	n/a	n/a	n/a	-65%	0.00 **
	Median	-73%	-66%	-66%	-66%	-70%	-66%	-67%	-69%	-65%	-63%	-65%	n/a	n/a	n/a	n/a	-66%	0.00 *
	Std. Dev.	17%	16%	17%	18%	16%	16%	14%	18%	16%	13%	15%	n/a	n/a	n/a	n/a	16%	0.00
	Obs.	35,741	37,133	37,133	37,126	37,075	36,949	36,299	33,958	26,995	20,641	16,031	n/a	n/a	n/a	n/a	319,340	
All	**Mean**	-68%	n/a	-59%	-59%	n/a	-61%	n/a	-59%	-59%	n/a	-50%	n/a	n/a	n/a	n/a	-58%	0.00 **
	Median	-70%	n/a	-61%	-61%	n/a	-61%	n/a	-58%	-58%	n/a	-56%	n/a	n/a	n/a	n/a	-60%	0.00 **
	Std. Dev.	18%	n/a	19%	19%	n/a	18%	n/a	18%	19%	n/a	56%	n/a	n/a	n/a	n/a	25%	0.00 **
	Obs.	217,436	n/a	218,671	214,590	n/a	207,201	n/a	184,968	153,599	n/a	113,798	n/a	n/a	n/a	n/a	1,843,272	

Table 8: Correlation factor (VaR, 10 day, 99%) by index and order size

Table shows cross-sectional statistics of the correlation factor, which measures correlation between liquidity cost and mid-price return according to (11); min-column measures the effect at minimum spread level; all-column is average over all standardized order sizes, i.e. without minimum; size impact is the coefficient in 10^{-2} of size in a linear regression of the distribution statistic on size including an intercept; * indicates 10%, ** 5% and *** 1% confidence level of being different from zero based on a two-tailed test.

λ(q), VaR(1 day, 99%) in % of price risk		Order size (in thsd. Euro)																Size impact
		Min	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All	
DAX	**Mean**	3%	n/a	2%	2%	n/a	3%	n/a	5%	11%	n/a	30%	56%	68%	76%	79%	32%	0.82 ***
	Median	3%	n/a	2%	2%	n/a	2%	n/a	4%	7%	n/a	20%	32%	42%	61%	69%	9%	0.79 ***
	Std. Dev.	2%	n/a	1%	1%	n/a	2%	n/a	4%	11%	n/a	29%	56%	57%	54%	56%	47%	0.90 ***
	Obs.	42,129	n/a	42,650	42,650	n/a	42,650	n/a	42,650	42,646	n/a	42,604	41,716	39,970	38,225	36,343	412,104	
MDAX	**Mean**	7%	6%	8%	10%	14%	18%	28%	44%	68%	76%	77%	n/a	n/a	n/a	n/a	31%	0.62 ***
	Median	6%	5%	6%	7%	11%	13%	22%	37%	56%	66%	61%	n/a	n/a	n/a	n/a	15%	0.66 ***
	Std. Dev.	6%	9%	16%	12%	17%	18%	24%	31%	44%	51%	51%	n/a	n/a	n/a	n/a	39%	0.40 ***
	Obs.	69,578	73,902	73,697	73,053	72,414	71,794	70,480	67,645	59,972	52,777	46,239	n/a	n/a	n/a	n/a	661,973	
SDAX	**Mean**	30%	17%	27%	41%	60%	68%	75%	80%	70%	70%	96%	n/a	n/a	n/a	n/a	52%	0.33 ***
	Median	12%	13%	18%	32%	43%	49%	58%	56%	62%	51%	73%	n/a	n/a	n/a	n/a	33%	0.34 ***
	Std. Dev.	102%	17%	35%	48%	83%	88%	102%	118%	46%	54%	96%	n/a	n/a	n/a	n/a	76%	0.25 *
	Obs.	69,988	68,497	64,068	60,824	57,733	54,871	49,291	39,714	23,114	13,985	8,363	n/a	n/a	n/a	n/a	440,460	
TECDAX	**Mean**	11%	6%	8%	11%	17%	23%	32%	47%	62%	67%	66%	n/a	n/a	n/a	n/a	29%	0.60 ***
	Median	7%	5%	7%	10%	13%	17%	28%	42%	56%	61%	67%	n/a	n/a	n/a	n/a	14%	0.62 ***
	Std. Dev.	28%	3%	5%	8%	15%	25%	26%	36%	64%	47%	34%	n/a	n/a	n/a	n/a	37%	0.62 ***
	Obs.	35,741	37,133	37,133	37,126	37,075	36,949	36,299	33,958	26,995	20,641	16,031	n/a	n/a	n/a	n/a	319,340	
All	**Mean**	15%	n/a	13%	18%	n/a	29%	n/a	43%	52%	n/a	59%	n/a	n/a	n/a	n/a	36%	0.43 ***
	Median	7%	n/a	7%	9%	n/a	13%	n/a	33%	43%	n/a	53%	n/a	n/a	n/a	n/a	17%	0.61 ***
	Std. Dev.	60%	n/a	23%	31%	n/a	54%	n/a	65%	50%	n/a	53%	n/a	n/a	n/a	n/a	52%	0.22 *
	Obs.	217,436	n/a	217,548	213,653	n/a	206,264	n/a	183,967	152,727	n/a	113,237	n/a	n/a	n/a	n/a	1,833,877	

Table 9: Liquidity impact on risk (VaR, 1 day, 99%)

Table shows cross-sectional statistics of lambda, which is the liquidity impact on risk in percent of price risk according to equation (9); min-column measures risk at minimum spread level; all-column is average over all standardized order sizes, i.e. without minimum; size impact is the increase in risk in percentage points when doubling order size, measured as coefficient in 10^{-2} of log-size in a regression of the log distribution statistic on log-size including an intercept; * indicates 10%, ** 5% and *** 1% confidence level of being different from zero based on a two-tailed test.

$\lambda(q)$, ES(10 day, 99%) in % of price risk		Order size (in thsd. Euro)																Size impact
		Min	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All	
DAX	Mean	1%	n/a	1%	1%	n/a	1%	n/a	2%	3%	n/a	12%	17%	22%	23%	26%	9%	0.82 ***
	Median	1%	n/a	1%	1%	n/a	1%	n/a	1%	2%	n/a	4%	8%	12%	13%	16%	2%	0.70 ***
	Std. Dev.	1%	n/a	0%	0%	n/a	1%	n/a	1%	3%	n/a	33%	35%	36%	37%	38%	26%	1.12 ***
	Obs.	423	n/a	431	431	n/a	430	n/a	427	412	n/a	388	354	335	314	289	3,811	
MDAX	Mean	2%	2%	2%	3%	4%	5%	7%	9%	13%	13%	15%	n/a	n/a	n/a	n/a	6%	0.51 ***
	Median	1%	1%	1%	2%	3%	3%	5%	7%	9%	9%	11%	n/a	n/a	n/a	n/a	3%	0.53 ***
	Std. Dev.	2%	4%	3%	4%	7%	7%	13%	9%	13%	11%	11%	n/a	n/a	n/a	n/a	9%	0.30 ***
	Obs.	780	826	812	803	791	774	724	653	550	425	376	n/a	n/a	n/a	n/a	6,734	
SDAX	Mean	5%	5%	6%	7%	9%	13%	14%	14%	21%	25%	25%	n/a	n/a	n/a	n/a	10%	0.40 ***
	Median	2%	2%	3%	4%	5%	7%	9%	12%	17%	22%	22%	n/a	n/a	n/a	n/a	6%	0.54 ***
	Std. Dev.	13%	6%	12%	8%	11%	26%	29%	13%	16%	22%	16%	n/a	n/a	n/a	n/a	17%	0.20 *
	Obs.	608	564	499	431	397	370	319	234	154	98	55	n/a	n/a	n/a	n/a	3,121	
TECDAX	Mean	2%	1%	2%	2%	3%	4%	6%	9%	12%	15%	19%	n/a	n/a	n/a	n/a	6%	0.63 ***
	Median	1%	1%	2%	2%	3%	3%	4%	6%	9%	14%	14%	n/a	n/a	n/a	n/a	3%	0.59 ***
	Std. Dev.	2%	1%	1%	2%	3%	4%	6%	8%	11%	16%	27%	n/a	n/a	n/a	n/a	9%	0.77 ***
	Obs.	169	347	333	329	334	313	300	261	196	153	107	n/a	n/a	n/a	n/a	2,673	
All	Mean	3%	n/a	3%	3%	n/a	5%	n/a	8%	11%	n/a	15%	n/a	n/a	n/a	n/a	8%	0.48 ***
	Median	1%	n/a	1%	2%	n/a	3%	n/a	5%	7%	n/a	10%	n/a	n/a	n/a	n/a	3%	0.52 ***
	Std. Dev.	7%	n/a	7%	5%	n/a	13%	n/a	9%	12%	n/a	25%	n/a	n/a	n/a	n/a	16%	0.34 **
	Obs.	1,980	n/a	2,075	1,994	n/a	1,887	n/a	1,575	1,312	n/a	926	n/a	n/a	n/a	n/a	16,339	

Table 10: Liquidity-adjusted total expected shortfall (ES, 10 day, 99%)

Table shows cross-sectional statistics of lambda, which is liquidity impact in percent of price risk according to equations (9) and (13); Min-column measures risk at minimum spread level; all-column is average over all standardized order sizes, i.e. without minimum; size impact is the coefficient in 10^2 of log-size in a regression of the log distribution statistic on log-size including an intercept; * indicates 10%, ** 5% and *** 1% confidence level of being different from zero based on a two-tailed test.

Avg. λ(q), VaR(10d, 99%) in % of price risk		Order size (in thsd. Euro)																Size impact
		Min	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All	
DAX	**2002**	1%	n/a	1%	1%	n/a	1%	n/a	2%	4%	n/a	10%	18%	17%	17%	18%	8%	0.77 ***
	2003	1%	n/a	1%	1%	n/a	1%	n/a	2%	4%	n/a	9%	19%	23%	25%	25%	10%	0.82 ***
	2004	1%	n/a	1%	1%	n/a	1%	n/a	1%	2%	n/a	8%	15%	21%	25%	29%	10%	0.83 ***
	2005	1%	n/a	1%	1%	n/a	1%	n/a	1%	2%	n/a	3%	7%	11%	14%	19%	6%	0.70 ***
	2006	0%	n/a	0%	1%	n/a	1%	n/a	1%	2%	n/a	3%	5%	7%	10%	14%	4%	0.65 ***
	2007	0%	n/a	0%	0%	n/a	1%	n/a	1%	1%	n/a	2%	4%	5%	7%	9%	3%	0.60 ***
	2008	0%	n/a	0%	0%	n/a	0%	n/a	1%	1%	n/a	2%	3%	5%	7%	8%	3%	0.63 ***
	All	1%	n/a	1%	1%	n/a	1%	n/a	1%	2%	n/a	5%	10%	13%	16%	18%	6%	0.74 ***
	Δ2002-2008 [a.]	-1%	n/a	0%	0%	n/a	0%	n/a	-1%	-1%	n/a	-4%	-8%	-4%	-2%	0%	-1%	
MDAX	**2002**	4%	6%	7%	8%	10%	12%	16%	19%	28%	31%	30%	n/a	n/a	n/a	n/a	12%	0.42 ***
	2003	3%	3%	4%	5%	7%	10%	16%	24%	35%	40%	37%	n/a	n/a	n/a	n/a	14%	0.63 ***
	2004	2%	2%	3%	4%	5%	6%	11%	18%	32%	35%	37%	n/a	n/a	n/a	n/a	13%	0.73 ***
	2005	2%	2%	2%	3%	3%	4%	6%	10%	21%	29%	34%	n/a	n/a	n/a	n/a	10%	0.74 ***
	2006	1%	1%	1%	2%	2%	3%	4%	6%	12%	17%	22%	n/a	n/a	n/a	n/a	7%	0.71 ***
	2007	1%	1%	1%	2%	2%	2%	3%	4%	8%	12%	17%	n/a	n/a	n/a	n/a	5%	0.65 ***
	2008	1%	1%	1%	1%	1%	2%	2%	3%	6%	10%	13%	n/a	n/a	n/a	n/a	4%	0.67 ***
	All	2%	2%	3%	3%	4%	6%	8%	12%	20%	24%	26%	n/a	n/a	n/a	n/a	10%	0.61 ***
	Δ2002-2008 [a.]	-2%	-3%	-4%	-4%	-5%	-6%	-7%	-7%	-8%	-6%	-4%	n/a	n/a	n/a	n/a	-2%	
SDAX	**2002**	13%	12%	27%	61%	91%	120%	153%	115%	79%	n/a	n/a	n/a	n/a	n/a	n/a	36%	0.56 **
	2003	10%	12%	17%	26%	36%	42%	40%	41%	40%	51%	64%	n/a	n/a	n/a	n/a	27%	0.32 ***
	2004	6%	7%	11%	20%	27%	35%	40%	52%	46%	38%	120%	n/a	n/a	n/a	n/a	24%	0.49 ***
	2005	6%	5%	7%	11%	15%	19%	24%	29%	32%	41%	37%	n/a	n/a	n/a	n/a	17%	0.46 ***
	2006	3%	4%	5%	7%	10%	13%	19%	28%	37%	36%	37%	n/a	n/a	n/a	n/a	15%	0.56 ***
	2007	2%	2%	3%	4%	5%	7%	10%	17%	31%	38%	43%	n/a	n/a	n/a	n/a	13%	0.71 ***
	2008	2%	2%	2%	3%	5%	6%	10%	16%	26%	32%	39%	n/a	n/a	n/a	n/a	10%	0.75 ***
	All	6%	6%	9%	14%	17%	21%	23%	28%	34%	38%	43%	n/a	n/a	n/a	n/a	18%	0.42 ***
	Δ2002-2008 [a.]	-7%	-6%	-18%	-47%	-74%	-99%	-130%	-86%	-45%	-13%	-21%	n/a	n/a	n/a	n/a	-18%	
TecDAX	**2002**	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	
	2003	2%	3%	4%	5%	8%	11%	15%	21%	25%	28%	31%	n/a	n/a	n/a	n/a	11%	0.57 ***
	2004	2%	3%	4%	5%	8%	12%	18%	26%	26%	29%	31%	n/a	n/a	n/a	n/a	13%	0.60 ***
	2005	2%	2%	3%	5%	6%	8%	11%	17%	27%	38%	41%	n/a	n/a	n/a	n/a	12%	0.67 ***
	2006	2%	2%	2%	3%	4%	5%	7%	13%	22%	27%	31%	n/a	n/a	n/a	n/a	10%	0.68 ***
	2007	1%	1%	2%	2%	3%	4%	5%	8%	15%	19%	23%	n/a	n/a	n/a	n/a	8%	0.66 ***
	2008	1%	1%	1%	2%	2%	3%	4%	6%	11%	15%	16%	n/a	n/a	n/a	n/a	5%	0.65 ***
	All	2%	2%	3%	4%	6%	8%	11%	16%	22%	27%	29%	n/a	n/a	n/a	n/a	11%	0.62 ***
	Δ2002-2008 [a.]	0%	-1%	-1%	-1%	-2%	-4%	-4%	-5%	-3%	-1%	-1%	n/a	n/a	n/a	n/a	-1%	
All	**2002**	6%	n/a	8%	10%	n/a	13%	n/a	12%	14%	n/a	15%	n/a	n/a	n/a	n/a	13%	0.14 ***
	2003	5%	n/a	7%	9%	n/a	14%	n/a	19%	21%	n/a	21%	n/a	n/a	n/a	n/a	15%	0.31 ***
	2004	3%	n/a	5%	8%	n/a	14%	n/a	20%	22%	n/a	23%	n/a	n/a	n/a	n/a	15%	0.42 ***
	2005	3%	n/a	4%	5%	n/a	9%	n/a	14%	19%	n/a	24%	n/a	n/a	n/a	n/a	11%	0.53 ***
	2006	2%	n/a	2%	3%	n/a	6%	n/a	12%	16%	n/a	19%	n/a	n/a	n/a	n/a	9%	0.59 ***
	2007	1%	n/a	2%	2%	n/a	4%	n/a	8%	14%	n/a	19%	n/a	n/a	n/a	n/a	7%	0.70 ***
	2008	1%	n/a	1%	2%	n/a	3%	n/a	7%	11%	n/a	13%	n/a	n/a	n/a	n/a	6%	0.69 ***
	All	3%	n/a	4%	6%	n/a	9%	n/a	14%	17%	n/a	20%	n/a	n/a	n/a	n/a	11%	0.45 ***
	Δ2002-2008 [a.]	-3%	n/a	-4%	-4%	n/a	-4%	n/a	2%	3%	n/a	5%	n/a	n/a	n/a	n/a	-2%	

Table 11: Liquidity impact on risk (rolling VaR, 10-day, 99%)

Table shows mean lambda, which is liquidity impact in percent of price risk by sub-sample calculated with a rolling mean-variance estimation of Value-at-Risk (10-day, 99%) according to (9) based on (15); a. Statistic shows absolute change between 2003 and 2008 when 2002 number not available; min-column measures risk at minimum spread level; all-column is average over all standardized order sizes, i.e. without minimum; size impact is the coefficient in 10^{-2} of log-size in a regression of the log distribution statistic on log-size including an intercept; * indicates 10%, ** 5% and *** 1% confidence level of being different from zero based on a two-tailed test.

λ(q) VaR(10 day, 99%)		Order size (in thsd. Euro)																Size
in % of liquidity risk		Min	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All	impact
DAX	**Estimate**	1%	n/a	1%	1%	n/a	1%	n/a	1%	2%	n/a	12%	22%	18%	25%	27%	10%	0.84 ***
	Obs.	42,129	n/a	42,710	42,710	n/a	42,710	n/a	42,710	42,706	n/a	42,663	41,759	40,002	38,294	36,388	412,652	
MDAX	**Estimate**	2%	2%	2%	2%	3%	5%	9%	5%	13%	21%	20%	n/a	n/a	n/a	n/a	7%	0.60 ***
	Obs.	73,279	74,858	74,679	74,040	73,365	72,737	71,409	68,543	60,788	53,501	46,793	n/a	n/a	n/a	n/a	670,713	
SDAX	**Estimate**	5%	32%	15%	27%	32%	52%	56%	26%	39%	45%	54%	n/a	n/a	n/a	n/a	35%	0.16 *
	Obs.	70,048	69,197	64,614	61,119	57,938	55,000	49,410	39,920	23,442	14,435	9,112	n/a	n/a	n/a	n/a	444,187	
TECDAX	**Estimate**	1%	1%	2%	2%	3%	4%	5%	8%	13%	15%	17%	n/a	n/a	n/a	n/a	6%	0.61 ***
	Obs.	36,980	37,157	37,157	37,150	37,099	36,973	36,323	34,028	27,077	20,868	16,291	n/a	n/a	n/a	n/a	320,123	
All stocks	**Estimate**	3%	n/a	2%	2%	n/a	3%	n/a	4%	5%	n/a	20%	n/a	n/a	n/a	n/a	6%	0.55 **
	Obs.	181,212	n/a	219,160	215,019	n/a	207,420	n/a	185,201	154,013	n/a	114,859	n/a	n/a	n/a	n/a	1,847,675	

Table 12: Liquidity impact $\lambda(q)$ (VaR, 10 day, 99%) by (index) portfolio and order size

Table shows portfolio statistics of lambda, which is the impact of liquidity in percent of price risk according to (9); min-column measures risk at minimum spread level; all-column is average over all standardized order sizes, i.e. without minimum; size impact is the coefficient of log-size regressed on the log distribution statistic including an intercept; * indicates 10%, ** 5% and *** 1% confidence level of being different from zero based on a two-tailed test.