Using Structural Models for Default Prediction [⋆]

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Abstract

I propose a new procedure for extracting probabilities of default from structural credit risk models based on virtual credit spreads (VCS) and implement this approach assuming a simple Merton (1974) model of capital structure. VCS are derived from the increase in the payout to debtholders necessary to offset the impact of an increase in asset variance on the option value of debt and equity. In contrast to real-world credit spreads, VCS do not contain risk premia for default timing and recovery uncertainty, thus yielding a purer estimate of physical default probabilities. Relative to the Merton distance to default (DD) measure, my measure (i) predicts higher credit risk for safe firms and lower credit risk for firms with high volatility and leverage (ii) requires fewer parameter assumptions (iii) clearly outperforms the DD measure when used to predict corporate default.

Key words: Structural Credit Risk Models, Bankruptcy Prediction, Risk-Neutral Pricing

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1 Introduction

Recent research has reported deficiencies of structural models employed for quantifying credit risk. Eom et al. (2004) use five structural models for bond pricing and conclude that these tend to underestimate spreads of safe bonds while overstating credit spreads for bond issues of firms with high asset volatility and leverage. Bharath and Shumway (2008) construct a naïve bankruptcy predictor as an alternative to the classical Merton distance to default (DD) model which outperforms the original. They reason that "if the predictive power of our naïve probability is comparable to that of [the original model], then presumably a more carefully constructed probability that captures the same information should have superior power." Campbell et al. (forthcoming) construct the current state-of-the-art statistical model for bankruptcy prediction using simple market and accounting variables. They demonstrate a substantial underperformance of the Merton DD model relative to theirs in terms of Pseudo-R^2s and conclude that summarizing default estimates in a single predictor, as done in the DD model, is not feasible.

Addressing these concerns about the ability of structural models – particularly the DD model – to appropriately capture credit risk, I propose a new risk-neutral default measure based on virtual credit spreads. Virtual credit spreads are derived from the increase in the payout to debtholders necessary to offset the impact of an increase in asset variance on the option values of debt and equity. The intuition underlying my approach can be applied to any framework in which debt and equity are regarded as one or multiple options and valued as a function of asset risk and payout- ratio. This paper focuses on the application to Merton's DD model, comparing properties and explanatory power of default probabilities estimated based on virtual credit spreads and assuming a Merton model of capital structure (π_{VCS}) to default probabilities estimated using the original Merton DD model itself (π_{DD}).

My approach has several advantages over the DD measure. First, π_{VCS} is higher than π_{DD} for relatively safe firms and lower for firms with high leverage and volatility, as requested by Eom et al. (2004). Second, the estimation of my measure requires fewer parameter assumptions than the DD model. Specifically, no assumptions about the physical growth in asset values or the cash payout to debtholders are needed. Third, π_{VCS} clearly outperforms π_{DD} when applied to bankruptcy prediction. By improving the derivation of the asset volatility parameter, I am able to augment the Pseudo-R^2 of a regression of π_{DD} on corporate failure from approximately 15% reported by Campbell et al. (forthcoming) and confirmed for my sample to 16.9% in the standard model and 20.1% in the best model. When running the same regression using the π_{VCS} measure, Pseudo-R^2s increase to 24.9% in the standard model and 27.2% in the best model. The VCS approach thus seems a promising alternative for estimating credit risk based on structural default models.

The paper is organized as follows. Section 2 reviews previous literature on corporate failure. Section 3 comments on common statistical models used for bankruptcy prediction. Section 4 explains existing approaches to estimating default probabilities (4.1 and 4.2), and introduces the new risk-neutral default measure π_{VCS} (4.3). Section 5 presents results of a numerical sensitivity analysis, comparing properties of π_{VCS} and π_{DD}. Section 6 compares the measures' ability to predict corporate default in an empirical setting. Section 7 concludes.

2 Previous Literature

Theoretical bond pricing models can be assigned to two broad categories. *Reduced-form approaches* rely on jump processes to model default as an unexpected event. In contrast, *structural models* take on an option-based view of capital structure and model the firm's inability to serve its obligations explicitly. In doing so, most structural models compare the values of a firm's assets and liabilities and assume that default happens if the former fall below a certain threshold. The following summary of previous studies focuses on structural models. [1]

[1] For a brief technical introduction to reduced-form pricing, see Duffie and Singleton (2003), pp.106-111.

Under the model of capital structure by Black and Scholes (1973) and Merton (1974) (BSM), a firm defaults if the value of its assets is below the value of debt at the expiration date of current debt contracts. In this framework, equity owners are assumed to own a call option on the value of the firm, while debt holders hold a short put and a risk-free bond. This intuition is due to the payoff structure of both claims. While shareholders participate in the firm's entire upside potential, their liability is limited. To the contrary, while the payoff to debt holders does not increase in case of a positive firm development, they can lose money if the firm becomes insolvent.

Numerous variations of this classical model exist. Black and Cox (1976) introduce first-passage models, arguing that default can not only occur at debt maturity, but any time before. In their model, a firm defaults as soon as the value of the firm falls below a predefined default boundary. Recent research uses barrier option pricing to account for this path dependency of equity.[2] Using compound option pricing, Geske (1977) generalizes the BSM model to cases where the firm is financed with coupon-paying debt or with debt maturing at different dates. At each payment date, shareholders decide either to meet their obligation or to discontinue firm operations and leave firm assets to debt holders, thereby creating future options or not. Longstaff and Schwartz (1995) relax two assumptions underlying the BSM framework. First, they allow for time-varying interest rates, incorporating default and interest-rate risk. Second, they depart from the assumption of strict absolute priority rules.[3] Amongst others, they predict a significant impact of the correlation between asset values and interest rates over time on credit spreads, as well as a negative relation between interest rates and credit spreads, and find empirical support for their predictions. Assuming debt with infinite maturity, Leland (1994) derives a closed form solution for value maximizing

[2] Brockman and Turtle (2003) use a barrier option framework to predict bankruptcy, while Purnanandam (forthcoming) propose a theory for optimal risk management in financially distressed firms based on barrier options.

[3] Under this assumption, the claims of debt holders are served according to their seniority, with claimants in highest seniority debt being fully paid off before other claimants. Eberhardt et al. (1990) and Weiss (1990), amongst others, show that this assumption contradicts empirical evidence.

capital structure under taxes, bankruptcy cost and protective covenants. Leland and Toft (1996) extend his work by introducing finite debt maturities. In contrast to other studies implementing arbitrary debt maturity,[4] they assume bankruptcy to be endogenous in the sense that it is triggered by shareholders' decision to default on debt. To them, the capital structure decision represents a tradeoff between tax benefits, agency problems and bankruptcy costs. Along these lines, they argue that shareholders will choose to reduce the default barrier in highly-levered firms financed with junk bonds to provide an additional cushion to debt and augments its value. Vice versa, Grass (2008) argues that under exogenous bankruptcy, shareholders of financially distressed firms subject to strict bankruptcy codes and thus high default barriers can profit from a reduction in firm risk. Collin-Dufresne and Goldstein (2001) depart from the assumption of constant debt levels and assume mean reverting leverage ratios. In line with empirical evidence, they predict an upward-sloping term structure of credit spreads for low-grade debt and a weaker relation between changes in credit spreads and firm value than commonly assumed.

In summary, the restrictive model of BSM has been modified in numerous ways to allow for more realistic debt-specific and general characteristics. However, this paper limits its view to the simple BSM framework for the following reasons. First, as both the proposed concept *and* its benchmark are based on the BSM model, its restrictive assumptions affect the explanatory power of both frameworks. Second, the paper aims at introducing a new concept for deriving improved estimates of physical default probabilities. Rather than including recent improvements in model features, the BSM framework allows to focus on the comparison between conventional and proposed default measure in a simple setting. Future research may well extend the conceptual insights of this paper to extended frameworks, some of which are suggested throughout the paper.[5] Third, as outlined in the following, not all features which seem appealing in theory prove to add value in an empirical setting.

[4] See, for example, Kim et al. (1993) and Nielsen et al. (1993).

[5] One of the advantages of the intuition presented in this article is its general applicability to any framework in which debt and equity are regarded as one or multiple options and valued as a function of asset risk and payout-ratio. For example, the concept can easily be applied to the Geske (1977) or Collin-Dufresne and Goldstein (2001) model, as well as Eom et al. (2004)'s extension of the Merton model.

2.2 Empirical Studies

Empirical tests of structural models exist for both, the application to the explanation of variations in credit spreads and to bankruptcy prediction.

Few studies used structural models to predict credit spreads. Early studies assessing the empirical performance of theoretical bond pricing models include those by Jones et al. (1984) and Odgen (1987). Using a sample of only 27 observations, Jones et al. (1984) test the predictive power of contingent claims analysis for bond pricing and conclude that – while being a significant improvement over their benchmark approach for high yield bonds – such analysis substantially underpredicts credit spreads. This finding is confirmed by the latter for a larger sample. Lyden and Sariniti (2001) implement the Merton and the Longstaff and Schwartz (1995) model using data on 56 noncallable bonds and find underestimated yield spreads for both models. In addition, they report that some key features added in the latter seem not to add value in an empirical setting. Particularly, they argue that the possibility of default prior to maturity, as well as the incorporation of stochastic interest rates have little impact on results. Furthermore, they show that adding industry-specific recovery rates significantly decreases model fit. Overall, the authors demonstrate that – for their sample – the Merton model dominated the Longstaff and Schwartz (1995) model.

The most comprehensive empirical study to this date has been conducted by Eom et al. (2004), who provide a thorough empirical comparison of five structural models for bond pricing using data on noncallable and nonputable bonds. More specifically, they contrast the performance of the models by Merton (1974), Geske (1977), Longstaff and Schwartz (1995), Leland and Toft (1996), and Collin-Dufresne and Goldstein (2001). The authors are unable to confirm the notion of previous research that structural models tend to underestimate spreads on average. Rather, they report underestimation of bond spreads only for the Merton and Geske model. Assuming endogenous default in the latter helps to further increase spreads. Furthermore, they find overestimated spreads for the Longstaff and Schwartz and the Collin-Dufresne and Goldstein model. Their introduction of stochastic interest rates and correlation between firm values and interest rates turns out to add only limited value in an empirical setting. Specifically, stochastic interest rates do raise spreads but also increase model uncertainty, as results are very sensitive to the assumed interest rate variance. Correlating firm values and interest rates only has a marginal impact on predicted spreads. The added feature of mean reverting leverage ratios in the Collin-Dufresne and Goldstein is problematic in that model uncertainty increases due to additionally required assumptions. Overall, Eom et al. (2004) conclude that "the focus of future research should be on raising spreads on the safer bonds without raising them too much for the riskiest bonds [...] by overstating the risks associated with leverage, volatility, or coupon". The concept presented in this paper addresses this request, as demonstrated in Section 5.

More recently, a number of researchers have employed structural models for predicting corporate failure. The most common model in this context is the Merton DD model, which is also assumed by Moody's KMV in a more general form.[6] More specifically, Moody's uses the proprietary Kealhofer-Vasicek model, which allows for different debt instruments and incorporates maturity structures.[7] As probabilities of default computed from the model only approximate physical default probabilities, Moody's uses a comprehensive database to map expected default frequencies from the model to probabilities of default which have been observed in the past. Furthermore, Moody's employs proprietary adjustments to the parameters strike price and asset volatility which cannot be replicated by academics. Bharath and Shumway (2008) follow Vassalou and Xing (2004) and Duffie et al. (2007) to approximate the KMV model as good as possible and compare its ability to forecast default to the one of a predictor derived in a similar manner but using naïve parameter estimates. They find that, despite its simple derivation, their naïve predictor outperforms the KMV-like model and conclude that, most likely, better ways of constructing a default predictor based on the same set of information they use exist. In line with their research, Campbell et al. (forthcoming) report that the KMV-like predictor contributes only marginally to the statistical power of a hazard model estimated using a set of simple

[6] One of the few exceptions is the study of Brockman and Turtle (2003), forecasting bankruptcy assuming equity to be a barrier option on firm assets.
[7] See Crosbie and Bohn (2003) and Bharath and Shumway (2008).

market and accounting variables.

3 Statistical Models for Default Prediction

The previous section gave an overview of theoretical debt pricing models and their ability to explain bond yields and to predict default. This section provides a brief and non-technical review of some statistical methods for assessing the adequacy of structural models for bankruptcy prediction in an empirical setting. This overview includes discriminant analysis, probit and logit models, as well as hazard models.

These statistical methods attempt to predict whether a firm will fail during a specified period in the future (for example the next 12 months) conditioned on current information about a firm and its market environment. In this context it is common to regress on a dichotomous dependent variable, with ones indicating default or failure and zeros indicating survival (or vice versa).

One of the most prominent statistical measures for bankruptcy prediction, the Altman (1968) Z-score, is based on discriminant analysis. Discriminant analysis aims at finding a linear combination of independent variables that best separates all observations into two specified groups such as future failure or survival. For this purpose, coefficients of a discriminant function are estimated by a procedure similar to multiple regression analysis. The expected values obtained from this function for the set of independent variables are the Z-score, according to which each observation is classified.

A problem with such a linear probability model is that its expected values cannot be restricted to the zero to one interval, while the dependent variable is. For example, Altman's Z-score can take on values of more than three. Beyond yielding unrealistic predictions (the probability of default can not equal 300%), such a model can thus produce negative variances for the disturbance term. To overcome this problem, probit and logit models transform the model's prediction using normal and logistic probability distributions, respectively. Given the similarity of logistic and normal cumulative distribution functions, both models yield similar results for most applications.[8] An important contribution in the context of bankruptcy prediction with such qualitative-response models is due to Ohlson (1980).

Shumway (2001) criticizes probit and logit models for not taking into account the duration of survival if applied to single-period data and proposes a hazard model instead. He demonstrates that his model is substantially more accurate in bankruptcy prediction than the discriminant analysis by Altman (1968), using the same set of variables as predictors. The superior performance of his model is confirmed for a larger data set by Chava and Jarrow (2004). Other studies applying hazard models for bankruptcy prediction include Bharath and Shumway (2008) and Campbell et al. (forthcoming). While hazard models appear to be a recent trend in the literature on corporate default, it is important to note that they can be and have been employed using simple logit programs and adjusting overall statistics. Shumway (2001) admits that "estimating hazard models with a logit program is so simple and intuitive that it has been done by academics and researchers without a hazard model justification." The main difference between an adjusted logit analysis and a hazard model is the way how firm age can be accounted for as a potential factor driving the probability of bankruptcy. While in a logit model, some function of firm age can be simply included as additional explanatory variable, it enters the estimation of the hazard model more elegantly via the baseline hazard function. However, given the insignificance of firm age for predicting bankruptcy reported by Shumway (2001), I follow Chava and Jarrow (2004) and Campbell et al. (forthcoming) and estimate a hazard model using a logit program excluding firm age as explanatory variable. The marginal probability of failure (or bankruptcy) over the next period is

$$P_t(Y_{t+1} = 1) = \frac{1}{1 + e^{-\alpha - \beta \times X_t}}, \tag{1}$$

where Y_t equals one if failure (or bankruptcy) occurs at time t and zero otherwise, α and β are model parameters and X_t is a matrix of time-varying covariates. Higher values of $\alpha + \beta \times X_t$ imply a higher probability of failure (or bankruptcy).

[8] For technical details, see Greene (2008), pp.772-774. For a comparison of the performance of discriminant, probit and logit analysis in bankruptcy prediction, see Lennox (1999).

4 Estimating Probabilities of Default

No method for computing physical probabilities of default exists until today. In *structural models*, they are approximated as the probability that debt as a put option on firm assets will be exercised in a risk-neutral valuation framework. Market participants' default estimates implied in *credit spreads* are polluted by risk premia. On average, both approaches overstate the physical risk of default given no further adjustments. After reviewing the two frameworks, I propose a new risk-neutral default measure based on virtual credit spreads. My measure does neither require assumptions about growth in asset values nor about payouts to debtholders and is not overstated due to risk premia.

4.1 The Merton Distance to Default Measure

The BSM model assumes that shareholders hold a call option on firm assets while debtholders own risk-free debt and are short a put option on the value of the firm. This is due to their characteristic payout profiles. Shareholders profit from positive firm developments but have a limited downside risk. Once the value of the firm falls below the face value of total debt, the equity claim is worthless and bondholders start losing money. In contrast to owners of equity, however, their upside is limited. Given a positive development of the firm, they will simply receive the pre-agreed payoff at maturity.

In this framework, the value of the firm follows a geometric Brownian Motion:

$$\frac{d\mathrm{V}}{\mathrm{V}} = (\mu - \delta)\, dt + \sigma_{\mathrm{V}} d\mathrm{W}, \tag{2}$$

where V is firm value with a drift rate $\mu - \delta$ and a volatility σ_V. μ is the average rate of return on assets and δ denotes the combined payout ratio to debt and equity holders. $d\mathrm{W}$ is a standard Wiener process.

The value of equity as a call option equals

$$E = Ve^{-\delta T} N(d_1) - De^{-rT} N(d_2), \tag{3}$$

where D is the face value of debt, r the risk-free rate and T the time to debt maturity. $N(\cdot)$ denotes the function describing the standard cumulative probability density,

$$d_1 = \frac{\left(\ln\left(\frac{V}{D}\right) + (r - \delta + .5\,{\sigma_V}^2)\,T\right)}{\sigma_V \sqrt{T}} \tag{4}$$

and

$$d_2 = d_1 - \sigma_V \sqrt{T}. \tag{5}$$

In this framework, a firm's probability of default is the probability that firm value is below the face value of debt at expiration. It is thus a function of Merton's DD measure, defined as the number of standard deviations by which assets exceed liabilities:

$$DD = \frac{\left(\ln\left(\frac{V}{D}\right) + (\mu - \delta + .5\,{\sigma_V}^2)\,T\right)}{\sigma_V \sqrt{T}}. \tag{6}$$

The DD measure is, of course, equal to d_1, except that the DD measure's drift rate before payouts equals the expected growth rate in firm value μ, and not the risk-free rate. Assuming firm values to grow faster than risk-free assets on average, probabilities of default derived using risk-free drift rates thus overstate the true risk of bankruptcy.

The probability of default is then defined as

$$\pi_{DD} = N\left(-DD\right). \tag{7}$$ [9]

If the true future growth rate in firm value was known and all other model parameters estimated correctly, π_{DD} would reflect the true physical probability of default. However, as becomes clear from the discussion in Section 6.2, estimating future asset growth – at least to some extent – is an arbitrary exercise.

[9] For a thorough derivation, see Vassalou and Xing (2004), pp.836f.

4.2 Real-World Credit Spreads

The level of real-world credit spreads depends – amongst others – on the physical probability of default and the recovery rate on the one hand, and risk premia on the other. Thus, even if default risk and recovery rate of a bond stay constant, credit spreads can vary over time due to changes in investors' risk aversion. While credit spreads observed at markets do contain information about physical default risk, it is not possible to extract this information without additional assumptions. Still, computing risk-neutral probabilities of default from credit spreads is a straightforward task.

For example, consider a bond valued at par which in one year either pays 110 or defaults and leaves the investor with 50% of his initial investment. By simply discounting these payoffs using the risk-free rate – say 5% – we can solve $100 = 1/1.05\,[\pi_{RN} \times 110 + (1 - \pi_{RN}) \times 50]$ for the risk-neutral probability π_{RN}. Risk-neutrality in this case refers to the risk preference of investors and implies these are neither risk-averse nor risk-loving. The above equation can thus as well be read as $105 = [\pi_{RN} \times 110 + (1 - \pi_{RN}) \times 50]$, reflecting investors' indifference between receiving the risk-free rate of return or investing in the bond with the above properties and a default probability π_{RN}.[10]

In reality, however, investors are risk-averse in most cases. They will thus demand premia for taking risks, increasing their discount rate and decreasing today's bond value below par. Vice versa, if the above bond is valued at par using a discount rate higher than the risk-free rate, its default risk thus has to be below π_{RN}. Given the risk-premia in real-world credit spreads, risk-neutral probabilities of default derived from these spreads thus overstate physical default probabilities in most cases.[11]

[10] A generalized form of this equation is presented in the following section.
[11] Compare Duffie and Singleton (2003), pp.104f.

4.3 Virtual Credit Spreads

Assume that creditors could adjust the cost of debt instantaneously and at all points in time knew the exact firm risk, measured as the standard deviation of asset returns. They could price any marginal change in asset – and thus default – risk by adjusting their required rate of return such that the present value of their claim would not be affected.[12] Assuming firm value to be exogenous, that is, ignoring taxes and the cost of financial distress, the value of the equity claim would be left unchanged as well.[13] There thus exists a "fair" change in the cost of debt that offsets the wealth transfer between claimholders induced by a change in a firm's asset and default risk. This adjustment in the cost of debt is at the center of my framework's intuition.

Under this assumption of a "zero sum game", the value of the firm in a contingent claims framework is the sum of the option value of debt, the present value of future payouts to debtholders, the option value of equity and the present value of future payouts to shareholders.[14] While it seems intuitively appealing to look at the option value of debt in the given context, I instead examine option values of equity for two reasons. First, under exogenous firm values it leads to exactly the same results. Second, it is analytically more straightforward, as changes in bondholder value due to adjustments in the cost of debt and related changes in the present value of future interest payments do not enter calculations. Adjustments in the cost of debt do not affect the present value of future dividend streams to shareholders, but only the value of their option. The subsequent line of argumentation is therefore based on the option value of equity instead of debt.

The rate of return required by bondholders enters the valuation of equity and debt as contingent claims via the payout ratio δ, corresponding to the dividend yield for the

[12] Note that these assumptions are not required for the procedure to be valid but are used for explanation.
[13] Taxes and market frictions can be implemented but are ignored in the following.
[14] The Black and Scholes (1973) framework for pricing financial options assumes the holder of the option to be different from the owner of the underlying. Higher dividend yields thus lead to lower call option and higher put option values, as the value of the underlying after payouts declines. In contrast, when valuing debt and equity as options, payouts of the underlying are made to the very holder of these options.

case of stock options. For pricing equity as a call option, δ is defined as the weighted average of equity and debt payouts:

$$\delta = \left(1 - \frac{\mathrm{D}}{\mathrm{V}}\right) \mathrm{DIV} + \frac{\mathrm{D}}{\mathrm{V}} (\mathrm{r} + \mathrm{p}), \tag{8}$$

where D denotes total debt, V firm value, DIV the dividend yield on equity, r the risk-free rate, and p a premium paid to creditors on top of r.[15]

Following the previously described intuition, consider the example presented in Figure 1, displaying isolines of call option values as a function of payout ratios and volatility. A levered firm with an initial asset variance of 20% shifts its risk to 40%. The value of equity as a contingent claim increases from 51.76 to 66.55 (dotted arrow), the value of debt decreases analogously. In order to offset this wealth transfer, creditors would need to raise p up to a level where the value of equity is reduced to the initial 51.76 (solid arrow) and where the value of their claim is thus fully restored. This fair increase in p would compensate debt holders for the higher risk of default.

[Fig. 1 about here.]

This example can be generalized to any shift in asset risk. One specific case is the (hypothetical) increase in σ_V from zero to the actual level of firm risk. In this case, the increase in p from zero to λ reflects the entire default risk born by debt holders.[16] However, it does not contain *premia* for default, liquidity, or other risks. As pointed out by Duffie and Singleton (2003), default-risk premia in real-world credit spreads reflect aversion to default timing risk and recovery uncertainty.[17]

To clarify the difference between the compensation for default risk on the one hand and default risk premia on the other, consider the following case. An investor can buy a corporate bond for $100 that either pays the amount PO after one year with 90% probability or defaults with 10% probability. In case of default, the investor recovers $50 of his investment. By investing his money in a risk-free asset, the investor

[15] Note that in fact, δ is stochastic, as it depends on firm value V. However, in the empirical analyses discussed later on, δ is assumed to be stationary for the purpose of simplification.
[16] Interpretation and calculation of λ are discussed in an instant.
[17] See Duffie and Singleton (2003), p.102.

can obtain \$105 after one year with certainty. In order to offset the 10% chance of receiving only \$50, the investor would ask for a payoff PO of \$111.11 (obtained by solving $105 = .1 \times 50 + .9 \times PO$). The \$6.11 are thus a *compensation for default risk*. However, the risk-free investment still offers a higher utility to a risk-averse investor. In order to make the risky investment attractive to such an investor, PO needs to be further increased to \$111.11 + DP, where DP is a *default risk premium*, compensating the investor for his utility loss of bearing uncertainty. λ does not contain such default risk premia.

Therefore, λ can be regarded as a "virtual credit spread" containing pure information on the annual default probability. Another way to interpret λ is as *default intensity*. In a simple framework, default intensities are the constant mean arrival rate of default in a Poisson process. Extensions to time varying default intensities can be implemented in a straightforward manner using Bayes' rule. While this is beyond the scope of this article, such extended models allow accounting for the fact that default intensities vary as new information arrives. [18]

Following the previously illustrated intuition, λ can be obtained numerically by solving Equation 9 for δ_V and plugging the results into Equation 8:

$$\mathrm{c}\left(\mathrm{V}, \mathrm{D}, \sigma_0, \mathrm{T}, \delta_0, \mathrm{r}\right) = \mathrm{c}\left(\mathrm{V}, \mathrm{D}, \sigma_\mathrm{V}, \mathrm{T}, \delta_\mathrm{V}, \mathrm{r}\right) \tag{9}$$

where $c(\cdot)$ denotes the value of a European call depending on spot price, strike price, variance of the underlying, time to maturity, payout ratio and the risk-free rate, respectively. σ_V denotes the annualized standard deviation of asset returns, T the years to debt maturity, and δ_V the target payout ratio. σ_0 equals zero and δ_0 is computed from Equation 8 setting p_0=0. The level of tolerance for numerical convergance used throughout the entire analysis is E-10.

A risk-neutral probability of default can be extracted from a virtual credit spread λ

[18] For a summary of properties of default intensities and survival probabilities in a simple model, and a model allowing for time-varying default intensities, see (Duffie and Singleton, 2003), pp.59-62.

for a loss given default (LGD) as

$$\pi_{VCS} = \frac{1 - e^{-\lambda}}{LGD}.^{19} \tag{10}$$

Analogously to Section 4.2, Equation 19 is based on the assumption of risk-neutral investors, being indifferent between receiving the risk-free rate with certainty, or receiving the risk-free rate plus a mark-up: $\mathrm{e^r} = (1 - \pi_{\mathrm{VCS}})\mathrm{e}^{\mathrm{r}+\lambda} + \pi_{\mathrm{VCS}}(1 - \mathrm{LGD})\mathrm{e}^{\mathrm{r}+\lambda}$. The mark-up compensates investors for defaults such that the expected rate of return accounting for defaults equals the risk-free rate. However, as outlined before, the "virtual credit spread" calculated according to Equation 9 is a pure default spread, only reflecting the mark-up required to keep the value of debt constant for increasing asset (and thus default) risk. It does not include any kind of risk-premia or any other noise polluting historical credit spreads. Values for π_{VCS} extracted from λ therefore do not exhibit the upward bias observed for risk-neutral probabilities of default computed from real- world credit spreads.

5 Sensitivity Analysis

Before applying the presented concepts to bankruptcy prediction in an empirical setting, a brief numerical analysis of the sensitivity of π_{VCS} and π_{DD} to changes in input parameters helps to understand differences in the two underlying approaches.[20]

Figure 2 displays the relationship between leverage and default probabilities π_{VCS} and π_{DD} for low-risk ($\sigma_V = .2$), average-risk ($\sigma_V = .4$) and high-risk ($\sigma_V = .8$) firms. For any level of firm risk, π_{VCS} is higher than π_{DD} for firms with low leverage and lower than π_{DD} for firms with high leverage. The higher firm risk, the faster the two functions cross each other. For the firms with the highest asset variance displayed, π_{VCS} is clearly above zero even for low-levered firms, while π_{DD} is virtually equal to

[19] Following Berndt et al. (2005) and Bharath and Shumway (2008), I ignore the correlation between default and recovery rates documented in Altman et al. (2003) and assume a constant LGD of .75 throughout the paper.
[20] Note that the time to maturity and drift rates assumed in the two models differ. The underlying reasoning is discussed in detail in Section 6.2.

zero.[21]

[Fig. 2 about here.]

Figure 3 shows π_{VCS} and π_{DD} as a function of asset volatility σ_V for different leverage ratios. A similar effect as outlined before is observable. For low levels of asset risk, π_{VCS} is higher than π_{DD}, while for high levels of asset risk it is lower. Analogous to Figure 2, the higher the leverage, the lower σ_V at the intersection of the two functions.

[Fig. 3 about here.]

For completeness, Figure 4 presents these relationships in three dimensions.

[Fig. 4 about here.]

In the light of precedent empirical evidence, the observed behavior of π_{VCS} relative to π_{DD} is appealing. Taken together, default estimates derived from virtual credit spreads are higher for relatively safe firms and lower for the riskiest firms. Recalling the previously cited conclusion by Eom et al. (2004), according to which structural models underestimate (real-world) credit spreads of safe bonds with low leverage and volatility, a future application of virtual credit spreads to bond pricing seems promising.

Both π_{VCS} and π_{DD} are a decreasing function of the risk-free rate r, as shown in Figure 5. The reason is, of course, that r is the – or part of the – drift rate of the

[21] No difference in the definition of leverage between the two measures is assumed in the numerical analysis. However, as outlined in Section 6.2, leverage ratios used as input parameter for the Merton model are typically below those used for computing virtual credit spreads, as 50% of long-term debt is simply ignored. Accounting for this difference implies "stretching" π_{DD} as a function of leverage along the x-axis. For example, if a firm is financed with 50% short-term and 50% long-term debt, leverage ratios in the Merton model are 25% below leverage ratios based on total debt. In this case, the function needs to be stretched by 1/.75=1.33. As the magnitude of the difference in leverage ratios depends on assumptions about the maturity structure of debt, it is ignored in this context for the sake of simplification. Accounting for the difference does not change the main takeaways of this section. In fact, stretching π_{DD} as a function of leverage along the x-axis would even increase the difference to π_{VCS} for firms with little total debt on their balance sheet and thus strengthen the potential advantages of π_{VCS} over π_{DD} discussed in the following.

stochastic process assumed in the pricing framework.[22] The higher this drift rate, the faster the call option moves deep into the money and the safer the debt. For firms with relatively low asset risk and leverage, π_{VCS} is more sensitive to changes in r than π_{DD}. For higher numbers, the two functions are more parallel.[23]

[Fig. 5 about here.]

As outlined in Section 6.2, when calculating π_{VCS} (π_{DD}), an equal time to maturity of 6 years (1 year) will be assumed for all observations. While a sensitivity analysis in this case does not tell anything about variations in the two measures across observations and along time, it is useful to understand the term structures of default implied by the two approaches. Those are depicted in Figure 6 for different variance-leverage combinations. The shapes of all functions share two common characteristics typical for structural models. First, all functions start in the origin. This implies that – even for high firm risk and leverage – the instantaneous probability of default is predicted to be zero. This unsatisfactory feature can be improved by introducing jumps in the stochastic process underlying the model. Second, the probability of default converges towards zero over very long-term horizons (not displayed), assuming positive drift rates and reasonable asset variances. The reason is that under these assumptions and given constant debt levels, the distribution of asset values moves away from the strike price over time. Incorporating mean-reverting leverage ratios, as done by Collin-Dufresne and Goldstein (2001), allows for non-zero default probabilities over infinite time horizons.[24]

[Fig. 6 about here.]

[22] As explained later, the drift rate assumed in the Merton model equals r plus an equity premium of 6%.

[23] Only moderate levels of firm risk and leverage are shown for displaying purposes.

[24] For details on the term structure of credit risk in structural and reduced form models, see (Duffie and Singleton, 2003), pp.114-116.

6 Empirical Analysis

6.1 Data and Sample Selection

Empirical analyses are conducted using data from three main sources. Balance sheet items are from the Compustat Annual Industrial File. These include total assets, current liabilities, total long-term debt, common dividends, total liabilities, net income and the primary SIC code (Compustat items AT, TLC, DLTT, DVC, LT, NI, and SIC respectively). Stock market data is from the Center of Research in Security Prices (CRSP) Monthly File and includes stock prices, stock returns, and the number of shares outstanding (CRSP items PRC, RET, and SHROUT), as well as returns on the value weighted S&P500 index and its total market capitalization (items VWRETD and TOTVAL). Information about delistings is from the CRSP Daily Event File and includes delisting code and delisting date (items DLSTCD and DLSTDT). CRSP and Compustat data are matched using the CRSP Compustat Merged database. Furthermore, I obtain interest rates on constant maturity treasury securities from the Federal Reserve Board of Governors.[25] Data on implied volatilities of long term call options with 547 days maturity used for robustness checks is from the OptionMetrics IVY Database with time series starting in 1996. Monthly levels of the VIX volatility index are obtained from the Global Insight Financial Market Indexes database (item CBOEVIXC).

Due to limited availability of Compustat data before 1960, I restrict my analysis to the subsequent years, forecasting corporate failures and defaults which occurred in the years 1961 to 2007 using accounting and market data from the years 1960 to 2006. An exception is made for computing firm risk estimates from 1960-2006, which

[25] Taken from the H.15 file which can be downloaded at www.federalreserve.gov.

are computed in a regression using Compustat and CRSP data from 1950 to 2006.[26]

Following standard methodology, I exclude all firms active in the financial services and utilities sector (SIC codes 6000 to 6999 and 4900 to 4999). The reasons are twofold. First, due to different accounting standards, balance sheet items of such firms are not fully comparable to those of other firms. Second, due to their socio-economic importance for a country, firms in these industries are likely to behave differently in financial distress, as they may receive some kind of governmental support or face additional restrictions when approaching default.

Closely related to Dichev (1998) and Brockman and Turtle (2003), I define corporate failure as delistings due to bankruptcy, liquidation, or poor performance. My definition deviates from theirs in that I do not include observations with CRSP delisting code 573 in the sample, defined as delistings requested by companies going private. My definition of failure thus comprises the CRSP delisting codes 400 and 500 to 585, except code 573.[27] In line with Dichev, corporate defaults are defined more narrowly as all delistings due to bankruptcy or liquidation (codes 400, 572, 574).

After additionally excluding all observations for which not all data required for parameter estimation is available, my sample includes 4,045 failures, 293 bankruptcies and 151,451 firm-year observations. Table 1 displays descriptive sample statistics for the control variables used in the subsequent regression analysis. Parameter descriptives are reported separately later on, in Table 3.

[Table 1 about here.]

The solid lines in Figure 7 display the number of failures and bankruptcies per year in my sample. A tremendous increase over time can be observed. This rise is not mainly

[26] Several time periods for default prediction using Compustat data exist in the literature. For instance, Vassalou and Xing (2004) argue that prior to 1971, debt items are only scarcely available and restrict their analysis to the years thereafter. In contrast, Campbell et al. (forthcoming) consider a broader time period starting in 1963 with the availability of their failure indicator. Other studies, for example by Bharath and Shumway (2008) or Brockman and Turtle (2003), look at shorter time periods.

[27] This deviation reduces the sample of bankruptcies by less than one percent and thus is not expected to have any significant impact on the results.

due to a lower number of firms listed in the earlier periods – the same trend can be observed when plotting these numbers relative to the number of firms entering the sample per year (not reported here). Campbell et al. (forthcoming) document the same phenomenon for different datasets on corporate bankruptcy and failure. They explain the observable trend with changes in bankruptcy law in the 70s, financial innovations in the 80s and 90s, as well as overall changes in firms' capital structure and risk-taking. The spike in corporate failures in the late 90s and *before* the burst of the dot-com bubble in 2000-2003 is attributed to numerous listings of small firms which occurred during the stock market boom of the late 90s.

[Fig. 7 about here.]

The dashed line in Figure 7 corresponds to the yearly number of failures in the sample of Campbell et al. (forthcoming). The high correlation to the numbers of failures in my sample indicates the comparability of the data used in their and this study. Except for the very last years, my definition of failure is broader than theirs, while my definition of bankruptcy is clearly more narrow.

6.2 Parametrization

Similar sets of parameters need to be estimated for calculating probabilities of default based on the DD measure (π_{DD}) and virtual credit spreads (π_{VCS}). The assumptions underlying my estimates, their impact on the results, as well as alternative definitions used in related literature are discussed in the following. Asset variance is of particular importance for the estimation of π_{VCS} and thus discussed extensively. An overview of the definitions used in different studies is given in Table 2. Descriptive statistics of the model parameters are shown in Table 3. The exact definition of all input parameters is outlined in the following.

[Table 2 about here.]

[Table 3 about here.]

Strike Price Equivalents In a firm financed with equity and one zero coupon bond, equity is a call option on the firm with the strike price equal to the face value of total debt. When debt matures, stockholders can choose to either buy the entire firm from bondholders by paying them off the face value, or to roll over debt for another term. However, capital structures are more complex in reality. The debt of a firm typically is not homogeneous but usually a combination of numerous financing instruments with different maturities, seniorities, embedded options and so forth. The right choice of the strike price parameter is thus less trivial than it appears at first sight and depends on the perspective taken in a model.

When assessing a firm's risk of defaulting during the next year using the DD model, it makes sense not to consider the full amount of debt on the balance sheet. As pointed out by Crosbie and Bohn (2003) in their description of Moody's KMV model, long-term debt provides a firm in financial distress with additional breathing space, as the firm does not have to raise the cash for paying off this debt in the near future. For the 1-year DD measure it thus seems appealing to ignore long-term debt and only examine how likely it is that a firm cannot service its short term obligations in the next year. However, numerous authors define the strike price of equity as a call option as the amount of short-term debt plus 50% of long-term debt. [28] The main reason is that having long-term debt on the balance sheet makes it harder for a firm approaching distress to roll-over short-term debt at similar conditions. I thus define the spot price for the DD measure as current liabilities (Compustat Item LCT) + 0.5 × long-term debt (Compustat Item DLTT). [29]

In contrast, long-term debt does matter when assessing the impact of changes in asset risk on the option value of equity in the derivation of π_{VCS}. In fact, when estimating this impact in the context of real-world asset substitution phenomena, long-term debt

[28] For some studies, see Table 2.

[29] Note that in doing so I deviate from the definition of short-term debt used in the studies of Bharath and Shumway (2008) and Vassalou and Xing (2004), who only use debt in current liabilities (Compustat item DLC). My definition thus additionally includes the items accounts payable, income taxes payable and other current liabilities. Studies defining total debt more broadly as total liabilities include Brockman and Turtle (2003) and Campbell et al. (forthcoming). Using current and total liabilities instead of short-term and total debt substantially augments the fit of both, π_{DD} and π_{VCS} to the data for my sample.

is even more important than short-term debt, since the effects of asset substitution disappear over time as debt contracts are renegotiated.[30] To derive π_{VCS}, I estimate by how much bondholders would have to raise the interest on their claims to offset a hypothetical value increase in equity due to an – also hypothetical – instantaneous rise in implied asset volatility from zero to current levels. Such a shift in asset risk would shift value from every single debt claim to stockholders. Of course, the value of short-term debt would be less affected than the value of long-term debt. Ideally, debt should therefore be modeled as a portfolio of claims with different maturities. However, due to limited availability of information on the maturity structure of debt and – more importantly – for the sake of reducing model complexity and increasing transparency, I simply define the strike price of equity as a call option as total liabilities (Compustat Item LT).

Spot Price Equivalents The spot price equals the – unobservable – market value of firm assets which in turn is the sum of the market value of equity and the market value of debt. A simple way of defining the spot price is thus as strike price plus the market value of equity. However, as the strike price is correctly defined using book values, this definition can overstate the market value of debt and thus the spot price, which is particularly likely for firms with deteriorating credit quality. Attempting to address this problem, authors of recent studies have used a different definition of spot prices derived by using the contingent claims framework. They measure asset values as they are implied by observable equity values. Assuming that Equations 3-5 hold, the market value of assets can be inferred numerically as the value that solves this set of equations for the correct market value of equity. In other words, instead of computing the value of an option based on the spot price of the underlying, the spot price of the underlying is derived from the value of the option.

I have two concerns about this approach. First and as discussed in the next paragraph, the resulting values are sensitive to the underlying assumptions about the time to maturity of the option. Assuming a time to maturity of one year consistent with the short-term view taken in the DD model, the time values of equity and debt

[30] See Grass (2008).

are relatively low such that this procedure yields estimates fairly close to the simple definition named above while complicating the procedure. Second and more important, this approach by definition assumes book values of debt to be upward-biased estimates of market values of debt if cash-payouts to bondholders are ignored, as they have been in recent research. However, whether or not book values over- or understate market values of debt depends on the conditions under which debt contracts were negotiated and on unexpected changes in the discount factor since then.

Due to these two reasons, and since I observe a slightly reduced model fit when using implied spot prices, I follow Eom et al. (2004) and define spot prices as the sum of strike price and the market value of equity. The latter is computed as the product of the CRSP items PRC and NOSH at fiscal year end.

Asset Variance The previously outlined intuition underlying the calculation of π_{VCS} assumes a hypothetical shift in asset variance from zero to the current level. Thus, obtaining a precise estimate of the asset variance used for pricing equity as a call option is the most crucial part of model parametrization. Approaches used in past studies and their underlying assumptions are therefore assessed in greater detail in the following, before proposing a new estimation of asset risk.

Several ways of estimating asset variance exist in the literature.[31] Most of them use the observable volatility of past stock returns as a starting point and derive estimates for asset risk based on assumptions about the relation between equity and firm returns. As asset volatility can be computed as the portfolio variance of a stock and bond investment, it is driven by the volatility of stock and bond returns, as well as their correlation. Given stock returns, more or less explicit assumptions about the two other factors are required for the measures discussed subsequently.

In the most simple case, asset variance is defined as unlevered equity variance using

[31] The terms asset risk, asset variance, asset volatility and firm risk are used as equivalents in the following.

the equation

$$\sigma_V = \frac{\sigma_E}{\left(1 + \frac{D}{E}\right)}. \tag{11}$$

This definition assumes constant bondholder value and thus zero risk of debt and no correlation between stock and bond value. Clearly, this simplification is particularly unrealistic for firms in financial distress, which are exactly the firms of interest in bankruptcy prediction. Ignoring the risk of debt, firm risk computed using Equation 11 underestimates the true level of risk and leads to downward-biased estimates of π_{DD} and π_{VCS}.

One approach accounting for the riskiness of distressed debt uses the "optimal hedge equation" known from delta-hedging to describe the relationship between equity and firm risk:

$$\sigma_V = \frac{\sigma_E}{\left(1 + \frac{D}{E}\right) N\left(d_1\right)}, \tag{12}$$

where $N\left(d_1\right)$ is defined according to Equation 4. [32] It is the first derivative of the value of a call option as a function of the value of its underlying (also known as delta), reflecting the sensitivity of option values to changes in the underlying. Roughly speaking and ignoring drift rates, $N\left(d_1\right)$ is close to .5 for at the money options – corresponding to firms in or close to financial distress – and approaches 1 as moneyness increases. Adding the inverse of this factor to Equation 11 thus induces an assumption about the riskiness of debt consistent with the contingent claims framework. It implies that the value of debt behaves just as the value of a short put on firm assets.

One concern about the application of this relationship is its stationarity. As we know from delta hedging, the relationship described by Equation 12 only holds instantaneously. As soon as the value of the underlying changes, deltas change as well – particularly rapid for at the money options. [33] Crosbie and Bohn (2003) therefore conclude that "in practice, the market leverage moves around far too much to provide reasonable results" and propose the following iterative estimation procedure adopted in several studies.

[32] Note that, as $N\left(d_1\right)$ itself depends on σ_V, Equation 12 requires a numerical solution.
[33] See Figure 7, displaying delta for out of the money (D/V>1), at the money (D/V=1) and in the money (D/V<1) options.

As pointed out previously, implied asset values can be obtained using Equations 3-5 given the value of equity and all necessary pricing parameters except for firm value but including asset variance. Starting with a naïve value for asset variance, for example derived according to Equation 11, a time series of historical firm values can be computed. This time series allows the estimation of a new, improved starting value for asset variance and again the estimation of a new time series of asset values. After repeating this iteration several times, the estimates for asset risk derived in this manner converge. The iteration's last asset variance is then used as an input parameter to the default model. In Table 2 I refer to this estimation procedure as "KMV-like". Using Bayesian adjustments, Moody's combines this estimate with country, size and industry averages (labeled "KMV" in Table 2).

While this procedure yields more constant parameter values over time, Bharath and Shumway (2008) label the procedure "complicated" and – for their simplified model – define asset variance as

$$\sigma_V = \left(\frac{E}{D+E}\right) \times \sigma_E + \left(\frac{D}{D+E}\right) \times \left(.05 + .25 \times \sigma_E\right), \tag{13}$$

where the last term in parentheses reflects a naïve variance estimate of debt. While the choice of this parameter is arbitrary, it yields estimates of firm risk far closer to the ones used by Moody's KMV than the estimates derived from the iterative procedure.[34] Furthermore, the authors show that estimating the Merton model with this and other naïve parameters yields superior predictions relative to a model based on more complicated parameter estimates. They conclude that "the iterative procedure used to solve the Merton model for default probability does not appear to be useful."

Eom et al. (2004) note that the resulting values for asset variance actually do not differ substantially from the ones computed using Equation 12. I argue that the measures have to be similar by definition if assuming a short-term perspective with option lives of only 1 year, as done in studies implementing the measure: Option values are the sum of an inner value component, representing the option's payoff if exercised immediately, and a time-value component, reflecting the option's chance of

[34] See Bharath and Shumway (2008), Table 2.

moving deeper into the money. Options with a low time-to-maturity can only have a limited time value by definition. The variance of returns on debt as a short put option on firm assets is only due to changes in the value of its time component, as its inner value is constant when in the money (that is, not bankrupt). If the share of time value in total debt value is relatively low, so are the changes in total debt value. When decreasing the time to maturity assumed in the iterative procedure towards zero, asset risk actually converges towards the definition reflected in Equation 11.

The procedure's shortcoming can be overcome by increasing the assumed time to maturity. When doing so, precise estimates of the payout ratio to debt holders are required to avoid the misspecification of debt-values. In line with the previous argumentation, the iterative estimate of asset risk proves to be highly sensitive to changes in the assumed option life and payout ratio. It remains unclear which time to maturity best reflects the time value component inherent in each equity option and how to precisely approximate debt payouts.

Overall, the estimates of asset variance (excluding Bharath and Shumway (2008)'s naïve but arbitrary measure) discussed so far share one common disadvantage. The average asset risk of firms decreases heavily in leverage. Using any of these estimates frequently yields low values of 10% to 20% asset variance for highly levered firms, which is clearly below the average asset risk of firms with a low leverage, amounting to over 40%. Of course, one reason for this phenomenon is that less risky firms can obtain cheaper debt financing. Another potential explanation is the underestimation of the riskiness of distressed debt in all procedures. For example, consider a firm with 90% leverage and 100% equity volatility. Equation 11 predicts an asset volatility of 10%, Equation 12 an asset volatility of less than 20%, (assuming non-negative drift rates) and the iterative procedure a volatility of roughly 12% for a one year time to maturity, depending on the underlying assumptions. Intuitively, none of these values seems to adequately reflect firm risk in the example. Interestingly, the naïve measure defined in Equation 13 yields 37% asset variance.

Concluding, while a final assessment of the precision of each of the approaches is beyond the scope of this study, good reasons exist for questioning their ability to estimate asset risk for the most important group of firms – those with a high likelihood of financial distress.[35] In the following, an entirely new approach is presented, which avoids explicit assumptions about the relation between debt and equity returns in highly levered firms.

As known from the previous discussion, the relationship between equity and debt returns is described with a high degree of precision by both Equation 11 and Equation 12 when firms with very low leverage are considered. The reason is that in a contingent claims framework the debt of such firms is almost riskless and – assuming some degree of tolerance – the optimal hedge equation does not only hold stationary. Figure 8 displays equity's option delta $N(d_1)$ as a function of leverage for asset values moving from 0 to 1,000 and the level of debt remaining constant at 100. The problem stemming from the stationarity assumption underlying Equation 12 becomes obvious when comparing the two dashed lines in the graph, which represents the tangents to the delta function for a leverage of roughly 90% (V=111) and a leverage of 20% (V=500). While the linear tangent to the delta function is a good approximation of the function for low leverage ratios, this is not at all true for higher leverage ratios.

[Fig. 8 about here.]

It is thus possible to compute good proxies for asset risk for the subsample of firms with a low leverage, while the asset variances of the – more relevant – subsample of highly levered firms remain unknown.[36] However, using the former subsample, it is possible to analyze the determinants of asset risk using a plain regression model of the form:

$$\sigma_{V[LEV<.25]} = \beta_{[LEV<.25]} \times X_{[LEV<.25]} + \varepsilon, \tag{14}$$

[35] An assessment of all described approaches can be done for firms with available market prices of debt by comparing the estimates to fluctuations in aggregate debt and equity market values.

[36] For firms with low leverage ratios, any of the previously outlined estimates (excluding Bharath and Shumway (2008)'s naïve measure) yields good approximations. I use the one based on the optimal hedge equation and given by Equation 12. I compute equity risk as the standard deviation of monthly stock returns over the last 12 months. If no stock returns are reported for all 12 months of the previous fiscal year, I only consider the months for which data is available. If less than 9 months of data are given, the observation is dropped.

where $\sigma_{V[LEV<.25]}$ is a vector of asset variances of firms with leverage ratios below 25%, $\beta_{[LEV<.25]}$ is a vector of coefficients and $X_{[LEV<.25]}$ is a matrix containing a set of independent variables for these firms. The relationships observed for the small sample and captured in $\beta_{[LEV<.25]}$ can then be extrapolated to the entire sample, including firms with higher leverage. The new estimate of asset variance is the regression's expected, or fitted, value:

$$\sigma_{V[ALL]} = \beta_{[LEV<.25]} \times X_{[ALL]}, \tag{15}$$

where $\sigma_{V[ALL]}$ are the variances and $X_{[ALL]}$ is the set of independent variables for the entire sample. [37]

When identifying a set of independent variables determining the asset variance used to derive π_{DD} and π_{VCS}, also factors which relate to uncertainty are worth considering. As a detailed analysis of factors driving asset risk and uncertainty is beyond the scope of this paper, I rely on a small set of intuitively appealing variables: The ratio of book-to-market values of equity (BM), relative firm size measured as the logarithm of the market value of a firm's equity relative to the market capitalization of the S&P500 (RSIZE), the ratio of total liabilities to the market value of assets (TLMTA), the ratio of net income to the market value of total assets (NIMTA), as well as the logarithm of the yearly average standard deviation of equity returns (LOGAVGSDE). A significant part of the value of firms with low book-to-market ratios is intangible, and thus represents uncertainty or risk. Firm size is arguably the most obvious proxy for asset risk; big firms are expected to exhibit lower risk than small firms. Risky firms are expected to operate with lower leverage ratios, as the indirect costs of financial distress are too high, blocking the access to reasonably priced debt. Accounting for leverage in the model is particularly important as model coefficients from a regression including only firms with relatively low leverage ratios are extrapolated to all firms. The logged average standard deviation of stock returns controls for yearly market-

[37] Of course, choosing a leverage ratio of 25% is arbitrary. While estimating regression coefficients only based on all-equity firm-year observations seems even more appealing, it would not allow capturing the negative relation between leverage and asset risk which most likely increases the accuracy of firm risk estimates when applied out-of-sample to firms with higher leverage.

wide variations in risk. The fourth explanatory variable is the ratio of net income to the market value of total assets (NIMTA), capturing uncertainty about the future distribution of asset returns. Firms with a low profitability are more likely to undergo fundamental changes in their business strategy or even comprehensive restructuring. Outliers are eliminated from the variables BM, RSIZE and NIMTA by Winsorizing them at the 1^{st} and 99^{th} percentile.[38]

Finally, instead of regressing on asset variance directly, I regress on its log for two reasons. First, doing so strongly increases the linearity of the relationship captured in the regression model and reduces heteroskedasticity. Second, the resulting distribution of asset variances is not symmetrical but close to log-normal, which is clearly more consistent with empirical evidence on distributions of asset returns. The regression model is estimated for each year using all data available since 1950 until the year for which asset variances are estimated. Given limited data availability in the first years of the sample period, the explanatory power of the regression remains relatively low until 1970 (with an average R^2 of 25.19%) and substantially increases afterwards (with R^2 moving between 45% and 55%). As a robustness check, I calculate asset risk using coefficients from a regression including all data. While this approach slightly improves the predictive power of π_{VCS} and π_{DD}, it can of course be criticized for predicting corporate failure using future data.

For simplification, I do not report the 47 sets of coefficient estimates for the years 1960-2006 but only the coefficients of the pooled regression analysis. Coefficient signs and significance are consistent among all regressions conducted.

My measure of asset variance based on the pooled regression is defined as

$$\sigma_V = e^{-1.026-.329\times BM-.107\times RSIZE-1.965\times TLMTA+.701\times LOGAVGSDE-.962\times NIMTA}. \quad (16)$$

As shown in column 1 of Table 4, all variables enter the equation significantly and

[38] Winsorizing implies setting all values below (above) the x^{th} lowest (highest) percentile(s) of a distribution equal to the x^{th} percentile. The definition of the variables TLMTA, RSIZE and NIMTA is close to the one used in Campbell et al. (forthcoming).

with the expected sign.

[Table 4 about here.]

The use of implied volatilities in structural models has been discussed and implemented in several studies.[39] The VIX proves to be a clearly better predictor of the number of failures occurring in the next year than the historical volatility of returns on the S&P500 for the data in my sample. Table 5 displays correlations between time-series of monthly historical S&P500 standard deviations, the VIX as well as the number of defaults occurring over the next 12 months.

[Table 5 about here.]

Attempting to benefit from the possibly improved variance estimates implied in option prices, I compute an estimate of implied firm risk as a robustness check. Again, the measure is computed as the expected value from a regression model. This time, instead of fitting the model using historical equity volatilities as dependant variable, I use the implied volatilities of long-term call options with 547 days to maturity which I obtain from the OptionMetrics database.[40] As no data is available before 1996, I use the coefficients from a regression based on all observations between 1996 and 2006 to compute firm risk for the years 1960-2006. Results are reported in column 2 of Table 4. All coefficients are similar in sign and significance, except for BM, which is positively correlated with firm risk. This counterintuitive finding may be caused by the unrepresentative selection of a sample including only firms with long-term options outstanding and spanning over years of extreme market movements. As reported in the following Section, variance estimates based on implied volatilities yield poorer values of π_{VCS} and π_{DD} in most cases.

[39] For instance, Bharath and Shumway (2008) calibrate the Merton model using implied volatilities from option markets and Collin-Dufresne et al. (2001) explain changes in bond-spreads over time using changes in the VIX volatility index.

[40] To reduce noise in the data, I use the average of daily implied volatilities of the last months in a firm's fiscal year instead the last daily volatility of a fiscal year.

Time To Maturity In the simple case where a firm is financed by equity and one zero coupon bond, the most intuitive way of defining the time to maturity of equity as a call option is as the maturity of the bond. At the time the bond expires – and not earlier – shareholders have the possibility not to roll the bond over, but to acquire the entire firm by paying off the entire debt. In reality, capital structures are more complex and consist of instruments with different maturities, coupon payments before maturity and embedded options that allow for early redemption. Attempting to capture at least a part of this complexity, Eom et al. (2004) price bonds with the Merton model by considering each payment – including coupon payments and the final payment of the principal – as an individual bond with maturity equal to the date of the payment. As described in Crosbie and Bohn (2003), a different view underlies Moody's KMV model. Given their previously described short-term perspective, where they consider short-term debt plus a fraction of long-term debt only, they simply set the time to maturity to one when computing the one-year DD measure. Given the measure's underlying concept in which total asset value moves randomly towards or away from liabilities (and thus default) over time, this is a straightforward choice. Several other recent studies do the same. For Merton's DD model I therefore adopt a one year time to maturity as well.

The intuition underlying π_{VCS} requires a different view. The measure is driven by call option sensitivity to changes in volatility ($Vega$) and in the payout ratio (Rho_D). Simply assuming a short-term horizon of one year despite of a far longer actual option term yields unrealistic sensitivities and thus unprecise default estimates. Given limited availability of information about the term structure of firms' liabilities, I simply set the time to maturity to six and ten years.[41] An alternative approach involves using a weighted average measure of debt maturity based on the Compustat items debt due in year one to five, as done in Grass (2008). However, these items are less frequently available and less consistent with data on total liabilities for the early years of the sample period.

[41] See, for example, Brockman and Turtle (2003).

Drift Rate For both, the DD and the VCS approach, the drift of the stochastic process underlying the equity option is defined as the difference between the assumed growth rate of the stochastic process and the weighted payout to share- and debtholders.

In the DD model, the drift rate is an important parameter as it determines how fast the distribution of assets moves towards or away from the bankruptcy threshold. However, as shown in Table 2, recent literature does not agree about the correct estimation of this parameter. Some studies suggested the use of past stock returns or the risk-free rate plus equity premium, while others use only the risk-free rate, or even assume a zero drift in asset values.[42] Furthermore, cash payouts to share- and debtholders are neglected in most studies. The scope of this problem is clearly reduced by looking at short-term horizons of one year as done in various studies, but it still is an important driver of results.[43] Given empirical evidence that underperforming firms are more likely to default it is tempting to use past asset or equity returns as the future drift rate. However, I follow Campbell et al. (forthcoming) and use the risk free rate plus an equity premium of .06 as growth rate instead. The simple reason is that projecting past under- or overperformance into the future implicitly assumes a misvaluation of firm assets today. Extending their definition, I reduce asset growth by a cash payout rate equal to the weighted average of dividend payments (Compustat Item DVC) and interest rate payments (assumed to be equal to the 1-year risk-free rate plus a flat credit risk premium of .02), as described in Equation 8.[44] In order to control for the explanatory power of past returns for predicting corporate default, I follow Bharath and Shumway (2008) and Campbell et al. (forthcoming) and include stock excess return as an explanatory variable in the regression model described subsequently.

[42] See Crosbie and Bohn (2003), Vassalou and Xing (2004), Bharath and Shumway (2008) and Campbell et al. (forthcoming), as well as Brockman and Turtle (2003) and Brown et al. (1995), respectively.

[43] Among the studies cited in Table 2, only Eom et al. (2004) use the actual time to maturity. They are also the only ones capturing the impact of cash payouts in their model.

[44] Alternatively, payouts to debtholders can be approximated as accounting interest payments, which is avoided due to limited data availability in the early years of the sample period.

Fewer assumptions are needed when computing π_{VCS}. First, the physical drift rate in assets is not required as an input parameter, since the risk-neutral valuation framework need not be abandoned. Instead, the assumed asset growth is risk-neutral and equal to the risk-free rate matched to the maturity of the option. As no 6-year rates on constant maturity treasuries are available, I construct a yield curve using cubic spline interpolation to obtain an estimate of the 6-year risk-free rate. [45] Second, no assumptions are required about the payouts to debt holders. Rather, as described more extensively in Section 4.3, hypothetical payouts are derived that offset the reduction in bondholder wealth due to a positive asset risk. As for the DD model, dividend payments are taken from Compustat.

6.2.1 Results

In the following, the measures' ability of forecasting corporate failure are assessed in a bivariate and multivariate setting. As outlined previously, corporate failure is defined as a delisting due to bankruptcy, liquidation, or poor performance. A far more narrow definition of failure is included merely as an additional robustness check and should not be overstated in its relevance, as the low number of events reduces statistical power and is uncommon in the literature on bankruptcy prediction.

Empirical results are reported for three slightly different parameter combinations. The first one is the standard model (labeled "Standard" in the subsequent tables), where asset variance is computed backward-looking. As described previously, it thus only incorporates information available up to the date for which default probabilities are estimated. The set of parameters yielding the best fit to the data of both, the π_{VCS} and π_{DD} measure (labeled "Best"), includes asset variance defined based on the coefficients of a regression including all years, which apparently yields more reliable estimates particularly in the first years of the sample period. Furthermore, instead of using historical risk-free rates for computing drift rates, the risk-free rate is simply set to 5% for all years. One possible reason for the improvement of model fit due to this adoption is implied by the unrealistic assumption of constant debt underlying both

[45] For details on cubic spline interpolation for yield curve construction, see Ron (2000).

approaches. In times of high interests and thus high drift rates, the average value of firm assets is assumed to move away rapidly from the bankruptcy threshold, that is, the book value of debt. In reality, however, firms frequently adjust their leverage ratios, as modeled by Collin-Dufresne and Goldstein (2001). Assuming a constant risk-free rate of five percent appears to offer a simplified way to account for this fact. Finally, the third model (labeled "IV") is equivalent to the standard model, except for the definition of asset variances, which are derived using implied volatilities as previously described.

Table 6 reports the results of a bivariate and non-parametric test of each measure's ability to forecast failure and bankruptcy. To create this overview, I sort all firm-year observations by their π-value in a first step. In a second step, I count the number of defaults observed over the next fiscal year for each firm-year observation. The reported numbers are the percentages of all 4,045 failures (Panel 6(a)) and 293 bankruptcies (Panel 6(b)) assigned to each decile in that way.

[Table 6 about here.]

For both, the DD and VCS approach, the best parameter set proves to be slightly superior to the standard one. The estimates based on implied volatilities perform clearly worse. In this bivariate setting, π_{VCS} seems to perform slightly better than π_{DD}, as the first decile contains the higher share of failures in the standard model. For the narrower definition of default, the DD model performs somewhat better.

The superior performance of the π_{VCS} measure becomes clearer in a multivariate setting. Table 7 reports results of a dynamic logit regression on corporate failure using different combinations of independent variables.

[Table 7 about here.]

The ability of both approaches to predict corporate failure is confirmed in the regression, as both π_{VCS} and π_{DD} enter the simple models (1) and (2) highly significantly and with a positive sign. Using standard parameters, the model including only π_{VCS} as an explanatory variable has a Pseudo R^2 of 24.9%, which is clearly above the 16.9%

R^2 of the second model, which only includes π_{DD}. Adding π_{DD} to the first model only marginally increases model fit to an R^2 of 25.3%. Interestingly, π_{DD} enters statistically significantly but with a negative sign. Both approaches perform better than models derived based on different definitions of asset variance.[46] This indicates the usefulness of my estimate of asset risk as an input parameter for structural models. In order to test whether the information used to derive asset risk causes the observable dominance of my measure over the DD measure, I include the variables used as predictors in the regression on asset variance (TLMTA, RSIZE, BM, NIMTA, LOGAVGSDE). Furthermore, as explained previously, I include a stock's excess return EXRET, defined as the difference between the continuous return of the stock and the S&P500 over the last 12 months, Winsorized at the 1^{st} and 99^{th} percentile.[47] The significance of both π_{VCS} and π_{DD} prevails in models 4 and 5. However, including both variables in an aggregate model, only the π_{VCS} coefficient is significant. The previously documented negative significance of the π_{DD} coefficient disappears.

Table 8, 9, and 10 report the results of several robustness checks. When applied to the more narrow definition of default with only 293 events during the years 1960 to 2006, both measures enter the regression significantly and with the expected sign. However, as indicated by a lower Pseudo-R^2 and disappearing significance in the augmented models including control variables, the quality of the π_{VCS} measure is clearly reduced.

[Table 8 about here.]

Using the best set of parameters, model fit of both approaches can be increased further. The dominance of the VCS measure over the DD measure prevails, as indicated by a difference in Pseudo-R^2s of 7.01% between model 1 and 2, as well as lower z-values of π_{DD} in regressions combining π_{VCS} and π_{DD}. The negative coefficient of

[46] Campbell et al. (forthcoming) report a Pseudo R^2 of approximately 15% for the model including only the DD measure with input parameters based on the iterative procedure described previously. I confirm this number defining asset variance based on the optimal hedge equation instead. Results are not reported.

[47] If no stock returns are available for all 12 months of the previous fiscal year, I only consider the months for which data is given. If less than 9 months of data are available, the observation is dropped.

π_{DD} in model 6 is unexpected but has a low magnitude and a low z-value.

[Table 9 about here.]

In line with the results of the bivariate analysis reported previously, probabilities of default estimated based on an asset variance derived using implied volatilities prove to be less useful for predicting bankruptcy. Pseudo R^2s are clearly lower than those documented above and neither π_{VCS} nor π_{DD} enter the regression models including the set of control variables significantly.

[Table 10 about here.]

7 Conclusion

The contribution of this paper is threefold.

First, I propose a new method for extracting probability of default estimates from structural credit risk models. The method is applicable to all models assuming a contingent claims perspective on debt and equity and value claims as a function of asset risk and payout-ratio. The estimation procedure consists of two parts. In a first step, I numerically derive virtual credit spreads (VCS) from the increase in the payout to debtholders necessary to offset the impact of an increase in asset variance on the option value of debt and equity. In a second step, I calculate a risk-neutral probability of default from VCS in a similar way as default estimates are derived from credit spreads observed at markets. In contrast to real-world credit spreads, VCS do not contain risk premia for default timing and recovery uncertainty, thus yielding a purer estimate of physical default probabilities.

Second, I compare the properties of VCS default estimates derived assuming a simple Merton model of capital strucure to the expected default frequency from the Merton distance to default (DD) measure in a numerical analysis. Addressing the needs highlighted in previous research, the proposed estimate takes on higher values for safe firms and lower values for firms with high leverage and asset volatility.

Third, I assess my measure's ability to predict corporate failure relative to the DD measure in an empirical setting. Given doubts about the usefulness of the iterative procedure frequently employed to compute input parameters for the Merton model, I propose a new approach for deriving asset volatility. Using this new parameter estimate increases the explanatory power of the DD model from a Pseudo-R^2 of approximately 15% documented in previous research and confirmed for my sample to 16.9% for the standard model and 20.1% for the best model. Employing my VCS-measure in the same context yields clearly superior results, with R^2s increasing to 24.9% for the standard model and 27.2% for the best model. Adding the DD measure to regressions including the VCS measure only marginally increases the fit.

While the measure is not able to perform as well as the current state-of-the-art model for bankruptcy prediction presented in Campbell et al. (forthcoming), my results cast doubt on their conclusion that summarizing default risk in a single predictor is not feasible. Future research should further explore the potential of the VCS approach by applying it to more advanced structural models and by using it for the pricing of corporate bonds.

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List of Figures

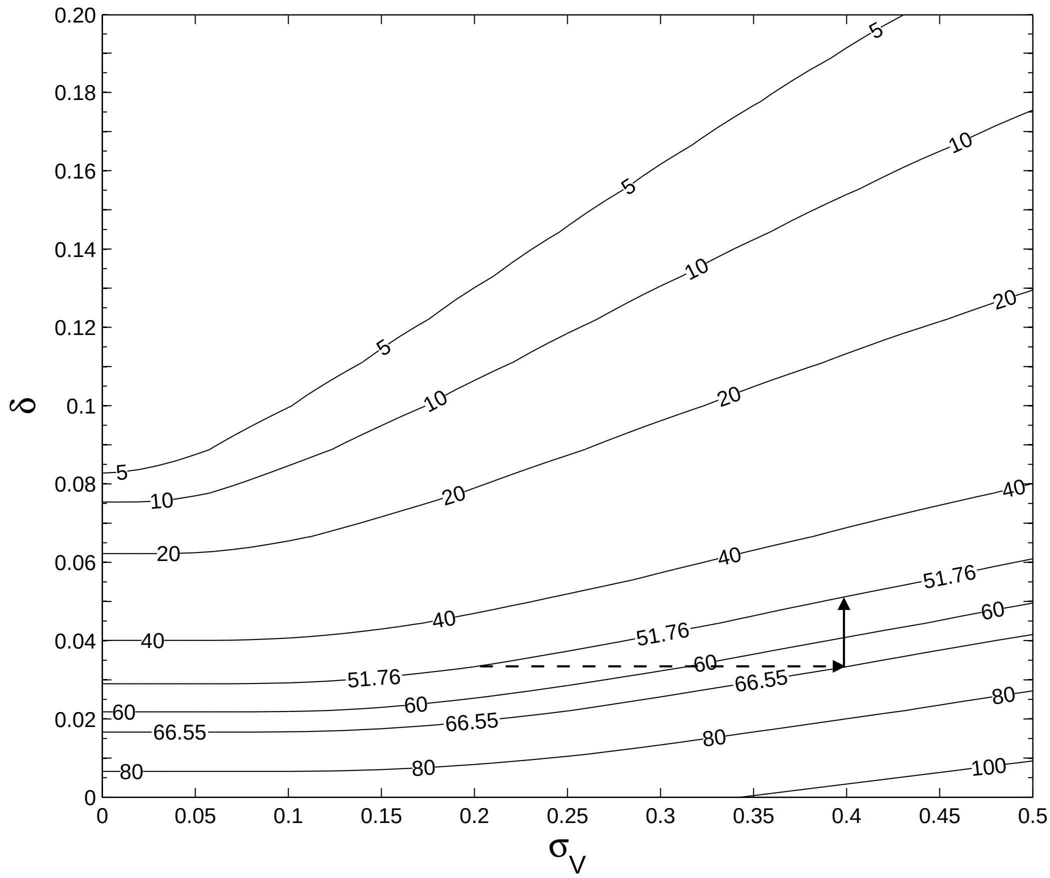


Fig. 1. Option value as a function of asset volatility σ_V and payout ratio δ. ($V = 150$, $D = 100$, $r = .05$, $T = 10$).

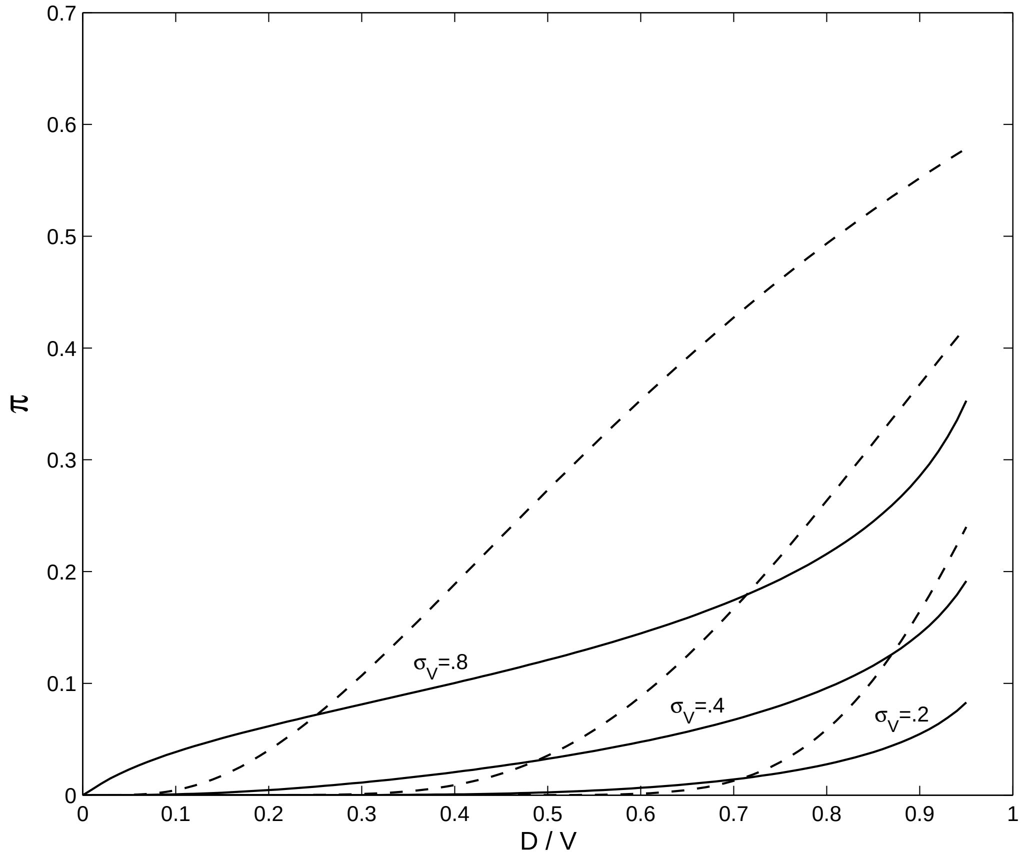


Fig. 2. Estimated default probabilities as a function of leverage D / V for different levels of asset variance σ_V. The solid and dashed lines are probabilities of default estimated based on virtual credit spreads (π_{VCS}) and the Merton distance to default model (π_{DD}), respectively. ($V = 100$, $r = .05$, $T_{VCS} = 6$, $T_{DD} = 1$, μ_{DD}=r+.06, δ=0, LGD=.75).

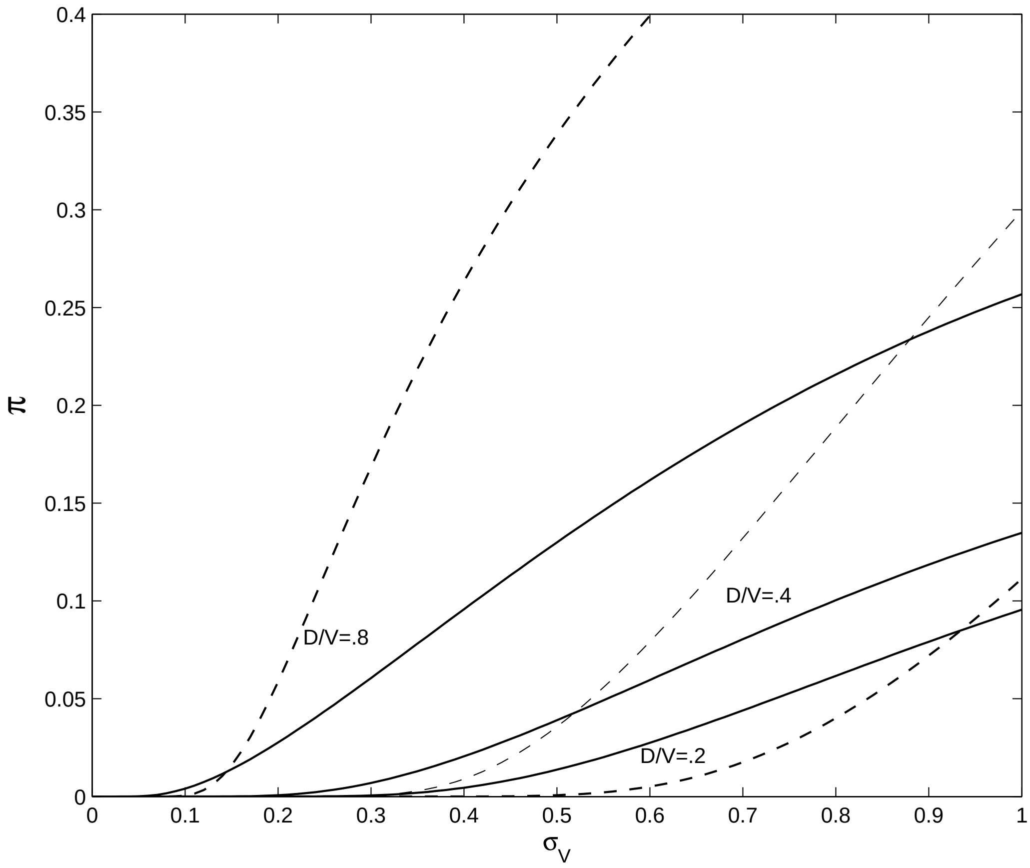
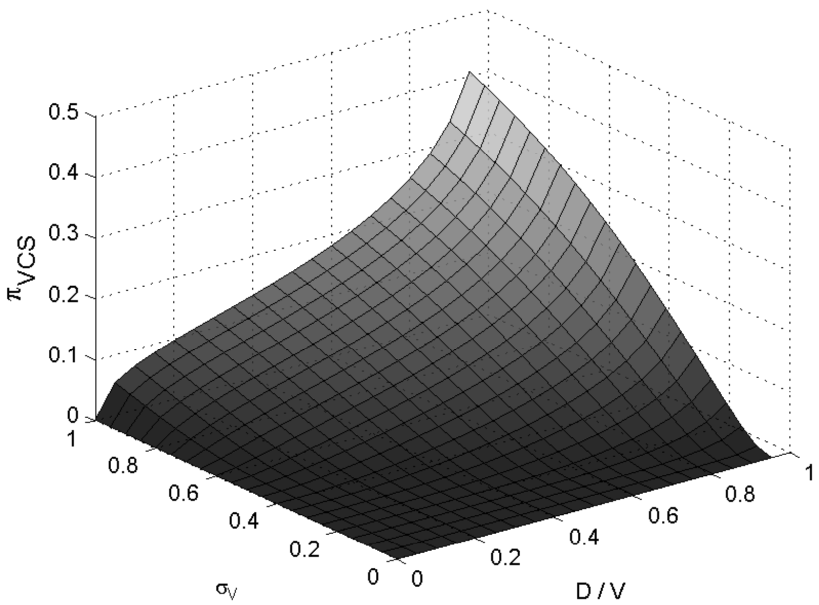


Fig. 3. Estimated default probabilities as a function of asset volatility σ_V for different levels of leverage D/V. The solid and dashed lines are probabilities of default estimated based on virtual credit spreads (π_{VCS}) and the Merton distance to default model (π_{DD}), respectively. The graph is cut-off at π=.4 for displaying purposes.
($V = 100$, $r = .05$, $T_{VCS} = 6$, $T_{DD} = 1$, μ_{DD}=r+.06, δ=0, LGD=.75).



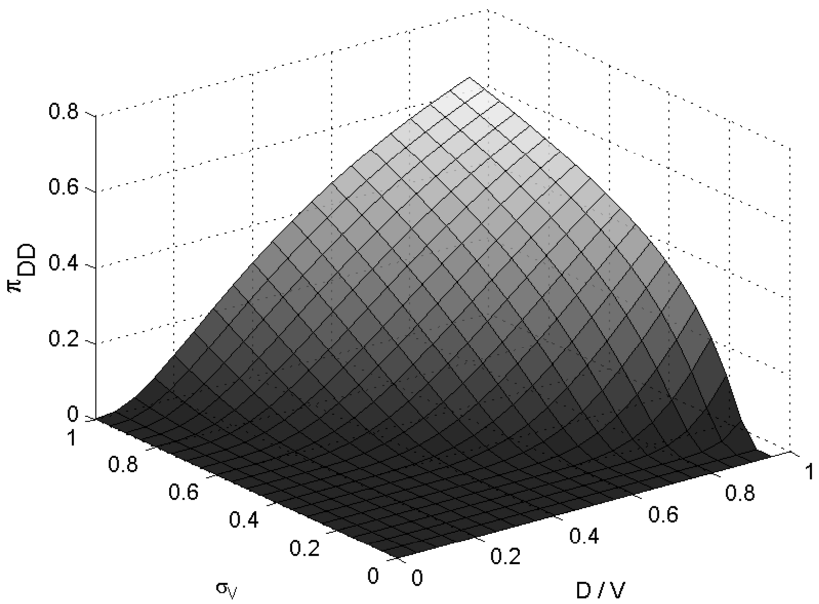


Fig. 4. Estimated default probabilities as a function of asset volatility σ_V and leverage D/V. The upper and the lower graph display probabilities of default estimated based on virtual credit spreads (π_{VCS}) and the Merton distance to default model (π_{DD}), respectively. ($V = 100$, $r = .05$, $T_{VCS} = 6$, $T_{DD} = 1$, μ_{DD}=r+.06, δ=0, LGD=.75).

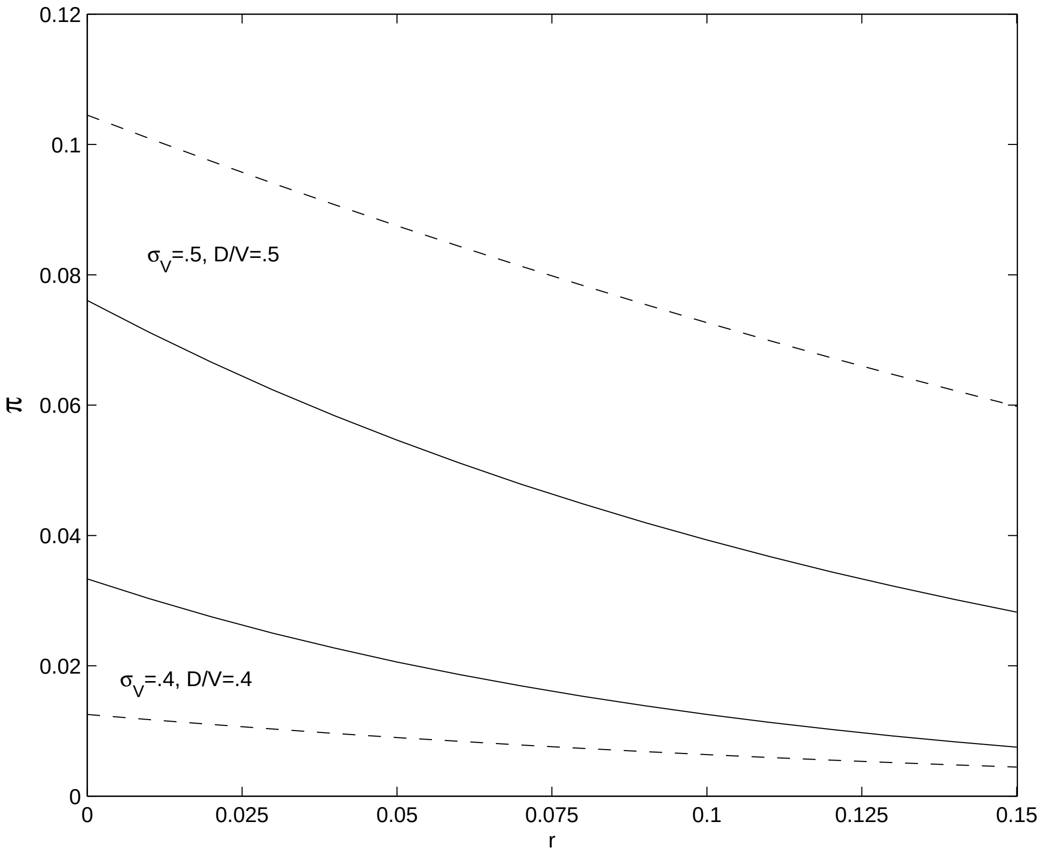


Fig. 5. Estimated default probabilities as a function of risk-free rate r for different levels of asset volatility σ_V and leverage D/V. The solid and dashed lines are probabilities of default estimated based on virtual credit spreads (π_{VCS}) and the Merton distance to default model (π_{DD}), respectively.
($V = 100$, $T_{VCS} = 6$, $T_{DD} = 1$, μ_{DD}=r+.06, δ=0, LGD=.75).

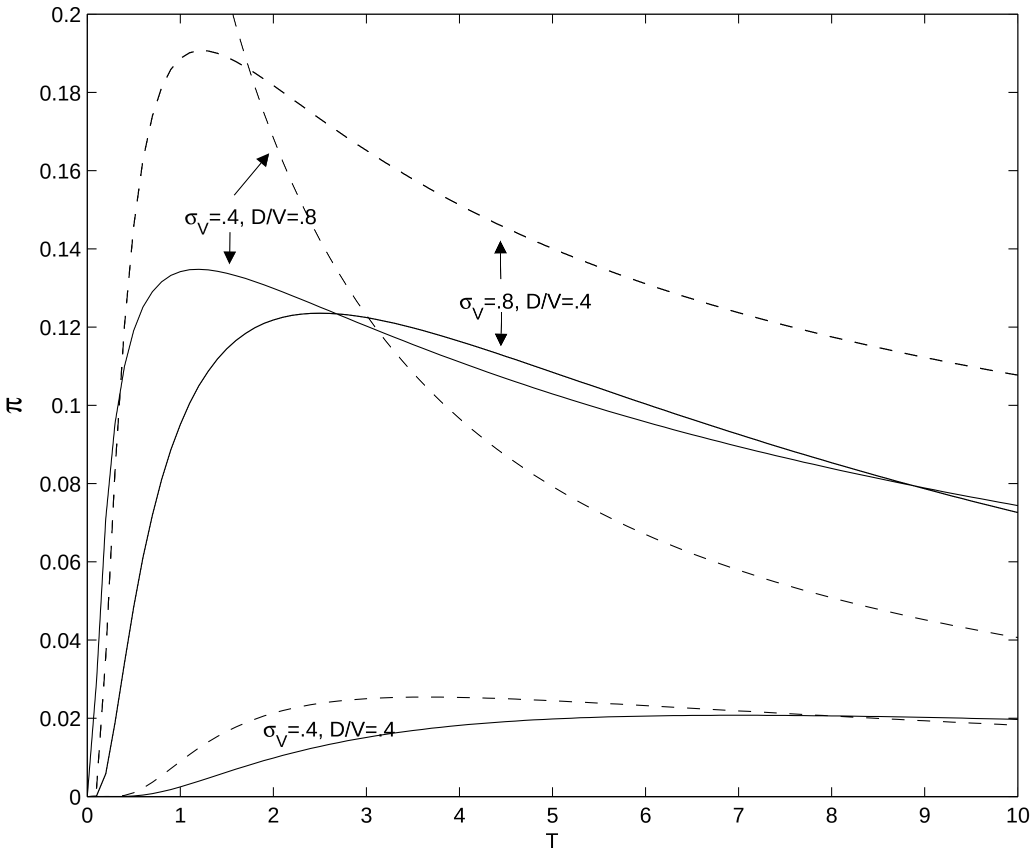


Fig. 6. Estimated default probabilities as a function of time to debt maturity for different levels of asset volatility σ_V and leverage D/V. The solid and dashed lines are probabilities of default estimated based on virtual credit spreads (π_{VCS}) and the Merton distance to default model (π_{DD}), respectively. The graph is cut-off at π=.2 for displaying purposes. (V = 100, r = .05, μ_{DD}=r+.06, δ=0, LGD=.75).

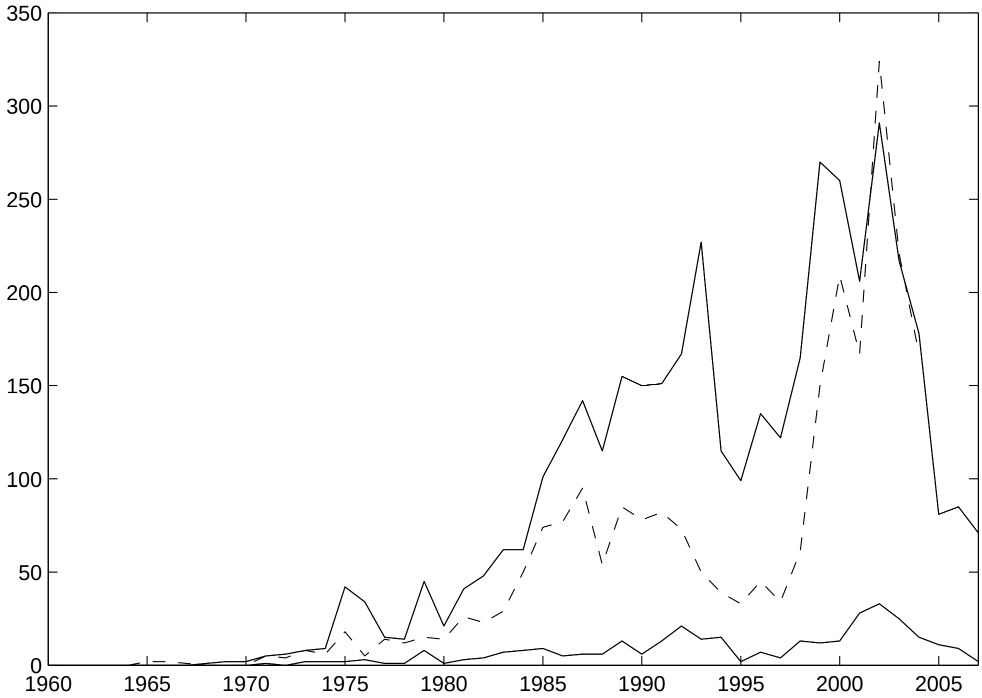


Fig. 7. The development of corporate failure in the years 1961 through 2007.
The higher (lower) solid line displays the sample's total number of corporate failures (bankruptcies) per year, as defined by CRSP delisting codes. The dashed line displays the total number of failures observed by Campbell et al. (forthcoming), using a slightly different definition.

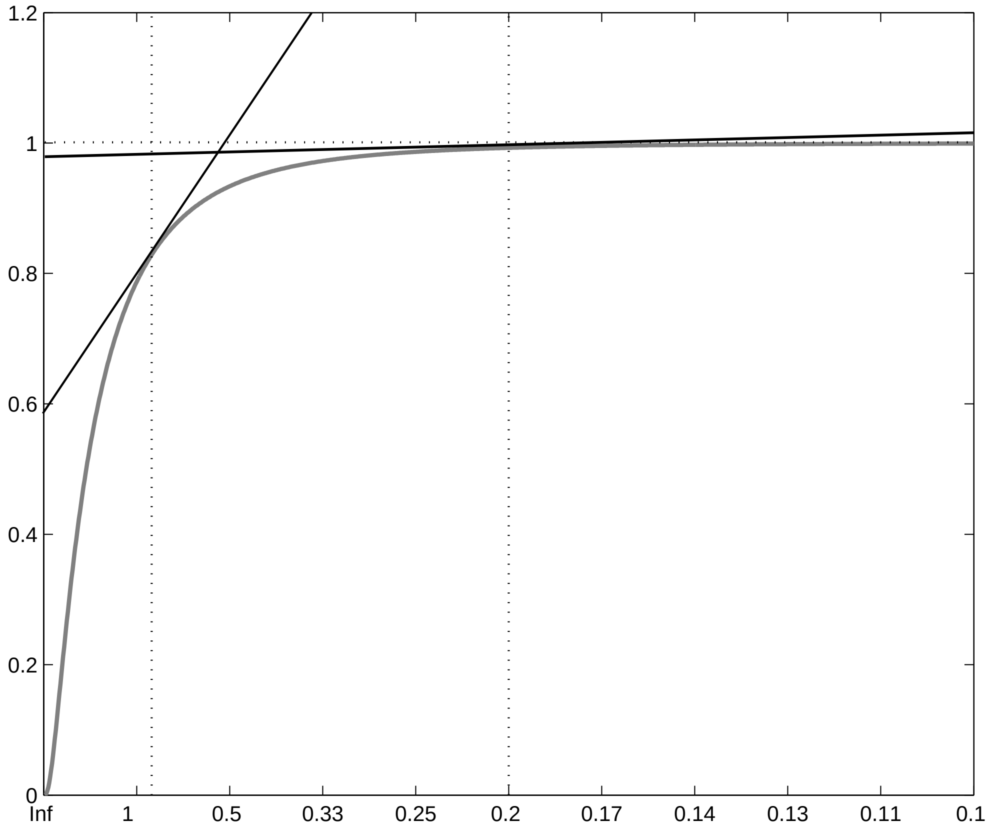


Fig. 8. Equity's option delta $N(d_1)$ as a function of leverage (D/V).
V = 0 to 1000, D = 100, σ_V=.4, r=.05, δ=0.

List of Tables

Table 1
Descriptive sample statistics.
Market leverage (TLMTA) is measured as the ratio of total liabilities to the market value of assets (the sum of the market value of equity and the book value of liabilities), relative firm size (RSIZE) as the logarithm of the market value of a firm's equity relative to the market capitalization of the S&P500, book-to-market (BM) as the ratio of equity's book and market value, profitability (NIMTA) as the ratio of net income to the market value of total assets, market wide equity risk (LOGAVGSDE) as the natural logarithm of the yearly average standard deviation of equity returns, and excess return EXRET as the difference between the continuous return of the stock and the S&P500 over the last 12 months. All variables except π_{VCS} and π_{DD} and LOGAVGSDE are Winsorized at the 1^{st} and 99^{th} percentile.

(a) All observations

	Median	Mean	Quartiles		STD
			First	Third	
TLMTA	0.358	0.384	0.172	0.571	0.247
RSIZE	−10.563	−10.431	−11.874	−9.127	2.025
BM	0.598	0.816	0.316	1.068	0.775
NIMTA	0.029	−0.007	−0.018	0.053	0.125
LOGAVGSDE	−0.795	−0.786	−0.872	−0.684	0.191
EXRET	−0.006	−0.011	−0.034	0.017	0.048
Observations					151,451

(b) Subgroup of firms failing in the next 12 months

	Median	Mean	Quartiles		STD
			First	Third	
TLMTA	0.560	0.537	0.277	0.807	0.298
RSIZE	−13.257	−13.105	−14.180	−12.312	1.254
BM	0.298	0.707	0.018	1.053	1.142
NIMTA	−0.205	−0.258	−0.424	−0.079	0.228
LOGAVGSDE	−0.752	−0.704	−0.811	−0.593	0.175
EXRET	−0.068	−0.070	−0.121	−0.025	0.064
Observations					4,045

(c) Subgroup of firms declaring bankruptcy in the next 12 months

	Median	Mean	Quartiles		STD
			First	Third	
TLMTA	0.843	0.777	0.697	0.937	0.212
RSIZE	−12.347	−12.321	−13.267	−11.444	1.365
BM	0.347	0.800	−0.386	1.289	1.408
NIMTA	−0.196	−0.251	−0.389	−0.081	0.214
LOGAVGSDE	−0.684	−0.669	−0.798	−0.514	0.183
EXRET	−0.081	−0.082	−0.140	−0.035	0.062
Observations					293

Table 2. *Parameters used in different studies for the estimation of the Merton Model.*

	Strike Price	Spot Price	Asset Variance	Time to Maturity	Drift Rate
Crosbie and Bohn (2003) / KMV	ST+.5 LT debt	Implied	KMV	1YR	-
Vassalou and Xing (2004)	ST+.5 LT debt	Implied	KMV-like	-	Past Asset Return
Bharath and Shumway (2008)	ST+.5 LT debt	Implied	KMV-like	1YR	Past Asset Return
Bharath and Shumway (2008) Naïve	TD	MVE+TD	Naïve function of SDE	1YR	Past Equity Return
Campbell et al. (forthcoming)	ST+.5 LT debt	Implied	KMV-like	1YR	ST TR + .06
Eom et al. (2004)	TD	MVE+TD	Optimal hedge	Bond TTM	TR - Asset Payout
π_{DD}	ST+.5 LT debt	MVE+TD	Regression-based	1 YR	ST TR+.06
π_{VCS}	TD	MVE+TD	Regression-based	6 YRS	LT TR

The upper part of the table displays parameters used to estimate the Merton model for default prediction (upper five entries) and bond pricing (Eom et al. (2004)) in previous research. The bottom part of the table shows the parameters used in this study.
The following abbreviations are used only in this table: ST = short-term, LT = long-term, TD = total debt, MVE = market value of equity, SDE = standard deviation of stock returns, YR= year, TTM = time to maturity, TR = treasury rate
Furthermore, *Implied* refers to total firm value as implied by the market value of equity in a contingent claims framework. *KMV* refers to Moody's procedure for estimating asset volatility using an iterative procedure and additional statistical analyses; *KMV-like* to a simplified adoption of this procedure. *Optimal hedge* stands for equity variance unlevered using the optimal hedge equation and *Regression-based* for the variance derived as the expected value in a regression model described in this study. In case a cell does not contain any entry, no exact definition was found in the article.

Table 3

Model parameters for estimating default probabilities.

Leverage D/V is the inverse of option moneyness and computed as the ratio of book debt to the sum of book debt and the market value of equity. For the VCS (DD) approach, debt is defined as total debt (short-term debt + .5 long-term debt). σ_V is the asset variance estimate from a regression model, DIV the dividend yield, δ_{DD} the cash payout ratio (only required for the DD model), and r_{VCS} (r_{DD}) the 6- and 1-year risk-free rate on constant maturity treasuries.

(a) All observations

	Median	Mean	Quartiles		STD
			First	Third	
D_{VCS}/V_{VCS}	0.358	0.384	0.172	0.571	0.247
D_{DD}/V_{DD}	0.284	0.328	0.137	0.483	0.230
σ_V	0.274	0.318	0.193	0.407	0.169
DIV	0.000	0.012	0.000	0.020	0.021
δ_{DD}	0.031	0.036	0.013	0.053	0.027
r_{VCS}	0.065	0.071	0.054	0.083	0.025
r_{DD}	0.058	0.063	0.046	0.077	0.029
Observations					151,451

(b) Subgroup of firms failing in the next 12 months

	Median	Mean	Quartiles		STD
			First	Third	
D_{VCS}/V_{VCS}	0.560	0.537	0.277	0.807	0.298
D_{DD}/V_{DD}	0.505	0.501	0.238	0.765	0.294
σ_V	0.418	0.451	0.248	0.650	0.240
DIV	0.000	0.001	0.000	0.000	0.008
δ_{DD}	0.036	0.040	0.018	0.059	0.028
r_{VCS}	0.063	0.069	0.052	0.081	0.024
r_{DD}	0.056	0.060	0.045	0.073	0.026
Observations					4,045

(c) Subgroup of firms declaring bankruptcy in the next 12 months

	Median	Mean	Quartiles		STD
			First	Third	
D_{VCS}/V_{VCS}	0.843	0.777	0.697	0.937	0.212
D_{DD}/V_{DD}	0.778	0.727	0.618	0.899	0.220
σ_V	0.253	0.295	0.158	0.391	0.183
DIV	0.000	0.004	0.000	0.000	0.015
δ_{DD}	0.056	0.057	0.037	0.074	0.026
r_{VCS}	0.063	0.067	0.050	0.079	0.024
r_{DD}	0.056	0.058	0.043	0.071	0.028
Observations					293

Table 4

Estimates of a pooled linear regression on firm risk.

This table displays the results of a linear regression on historical firm risk based on data from 1950-2006 and implied firm risk based on data from 1996-2006. Firm risk σ_V is defined as the unlevered volatility of monthly stock returns, implied firm risk σ_{V^*} as the unlevered implied volatility of long-term (547 day) call options on a firm's equity. Volatilities are unlevered using the optimal hedge equation. Book-to-market (BM) is measured as the ratio of equity's book and market value, relative firm size (RSIZE) as the logarithm of the market value of a firm's equity relative to the market capitalization of the S&P500, market leverage (TLMTA) as the ratio of total liabilities to the market value of assets, profitability (NIMTA) as the ratio of net income to the market value of total assets, and market wide equity risk (LOGAVGSDE) as the natural logarithm of the yearly average standard deviation of equity returns. The variables BM, RSIZE and TLMTA are Winsorized at the 1^{st} and 99^{th} percentile. The samples only includes firms with leverage ratios below 25%.

	σ_V	σ_{V^*}
(Intercept)	-1.026 ***	-1.029 ***
	(.018)	(.035)
BM	$-.329$ ***	.174 ***
	(.006)	(.042)
RSIZE	$-.107$ ***	$-.130$ ***
	(.001)	(.005)
TLMTA	-1.965 ***	-2.588 ***
	(.031)	(.095)
NIMTA	$-.962$ ***	-1.068 ***
	(.019)	(.102)
LOGAVGSDE	.701 ***	.891 ***
	(.009)	(.025)
AdjustedR^2	.468	.699
Observations	49,044	1,429

$***$ indicates significance at the 1% Level. Standard errors are in parentheses.

Table 5

Historical and implied volatility versus default risk.

This table displays the correlation between monthly time series of the 12 months backward-looking S&P500 standard deviation, the VIX and the number of failures occurring in the next 12 months for the period 1990-2006.

	$\sigma_{S\&P500}$	VIX	$Failures$
$\sigma_{S\&P500}$	1.00	0.668	0.568
VIX		1.000	0.680
$Failures$			1.000

Table 6

Failures and Bankruptcy per π-Decile.

This table assesses the ability of different default measures to forecast failure and bankruptcy. Firm-year observations are sorted according to their π-value. The reported numbers are the percentages of all 4,045 failures (Panel 6(a)) and 293 bankruptcies (Panel 6(b)) occurring in the subsequent fiscal year.

(a) Percentage of 4,045 Failures per π-Decile

	Standard		Best		IV	
Decile	π_{VCS}	π_{DD}	π_{VCS}	π_{DD}	π_{VCS}	π_{DD}
1	0.63	0.56	0.67	0.65	0.47	0.54
2	0.15	0.18	0.14	0.15	0.22	0.21
3	0.07	0.09	0.06	0.07	0.13	0.10
4	0.05	0.05	0.04	0.04	0.08	0.06
5	0.03	0.04	0.03	0.02	0.04	0.03
6-10	0.07	0.08	0.06	0.07	0.06	0.06

(b) Percentage of 293 Bankruptcies per π-Decile

	Standard		Best		IV	
Decile	π_{VCS}	π_{DD}	π_{VCS}	π_{DD}	π_{VCS}	π_{DD}
1	0.67	0.71	0.65	0.73	0.41	0.59
2	0.13	0.11	0.16	0.12	0.23	0.21
3	0.06	0.05	0.06	0.06	0.15	0.07
4	0.06	0.05	0.05	0.02	0.08	0.03
5	0.04	0.01	0.04	0.03	0.06	0.03
6-10	0.04	0.05	0.04	0.04	0.08	0.06

Table 7

Estimates of a logit regression on corporate failure.

This table displays the results of a dynamic logit regression on corporate failure with π values computed using the standard set of parameters. Corporate failure is defined as any delisting due to bankruptcy, liquidation, or poor performance. π_{VCS} and π_{DD} are default probabilities estimated based on virtual credit spreads and the distance to default measure. Market leverage (TLMTA) is measured as the ratio of total liabilities to the market value of assets (the sum of the market value of equity and the book value of liabilities), relative firm size (RSIZE) as the logarithm of the market value of a firm's equity relative to the market capitalization of the S&P500, book-to-market (BM) as the ratio of equity's book and market value, profitability (NIMTA) as the ratio of net income to the market value of total assets, market wide equity risk (LOGAVGSDE) as the natural logarithm of the yearly average standard deviation of equity returns, and excess return EXRET as the difference between the continuous return of the stock and the S&P500 over the last 12 months. All variables except π_{VCS} and π_{DD} and LOGAVGSDE are Winsorized at the 1^{st} and 99^{th} percentile.

	(1)	(2)	(3)	(4)	(5)	(6)
(Intercept)	−4.636 ***	−4.072 ***	−4.729 ***	−12.887 ***	−12.840 ***	−12.851 ***
	(0.025)	(0.02)	(0.027)	(0.218)	(0.219)	(0.22)
π_{VCS}	53.632 ***		64.961 ***	8.295 ***		6.684 ***
	(0.55)		(1.124)	(1.269)		(1.877)
π_{DD}		14.824 ***	−4.284 ***		1.949 ***	0.602
		(0.172)	(0.372)		(0.351)	(0.515)
TLMTA				1.277 ***	1.237 ***	1.222 ***
				(0.089)	(0.101)	(0.101)
RSIZE				−0.730 ***	−0.748 ***	−0.732 ***
				(0.016)	(0.016)	(0.016)
BM				−0.451 ***	−0.511 ***	−0.453 ***
				(0.03)	(0.025)	(0.03)
NIMTA				−2.603 ***	−2.867 ***	−2.629 ***
				(0.122)	(0.105)	(0.124)
LOGAVGSDE				0.220 **	0.352 ***	0.242 **
				(0.109)	(0.106)	(0.11)
EXRET				−4.313 ***	−4.224 ***	−4.293 ***
				(0.363)	(0.364)	(0.364)
Pseudo R^2	0.249	0.169	0.253	0.379	0.379	0.379
N	151,451	151,451	151,451	151,451	151,451	151,451

$***$ indicates significance at the 1% Level. Standard errors are reported in parentheses.

Table 8

Estimates of a logit regression on bankruptcy.

This table displays the results of a dynamic logit regression on bankruptcy with π values computed using the standard set of parameters. Bankruptcy is defined narrowly in line with the CRSP delisting codes. π_{VCS} and π_{DD} are default probabilities estimated based on virtual credit spreads and the distance to default measure. Market leverage (TLMTA) is measured as the ratio of total liabilities to the market value of assets (the sum of the market value of equity and the book value of liabilities), relative firm size (RSIZE) as the logarithm of the market value of a firm's equity relative to the market capitalization of the S&P500, book-to-market (BM) as the ratio of equity's book and market value, profitability (NIMTA) as the ratio of net income to the market value of total assets, market wide equity risk (LOGAVGSDE) as the natural logarithm of the yearly average standard deviation of equity returns, and excess return EXRET as the difference between the continuous return of the stock and the S&P500 over the last 12 months. All variables except π_{VCS} and π_{DD} and LOGAVGSDE are Winsorized at the 1^{st} and 99^{th} percentile.

	(1)	(2)	(3)	(4)	(5)	(6)
(Intercept)	−7.395 ***	−6.995 ***	−7.265 ***	−8.905 ***	−9.081 ***	−9.088 ***
	(0.096)	(0.084)	(0.098)	(0.591)	(0.612)	(0.613)
π_{VCS}	50.466 ***		32.049 ***	3.123		8.511
	(1.614)		(4.501)	(3.778)		(5.763)
π_{DD}		15.674 ***	6.266 ***		−0.126	−1.807
		(0.483)	(1.399)		(0.988)	(1.481)
TLMTA				5.957 ***	6.186 ***	6.102 ***
				(0.415)	(0.432)	(0.43)
RSIZE				−0.004	−0.018	−0.006
				(0.043)	(0.042)	(0.043)
BM				−0.273 ***	−0.324 ***	−0.265 ***
				(0.075)	(0.065)	(0.075)
NIMTA				−2.645 ***	−2.795 ***	−2.616 ***
				(0.378)	(0.358)	(0.378)
LOGAVGSDE				1.684 ***	1.694 ***	1.654 ***
				(0.361)	(0.36)	(0.362)
EXRET				−6.893 ***	−6.907 ***	−6.894 ***
				(1.307)	(1.308)	(1.303)
Pseudo R^2	0.175	0.170	0.180	0.280	0.280	0.280
N	151,451	151,451	151,451	151,451	151,451	151,451

$***$ indicates significance at the 1% Level. Standard errors are reported in parentheses.

Table 9

Estimates of a logit regression on corporate failure.

This table displays the results of a dynamic logit regression on corporate failure with π values computed using the best set of parameters. Corporate failure is defined as any delisting due to bankruptcy, liquidation, or poor performance. π_{VCS} and π_{DD} are default probabilities estimated based on virtual credit spreads and the distance to default measure. Market leverage (TLMTA) is measured as the ratio of total liabilities to the market value of assets (the sum of the market value of equity and the book value of liabilities), relative firm size (RSIZE) as the logarithm of the market value of a firm's equity relative to the market capitalization of the S&P500, book-to-market (BM) as the ratio of equity's book and market value, profitability (NIMTA) as the ratio of net income to the market value of total assets, market wide equity risk (LOGAVGSDE) as the natural logarithm of the yearly average standard deviation of equity returns, and excess return EXRET as the difference between the continuous return of the stock and the S&P500 over the last 12 months. All variables except π_{VCS} and π_{DD} and LOGAVGSDE are Winsorized at the 1^{st} and 99^{th} percentile.

	(1)	(2)	(3)	(4)	(5)	(6)
(Intercept)	-4.624 ***	-4.052 ***	-4.578 ***	-12.974 ***	-12.985 ***	-13.039 ***
	(0.025)	(0.02)	(0.025)	(0.216)	(0.217)	(0.218)
π_{VCS}	72.063 ***		63.934 ***	15.340 ***		17.979 ***
	(0.712)		(1.155)	(1.873)		(2.23)
π_{DD}		30.723 ***	5.194 ***		1.724 ***	-1.671 **
		(0.338)	(0.574)		(0.65)	(0.775)
TLMTA				1.436 ***	1.483 ***	1.555 ***
				(0.074)	(0.091)	(0.092)
RSIZE				-0.707 ***	-0.753 ***	-0.706 ***
				(0.017)	(0.016)	(0.017)
BM				-0.387 ***	-0.550 ***	-0.393 ***
				(0.032)	(0.027)	(0.032)
NIMTA				-2.268 ***	-2.942 ***	-2.264 ***
				(0.139)	(0.111)	(0.139)
LOGAVGSDE				-0.083	0.310 ***	-0.092
				(0.12)	(0.109)	(0.119)
EXRET				-4.160 ***	-4.219 ***	-4.175 ***
				(0.364)	(0.364)	(0.364)
Pseudo R^2	0.272	0.201	0.274	0.380	0.378	0.380
N	151,451	151,451	151,451	151,451	151,451	151,451

$***$ indicates significance at the 1% Level. Standard errors are reported in parentheses.

Table 10

Estimates of a logit regression on corporate failure.

This table displays the results of a dynamic logit regression on corporate failure with π values computed using an asset variance estimate derived using implied volatilities. Corporate failure is defined as any delisting due to bankruptcy, liquidation, or poor performance. π_{VCS} and π_{DD} are default probabilities estimated based on virtual credit spreads and the distance to default measure. Market leverage (TLMTA) is measured as the ratio of total liabilities to the market value of assets (the sum of the market value of equity and the book value of liabilities), relative firm size (RSIZE) as the logarithm of the market value of a firm's equity relative to the market capitalization of the S&P500, book-to-market (BM) as the ratio of equity's book and market value, profitability (NIMTA) as the ratio of net income to the market value of total assets, market wide equity risk (LOGAVGSDE) as the natural logarithm of the yearly average standard deviation of equity returns, and excess return EXRET as the difference between the continuous return of the stock and the S&P500 over the last 12 months. All variables except π_{VCS} and π_{DD} and LOGAVGSDE are Winsorized at the 1^{st} and 99^{th} percentile.

	(1)	(2)	(3)	(4)	(5)	(6)
(Intercept)	-4.402 ***	-4.100 ***	-4.308 ***	-13.086 ***	-13.105 ***	-13.088 ***
	(0.023)	(0.02)	(0.023)	(0.216)	(0.218)	(0.219)
π_{VCS}	30.399 ***		16.615 ***	-1.904		-1.855
	(0.384)		(0.73)	(1.194)		(1.529)
π_{DD}		13.030 ***	7.202 ***		-0.372	-0.024
		(0.158)	(0.305)		(0.359)	(0.46)
TLMTA				1.559 ***	1.657 ***	1.562 ***
				(0.086)	(0.072)	(0.106)
RSIZE				-0.777 ***	-0.768 ***	-0.777 ***
				(0.018)	(0.016)	(0.018)
BM				-0.571 ***	-0.583 ***	-0.571 ***
				(0.025)	(0.023)	(0.025)
NIMTA				-3.252 ***	-3.161 ***	-3.253 ***
				(0.144)	(0.123)	(0.144)
LOGAVGSDE				0.521 ***	0.435 ***	0.520 ***
				(0.136)	(0.116)	(0.136)
EXRET				-4.261 ***	-4.257 ***	-4.261 ***
				(0.363)	(0.363)	(0.363)
Pseudo R^2	0.151	0.154	0.168	0.378	0.378	0.378
N	151,451	151,451	151,451	151,451	151,451	151,451

$***$ indicates significance at the 1% Level. Standard errors are reported in parentheses.