The Role of Jumps in Foreign Exchange Rates

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Abstract

This paper analyzes the nature and pricing implications of jumps in foreign exchange rate processes. I propose a general stochastic-volatility jump-diffusion model of exchange rate dynamics that contains several popular models as its special cases. I use the efficient method of moments to estimate the model parameters from the spot exchange rates of Euro, British Pound, Japanese Yen and Swiss Franc with respect to the U.S. Dollar. The results indicate that any reasonably descriptive continuous-time model must allow for jumps with a bimodal distribution of jump sizes, in addition to stochastic volatility. Finally, I investigate the option pricing implications of jumps. Although the ex-post estimates of jump probabilities show that jumps occur irregularly and rarely, the jump component is important for explaining the shapes of implied volatility "smiles". The risk premia implicit in the cross-sectional currency options data suggest that the exchange-rate jump risk appears to be priced by the market.

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1 Introduction

Our knowledge about the complexity of underlying risk factors in exchange rate processes parallels the increase in the number of studies on time series and option prices. The complexity suggests that investment decisions in currency markets will be adequate only if they build upon fairly reasonable specifications of the exchange rate dynamics. Specifically, currency derivatives such as forward rates, options or currency swaps will be very sensitive to volatility dynamics and to higher moments of return distributions.

It is now widely accepted that the exchange rate volatility is time-varying and that the distributions of returns are fat-tailed (see, for example, Bates (1996a,b) and the references cited therein). Figure 1, for example, displays the daily relative changes of the exchange rate of Euro with respect to U.S. Dollar, from January 2005 to September 2008. The time-varying nature of volatility is responsible for the interchanging periods of high and low variations in returns. On the other hand, the outliers are manifested through relatively rare but large spikes, or "jumps". The presence of outliers and the extent of skewness are critical for derivatives pricing, as well as hedging and risk management decisions.

Bates (1988) and Jorion (1988) were among the first to assert that the outliers in exchange rate series can be accounted for by combining a continuous- and a discrete-time process. Many studies have later documented the statistical significance of jumps in exchange rates. Bates (1996b), Jiang (1998), Craine *et al.* (2000) and Doffou & Hilliard (2001) find that jumps are important components of the currency exchange rate dynamics, even when conditional heteroskedasticity is taken into account. Moreover, several authors had reported that neglecting one of the exchange rate properties

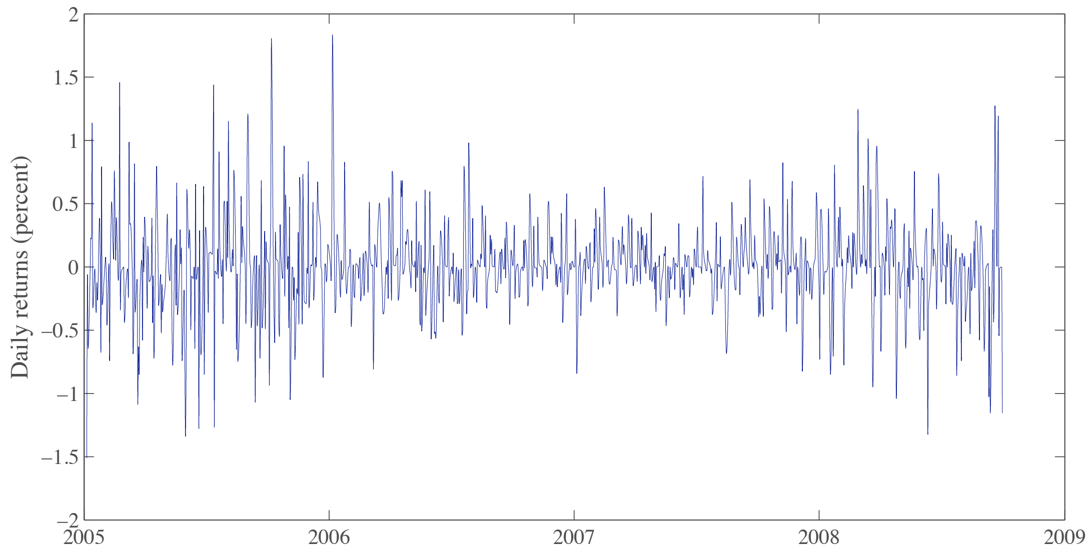

Figure 1: **EUR/USD exchange rate.** Daily returns for January 2005–September 2008.

usually leads to a significant overestimation of importance of another risk factor (see Jiang (1998) for a discussion). A number of empirical studies revealed other important stylized facts about the exchange rates. For example, Guillaume *et al.* (1997) show that exchange-rate returns in general exhibit non-stable, symmetric, fat-tailed distributions with finite variance and negative first-order autocorrelation and heteroskedasticity.[1]

This paper studies the nature of jumps in foreign exchange rates, as well as their implications to the option pricing. I propose a general continuous-time stochastic volatility model with Poisson jumps of time-varying intensity. The model conveniently captures all the stylized facts known to the literature. The special cases of the model are several popular benchmarks, such as the Black & Scholes (1973) model, the Merton (1976) model, the stochastic volatility model of Taylor (1986) and the stochastic-volatility jump-diffusion model of Bates (1996b). To estimate the model parameters,

[1]The economic literature dealing with jump processes and their pricing implications has been growing ever since the seminal work of Merton (1976). Examples include Ball & Torous (1985), Bates (1991), Bates (1996a), Bates (1996b), Chernov *et al.* (1999), Pan (2002) and Andersen *et al.* (2002).

I use daily interbank spot exchange rates of Euro, British Pound, Japanese Yen and Swiss Franc with respect to the U.S. Dollar, the four most important exchange rates in terms of currency turnover. The inference framework is based on the efficient method of moments procedure of Gallant & Tauchen (1996).

The results confirm that both stochastic volatility and jumps play a critical role in the exchange rate dynamics. Moreover, a correctly specified model should include a bimodal distribution of jump sizes. Depending on the exchange rate, a model with the volatility-dependent jump intensity may outperform a model with a constant intensity. The proposed general model also allows for a closed-form solution for the price of European-style currency options. It is capable to accommodate the shapes of Black-Scholes implied volatilities observed in the actual data. This indicates that the dominant empirical characteristics of exchange rate processes seem to be priced by the market.

The remainder of the paper is organized as follows: Section 2 develops a model specification for exchange rates and describes the estimation methodology. Section 3 describes the data and provides the estimation results. Section 4 considers the option pricing implications of jumps. Concluding remarks are given in Section 5.

2 Model Specification and Estimation Methodology

2.1 Model

The model is constructed to capture the salient features of exchange rate dynamics and incorporate the majority of popular models used in the literature as its special cases. I will assume that the instantaneous exchange rate S_t solves

$$\frac{\mathrm{d}S_t}{S_t} = \mu \mathrm{d}t + \sqrt{V_t}\, \mathrm{d}W_{1,t} + \left(e^{u_t} - 1\right) \mathrm{d}q_t - \lambda_t \bar{k} \mathrm{d}t, \tag{1}$$

where the instantaneous variance V_t follows a mean-reverting diffusion given by the "square-root" specification of Heston (1993):

$$\mathrm{d}V_t = \left(\alpha - \beta V_t\right) \mathrm{d}t + \sigma \sqrt{V_t}\, \mathrm{d}W_{2,t}. \tag{2}$$

The stochastic processes $W_{1,t}$ and $W_{2,t}$ are standard Brownian motions on the usual probability-space triple $(\Omega, \mathcal{F}_t, \mathbb{P})$, where $\mathbb{P}$ is the "physical", or the data-generating measure. The correlation between $W_{1,t}$ and $W_{2,t}$ is ρ, which can be written as

$$\mathrm{d}W_{1,t} \mathrm{d}W_{2,t} = \rho \mathrm{d}t. \tag{3}$$

The term $\left(e^{u_t} - 1\right) \mathrm{d}q_t$ in equation (1) is the jump component. The returns jump at t if the Poisson counter (or jump "flag") $\mathrm{d}q_t$ is equal to one, which happens with probability $\lambda_t \mathrm{d}t$. Jump intensity λ_t may change over time. In particular, jumps may be more likely in periods of high volatility. I will therefore allow the intensity to be a linear function of the instantaneous variance,

$$\lambda_t = \lambda_0 + \lambda_1 V_t. \tag{4}$$

The random variable u_t in equation (1) determines the relative magnitude of a jump. The processes $\mathrm{d}q_t$ and u_t are independent, both are serially uncorrelated, and both are uncorrelated with diffusions $\mathrm{d}W_{1,t}$ and $\mathrm{d}W_{2,t}$. Also, neither $\mathrm{d}q_t$ nor u_t are measurable with respect to $\mathcal{F}_t$.

It is reasonable to assume that distribution of jump sizes is *not* concentrated around zero. This is actually not the case in most of the jump-diffusion specifications in the literature: jump sizes are usually modeled as random variables from a unimodal distribution. Since jumps can be both positive and negative, their unconditional expected size is typically close to zero. Unimodal jump-size distributions imply that majority of jumps will be relatively small in magnitude, which is exactly the opposite of their nature. They will also tend to increase kurtosis by adding more mass at the center of the return distribution instead of adding it to the tails. In this way, the effect of fat tails is achieved through normalization of the probability density function. In such specifications, most of the jumps are difficult to distinguish from returns generated by diffusion, which may lead to an overestimation of jump frequencies. Johannes (2004), for example, estimates a jump-diffusion interest rate model and finds jump intensities that are between 0.05 and 0.10, but detects only 5 jumps per year, which corresponds to an intensity of around 0.02.

I will therefore assume that the variable u_t, which determines the size of the jump, comes from a mixture of two normal distributions, one centered around a positive value, the other around a negative value:

$$u_t \;\sim\; p\,\mathcal{N}\left(\ln(1+k)-\omega^2/2,\ \omega^2\right) \;+\; (1-p)\,\mathcal{N}\left(\ln(1-k)-\omega^2/2,\ \omega^2\right). \tag{5}$$

Hence, p has the meaning of the probability that the jump is positive, k is the expected size of a positive jump, while $-k$ is the expected size of a negative jump. At time t, the expected contribution of jumps to return $\mathrm{d}S_t/S_t$ is

$$\mathbb{E}_t\left[\left(e^{u_t}-1\right)\mathrm{d}q_t\right] = \lambda_t\bar{k}\mathrm{d}t,$$

where

$$1+\bar{k} \equiv p(1+k)+(1-p)(1-k).$$

Therefore, the return process is constructed such that the jumps are on average compensated by the last term in equation (1). I use $\mathbb{E}_t(\cdot)$ to denote the conditional expectation given the information available at time t, instead of a more cumbersome $\mathbb{E}(\cdot|\mathcal{F}_t)$.

The outlined model specification has a form of a stochastic volatility jump-diffusion process with bimodal distribution of jump sizes (hereafter: SVJD-B).[2] It has a convenient feature that it contains several popular jump- and pure-diffusion benchmark models as its special cases. For example, by setting $p=1$ and $\lambda_1=0$ we obtain the usual SVJD specification of Bates (1996b) or Bates (2000). A stochastic volatility model without jumps (SV) of Taylor (1986) is obtained by setting all jump parameters (λ_0, λ_1, p, k and ω) to zero. Merton (1976) diffusion model with constant variance is obtained by setting all stochastic-volatility parameters (α, β, ρ, λ_1) to zero, introducing a constant jump intensity ($\lambda_t=\lambda_0$, $\lambda_1=0$) and constraining the distribution of

[2]To the best of my knowledge, the bimodal assumption for the distribution of jump sizes was previously used only in a numerical valuation of real options in Dias & Rocha (2001).

jump sizes to be unimodal ($p = 1$). Finally, the Black & Scholes (1973) model (BS) is obtained by setting all jump parameters to zero, α, β and ρ to zero, and (with a slight abuse of notation) by fixing $V_t = \sigma^2$.

2.2 Estimation Methodology

Estimation of a continuous-time model, such as one given by equations (1)–(2), is never straightforward when we bring it to discretely sampled data. The main difficulty lies in the fact that closed-form expressions for a discrete transition density are seldom available. The presence of unobservable state variables, such as stochastic volatility, makes this task even more arduous. In principle, some form of maximum likelihood estimation might be feasible (see, for example, Lo (1988)), but it is based on computationally very demanding numerical procedures that involve integration of latent variables out of the likelihood function. The problem becomes even more difficult when jumps are introduced into the model.

A number of alternatives to the maximum likelihood technique have been proposed to overcome the issue of computational inefficiency. Examples of simulation-based inference for jump-diffusion models can be found in Andersen *et al.* (2002), Duffie *et al.* (2000) and Chernov *et al.* (1999). Simulation approaches based on the method of moments are a useful tool whenever it is possible to alleviate the problem of inefficient inference, which can be done by careful selection of moment conditions. For example, Pan (2002) uses the simulated method of moments (SMM) of Duffie & Singleton (1993) and matches sample moments with the simulated ones to estimate risk premia embedded in options on a stock market index. The efficient method of moments (EMM) of Gallant & Tauchen (1996) refines the SMM approach by a convenient choice of moment conditions: they are obtained from the expected score of the auxiliary model. The auxiliary model is a discrete-time model whose purpose is to approximate the sample distribution. Hence, there are at least two good features of the EMM approach: first, it will achieve the efficiency of the maximum likelihood technique under reasonable assumptions, and second, the objective function can be used to test for overidentifying restrictions, as with an ordinary generalized method of moments.

Several jump-diffusion models were developed to describe the exchange rate dynamics. Bates (1996b), for example, estimates the parameters of an SVJD model from the prices of Deutsche Mark options traded on the Philadelphia Stock Exchange. More recently, Maheu & McCurdy (2006) proposed a discrete-time model of foreign exchange rate returns with jumps. Their estimation is based on a Markov Chain Monte Carlo technique. Although this method is a powerful inference tool, its implementation always has to be tailored for a particular choice of model, making it difficult to compare with other specifications.

I use the EMM to estimate the proposed SVJD-B model (1)–(2) and to compare it with the alternatives. As pointed out by Andersen *et al.* (2002), the EMM procedure critically relies on the correct specification of the auxiliary model. The auxiliary model should approximate the conditional distribution of the return process as close as possible. If the score of the auxiliary model asymptotically spans the score of the true model, the EMM will be asymptotically efficient (see Gallant & Long (1997) for the proof). Therefore, any auxiliary model should capture the dominant features of the return dynamics in a discrete-time series. Specifically, it should be able to take into account the presence of autocorrelation and heteroskedasticity, as well as to model any excess skewness and kurtosis. A semi-nonparametric (SNP) specification for the auxiliary model by Gallant & Nychka (1987) is based on the notion that higher-order moments of distribution can be captured with a polynomial expansion.

Given that a set of data is stationary, an ARMA term is sufficient to describe the conditional mean, while an ARCH-type term should be able to filter out conditional heteroskedasticity. I choose the EGARCH model of Nelson (1991) in order to capture both heteroskedasticity and potential presence of asymmetric responses of conditional variance to positive and negative returns. Finally, to accommodate the presence of fat tails in the return distribution, I augment the conditional probability density function of the auxiliary model by a polynomial in standardized returns.

The semi-nonparametric (SNP) estimation step is performed via quasi-maximum likelihood technique on the fully specified auxiliary model. I follow Andersen *et al.* (2002) and assume that auxiliary model follows an ARMA(r,m)-EGARCH(p,q)-Kz(K_z)-Kx(K_x) process with a probability distribution function of the form:

$$f_K(y_t|\mathcal{F}_{t-1};\varphi) = \frac{[P_K(z_t,x_t)]^2}{\int[P_K(z,x)]^2\phi(z)\mathrm{d}z}\frac{\phi(z_t)}{\sqrt{h_t}} \tag{6}$$

where $y_t \equiv \ln(S_t/S_{t-1})$ is a vector of log-returns that follows an ARMA(r,m) process

$$y_t = \mu + \sum_{i=1}^{r} b_i y_{t-i} + \varepsilon_t + \sum_{i=1}^{m} c_i \varepsilon_{t-i}. \tag{7}$$

The residuals ε_t are assumed to be normally distributed conditionally on the information available one time step before:

$$\varepsilon_t|\mathcal{F}_{t-1} \ \sim \ \mathcal{N}(0,h_t). \tag{8}$$

The corresponding standardized residuals are $z_t = \varepsilon_t/\sqrt{h_t}$, and x_t is the vector of their lags. The standard normal probability density function is labeled by $\phi(\cdot)$. The conditional variance h_t follows an EGARCH(p,q) process of the form

$$\ln h_t = \omega + \sum_{i=1}^{p} \beta_i \ln h_{t-i} + \sum_{j=1}^{q} \alpha_j \left(|z_{t-j}| - \sqrt{\frac{2}{\pi}} \right) + \sum_{j=1}^{q} \theta_j z_{t-j}. \tag{9}$$

In equation (6), the full set of parameters is labeled by φ. Finally, $P_K(\cdot)$ is a non-parametric polynomial expansion given by

$$P_K(z,x) = \sum_{i=0}^{K_z} \sum_{j=0}^{K_x} \left(a_{ij} x^j \right) z^i, \quad a_{00} = 1. \tag{10}$$

Here, as in Andersen *et al.* (2002), the coefficients in expansion depend on lags x. This expansion is designed to capture any excess kurtosis in returns, but also to accommodate additional skewness that has not already been represented by the EGARCH term. I use the Bayesian information criterion (BIC) to select the best fitting model for each series.

The EMM estimation step works in the following way. Given the set of parameters

$$\psi = \{\mu, \alpha, \beta, \sigma, \rho, \lambda_0, \lambda_1, p, k, \omega\},$$

I simulate the sample of exchange rates $\{\widetilde{S}_t\}_{t=1}^{T_{\text{sim}}}$ and instantaneous variances $\{\widetilde{V}_t\}_{t=1}^{T_{\text{sim}}}$ using the specification given by the continuous-time model (1)–(2). The EMM estimator of model parameters ψ is defined as

$$\widehat{\psi} = \arg\min_{\psi} \ \mathbf{m}(\psi, \widehat{\varphi})' \ \mathbf{W} \ \mathbf{m}(\psi, \widehat{\varphi}), \tag{11}$$

where $\mathbf{m}(\psi, \widehat{\varphi})$ is the expectation of the score function and $\widehat{\varphi}$ is the quasi-maximum likelihood estimate of the set of SNP parameters. The expectation of the score is evaluated as the sample mean across simulations,

$$\mathbf{m}(\psi, \widehat{\varphi}) = \frac{1}{T_{\text{sim}}} \sum_{t=1}^{T_{\text{sim}}} \frac{\partial \ln f_K(\widetilde{y}_t | \mathcal{F}_{t-1}; \widehat{\varphi})}{\partial \varphi},$$

where $\widetilde{y}_t \equiv \ln(\widetilde{S}_t / \widetilde{S}_{t-1})$. The weighting matrix $\mathbf{W}$ is a consistent estimate of the inverse asymptotic covariance matrix of the auxiliary score.

To reduce the effects of discretization, I sample at time intervals of 1/10 of a day. At each run, two antithetic samples were created for the purpose of variance reduction, each of length $100,000 \times 10 + 20,000$. To eliminate the effects of initial conditions, I discard the "burn-in" period of the first 20,000 simulated points. The final sample of $T_{\text{sim}} = 100,000$ daily log-returns, $\{\widetilde{y}_t\}_{t=1}^{T_{\text{sim}}}$, was obtained by adding up the groups of 10 elements in the simulated sample.

3 Estimation Results

3.1 Data

The results are based on average daily interbank spot exchange rates of Euro, British Pound, Japanese Yen and Swiss Franc with respect to the U.S. Dollar, from January 4, 1999 to September 30, 2008, a sample of 2542 observations. All four time series, obtained from Thomson Financial's Datastream, are shown in Figure 2. The JPY/USD exchange rate is expressed per 100 Yens. Table 1 provides summary statistics for the exchange rate levels S_t and the corresponding daily returns, computed as $y_t = \ln(S_t / S_{t-1})$. Daily sampling is chosen in order to capture high-frequency fluctuations in return processes that may be critical for identification of jump components, while avoiding to model the intraday return dynamics, abundant with spurious market microstructure distortions and trading frictions.

Table 1: **Summary Statistics**
Daily interbank spot exchange rates of Euro, British Pound, Japanese Yen and Swiss Franc with respect to the U.S. Dollar, from January 4, 1999 to September 30, 2008 (2542 observations).

Panel A: Daily exchange rate levels				
Currency	Mean	Variance	Skewness	Kurtosis
EUR	1.1511	0.0376	0.2234	2.1992
GBP	1.7103	0.0376	0.0661	1.7385
JPY	0.8774	0.0030	−0.0339	2.2511
CHF	0.7380	0.0121	0.1375	2.1955

Panel B: Daily returns (percent)				
Currency	Mean	Variance	Skewness	Kurtosis
EUR	0.0084	0.3539	−0.0267	4.5420
GBP	0.0040	0.2338	0.0757	4.1778
JPY	0.0026	0.3493	0.2267	4.8656
CHF	0.0088	0.4012	0.1411	4.2532

I perform several preliminary test on the data. The values of skewness and kurtosis in Table 1 indicate that both the levels and returns deviate from normality. This is also confirmed by Jarque-Bera and Kolmogorov-Smirnov tests (not reported), whose p-values are at most of the order of 10^{-3}. Table 2 shows the results of Ljung-Box test for the autocorrelation of returns, up to order 10 (Panel A). The null hypotheses of no autocorrelation in returns cannot be rejected. The absence of a significant short-run return predictability is consistent with high efficiency of the currency market. The autocorrelation in the squared returns is, on the other hand, highly significant in all four series, indicating the presence of heteroskedasticity (Panel B). The correlation coefficients between squared returns and their lags (not reported) are all positive, confirming the notion of clustering – the periods of high volatility are likely to be

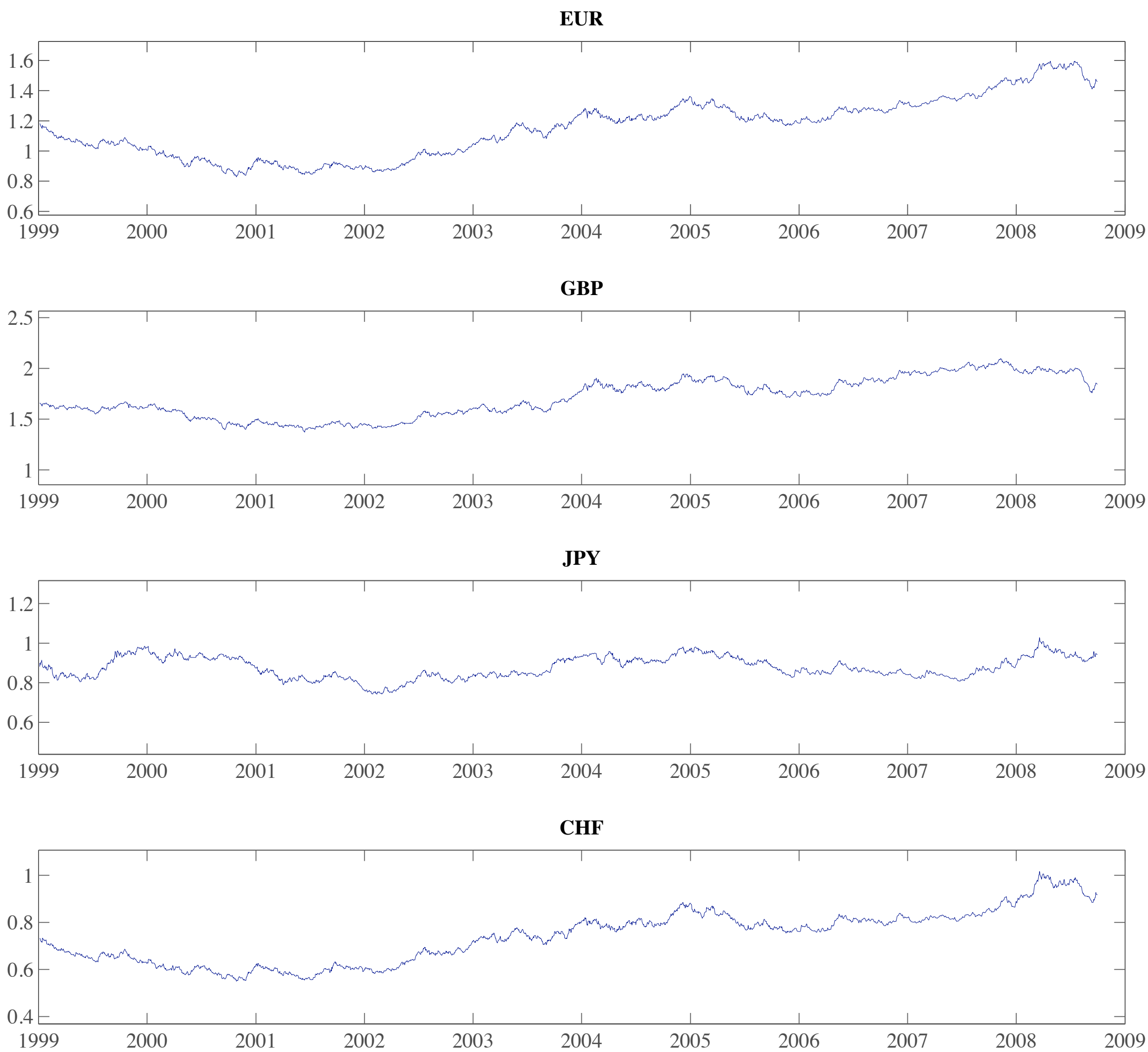

Figure 2: **Daily exchange rate levels:** January 4, 1999 to September 30, 2008. The JPY/USD rate is expressed per 100 Yens.

followed by high volatility.

Table 3 reports the results of the unit root tests. The values of the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) statistics indicate that the unit root hypothesis is convincingly rejected in favor of stationary returns (the critical values of ADF and PP statistics at 5 and 1 percent confidence are -3.41 and -3.96, respectively). The stationarity is a prerequisite for any method of moments approach that

Table 2: **Autocorrelation**
Ljung-Box test for autocorrelation of returns and squared returns up to 10^{th} lag.

Panel A: Autocorrelation of returns		
Currency	Q statistic	p-value
EUR	3.9867	0.9479
GBP	9.4858	0.4867
JPY	6.8611	0.7385
CHF	12.7326	0.2390

Panel B: Autocorrelation of squared returns		
Currency	Q statistic	p-value
EUR	111.5435	$< 10^{-5}$
GBP	105.7946	$< 10^{-5}$
JPY	81.5108	$< 10^{-5}$
CHF	42.7107	$< 10^{-5}$

Table 3: **Stationarity**
Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests for the presence of unit roots, based on the regression

$$y_t = c + \delta t + \phi y_{t-1} + \sum_{L=1}^{10} b_L \Delta y_{t-L} + \varepsilon_t,$$
$$\mathrm{H}_0 : \phi = 1,\ \delta = 0.$$

Currency	ADF statistic	PP statistic
EUR	−15.8257	−50.4768
GBP	−15.2648	−48.0508
JPY	−14.6802	−49.2301
CHF	−15.6345	−50.9751
5% crit. value	−3.41	−3.41
1% crit. value	−3.96	−3.96

is critically relying on stability of the data-generating process.

Finally, I report the results of the Jiang & Oomen (2008) swap-variance test for detection of jumps in returns and squared returns, Table 4. The swap-variance test exploits the impact of jumps on the third and higher order moments of asset returns. The test is based on the statistic

$$\frac{T}{\sqrt{\Omega}}\left(SwV_T - RV_T\right) \;\sim\; \mathcal{N}(0,1),$$

where

$$SwV_T = 2\sum_{t=2}^{T}\left(R_t - y_t\right)$$

is twice the accumulated difference between discretely and continuously compounded returns, $R_t = S_t/S_{t-1} - 1$ and $y_t = \ln(S_t/S_{t-1})$, respectively,

$$RV_T = \sum_{t=2}^{T} y_t^2$$

is the realized variance, and Ω is the asymptotic variance of the test statistic. The robust estimator of Ω is given by

$$\widehat{\Omega} = \frac{1}{9}\frac{\mu_6}{\mu_1^6}\frac{T^3}{T-5}\sum_{s=1}^{T-6}\prod_{t=1}^{6}|y_{s+t}|,$$

with $\mu_k = \mathbb{E}(|y|^k)$. The null hypothesis of the swap-variance test is that S_t follows a process without jumps. Intuitively, the test statistic reflects the cumulative gain of a variance swap replication strategy which is known to be minimal in the absence of jumps but substantial in the presence of jumps. If the underlying process is continuous, the difference between SwV_T and RV_T should asymptotically go to zero. The results of the test indicate that in all four series the jumps in returns are highly significant (Panel A), whereas the jumps in squared returns are not (Panel B). This implies that it is not necessary to overparameterize the model by introducing discontinuities into the process for conditional variance.

Table 4: **Presence of jumps**

Swap-variance jump test, based on the test statistics

$$\frac{T}{\sqrt{\widehat{\Omega}}}\left(SwV_T - RV_T\right) \ \sim\ \mathcal{N}(0,1),$$

where

$$SwV_T = 2\sum_{t=2}^{T}\left(R_t - y_t\right), \quad RV_T = \sum_{t=2}^{T} y_t^2,$$
$$R_t = S_t/S_{t-1} - 1, \quad y_t = \ln(S_t/S_{t-1}),$$
$$\widehat{\Omega} = \frac{1}{9}\frac{\mu_6}{\mu_1^6}\frac{T^3}{T-5}\sum_{s=1}^{T-6}\prod_{t=1}^{6}|y_{s+t}|, \quad \mu_k = \mathbb{E}(|y|^k),$$
$$\mathrm{H}_0 : S_t \text{ follows a process without jumps.}$$

Panel A: Jumps in returns		
Currency	Swap-var stat.	p-value
EUR	-6.9228	$< 10^{-5}$
GBP	-10.9712	$< 10^{-5}$
JPY	-5.9042	$< 10^{-5}$
CHF	-7.9779	$< 10^{-5}$

Panel B: Jumps in squared returns		
Currency	Swap-var stat.	p-value
EUR	-0.0254	0.9797
GBP	-0.0259	0.9794
JPY	-0.0220	0.9824
CHF	-0.0210	0.9833

3.2 Estimation of the auxiliary model

Table 5 reports the results of the SNP step. It shows the quasi-maximum likelihood parameter estimates $\widehat{\varphi}$, along with their standard errors. I ran the estimations across the possible combinations (r, m, p, q, K_x, K_z), allowing each of the parameters to take values between 0 and 10. The selection criterion based on the BIC indicates that the best-fitting auxiliary models have the form ARMA(0,0)-EGARCH(1,1)-Kz(K_z)-Kx(0), with K_z being 8, 7, 6 and 7 for the Euro, Pound, Yen and Franc exchange rate, respectively.[3] Table 5 also shows the total number of SNP parameters n, as well as the optimal values of log-likelihood functions, LL. The absence of the ARMA term is not surprising given that the data exhibit no significant autocorrelation. Also, in all four cases heteroskedasticity is entirely captured by the first lags of conditional variance and return innovations in the EGARCH terms, as $p = 1$ and $q = 1$. The values of EGARCH parameter β governing the persistence are close to the boundary of covariance stationary region, but still significantly within the boundaries. The parameter θ is relatively small and – with the exception of Swiss Franc – statistically insignificant. This indicates that the "leverage" effect does not play such an important role in dynamics of exchange rates. The terms Kx that should take into account heterogeneity in the polynomial expansion are always insignificant ($K_x = 0$), which indicates that the EGARCH leading terms pick up all the serial dependence in the returns.

[3]The actual values of BIC are not reported, but are available upon request.

Table 5: **Estimates of the Auxiliary Model**

	EUR	GBP	JPY	CHF
μ	0.0558	0.0844	−0.0588	0.0737
	(0.0350)	(0.0493)	(0.0443)	(0.0434)
ω	−0.0065	−0.0437	−0.0263	−0.0021
	(0.0034)	(0.0162)	(0.0242)	(0.0035)
β	0.9945	0.9799	0.9731	0.9969
	(0.0018)	(0.0078)	(0.0093)	(0.0016)
α	0.0675	0.0583	0.1239	0.0401
	(0.0119)	(0.0149)	(0.0288)	(0.0096)
θ	0.0047	−0.0023	0.0203	0.0161
	(0.0063)	(0.0080)	(0.0144)	(0.0064)
a_{10}	−0.1275	−0.1671	0.0684	−0.1815
	(0.0588)	(0.0776)	(0.0646)	(0.0606)
a_{20}	−0.1274	−0.0274	−0.1407	−0.0980
	(0.0521)	(0.0582)	(0.1172)	(0.0632)
a_{30}	0.0743	0.0701	−0.0187	0.0852
	(0.0240)	(0.0206)	(0.0161)	(0.0206)
a_{40}	0.0330	0.0119	0.0237	0.0205
	(0.0184)	(0.0122)	(0.0237)	(0.0130)
a_{50}	−0.0141	−0.0130	0.0024	−0.0126
	(0.0039)	(0.0033)	(0.0014)	(0.0035)
a_{60}	−0.0013	0.0004	−0.0004	−0.0002
	(0.0024)	(0.0009)	(0.0017)	(0.0009)
a_{70}	0.0008	0.0006		0.0005
	(0.0002)	(0.0002)		(0.0002)
a_{80}	0.0000			
	(0.0001)			
Model	001180	001170	001160	001170
n	13	12	11	12
LL	9597.85	10048.37	9568.58	9402.97

(Standard errors in parentheses.)

3.3 EMM estimation

Once we have the optimal score parameters $\widehat{\varphi}$ obtained from the SNP model, we can estimate the main step of the EMM procedure. The EMM parameter estimates $\widehat{\psi}$ obtained from the SVJD-B and the competing models are summarized in Tables 6–9. Standard errors are given in the parentheses. Tables also report the results of Hansen's test of overidentifying restrictions: chi-squares, degrees of freedom and p-values.

We can draw several important conclusions from these estimates. First, as expected, stochastic volatility is important: the constant-volatility Merton and Black-Scholes models can be overwhelmingly rejected in all four cases. Second, jumps are statistically significant, since SVJD-B and SVJD specifications outperform the SV model without jumps. The SV model is also strongly rejected at any reasonable level of significance for Euro and Franc. Third, the usual SVJD specification is outperformed by the SVJD-B model with bimodal distribution of jump sizes given by equation (5). The SVJD model may as well be rejected at significance levels less than 0.05. Fourth, the dependence of jump intensity on volatility levels as given by the affine specification (4) is important, but the alternative of constant jump intensity ($\lambda_1 = 0$) cannot be easily rejected. For example, for the Yen exchange rate the restricted model is significant at 0.05 level, while the fully specified SVJD-B model is not. The constant term λ_0 is by an order of magnitude greater than the affine coefficient λ_1. Finally, the correlation between return and volatility is important: the estimated values of ρ are significant and negative. The restriction $\rho = 0$ may not be rejected only for the Euro exchange rate. As suggested by Andersen *et al.* (2002), a negative correlation between return diffusion and volatility can explain part of the skewness in returns.

Table 6: **EMM Estimates: EUR**

Parameter	SVJD-B	SVJD-B $\lambda_1 = 0$	SVJD-B $\rho = 0$	SVJD	SV	Merton	BS
μ	0.0239	0.0287	0.0270	0.0516	0.0573	0.0558	0.0558
	(0.0097)	(0.0029)	(0.0044)	(0.0022)	(0.0025)	(0.0031)	(0.0043)
α	0.0021	0.0021	0.0002	0.0002	0.0019		
	(0.0033)	(0.0061)	(0.0136)	(0.0069)	(0.0019)		
β	0.0063	0.0055	0.0006	0.0018	0.0055		
	(0.0035)	(0.0025)	(0.0041)	(0.0031)	(0.0078)		
σ	0.1078	0.1070	0.0683	0.8174	0.0992	0.4418	0.4453
	(0.0063)	(0.0026)	(0.0049)	(0.0026)	(0.0033)	(0.0044)	(0.0050)
ρ	-0.0255	-0.0262		-0.0236	-0.0262		
	(0.0044)	(0.0044)		(0.0044)	(0.0021)		
λ_0	0.0298	0.0308	0.0248	0.0247		0.0308	
	(0.0053)	(0.0037)	(0.0038)	(0.0060)		(0.0043)	
λ_1	0.0024		0.0000				
	(0.0066)		(0.0136)				
p	0.5227	0.4813	0.3844				
	(0.0153)	(0.0165)	(0.0025)				
k	1.4834	1.4958	1.1948	1.2596		-0.0582	
	(0.0142)	(0.0038)	(0.0045)	(0.0062)		(0.0073)	
ω	0.4413	0.4876	0.3771	0.5271		0.4714	
	(0.0030)	(0.0043)	(0.0022)	(0.0029)		(0.0035)	
χ^2 [$d.f.$]	7.5852 [3]	10.5561 [4]	9.0277 [4]	13.3386 [5]	23.3928 [8]	22.5033 [8]	96.5893 [11]
(p-value)	(0.0554)	(0.0320)	(0.0604)	(0.0204)	(0.0029)	(0.0041)	$< 10^{-5}$

(Standard errors in parentheses.)

Table 7: **EMM Estimates: GBP**

Parameter	SVJD-B	SVJD-B $\lambda_1 = 0$	SVJD-B $\rho = 0$	SVJD	SV	Merton	BS
μ	0.0198	0.0174	0.0277	0.0180	0.1206	0.0844	0.0844
	(0.0138)	(0.0049)	(0.0035)	(0.0123)	(0.0021)	(0.0021)	(0.0026)
α	0.0011	0.0026	0.0019	0.0039	0.0034		
	(0.0099)	(0.0039)	(0.0065)	(0.0108)	(0.0029)		
β	0.0139	0.0263	0.0182	0.0534	0.0448		
	(0.0187)	(0.0061)	(0.0075)	(0.0082)	(0.0025)		
σ	0.2099	0.4469	0.3699	0.7219	0.2027	0.2167	0.2179
	(0.0195)	(0.0067)	(0.0028)	(0.0047)	(0.0026)	(0.0032)	(0.0043)
ρ	-0.0140	-0.0157		-0.0151	-0.0157		
	(0.0205)	(0.0082)		(0.0038)	(0.0016)		
λ_0	0.0300	0.0291	0.0280	0.0312		0.0324	
	(0.0209)	(0.0082)	(0.0082)	(0.0023)		(0.0030)	
λ_1	0.0037		0.0038				
	(0.0091)		(0.0046)				
p	0.3746	0.4760	0.4513				
	(0.0202)	(0.0205)	(0.0030)				
k	1.4241	1.1564	1.1884	1.0728		-0.1800	
	(0.0057)	(0.0060)	(0.0045)	(0.0063)		(0.0124)	
ω	0.3337	0.3516	0.2769	0.3147		0.3192	
	(0.0036)	(0.0034)	(0.0032)	(0.0036)		(0.0038)	
χ^2 [$d.f.$]	4.4774 [2]	7.1153 [3]	14.1125 [3]	10.3601 [4]	17.1286 [7]	32.0357 [7]	28.1604 [10]
(p-value)	(0.1066)	(0.0683)	(0.0028)	(0.0348)	(0.0166)	$< 10^{-5}$	(0.0017)

(Standard errors in parentheses.)

Table 8: **EMM Estimates: JPY**

Parameter	SVJ-B	SVJD-B $\lambda_1 = 0$	SVJD-B $\rho = 0$	SVJD	SV	Merton	BS
μ	0.0128	-0.0048	0.0459	-0.0700	-0.0305	-0.0588	-0.0589
	(0.0215)	(0.0104)	(0.0900)	(0.0500)	(0.0240)	(0.0250)	(0.0280)
α	0.0112	0.0044	0.0126	0.0127	0.0030		
	(0.0181)	(0.0088)	(0.0025)	(0.0047)	(0.0045)		
β	0.0880	0.0446	0.0466	0.0931	0.0146		
	(0.0182)	(0.0081)	(0.0038)	(0.0036)	(0.0030)		
σ	0.3473	0.1729	0.2114	0.3656	0.0779	0.3064	0.3068
	(0.0106)	(0.0075)	(0.0062)	(0.0034)	(0.0024)	(0.0026)	(0.0028)
ρ	-0.0276	-0.0287		-0.0278	-0.0263		
	(0.0115)	(0.0082)		(0.0037)	(0.0033)		
λ_0	0.0263	0.0242	0.0262	0.0261		0.0260	
	(0.0059)	(0.0078)	(0.0085)	(0.0036)		(0.0090)	
λ_1	0.0045		0.0021				
	(0.0208)		(0.0081)				
p	0.6754	0.6616	0.5810				
	(0.0488)	(0.0519)	(0.0121)				
k	1.5733	1.6358	1.8133	1.7183		0.3912	
	(0.0270)	(0.0044)	(0.0057)	(0.0065)		(0.0061)	
ω	0.5099	0.4851	0.4355	0.4008		0.4535	
	(0.0102)	(0.0262)	(0.0128)	(0.0143)		(0.0134)	
χ^2 $[d.f.]$	4.4367 [1]	5.3276 [2]	9.4297 [2]	9.2045 [3]	14.3776 [6]	17.6237 [6]	24.3500 [9]
(p-value)	(0.0352)	(0.0697)	(0.0090)	(0.0267)	(0.0257)	(0.0072)	(0.0038)

(Standard errors in parentheses.)

Table 9: **EMM Estimates: CHF**

Parameter	SVJD-B	SVJD-B $\lambda_1 = 0$	SVJD-B $\rho = 0$	SVJD	SV	Merton	BS
μ	0.0077	0.0190	0.0424	0.0056	0.0259	0.0737	0.0743
	(0.0354)	(0.0038)	(0.0054)	(0.0051)	(0.0019)	(0.0020)	(0.0022)
α	0.0042	0.0012	0.0009	0.0007	0.0011		
	(0.0108)	(0.0055)	(0.0051)	(0.0028)	(0.0042)		
β	0.0147	0.0100	0.0115	0.0017	0.0027		
	(0.0128)	(0.0088)	(0.0035)	(0.0088)	(0.0046)		
σ	0.6100	0.4101	0.0851	0.1175	0.0935	0.1118	0.1128
	(0.0193)	(0.0060)	(0.0028)	(0.0037)	(0.0036)	(0.0039)	(0.0041)
ρ	-0.0216	-0.0215		-0.0210	-0.0220		
	(0.0093)	(0.0048)		(0.0027)	(0.0026)		
λ_0	0.0356	0.0361	0.0324	0.0310		0.0344	
	(0.0204)	(0.0040)	(0.0042)	(0.0026)		(0.0043)	
λ_1	0.0059		0.0041				
	(0.0365)		(0.0022)				
p	0.5625	0.6271	0.5249				
	(0.0592)	(0.0641)	(0.0038)				
k	1.5652	1.5771	1.6466	1.5651		0.1941	
	(0.0050)	(0.0049)	(0.0070)	(0.0074)		(0.0074)	
ω	0.4053	0.4192	0.4230	0.3967		0.4408	
	(0.0355)	(0.0058)	(0.0057)	(0.0062)		(0.0061)	
χ^2 [$d.f.$]	5.1439 [2]	9.4681 [3]	15.2722 [3]	10.3886 [4]	25.5051 [7]	20.8516 [7]	47.2409 [10]
(p-value)	(0.0764)	(0.0237)	(0.0016)	(0.0344)	(0.0006)	(0.0040)	$< 10^{-5}$

(Standard errors in parentheses.)

The values of the leading intensity term λ_0 roughly indicate that jumps should on average occur between 7 and 10 times per year, depending on the exchange rate. Although jumps are rare, their significance implies that they cannot be ruled out. Positive jumps are more likely on average, with the exception of the British Pound, where about 63 percent of jumps are negative. The unconditional mean of jump sizes, $\bar{k} = (2p-1)k$, is close to zero and positive, except for the Pound, where $p < 0.5$. This asymmetry captures a part of the skewness of the unconditional return distribution. The confidence bounds for jump sizes can be obtained from the values of the standard deviation ω. For example, positive jumps in the Euro exchange rate happen with probability 0.52 and have magnitudes that are in the 95-percent confidence interval of $[0.61,\ 2.36]$ percent.

Using the estimated parameters, we can infer the ex-post probability of a jump on a given date implied by the actual data. Following Johannes (2004), I use a Gibbs sampling technique to compute the filtering distribution of jump times and jump sizes. The Gibbs sampler iteratively samples from the filtering distribution of variances

$$\pi(V_{t+\Delta t}|V_t, q_{t+\Delta t}, u_{t+\Delta t}, y_{t+\Delta t}, y_t; \widehat{\psi}),$$

the filtering distribution of jump times

$$\pi(q_{t+\Delta t}|u_{t+\Delta t}, y_{t+\Delta t}, y_t, V_{t+\Delta t}, V_t; \widehat{\psi}),$$

and the filtering distribution of jump sizes

$$\pi(u_{t+\Delta t}|q_{t+\Delta t}, y_{t+\Delta t}, y_t, V_{t+\Delta t}, V_t; \widehat{\psi}),$$

all of which are know distributions, where $\{y_t\}_{t=1}^{T}$ is the observed time series of daily returns and $\widehat{\psi}$ are the estimated SVJD-B parameters. In each iteration j, the algorithm produces a sequence $\{\{V_t^{(j)}\}_{t=1}^T,\ \{q_t^{(j)}\}_{t=1}^T,\ \{u_t^{(j)}\}_{t=1}^T\}$ of conditional variances, jump flags and jump sizes, which are draws from the joint distribution $\pi(V_{t+\Delta t}, q_{t+\Delta t}, u_{t+\Delta t}|y_{t+\Delta t}, y_t, V_t; \widehat{\psi})$. The algorithm converges quickly since there is no parameter uncertainty. Hence, I work with at most $10,000$ iteration steps and discard the "burn-in" period of the first 2,000 iterations.

Figures 3–6 display the results. They show daily returns y_t (top panel), jump probabilities (middle panel) and ex-post jump sizes (bottom panel), for the four exchange rates between January 3, 2005 and September 30, 2008. The algorithm identified numerous observations that have a high probability of being a jump. The average number of events with jump probability over 0.5 is roughly between 8 per year (for GBP) and 11 per year (for EUR), which is close to the values obtained from the EMM estimates over the full samples. The bimodal nature of the jump size distribution in the SVJD-B model guarantees that most of the identified jumps will be significant in size. This is an important feature of the model. For example, when the probability of a jump in the Euro exchange rate is greater than 0.5, the expected sizes fall within two bounds: the negative one, $[-1.45, -0.41]$ percent, and the positive one, $[0.53, 1.72]$ percent. In the usual SVJD specification with unimodal distribution of jump sizes most of the jumps are difficult to identify. This is because majority of them have a magnitude that is relatively close to the unconditional expectation, which is often very small.

Some jumps are isolated events, while others tend to cluster and lead to higher volatility and even more jumps. The highest concentration of jumps is in 2008, of which most coincide with the events related to the sub-prime mortgage crisis. Other jumps often coincide with the important news related to macroeconomy or asset markets. Consider, for example, the Euro exchange rate (Figure 3). Eight jumps happened on the dates when the U.S. Commerce Department issued reports about trade balance, unemployment levels, retail sales or GDP growth. Five jumps coincide with the announcements by the U.S. Federal Reserve or the European Central Bank regarding monetary policy, and two of them with important fiscal policy moves made by the U.S. Senate. Ten jumps coincide with unusually large stock market movements in the United States or Europe, three with the unexpected earnings announcements by some of the major U.S. corporations, and one with the Société Générale \$7 billion trading fraud. The strong co-movement of the currency market and the stock market is consistent with the findings of Cao (2001). These results, although far from being conclusive, reinforce the intuition based on Merton (1976, 1990) that jumps provide a mechanism through which unanticipated information about the most important determinants of the underlying process enter the market.

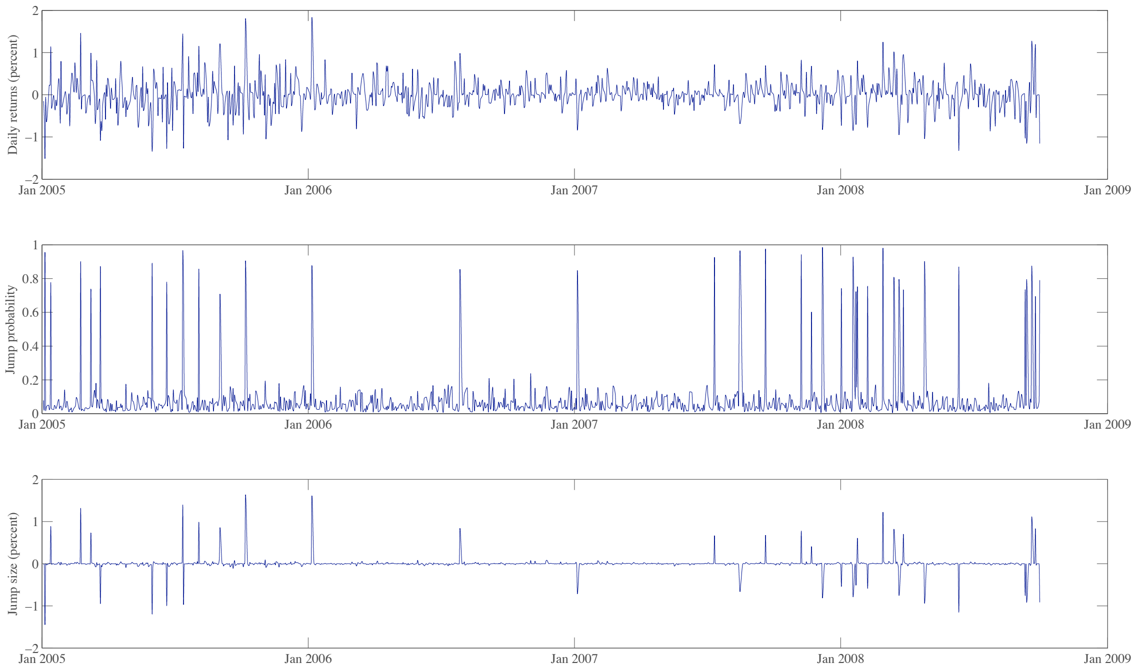

Figure 3: **EUR/USD exchange rate:** Returns, ex-post jump probabilities and expected jump sizes for January 2005–September 2008.

Figure 4: **GBP/USD exchange rate:** Returns, ex-post jump probabilities and expected jump sizes for January 2005–September 2008.

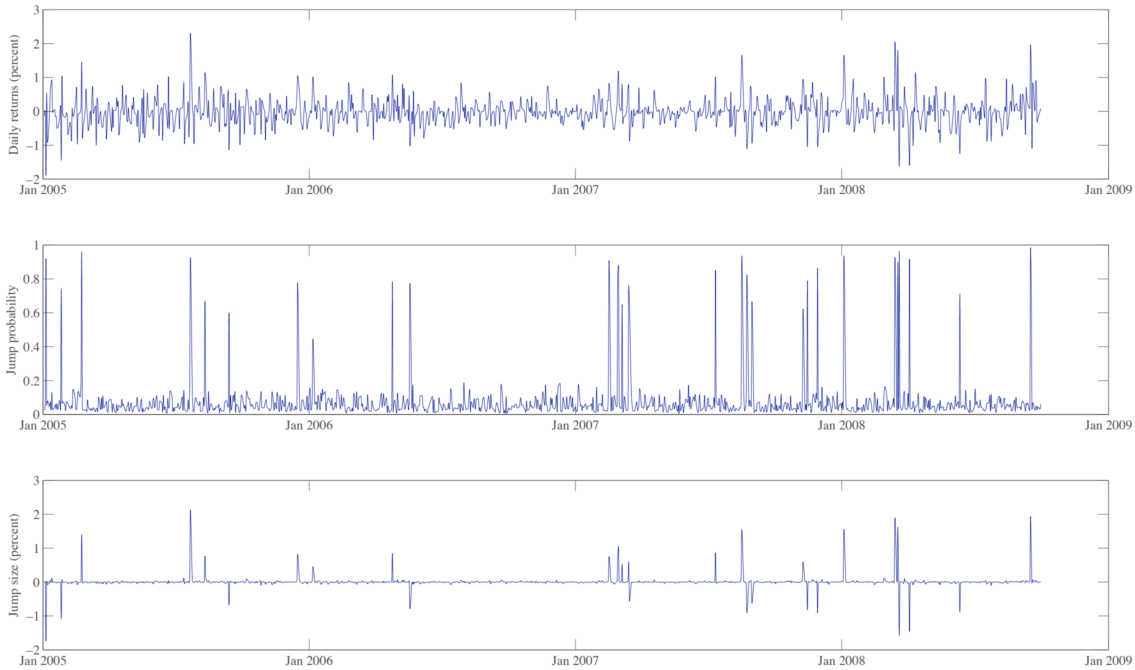

Figure 5: **JPY/USD exchange rate:** Returns, ex-post jump probabilities and expected jump sizes for January 2005–September 2008.

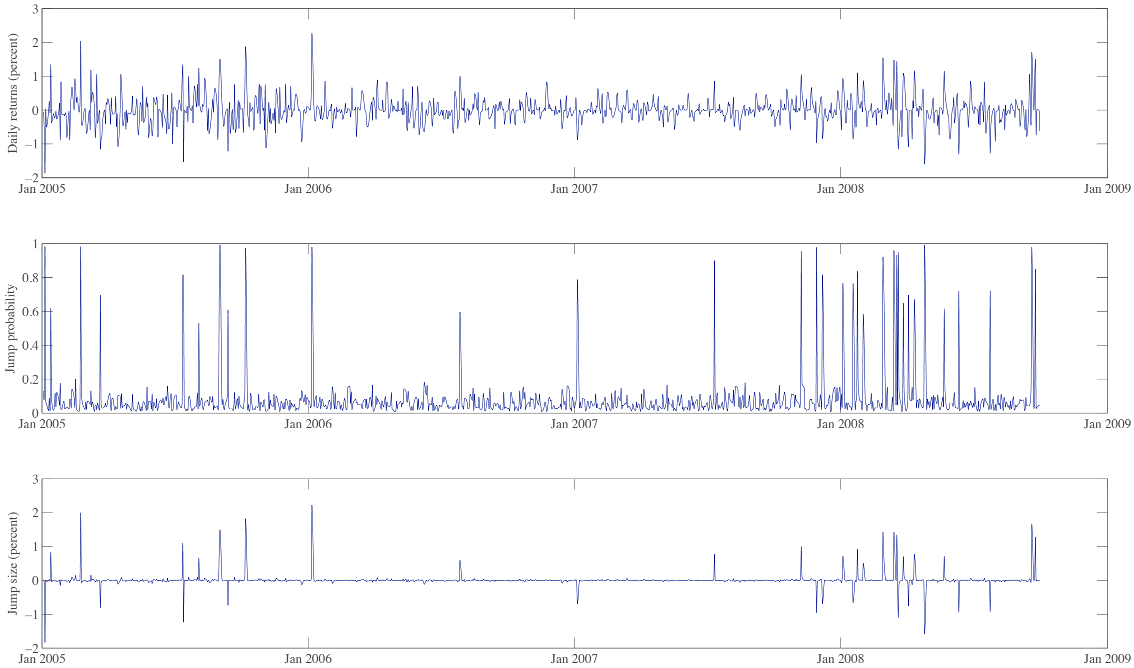

Figure 6: **CHF/USD exchange rate:** Returns, ex-post jump probabilities and expected jump sizes for January 2005–September 2008.

4 Option pricing implications

4.1 The impact of jumps on implied volatility patterns

The main empirical issue in option pricing is to find an appropriate model that will be consistent both with the observed dynamics of the underlying asset as well as with the observed option prices. The U-shaped patterns of implied volatilities, the so-called "volatility smiles", obtained from the actual data are difficult to reconcile with a great number of return models. This is also true for exchange rates, where any specification that does not allow for jumps fails to accommodate observed implied volatility patterns, even when stochastic nature of volatility is taken into account.

In this section I illustrate the effect of jumps on currency option prices. A suitable property of the SVJD-B model is that it can yield a closed-form solution for the price of European-style options. Options can be priced if the model specification is written in the risk-neutral form. Introducing the usual change of measure, the risk-neutral counterparts of the processes for the return and the instantaneous variance, equations (1) and (2), become

$$
\begin{aligned}
\frac{\mathrm{d}S_t}{S_t} &= \mu_t^* \mathrm{d}t + \sqrt{V_t}\,\mathrm{d}W_{1,t}^* + \left(e^{u_t} - 1\right)\mathrm{d}q_t - \lambda_t^* \bar{k}^* \mathrm{d}t, && (12) \\
\mathrm{d}V_t &= \left(\alpha - \beta_t^* V_t\right)\mathrm{d}t + \sigma\sqrt{V_t}\,\mathrm{d}W_{2,t}^*, && (13)
\end{aligned}
$$

where $\mu_t^* = r_t - r_t^f$ is the domestic-foreign interest rate differential. Stochastic processes $W_{1,t}^*$ and $W_{2,t}^*$ are now standard Brownian motions under the risk-neutral probability measure $\mathbb{P}^*$, having the same correlation coefficient as under the physical measure $\mathbb{P}$, that is $\mathrm{d}W_{1,t}^*\mathrm{d}W_{2,t}^* = \rho \mathrm{d}t$. The mean-reversion speed of the instantaneous variance β_t^* and the expected jump size $\lambda_t^* \bar{k}^* \mathrm{d}t$ depend on the market prices of volatility and jump risk, respectively. The explicit relationships are derived in Appendix A. The instantaneous risk premia are:

$$
\begin{aligned}
\text{premium for the return diffusion risk} &= \mu - \mu_t^*, \\
\text{premium for the volatility risk} &= (\beta - \beta_t^*)V_t, \\
\text{overall premium for the jump risk} &= \lambda_t \bar{k} - \lambda_t^* \bar{k}^*.
\end{aligned}
$$

The overall jump-risk premium consists of the combined premia for the uncertainty about the arrival of a jump and the uncertainty about the size of a jump.

At time t, the price of a European-style call option with the value of the underlying exchange rate equal to S_t, time to maturity τ and strike price X, is given by

$$C_t(S_t, V_t, \tau, X; \widehat{\psi}) = e^{-r_t^f \tau} S_t P_1(S_t, V_t, \tau, X; \widehat{\psi}) - e^{-r_t \tau} X P_2(S_t, V_t, \tau, X; \widehat{\psi}). \qquad (14)$$

Closed-form expressions for the functions P_1 and P_2 are given in Appendix B.

Various effects of stochastic volatility and jumps on option prices are illustrated in Figures 7–9. The graphs show generic examples, calculated for European-style call options on EUR/USD exchange rate. The curves represent the Black-Scholes implied volatilities

$$\sigma_{\text{imp}} = \text{BSImpVol}(S_t, C_t, r_t, r_t^f, \tau, X). \qquad (15)$$

The implied volatilities σ_{imp} were obtained numerically, by substituting the values of C_t calculated with the formula (14) into equation (15). The set of parameters $\widehat{\psi}$ in (14) are the EMM estimates given in Table 6. The independent variable in Figures 7–9 is the relative moneyness, defined as the ratio of intrinsic value of option to the underlying exchange rate, i.e. $(S_t - X)/S_t$. All option prices C_t are computed for $S_t = 1.1512$, the sample average of the EUR/USD exchange rate. The U.S. and the Eurozone risk-free interest rates are set to $r_t = 0.02$ and $r_t^f = 0.05$, respectively. The instantaneous volatility $\sqrt{V_t}$ is fixed at the annualized long-run mean of 11.1433 percent.

Figure 7 displays the pricing effect of stochastic volatility and jumps, when there is no premium for volatility and jump risk ($\beta_t^* = \beta$, $\lambda_t^* = \lambda_t$ and $\bar{k}^* = \bar{k}$). The SV model produces a "smirk" pattern (dashed line), which is more pronounced for shorter maturities. This is indicative of a model in which the probability that the call option price will change significantly is low if the option is deep out of the money. The smirk effect wanes with maturity since the probability of moving towards higher prices increases with the remaining life of the option, while at the same time the probability of staying in the money decreases. In the SVJD-B model (full line), the jump component adds an upward tilt to the implied volatility, creating a familiar "smile" pattern. The smile virtually disappears at longer maturities. This effect has the following simple intuition. Jumps are not important for options with longer maturities, as they tend to be compensated in the long run. However, in the short run, the chance for a compensation is small. Therefore, jumps will make an impact on price as maturity date approaches: a deep-out-of-the-money option will have a non-negligible probability of ending up in the money only if the underlying exchange rate has a tendency to make sudden large jumps.

Figure 8 shows the effect of volatility risk premium implied by the SVJD-B model when jump risks premium is set to zero ($\lambda_t^* = \lambda_t$ and $\bar{k}^* = \bar{k}$). The instantaneous premium for volatility risk is measured by the difference between the speed of mean reversion β and its risk neutral counterpart β_t^*. I set the premium to 0 (full lines), 2 percent (dashed lines) and -2 percent (dotted lines). The graphs indicate that the volatility premium has little to no effect on short-maturity options. This is because unexpected changes of the underlying exchange rate over short time periods are mostly picked up by jumps, and if the jump risk premium is zero the exposure to the volatility risk alone has a negligible effect on option prices. At longer maturities, the exchange rate has more time to drift across the moneyness and hence the volatility risk becomes increasingly important. Positive premia decrease the long-run mean of the risk-neutral volatility, pushing the option prices down, and vice versa.

The impact of jump risk premium is shown in Figure 9. Now, the volatility premium implied by the SVJD-B model is set to zero ($\beta_t^* = \beta$), while the risk-neutral jump intensities take the values $\lambda_t^* = \lambda_t = 0.03$ (full line), $\lambda_t^* = 0.05$ (dashed line) and

$\lambda_t^* = 0.07$ (dotted line). The risk-neutral expected jump size is set equal to its "physical" value, $\bar{k}^* = \bar{k} = 0.067$ percent. These values imply annual jump risk premia of 0, 0.5 and 1.0 percent, respectively. Even with relatively small premia, the effects are significant: a change in the risk-neutral jump intensity produces the twists in volatility smiles. The twists are more pronounced at short option maturities and show an asymmetric behavior. First, they are directed upward for out-of-the-money options and downward for in-the-money options. Second, the increase in implied volatility of out-of-the-money options is greater than the decrease of in-the-money options. A positive jump risk premium implies that the buyers require to be compensated for holding an option that is in the money to account for the risk of a negative jump. At the same time, they are willing to pay more for an out-of-the-money option, since higher jumps probabilities increase the chance to profit.

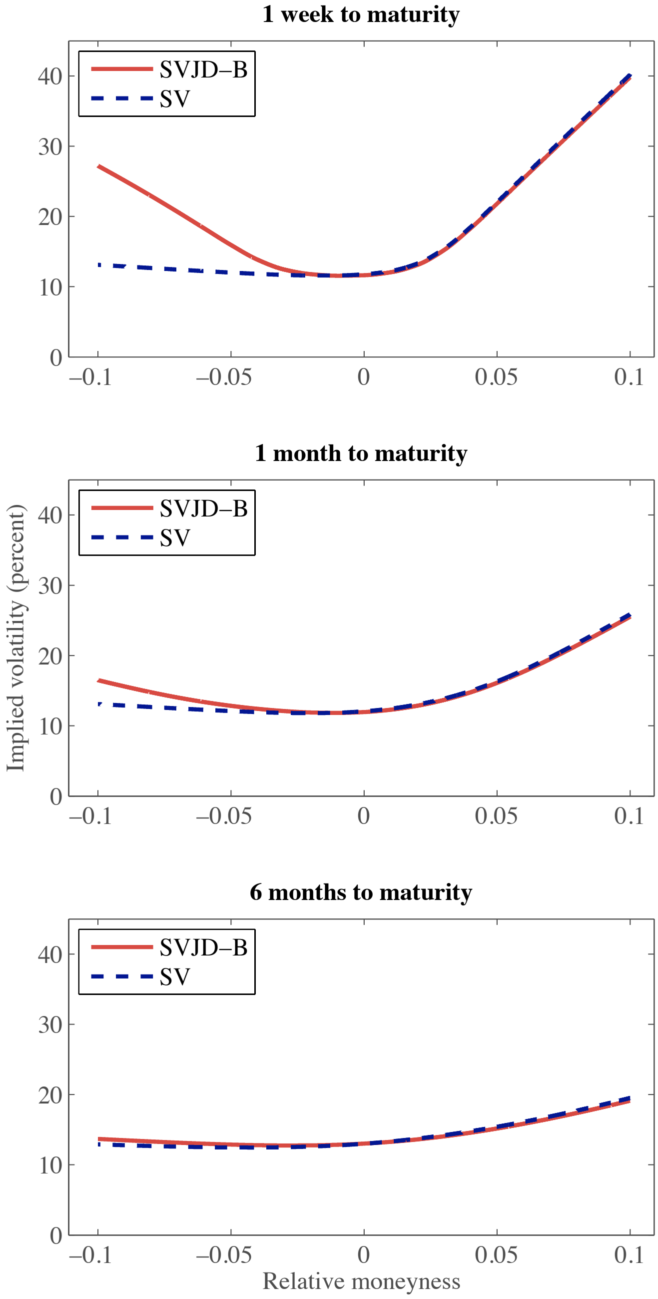

Figure 7: **The effect of stochastic volatility and jumps on option prices.** Black-Scholes implied volatilities are calculated from option prices generated by SVJD-B and SV models for the EUR/USD exchange rate. Model parameters are given in Table 6. The risk premia for the volatility and jump risks are set to zero. Panels display different times to maturity: 1 week, 1 month and 6 months.

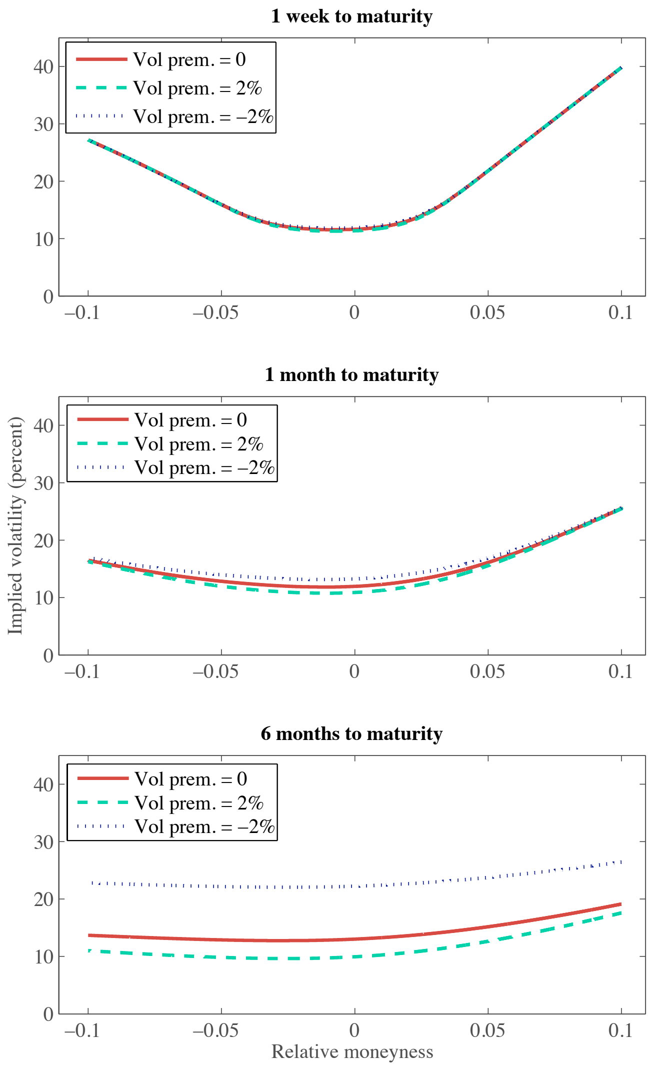

Figure 8: **The effect of volatility risk premium on option prices.** Black-Scholes implied volatilities are calculated from option prices generated by the SVJD-B model for the EUR/USD exchange rate. Model parameters are given in Table 6. Annual volatility risk premia are set to 0, 2 and −2 percent. Panels display different times to maturity: 1 week, 1 month and 6 months.

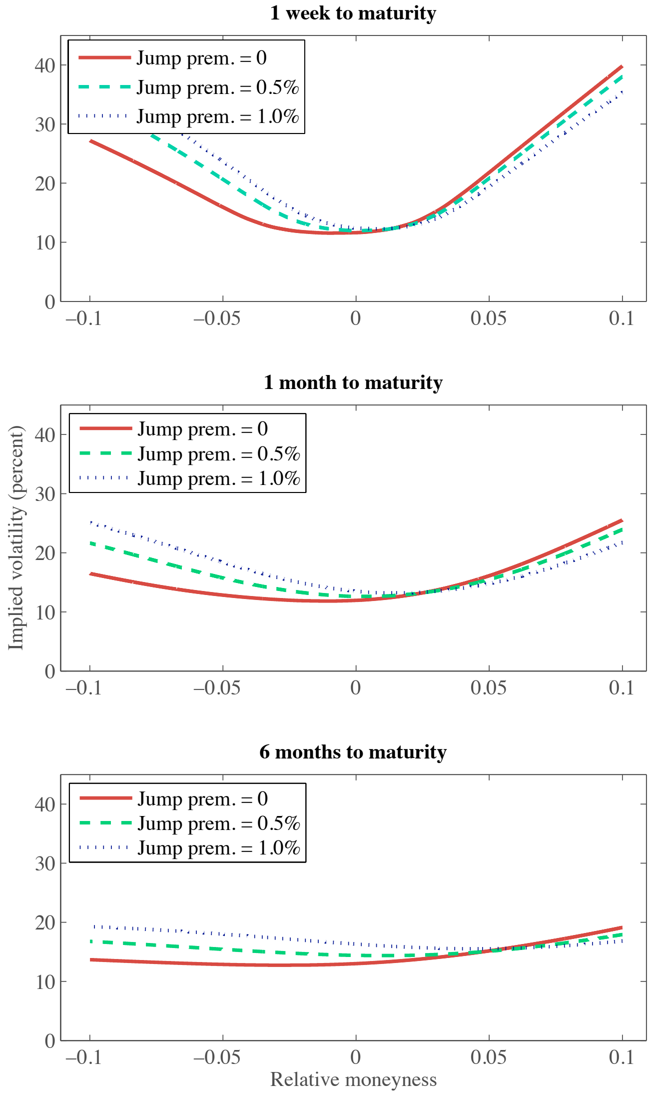

Figure 9: **The effect of jump risk premium on option prices.** Black-Scholes implied volatilities are calculated from option prices generated by the SVJD-B model for the EUR/USD exchange rate. Model parameters are given in Table 6. Annual jump risk premia are set to 0, 0.5 and 1.0 percent. Panels display different times to maturity: 1 week, 1 month and 6 months.

4.2 Risk premia and volatility smiles implicit in the cross-sectional currency options data

The SVJD-B model can fully accommodate the implied volatility patterns observed in the actual data. As an illustration, I use a cross section of European-style call options on Euro that were traded on the Philadelphia Stock Exchange (PHLX) on August 6, 2008. The PHLX currency options are settled in U.S. Dollars and expire on Saturday following the third Friday of the month. There were six available maturities: August 2008, September 2008, October 2008, December 2008, March 2009 and June 2009. The underlying exchange rate was $S_t = 1.5409$ and the available strikes went from 1.2700 to 1.6600, in steps of 0.0050, although some strike/maturity combinations had no open interest. There were 247 options in the cross section in total.

In order to match the model-implied options prices with the observed ones we need the risk-neutral parameter estimates. I use the yield on 3-month Treasury bill as a proxy for the U.S. risk-free rate and the 3-month Euribor as a proxy for the Eurozone risk-free rate. Their respective values on August 6, 2008 were $r_t = 1.4800$ percent and $r_t^f = 5.0289$ percent. Hence, the annualized risk-neutral drift rate was $\mu_t^* = -3.5489$ percent. This implies an annual premium for the return diffusion risk of 12.28 percent.

The remaining risk-neutral parameters, β_t^*, λ_0^*, λ_1^*, $\bar{k}^*$, as well as the instantaneous variance, V_t, can be obtained by solving

$$\begin{aligned} &\min_{\{V_t,\beta_t^*,\lambda_0^*,\lambda_1^*,\bar{k}^*\}} \sum_i w_i(\text{BSImpVol}_i^{\text{model}} - \text{BSImpVol}_i^{\text{data}})^2, \\ &w_i = \left(C_i^{\text{ask}} - C_i^{\text{bid}}\right)^{-1}. \end{aligned} \tag{16}$$

The estimator is designed to minimize the weighted squared difference between the Black-Scholes implied volatilities obtained from the data and the SVJD-B model. For every contract i, the point estimates of $\mathrm{BSImpVol}_i^{\mathrm{data}}$ are obtained from the average values of volatilities implied by the bid and the ask price. To account for the differences in liquidity, the weights w_i are set equal to the reciprocal of the bid-ask spread of a given option contract. In this way, the contracts with higher liquidity will carry more weight in the estimation. The results of the optimization (16) are given in the left panel of Table 10. The Pearson's chi-square statistic indicates that the fit is highly significant. Figure 10 displays the market- and model-implied volatilities for four selected maturities. The error bars correspond to implied volatilities calculated from the bid and ask market prices, while the smooth lines are obtained from the SVJD-B model using the parameter estimates given in Tables 6 and 10. Parameter values imply annual risk premia of −2.30 and 0.16 percent for the volatility and jump risk, respectively (see the right panel of Table 10).

Table 10: **Option-implied parameters**
The left panel shows the instantaneous variance and risk-neutral parameters estimated from the cross section of currency option prices that were traded on PHLX on August 6, 2008. The right panel shows the corresponding risk premia.

Parameter	Value
V_t	0.0106
	(0.0015)
β_t^*	0.0248
	(0.0047)
λ_0^*	0.0332
	(0.0017)
λ_1^*	0.0027
	(0.0002)
$\bar{k}^*$	0.0007
	(0.0001)
$\chi^2[246]$	0.2973

(Standard errors in parentheses.)

Premium	Value (%)
Return diff. risk	12.28
	(4.98)
Volatility risk	−2.30
	(0.76)
Jump risk	0.16
	(0.03)

(Standard errors in parentheses.)

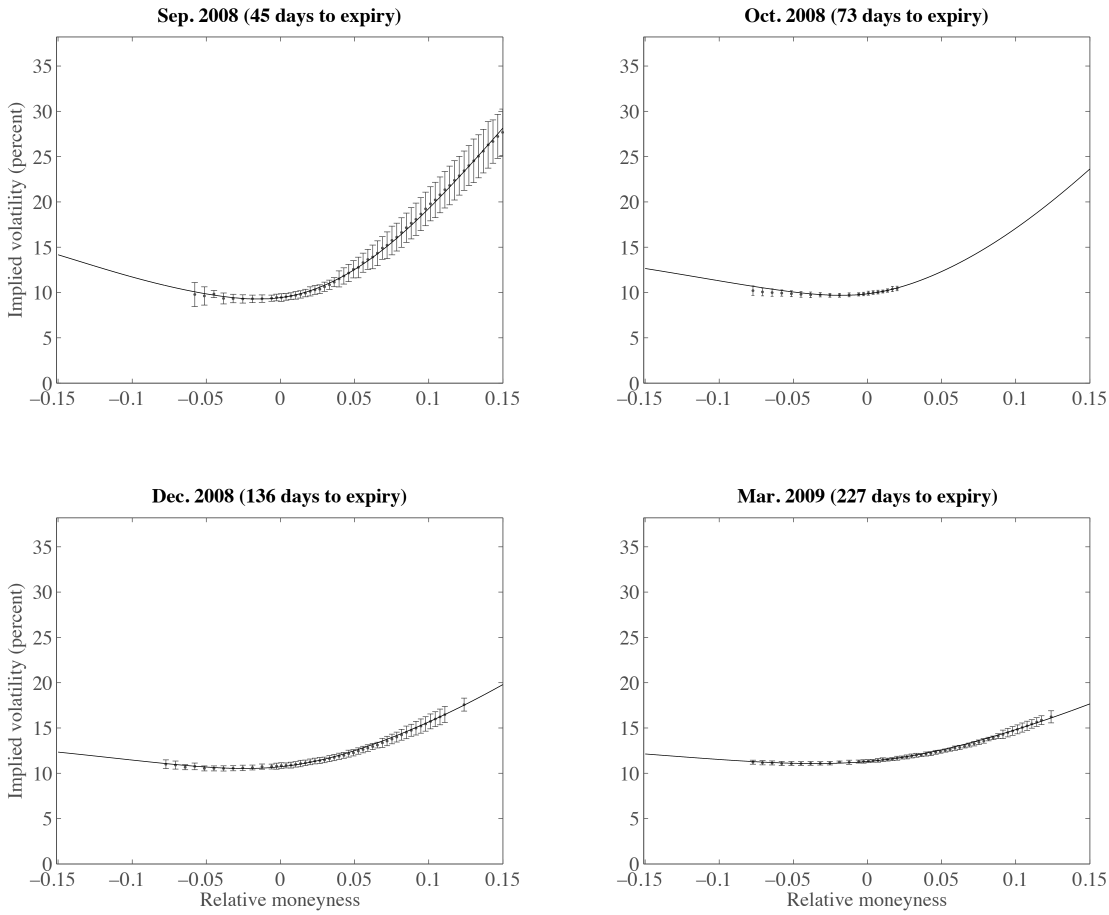

Figure 10: **Black-Scholes market- and model-implied volatilities.** Four selected maturities of European-style call option contracts on Euro. The error bars correspond to implied volatilities calculated from the bid and ask market prices quoted on PHLX on August 6, 2008. The smooth lines are obtained from the proposed SVJD-B model with instantaneous variance and risk-neutral parameters given in Table 10. Parameter values imply a volatility risk premium of -2.30 percent and a jump risk premium of 0.16 percent.

The premium for the return diffusion risk has the highest absolute value of the three, which is plausible given that the diffusion is responsible for most of the everyday changes. Volatility risk premium is negative and significant. The negative premium is a sign that investors are willing to pay more for exposure to the volatility uncertainty, which is reasonable given that higher volatility increases the option premium. The negative volatility risk premium is consistent with the findings of Bates (1996b). It is also implied in the prices of options on stock market indices (see, for example, Chernov & Ghysels (2000) or Pan (2002)). Finally, the jump risk premium is positive and significant, although an order of magnitude smaller than the volatility premium. Since jumps are very rare this is not surprising. However, the statistical significance of the jump risk premium indicates that the fear of jumps is important and seems to be priced by the market.

5 Conclusion

This paper confirms the crucial role of stochastic volatility and jumps in exchange rate processes, at least in the four major U.S. Dollar-based spot exchange rates. The inference procedure based on the efficient method of moments shows that all pure-diffusion models are misspecified. These models are not able to capture the events in the tails of return distributions nor to accommodate the implied volatility patterns obtained from the actual options data. A stochastic volatility model with jump sizes from a bimodal distribution is able to fully remove the misspecification and yield an option pricing formula.

The filtering distributions of jump times inferred from the data indicate that jumps occur in irregular patterns, on average between eight and eleven times a year, depending on the exchange rate. In general, the jump probability weakly depends on volatility. On the other hand, jump events tend to coincide with the arrival of important news to the currency market. They also appear to be more frequent in the periods of turbulence in the stock market. This observation points to the importance of a deeper understanding of jumps in foreign exchange rates that goes beyond statistical significance.

Finally, jumps have a large impact on the prices of foreign currency options. They remove the distinct asymmetry of Black-Scholes implied volatility patterns characteristic for models without jumps. Moreover, the proposed general model is capable to accommodate the smile patterns observed in the actual data. Estimates of the risk-neutral model parameters obtained from the cross-sectional options data indicate that jump risk appears to be priced by the market.

Appendix A: The risk-neutral version of the model

Given that the diffusion and the jump process are independent of each other, we can split the return dynamics into the pure-diffusion part and the pure-jump part:

$$\frac{\mathrm{d}S_t}{S_t} = \left(\frac{\mathrm{d}S_t}{S_t}\right)_{\mathrm{diff}} + \mathrm{d}J_t, \tag{17}$$

where

$$\left(\frac{\mathrm{d}S_t}{S_t}\right)_{\mathrm{diff}} = \mu \mathrm{d}t + \sqrt{V_t}\ \mathrm{d}W_{1,t} \tag{18}$$

and

$$\mathrm{d}J_t = \left(e^{u_t} - 1\right)\mathrm{d}q_t - \lambda_t \bar{k}\mathrm{d}t. \tag{19}$$

Let us focus on the diffusion part first. Pure-diffusion return (18) and the instantaneous volatility V_t follow a joint Brownian diffusion, since W_1 and W_2 are correlated. Define

$$\mathrm{d}\mathbf{W}_t = \begin{bmatrix} \sqrt{V_t}\,\mathrm{d}W_{1,t} \\ \sigma\sqrt{V_t}\,\mathrm{d}W_{2,t} \end{bmatrix}, \tag{20}$$

for all t. To find the risk-neutral equivalent $\mathrm{d}\mathbf{W}^*$ of (20) that would be a martingale under an equivalent measure $\mathbb{P}^*$, we first write the Radon-Nikodým derivative of $\mathbb{P}^*$ with respect to the physical measure $\mathbb{P}$:

$$\frac{\mathrm{d}\mathbb{P}^*}{\mathrm{d}\mathbb{P}} = \exp\left[-\int_0^t \boldsymbol{\xi}_s \cdot \mathrm{d}\mathbf{W}_s - \frac{1}{2}\int_0^t \left(\boldsymbol{\xi}_s \cdot \mathrm{d}\mathbf{W}_s\right)\left(\mathrm{d}\mathbf{W}_s \cdot \boldsymbol{\xi}_s\right)\right],$$

where

$$\boldsymbol{\xi}_s = \begin{bmatrix} \xi_{1,s} \\ \xi_{2,s} \end{bmatrix}$$

is predictable at s (see Bingham & Kiesel (2004)). Then, by Girsanov's theorem, a $\mathbb{P}^*$-Brownian motion has the form

$$\mathrm{d}\mathbf{W}_t^* = \mathrm{d}\mathbf{W}_t\left(1 + \mathrm{d}\mathbf{W}_t \cdot \boldsymbol{\xi}_t\right).$$

Therefore,

$$\mathrm{d}\mathbf{W}_t \quad = \quad \mathrm{d}\mathbf{W}_t^* - \begin{bmatrix} 1 & \rho\sigma \\ \rho\sigma & \sigma^2 \end{bmatrix}\begin{bmatrix} \xi_{1,t} \\ \xi_{2,t} \end{bmatrix} V_t\mathrm{d}t,$$

which implies that we can substitute

$$\begin{bmatrix} \sqrt{V_t}\ \mathrm{d}W_{1,t} \\ \sigma\sqrt{V_t}\ \mathrm{d}W_{2,t} \end{bmatrix} = \begin{bmatrix} \sqrt{V_t}\ \mathrm{d}W_{1,t}^* - (\xi_{1,t} + \rho\sigma\xi_{2,t})\, V_t \mathrm{d}t \\ \sigma\sqrt{V_t}\ \mathrm{d}W_{2,t}^* - (\rho\sigma\xi_{1,t} + \sigma^2\xi_{2,t})\, V_t \mathrm{d}t \end{bmatrix}$$

into (1) and (2). Hence, the processes

$$\left(\frac{\mathrm{d}S_t}{S_t}\right)_{\mathrm{diff}} = \mu_t^* \mathrm{d}t + \sqrt{V_t}\ \mathrm{d}W_{1,t}^*$$

and

$$(\alpha - \beta_t^* V_t)\, \mathrm{d}t + \sigma\sqrt{V_t}\ \mathrm{d}W_{2,t}^*$$

both contain diffusions that are (jointly) martingales under $\mathbb{P}^*$, as long as

$$\mu_t^* = \mu - (\xi_{1,t} + \rho\sigma\xi_{2,t})\, V_t$$

and

$$\beta_t^* = \beta - \xi_t,$$

where $\xi_t \equiv \rho\sigma\xi_{1,t} + \sigma^2\xi_{2,t}$. The no-arbitrage argument in the form of covered interest parity requires that $\mu_t^* \mathrm{d}t = \mathbb{E}_t^*(\mathrm{d}S_t/S_t) = (r_t - r_t^f)\mathrm{d}t$. This constraint implies that at each t, $\xi_{1,t}$ and $\xi_{2,t}$ will not be independent given the values of the interest rates. A common assumption of constant elasticity of substitution in the utility function of the representative agent, as in Bates (1996b), will correspond to the case where ξ_t is constant in time.

The jump component in equation (19) is a $\mathbb{P}$-martingale by construction:

$$\begin{aligned} \mathbb{E}_t(\mathrm{d}J_t) &= \mathbb{E}_t\left[(e^{u_t} - 1)\mathrm{d}q_t\right] - \lambda_t \bar{k}\mathrm{d}t \\ &= \mathbb{E}_t(e^{u_t} - 1)\lambda_t \mathrm{d}t - \lambda_t \bar{k}\mathrm{d}t \end{aligned}$$

$$= \quad 0.$$

The second equality follows from measurability of V_t with respect to $\mathcal{F}_t$. Define

$$\mathrm{d}N_t = \mathrm{d}q_t - \lambda_t \mathrm{d}t.$$

By applying Girsanov's theorem for point processes (Elliot & Kopp (2005)), the risk-neutral version of $\mathrm{d}N$ will be

$$\begin{aligned} \mathrm{d}N_t^* &= \mathrm{d}N_t - \mathbb{E}_t\left[\frac{e^{a+bu_t}}{\mathbb{E}_t(e^{bu_t})} - 1\right]\lambda_t \mathrm{d}t \\ &= \mathrm{d}N_t - (e^a - 1)\lambda_t \mathrm{d}t \\ &= \mathrm{d}q_t - \lambda_t^* \mathrm{d}t, \end{aligned}$$

where the market prices of jump risk a and b are measurable with respect to $\mathcal{F}_t$, and $\lambda_t^* \equiv e^a \lambda_t$. Girsanov's theorem applied to $\mathrm{d}J$ then yields

$$\begin{aligned} \mathrm{d}J_t^* &= \mathrm{d}J_t - \mathbb{E}_t\left[\left(e^a \frac{e^{bu_t}}{\mathbb{E}_t(e^{bu_t})} - 1\right)(e^{u_t} - 1)\right]\lambda_t \mathrm{d}t \\ &= (e^{u_t} - 1)\mathrm{d}q_t - \lambda_t^*\left[e^{b\omega^2}\frac{Q(b+1)}{Q(b)} - 1\right]\mathrm{d}t, \end{aligned}$$

where

$$Q(\Phi) = p(1+k)^{\Phi} + (1-p)(1-k)^{\Phi}.$$

Therefore, the process

$$\mathrm{d}J^* = (e^{u_t} - 1)\mathrm{d}q_t - \lambda_t^* \bar{k}^* \mathrm{d}t$$

will be a martingale under $\mathbb{P}^*$ as long as

$$\bar{k}^* = e^{b\omega^2}\frac{Q(b+1)}{Q(b)} - 1.$$

Parameter a captures the inability of the market to time the arrival of jumps, while b measures the uncertainty related to the jump size and, possibly, the model uncertainty. Liu *et al.* (2005) also argue that a significant part of the jump risk premium should come from the uncertainty aversion in the sense of Knight (1921) and Ellsberg (1961).

Putting everything together, the processes

$$
\begin{aligned}
\frac{\mathrm{d}S_t}{S_t} &= \mu_t^* \mathrm{d}t + \sqrt{V_t}\mathrm{d}W_{1,t}^* + \left(e^{u_t} - 1\right)\mathrm{d}q_t - \lambda_t^* \bar{k}^* \mathrm{d}t, \\
\mathrm{d}V_t &= \left(\alpha - \beta_t^* V_t\right)\mathrm{d}t + \sigma\sqrt{V_t}\mathrm{d}W_{2,t}^*,
\end{aligned}
$$

with $\mathrm{d}W_{1,t}^*\mathrm{d}W_{2,t}^* = \rho \mathrm{d}t$, represent the risk-neutral equivalents of (1) and (2). The market risk premia are the following:

$$
\begin{aligned}
\text{premium for the return diffusion risk} &= \mu - \mu_t^* \\
\text{premium for the volatility risk} &= (\beta - \beta_t^*)V_t \;=\; -\xi_t V_t \\
\text{overall premium for the jump risk} &= \lambda_t \bar{k} - \lambda_t^* \bar{k}^*.
\end{aligned}
$$

Appendix B: Closed-form solution for the price of a European currency option

Given the risk-adjusted model (12)–(13), the price at t of a European call option with residual maturity $\tau = T - t$ and strike price X is given by

$$
C_t(S_t, V_t, \tau, X; \widehat{\psi}) \;=\; e^{-r_t \tau}\mathbb{E}_t^*\left[\max\left(S_T - X, 0\right)\right]
$$

$$= e^{-r_t^f\tau} S_t P_1 - e^{-r_t\tau} X P_2,$$

where $\mathbb{E}_t^*(\cdot)$ denotes the expectation with respect to the risk-neutral probability measure $\mathbb{P}^*$ and conditional on the sigma-algebra $\mathcal{F}_t$. P_1 and P_2 have the usual Black-Scholes interpretation of the expected value of the underlying asset conditionally on the option being in the money, and probability of being in the money, respectively. The closed-form expressions for P_1 and P_2 can be obtained by following the calculation steps similar to those in Bates (1996b). The results are

$$\begin{aligned}
P_j &= \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \frac{\mathrm{imag}\left(F_j(i\Phi)e^{-i\Phi x}\right)}{\Phi}\mathrm{d}\Phi, \\
F_j(\Phi; V, \tau) &= \exp\left\{A_j(\tau;\Phi) + B_j(\tau;\Phi)V + \lambda_0^*\tau C_j(\Phi)\right\}, \\
A_j(\tau;\Phi) &= \mu_t^*\tau\Phi - \frac{\alpha\tau}{\sigma^2}(\rho\sigma\Phi - \beta_j - \gamma_j) \\
&\quad -\frac{2\alpha}{\sigma^2}\ln\left[1 + \frac{1}{2}(\rho\sigma\Phi - \beta_j - \gamma_j)\frac{1-e^{\gamma_j\tau}}{\gamma_j}\right], \\
B_j(\tau;\Phi) &= -\frac{\Phi^2 + (3-2j)\Phi + 2\lambda_1^* C_j(\Phi)}{\rho\sigma\Phi - \beta_j + \gamma_j(1+e^{\gamma_j\tau})/(1-e^{\gamma_j\tau})}, \\
C_j(\Phi) &= (1+\bar{k}^*)^{2-j}\left[Q(\Phi;p,k^*)e^{(1/2)\omega^2(\Phi^2+(3-2j)\Phi)} - 1\right] - \bar{k}^*\Phi, \\
\gamma_j &= \sqrt{(\rho\sigma\Phi-\beta_j)^2 - \sigma^2\left[\Phi^2 + (3-2j)\Phi + 2\lambda_1^* C_j(\Phi)\right]}, \\
\beta_j &= \beta_t^* + \rho\sigma(j-2), \\
Q(\Phi;p,k^*) &= p(1+k)^\Phi + (1-p)(1-k)^\Phi, \\
x &= \ln(X/S_t), \\
\mu_t^* &= r_t - r_t^f,
\end{aligned}$$

for $j = 1, 2$. By setting $p = 1$ and $\lambda_1^* = 0$ we obtain the option pricing formula given in Bates (1996b) for currency options, or in Bates (2000) for options on a stock market index.

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