Asset Allocation with Option-Implied Distributions: A Forward-Looking Approach*

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Abstract

We address the empirical implementation of the static asset allocation problem by employing forward-looking information from market option prices. To this end, constant maturity one-month S&P 500 implied distributions are extracted and subsequently transformed to the corresponding risk-adjusted ones. Then, optimal portfolios consisting of a risky and a risk-free asset are formed and their out-of-sample performance is evaluated. We find that the use of risk-adjusted implied distributions makes the investor significantly better off compared with the case where she uses the historical distribution of returns to calculate her optimal strategy. The results hold under a number of evaluation metrics and utility functions and carry through even when transaction costs are taken into account.

JEL Classification: C13, G10, G11, G13.

Keywords: Asset allocation, Option-implied distributions, Performance evaluation, Portfolio Choice, Risk aversion.

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The standard static optimal portfolio selection problem boils down to maximizing the expected utility derived by the one-period-ahead wealth. Maximisation of expected utility can be carried out in two alternative ways. The first obvious one is by performing a direct utility maximisation (e.g., Adler and Kritzman (2007) and Sharpe (2007)). The second is by maximising a Taylor series expansion up to a certain order that approximates expected utility (e.g., Levy and Markowitz (1979), Puley (1981), Kroll, Levy and Markowitz (1984), Jondeau and Rockinger (2006), and Guidolin and Timmermann (2008)). This approach results in portfolio choice based on some moments of the returns' distribution; the mean-variance optimization à la Markowitz is the most popular example. Implementation of the two routes requires estimation of the portfolio returns probability density function (PDF) and its moments, respectively. To this end, the literature has so far used historical data (*backward-looking approach*, see e.g., DeMiguel, Garlappi and Uppal (2009), for a review of various historical estimators). This implicitly assumes that the past is going to repeat itself, which often is not the case. As a result, the issue of estimation error in the inputs of expected utility maximisation arises (e.g., Merton (1980), Chan, Karceski and Lakonishok (1999)) and the optimal portfolio may be mis-calculated (e.g., Klein and Bawa (1976), Best and Grauer (1991), and Chopra and Ziemba (1993), Kan and Zhou (2007)). Mis-calculation of the optimal portfolio reduces investor's utility.

To avoid the use of historical distributions, this paper takes a very different approach and develops an empirical procedure to using stock index implied distributions as inputs to calculate the optimal portfolio. By definition, implied distributions are extracted from the market option prices that reflect the market participants' expectations; they refer to the distribution of the asset price that serves as underlying to the option. The horizon of the distribution matches the expiry date of the option. Therefore, the appeal of the suggested approach is that implied distributions are inherently forward-looking and may serve as more accurate estimates of the distribution/moments in an asset allocation problem where the optimal portfolio needs to be calculated. The suggested *forward-looking approach* can be viewed as a generalisation of the literature that suggests forecasting volatility by the implied volatility (the second moment of the implied distribution) rather than backward-looking measures of volatility that use historical data (see Poon and Granger (2003) for a review of this literature). It can also be viewed as part of the literature that suggests using information from option prices rather than historical data to estimate parameters that are of crucial importance to quantify risk and perform asset allocation such as the beta (Christoffersen, Jacobs and Vainberg (2008)) and correlation coefficients (see e.g., Driessen, Maenhout and Vilkov (2008)), as well as to forecast future returns of the underlying assets (Cremers and Weinbaum, (2008), Xing, Zhang and Zhao, (2008)).

There is already a significant literature on methods to extract implied PDFs as well as their potential applications to policy-making (see e.g., Söderlind and Svensson (1997)), option pricing and risk management (Ait-Sahalia and Lo (2000), Panigirtzoglou and Skiadopoulos (2004), Alentorn and Markose (2008)) and forecasting the future value of the underlying asset (Bliss and Panigirtzoglou (2004), Anagnou-Basioudis, Bedendo, Hodges and Tompkins (2005), Kang and Kim (2006), and Liu, Shackleton, Taylor and Xu (2007)). Jackwerth (2004) also provides an excellent review of the applications of implied distributions. However, to the best of our knowledge, their use for asset allocation purposes has not yet been considerably explored. Concurrently, but independently, Ait-Sahalia and Brandt (AB, 2008) propose a methodology that uses implied PDFs to solve the intertemporal consumption and portfolio choice problem and then examine the properties of the derived optimal consumption and portfolio weights' paths. Our study is distinct from theirs in two aspects. First, we propose an alternative empirical procedure to using implied PDFs for asset allocation purposes. The proposed methodology uses option-implied distributions to estimate the degree of risk aversion of the representative investor and then extracts the corresponding risk-adjusted probability distribution of returns.[1] The latter is used to calculate the optimal portfolio. Instead, the focus of AB is rather different; they use risk-neutral option-implied PDFs per se to determine the relative prices of consumption across various states in an intertemporal setting. Second, we compare the *out-of-sample* performance of the optimal portfolio strategy based on information derived by option prices to that of a portfolio solely based on historical information; AB do not address this issue. [2]

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[1]The term "risk-adjusted" is used to remind that risk-preferences are embedded and to distinguish it from the term "historical distribution"; the latter is used to define the PDF estimated solely from time series of asset prices.

[2]Jabbour, Pena, Vera and Zuluaga (2008) also use information from option prices to construct optimal portfolios. However, their definition of optimality is not in terms of maximising expected utility. Instead, the optimal portfolio is defined as the one that minimises the Conditional Value-at-Risk. This

In particular, we consider an asset universe that consists of a risky (the S&P 500 index) and a riskless asset. This setup has been commonly used in the literature (see e.g., Wachter (2002) and Chacko and Viceira (2005)). First, constant maturity one-month S&P 500 implied PDFs are extracted by applying the method of Bliss and Panigirtzoglou (2002) that has been found to be robust to the presence of measurement errors in the data. Then, they are converted to the corresponding risk-adjusted ones by employing the approach of Bliss and Panigirtzoglou (2004). This transformation is necessary because the implied distributions are measured under the risk-neutral probability measure and therefore their mean equals the risk-free rate. Hence, they cannot be used per se in the expected utility maximisation problem; in this case, the asset allocation problem is trivial since a risk-averse agent will place all her wealth in the risk-free asset (Arrow (1971)). Next, the risk-adjusted S&P 500 implied distributions are used to calculate the optimal portfolio. Finally, we compare the out-of-sample performance of the derived optimal strategies based on the risk-adjusted implied distributions/moments with that of the optimal strategies based on historical distributions/moments.

To check the robustness of the obtained results and shed light on whether implied distributions should be preferred to backward-looking ones for asset allocation purposes, a number of robustness tests are conducted. First, the risk-adjustment of implied distributions is performed by assuming alternative utility functions (exponential and power) for the representative (average) agent. Second, the optimal portfolios are calculated by maximising the expected utility per se and its truncated Taylor series expansion, separately. This is to check whether the use of a moment-based rule (e.g., the popular mean-variance analysis) will affect the properties of the derived optimal portfolios (see e.g., Jondeau and Rockinger (2006) for a comparison of the optimal portfolios derived by direct and Taylor series expansion maximisation in an in-sample historical estimators setting). Moreover, various utility/value functions and degrees of risk aversion that describe the preferences of the marginal (individual) investor are employed. The rationale justifying these partial-equilibrium exercises is that there exists a marginal investor who is price-taker, i.e. takes these already extracted

definition may be restrictive since it does not capture all of the characteristics of the utility function of the investor. In addition, their study focuses on the properties of the suggested algorithm and does not provide further tests on its out-of-sample performance relative to a method that uses historical data to calculate the optimal portfolio.

distributions as exogenously given and maximizes her own utility without affecting market prices because she only holds a small portion of the market wealth. In line with the existing asset allocation literature, the marginal investor, whose portfolio choice we examine, is distinct from the representative agent. Standard and behavioral utility functions are used. In particular, exponential and power utility functions as well as the disappointment aversion setting introduced by Gul (1991) are used. The latter has been used to explain investors' behavior with respect to their stock holdings (see e.g., Barberis, Huang and Santos (2001) and Ang, Bekaert and Liu (2005)) and option holdings (see Driessen and Maenhout (2007)). In particular, we employ a kinked value function to examine whether our results are robust in the presence of loss aversion. Finally, a number of measures (Sharpe ratio, opportunity cost, portfolio turnover and risk-adjusted returns net of transaction costs) are used to assess the optimal portfolio's performance.

The rest of the paper is structured as follows. Section I outlines the methodology to find the optimal portfolio by direct maximisation and truncated Taylor series expansion. Section II describes the data sets, the method to extract the implied distributions, and how their risk-adjusted analogues are derived. The following Section explains the implementation of the forward and backward-looking approach and discusses their relative performance under a number of metrics. Sections IV and V investigate the effect of loss aversion and sources for the discrepancy in the performance of the two approaches, respectively. The last Section concludes and presents the implications of this study, as well as, suggestions for future research.

I. Calculating the Optimal Portfolio

Consider a risk-averse investor with utility function $U(W)$ where $U''(W) < 0 \;\forall\; W$. At any point in time t, the investor decides about her optimal allocation of wealth W_t between a risky and a riskless asset over the period $[t, t+1]$ (static allocation problem). To fix ideas, let the return of the risky and the riskless asset from time t to $t+1$ be r_{t+1} and $r_{f,t+1}$ respectively. Let also the weights of wealth invested in the risky and the riskless asset at time t over the next period be α_t and α_t^f, respectively, where $\alpha_t + \alpha_t^f = 1$. Then, the optimal portfolio at time t is constructed by maximising the expected utility of wealth at time $t+1$ with respect to the portfolio weights, i.e.

$$\max_{\alpha_t} E[U(W_{t+1})] \tag{1}$$

where

$$W_{t+1} = W_t(1 + \alpha_t r_{t+1} + \alpha_t^f r_{f,t+1}) \tag{2}$$

Without loss of generality, initial wealth is normalised to one, i.e. W_t=1. Therefore,

$$W_{t+1} = 1 + \alpha_t r_{t+1} + \alpha_t^f r_{f,t+1} \tag{3}$$

This Section sets up the notation and describes the two alternative approaches (direct maximisation and maximisation of a truncated Taylor series expansion) to determine the optimal portfolios derived from equation (1).

A. Optimal Portfolio: Direct Maximization

At any point in time *t*, the problem of the direct maximization of the expected utility is defined as:

$$\begin{aligned}\max_{\alpha_t} E[U(W_{t+1})] &= \max_{\alpha_t} E[U(1 + \alpha_t r_{t+1} + \alpha_t^f r_{f,\,t+1})] \\ &= \max_{\alpha_t} \int U(1 + \alpha_t r_{t+1} + \alpha_t^f r_{f,t+1}) dF(r_{t+1})\end{aligned} \tag{4}$$

$$s.t.\ \alpha_t + \alpha_t^f = 1 \tag{5}$$

where $F(\bullet)$ is the cumulative conditional distribution function (CDF) of the return of the risky asset r_{t+1} at time t+1; the CDF depends only on the return of the risky asset, since $r_{f,t+1}$ is known ex ante (at time *t*). The first order condition (FOC) of this problem is given by:

$$\frac{\partial E(U(W_{t+1}))}{\partial \alpha_t} = E[U^{'}(W_{t+1})(r_{t+1} - r_{f,t+1})] = 0 \tag{6}$$

B. Optimal Portfolio: Truncated Taylor Series Expansion

Let the mean value $\overline{W}_{t+1}$ of the future wealth defined by equation (3) be

$$\overline{W}_{t+1} = E_t(W_{t+1}) = 1 + \alpha_t \mu_{t+1} + \alpha_t^f r_{f,t+1} \tag{7}$$

where $\mu_{t+1} = E_t(r_{t+1})$. The one-period portfolio return $r_{p,t+1}$ at time t+1 is given by

$$r_{p,t+1} = \alpha_t r_{t+1} + (1-\alpha_t) r_{f,t+1} = r_{f,t+1} + \alpha_t (r_{t+1} - r_{f,t+1}) \tag{8}$$

Then, at any point in time t, the expected utility approximated by an infinite order Taylor series expansion around $\overline{W}_{t+1}$ is given by

$$E[U(W_{t+1})] = E[\sum_{k=0}^{\infty} \frac{U^{(k)}(\overline{W}_{t+1})(W_{t+1} - \overline{W}_{t+1})^k}{k!}] \tag{9}$$

Equation (9) can be re-written, under certain assumptions (see Garlappi and Skoulakis (2008) and the references therein) as:

$$E[U(W_{t+1})] = \sum_{k=0}^{\infty} \frac{U^{(k)}(\overline{W}_{t+1})}{k!} E[(W_{t+1} - \overline{W}_{t+1})^k] \tag{10}$$

For the purposes of our analysis, we will calculate the optimal portfolios for k=2,4 and compare them with the ones derived from direct maximisation of expected utility. This will enable us to understand the features of the suggested forward-looking approach in a moments-based portfolio formation setting that is used widely by academics and practitioners. The case of k=2 corresponds to the familiar mean-variance Markowitz analysis while k=4 incorporates also the skewness and kurtosis of the returns distribution and has been extensively used in the literature (see e.g., Jondeau and Rockinger (2006) and the references therein). This is:

$$\begin{aligned} E[U(W_{t+1})] &\approx U(\overline{W}_{t+1}) + \frac{U^{(2)}(\overline{W}_{t+1})}{2!} E[(W_{t+1} - \overline{W}_{t+1})^2] + \\ &+ \frac{U^{(3)}(\overline{W}_{t+1})}{3!} E[(W_{t+1} - \overline{W}_{t+1})^3] + \frac{U^{(4)}(\overline{W}_{t+1})}{4!} E[(W_{t+1} - \overline{W}_{t+1})^4] \end{aligned} \tag{11}$$

Equation (11) can be re-written in terms of the first four moments of the distribution of the asset returns. This is possible because $\mu_{p,t+1}$ at time t+1 is given by:

$$\mu_{p,t+1} = r_{f,t+1} + \alpha_t(\mu_{t+1} - r_{f,t+1}) \tag{12}$$

Hence,

$$r_{p,t+1} - \mu_{p,t+1} = \alpha_t (r_{t+1} - \mu_{t+1}) \tag{13}$$

Subtracting equation (7) from equation (3) yields:

$$W_{t+1} - \overline{W}_{t+1} = \alpha_t(r_{t+1} - \mu_{t+1}) \tag{14}$$

Therefore,

$$W_{t+1} - \overline{W}_{t+1} = r_{p,t+1} - \mu_{p,t+1} = \alpha_t(r_{t+1} - \mu_{t+1}) \tag{15}$$

Let $M_{i,t+1}$ denote the ith central moment at time t+1, i=1,2,3,4, where

$$M_{i,t+1} \equiv E[(r_{t+1} - \mu_{t+1})^i],\ i = 2,3,4. \tag{16}$$

Therefore, equation (11) can be re-written as:

$$E[U(W_{t+1})] = U(\overline{W}_{t+1}) + \frac{U^{(2)}(\overline{W}_{t+1})}{2!}\sigma^2_{p,t+1} + \frac{U^{(3)}(\overline{W}_{t+1})}{3!}s^3_{p,t+1} + \frac{U^{(4)}(\overline{W}_{t+1})}{4!}k^4_{p,t+1} \tag{17}$$

where

$$\sigma^2_{p,t+1} \equiv E[(r_{p,t+1} - \mu_{p,t+1})^2] = E[(\alpha_t(r_{t+1} - \mu_{t+1}))^2] = \alpha_t^2 M_{2,t+1} \tag{18}$$

$$s^3_{p,t+1} \equiv E[(r_{p,t+1} - \mu_{p,t+1})^3] = E[(\alpha_t(r_{t+1} - \mu_{t+1}))^3] = \alpha_t^3 M_{3,t+1} \tag{19}$$

$$k^4_{p,t+1} \equiv E[(r_{p,t+1} - \mu_{p,t+1})^4] = E[(\alpha_t(r_{t+1} - \mu_{t+1}))^4] = \alpha_t^4 M_{4,t+1} \tag{20}$$

II. The Dataset

The data set consists of S&P 500 futures options monthly closing prices (January 1986 through May 2002) traded on the Chicago Mercantile Exchange (CME). This is a period that includes both bearish and bullish regimes. The CME S&P 500 options contract is an American style futures option; the underlying future is the CME S&P 500 futures contract. The expiry dates of the S&P 500 options coincide with those of the futures contracts; these trade out to one year with expiries in March, June, September, and December. In addition, there are monthly serial options contracts out to one quarter; these were introduced in 1987. Options and futures expire on the third Friday of the expiry month. For serial months there is no futures expiry and the options settle to the closing price on the option expiry date of the next maturing S&P 500 futures contract. The associated value of the underlying is the settlement price of the S&P 500 futures contract maturing on or just after the option expiry date. The risk-free rate used in this study is the one-month LIBOR rate taken from Bloomberg. The dividend yield is calculated as the twelve-month rolling dividends divided by the stock index price obtained from Datastream.

A. Extracting the Implied Distribution

We estimate the implied PDFs using the non-parametric method suggested by Bliss and Panigirtzoglou (2002) and currently used by the Bank of England. This method is chosen because they have shown that it generates PDFs that are robust to quite significant measurement errors in the quoted option prices. The technique makes use of Breeden and Litzenberger (1978) non-parametric result and uses a natural spline to fit implied volatilities as a function of the deltas of the options in the sample.

In particular, Breeden and Litzenberger (1978) showed that assuming that option prices are observed across a continuum of strikes, the second derivative of a European call price with respect to the strike price delivers the risk neutral PDF. However, in practice, available option quotes do not provide a continuous call price function. To construct such a function, a smoothing function (natural spline) is fitted to implied volatilities.[3] Implied volatilities are calculated from option prices by using the analytical quadratic approximation of Barone-Adesi and Whaley (BAW, 1987). This is an accurate and computationally efficient modification of the Black-Scholes formula that captures the early exercise premium of the American-style S&P 500 futures options. In addition, the implied volatility calculated via the BAW formula can be inserted in Black's (1976) formula to calculate the European option prices (see BAW, 1987, for a discussion). Hence, Breeden and Litzenberger's result can also be applied to our American option data set despite the fact that was derived for European

[3]To fit the natural spline to implied volatilities, a value for the smoothing parameter of the spline needs to be chosen (see Bliss and Panigirtzoglou (2002), (2004)) for an extensive discussion). The parameter is constrained to be constant across days so as any change in the implied PDF over time does not reflect the change in the smoothing parameter. Hence, techniques that can determine the smoothing parameter (e.g., cross validation) cannot be applied, and the choice of the parameter is subjective. The choice is determined by the trade-off between a smooth shape of the density and the goodness-of-fit to option prices. Choosing a very high value for the smoothing parameter will overfit option prices and will lead to contorted PDFs. On the other hand, a very low value of the parameter will not fit option prices well. We choose a value of 0.99 that yields well-behaved PDFs and provides a good fit to option prices. Moreover, Bliss and Panigirtzoglou (2004) and Kang and Kim (2006) find that the forecasting performance of the implied PDF does not depend on the smoothing parameter for a wide range of values.

options.[4] The delta metric is constructed by converting strikes into their corresponding call deltas by using the at-the-money implied volatility.[5] Hence, a set of implied volatilities and corresponding deltas is constructed for each available contract.

For the purposes of calculating the implied volatilities, the standard filtering constraints were imposed. Only at-the-money and out-of-the-money options were used because they are more liquid than in-the-money. Hence, measurement errors in the calculation of implied volatilities due to bid-ask spreads and non-synchroneous trading (Harvey and Whaley (1991)) are less likely to occur. Option prices that violate Merton's (1973) arbitrage bounds were discarded. Option prices with less than five working days to maturity were also excluded; these prices are excessively volatile as market participants close their positions.

Implied volatilities of deltas greater than 0.99 or less than 0.01, were also eliminated. These volatilities correspond to far out-of-the-money call and put prices, which have generally low liquidity. An implied volatility smile is constructed if there are at least three implied volatilities, with the lowest delta being less than or equal to 0.25 and the highest delta being greater than or equal to 0.75. This ensures that the available strikes cover a wide range of the PDF available outcomes. In the case that the range of strikes does not spread along the required interval, no PDF is extracted. Once the spline is fitted, 5,000 points along the

[4]Inserting the BAW implied volatilities in Black's (1976) rather than in BAW model does not affect the derived probabilities. This is because the size of the early exercise premium is very small in our case, since only short maturity (less than six months), out-of-the money options are used, and the cost of carry of the underlying asset is zero. BAW (1987) illustrate that out-of-the money options have very small early exercise premiums of the order of 0.01 (see Tables II and III in their paper, pages 313 and 314, respectively). This small size becomes even more insignificant when compared with the tick size error (0.05 for the S&P 500 futures options used in the paper). Moreover, in the case that the cost of carry is zero (Table III) the early exercise premium is smaller as compared to a 4% cost of carry case (Table II). They also show that the early exercise premium decreases as the time-to-maturity decreases. Therefore, the effect of the adjustment is very small on the option prices, and hence on the derived probabilities.

[5]The (call) delta metric is preferred to strike (or moneyness metric) because it takes values between zero and one irrespectively of the maturity of the contract in contrast to the range of strikes that varies with the maturity widely. In addition, it is well known that the interpolated implied volatilities are more stable under a delta than a strike metric. A small delta corresponds to a high strike (i.e. out-of-the-money calls), while a large delta corresponds to a low strike (i.e. in-the-money calls). Black's (1976) model is used to calculate deltas. In line with Bliss and Panigirtzoglou (2002, 2004) and Liu, Shackleton, Taylor and Xu (2007), we use the at-the-money implied volatility so as the ordering of deltas is the same as that of the strikes. Using the implied volatilities that correspond to each strike could change the ordering in the delta space in cases where steep volatility skews are observed. This would result in generating volatility smiles with artificially created kinks.

function are converted back to option price/strike space using Black's (1976) model. The 5,000 call price/strike data points are used to differentiate numerically the call price function so as to obtain the estimated PDF for the cross-section (see also Bliss and Panigirtzoglou (2004) for a more detailed description of the method to filter the data and extract the implied PDFs).

For the purposes of our analysis, constant one-month maturity implied PDFs are constructed using the methodology described in Panigirtzoglou and Skiadopoulos (2004). This is done as follows. First, the implied volatility smile of a synthetic constant one-month maturity option contract is constructed. This is done in three steps. First, for each expiry contract, a spline interpolation (smoothing parameter of 0.99) is performed across implied volatilities as a function of delta. Implied volatilities corresponding to nine values of delta (ranging from 0.1 to 0.9) are retained. Next, spline interpolation is applied across the implied volatilities of contracts with different maturities for any one of the nine values of delta; the one-month maturity implied volatilities are picked. In the final step, once this discrete constant one-month smile has been obtained (nine implied volatility points corresponding to nine deltas), a continuous implied volatility function is constructed by spline interpolating across these nine deltas. Finally, the constant one-month maturity implied PDF is backed out by following the already described Bliss and Panigirtzoglou (2002) method. This exercise is repeated at the end of each month.

A final point to be taken into account is that in the case of the S&P 500 futures options, the extracted implied distributions are measured in the space of the variable

$$x = \frac{F_{T,T}}{F_{t,T}} - 1 = \frac{S_T}{F_{t,T}} - 1 \tag{21}$$

where $F_{t,T}$ is the price at time t of the futures contract on the S&P 500 that matures at T=1 month. However, for the purposes of our analysis, we are interested in measuring implied distributions in the space

$$y = \frac{S_T}{S_t} - 1 \tag{22}$$

To switch from the x to y consistently, we use the no-arbitrage formula

$$S_t = \frac{F_{t,T}}{1+(r_{f,t+1}-d_t)\times 1/12} \tag{23}$$

where d_t is the dividend yield at time t. Plugging equation (23) in equation (22) yields

$$y = (1+x)\times[1+(r_{f,t+1}-d_t)\times 1/12]-1 \tag{24}$$

Hence, the new variable y is a linear function of the original one.

B. Risk-adjusting the Implied Distributions

There is a subtle point in the case where risk-neutral densities are used for asset allocation purposes. Option implied distributions are formed under the risk-neutral measure. Therefore, the mean of the implied distribution equals the risk-free rate. Consequently, the implied distribution cannot be used per se in an asset allocation setting since the risk-averse static optimizer will hold only the risk-free asset. This is because there is no risk-premium for holding the risky asset (Arrow (1971)). Hence, the option risk-neutral densities need to be risk-adjusted so as to be converted to the corresponding statistical distributions. This transformation will reveal the risk premium, as well. The transformation uses the well-known link between the measured at time t risk-neutral distribution $q_t(S_T)$ and statistical distribution $p_t(S_T)$ of the asset price S_T at time T ($t \leq T$). To fix ideas, assume that a representative agent with utility function $U(.)$ exists. Then,

$$q_t(S_T) = \zeta(S_T) \times p_t(S_T) \tag{25}$$

where

$$\zeta(S_T) \equiv \exp[-r(T-t)]\frac{U^{'}(S_T)}{U^{'}(S_t)} \tag{26}$$

$\zeta(S_T)$ is the so-called pricing kernel. Equation (26) is derived by the first-order condition of the intertemporal expected utility maximisation problem of the representative agent (see also Ait-Sahalia and Lo (2000) for a detailed discussion). Equation (25) shows that given a utility function and the risk-neutral probabilities for the asset price returns, the corresponding risk-adjusted probabilities can be derived; the adjustment is non-linear and hence cannot be done by simply adding an econometrically estimated risk premium to every point of the implied PDF. The resulting risk-adjusted density function must be normalised to integrate to one.

Hence, equations (25) and (26) yield

$$p_t(S_T) = \frac{\dfrac{q_t(S_T)}{\zeta(S_T)}}{\int \dfrac{q_t(x)}{\zeta(x)}dx} = \frac{\dfrac{q_t(S_T)}{U'(S_T)}}{\int \dfrac{q_t(x)}{U'(x)}dx} \tag{27}$$

To risk-adjust the risk-neutral densities [equation (27)] an assumption about the utility function of the representative agent needs to be made. We assume either one of the two most commonly used in the finance literature utility functions: (1) the negative exponential utility function, and (2) the power utility function.[6] The negative exponential utility function is defined as

$$U(W) = -exp(-\eta W)/\eta \tag{28}$$

where η is the coefficient of absolute risk aversion (ARA). The power utility function is defined as

$$U(W) = \frac{W^{1-\gamma} - 1}{1-\gamma}, \quad \gamma \neq 1 \tag{29}$$

where γ is the coefficient of constant relative risk aversion (RRA).

Both utility functions and thus the corresponding risk-adjusted densities depend on the value of the single parameter η (γ) that has an economic interpretation. We follow Bliss and Panigirtzoglou (2004) to determine this parameter in a three-step procedure. First, a sample of one-month constant maturity risk-neutral PDFs is extracted from the market option prices as explained in section II.A. Then, the extracted constant maturity risk-neutral PDFs are converted to the corresponding subjective risk-adjusted PDFs for any given value of the single parameter η (γ). Finally, we find the value η^* (γ^*) of the risk aversion parameter that maximizes the forecasting ability of the risk-adjusted PDFs with respect to future realizations of the underlying index, i.e. the *p*-value of Berkowitz (2001) likelihood ratio statistic. This

[6]More flexible functional forms may be alternatively used for the utility function of the representative agent (see e.g. Kang and Kim (2006)). Equivalently, a more flexible specification for the pricing kernel may be adopted (see e.g., Rosenberg and Engle (2002)). However, these specifications have not been used in an asset allocation setting partly because the economic interpretation of their extra parameters is not obvious. Therefore, we employ the widely used power and exponential utility functions to risk-adjust option implied distributions as in Bliss and Panigirtzoglou (2004).

optimal value determines the (implied) risk aversion coefficients.[7] The coefficient η^* (γ^*) can be interpreted as the "average market" risk-aversion parameter for the sample time period considered.

For the purposes of our analysis, we derive a time series of η^* (γ^*). This is done by repeating the above three-step procedure on a monthly basis using a rolling window of K extracted risk-neutral PDFs and monthly realizations of the underlying index. That is, at each point in time t, we employ a time series of K monthly one-month constant maturity risk-neutral PDFs (extracted on the dates from t - K to t -1) and their corresponding index realizations to estimate η^* (γ^*). Then, we use this estimated value to risk-adjust the constant one month-maturity risk-neutral density extracted at time t; this derives the (risk-adjusted) subjective PDF over the t to t+1 horizon that will be used for the direct expected utility maximization [equation (4)], as well as for the calculation of the relevant moments to be replaced in the Taylor series expansions [equation (17)].

Our methodology ensures that only information known to investors up to time t is employed to derive the risk-adjustment parameter η^* (γ^*) and only the most recent, t-K to t, information to adjust the risk-neutral density over the period between t and t +1 is used. This will enable the subsequent evaluation of the suggested forward-looking asset allocation approach in an out-of-sample setting. The resulting time series of η^* (γ^*) is calculated by using alternative rolling windows of K=36,48,60, 72 monthly observations until we exhaust the whole sample. We consider alternative rolling windows of different sizes so as to check whether our subsequent results will be robust to the choice of the rolling window that will be used to derive the risk-adjusted PDF.

[7] In general, the risk-neutral PDF, the physical one, and the (differentiable) utility function of the representative agent are linked; the knowledge of any two of the three quantities delivers the third one. Therefore, the implied risk aversion can also be derived by knowledge of the risk-neutral PDF and the physical one (see e.g., Ait-Sahalia and Lo (2000) and Jackwerth (2004)). However, this approach is not applicable in our case since we are in search of the risk-adjusted physical PDF. Hence, we use the implied distribution and an assumed utility function in order to extract the corresponding risk-adjusted physical PDF.

III. Optimal Portfolios: Historical versus Implied Distributions

A. Implementation

In the case of direct maximisation (4), the CDF $F(r_{t+1})$ of the risky asset returns needs to be estimated to determine the optimal α_t at any point in time t. Two alternative "estimators" are compared: the unconditional empirical distribution estimated from monthly historical data up to time t (termed historical distribution), and the risk-adjusted implied distribution extracted from option prices at time t with expiry date at time t+1 -i.e. one month ahead expiry. Following Ait-Sahalia and Lo (2000), the historical distribution is estimated by means of a Gaussian kernel. To solve equation (6), a grid search is performed. In the case of the truncated Taylor series expansion [equation (17)], the central moments M_t need to be estimated. These are alternatively extracted from the estimated historical distribution (sample historical moments, see also Jondeau and Rockinger (2006)) and the risk-adjusted implied distribution.

Then, a "rolling-window" procedure is followed to compare the *out-of-sample* performance of the forward-looking approach to asset allocation with the backward-looking one. At any given point in time t, the optimal portfolio weights are determined by the forward and backward-looking estimators separately by maximising the expected utility; in the case of the backward-looking estimator, K=36,48,60, 72 monthly historical data up to time t are used. Next, the corresponding optimal portfolios are formed and the out-of-sample portfolio monthly return over the period [t, t+1] is calculated. This process is repeated (i.e. we rebalance the portfolio) until the end of the data set is reached; again, in the case of the historical estimator, a moving window of K monthly historical data is used so as to re-calculate the central moments of the updated dataset. Eventually, a time series of one-month out-of-sample portfolio returns is generated based on any given approach to estimating the required inputs to maximise expected utility.

B. Utility Function of the Marginal Investor

An assumption about the utility function that describes the preferences of the marginal investor needs to be made in order to find the optimal portfolio. Two alternative standard utility functions are considered: the negative exponential utility function and the power utility function [equations (28) and (29), respectively]. In line with Jondeau and Rockinger (2006), a grid search over possible values of the risky and risk-free asset weights is employed to perform the direct maximisation [equation (4)]. In the case where the truncated Taylor series expansion is used to determine the optimal portfolio, truncation up to second and fourth order are performed, separately. At any point in time *t*, the fourth order truncated Taylor series expansion [equation (17)] of the negative exponential utility function is given by:

$$E[U(W_{t+1})] \approx -\frac{1}{\eta} exp(-\eta \overline{W}_{t+1})(1 + \frac{\eta^2}{2}\sigma^2_{p,t+1} - \frac{\eta^3}{6}s^3_{p,t+1} + \frac{\eta^4}{24}k^4_{p,t+1}) \tag{30}$$

In the case of the power utility function, the fourth order Taylor series expansion is given by:

$$E[U(W_{t+1})] \approx \frac{\overline{W}^{1-\gamma}_{t+1} - 1}{1-\gamma} - \frac{\gamma}{2}\overline{W}^{(-\gamma-1)}_{t+1}\sigma^2_{p,t+1} + \frac{\gamma(\gamma+1)}{6}\overline{W}^{(-\gamma-2)}_{t+1}s^3_{p,t+1} - \frac{\gamma(\gamma+1)(\gamma+2)\overline{W}^{(-\gamma-3)}_{t+1}}{24}k^4_{p,t+1} \tag{31}$$

where $\overline{W}_{t+1}$ is defined by equation (7). Equations (30) and (31) are maximised with respect to α_t to obtain the optimal portfolio choice α^*_t; a grid search over possible values of the risky and risk-free asset weights is performed again.

C. Evaluation Metrics

The alternative methodologies (i.e. option-implied moments/distribution versus historical moments/distribution) are evaluated in terms of certain characteristics of the respective optimal portfolios that have been obtained out-of-sample. To this end, the Sharpe ratio (SR), the concept of opportunity cost, the portfolio turnover and a measure of the portfolio risk-adjusted returns net of transaction costs are used. The comparison of the backward and forward-looking approaches is carried out for any given expected utility function to be maximised when the risk-adjustment has been preformed by the given utility function.

The SR is used to compare the risk-adjusted performance of the alternative investments during the whole time period (from *t*=1 to *T*) in line with the finance industry practice. The concept of opportunity cost has been introduced by Simaan (1993) to assess the economic significance of the difference in the performance of the best and second best strategies (see also Jondeau and Rockinger (2006)). To fix ideas, let α^{imp} be the optimal portfolio choice derived by using the implied distribution approach. Similarly, let α^{hist} be the optimal portfolio choice that is obtained by employing the historical distribution. Denote by r_p^{imp} and r_p^{hist} the corresponding realized portfolio returns. The opportunity cost c is defined to be the return that needs to be added (or subtracted) to the one obtained by the strategy based on the historical distribution so as the investor becomes indifferent (in utility terms) between the two strategies, i.e.

$$E[U(1 + r_p^{hist} + c)] = E[U(1 + r_p^{imp})] \tag{32}$$

Therefore, in the case where the opportunity cost is positive (negative) the investor will be better (worse) off by adopting the risk-adjusted implied rather than the historical distribution as an input to calculate her optimal portfolio. Note that there is not necessarily a one-to-one correspondence between the SR and the opportunity cost. This is because the SR is a mean-variance measure while the opportunity cost is based on the assumed utility function and, hence, it takes into account the higher order moments of the portfolio returns distribution too [see equation (17)].

The portfolio turnover (*PT*) is computed so as to get a feel of the degree of rebalancing required to implement each one of the two strategies. In line with DeMiguel, Garlappi and Uppal (2009), for any portfolio strategy k, PT_k is defined as the average absolute change in the weights over the T-1 rebalancing points in time and across the N available assets (two in our case), i.e.

$$PT_k = \frac{1}{T-1}\sum_{t=1}^{T}\sum_{j=1}^{N}\left(\left|a_{k,j,t+1} - a_{k,j,t^+}\right|\right) \tag{33}$$

where $a_{k,j,t}$ is the portfolio weight in asset j at time t under strategy k, $a_{k,j,t+1}$ is the desired (based on the optimisation of expected utility) portfolio weight in asset j at time t+1 under strategy k, and a_{k,j,t^+} is the portfolio weight *before* rebalancing at t+1. For example, in the case of the 1/N strategy (i.e. 50% of the wealth invested in the risky asset and 50% of the wealth invested in the riskless asset), $a_{j,t} = a_{j,t+1} = 1/N$, but a_{k,j,t^+} may be different due to changes in asset prices between t and t+1. The *PT* quantity defined above can be interpreted as the average fraction (in percentage terms) of the portfolio value that has to be reallocated over the whole period.

Finally, the historical and implied distributions strategies are evaluated under the risk-adjusted, net of transaction costs, return-loss measure of DeMiguel, Garlappi and Uppal (2009). This measure provides an economic interpretation of the *PT* metric; it shows how the proportional transaction costs generated by the portfolio turnover affect the returns from any given strategy. To fix ideas, let *pc* be the proportional transaction cost. In the case where the portfolio is rebalanced, the total proportional cost is given by $pc \times \sum_{j=1}^{N}(\left|a_{k,\,j,\,t+1} - a_{k,\,j,\,t^+}\right|)]$. The evolution of the net of transaction costs wealth (NW_k) for strategy *k* is given by:

$$NW_{k,t+1} = NW_{k,t}(1 + r_{k,p,t+1})[1 - pc \times \sum_{j=1}^{N}\left|a_{k,j,t+1} - a_{k,j,t^+}\right|] \qquad (34)$$

Then, the Return Net of Transaction Costs $RNTC_{k,t+1}$ for strategy *k* at time *t*+1 is given by:

$$RNTC_{k,t+1} = \frac{NW_{k,t+1}}{NW_{k,t}} - 1 \qquad (35)$$

To calculate $NW_{k,t+1}$ the proportional transaction cost *pc* for the S&P 500 (risky asset) is assumed to be equal to 50 basis points per transaction, as assumed in DeMiguel, Garlappi and Uppal (2009) and documented in the references therein. On the other hand, *pc* is set equal to zero for the risk-free asset; this is a legitimate assumption since in practice no transaction fees are charged in the case where the investor deposits or withdraws an amount from the risk-free savings account.

The return-loss measure is calculated with respect to the implied distribution based strategy; it is defined as the additional return needed for the historical distribution based strategy to perform as well as the implied distribution based strategy. Let μ_{imp} and σ_{imp} be the monthly *out-of-sample* mean and standard deviation of *RNTC* from the implied distribution based strategy, and μ_{hist} and σ_{hist} be the corresponding quantities for the historical distribution based strategy. Then, the return-loss from the historical distribution based strategy is given by:

$$return - loss = \frac{\mu_{imp}}{\sigma_{imp}} \times \sigma_{hist} - \mu_{hist} \qquad (36)$$

In the simplest case where σ_{imp}=σ_{hist} the return-loss measure amounts to the difference in the mean returns obtained under the two strategies.

D. Direct Maximisation: Results and Discussion

Table I shows the annualised SRs of the forward (Panels A and C) and backward-looking (Panel B and D) based strategies formed by direct maximisation of expected utility over the period 31/03/1992 to 28/06/2002. The maximisation of expected utility and the risk-adjustment of implied distributions have been implemented under the same assumed utility function for the marginal and representative investor (i.e., exponential or power utility). The SRs are reported for different levels of absolute and relative risk aversion (ARA, RRA=2,4,6,8,10) and different sample sizes of the rolling window (36, 48, 60 and 72 observations, with corresponding SRs SR_36, SR_48, SR_60, and SR_72) used to risk-adjust the implied distribution. The *p*-values of Memmel's (2003) test are reported within parentheses. The null hypothesis is that for any given expected utility function to be maximised, the SRs obtained under the risk-adjusted implied and historical distributions based strategies are equal; the risk-adjustment of the implied distribution has been performed by the given utility function.

-Table I about here-

We can see that in the case where either the exponential or power utility function is maximised, the optimal portfolios formed based on the forward-looking approach yield statistically greater SRs than the corresponding portfolios based on historical distributions in most cases. This holds regardless of the degree of the investor's relative risk aversion and the employed window length. The greatest SR obtained by the risk-adjusted distribution is encountered in the case of η=2 and K=36 months (SR=0.72), while the corresponding SR obtained by the historical estimators is 0.56. The SRs derived by the forward-looking approach are relatively high as compared to the findings in the asset allocation literature (see e.g., Driessen and Maenhout (2007) and DeMiguel, Garlappi and Uppal (2009)).

Notice that for any given level of risk aversion, the SRs decrease as the sample size of the rolling window increases. This implies that the recently arrived information should be weighted more heavily. Furthermore, the optimal portfolios formed using the risk-adjusted distributions outperform the ones generated by the 1/*N* strategy that yields SR=0.33; the study by DeMiguel, Garlappi and Uppal (2009) has indicated that the performance of any novel methodology for asset allocation should be compared with the naive $1/N$ rule. Overall, the results confirm the superiority of the forward-looking approach and show that this does not depend on the choice of the utility function.

Table II shows the annualised opportunity cost over the period 31/03/1992 to 28/06/2002. Panels A and B show the results for the cases where the expected utility is maximised under an exponential and a power utility function, respectively. Results are reported for different sizes of the rolling window (36, 48, 60 and 72 observations) used to risk-adjust the implied distribution and estimate the historical distribution. The risk-adjustment has been performed by assuming that the utility function of the representative agent is exponential (Panel A) and power (Panel B).

-Table II about here-

We can see that the opportunity cost is positive in most cases regardless of the window of estimation and degree of risk aversion, i.e. the investor is better off by adopting the risk-adjusted implied rather than the historical distribution to obtain the optimal trading strategy. In particular, in the case where the marginal investor uses a negative exponential function to calculate the optimal portfolio, the opportunity cost is positive for K=36,48 months. This holds regardless of the level of his ARA; the opportunity cost becomes as high as 3.48% for the case of η=6 and K=48 months. In the case of the power utility investor, the magnitude of the opportunity costs is now even greater compared to the case of exponential utility, underlining the usefulness of option-implied distributions for the formation of optimal portfolios. In particular, the opportunity cost reported for the case of γ=10 and K=36 months is as high as 8.04%, while for γ=6 and K=48 months is 6.84%. The reported opportunity costs are of the same order as the ones reported by Jondeau and Rockinger (2006).

Nevertheless, there are some cases where the opportunity cost is negative. This occurs when the implied distributions are adjusted assuming an exponential utility function for the representative agent (for $\eta \geq 6$ and K=60,72 months). This finding requires further explanation. It should be reminded that unlike Sharpe ratios that take into account only the mean and the standard deviation of excess portfolio returns, the opportunity cost metric takes also into account the higher-order moments, as well. In particular, the Taylor expansions of the exponential and power utility function [equations (30) and (31), respectively] illustrate that portfolio returns with negative skewness and excess kurtosis induce severe penalties in utility terms. In fact, the greater the degree of risk aversion, the greater this penalty becomes. Unreported results show that there are a series of cases, especially when the implied distributions are risk-adjusted by means of an exponential utility function, where the portfolio returns exhibit a greater degree of negative skewness and excess kurtosis as compared to the returns of portfolios formed on the basis of historical distributions. As a result, the mean-variance superiority of the portfolios' returns that make use of option-implied distribution is offset in some cases, due to the properties of their higher moments; this leads to the negative opportunity costs reported in Panel A of Table II.

Table III shows the portfolio turnover results. Panels A and B (D and E) show the portfolio turnover for the cases where the expected utility is maximised under an exponential (power) utility function. Results are reported for various levels of risk aversion for the marginal investor and sizes of the rolling window (36, 48, 60 and 72 observations) used to risk-adjust the implied distribution and estimate the historical distribution. The risk-adjustment has been performed by assuming that the utility function of the representative agent is exponential (Panels A and B) and power (Panels D and E). Panels C and F show the ratio of the portfolio turnovers of the risk-adjusted implied distributions to the historical distribution-based strategies under an exponential and a power utility function, respectively. We can see that the portfolio turnover decreases as the risk aversion increases, as expected. In addition, the ratio of the portfolio turnovers of the implied to the historical distribution-based strategies is slightly greater than one. This indicates that the portfolio turnover is slightly greater in the case where the investor uses the risk-adjusted implied distributions as an input in her asset allocation formation.

-Table III about here-

Table IV (Panels A and B) shows the annualised return-loss in the case where the expected utility is maximised directly under an exponential and power utility function, respectively. Results are reported for the different sizes of the rolling window (36, 48, 60 and 72 observations) used to risk-adjust the implied distribution and estimate the historical distribution. The risk-adjustment has been performed by assuming that the utility function of the representative agent is exponential and power, respectively. In almost every case, the investor is 1%-3% per annum worse-off after deducting transaction costs, if she adopts the backward-looking approach. This implies that the greater transaction costs incurred by the forward-looking approach (arising from the fact that the portfolios based on the risk-adjusted implied distributions have greater turnover than the ones based on historical distributions) cannot offset the corresponding extra risk-adjusted returns of this approach. Therefore, the mean-variance superiority of portfolios derived from the risk-adjusted implied distributions is confirmed, even after deducting the incurred transaction costs.

-Table IV about here-

E. Truncated Taylor Series Expansion: Results and Discussion

The current section discusses the results referring to portfolios formed on the basis of a second and a fourth order Taylor series expansion, separately. Table V shows the annualised SRs obtained by maximisation of a second order Taylor series approximated expected utility function for the period 31/03/1992 to 28/06/2002 in analogy with Table I where direct maximisation was performed.

-Table V about here-

In almost every case, the optimal portfolios based on the forward-looking approach outperform those based on the historical approach. This holds regardless of the level of the risk aversion and the choice of the rolling window's length. Hence, the superiority of the proposed methodology is confirmed in the case of moments-based portfolio formation just as was the case with the optimal portfolios derived by direct maximisation.

Table VI shows the opportunity cost under the second order Taylor series expansion. As in the case of direct maximisation, in most of the cases the opportunity cost is positive; in general, the opportunity cost is greater under the power utility function. The results imply that the investor is better off by adopting the risk-adjusted implied distribution to form the optimal portfolio. Some exceptions occur in the case where the marginal investor forms her optimal portfolio under the negative exponential function; negative opportunity costs occur for high levels of risk aversion ($\eta \geq 6$). Similar to the case of direct maximization, this is due to the fact that the opportunity cost metric does take into account the properties of the higher moments of the portfolio returns. In addition, the use of a second order Taylor series expansion leads to the formation of portfolios that yield returns characterized by even greater negative skewness and excess kurtosis as compared to the direct maximisation case. As a result, the utility penalty may get even greater, especially as the degree of risk aversion increases.

-Table VI about here-

We have also assessed the performance of optimal portfolios in the case where the expected utility is approximated by a fourth order Taylor series expansion. Again, the superiority of portfolios formed by using the risk-adjusted implied distributions rather than historical ones is confirmed (results are not reported due to space limitations). Interestingly, the results are very similar to the ones obtained by direct maximisation of the expected utility (Table I). This confirms in our setting the argument of Jondeau and Rockinger (2006) that the four-moment optimization strategy provides a very good approximation of the full scale utility optimization approach.

Table VII shows the portfolio turnover results in analogy with Table III. We can see that the portfolio turnover decreases as the risk aversion increases. In addition, the ratio of the portfolio turnovers of the implied distribution to the historical distribution-based strategies is slightly greater than one just as was the case with the direct maximisation.

-Table VII about here-

However, despite the slightly greater portfolio turnover induced by the use of implied distributions, the corresponding transaction costs do not offset the superiority of the portfolios' returns in risk-adjusted terms. Table IV (Panels C and D) shows the annualised results for the return-loss metric in the case where the expected utility is maximised under a second order Taylor series expansion of the exponential and power utility function, respectively. These results confirm the enhancement in terms of risk-adjusted excess returns, net of transaction costs, that is accomplished by employing implied distributions. This enhancement is on average 2% p.a., highlighting the importance of the proposed methodology.

IV. The Effect of Loss Aversion: A Robustness Test

The popular literature of behavioral finance has documented that the carriers of value for an investor are the gains and losses relative to a reference point, not terminal wealth. This is a characteristic that cannot be captured by the standard utility functions that have been considered in the previous sections. In particular, starting from Kahneman and Tversky (1979), a series of experimental studies have found that loss aversion is a dominant characteristic of individuals' behavior; the investor is much more sensitive to reductions in her financial wealth than to increases. Loss aversion implies that the value function that describes investors' preferences is steeper in the domain of losses than in the region of gains. In addition, loss aversion may explain some stylised facts such as non-participation (i.e. zero investment in the risky asset) and the success of capital-guarantee products. This section investigates whether implied distributions still outperform historical distributions as an input in the formation of optimal portfolios in the case where the marginal investor is loss averse. To this end, the preferences of the marginal investor are assumed to be described by a disappointment aversion (DA) setting, firstly introduced by Gul (1991).

The DA setting increases sensitivity to bad events (disappointments). It scales up the probabilities of all bad events by the same factor and scales down the probabilities of good events by a complementary factor, with good and bad defined as better and worse than a reference point, respectively. This framework has been employed in recent asset allocation studies so as to capture the presence of loss aversion. For instance, Ang, Bekaert and Liu (2005) have found that it can generate equity holdings that are consistent with the empirical evidence of non-participation. Driessen and Maenhout (2007) have also used it to address asset allocation questions for portfolios of stock and options. In addition, this setting is firmly grounded in decision theory and it is very similar to expected utility; it retains all the axioms underlying expected utility but the independence axiom that is replaced by a weaker version so as to accommodate the Allais paradox (see Gul (1991) and Ang, Bekaert and Liu (2005) for a discussion). In line with Driessen and Maenhout (2007), a DA value function $V(W_T)$ based on a power utility function is employed, i.e.:

$$V(W_T) = \begin{cases} \frac{W_T^{1-\gamma}-1}{1-\gamma} & \text{if } W_T > \mu_W \\ \frac{W_T^{1-\gamma}-1}{1-\gamma} - (\frac{1}{A} - 1)[\frac{\mu_W^{1-\gamma}-1}{1-\gamma} - \frac{W_T^{1-\gamma}-1}{1-\gamma}] & \text{if } W_T \leq \mu_W \end{cases} \quad (37)$$

where μ_W is the reference point relative to which gains or losses are measured, γ the RRA coefficient that controls the concavity of the value function in each region, and $A \leq 1$ is the coefficient of DA that controls the relative steepness of the value function in the region of gains versus the region of losses. The loss aversion decreases as A increases; A=1, corresponds to the case of the standard power utility function where there is no loss aversion. The main modelling advantage of this value function is that it is a one-parameter extension of the power utility function; hence, it nests the latter as a special case and inherits its attractive features. We follow Driessen and Maenhout (2007) and employ two values for A=0.6, 0.8 so as to consider the effect of DA; the weight of the risky asset will decrease as the DA increases.

To maximise the expected value of the DA function [equation (37)], μ_W has to be defined first. Given that only static asset allocation is considered, we assume that μ_W equals the initial wealth invested at the risk-free rate, i.e. $\mu_W = W_t(1 + r^f)$ (see Grant, Kajii and Polac (2001) for a review of alternative choices of the reference point within a DA setting). This choice of the reference point is in line with Barberis, Huang and Santos (2001) and implies that the investor uses the risk-free rate as a benchmark to code a gain or a loss. For instance, if the riskless rate is 4 percent, the investor will be disappointed if her stock market investment returns only 3 percent. In fact, this is a realistic assumption. Veld and Veld-Merkoulova (2008) conducted a study on investors' behavior and found that a significant portion of investors use the risk-free rate as a reference point to distinguish between losses and gains.

Table VIII shows the annualised SRs of the forward and backward-looking strategies obtained by direct maximisation of the DA value function of the marginal investor (maximisation of a Taylor series expansion is not possible since the DA value function is not differentiable). The SRs are reported for different levels of relative risk aversion and different values of DA (A=0.6 and A=0.8) for the marginal investor.

-Table VIII about here-

We can see that the portfolios based on the risk-adjusted implied distributions yield greater SRs compared to the ones formed on the basis of historical distributions. This finding holds for any given level of RRA, any degree of DA, and any choice of the rolling window length. These results confirm the conclusions of the previous sections that the use of forward-looking option-implied distributions may prove extremely beneficial. Furthermore, the risk-adjusted performance of the portfolios based on the forward-looking methodology was superior to the performance of the portfolios based on the naive $1/N$ strategy.

There are few additional observations to make. First, the SRs increase from A=0.6 to A=0.8. This is because the participation of the investor to the risky asset increases as the loss aversion of the investor decreases and this enables her to reap the realised risk premium. Second, the SRs decrease as the length of the rolling window increases, regardless of the employed methodology (forward or backward-looking). This is consistent with the findings of the previous sections. Third, we can see that in most cases, the disappointment averse investor would be better off by following the naive $1/N$ asset allocation strategy rather than forming portfolios on the basis of historical distributions (Panel C). This result reinforces the argument that making use of historical returns for asset allocation may lead to worse out-of-sample performance.

Table IX shows the opportunity costs for the cases where the DA value function is maximised. The risk-adjustment has been performed by assuming that the utility function of the representative agent is exponential (Panel A) and power (Panel B). Entries in each panel are reported for values of the parameter A=0.6,0.8 of the DA value function for the marginal investor.

-Table IX about here-

The results reported in Panel A are mixed. In particular, the opportunity cost is positive (negative) in the case where a rolling window of 36 and 48 (60 and 72) observations is used. Therefore, the mean-variance superiority of the portfolios based on implied distributions that have been risk-adjusted by a negative exponential utility function cannot be unambiguously confirmed in the case where the opportunity cost is employed as an evaluation metric by a disappointment averse investor. The explanation for this finding lies again in the higher moments of the portfolio returns' distributions. In particular, the portfolio returns derived by implied distributions that are risk-adjusted by an exponential utility function are, in some cases, characterized by a greater degree of negative skewness and excess kurtosis as compared to the corresponding returns of portfolios formed using historical distributions. Given that the opportunity cost is now calculated using the DA value function, negative skewness and excess kurtosis are severely penalized, leading to the reported negative opportunity costs. On the other hand, in the case where the power utility function is employed to risk-adjust the implied distributions (Panel B), the opportunity cost is positive in almost every case we examine. Hence, the superiority of the optimal portfolios formed on the basis of implied distributions that are risk-adjusted by means of a power utility function is confirmed under the opportunity cost metric.

Panels A and B of Table X show the ratio of the portfolio turnovers of the risk-adjusted implied distribution to the historical distribution-based strategies. The strategies are obtained by maximising a DA value function. We can see that the ratio is less than one in most of the cases. This is in contrast to the portfolio turnover obtained under the exponential and power utility functions that was greater than one. The results imply that the use of implied distributions is preferable to that of historical distributions in terms of the portfolio turnover. Table IV (Panels E and F) shows the annualised results under the return-loss metric in the case where the DA value function is maximised for various degrees of DA (A=0.6, 0.8, respectively) and risk aversion, as well as for different window lengths. The investor achieves an enhancement of up to 5% p.a. in terms of risk-adjusted, net of transaction costs, excess returns when she utilizes option-implied distributions for asset allocation purposes. Therefore, the mean-variance superiority of the forward-looking approach in the presence of DA is confirmed even when transaction costs are taken into account, just as was the case with the standard utility functions.

-Table X about here-

V. Sources of outperformance

Given that the forward-looking approach is found to be superior to the backward-looking one, we proceed to identify the source of its superiority. An implication of the previously reported results is that the use of information from option markets allows the investor to time the market more effectively. We employ the Treynor-Mazuy (1966) model to formally test the market timing ability of the proposed forward-looking approach; unreported results confirm our conjecture. Next, we use the following procedure to identify which one of the forward-looking risk-adjusted moments accounts for the reported outperformance. We calculate SRs of optimal strategies based on maximising the expected utility of the individual investor by means of a Taylor series expansion of order four by substituting repeatedly one central moment with the value of the corresponding risk-adjusted moment and the remaining three moments with the corresponding values of the central moments obtained from the historical PDF [see equations (30) and (31)]. This exercise is performed for the exponential and power utility function separately, and repeated four times so as to check whether the outperformance of the forward-looking approach stems from either the risk-adjusted mean, variance, skewness or kurtosis (i.e., in the first round, maximisation is implemented by using the risk-adjusted mean and the 'historical' variance, skewness and kurtosis. Then, in the second round, maximisation is implemented by using the risk-adjusted variance and the 'historical' mean, skewness and kurtosis, and so on). The obtained SRs are compared with the corresponding ones obtained by maximising expected utility through a 4th-order Taylor series expansion using only historical moments as inputs (these are almost identical to the ones obtained by direct maximisation using as input the historical PDF, as mentioned in Section III.E.).

Table XI reports the annualised SRs obtained from the described above exercise in the case where an exponential utility function describes the preferences of the marginal investor; Panels A to D tabulate the SRs using as input the first four forward-looking moments, respectively. A comparison with the SRs obtained by direct maximisation using as input the historical PDF (Panel B of Table I) shows that the outperformance of the forward-looking approach is due to the use of the forward-looking mean since this delivers the highest SRs; the use of the other three forward-looking moments leads to SRs of similar magnitude as compared to the "historical" ones. Similar results are also obtained in the case where a power utility function is assumed to describe the preferences of the individual investor.

-Table XI about here-

VI. Conclusions

This paper has taken a forward-looking approach to implementing static asset allocation by suggesting a way of using information from market option prices (option-implied distributions) to calculate the optimal portfolio. The motivation for doing so is that by their nature, implied distributions are forward-looking. Therefore, they are expected to capture the true unknown distribution/moments of asset returns that are required in any asset allocation problem more precisely than a backward-looking approach (i.e. based on historical distributions) does. Next, the validity of our hypothesis has been tested by comparing the *out-of-sample* performance of the forward-looking approach to that of a typical backward-looking one.

The commonly used asset space of a risky and a risk-free asset has been considered. Implied distributions have been extracted from the S&P 500 futures options and subsequently converted to the corresponding risk-adjusted ones. The risk-adjustment has been performed by backing out the coefficient of (absolute, relative) risk-aversion on a rolling window basis; to this end, alternative widely used in the finance literature utility functions have been employed to describe the preferences of the representative agent. Optimal portfolios were obtained and compared to the ones derived by historical distributions. To check the robustness of the results, maximisation of the assumed utility function per se (direct maximisation) and its Taylor series approximation has been performed alternatively; the maximisation has been performed by assuming a number of utility functions and levels of risk aversion for the marginal investor. The effect of loss aversion has also been investigated. Furthermore, a number of criteria have been used to assess the out-of-sample performance of the optimal portfolios.

We found that using option-implied information increases the investor's obtained risk-adjusted returns and makes her significantly better off compared with the case where she uses only historical distributions. The results hold regardless of the performance measure, specification of the utility function and objective value function to be maximised. Most importantly, the superiority of the forward-looking approach for asset allocation purposes is also confirmed in the case where transaction costs are taken into account.

Our results imply that the use of information from option markets allows the investor to time the market more effectively. In particular, we found that the use of the forward-looking mean drives the outperformance of the suggested approach. Hence, the accurate estimation of expected returns is crucial for asset allocation purposes, endorsing the conclusions of previous studies (e.g., Merton (1980) and Chopra and Ziemba (1993)) that documented that the accurate estimation of the mean return is more important than that of the variance. The reported findings imply that it is also more important than estimating higher order moments accurately; this is in line with the findings of Jondeau and Rockinger (2006) who found that, for moderate values of risk aversion, the formation of optimal portfolios is not considerably affected by departing from a mean-variance setting.

The presented framework for asset allocation opens up at least four avenues for future research. First, the benefits from using risk-adjusted implied distributions to form optimal portfolios should be explored for alternative risky assets. This can be done by extracting implied distributions from other option markets too. Second, given the vast literature on alternative methods to extract implied distributions, these may be extracted by an alternative method to the one employed in this paper. The risk-adjustment of the implied distribution may also be performed by other methods than the Bliss and Panigirtzoglou (2004) one (see e.g., Liu, Shackleton, Taylor and Xu (2007)). Third, alternative estimators may be used to estimate the historical distributions and its moments (see e.g., Jondeau and Rockinger (2003) for GARCH-type estimators for conditional skewness and kurtosis). Finally, the asset allocation problem should be investigated in the case where there are more than one risky assets in the investor's portfolio. To this end, data of options on baskets of assets should be used instead to capture the correlation structure of assets; unfortunately, at the moment these are traded over-the-counter with limited liquidity. Alternatively, the multivariate implied distribution could be recovered by using copulas as in Ait-Sahalia and Brandt (2008).

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List of Tables and Figures

Table I

Sharpe Ratios obtained by Direct Maximisation of Expected Utility

Annualised Sharpe Ratios (SRs) for the period 31/03/1992 to 28/06/2002. Panels A and C report the SRs obtained by the optimal strategy based on the risk-adjusted implied distributions. The risk-adjustment (maximisation) has been performed assuming that the representative (marginal) agent has an exponential and a power utility function, respectively. Panels B and D report the SRs obtained by the optimal strategy based on the historical distributions where maximisation of the exponential and power utility function has been performed, respectively. The SRs are reported for different levels of absolute and relative risk aversion (ARA, RRA=2, 4, 6 ,8, 10) and different sizes of the rolling window (36, 48, 60 and 72 observations, with corresponding SRs SR_36, SR_48, SR_60, and SR_72) used to risk-adjust the implied distribution. The *p*-values of Memmel's (2003) test are reported within parentheses; the null hypothesis is that the SRs obtained under risk-adjusted implied distributions and historical distributions based strategies are equal in each case examined.

Panel A: Risk-Adjusted Implied Distributions & Exponential Utility function					
	ARA=2	ARA=4	ARA=6	ARA=8	ARA=10
Sharpe Ratio_36	0.72	0.66	0.59	0.59	0.59
	(0.0345)	(0.0514)	(0.1956)	(0.1689)	(0.1702)
Sharpe Ratio_48	0.6	0.6	0.54	0.54	0.54
	(0.0483)	(0.0677)	(0.0271)	(0.0271)	(0.027)
Sharpe Ratio_60	0.46	0.49	0.44	0.44	0.44
	(0.1043)	(0.1021)	(0.1634)	(0.1631)	(0.1641)
Sharpe Ratio_72	0.39	0.42	0.4	0.4	0.4
	(0.0327)	(0.0177)	(0.0225)	(0.0224)	(0.0226)
Panel B: Historical Distributions & Exponential Utility function					
	ARA=2	ARA=4	ARA=6	ARA=8	ARA=10
Sharpe Ratio_36	0.56	0.52	0.5	0.48	0.48
Sharpe Ratio_48	0.47	0.49	0.36	0.36	0.36
Sharpe Ratio_60	0.37	0.4	0.35	0.35	0.35
Sharpe Ratio_72	0.28	0.27	0.24	0.24	0.24
Panel C: Risk-Adjusted Implied Distributions & Power Utility function					
	RRA=2	RRA=4	RRA=6	RRA=8	RRA=10
Sharpe Ratio_36	0.75	0.71	0.72	0.73	0.73
	(0.03)	(0.0137)	(0.0072)	(0.0092)	(0.017)
Sharpe Ratio_48	0.64	0.6	0.61	0.62	0.63
	(0.039)	(0.0487)	(0.0026)	(0.0059)	(0.0121)
Sharpe Ratio_60	0.46	0.5	0.51	0.53	0.54
	(0.1233)	(0.0732)	(0.0279)	(0.0418)	(0.0614)
Sharpe Ratio_72	0.37	0.39	0.41	0.43	0.44
	(0.1289)	(0.0386)	(0.02)	(0.0305)	(0.0458)
Panel D: Historical Distributions & Power Utility function					
	RRA=2	RRA=4	RRA=6	RRA=8	RRA=10
Sharpe Ratio_36	0.56	0.52	0.5	0.48	0.48
Sharpe Ratio_48	0.47	0.49	0.36	0.36	0.36
Sharpe Ratio_60	0.38	0.4	0.35	0.35	0.35
Sharpe Ratio_72	0.28	0.27	0.24	0.24	0.24

Table II

Direct Maximisation of Expected Utility: Annualised Opportunity Cost over the Period 31/03/1992 to 28/06/2002

Panels A and B show the opportunity cost (how much worse off the investor is in return terms by adopting the historical distribution rather than the risk-adjusted implied distribution to obtain the optimal trading strategy) for the cases where the expected utility is maximised under an exponential and power utility function, respectively. Results are reported for different sizes of the rolling window (36, 48, 60 and 72 observations) used to risk-adjust the implied distribution and estimate the historical distribution by means of a Gaussian kernel estimator. The risk-adjustment has been performed by assuming that the representative agent has an exponential (Panel A) and a power (Panel B) utility function.

Panel A: Exponential Utility function					
	ARA=2	ARA=4	ARA=6	ARA=8	ARA=10
36_Obs	3.12%	3.36%	1.56%	1.56%	1.20%
48_Obs	3.12%	2.16%	3.48%	2.64%	2.04%
60_Obs	2.40%	1.44%	-0.60%	-0.48%	-0.36%
72_Obs	3.00%	1.92%	-0.48%	-0.36%	-0.24%
Panel B: Power Utility function					
	RRA=2	RRA=4	RRA=6	RRA=8	RRA=10
36_Obs	3.72%	3.84%	4.80%	3.96%	8.04%
48_Obs	3.84%	2.40%	6.84%	5.04%	3.96%
60_Obs	2.16%	2.88%	3.36%	2.64%	2.16%
72_Obs	2.28%	3.24%	3.24%	2.52%	2.04%

Table III

Direct Maximisation of Expected Utility: Portfolio Turnover over the Period 31/03/1992 to 28/06/2002

Panels A and B (D and E) show the portfolio turnover for the cases where the expected utility is maximised under an exponential (power) utility function. Results are reported for different sizes of the rolling window (36, 48, 60 and 72 observations) used to risk-adjust the implied distribution and estimate the historical distribution by means of a Gaussian kernel estimator. The risk-adjustment has been performed by assuming that the representative agent has an exponential (Panels A and B) and a power (Panels D and E) utility function. Panels C and F show the ratio of the turnover generated by the strategy based on risk-adjusted implied distributions relative to that generated by the strategy based on historical distributions under an exponential and power utility function, respectively.

Panel A: Risk-adjusted Implied Distributions & Exponential Utility function					
	ARA=2	ARA=4	ARA=6	ARA=8	ARA=10
turnover_36	45.33%	31.48%	21.87%	16.49%	13.43%
turnover_48	37.28%	25.73%	19.47%	14.50%	11.83%
turnover_60	35.55%	24.45%	19.55%	14.69%	12.07%
turnover_72	35.05%	24.74%	17.95%	13.38%	10.99%
Panel B: Historical Distributions & Exponential Utility function					
	ARA=2	ARA=4	ARA=6	ARA=8	ARA=10
turnover_36	42.17%	34.45%	27.89%	21.30%	16.43%
turnover_48	33.14%	25.62%	21.48%	15.07%	11.62%
turnover_60	25.32%	23.67%	16.77%	11.78%	09.10%
turnover_72	25.61%	24.41%	15.63%	10.97%	08.44%
Panel C: Turnover Ratio: Risk-adjusted Implied Distributions/Historical distributions					
	ARA=2	ARA=4	ARA=6	ARA=8	ARA=10
ratio_36	1.08	0.91	0.78	0.77	0.82
ratio _48	1.12	1.00	0.91	0.96	1.02
ratio _60	1.40	1.03	1.17	1.25	1.33
ratio _72	1.37	1.01	1.15	1.22	1.30
Panel D: Risk-adjusted Implied Distributions & Power Utility function					
	RRA=2	RRA=4	RRA=6	RRA=8	RRA=10
turnover_36	49.44%	34.78%	26.85%	21.91%	18.77%
turnover_48	43.65%	30.51%	22.62%	19.03%	16.63%
turnover_60	35.38%	25.65%	19.88%	16.60%	14.64%
turnover_72	37.29%	23.73%	18.09%	15.33%	13.68%
Panel E: Historical Distributions & Power Utility function					
	RRA=2	RRA=4	RRA=6	RRA=8	RRA=10
turnover_36	42.44%	34.19%	27.79%	21.20%	16.40%
turnover_48	33.24%	25.26%	21.25%	14.98%	11.59%
turnover_60	25.15%	23.00%	16.54%	11.68%	09.06%
turnover_72	25.47%	23.81%	15.45%	10.90%	08.41%
Panel F: Turnover Ratio: Risk-adjusted Implied Distributions/Historical distributions					
	RRA=2	RRA=4	RRA=6	RRA=8	RRA=10
ratio _36	1.17	1.02	0.97	1.03	1.14
ratio _48	1.31	1.21	1.06	1.27	1.44
ratio _60	1.41	1.12	1.20	1.42	1.62
ratio_72	1.46	0.99	1.17	1.41	1.63

Table IV

Return-Loss for Direct Maximisation, Maximisation of the 2nd order Taylor Series Expansion, and the Disappointment Aversion Value Function

Panels A and B (C and D) show the annualised return-loss in the case where the expected utility is maximised directly (by means of a second order Taylor series expansion) under an exponential and power utility function, respectively. Results are reported for different sizes of the rolling window (36, 48, 60 and 72 observations) used to risk-adjust the implied distribution and estimate the historical distribution by means of a Gaussian kernel estimator. The risk-adjustment has been performed by assuming that the representative agent has an exponential (Panels A and C) and a power (Panels B and D) utility function. Panels E and F show the annualised return-loss in the case where the disappointment aversion value function is maximised and the risk-adjustment of the implied distributions has been performed under the assumption of an exponential (Panel E) or a power (Panel F) utility function.

Panel A: Return-Loss for direct maximization of Exponential Utility function					
	ARA=2	ARA=4	ARA=6	ARA=8	ARA=10
return-loss_36	3.33%	2.83%	1.59%	1.42%	1.10%
return-loss_48	3.10%	2.21%	2.90%	2.16%	1.71%
return-loss_60	2.00%	1.81%	1.23%	0.91%	0.70%
return-loss_72	2.65%	3.02%	2.20%	1.63%	1.29%
Panel B: Return-Loss for direct maximization of Power Utility function					
	RRA=2	RRA=4	RRA=6	RRA=8	RRA=10
return-loss_36	3.90%	3.60%	3.53%	2.97%	2.39%
return-loss_48	3.64%	2.12%	3.76%	2.87%	2.32%
return-loss_60	1.67%	1.73%	2.07%	1.64%	1.37%
return-loss_72	1.72%	2.27%	2.21%	1.76%	1.48%
Panel C: Return-Loss for Taylor series maximisation of Exponential Utility function					
	ARA=2	ARA=4	ARA=6	ARA=8	ARA=10
return-loss_36	3.36%	3.15%	2.09%	0.46%	-0.01%
return-loss_48	3.03%	1.95%	2.87%	2.29%	1.22%
return-loss_60	2.14%	1.96%	2.22%	0.86%	0.28%
return-loss_72	2.61%	2.94%	2.79%	1.80%	1.19%
Panel D: Return-Loss for Taylor series maximisation of Power Utility function					
	RRA=2	RRA=4	RRA=6	RRA=8	RRA=10
return-loss_36	3.39%	3.70%	3.30%	2.63%	2.58%
return-loss_48	3.62%	2.48%	2.93%	2.83%	2.62%
return-loss_60	1.57%	1.80%	2.45%	2.25%	2.02%
return-loss_72	1.76%	2.73%	2.40%	2.22%	1.99%
Panel E: Return-Loss for the Disappointment Aversion value function (Exponential utility risk-adjustment)					
A=0.6	RRA=2	RRA=4	RRA=6	RRA=8	RRA=10
return-loss_36	3.60%	1.24%	1.05%	0.83%	0.69%
return-loss_48	5.17%	3.50%	2.42%	1.85%	1.49%
return-loss_60	5.60%	2.11%	1.47%	1.13%	0.92%
return-loss_72	5.32%	2.46%	1.69%	1.29%	1.05%
A=0.8	RRA=2	RRA=4	RRA=6	RRA=8	RRA=10
return-loss_36	3.02%	1.85%	1.28%	1.02%	0.84%
return-loss_48	2.39%	3.13%	2.65%	2.04%	1.66%
return-loss_60	0.92%	1.95%	1.19%	0.95%	0.80%
return-loss_72	2.67%	3.21%	2.13%	1.67%	1.38%
Panel F: Return-Loss for the Disappointment Aversion value function (Power utility risk-adjustment)					
A=0.6	RRA=2	RRA=4	RRA=6	RRA=8	RRA=10
return-loss_36	4.23%	3.20%	2.32%	1.76%	1.41%
return-loss_48	5.26%	4.09%	2.79%	2.11%	1.68%
return-loss_60	5.46%	2.84%	1.94%	1.45%	1.15%
return-loss_72	4.91%	2.61%	1.79%	1.35%	1.09%
A=0.8	RRA=2	RRA=4	RRA=6	RRA=8	RRA=10
return-loss_36	3.99%	4.04%	3.03%	2.33%	1.87%
return-loss_48	3.18%	3.84%	3.42%	2.61%	2.09%
return-loss_60	2.19%	3.15%	2.21%	1.69%	1.37%
return-loss_72	3.17%	3.27%	2.32%	1.81%	1.49%

Table V

Sharpe Ratios obtained by Maximization of the 2^{nd} order Taylor Series Expansion of Expected Utility

Annualised Sharpe Ratios (SRs) obtained by maximisation of a second order Taylor series expansion of expected utility over the period 31/03/1992 to 28/06/2002. Panels A and C report the SRs obtained by the optimal strategy based on the risk-adjusted implied distributions; the risk-adjustment (maximisation) has been performed assuming that the representative (marginal) agent has an exponential and a power utility function, respectively. Panels B and D report the SRs obtained by the optimal strategy based on the historical distributions. The SRs are reported for different levels of absolute and relative risk aversion (ARA, RRA=2, 4, 6, 8, 10) and different sizes of the rolling window (36, 48, 60, and 72 observations, with corresponding SRs SR_36, SR_48, SR_60, and SR_72) used to risk-adjust the implied distribution. The *p*-values of Memmel's (2003) test are reported within parentheses; the null hypothesis is that the SRs obtained under the risk-adjusted implied and historical distributions based strategies are equal in each case examined.

Panel A: Risk-Adjusted Implied Distributions & Exponential Utility					
	ARA=2	ARA=4	ARA=6	ARA=8	ARA=10
Sharpe Ratio_36	0.72	0.69	0.60	0.53	0.47
	(0.03)	(0.02)	(0.11)	(0.42)	(0.52)
Sharpe Ratio_48	0.59	0.61	0.57	0.51	0.46
	(0.05)	(0.06)	(0.02)	(0.04)	(0.14)
Sharpe Ratio_60	0.46	0.51	0.48	0.42	0.38
	(0.09)	(0.05)	(0.03)	(0.21)	(0.35)
Sharpe Ratio_72	0.39	0.41	0.41	0.39	0.37
	(0.03)	(0.01)	(0.01)	(0.03)	(0.08)
Panel B: Historical Distributions & Exponential Utility					
	ARA=2	ARA=4	ARA=6	ARA=8	ARA=10
Sharpe Ratio_36	0.56	0.54	0.48	0.50	0.48
Sharpe Ratio_48	0.46	0.52	0.40	0.34	0.34
Sharpe Ratio_60	0.36	0.42	0.34	0.34	0.34
Sharpe Ratio_72	0.27	0.27	0.23	0.23	0.23
Panel C: Risk-Adjusted Implied Distributions & Power Utility					
	RRA=2	RRA=4	RRA=6	RRA=8	RRA=10
Sharpe Ratio_36	0.73	0.73	0.66	0.68	0.69
	(0.04)	(0.01)	(0.01)	(0.04)	(0.04)
Sharpe Ratio_48	0.62	0.64	0.58	0.55	0.59
	(0.04)	(0.02)	(0.01)	(0.01)	(0.01)
Sharpe Ratio_60	0.43	0.52	0.49	0.54	0.57
	(0.13)	(0.05)	(0.02)	(0.02)	(0.03)
Sharpe Ratio_72	0.36	0.41	0.39	0.44	0.47
	(0.13)	(0.02)	(0.02)	(0.02)	(0.03)
Panel D: Historical Distributions & Power Utility					
	RRA=2	RRA=4	RRA=6	RRA=8	RRA=10
Sharpe Ratio_36	0.57	0.54	0.47	0.50	0.48
Sharpe Ratio_48	0.45	0.52	0.41	0.33	0.34
Sharpe Ratio_60	0.35	0.43	0.33	0.34	0.34
Sharpe Ratio_72	0.27	0.27	0.22	0.22	0.22

Table VI

Maximisation of the 2nd order Taylor Series Expansion of Expected Utility: Annualised Opportunity Cost over the Period 31/03/1992 to 28/06/2002

Panels A and B show the opportunity cost (how much worse off the investor is in return terms by adopting the historical distribution rather than the risk-adjusted implied distribution to obtain the optimal trading strategy) for the cases where the second order Taylor series expansion of expected utility is maximised under an exponential and power utility function, respectively. Results are reported for different sizes of the rolling window used to transform the implied distribution to the risk-adjusted one and estimate the historical distribution. The risk-adjustment has been performed by assuming that the representative agent has an exponential (Panel A) and a power (Panel B) utility function.

Panel A: Exponential Utility function					
	ARA=2	ARA=4	ARA=6	ARA=8	ARA=10
36_obs	3.24%	3.12%	1.68%	-4.08%	-10.68%
48_obs	3.12%	2.04%	2.16%	-1.44%	-9.36%
60_obs	2.52%	1.68%	-1.20%	-7.56%	-14.16%
72_obs	3.00%	1.68%	-2.04%	-7.56%	-13.92%
Panel B: Power Utility function					
	RRA=2	RRA=4	RRA=6	RRA=8	RRA=10
36_obs	3.36%	3.72%	3.24%	1.08%	1.20%
48_obs	3.84%	2.76%	2.64%	1.20%	1.68%
60_obs	2.16%	2.88%	3.72%	1.92%	1.80%
72_obs	2.40%	2.52%	-0.12%	0.60%	0.84%

Table VII

Maximisation of the 2nd order Taylor Series Expansion of Expected Utility: Portfolio Turnover over the Period 31/03/1992 to 28/06/2002

Panels A and B (D and E) show the portfolio turnover induced by the forward-looking and backward-looking approaches, respectively, in the case where expected utility is maximised under a second order Taylor series expansion of the exponential (power) utility function. Results are reported for different sizes of the rolling window (36, 48, 60 and 72 observations) used to risk-adjust the implied distribution and estimate the historical distribution by means of a Gaussian kernel estimator. The risk-adjustment has been performed by assuming that the representative agent has an exponential (Panels A and B) and a power (Panels D and E) utility function. Panels C and F show the ratio of the turnover generated by the strategy based on risk-adjusted implied distributions relative to that generated by the strategy based on historical distributions under an exponential and power utility function, respectively.

Panel A: Risk-adjusted Implied Distributions & Exponential Utility function					
	ARA=2	ARA=4	ARA=6	ARA=8	ARA=10
Turnover_36	45.87%	33.34%	28.21%	24.42%	22.93%
Turnover_48	38.39%	28.76%	22.83%	22.45%	23.83%
Turnover_60	36.51%	26.00%	23.27%	24.04%	26.89%
Turnover_72	36.53%	26.96%	23.28%	22.92%	24.04%
Panel B: Historical Distributions & Exponential Utility function					
	ARA=2	ARA=4	ARA=6	ARA=8	ARA=10
Turnover_36	41.87%	36.24%	29.90%	24.65%	20.13%
Turnover_48	33.25%	25.49%	23.93%	18.33%	14.02%
Turnover_60	25.71%	24.83%	20.35%	14.15%	10.82%
Turnover_72	26.05%	27.20%	18.00%	12.53%	9.65%
Panel C: Turnover Ratio: Risk-adjusted Implied Distributions/Historical distributions					
	ARA=2	ARA=4	ARA=6	ARA=8	ARA=10
ratio_36	1.09	0.92	0.94	0.99	1.14
ratio_48	1.15	1.13	0.95	1.22	1.70
ratio _60	1.42	1.05	1.14	1.70	2.48
ratio _72	1.40	0.99	1.29	1.83	2.49
Panel D: Risk-adjusted Implied Distributions & Power Utility function					
	RRA=2	RRA=4	RRA=6	RRA=8	RRA=10
Turnover_36	51.33%	37.96%	30.76%	26.09%	25.48%
Turnover_48	46.26%	30.75%	29.70%	28.96%	26.64%
Turnover_60	38.07%	28.41%	28.36%	25.34%	22.86%
Turnover_72	39.66%	30.36%	25.68%	22.14%	19.62%
Panel E: Historical Distributions & Power Utility function					
	RRA=2	RRA=4	RRA=6	RRA=8	RRA=10
Turnover_36	40.37%	35.91%	30.09%	24.57%	20.64%
Turnover_48	33.37%	25.51%	24.87%	19.26%	14.58%
Turnover_60	25.86%	24.38%	21.44%	14.72%	11.17%
Turnover_72	26.26%	27.63%	18.80%	12.95%	9.90%
Panel F: Turnover Ratio: Risk-adjusted Implied Distributions/Historical distributions					
	RRA=2	RRA=4	RRA=6	RRA=8	RRA=10
ratio _36	1.27	1.06	1.02	1.06	1.23
ratio _48	1.39	1.21	1.19	1.50	1.83
ratio _60	1.47	1.17	1.32	1.72	2.05
ratio _72	1.51	1.09	1.37	1.71	1.98

Table VIII

Sharpe Ratios obtained by Direct Maximisation of the Disappointment Aversion Value Function

Entries report the annualised Sharpe Ratios (SRs) for the period 31/03/1992 to 28/06/2002. Panels A and B report the SRs obtained by the optimal strategy based on the risk-adjusted implied distributions derived by assuming that the representative agent has an exponential and a power utility function, respectively. Panel C reports the SRs obtained by the optimal strategy based on the historical distributions. The SRs are reported for different levels of relative risk aversion (RRA=2,4,6,8,10) and different sizes of the rolling window (36, 48, 60 and 72 observations with corresponding SRs SR_36, SR_48, SR_60, and SR_72) used to risk-adjust the implied distribution. Entries in each panel are reported for values of the parameter *A*=0.6,0.8 of the disappointment aversion utility function. The *p*-values of Memmel's (2003) test are reported within parentheses; the null hypothesis is that for any given utility function, the SRs obtained under the risk-adjusted implied and historical distributions based strategies are equal in each case examined.

Panel A: Risk-Adjusted Implied Distributions by Exponential Utility function					
A=0.6	RRA=2	RRA=4	RRA=6	RRA=8	RRA=10
Sharpe Ratio_36	0.52	0.41	0.43	0.44	0.45
	(0.02)	(0.26)	(0.22)	(0.23)	(0.24)
Sharpe Ratio_48	0.56	0.45	0.47	0.49	0.49
	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)
Sharpe Ratio_60	0.51	0.40	0.42	0.44	0.45
	(0.00)	(0.00)	(0.01)	(0.01)	(0.02)
Sharpe Ratio_72	0.41	0.37	0.40	0.41	0.43
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
A=0.8	RRA=2	RRA=4	RRA=6	RRA=8	RRA=10
Sharpe Ratio_36	0.66	0.53	0.52	0.53	0.54
	(0.05)	(0.15)	(0.23)	(0.23)	(0.24)
Sharpe Ratio_48	0.58	0.51	0.50	0.51	0.52
	(0.04)	(0.02)	(0.01)	(0.02)	(0.03)
Sharpe Ratio_60	0.48	0.43	0.42	0.44	0.45
	(0.28)	(0.06)	(0.10)	(0.11)	(0.12)
Sharpe Ratio_72	0.35	0.37	0.37	0.38	0.40
	(0.05)	(0.00)	(0.01)	(0.01)	(0.02)
Panel B: Risk-Adjusted Implied Distributions by Power Utility function					
A=0.6	RRA=2	RRA=4	RRA=6	RRA=8	RRA=10
Sharpe Ratio_36	0.56	0.59	0.60	0.61	0.62
	(0.01)	(0.01)	(0.02)	(0.03)	(0.05)
Sharpe Ratio_48	0.57	0.53	0.55	0.55	0.56
	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)
Sharpe Ratio_60	0.52	0.55	0.56	0.57	0.56
	(0.00)	(0.00)	(0.00)	(0.01)	(0.02)
Sharpe Ratio_72	0.39	0.43	0.45	0.46	0.47
	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)
A=0.8	RRA=2	RRA=4	RRA=6	RRA=8	RRA=10
Sharpe Ratio_36	0.71	0.66	0.67	0.68	0.68
	(0.03)	(0.01)	(0.01)	(0.02)	(0.03)
Sharpe Ratio_48	0.61	0.55	0.57	0.58	0.59
	(0.01)	(0.00)	(0.00)	(0.01)	(0.01)
Sharpe Ratio_60	0.54	0.52	0.54	0.55	0.56
	(0.05)	(0.01)	(0.01)	(0.03)	(0.04)
Sharpe Ratio_72	0.36	0.37	0.40	0.41	0.43
	(0.02)	(0.00)	(0.01)	(0.02)	(0.03)
Panel C: Historical Distributions					
A=0.6	RRA=2	RRA=4	RRA=6	RRA=8	RRA=10
Sharpe Ratio_36	0.30	0.30	0.29	0.29	0.29
Sharpe Ratio_48	0.22	0.08	0.08	0.08	0.08
Sharpe Ratio_60	0.07	0.07	0.07	0.07	0.07
Sharpe Ratio_72	-0.13	-0.13	-0.13	-0.13	-0.13
A=0.8	RRA=2	RRA=4	RRA=6	RRA=8	RRA=10
Sharpe Ratio_36	0.52	0.43	0.42	0.42	0.42
Sharpe Ratio_48	0.47	0.32	0.27	0.27	0.27
Sharpe Ratio_60	0.44	0.30	0.30	0.30	0.30
Sharpe Ratio_72	0.23	0.14	0.14	0.14	0.14

Table IX

Direct Maximisation of the Disappointment Aversion Value Function: Annualised Opportunity Cost over the Period 31/03/1992 to 28/06/2002

Results are reported for different sizes of the rolling window (36, 48, 60 and 72 observations) used to risk-adjust the implied distribution and estimate the historical distribution by means of a Gaussian kernel estimator. The risk-adjustment has been performed by assuming that the representative agent has an exponential (Panel A) and a power (Panel B) utility function. Entries in each panel are reported for both values of the disappointment aversion parameter (*A*=0.6, 0.8) employed in this study.

Panel A: Risk-Adjusted Implied Distributions by Exponential Utility function					
A=0.6	RRA=2	RRA=4	RRA=6	RRA=8	RRA=10
36_obs	3.48%	-0.48%	0.00%	2.04%	0.12%
48_obs	4.08%	0.24%	-2.28%	0.24%	0.24%
60_obs	2.52%	-3.00%	-1.92%	-1.44%	-1.08%
72_obs	-0.72%	-3.84%	-2.52%	-4.44%	-1.44%
A=0.8	RRA=2	RRA=4	RRA=6	RRA=8	RRA=10
36_obs	3.36%	1.92%	1.20%	1.80%	-0.60%
48_obs	2.64%	2.52%	4.80%	1.68%	1.44%
60_obs	0.96%	-0.84%	-1.44%	-1.08%	-0.84%
72_obs	1.80%	-0.96%	-1.56%	-1.08%	-0.84%
Panel B: Risk-Adjusted Implied Distributions by Power Utility function					
A=0.6	RRA=2	RRA=4	RRA=6	RRA=8	RRA=10
36_obs	4.56%	3.00%	2.16%	1.68%	1.44%
48_obs	4.68%	3.72%	-0.84%	2.04%	1.68%
60_obs	4.08%	2.16%	1.68%	1.32%	1.08%
72_obs	1.32%	0.96%	0.84%	0.72%	0.60%
A=0.8	RRA=2	RRA=4	RRA=6	RRA=8	RRA=10
36_obs	4.20%	5.04%	3.60%	2.76%	0.96%
48_obs	3.24%	4.44%	4.68%	3.48%	2.88%
60_obs	2.52%	3.60%	2.52%	2.04%	1.68%
72_obs	2.88%	2.88%	2.04%	1.68%	1.44%

Table X

Direct Maximisation of the Disappointment Aversion Value Function: Portfolio Turnover over the Period 31/03/1992 to 28/06/2002

Panels A and B show the ratio of the portfolio turnovers of the risk-adjusted implied distribution to the historical distribution based strategies. The strategies are obtained by maximising a disappointment aversion value function. Results are reported for different sizes of the rolling window (36, 48, 60 and 72 observations) used to risk-adjust the implied distribution and estimate the historical distribution by means of a Gaussian kernel estimator. The risk-adjustment has been performed by assuming that the representative agent has an exponential (Panel A) and a power (Panel B) utility function. Entries in each panel are reported for both values of the disappointment aversion parameter (*A*=0.6,0.8) employed in this study.

Panel A: Turnover Ratio: Risk-adjusted Implied Distributions by Exponential Utility/Historical distributions					
A=0.6	RRA=2	RRA=4	RRA=6	RRA=8	RRA=10
ratio_36	0.93	0.92	1.14	1.34	1.52
ratio _48	0.89	1.35	1.69	2.02	2.34
ratio _60	0.85	1.78	2.21	2.61	2.98
ratio _72	0.90	1.64	2.03	2.42	2.78
A=0.8	RRA=2	RRA=4	RRA=6	RRA=8	RRA=10
ratio _36	0.79	0.79	0.77	0.92	1.05
ratio _48	0.93	0.87	0.98	1.18	1.37
ratio _60	0.95	1.03	1.29	1.56	1.81
ratio _72	0.92	1.07	1.25	1.51	1.75
Panel B: Turnover Ratio: Risk-adjusted Implied Distributions by Power Utility/Historical distributions					
A=0.6	RRA=2	RRA=4	RRA=6	RRA=8	RRA=10
ratio_36	0.96	1.09	1.37	1.58	1.75
ratio_48	0.91	1.44	1.84	2.17	2.48
ratio_60	1.18	1.84	2.27	2.63	2.97
ratio_72	1.30	1.74	2.13	2.47	2.81
A=0.8	RRA=2	RRA=4	RRA=6	RRA=8	RRA=10
ratio_36	0.93	0.89	0.97	1.13	1.25
ratio_48	0.82	0.88	1.06	1.27	1.45
ratio_60	0.89	0.98	1.28	1.52	1.75
ratio_72	0.67	0.81	1.09	1.31	1.53

Table XI

Sources of Outperformance of the Forward-looking Approach: The Exponential Utility Case

Panels A to D report the annualised Sharpe Ratios (SRs) obtained by maximisation of a fourth order Taylor series expansion of expected utility over the period 31/03/1992 to 28/06/2002 by substituting repeatedly one central moment with the value of the corresponding forward-looking risk-adjusted moment and the remaining three with the corresponding values of the central moments obtained from the historical PDF. For example Panel A reports the SRs obtained by using the risk-adjusted mean and the 'historical' variance, skewness and kurtosis. Panel B reports the SRs obtained by using the risk-adjusted variance and the 'historical' mean, skewness and kurtosis, and so on. The risk-adjustment (maximisation) has been performed assuming that the representative (marginal) agent has an exponential utility function. The SRs are reported for different levels of absolute and relative risk aversion (ARA, RRA=2, 4, 6, 8, 10) and different sizes of the rolling window (36, 48, 60, and 72 observations, with corresponding SRs SR_36, SR_48, SR_60, and SR_72) used to risk-adjust the implied distribution.

Panel A: Forward-looking mean and historical variance, skewness and kurtosis					
	ARA=2	ARA=4	ARA=6	ARA=8	ARA=10
Sharpe Ratio_36	0.71	0.67	0.65	0.61	0.56
Sharpe Ratio_48	0.57	0.59	0.62	0.61	0.57
Sharpe Ratio_60	0.42	0.51	0.55	0.54	0.53
Sharpe Ratio_72	0.35	0.40	0.45	0.48	0.47
Panel B: Forward-looking variance and historical mean, skewness and kurtosis					
	ARA=2	ARA=4	ARA=6	ARA=8	ARA=10
Sharpe Ratio_36	0.51	0.38	0.36	0.35	0.35
Sharpe Ratio_48	0.36	0.26	0.26	0.26	0.26
Sharpe Ratio_60	0.23	0.22	0.21	0.20	0.21
Sharpe Ratio_72	0.17	0.12	0.11	0.11	0.11
Panel C: Forward-looking skewness and historical mean, variance,and kurtosis					
	RRA=2	RRA=4	RRA=6	RRA=8	RRA=10
Sharpe Ratio_36	0.56	0.49	0.50	0.49	0.49
Sharpe Ratio_48	0.29	0.36	0.29	0.29	0.29
Sharpe Ratio_60	0.37	0.35	0.34	0.34	0.34
Sharpe Ratio_72	0.28	0.23	0.23	0.23	0.23
Panel D: Forward-looking kurtosis and historical mean, variance, and skewness					
	RRA=2	RRA=4	RRA=6	RRA=8	RRA=10
Sharpe Ratio_36	0.56	0.50	0.50	0.48	0.48
Sharpe Ratio_48	0.47	0.44	0.38	0.39	0.38
Sharpe Ratio_60	0.37	0.36	0.35	0.35	0.35
Sharpe Ratio_72	0.28	0.24	0.24	0.24	0.24