Dynamic Asset Allocation with Distorted Beliefs

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Abstract

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Keywords: Distorted Beliefs, Investor Sentiment, Dynamic Asset Allocation, Markov-Switching Vector Autoregressive Model, Return Predictability.

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1 Introduction

According to the classical financial theory, through arbitrage, rational investors can benefit from the price correction that makes irrational investors, who misperceive asset returns, buy high and sell low, causing them to lose their wealth; see for example, Sandroni (2001) and Blume and Easley (2006). However, recent researches, such as De Long, Shleifer, Summers and Waldman (1991), and Kogan, Ross, Wang and Westerfield (2006) advocated that investors with biased beliefs might form portfolio holdings performing a higher growth rates that outgrow that of rational investors. For example, the optimistic trader bets on good states of the economy (with under-diversification), massively investing in the contingent claims of good states. It would cause rational investors switching to buy the contingent claims of bad states and give up those of good states, including those with high probabilities. As a result, the optimistic trader may end up with more wealth.

It appears that the outperformance in investment for irrational investors is possible in terms of theoretical modelling. It motivates this study to empirically investigate the performance of the portfolio allocations that calculated by the investors with various belief formation algorithms. Specifically, we examine whether the investment performances of the irrational investors with psychological biases in beliefs are superior to that of the Bayesian investor from the asset allocation perspective.

To address this issue and make a meaningful comparison without the internal inconsistency, we propose a tangible dynamic asset allocation model. We assume that investors have identical information on the investment opportunity set, identical investment horizon, identical risk-aversion coefficient and identical utility specification. They fail to correctly calculate or update their own beliefs because of psychological biases. Specifically, the investors with an Epstein-Zin recursive utility attempt to allocate their wealth into several risky assets, savings and consumption. The dynamics of the investment opportunity set described by a vector of risky asset returns and predictors are assumed to be driven by a discrete and unobserved regime variable that determines the levels, volatility and correlations of the corresponding joint distribution. It would cause the investor's optimal investment strategy associated with the beliefs of regime predictions. In effect, predicting the state of regime reflects the investor's subjective perspective about the forward outlook of the economy. In this study, we specify seven regime predicting/updating algorithms that are related to the psychological biases the most relevant and interesting in financial literature. These algorithms are termed as the belief formation mechanisms and consistent with that of Barberis, Shleifer, and Vishny (1998), Cecchetti, Lam and Mark (2000) and Brandt, Zeng, and Zhang (2004).

This paper contributes the literature in several aspects. First, we extend the framework of

Campbell, Chan and Viceira (2003) and incorporate (i) a regime-switching investment opportunity set; (ii) the psychological-based belief formation mechanisms. This is important because our proposed model offer tractable (analytical) solutions for the optimal portfolio weights that can be implemented easily under an economy with multiple assets and incomplete information. Second, we conduct the empirical analysis to examine (i) the effects of the distorted beliefs on the investor's asset allocation decision; (ii) the empirical performances of the portfolio allocations calibrated using the real observations and a variety of psychological-biased belief algorithms. Our calibration exercises are based on three investment opportunity sets, including five stock portfolios, bond and predictors, such that our findings are supposed to be robust.

For the researches during past two decades in retrospect, investigating the relationship between the investor's psychological biases (or investor sentiment) and the dynamic patterns of stock returns is not new. Barberis, Shleifer and Vishny (1998), for example, proposed a model of investor sentiment which is characterized by distorted predictions about the regime-shift stock fundamentals. The short/long run patterns of stock returns can be linked to the shifts in distorted beliefs. Cecchetti, Lam and Mark (2000) provided a similar model to link distorted beliefs with equity premium puzzle and excessive volatility, both of which have been long advocated in financial literature. Baker and Wurgler (2006) and Kumar and Lee (2006) pronounced that investor sentiment can predict the cross-section of stock returns. Although there are mounting studies that address how psychological biases affect asset returns, they didn't explore how to influence the investor's asset allocation decision.

In light of the asset allocation theory, in order to tackle the time-variant investment opportunity set with predictability patterns, long-term investors are supposed to look for the portfolio strategies to optimally trade off between the risk and the reward and to hedge the risk from the negative variation in the investment opportunity set. For example, Brennan, Schwartz and Lagnado (1997) , Lynch (2001), Wachter (2002) and Jurek and Viceira (2006) examined how the return predictability is related to the intertemporal hedging demands and the horizon effect; Liu (2007) and Liu, Longstaff and Pan (2003) explored that the volatility and jump risks in the investment opportunity may give rise to a substantial intertemporal hedging motivation; Ang and Bekaert (2002) and Guidolin and Timmermann (2007, 2008) demonstrated the importance of regimes in the joint distribution of asset returns that would cause a stronger long-term effect. In comparison with the literature, our proposed model address the issue that how the biased belief mechanisms about the state of regime affect the portfolio allocations. Our findings are as follows. (i) as compared to the Bayesian investor, the optimistic one would like to bet on good states of the economy, more aggressively chasing /shorting the assets with higher rewards in good/bad times. In contrast, the pessimistic investor would take the opposite positions to the optimistic one; (ii) the optimistic investor has the best empirical performance of the portfolio allocation and in turn outperforms the Bayesian one. The other irrational investors underperform the Bayesian one in most cases; (iii) while the predictability in the investment opportunity set is removed, the outperformance of the optimistic investor disappears. It suggests that the benefit from the intertemporal hedging induced by the return predictability is a potential explanation to the domination of the optimistic investor in our analysis.

In section 2, we propose a regime-switching investment opportunity set and describe the investor's dynamic optimization problem. Section 3 derives the approximate solutions for the optimal portfolio weights and the consumption-wealth ratio, and introduces seven kinds of belief formation algorithms. The empirical results of the MS-VAR model are shown in section 4. In section 5, we discuss the effects of the distorted beliefs, the return predictability, and the stock characteristics on the investor's optimal portfolio rules via the numerical analysis. Section 6 conducts the empirical analysis of the performance of the portfolio allocations associated with various belief formation algorithms using the real observations. Section 6 concludes.

2 The Model

We consider a discrete-time, incomplete-information dynamic asset allocation decision. Broadly speaking, an infinitely long-lived investor characterized by an Epstein-Zin recursive utility defined over a stream of consumption attempts to allocate her wealth into risky assets, savings and consumption. The dynamics of the investment opportunity set described by a vector of risky asset returns and predictors are assumed to be driven by a discrete and unobserved regime variable that determines the levels, volatility and correlations of the corresponding joint distribution. As a result, the investor's optimal investment strategies would be associated with the investor's predictions of the states of regime. In effect, predicting the states of regime reflects the investor's subjective perspective about the forward market outlook. Our model provides a belief-dependent optimal investment strategy. Moreover, the investor's beliefs may be distorted while the mechanisms of predicting regimes are contaminated by psychological biases. In comparison to Campbell, Chan and Viceira (2003), we substantially extend their framework into a more general one by involving regime shifts into the variation of the investment opportunity set and considering the various mechanisms of forming investor's beliefs.

2.1 Investment Opportunity Set

We assume that the investment opportunity set for the investor consists of $n+1$ tradable assets, including n risky assets and a riskfree one, and m state variables, used to predict the future returns of the risky assets. Many empirical articles in recent financial literature point out that the levels, volatility and correlations of the asset returns distribution may vary with regimes, see

Hamilton and Susmel (1994), Gray (1996), Ang and Chen (2002) and Perez-Quirors and Timmermann (2000). To the end of characterizing a more realistic and flexible specification for the investment opportunity set, we postulate a Markov switching first-order vector autoregressive (MS-VAR(1)) model. The Markov-switching (MS) specification allows regime-dependent levels, volatility and correlations across assets. The vector autoregressive specification (VAR(1)) conveniently captures the predictability of the expected returns by lagged returns and predictors. Since the state of regime changes with time, the expected returns, volatility, correlations and predictability are in turn time-varying. Moreover, as advocated by Timmermann (2000), a Markov switching model is able to generate an asset return distribution with complicated forms of heteroskedasticity, serial correlations, skews and fat tails. In sum, the MS-VAR(1) model can reconcile the consistent features implied by the realistic observations of asset returns.

Denote $r_{j,t+1}$ and $\bar{r}_{t+1}$ as the log returns of the risky asset j and the riskfree one, $\mathbf{x}_{t+1}$ as a $m \times 1$ vector of predictors, and

$$\mathbf{z}_{t+1} = \begin{bmatrix} \mathbf{r}_{t+1} - \bar{r}_{t+1} \cdot \boldsymbol{\vartheta} \\ \mathbf{x}_{t+1} \end{bmatrix},$$

where $\mathbf{r}_{t+1} = (r_{1,t+1}, \ldots, r_{n,t+1})'$ is the vector of n risky assets and $\boldsymbol{\vartheta}$ is the $n \times 1$ vector of ones. Specifically, the dynamics of $\mathbf{z}_{t+1}$ is given by

$$\mathbf{z}_{t+1} = \boldsymbol{\Phi}_{\circ}(s_{t+1}) + \boldsymbol{\Phi}_1(s_{t+1})\mathbf{z}_t + \boldsymbol{\varepsilon}_{t+1}, \tag{1}$$

where s_t is the latent regime variable, $\boldsymbol{\Phi}_{\circ}(s_{t+1})$ is the $(n+m) \times 1$ regime-dependent vector of the intercepts, $\boldsymbol{\Phi}_1(s_{t+1})$ is the $(n+m) \times (n+m)$ regime-dependent matrix of the autoregressive coefficients, and $\boldsymbol{\varepsilon}_{t+1}$ is the $(n+m) \times 1$ vector of the random shocks with the following distribution assumptions:

$$\begin{aligned} \boldsymbol{\varepsilon}_{t+1} &\overset{i.i.d.}{\sim} \mathcal{N}\Big(\mathbf{0}, \boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}(s_{t+1})\Big), \\ \boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}(s_{t+1}) &= \begin{bmatrix} \boldsymbol{\Sigma}_{\boldsymbol{\delta}}(s_{t+1}) & \boldsymbol{\Sigma}'_{\boldsymbol{\delta}\mathbf{x}}(s_{t+1}) \\ \boldsymbol{\Sigma}_{\boldsymbol{\delta}\mathbf{x}}(s_{t+1}) & \boldsymbol{\Sigma}_{\mathbf{x}}(s_{t+1}) \end{bmatrix}, \end{aligned} \tag{2}$$

where $\boldsymbol{\Sigma}_{\boldsymbol{\delta}}(s_{t+1})$ is the variance-covariance matrix of the log excess returns, $\boldsymbol{\Sigma}_{\boldsymbol{\delta}\mathbf{x}}(s_{t+1})$ is the covariance matrix between the log excess returns and the predictors vector, and $\boldsymbol{\Sigma}_{\mathbf{x}}(s_{t+1})$ is the variance-covariance matrix of the predictors vector $\mathbf{x}_{t+1}$. The latent regime variable s_t follows a J-state first-order Markov chain with transition probabilities: $p_{ji} = \text{Prob}(s_{t+1} = i | s_t = j)$, $i, j \in \{1, 2, \ldots, J\}$, and $\sum_{i=1}^{J} p_{ji} = 1$. To better illustrate how the investor's beliefs affect the investor's optimal investment strategy, we adopt a parsimonious model setup by setting $J = 2$ and terming regime 1 and 2 as the bull and bear market regimes, respectively. (1) can be reduced to that of Campbell, Chan and Viceira (2003), and Jurek and Viceira (2006) by imposing the number of regimes equal to one, $J = 1$.

2.2 Investor Preferences and Optimality Conditions

We assume that the investor has the Epstein-Zin recursive, non-expected utility function[1]

$$U_t = \left\{(1-\beta)C_t^{\frac{1-\gamma}{\theta}} + \beta\left[\mathcal{E}_t\left(U_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{\theta}}\right\}^{\frac{\theta}{1-\gamma}}, \tag{3}$$

where β is the rate of the time preference, γ is the relative risk aversion coefficient, ψ is the elasticity of intertemporal substitution and $\theta = (1-\gamma)/(1-\psi^{-1})$. This recursive utility (3) can be reduced to the time-separable power utility or the log utility if we set $\gamma = \psi^{-1}$or $\gamma = \psi^{-1} = 1$, respectively. $\mathcal{E}_t = \mathcal{E}(\cdot|\boldsymbol{\Omega}_t)$ is the investor's subjective expectation conditional on the available information $\boldsymbol{\Omega}_t$. In conjunction with the setting of the investment opportunity set, this conditional expectation specifically is associated with (i) the investor's subjective conditional density $f^\iota(\mathbf{z}_{t+1}|s_{t+1}, \boldsymbol{\Omega}_t)$; (ii) the subjective transition probabilities of regimes $\text{Prob}^\iota(s_{t+1}|s_t)$; (iii) the subjective filtering probabilities of regimes $\pi^\iota_{t|t} = \text{Prob}^\iota(s_t|\boldsymbol{\Omega}_t)$, where ι refers to the type of the investor's subjectivity. In the end, if this conditional expectation is calculated underlying the true probability law, (3) can be reduced to that of Campbell and Viceira (1999, 2002) and Campbell, Chan and Viceira (2003).

According to the psychological experiment evidence, the investor's subjective expectation is affected by her sentiment, such as optimism, pessimism and etc. While the new information arrives, psychological biases would contaminate the investor's updating or predicting in the current and subsequent states of regime. In the behavioral finance literature, the psychological-biased beliefs may persist for a long period of time, and it in turn causes the stock price departing from its fundamental value in general equilibrium analysis, see for example, De Long, Bradford, Shleifer and Summers (1990) and Lee, Shleifer and Thaler (1991).

Specifically, at time $t+1$, the investor's budget constraint is given by

$$W_{t+1} = R_{p,t+1}(W_t - C_t), \tag{4}$$

where

$$R_{p,t+1} = \sum_{j=1}^{n} \alpha_{j,t}\left(R_{j,t+1} - \bar{R}_{t+1}\right) + \bar{R}_t, \tag{5}$$

$R_{p,t+1}$ and $R_{j,t+1}$ are the gross returns for the portfolio and risky asset j, $\bar{R}_{t+1}$ is the return for the riskfree one, $\alpha_{j,t}$ is the proportion of the investor's wealth invested in the risky asset j at

[1]This special utility, proposed by Epstein and Zin (1989), achieve the separation between the risk aversion coefficient and the elasticity of intertemporal substitution. Note that the risk aversion describes the investor's reluctance in facing the uncertainty over various states of the world, whereas the elasticity of intertemporal substitution depicts the investor's reluctance in changing consumptions over time. It turns out that the power utility is a pretty restrictive one since it performs a reciprocal relation between the risk aversion coefficient and the elasticity of intertemporal substitution.

time t, C_t and W_t are the investor's consumption expenditure and wealth at time t. Given the budget constraint (4) and (5), the utility maximization yields a set of the Euler equations:

$$\mathcal{E}_t \left\{ \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \right]^{\theta} R_{p,t+1}^{(\theta-1)} R_{i,t+1} \right\} = 1, \tag{6}$$

and

$$\mathcal{E}_t \left\{ \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} R_{p,t+1} \right]^{\theta} \right\} = 1. \tag{7}$$

Note that whether the investment opportunity set is constant or not, and whether the investors have the complete information or not, the optimal consumption and investment strategy ought to satisfy the Euler equations (6) and (7).

3 Solving the Model

We follow the Dothan and Feldman (1986), Detemple (1986), Genotte (1968), Detemple and Murphy (1994), and Xia (2000) to separate the investor's asset allocation decision into two parts under the incomplete-information framework: the inference problem and the optimization problem. It is called the separation principle. The former one is projecting the latent variables using the all available information, and the latter one is computing the optimal investment strategy based on the projected latent variables. In relation to our proposed model specification, several things need to be sorted out. First, the inference problem indicates calculating the filtering and prediction probabilities of the current and subsequent states of regime. The corresponding filtering algorithm is supposed to be exogenous to the investor's optimization. For example, the Hamilton's algorithm is the optimal filtering one in terms of statistics. Second, given the inferred probabilities, we turn to compute the optimal portfolio weights and consumption. In order to obtain the analytical solutions, we follow Campbell and Viceira (1999) and Campbell, Chan and Viceira (2003) and log-linearize the Euler equations and the budget constraint to solve the approximate portfolio weights and consumption-wealth ratio. Third, while the investor adopts the alternative filtering algorithm to calculate the filtering and prediction probabilities of regimes, the "optimal" portfolio weights and consumption-wealth ratio are still available with respect to those probabilities. Forth, we don't consider the parameter uncertainty that implies the parameter coefficients in (1) are random. Although evaluating the parameter uncertainty can be the other channel to reflect the investor's beliefs, it goes beyond our scope.

3.1 The Optimal Filtering Algorithm

The Hamilton's (1989) algorithm is described as follows. At time t, the investor's beliefs on regimes (filtering probabilities) are given by $\pi^B_{t|t}(s_t) = \text{Prob}^B(s_t|\mathbf{\Omega}_t)$, where $\mathbf{\Omega}_t$ denotes the filtration generated by the stream of the observations up to time t, $\{\mathbf{z}_i\}_{i=1}^t$. The probability of predicting regime $s_t = j$ switching to regime $s_{t+1} = i$ (via the transition probabilities p_{ji}) is defined as:

$$\pi^B_{t+1|t}(s_{t+1} = i) = \text{Prob}^B(s_{t+1} = i|\mathbf{\Omega}_t) = \sum_{j=1}^{2} p_{ji} \cdot \pi^B_{t|t}(s_t = j). \tag{8}$$

While the new observation $\mathbf{z}_{t+1}$ comes out, the investor will revise her beliefs by:

$$\begin{aligned} \pi^B_{t+1|t+1}(s_{t+1} = i) &= \text{Prob}^B(s_{t+1} = i|\mathbf{\Omega}_{t+1}), \\ &= \frac{f(\mathbf{z}_{t+1}|s_{t+1} = i, \mathbf{\Omega}_t) \cdot \pi^B_{t+1|t}(s_{t+1} = i)}{\sum_{i=1}^{2} f(\mathbf{z}_{t+1}|s_{t+1} = i, \mathbf{\Omega}_t) \cdot \pi^B_{t+1|t}(s_{t+1} = i)}, \end{aligned} \tag{9}$$

where $f(\mathbf{z}_{t+1}|s_{t+1} = i, \mathbf{\Omega}_t)$ is the conditional density of the MS-VAR(1) process. In fact, (9) is calculated based on the Bayesian rule and we in turn call the investor who uses this filtering algorithm as the Bayesian one.

3.2 Log-Linearizing the Euler Equations

Before proceeding to solve the optimal portfolio weights for the risky assets, we need to derive the approximate expressions of the Euler equations in terms of log returns using the log-linearization technique of Campbell (1993, 1996) and Campbell and Viceira (1999, 2001). The log-linearized approximation to the budget constraint (4) and the return on the portfolio (5) can be expressed as[2]

$$\Delta c_{t+1} = r_{p,t+1} + (c_{t+1} - w_{t+1}) - \frac{1}{\rho}(c_t - w_t) + \kappa, \tag{10}$$

and

$$r_{p,t+1} = \bar{r} + \boldsymbol{\alpha}_t' \boldsymbol{\delta}_{t+1} + \frac{1}{2}\left(\boldsymbol{\alpha}_t' \boldsymbol{\sigma}^2_{\boldsymbol{\delta}}(s_{t+1}) - \boldsymbol{\alpha}_t' \mathbf{\Sigma}_{\boldsymbol{\delta}}(s_{t+1}) \boldsymbol{\alpha}_t\right), \tag{11}$$

where $\boldsymbol{\delta}_{t+1} = \mathbf{r}_{t+1} - \bar{r}_{t+1} \cdot \boldsymbol{\iota}$ is the vector of the log excess returns, $\boldsymbol{\sigma}^2_{\boldsymbol{\delta}}(s_t) \equiv \text{diag}(\mathbf{\Sigma}_{\boldsymbol{\delta}}(s_t))$ is the vector of the diagonal elements of $\mathbf{\Sigma}_{\boldsymbol{\delta}}(s_t)$, $c_t - w_t = \log(C_t/W_t)$ is the log consumption-wealth ratio, and $\Delta c_{t+1} = \log(C_{t+1}/C_t)$ is the log consumption growth. $\rho = 1 - \exp[\text{E}(c_t - w_t)]$ and

[2]Following Campbell (1993, 1996), log-linearizing the budget constraint (4) yields

$$\Delta w_{t+1} \approx r_{p,t+1} + \left(1 - \frac{1}{\rho}\right)(c_{t+1} - w_{t+1}) + \kappa,$$

and then we replace Δw_{t+1} with $\Delta c_{t+1} + (c_t - w_t) - (c_{t+1} - w_{t+1})$ to obtain (10). The derivation of (11) can refer to Campbell and Viceira (2002), or Campbell, Chan, and Viceira (2003).

$\kappa = \log\rho + (1-\rho)\log(1-\rho)/\rho$ are linearization parameters, respectively. In effect, the parameter ρ is an endogenous variable since it is a function of the expected optimal consumption-wealth ratio[3].

Now, we log-linearize the Euler equations. The fraction of the Euler equation (6) for the risky asset k and the riskfree asset can be written by:

$$\frac{\mathcal{E}\left\{\exp\left[\theta\log\beta - \left(\frac{\theta}{\psi}\right)\Delta c_{t+1} + (\theta-1)r_{p,t+1} + r_{k,t+1}\right]\middle|\mathbf{\Omega}_t\right\}}{\mathcal{E}\left\{\exp\left[\theta\log\beta - \left(\frac{\theta}{\psi}\right)\Delta c_{t+1} + (\theta-1)r_{p,t+1} + \bar{r}\right]\middle|\mathbf{\Omega}_t\right\}} = 1.$$

Applying the law of iterated expectation yields

$$\frac{\sum_{j=1}^{J}\sum_{i=1}^{J}\text{Prob}^{\iota}(s_{t+1}=i, s_t=j|\mathbf{\Omega}_t)\widehat{\mathcal{E}}_t^{(j,i)}\left\{\exp\left[\theta\log\beta - \left(\frac{\theta}{\psi}\right)\Delta c_{t+1} + (\theta-1)r_{p,t+1} + r_{k,t+1}\right]\right\}}{\sum_{j=1}^{J}\sum_{i=1}^{J}\text{Prob}^{\iota}(s_{t+1}=i, s_t=j|\mathbf{\Omega}_t)\widehat{\mathcal{E}}_t^{(j,i)}\left\{\exp\left[\theta\log\beta - \left(\frac{\theta}{\psi}\right)\Delta c_{t+1} + (\theta-1)r_{p,t+1} + \bar{r}\right]\right\}} = 1, \quad (12)$$

where $\widehat{\mathcal{E}}_t^{(j,i)} \equiv \mathcal{E}(\,\cdot\,|s_{t+1}=i, s_t=j, \mathbf{\Omega}_t)$ and $\widehat{\mathcal{VAR}}_t^{(j,i)} \equiv \mathcal{VAR}(\,\cdot\,|s_{t+1}=i, s_t=j, \mathbf{\Omega}_t)$ are the investor's subjective expectation and variance operators conditional on the current regime s_t, the subsequent one s_{t+1} and the information set $\mathbf{\Omega}_t$. $\text{Prob}^{\iota}(s_{t+1}=i, s_t=j|\mathbf{\Omega}_t)$ is the subjective joint probability of the current regime s_t and the subsequent one s_{t+1} conditional on the information set $\mathbf{\Omega}_t$. Specifically, it can be decomposed by

$$\text{Prob}^{\iota}(s_{t+1}=i, s_t=j|\mathbf{\Omega}_t) = \pi_{t|t}^{\iota}(s_t=j)\cdot\text{Prob}^{\iota}(s_{t+1}=i|s_t=j).$$

Given the conditional log-normality assumption imposed by the investment opportunity set specification, applying second-order Taylor expansion to the numerator and denominator of (12) can obtain the following expression:

$$\sum_{j=1}^{J}\sum_{i=1}^{J}\text{Prob}^{\iota}(s_{t+1}=i, s_t=j|\mathbf{\Omega}_t)\left[\widehat{\mathcal{E}}_t^{(j,i)}(\delta_{k,t+1}) + \frac{1}{2}\widehat{\mathcal{VAR}}_t^{(j,i)}(\delta_{k,t+1})\right] \quad (13)$$

$$= \sum_{j=1}^{J}\sum_{i=1}^{J}\text{Prob}^{\iota}(s_{t+1}=i, s_t=j|\mathbf{\Omega}_t)\left[\left(\frac{\theta}{\psi}\right)\widehat{\mathcal{COV}}_t^{(j,i)}(\Delta c_{t+1}, \delta_{k,t+1}) - (\theta-1)\widehat{\mathcal{COV}}_t^{(j,i)}(r_{p,t+1}, \delta_{k,t+1})\right].$$

The left-hand side of equation (13) is the expected risk premium on risky asset k, adjusted to the Jensen's inequality by adding one-half the variance of the excess return. It connects the asset k's expected risk premium with its expected covariances between the log consumption growth and the log portfolio return. If $r_{p,t+1}$ and Δc_{t+1} are assumed to be the exogenous market return and the exogenous aggregate consumption growth, (13) can be interpreted as

[3]This is one of the convergence conditions to determine the optimal portfolio weights in the numerical procedure.

a two-factor intertemporal asset pricing formula[4], in which the covariance $\widehat{\mathcal{COV}}_t(r_{p,t+1}, \delta_{k,t+1})$ measures the systematic risk and the covariance $\widehat{\mathcal{COV}}_t(\Delta c_{t+1}, \delta_{k,t+1})$ measures the risk of the change in future investment opportunities. In comparison to Giovannini and Jorian (1989), Giovannini and Weil (1989), (13) turns out to be a more general result since it is associated with the market beliefs about the states of regime.

3.3 Solving the Optimal Portfolio Weights and Consumption-Wealth Ratio

To solve the optimal portfolio weights for the risky assets and the optimal consumption-wealth ratio, we follow Campbell and Viceira (1999, 2002) and Campbell, Chan and Viceira (2003) and perform the undetermined coefficient method. That is, we conjecture the functional forms of those solutions and then insert them into the log-linearized Euler equations (13) and (16), verifying the solutions afterward. Specifically, the conjectured functional forms for the optimal portfolio weights and consumption-wealth ratio are given by

$$\begin{aligned} \boldsymbol{\alpha}_t &= \overline{\mathbf{A}}_\circ + \overline{\mathbf{A}}_1 \mathbf{z}_t, \\ c_t - w_t &= b_\circ + \mathbf{b}_1' \mathbf{z}_t + \mathbf{z}_t' \mathbf{B}_2 \mathbf{z}_t, \end{aligned} \tag{14}$$

where the optimal portfolio weights for the risky assets are an affine function of $\mathbf{z}_t$ and the optimal consumption-wealth ratio is an quadratic function of $\mathbf{z}_t$. $\overline{\mathbf{A}}_\circ, \overline{\mathbf{A}}_1, b_\circ, \mathbf{b}_1$ and $\mathbf{B}_2$ are the coefficient matrices with dimensions $n \times 1, n \times (n+m), 1 \times 1, (n+m) \times 1$ and $(n+m) \times (n+m)$, respectively.

Using the above conjectured solutions, we can solve the log-linearized Euler equation (13) and obtain the following analytical solution for the optimal portfolio weights for the risky assets (the derivation is given in the appendix):

$$\begin{aligned} \boldsymbol{\alpha}_t(\pi^\iota_{t|t}) &= \frac{1}{\gamma} \overline{\boldsymbol{\Sigma}}_{\boldsymbol{\delta}}^{-1} \sum_{j=1}^{J} \sum_{i=1}^{J} \pi^\iota_{t|t}(j) \mathrm{Prob}^\iota(s_{t+1} = i | s_t = j) \Big[\mathbf{H}_{\boldsymbol{\delta}} \Big(\boldsymbol{\Phi}_\circ(i) + \boldsymbol{\Phi}_1(i) \mathbf{z}_t \Big) + \frac{1}{2} \boldsymbol{\sigma}^2_{\boldsymbol{\delta}}(i) \Big] \\ &\quad + \left(\frac{\gamma - 1}{\gamma} \right) \left(\frac{1}{\psi - 1} \right) \overline{\boldsymbol{\Sigma}}_{\boldsymbol{\delta}}^{-1} \sum_{j=1}^{J} \sum_{i=1}^{J} \pi^\iota_{t|t}(j) \mathrm{Prob}^\iota(s_{t+1} = i | s_t = j) \widehat{\mathcal{COV}}_t^{(j,i)}(\boldsymbol{\delta}_{t+1}, c_{t+1} - w_{t+1}), \\ &= \overline{\mathbf{A}}_\circ(\pi^\iota_{t|t}) + \overline{\mathbf{A}}_1(\pi^\iota_{t|t}) \mathbf{z}_t, \end{aligned} \tag{15}$$

It is seen that, in the first equality of (15), the optimal portfolio weights is a sum of two components. The former one is called the myopic demand, the vector of the expected market prices of risk scaled by the reciprocal of the coefficient of relative risk aversion. The market price

[4]Note that an investor's asset allocation problem is just a partial equilibrium in which the return on portfolio $r_{p,t+1}$ and consumption c_{t+1} are endogenous; however, the two-factor intertemporal asset pricing formula is a general equilibrium result in which the portfolio and consumption strategies have been determined by market clear conditions.

of risk measures market compensation for the risk within the current opportunities. Current market condition, thus, is able to directly influences the investor's optimal investment strategy. The latter one is called the intertemporal hedging demand, introduced by Merton (1969, 1971). Since the investment opportunity set is time-varying and switches with regimes, the investor would like to hedge the negative change in future investment opportunities. This component is the main difference between the long- and short-term asset demands.

In addition, log-linearizing Euler equation (7) yields

$$\begin{aligned}1 &= \mathcal{E}\left\{\exp\left[\theta\log\beta - \left(\frac{\theta}{\psi}\right)\Delta c_{t+1} + \theta r_{p,t+1}\right]\middle|\boldsymbol{\Omega}_t\right\},\\ &= \sum_{j=1}^{J}\sum_{i=1}^{J}\mathrm{Prob}^{\iota}(s_{t+1}=i, s_t=j|\boldsymbol{\Omega}_t)\widehat{\mathcal{E}}_t^{(j,i)}\Bigg\{\exp\Bigg[\theta\log\beta - \left(\frac{\theta}{\psi}\right)(c_{t+1}-w_{t+1}).\\ &\qquad + \left(\frac{\theta}{\rho\psi}\right)(c_t - w_t) + (1-\gamma)r_{p,t+1} - \frac{\kappa\theta}{\psi}\Bigg]\Bigg\}.\end{aligned}$$

The second equality above is available by inserting the log-linearized budget constraint (10) into the Euler equation and in turn applying the law of iterated expectation. By taking a second-order Taylor expansion and conditional expectation, we acquire a difference equation for $c_t - w_t$:

$$\begin{aligned}c_t - w_t &= \sum_{j=1}^{J}\sum_{i=1}^{J}\mathrm{Prob}^{\iota}(s_{t+1}=i, s_t=j|\boldsymbol{\Omega}_t)\Bigg[-\rho\psi\log\beta + \rho\kappa + \rho\widehat{\mathcal{E}}_t^{(j,i)}(c_{t+1}-w_{t+1})\\ &+ \rho(1-\psi)\widehat{\mathcal{E}}_t^{(j,i)}(r_{p,t+1}) - \frac{1}{2}\left(\frac{\rho\theta}{\psi}\right)\widehat{\mathcal{VAR}}_t^{(j,i)}\Big((c_{t+1}-w_{t+1}) + (1-\psi)r_{p,t+1}\Big)\Bigg],\end{aligned} \tag{16}$$

in which the optimal consumption-wealth ratio is associated with the expected portfolio return, the expected future consumption-wealth ratio and the precautionary saving motivation[5]. In conjunction with the conjectured quadratic form for the consumption-wealth ratio, (16) can solve the optimal consumption-wealth ratio:

$$c_t - w_t = b_\circ(\pi_{t|t}^{\iota}) + \mathbf{b}_1'(\pi_{t|t}^{\iota})\mathbf{z}_t + \mathbf{z}_t'\mathbf{B}_2(\pi_{t|t}^{\iota})\mathbf{z}_t, \tag{17}$$

where $b_\circ(\pi_{t|t}^{\iota})$, $\mathbf{b}_1'(\pi_{t|t}^{\iota})$ and $\mathbf{B}_2(\pi_{t|t}^{\iota})$ are given in the Appendix.

(15) and (17) show that the verified conjectured functions for the optimal portfolio weights and consumption-wealth ratio are functions of the investor's beliefs $\pi_{t|t}^{\iota}(s_t)$. The belief-based coefficient matrices give rise to nonconstant responses with respect to the variation in $\mathbf{z}_t$ since any change in $\mathbf{z}_t$ (new information arrives) would cause synchronous changes in forming beliefs and in turn alter the corresponding coefficient matrices in the solutions of the optimal portfolio weights and consumption-wealth ratio. In comparison to the CCV model that has constant

[5] Precautionary saving is the economic intuition of the conditional variance term $\widehat{\mathcal{VAR}}_t^{(j,i)}\Big((c_{t+1}-w_{t+1}) + (1-\psi)r_{p,t+1}\Big)$.

responses to the variation in $\mathbf{z}_t$, our *belief-dependent* solutions manifest the flexibility. Besides, our proposed solutions are also more general than that of Honda (2003) since we allow the regime-dependent predictability in the investment opportunity set. In comparison with that of Ang and Bekaert (2002), and Guidolin and Timmermann (2007, 2008), our proposed optimal solutions have analytical convenience.

3.4 The Psychological-Biased Filtering Algorithms

In this study, we introduce several psychological biases the most relevant and interesting in the financial literature that may distort predicting and updating regimes. These psychological-distorted beliefs implies that the investor fails to correct their psychological biases. Our proposed beliefs are consistent with that of Barberis, Shleifer and Vishny (1998), Cecchetti, Lam and Mark (2000) and Brandt, Zeng and Zhang (2004).

I. Optimistic and Pessimistic Beliefs: the upbeat investors perform their optimism by making the systematic deviations on their beliefs in overrating the probabilities on the bull regime. They always display incredibly a rosy perspective about the future market outlook. Specifically, optimism gives rise to a distorted transition probability matrix

$$\mathbf{Q}^O = \begin{bmatrix} 1 & \omega \\ 0 & 1-\omega \end{bmatrix} \begin{bmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{bmatrix} = \begin{bmatrix} p_{11}+\omega p_{12} & p_{21}+\omega p_{22} \\ (1-\omega)p_{12} & (1-\omega)p_{22} \end{bmatrix},$$

in which the optimistic investors shift ω proportion of the bear regime's probabilities to the bull regime. On the contrary, the pessimistic investors move ω proportion of the bull regime's probabilities to the bear regime because of their gloomy perspectives about the future market conditions. The corresponding distorted transition probability matrix yields

$$\mathbf{Q}^P = \begin{bmatrix} 1-\omega & 0 \\ \omega & 1 \end{bmatrix} \begin{bmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{bmatrix} = \begin{bmatrix} (1-\omega)p_{11} & (1-\omega)p_{21} \\ \omega p_{11}+p_{12} & \omega p_{21}+p_{22} \end{bmatrix}.$$

II. Dynamic Switching Beliefs: a more realistic specification of the belief formation is to allow investors switching their opinions as the state of regime changes. Specifically, they are supposed to have a subjective regime indicator $\tilde{s}_t$. They take an optimistic (pessimistic) perspective on economy when the subjective regime is a bull (bear) one. It turns out that using the optimistic or pessimistic transition probability matrix depends on the subjective regime variable,

$$\begin{cases} \mathbf{Q}^O, & \text{if } \tilde{s}_t = 1, \\ \mathbf{Q}^P, & \text{if } \tilde{s}_t = 2, \end{cases} \tag{18}$$

where $\tilde{s}_t$ follows a two-state first-order Markov chain with a symmetric transition probability matrix

$$\begin{bmatrix} \eta & 1-\eta \\ 1-\eta & \eta \end{bmatrix},$$

in which $\eta = \text{Prob}(\tilde{s}_t = 1|\tilde{s}_{t-1} = 1) = \text{Prob}(\tilde{s}_t = 2|\tilde{s}_{t-1} = 2)$ measures the persistence of the subjective regimes. We term the investors with high η as the momentum ones since they believe the state of the regime will persist. For example, the current bull regime will be followed by an another bull one and vice versa. Alternatively, we term the investors with low η (equivalently, high $1-\eta$) as the reversal ones since they believe the state of the regime will change rapidly. For example, the current bear regime will switch to the bull one in next period and vice versa.

To complete the specification and facilitate the derivation of the investor's optimal investment decision, we define $s_t^* = (s_t, \tilde{s}_t)$ and let

$$s_t^* = 1, \text{ if } (s_t, \tilde{s}_t) = (1,1),$$

$$s_t^* = 2, \text{ if } (s_t, \tilde{s}_t) = (1,2),$$

$$s_t^* = 3, \text{ if } (s_t, \tilde{s}_t) = (2,1),$$

$$s_t^* = 4, \text{ if } (s_t, \tilde{s}_t) = (2,2),$$

, and then obtain an augmented transition probability matrix for the new subjective regime indicator s_t^*

$$\mathbf{Q}^* = \begin{bmatrix} q_{11}^O\eta & q_{11}^P(1-\eta) & q_{21}^O\eta & q_{21}^P(1-\eta) \\ q_{11}^O(1-\eta) & q_{11}^P\eta & q_{21}^O(1-\eta) & q_{21}^P\eta \\ q_{12}^O\eta & q_{12}^P(1-\eta) & q_{22}^O\eta & q_{22}^P(1-\eta) \\ q_{12}^O(1-\eta) & q_{12}^P\eta & q_{22}^O(1-\eta) & q_{22}^P\eta \end{bmatrix}. \tag{19}$$

In the end, the conditional distribution of $\mathbf{z}_t$ under the subjective regime indicator, s_t^*, is given by $f(\mathbf{z}_t|s_t^*, \mathbf{\Omega}_{t-1})$.

III. Representative-biased and Over-Confident Beliefs: representativeness indicates that investors make their own subjective beliefs based on some representatives instead of the whole probability law of the process. i.e. they merely concentrate on today's realization of data rather than the whole historical sample. Tversky and Kahneman (1974) first documented this psychological phenomena on agent's decision making, and Rabin (2002) termed it as the "law of small numbers". Specifically, this belief formation can be expressed by

$$\pi^R_{t+1|t+1}(s_{t+1}) = (1-\omega)\pi^B_{t+1|t+1}(s_{t+1}) + \omega \left[\frac{f(\mathbf{z}_{t+1}|s_{t+1}, \mathbf{\Omega}_t)}{\sum_{s_{t+1}=1}^{2} f(\mathbf{z}_{t+1}|s_{t+1}, \mathbf{\Omega}_t)} \right], \tag{20}$$

where $f(\mathbf{z}_{t+1}|s_{t+1}, \mathbf{\Omega}_t)/\sum_{s_{t+1}} f(\mathbf{z}_{t+1}|s_{t+1}, \mathbf{\Omega}_t)$ denotes the information gain from new observations. The representative investors will assign a large weight on new information gain and a small one on the Baysian benchmark at time $t+1$.

Over-confidence indicates that investors have a tendency of underweighting latest information but concentrates on their prior (or even private) information. Edwards (1968) first identified this psychological phenomena in which the individual updates his posterior distribution in the right direction whereas he puts too small weight on the rational Bayesian benchmark[6]. Specifically, this belief formation can be expressed as

$$\pi^C_{t+1|t+1}(s_{t+1}) = (1-\omega)\pi^B_{t+1|t+1}(s_{t+1}) + \omega\pi^C_{t|t}(s_t), \tag{21}$$

where $\pi^C_{t|t}(s_t)$ that contains all available information up to time t reflects the investors' prior information at the beginning of time $t+1$. They will share a large weight in his subjective prior and a small one in the Baysian benchmark at time $t+1$.

4 Empirical Results

4.1 Data Description

The calibration exercises in this study are based on monthly returns of market portfolio, growth stocks, value stocks, small caps, large caps and bond. Given all common stocks listed on the NYSE, AMEX and NASDAQ, the portfolios of value stocks and growth stocks are formed by the top 30% and bottom 30% deciles sorted by the book-to-market ratio, whereas those of the small caps and large caps are made by the top 30% and bottom 30% deciles sorted by the size of market capitalization[7]. We use the Moody's AAA corporate bond yields[8] to compute bond returns. Following Campbell, Lo, and Mackinlay (1997), and Campbell, Chan,

[6]Edwards mentioned that "it turns out that opinion change is very orderly, and usually proportional to numbers calculated from the Bayes Theorem– but it is insufficient in amount."

[7]The sorted stocks cover NYSE, NASDAQ and AMEX stocks in the Center of Research in Security Price (CRSP). The portfolio returns are value-weighted and for which the corresponding weights are revised at end of June every year and held constant for the following twelve months. The portfolios for July of year t to June of year $t+1$ are stemmed from market equity data available for December of year $t-1$ to June of year t. Further details of the portfolio return sample are available in Ken French's web page.

[8]The data of the Moody's AAA corporate bond yields is available on the web page of Federal Reserve Bank of St. Louis are available on the web page of Federal Reserve Bank of St. Louis.

and Viceira (2003), we calculate the log return on the corporate bond using the log-linearized approximation:

$$r_{n,t+1} \approx D_n y_{n,t} - (D_n - 1) y_{n-1,t+1},$$

where n is the maturity of the bond, $Y_{n,t}$ is the bond yield, $y_{n,t} = \log(1 + Y_{n,t})$ is the log bond yield, and D_n is the duration of bond[9]. The riskfree rate is the yield on one-month Treasury bills. The data period starts from January 1953 to December 2007.

According to the financial literature, the term spread and the log dividend-price ratio are two popular state variables used to predict the expected mean and volatility of asset returns. The term spread is the yield difference between the 5 year Treasury bonds and the 90-days Treasury bills. Many empirical studies, such as Fama and French (1988, 1989), Campbell (1987), and Kiem and Stambaugh (1986), advocated that the term spread can predict the future macroeconomic conditions and future bond returns. To calculate the dividend-price ratio, we first construct the dividend payout series using the value-weighted return including dividends, and the price index series associated with the value-weighted return excluding dividends. Following the standard convention in the literature, we take the dividend series to be the sum of dividend payments over the past year. The log dividend-price ratio is obtained by the difference between the log dividend and the log price index. Ait-Sahalia and Brandt (2000) and Campbell, Chan and Viceira (2003) pointed out that the log dividend-price ratio can induce the substantial hedging demands for stocks since its predictive ability is able to help the investor hedge the variation of investment opportunity set over time. The data of the price index and bond yields are obtained from the Center for Research in Security Prices.

Table 1

Panel A of Table 1 reports the descriptive statistics, including sample mean, standard deviation, skewness, kurtosis, maximum and minimum, for the log returns of market portfolio, small caps, large caps, growth stocks, value stocks as well as bond. Panel B of Table 1 represents the sample correlations matrix of five stock portfolios, bond and two predictors. Several observations are in order. First, the sample means for small caps, large caps, growth stocks and value stocks are all substantially higher than that of market portfolio. Small caps (bond) has the highest (lowest) mean return. Value stocks and small caps have higher mean returns than growth stocks and large caps. It simply highlights the size and value effects. Next, growth stocks, value stocks and small caps perform higher standard deviations than market portfolio.

[9] In practice, as proposed by Campbell, Lo, and Mackinlay (1997), Campbell, Chan, and Viceira (2003), we set the maturity n to 20 years (240 months) and replace $y_{n-1,t+1}$ by $y_{n,t+1}$. The duration is calculated by

$$D_n = \frac{1 - (1 + Y_{n,t})^{-n}}{1 - (1 + Y_{n,t})^{-1}}.$$

Small caps (bond) remains the highest (lowest) one. Except market portfolio, all skewness are positive. Kurtosis exceeds three in all cases. Finally, stock portfolios and bond are all positively correlated but negatively (positively) correlated with the log dividend-price ratio (term spread).

4.2 The Empirical Results of the Investment Opportunity Sets

This study contains three calibration exercises related to three various specifications of the investment opportunity sets:

- Set A: $\mathbf{z}_t = \langle$market portfolio, bond, term spread, log dividend-price ratio$\rangle$,
- Set B: $\mathbf{z}_t = \langle$small caps, large caps, bond, term spread, log dividend-price ratio$\rangle$,
- Set C: $\mathbf{z}_t = \langle$growth stocks, value stocks, bond, term spread, log dividend-price ratio$\rangle$.

The first set attempts to investigate the investor's stock-bond allocation decision. The second and third sets explore the investor's allocation decision while the size and value effects are considered. To estimate, we implement these three investment opportunity sets by the maximum likelihood estimation approach. Given the parameter estimates and the corresponding updating and predicting probabilities, we can compute the smoothed probabilities that are used to infer the state of regime at each time point. It is a standard procedure and the details can refer to Hamilton (1989, 1994).

Figure 1

We now proceed to interpret the economic meanings of regimes. Figure 1 plots the smoothed probabilities of regime 2 on three investment opportunity sets. It is seen that regime 2 covers most of the bear market times (the green areas) between 1953 and 2007, including the concerns over Vietnam in the end of 1960s, the two oil shocks in the 1970s, the 1987's stock market crash, Gulf war in the beginning of 1990s, Russia defaults and Long Term Capital Management crashes in 1998, and the internet bubble burst and corporate malfeasance since 2000. In contrast, regime 1 captures most of the bull markets since 1960, such as two long bull market times in 80s and 90s. We also can find that the smoothed probability patterns in three cases are consistent, even though they are calculated by different sample.

Based on the smoothed probabilities, we can infer the time periods of regime 2 using the decision rule: the smoothed probability $\text{Prob}(s_t = 2|\mathbf{\Omega}_T) > 0.5$. It seems to be relevant since only few points lie between 0.3 and 0.7 in Figure 2. We can explore the characteristics of regimes using the inferred regime times. Panel C of Table 1 reports the summary statistics in accordance with the regime-sorted monthly returns of five stock portfolios and bond. It is seen that regime 1 (regime 2) is a bullish (bearish) state. Market portfolio has a higher mean return and a lower standard deviation in regime 1 than regime 2. However, small caps and growth stocks have substantially higher mean returns than large caps and value stocks in regime 2 since investors would like to ask larger risk premiums for holding stocks with small size and young age in a bearish state. It also suggests that regime 2 (regime 1) performs the typical size effect (value effect). An interesting finding is that the maximum returns in regime 2 are higher because the inferred times of regime 2 involve some big bear market rallies in 1987, 1998 and 2001.

Table 2A, 2B and 2C

Table 2A, 2B and 2C report the parameter estimates for three investment opportunity sets. The "Intercepts" and "Autoregressive Coefficients" panels represent the estimates of the vector $\mathbf{\Phi}_\circ$ and the autoregressive coefficient matrix, $\mathbf{\Phi}_1$, respectively. The diagonal elements of the "Volatility and Correlations" panel are the volatility estimates, and the correlation estimates are placed on the off-diagonal terms. The transition probability estimates are reported in the bottom panel. The standard deviations of the parameter estimates are in the parentheses.

In conjunction with the results of these three tables, we can obtain several empirical observations about the investment opportunity set. First, it is seen that the volatility estimates of all risky assets in regime 2 are higher than those in regime 1. In contrast, regime 2 is a less persistent state with short duration 2∼3 months[10] in comparison with regime 1 that is a persistent one with duration about 18 months.

Second, the term spread (log dividend-price ratio) can predict bond (five stock portfolios) returns. The predictive ability of these two predictors on stock portfolios or bond returns is regime-dependent. For example, the parameter coefficient estimate of the term spread associated with bond are positive and significant under regime 1, whereas it is an insignificant positive estimate under regime 2. In addition, the parameter coefficient estimates of the log dividend-price ratio associated with small caps, large caps, growth stocks and value stocks are significantly positive under regime 1, whereas those are insignificantly negative (positive) for small caps and growth stocks (large caps and value stocks) under regime 2.

Third, the results of the correlation estimates show that the innovations of the log dividend-price ratio are negatively correlated with the innovations of the returns for small caps, large caps, growth stocks and value stocks under regime 1, whereas the correlation estimates change the sign for small caps and growth stocks under regime 2. In contrast, the innovations of the term spread are positively correlated with the innovations of the bond returns under regime

[10]The expected duration is computed by $\sum_{k=1}^{\infty} p_{ii}^{(k-1)}(1-p_{ii}) = 1/(1-p_{ii})$.

1, but the sign of the corresponding correlation becomes negative under regime 2. Stambaugh (1999) pointed out that the small-sample bias in predictive regressions has the opposite sign to the sign of the correlation between the innovations of asset returns and the innovations of the state variables. It turns out that the regime-dependent correlation structure between the innovations of risky assets and the innovation to predictors, like the log dividend-price ratio or the term spread, would have profound influence on the conclusion on the return predictability. In fact, our estimated results on coefficients may be contaminated by the small-sample bias, but bias correction in the current specification is complicated and beyond the scope of this study. Instead, the estimated results are assumed to be given by investors and we will focus on exploring the implication of the investor's asset allocation decision.

5 Empirical Asset Allocations

In this section, we explore the effects of the distorted beliefs, predictors and stock characteristics on the investor's asset allocation decision. To calibrate the optimal portfolio weight (15) and the optimal consumption-wealth ratio (17), we fix $\psi = 0.99$, $\beta = 0.92$, $\gamma = 5$ (or 10) and $\omega = 0.9$ is set for the algorithm generating the distorted beliefs. In this study, we follow Campbell and Viceira (1999, 2002) and Campbell, Chan and Viceira (2003) and perform a simple numerical algorithm to calibrate the approximate closed-form solutions. The steps are as follows.

- **Step 1:** Calculate the current belief $\pi^{\iota}_{t|t}$ based on an investor's learning algorithm, for example, the Bayesian filtering one, using the observation of a investment opportunity set $\mathbf{z}_t$, given the estimated parameter values of the investment opportunity set and the previous belief $\pi^{\iota}_{t-1|t-1}$.

- **Step 2:** Calibrate the coefficient vectors and matrices $\overline{\mathbf{A}}_{\circ}, \overline{\mathbf{A}}_1, b_{\circ}, \mathbf{b}_1$ and $\mathbf{B}_2$ conditional on the belief $\pi^{\iota}_{t|t}$ using the numerical algorithm that is given in appendix B[11].

- **Step 3:** Compute the approximate closed-form solutions based on the calibrated $\overline{\mathbf{A}}_{\circ}(\pi^{\iota}_{t|t})$,$\overline{\mathbf{A}}_1(\pi^{\iota}_{t|t})$, $b_{\circ}(\pi^{\iota}_{t|t})$, $\mathbf{b}_1(\pi^{\iota}_{t|t})$, $\mathbf{B}_2(\pi^{\iota}_{t|t})$ and the observation $\mathbf{z}_t$.

By repeating the above steps from $t = 1$ to $t = T$, we can obtain the series of the optimal portfolio weights, the optimal consumption-wealth ratio and the corresponding investor's beliefs.

[11] In fact, we extend Campbell, Chan and Viceira's (2003) code, which is available from the Campbell's web page, to involve our generalizations. We verified our code by replicating Campbell's results using their information.

5.1 Asset Allocations under the Bayesain Beliefs

To obtain a preliminary understanding of the effects of the investor's beliefs on the asset demands, we focus on the investment opportunity set A and calibrate the optimal portfolio weights at the sample average of $\mathbf{z}_t$ (see Table 1), varying beliefs within the range $[0, 1]$[12]. Figure 2A illustrates the portfolio weights for stocks and bonds with respect to the investor's beliefs $\pi_{t|t}(s_t = 1)$. When $\gamma = 5$, demands for stocks and bonds are increasing with beliefs. In particular, the changes in demands for stocks and bonds with respect to the change in beliefs (the slopes of the demand curves for stocks and bonds) are raising. It suggests that the investor is inclined to pursuit more stocks and bonds while her beliefs $\pi_{t|t}(s_t = 1)$ is high, whereas she is reluctant to rapidly sell off them while $\pi_{t|t}(s_t = 1)$ is low. When $\gamma = 10$, the demands for stocks and bonds shift down. The changes in demands for stocks and bonds become less sensitive to the change in beliefs. However, these curves would not be flatted out while γ becomes large[13]. In sum, the investor's beliefs have substantial impacts on the investor's asset allocation decision.

Figure 2A and 2B

Figure 2B plot the calibration results based on the investment opportunity set B and C. First, the investor reduces the demands for small caps and growth stocks when her beliefs are bullish. It is understandable that small caps and growth stocks have higher mean returns in regime 2 than regime 1 since the risk premiums for holding them are expected to be high in a bearish state. Stocks with small size, young age and less collaterals will be asked for a larger risk compensation for holding them. Second, the demands for large caps is increasing with the investor's beliefs becoming bullish, whereas that of value stocks is rigid with respect to the investor's beliefs. Third, when γ increases, the investor would reduce small caps but raise large caps. In contrast, the demands for growth stocks and value stocks jointly decline when the investor becomes more risk-averse.

Since the optimal portfolio weights is a linear/nonlinear function of the state vector $\mathbf{z}_t$/belief $\pi^{\iota}_{t|t}(s_t)$, the optimal portfolio allocations are time-varying. In this study, we follow Campbell, Chan and Viceira (2000) and conduct the analysis for the level effects of the optimal portfolio weights under various distorted beliefs. We compute the time series averages for the optimal portfolio weights that are calibrated using real data. Table 3 reports this results.

Table 3

[12] To fix $\mathbf{z}_t = \bar{z}$, the sample average, and let the beliefs $\pi_{t|t}$ be determined exogenously, we calibrate the optimal portfolio weights only based on step 2 and 3.

[13] In our calibration exercise that set $\gamma = 200$, the demand curves for stocks and bonds still keep upward sloping.

Table 3 reports the time series averages of the optimal portfolio wights for three investment opportunity sets and seven types of belief formation algorithms. For each investment opportunity set, we present the mean portfolio weights that average for the whole sample period and two subsample periods sorted by regimes[14] over seven belief formation algorithms. The Bayesian investor attempts to time the market. For example, based on the investment opportunity set A, the Bayesian investor holds more stocks and bonds in the bullish regime since the market portfolio and bonds have higher Sharpe ratios during this period of time (see Panel C of Table 1). In the investment opportunity set B (set C), the Bayesian investor would like to buy less small caps (growth stocks) but more large caps and bonds (value stocks and bonds) in the bullish regime than the bearish one, chasing the assets with high Sharpe ratios.

The portfolio allocation to stocks portfolios (such as small caps and large caps/growth stocks and value stocks), bonds and cash are also understandable. An increase in expected asset returns represents an improvement in investment opportunity set. According to the parameter estimates of Table 2B and 2C, a positive correlation between the small caps returns and the term spread in regime 1 would cause a negative intertemporal hedging demand for small caps. Since a negative shock to small caps returns is correlated with a negative shock to the term spread in regime 1 that would persist for a long period of time and the expected returns of small caps decrease when the term spread declines, the investor will take a short position in small caps to hedge the corresponding intertemporal risk. In contrast, the expected returns of small caps increase when the term spread declines in regime 2, a positive correlation between small caps and the term spread gives rise to a positive intertemporal hedging demand for small caps. This result deepens the timing effects from Sharpe ratios. The returns of small caps are highly positive correlated with those of large caps in both regimes, the investor would pursue the asset with a high Sharpe ratio and short the other one for hedging. The negative demand for large caps can offset (or exceed) the positive demand for large caps caused by the positive Sharpe ratio. The bonds and cash demands are associated with the correlation between the small caps returns and the bond returns. A larger positive correlation in regime 2 would cause a greater negative intertemporal hedging demand for bonds that exceeds the small positive demand for bond caused by a small Sharpe ratio in regime 2. Similarly, the returns of growth stocks are highly positive correlated with those of value stocks in both regimes, the investor would chase value stocks in regime 1/growth stocks in regime 2 and short growth stocks in regime 1/value stocks in regime 2 for hedging, strengthening the timing effects from Sharpe ratios. A short position in bond in regime 2 is caused that a larger positive correlation between the growth stocks returns and the bond returns in regime 2 would induce a greater negative intertemporal

[14] The decision rule for identifying the state of regime is the smoothed probability Prob($s_t = 2|\mathbf{\Omega}_t$) above or below 0.5.

hedging demand for bonds that exceeds the small positive demand for bond caused by a small Sharpe ratio in regime 2. In the end, the Bayesian investor will take a leveraged portfolio strategy in regime 1 but an unleveraged one strategy in regime 2.

5.2 The Effects of the Distorted Beliefs

We proceed to explore the effects of distorted beliefs. Several observations are in orders. First, the optimistic investor attempts to bet on the good states of the economy, chasing the assets with high Sharpe ratios in the bullish regimes. For example, based on the investment opportunity set A, the portfolio positions of stocks and bonds for the optimistic investor in regime 2 are larger than those in regime 1, even though the Sharpe ratio of the market portfolio is a negative one (see Panel C of Table 1). In the investment opportunity set B (set C), the optimistic investor tilts toward small caps and bonds (value stocks and bonds) with large positions in both regimes although bonds have poor Sharpe ratio than that of small caps (growth stocks and value stocks). In comparison to the Bayesian investor, the optimistic one purchases more stocks and bonds in the investment opportunity set A to chase a high reward in the bullish regime. In the investment opportunity set B (set C), the optimistic investor holds more large caps and bonds but less small caps (more value stocks and bonds) in the bullish regime, whereas she takes larger long positions on small caps and bond (value stocks and bonds) and a short one on large caps (growth stocks) in the bearish regime. The optimistic investor will take a leveraged portfolio strategy in both regimes.

Third, in contrast to the optimistic investor, the pessimistic one will bet on the bad states of the economy, chasing the assets with high Sharpe ratios in the bearish regime. For example, the pessimistic investor takes short positions on stocks and buys less bonds in both regimes based on the investment opportunity set A since the market portfolio returns have a negative Sharpe ratio in regime 2. In the investment opportunity set B (set C), the pessimistic investor chases small caps (growth stocks), short sells bonds and holds cash in both regimes since Small caps and growth stocks have higher Sharpe ratios in the bearish regime. In comparison to the Bayesian investor, the portfolio allocation of the pessimistic investor tilts toward buying more small caps (value stocks) and short selling bonds.

Third, the demand patterns for the representative-biased investor are similar to those of the Bayesian investor and attempts to time the market. This is not a surprised result because the updating bias for the representative-biased investor can be alleviated when we allow predictability for asset returns; the representative-biased investor still adopts the unbiased transition matrix in calibrating portfolio weights. Computing mean portfolio weight can further average the bias and the level effect of the representative-biased investor in turn is supposed to be similar to that of Bayesian one. However, it doesn't mean that the Bayesian and representative-biased investors have similar behaviors. The representative-biased investor has larger variances of the optimal portfolio weights for risky assets than that of Bayesian one[15]. The representative-biased investor is supposed to be frequent-trading. On the other hand, the over-confident investor seems to perform similar mean portfolio allocation patterns to those of the Bayesian investor. However, in terms of the variance of the optimal portfolio weights, the over-confident investor is infrequent-trading.

Forth, the momentum investor has similar mean portfolio allocations to those of the Bayesian. This may not be a surprised finding. Although the transition matrix for the momentum investor is distorted, the biased direction is not too far away from the reality. The durations of regimes, in particular for the bullish regime, are not transient in accordance with the realistic transition matrix. Instead, they may persist for a period of time. As a result, the regime persistence assumption for the momentum investor turns out to be a fine one in this regard. However, the reversal investor believes a regime transient assumption and follows a reversal investment strategy. In the bearish (bullish) regime, her optimal portfolio allocation tilts toward the assets with good Sharpe ratios in the bullish (bearish) regime. For example, the reversal investor tilts toward value stocks and bonds (small caps and bonds) in the bearish regime, but short sells bonds and buys more growth stocks (shorts bonds and buys more small caps) in the bullish regime.

5.3 The Effects of Predictors on Asset Demands

We proceed to investigate the effects of predictors on investor's asset allocation decision. Ait-Sahalia and Brandt (2001) and Campbell, Chan, and Viceira (2003) find that the term spread and the log dividend-price ratio have large impacts on the investor's demands for stocks and bonds because of their predictive powers on stock and bond returns. To address this question, we conduct the exercise reported in Figure 3. We calibrate the optimal portfolio weights under the term spread (the log dividend-price ratio) that can deviate by one standard deviation from its sample mean, labelled by "High Term Spread (Ln(D/P))", "Low Term Spread (Ln(D/P))", and "Mean Term Spread (Ln(D/P))", respectively. The other variables in the investment opportunity set are set to be their sample means. The calibration steps are the same as those of Figure 2.

Figure 3

The top (bottom) panels of Figure 3 report the results for the log dividend-price ratio (term spread). It is apparent that the log dividend-price ratio only has significant influence on stocks,

[15]The results about variance of the portfolio weight are available if the authors are requested.

whereas the term spread has substantial impacts on both of stocks and bonds. A high log dividend-price ratio (term spread) drives up the demands for stocks (stocks and bonds). Since both of the term spread and log dividend-price ratio are persistent, higher values for them would bring up expected returns of stocks and bonds in the future, driving up the demands for stocks and bonds. Note that the effect of Ln(D/P)_t on the expected bond returns is closed to zero, thus it fails to influence the bond demands.

In unreported results, a high (low) log dividend-price ratio drives down (up) the demands for small caps and growth stocks, but bring up (down) large caps and value stocks. However, a high term spread raises the demands for small caps, large caps, value stocks and bonds, except for growth stocks that is neutral to the term spread. In sum, predictability from the term spread and the log dividend-price ratio uncovers the deeply influence on the demands for stocks and bonds.

6 Assessing the Asset Allocations under Distorted Beliefs

6.1 Performance Measures

Insofar as the calibration exercises we have done reveal that distorted beliefs may give rise to a substantial influence on the investor's demands for stocks and bonds, it is natural to calibrate the optimal portfolio weights using the real observations to examine the empirical performances over investors with different belief formation algorithms. To address this issue, we use several measures to evaluate the empirical performance of portfolio returns.

Graham and Harvey (1997) proposed risk-adjusted performance metrics that match the portfolio return volatility to that of the benchmark by adjusting expected return up or down to obtain a better measure of return for a given level of risk. For example, GH1 is the metric that adopt the market volatility as the benchmark risk. Specifically, it is expressed by

$$\text{GH1}^{\iota} = r^{\iota}_{p,t} - r^{*}_{p,t}, \tag{22}$$

where $r^{\iota}_{p,t}$ is the portfolio return calculated by the optimal portfolio weights associated with the belief algorithm ι, $r^{*}_{p,t} = r_{f,t} + \frac{\sigma^{\iota}_{p}}{\sigma_m}(r_{m,t} - r_{f,t})$ is the risk-adjusted portfolio return by the market's risk, $r_{m,t}, r_{f,t}$ are the market portfolio return and riskfree rate, and $\sigma^{\iota}_{p}, \sigma_m$ are the sample standard deviations of the investor ι's portfolio returns and market portfolio, respectively. GH SCORE is the sum of two metrics, GH1 and GH2[16].

Another risk-adjusted performance metric is the intercept estimate of the time series regression based on the Fama-French four factor model. Specifically,

$$r^{\iota}_{p,t} - r^{B}_{p,t} = \alpha_F + \beta_m \text{RMRF}_t + \beta_s \text{SMB}_t + \beta_h \text{HML}_t + \beta_u \text{UMD}_t + \epsilon_t, \tag{23}$$

[16] GH2 is a similar performance metric to GH1. The details can refer to Graham and Harvey's (1997) article.

where $\text{RMRF}_t, \text{SMB}_t, \text{HML}_t$ and UMD_t are the time t return spreads on zero-investment factor-mimicking portfolios for aggregate market proxy, size, book-to-market equity, and one-year momentum in stock returns, respectively. α_F is interpreted as the abnormal return associated with the belief ι relative to the Bayesian one.

Denote the metric for the long-run portfolio returns associated with the belief algorithm ι as

$$\text{LR}_t^\iota(\tau) = \sum_{i=1}^{\tau} r_{p,t+i-1}^\iota, \tag{24}$$

where τ is the forward periods. By controlling the long-run market risk, we obtain a risk-adjusted performance measure that is the intercept of the following time series regression

$$\text{LR}_t^\iota(\tau) - \text{LR}_t^B(\tau) = \alpha_L + \beta_M \sum_{i=1}^{\tau} \text{RMRF}_{t+i-1} + \epsilon_t, \tag{25}$$

where α_L can be interpreted as the risk-adjusted long-run abnormal return associated with the belief ι relative to the Bayesian one and $\sum_{i=1}^{\tau} \text{RMRF}_{t+i-1}$ is the proxy for the long-run market risk.

6.2 Empirical Performances of Asset Allocation under Distorted Beliefs

The above performance measures may give rise to several econometric issues, such as the non-normality, the serial correlation caused by the overlapped summation, which in turn could cast doubt about the traditional statistical inference. To address this concern, we use the bootstrap method to calculate the empirical distribution of the metric that we want to test, and in turn assess the significance of the metric computed by real data[17]. For example, if we want to test the significance of GH1^ι, $\bar{r_{p,t}}^\iota - \bar{r_{p,t}}^B$ $\text{GH1}^\iota - \text{GH1}^B$, or $\text{LR}_t^\iota(\tau) - \text{LR}_t^B(\tau)$, we resample the observations using the block bootstrap and calibrate the corresponding optimal portfolio weights and portfolio returns using the resampled data. The resulting empirical distribution of the target metric, say for example GH1^ι, based on the resampled data can indicate the significance of the metric calculated by real data.

To avoid the portfolio performance benefitting from the full-sample information, we set the observations from January 2006 to December 2007 as the out-of-sample period and re-implement three investment opportunity sets using the data from January 1953 to December 2005. We then calibrate the optimal portfolio weights using the out-of-sample observations and compute the corresponding portfolio returns as well as the values of the performance metrics.

Table 4A, and 4B

[17]The steps are as follows. We use the Politis and Romano (1994) stationary bootstrap to generate a re-sampled data. We calibrate the optimal portfolio weights using this re-sampled data and calculate the corresponding portfolio returns that can be used to compute the performance metrics. By repeating these steps 500 times, we obtain the corresponding empirical distribution of the metric. In the end, the significance result is available via the empirical distribution.

Panel A of Table 4A reports the in-sample and out-of-sample results for the average portfolio returns and the GH measures over seven belief formation algorithms. It is seen that the optimistic investor has the best performance in accordance with the values of the performance measures. In particular, the average portfolio return and the GH SCORE of the optimistic investor are significantly superior to those of the Bayesian one. This finding is valid in both of in-sample and out-of-sample results. In some cases, the performance of the representative-biased investor is indifferent to that of the Bayesian investor. The pessimistic investor consistently has the worst performance.

Panel B of Table 4A reports the intercept estimates of the Fama-French four-factor model. We can find that the optimistic investor continues outperforming the Bayesian one even if the common risk factors are controlled. The $\hat{\alpha}_F$ estimates of the optimistic investor are significantly positive. However, the other psychological-biased investors underperform the Bayesian one. In particular, the $\hat{\alpha}_F$ estimates for these irrational investors are all significantly negative in the out-of-sample periods.

Table 4B reports the results for the average long-run returns and the intercept estimates of the long-run return regression (25) given by three forward periods, 12, 36 and 60 months. It is apparent that the findings in Table 4B remain valid in most cases, except for the over-confident investor. According to the results of the investment opportunity set B and C, the over-confident investor has positive and significant intercept estimates $\hat{\alpha}_L$ and higher average long-run returns.

Figure 5A and 5B

In order to verify the robustness of our findings, we assess the empirical performances of portfolio allocations under various belief formation algorithms over different subsample periods. Table 5A and 5B report the four empirical performance results over four subsample periods, including 1970:1 1979:12 (Panel A of Table 5A), 1990:1 1999:12 (Panel B of Table 5A), 1953:1 1979:12 (Panel C of Table 5B) and 1980:1 2007:12 (Panel D of Table 5B). According to the results over these subsample periods, our findings from Table 4A and 4B continue to be valid in most cases. In sum, the optimistic investor has the best performance and in turn is superior to the Bayesian one, in particular for the Set B and Set C. In contrast, the pessimistic investor has the worst performance. Our findings are consistent with De Long, Shleifer, Summers and Waldman (1991), and Kogan, Ross, Wang and Westerfield (2006) that the optimistic investor might earn more wealth than the rational Bayesian one.

6.3 Implications of the Empirical Performance Results

Outperformance and Predictability In this subsection, we start off by offering a potential explanation for the outperformance of the optimistic investor. We argue that our above empirical findings concerning the effect of the predictability from the term spread and the log dividend-price ratio.

Table 6

To illustrate the benefit from the predictability, we compare the empirical performances of the portfolio allocations associated with the Bayesian and optimistic belief algorithms based on the restricted and unrestricted investment opportunity sets. Specifically, the restricted one that removes the predictors off is expressed as a simple Markov-switching (MS) model with regime-dependent drifts, whereas the unrestricted one is given by the MS-VAR(1) model of (1). Table 6 reports the results. "MS (MSVAR)" indicates the results based on the restricted (unrestricted) investment opportunity set. It is seen that when the predictability is off, the Bayesian investor significantly outperforms the optimistic one. In contrast, while the predictability is invoked, the optimistic investor turns the tables around. This result has a solid economic interpretation. The predictability can help investors improve their forecasts on the state of regime as well as the future stock returns. The duration and the inferred bullish regimes are supposed to larger than the those of the bearish regime. While the optimistic investor is inclined to bet on good states, chasing the assets with higher rewards in good times, it would cause higher expected returns for the optimistic investor (although she takes more risks).

The Effects of Four States In this study we impose a strong assumption of the number of states to simplify our analysis in distorted beliefs. Recent articles like Guidolin and Timmermann (2007) provide the empirical evidence that the most appropriate number of states is four, characterizing by crash, slow growth, bull and recovery states. In related to four-state investment opportunity set specification, our findings remain valid if the predictability from the predictors, such as the log dividend-price ratio and the term spread, continues. This is particularly true when the duration of the bull regime is longer. According to the results of Guidolin and Timmermann (2007), the predictor, the dividend yield, has the substantial predictive power and the duration of the bull (and slow growth) regime is longer. As a result, our findings are expected to be robust to the four-state framework.

7 Conclusion

The traditional finance theory advocates that rational investors will benefit from the price correction and irrational investors will lose their wealth by their sentiment-driven investment strategy. The behavioral finance theory declare that irrational investors with bullish sentiment would bear more risk than rational ones and may earn higher a higher expected return. It appears that a higher expected return in investment for irrational investors is possible on terms of theoretical modelling. In this study, we empirically investigate this issue by utilizing a dynamic asset allocation model to explore the empirical performance of the portfolio allocations associated with a variety of belief formation mechanisms.

Our findings are as follows.(i) in comparison with the Bayesian investor, the optimistic one would like to bet on good states of the economy, more aggressively chasing /shorting the assets with higher rewards in good/bad times. In contrast, the pessimistic investor would take the opposite positions to the optimistic one; (ii) the optimistic investor has the best empirical performance of the portfolio allocation and in turn outperforms the Bayesian one. The other irrational investors underperform the Bayesian one in most cases; (iii) while the predictability in the investment opportunity set is removed, the outperformance of the optimistic investor disappears. It suggests that the benefit from the intertemporal hedging induced by the return predictability is a potential explanation to the domination of the optimistic investor in our analysis.

For tractability, our analysis did not consider (i) the transaction cost; (ii) the borrowing constraints; (iii) short sell constraints; (iv) the parameter uncertainty; (v) taxes. We also assume a parsimonious two-state MS-VAR model for the investment opportunity set. Extension of our analysis can involve the above restrictions and may yield unexpected results. What is more important is that using the investor's asset allocation perspective to complement the researches in these behavioral issues is imperative.

8 Reference

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Appendix

A Derivations of (15) and (17)

Step 1: Based on the model specification of the MS-VAR process (1), log-linearized budget constraint (11) and the conjectured functional forms (14), we can obtain the following moments.

$$\widehat{\mathcal{E}}_t^{(j,i)}(\boldsymbol{\delta}_{t+1}) = \mathbf{H}_{\boldsymbol{\delta}}(\boldsymbol{\Phi}_\circ(i) + \boldsymbol{\Phi}_1(i)\mathbf{z}_t),$$
$$\widehat{\mathcal{VAR}}_t^{(j,i)}(\boldsymbol{\delta}_{t+1}) = \text{diag}(\boldsymbol{\Sigma}_{\boldsymbol{\delta}}(i)) = \boldsymbol{\sigma}^2_{\boldsymbol{\delta}}(i),$$
$$\widehat{\mathcal{COV}}_t^{(j,i)}\Big(\boldsymbol{\delta}_{t+1}, r_{p,t+1}\Big) = \boldsymbol{\Sigma}_{\boldsymbol{\delta}}(i)\boldsymbol{\alpha}_t,$$

and

$$\begin{aligned}\widehat{\mathcal{COV}}_t^{(j,i)}(\boldsymbol{\delta}_{t+1}, c_{t+1} - w_{t+1}) &= \mathbf{H}_{\boldsymbol{\delta}}\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}(i)\mathbf{b}_1(i) + \mathbf{H}_{\boldsymbol{\delta}}\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}(i)(\mathbf{B}_2(i) + \mathbf{B}_2(i)')(\boldsymbol{\Phi}_\circ(i) + \boldsymbol{\Phi}_1(i)\mathbf{z}_t),\\ &= \boldsymbol{\Lambda}_\circ(i) + \boldsymbol{\Lambda}_1(i)\mathbf{z}_t,\end{aligned}$$

where diag(A) indicates the vector of diagonal elements in matrix A and

$$\begin{aligned}\boldsymbol{\Lambda}_\circ(i) &= \mathbf{H}_{\boldsymbol{\delta}}\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}(i)\mathbf{b}_1(i) + \mathbf{H}_{\boldsymbol{\delta}}\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}(i)(\mathbf{B}_2(i) + \mathbf{B}_2(i)')\boldsymbol{\Phi}_\circ(i),\\ \boldsymbol{\Lambda}_1(i) &= \mathbf{H}_{\boldsymbol{\delta}}\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}(i)(\mathbf{B}_2(i) + \mathbf{B}_2(i)')\boldsymbol{\Phi}_1(i).\end{aligned}$$

Step 2: Plugging the log-linearized budget constraint (10) into the log-linearized Euler equation (13) and stacking up all risky assets being a $n \times 1$ vector, we have

$$\begin{aligned}\sum_{j=1}^{J}\sum_{i=1}^{J}\text{Prob}^\iota(s_{t+1} = i, s_t = j|\boldsymbol{\Omega}_t)\Bigg[&\widehat{\mathcal{E}}_t^{(j,i)}(\boldsymbol{\delta}_{t+1}) + \frac{1}{2}\widehat{\mathcal{VAR}}_t^{(j,i)}(\boldsymbol{\delta}_{t+1})\\ &- \left(\frac{\theta}{\psi}\right)\widehat{\mathcal{COV}}_t^{(j,i)}(\boldsymbol{\delta}_{t+1}, c_{t+1} - w_{t+1}) - \gamma\widehat{\mathcal{COV}}_t^{(j,i)}(\boldsymbol{\delta}_{t+1}, r_{p,t+1})\Bigg] = 0.\end{aligned}$$

Inserting the results of the moments computed in **Step 1** yields

$$\begin{aligned}&\sum_{j=1}^{J}\sum_{i=1}^{J}\text{Prob}^\iota(s_{t+1} = i, s_t = j|\boldsymbol{\Omega}_t)\left[\mathbf{H}_{\boldsymbol{\delta}}\Big(\boldsymbol{\Phi}_\circ(i) + \boldsymbol{\Phi}_1(i)\mathbf{z}_{t+1}\Big) + \frac{1}{2}\boldsymbol{\sigma}^2_{\boldsymbol{\delta}}(i)\right]\\ &\qquad = \sum_{j=1}^{J}\sum_{i=1}^{J}\text{Prob}^\iota(s_{t+1} = i, s_t = j|\boldsymbol{\Omega}_t)\left[\left(\frac{\theta}{\psi}\right)(\boldsymbol{\Lambda}_\circ(i) + \boldsymbol{\Lambda}_1(i)\mathbf{z}_t) + \gamma\boldsymbol{\Sigma}_{\boldsymbol{\delta}}(i)\boldsymbol{\alpha}_t\right].\end{aligned}$$

As a result, we can obtain the optimal portfolio weights

$$\boldsymbol{\alpha}_t(\pi^\iota_{t|t}) = \overline{\mathbf{A}}_\circ(\pi^\iota_{t|t}) + \overline{\mathbf{A}}_1(\pi^\iota_{t|t})\mathbf{z}_t, \tag{26}$$

where

$$\overline{\mathbf{A}}_\circ(\pi_{t|t}) = \frac{1}{\gamma}\overline{\boldsymbol{\Sigma}}_{\boldsymbol{\delta}}^{-1}\left[\sum_{j=1}^{J}\sum_{i=1}^{J}\pi^\iota_{t|t}(j)\cdot\text{Prob}^\iota(s_{t+1} = i|s_t = j)\left(\mathbf{H}_{\boldsymbol{\delta}}\boldsymbol{\Phi}_\circ(i) + \frac{1}{2}\boldsymbol{\sigma}^2_{\boldsymbol{\delta}}(i)\right)\right]$$

$$+\left(1-\frac{1}{\gamma}\right)\overline{\mathbf{\Sigma}}_{\boldsymbol{\delta}}^{-1}\left[\sum_{j=1}^{J}\sum_{i=1}^{J}\pi_{t|t}^{\iota}(j)\cdot\mathrm{Prob}^{\iota}(s_{t+1}=i|s_t=j)\left(\frac{-\mathbf{\Lambda}_{\circ}(i)}{1-\psi}\right)\right],$$

$$\overline{\mathbf{A}}_1(\pi_{t|t})=\frac{1}{\gamma}\overline{\mathbf{\Sigma}}_{\boldsymbol{\delta}}^{-1}\left[\sum_{j=1}^{J}\sum_{i=1}^{J}\pi_{t|t}^{\iota}(j)\cdot\mathrm{Prob}^{\iota}(s_{t+1}=i|s_t=j)\mathbf{H}_{\boldsymbol{\delta}}\mathbf{\Phi}_1(i)\right]$$

$$+\left(1-\frac{1}{\gamma}\right)\overline{\mathbf{\Sigma}}_{\boldsymbol{\delta}}^{-1}\left[\sum_{j=1}^{J}\sum_{i=1}^{J}\pi_{t|t}^{\iota}(j)\cdot\mathrm{Prob}^{\iota}(s_{t+1}=i|s_t=j)\left(\frac{-\mathbf{\Lambda}_1(i)}{1-\psi}\right)\right],$$

$$\overline{\mathbf{\Sigma}}_{\boldsymbol{\delta}}=\sum_{j=1}^{J}\sum_{i=1}^{J}\pi_{t|t}^{\iota}(j)\cdot\mathrm{Prob}^{\iota}(s_{t+1}=i|s_t=j)\mathbf{\Sigma}_{\boldsymbol{\delta}}(i).$$

Step 3: In order to compute the optimal log consumption-wealth ratio, we have to know the following two moments.

$$\begin{aligned}\widehat{\mathcal{E}}_t^{(j,i)}(r_{p,t+1})&=\bar{r}+\boldsymbol{\alpha}_t'\widehat{\mathcal{E}}_t^{(j,i)}(\boldsymbol{\delta}_{t+1})+\frac{1}{2}\Big(\boldsymbol{\alpha}_t'\boldsymbol{\sigma}_{\boldsymbol{\delta}}^2(i)-\boldsymbol{\alpha}_t'\mathbf{\Sigma}_{\boldsymbol{\delta}}(i)\boldsymbol{\alpha}_t\Big),\\&=\bar{r}+(\overline{\mathbf{A}}_{\circ}+\overline{\mathbf{A}}_1\mathbf{z}_t)'\mathbf{H}_{\boldsymbol{\delta}}(\mathbf{\Phi}_{\circ}(i)+\mathbf{\Phi}_1(i)\mathbf{z}_t)+\frac{1}{2}(\overline{\mathbf{A}}_{\circ}+\overline{\mathbf{A}}_1\mathbf{z}_t)'\boldsymbol{\sigma}_{\boldsymbol{\delta}}^2(i)\\&\qquad-\frac{1}{2}(\overline{\mathbf{A}}_{\circ}+\overline{\mathbf{A}}_1\mathbf{z}_t)'\mathbf{\Sigma}_{\boldsymbol{\delta}}(i)(\overline{\mathbf{A}}_{\circ}+\overline{\mathbf{A}}_1\mathbf{z}_t),\\&=\Gamma_{\circ}^{(i)}+\mathbf{\Gamma}_1^{(i)}\mathbf{z}_t+\mathbf{\Gamma}_2^{(i)}\mathrm{vec}(\mathbf{z}_t\mathbf{z}_t'),\end{aligned}$$

where

$$\begin{aligned}\mathbf{\Gamma}_{\circ}^{(i)}&=\bar{r}+\overline{\mathbf{A}}_{\circ}'\mathbf{H}_{\boldsymbol{\delta}}\mathbf{\Phi}_{\circ}(i)+\frac{1}{2}\overline{\mathbf{A}}_{\circ}'\boldsymbol{\sigma}_{\boldsymbol{\delta}}^2(i)-\frac{1}{2}\overline{\mathbf{A}}_{\circ}'\mathbf{\Sigma}_{\boldsymbol{\delta}}(i)\overline{\mathbf{A}}_{\circ},\\\mathbf{\Gamma}_1^{(i)}&=\overline{\mathbf{A}}_{\circ}'\mathbf{H}_{\boldsymbol{\delta}}\mathbf{\Phi}_1(i)+\mathbf{\Phi}_{\circ}(i)'\mathbf{H}_{\boldsymbol{\delta}}'\overline{\mathbf{A}}_1+\frac{1}{2}\boldsymbol{\sigma}_{\boldsymbol{\delta}}^2(i)\overline{\mathbf{A}}_1-\overline{\mathbf{A}}_{\circ}'\mathbf{\Sigma}_{\boldsymbol{\delta}}(i)\overline{\mathbf{A}}_1,\\\mathbf{\Gamma}_2^{(i)}&=\mathrm{vec}(\overline{\mathbf{A}}_1'\mathbf{H}_{\boldsymbol{\delta}}\mathbf{\Phi}_1(i))'-\frac{1}{2}\mathrm{vec}(\overline{\mathbf{A}}_1'\mathbf{\Sigma}_{\boldsymbol{\delta}}(i)\overline{\mathbf{A}}_1)'.\end{aligned}$$

and

$$\begin{aligned}&\widehat{\mathcal{VAR}}_t^{(j,i)}\left[(c_{t+1}-w_{t+1})+(1-\psi)r_{p,t+1}\right]\\&=\Big[\Pi_1^{(i)}\mathbf{\Sigma}_{\boldsymbol{\varepsilon}}(i)\Pi_1^{(i)'}+\mathrm{vec}(\mathbf{B}_2)'\widehat{\mathrm{Var}}_t^{(i)}\Big(\mathrm{vec}(\boldsymbol{\varepsilon}_{t+1}\boldsymbol{\varepsilon}_{t+1}')\Big)\mathrm{vec}(\mathbf{B}_2)\Big]\\&\qquad+2\Pi_1^{(i)'}\mathbf{\Sigma}_{\boldsymbol{\varepsilon}}(i)\Pi_2^{(i)'}\mathbf{z}_t+\mathrm{vec}\Big(\Pi_2^{(i)}\mathbf{\Sigma}_{\boldsymbol{\varepsilon}}(i)\Pi_2^{(i)'}\Big)'\mathrm{vec}(\mathbf{z}_t\mathbf{z}_t'),\\&=V_{\circ}^{(i)}+\mathbf{V}_1^{(i)}\mathbf{z}_t+\mathbf{V}_2^{(i)}\mathrm{vec}(\mathbf{z}_t\mathbf{z}_t').\end{aligned}$$

Step 4: Plugging the above two moments into the difference equation on log consumption-wealth ratio (16) yields

$$\begin{aligned}c_t-w_t=\sum_{j=1}^{J}\sum_{i=1}^{J}\mathrm{Prob}^{\iota}(s_{t+1}=i,s_t=j|\mathbf{\Omega}_t)\Big\{&\rho(1-\psi)\Big[\Gamma_{\circ}^{(i)}+\mathbf{\Gamma}_1^{(i)}\mathbf{z}_t+\mathbf{\Gamma}_2^{(i)}\mathrm{vec}(\mathbf{z}_t\mathbf{z}_t')\Big]\\&+\rho\widehat{\mathcal{E}}_t^{(j,i)}\Big(b_{\circ}+\mathbf{b}_1'\mathbf{z}_{t+1}+\mathrm{vec}(\mathbf{B}_2)'\mathrm{vec}(\mathbf{z}_{t+1}\mathbf{z}_{t+1}')\Big)-\rho\psi\log\beta+\rho\kappa\end{aligned}$$

$$-\frac{1}{2}\left(\frac{\rho\theta}{\psi}\right)\left[V_\circ^{(i)}+\mathbf{V}_1^{(i)}\mathbf{z}_t+\mathbf{V}_2^{(i)}\text{vec}(\mathbf{z}_t\mathbf{z}_t')\right]\Bigg\},$$

$$=\sum_{i=1}^{J}\overline{P}_i\left(\Xi_\circ^{(i)}+\Xi_1^{(i)}\mathbf{z}_t+\Xi_2^{(i)}\text{vec}(\mathbf{z}_t\mathbf{z}_t')\right),$$

where

$$\begin{aligned}
\Xi_\circ^{(i)} &= \rho\Bigg[-\psi\log\beta+\kappa-\frac{1}{2}\left(\frac{\theta}{\psi}\right)V_\circ^{(i)}+(1-\psi)\Gamma_\circ^{(i)}+b_\circ+\mathbf{b}_1'\mathbf{\Phi}_\circ(i)\\
&\quad+\text{vec}(\mathbf{B}_2)'\text{vec}(\mathbf{\Phi}_\circ(i)\mathbf{\Phi}_\circ(i)')+\text{vec}(\mathbf{B}_2)'\text{vec}(\mathbf{\Sigma}_{\boldsymbol{\varepsilon}}(i))\Bigg],\\
\Xi_1^{(i)} &= \rho\Bigg[-\frac{1}{2}\left(\frac{\theta}{\psi}\right)\mathbf{V}_1^{(i)}+(1-\psi)\mathbf{\Gamma}_1^{(i)}+\mathbf{b}_1'\mathbf{\Phi}_1(i)+2\mathbf{\Phi}_\circ(i)'(\mathbf{B}_2+\mathbf{B}_2')\mathbf{\Phi}_1(i)\Bigg],\\
\Xi_2^{(i)} &= \rho\Bigg[-\frac{1}{2}\left(\frac{\theta}{\psi}\right)\mathbf{V}_2^{(i)}+(1-\psi)\mathbf{\Gamma}_2^{(i)}+\text{vec}(\mathbf{\Phi}_1(i)'\mathbf{B}_2\mathbf{\Phi}_1(i))'\Bigg],\\
\overline{P}_i &= \sum_{j=1}^{J}\text{Prob}^\iota(s_{t+1}=i,s_t=j|\mathbf{\Omega}_t).
\end{aligned}$$

As a result, the parameter vectors and matrix $\{b_\circ,\mathbf{b}_1,\mathbf{B}_2\}$ in the log consumption-wealth ratio are given by

$$\begin{aligned}
b_\circ(\pi_{t|t}^\iota) &= \sum_{i=1}^{J}\overline{P}_i\Xi_\circ^{(i)},\\
\mathbf{b}_1(\pi_{t|t}^\iota) &= \sum_{i=1}^{J}\overline{P}_i\Xi_1^{(i)\prime},\\
\text{vec}(\mathbf{B}_2(\pi_{t|t}^\iota)) &= \sum_{i=1}^{J}\overline{P}_i\Xi_2^{(i)\prime}.
\end{aligned}$$

B Numerical Procedure

Thanks for the availability for the analytical solutions that make the numerical procedure quite simple. The Numerical procedure is identical to that of Cambell, Chan and Viceira (2003). To facilitate the implementation as well as achieve desirable convergent results, we follow Cambell, Chan and Viceira (2003) to set $\psi=1$. It gives rise to $\rho=\delta$, so that ρ is no longer the endogenous variable and choosing ρ is equivalent to choosing δ. Fixing the value of $\gamma,\beta,\delta,\omega$ and initial values $\{b_\circ^{(0)},\mathbf{B}_1^{(0)},\text{vec}(\mathbf{B}_2^{(0)})\}$, the numerical procedure is the following. We compute the corresponding $\{\overline{A}_\circ^{(1)},\overline{A}_1^{(1)}\}$, then we continue to compute the updated $\{b_\circ^{(1)},\mathbf{B}_1^{(1)},\text{vec}(\mathbf{B}_2^{(1)})\}$. Continues this iterative procedure until it accomplishes convergence. The convergence criterion is the norm $||\mathbf{B}_1^{(n+1)}-\mathbf{B}_1^{(n)}||+||\text{vec}(\mathbf{B}_2^{(n+1)})-\text{vec}(\mathbf{B}_2^{(n)})||<1e-5$.

Table 1:
Descriptive Statistics for the Stock Portfolios and Bond

This table reports summary statistic for monthly returns on market portfolio, bond, small caps, large caps, growth stocks and value stocks. The sample period is 1953:1-2007:12. Panel A and B reports descriptive statistics and correlation matrix based on the whole sample. Panel C is based on regime-sorted sample.

Panel A: Summary Statistics for the Portfolio Returns

	Market	Bond	Growth	Value	Small Caps	Large Caps
Mean	0.0047	0.0023	0.0088	0.0106	0.0140	0.0070
Standard Dev.	0.0426	0.0083	0.0438	0.0421	0.0538	0.0401
Skewness	-0.7722	0.1959	0.2179	0.3312	0.4881	0.0596
Kurtosis	6.0021	5.8673	3.8398	4.3570	4.1703	3.9143
Maximum	0.1489	0.0384	0.2123	0.2095	0.2532	0.1839
Minimum	-0.2631	-0.0405	-0.1090	-0.1035	-0.1091	-0.1086

Panel B: Correlation Matrix

Market	1							
Growth	0.6489	1						
Value	0.6574	0.7196	1					
Small Caps	0.4800	0.6306	0.7012	1				
Large Caps	0.7739	0.8690	0.7717	0.5762	1			
Bond	0.0961	0.1530	0.1960	0.0559	0.1882	1		
Term Spread	0.1056	0.0293	0.0825	0.0353	0.0777	0.1847	1	
Ln(D/P)	-0.0394	-0.0339	-0.0211	-0.0428	-0.0308	-0.0001	-0.0557	1

Panel C: Summary Statistics for the Regime-Sorted Portfolio Returns

Regime 1:Bullish Regime

	Market	Bond	Growth	Value	Small Caps	Large Caps
Mean	0.0076	0.0022	0.0060	0.0088	0.0102	0.0071
Standard Dev.	0.0371	0.0072	0.0398	0.0381	0.0478	0.0358
Skewness	-0.3307	-0.2764	-0.1413	-0.0880	0.1041	-0.1782
Kurtosis	4.2679	6.4286	3.0820	-2.9897	3.1862	3.2669
Maximum	0.1489	0.0300	0.1292	0.1294	0.1727	0.1196
Minimum	-0.1767	-0.0405	-0.1090	-0.1024	-0.1089	-0.1086

Regime 2:Bearish Regime

	Market	Bond	Growth	Value	Small Caps	Large Caps
Mean	-0.0093	0.0029	0.0408	0.0317	0.0519	0.0059
Standard Dev.	0.0612	0.0123	0.0696	0.0716	0.0871	0.0700
Skewness	-0.7522	0.5473	-0.0068	0.3785	0.1259	0.4160
Kurtosis	4.6960	4.3992	2.6102	3.0289	2.5874	-0.4463
Maximum	0.1273	0.0384	0.2123	0.2095	0.2532	0.1839
Minimum	-0.2631	-0.0232	-0.0862	-0.1035	-0.1091	-0.1023

Table 2A:

Parameter Estimates of the MS-VAR(1) Model for the Investment Opportunity Set A

This table reports parameter estimates of the MS-VAR(1) model for the investment opportunity set A:

$$\mathbf{z}_{t+1} = \mathbf{\Phi}_{\circ}(s_{t+1}) + \mathbf{\Phi}_1(s_{t+1})\mathbf{z}_t + \boldsymbol{\varepsilon}_{t+1}$$

where $\boldsymbol{\varepsilon}_{t+1} \sim \mathcal{N}(0, \mathbf{\Sigma}_{\boldsymbol{\varepsilon}}(s_{t+1}))$ and s_{t+1} is the unobserved regime variable governed by a two-state, first-order Markov Chain. The panels of **Intercepts** and **Autoregressive Coefficients** report the estimates of $\mathbf{\Phi}_{\circ}$ and $\mathbf{\Phi}_1$. The diagonals of the **Volatility and Correlation** panel are the volatility estimates and the off-diagonals are the correlation ones. The sample period is 1953:1-2007:12. Standard errors are in parentheses.

	$\mathbf{Market_t}$	$\mathbf{Bond_t}$	$\mathbf{Term\ Spread_t}$	$\mathbf{Ln(D/P)_t}$
Intercepts				
Regime 1	0.0163	0.0004	0.0001	-0.0038
	(0.0158)	(0.0027)	(0.0001)	(0.0070)
Regime 2	0.0858	0.0044	0.0000	-0.0369
	(0.0450)	(0.0088)	(0.0004)	(0.0201)
Autoregressive coefficients				
		Regime 1		
$\mathbf{Market_{t-1}}$	-0.0465	-0.0256	0.0002	0.0244
	(0.0436)	(0.0073)	(0.0003)	(0.0191)
$\mathbf{Bond_{t-1}}$	0.9169	0.3883	-0.0116	-0.4481
	(0.2172)	(0.0390)	(0.0020)	(0.0970)
$\mathbf{Term\ Spread_{t-1}}$	0.5308	1.1874	0.9976	-0.3426
	(1.7753)	(0.3125)	(0.0078)	(0.7950)
$\mathbf{Ln(D/P)_{t-1}}$	0.0067	0.0001	0.0001	0.9982
	(0.0101)	(0.0017)	(0.0001)	(0.0045)
		Regime 2		
$\mathbf{Market_{t-1}}$	0.1729	-0.0514	-0.0004	-0.0574
	(0.1017)	(0.0199)	(0.0009)	(0.0450)
$\mathbf{Bond_{t-1}}$	1.3296	0.3079	0.0201	-0.5636
	(0.4872)	(0.0983)	(0.0047)	(0.2166)
$\mathbf{Term\ Spread_{t-1}}$	2.8851	1.6811	0.8575	-0.2132
	(0.2530)	(0.9498)	(0.0395)	(0.5719)
$\mathbf{Ln(D/P)_{t-1}}$	0.0679	0.0032	-0.0001	0.9703
	(0.0300)	(0.0059)	(0.0003)	(0.0135)
Volatility and Correlations				
		Regime 1		
$\mathbf{Market_t}$	0.0335			
	(0.0014)			
$\mathbf{Bond_t}$	0.0434	0.0058		
	(0.0000)	(0.0006)		
$\mathbf{Term\ Spread_t}$	0.0688	-0.0275	0.0001	
	(0.0002)	(0.0000)	(0.0002)	
$\mathbf{Ln(D/P)_t}$	-0.9602	-0.0473	-0.1002	0.0150
	(0.0001)	(0.0022)	(0.0020)	0.0000
		Regime 2		
$\mathbf{Market_t}$	0.0612			
	(0.0003)			
$\mathbf{Bond_t}$	-0.0011	0.0121		
	(0.0002)	(0.0000)		
$\mathbf{Term\ Spread_t}$	0.0747	-0.0402	0.0005	
	(0.0002)	(0.0052)	(0.0011)	
$\mathbf{Ln(D/P)_t}$	-0.9471	-0.0079	-0.0796	0.0273
	(0.0008)	(0.0001)	(0.0008)	(0.0014)
Transition Matrix				
	Regime 1			
Regime 1	0.9091	0.0909		
	-	(0.0303)		
Regime 2	0.3945	0.6055		
	(0.0747)	-		

Table 2B:

Parameter Estimates of the MS-VAR(1) Model for the Investment Opportunity Set B

This table reports parameter estimates of the MS-VAR(1) model for the investment opportunity set B:

$$\mathbf{z}_{t+1} = \mathbf{\Phi}_\circ(s_{t+1}) + \mathbf{\Phi}_1(s_{t+1})\mathbf{z}_t + \varepsilon_{t+1}$$

where $\varepsilon_{t+1} \sim \mathcal{N}(0, \mathbf{\Sigma}_\varepsilon(s_{t+1}))$ and s_{t+1} is the unobserved regime variable governed by a two-state, first-order Markov Chain. The panels of **Intercepts** and **Autoregressive Coefficients** report the estimates of $\mathbf{\Phi}_\circ$ and $\mathbf{\Phi}_1$. The diagonals of the **Volatility and Correlation** panel are the volatility estimates and the off-diagonals are the correlation ones. The sample period is 1953:1-2007:12. Standard errors are in parentheses.

	Small Caps$_t$	**Large Caps$_t$**	**Bond$_t$**	**Term Spread$_t$**	**Ln(D/P)$_t$**
Intercepts					
Regime 1	0.0211	0.0181	-0.0010	0.0000	-0.0055
	(0.0184)	(0.0141)	(0.0025)	(0.0001)	(0.0063)
Regime 2	-0.0591	0.0393	0.0146	0.0007	-0.0292
	(0.0752)	(0.0605)	(0.0133)	(0.0005)	(0.0280)
Autoregressive coefficients					
			Regime 1		
Small Caps$_{t-1}$	0.1196	-0.0605	-0.0152	0.0000	0.0138
	(0.0509)	(0.0369)	(0.0065)	(0.0002)	(0.0164)
Large Caps$_{t-1}$	0.1006	0.0050	-0.0116	0.0001	0.0005
	(0.0671)	(0.0496)	(0.0087)	(0.0003)	(0.0219)
Bond$_{t-1}$	0.6647	0.7690	0.3915	-0.0098	-0.3750
	(0.2646)	(0.2012)	(0.0360)	(0.0012)	(0.0902)
Term Spread$_{t-1}$	2.5102	2.0933	1.3171	0.9847	-0.9531
	(2.1362)	(1.6321)	(0.2897)	(0.0094)	(0.7300)
Ln(D/P)$_{t-1}$	0.0117	0.0093	-0.0008	0.0000	0.9965
	(0.0018)	(0.0091)	(0.0016)	(0.0001)	(0.0041)
			Regime 2		
Small Caps$_{t-1}$	-0.4030	-0.0475	-0.0109	-0.0004	-0.0570
	(0.1547)	(0.1239)	(0.0273)	(0.0011)	(0.0574)
Large Caps$_{t-1}$	0.3613	0.2250	-0.0733	-0.0021	-0.0918
	(0.2493)	(0.2011)	(0.0450)	(0.0018)	(0.0917)
Bond$_{t-1}$	-0.0920	0.7071	0.3186	0.0327	-0.6663
	(0.8393)	(0.6806)	(0.1496)	(0.0062)	(0.3165)
Term Spread$_{t-1}$	-0.0288	2.1353	0.9607	0.8919	2.1547
	(8.6138)	(7.0339)	(1.5395)	(0.0611)	(3.2326)
Ln(D/P)$_{t-1}$	-0.0757	0.0245	0.0092	0.0005	0.9724
	(0.0509)	(0.0408)	(0.0090)	(0.0004)	(0.0189)
Volatility and Correlations					
			Regime 1		
Small Caps$_t$	0.0461				
	(0.0014)				
Large Caps$_t$	0.7670	0.0350			
	(0.4380)	(0.0000)			
Bond$_t$	0.0137	0.0845	0.0061		
	(0.0003)	(0.0000)	(0.0003)		
Term Spread$_t$	0.0328	-0.0062	0.0051	0.0002	
	(0.0000)	(0.0002)	(0.0001)	(0.0072)	
Ln(D/P)$_t$	-0.7977	-0.9526	-0.0669	-0.0207	0.0157
	(0.0058)	(0.0018)	(0.0001)	(0.0037)	(0.0013)
			Regime 2		
Small Caps$_t$	0.0800				
	(0.0012)				
Large Caps$_t$	0.2196	0.0655			
	(0.0005)	(0.0007			
Bond$_t$	0.0960	0.2386	0.0142		
	(0.0002)	(0.0000	(0.0002)		
Term Spread$_t$	0.0457	0.0657	-0.1171	0.0006	
	(0.0082)	(0.0018	(0.0001)	(0.0038)	
Ln(D/P)$_t$	0.0536	-0.2375	0.0149	-0.0801	0.0301
	(0.0001)	(0.0036	(0.0000)	(0.0036)	(0.0026)
	Regime 1	**Regime 2**			
Regime 1	0.9425	0.0575			
	-	(0.0212)			
Regime 2	0.5193	0.4807			
	(0.0866)	-			

Table 2C:

Parameter Estimates of the MS-VAR(1) Model for the Investment Opportunity Set C

This table reports parameter estimates of the MS-VAR(1) model for the investment opportunity set C:

$$\mathbf{z}_{t+1} = \mathbf{\Phi}_{\circ}(s_{t+1}) + \mathbf{\Phi}_1(s_{t+1})\mathbf{z}_t + \boldsymbol{\varepsilon}_{t+1}$$

where $\boldsymbol{\varepsilon}_{t+1} \sim \mathcal{N}(0, \mathbf{\Sigma}_{\varepsilon}(s_{t+1}))$ and s_{t+1} is the unobserved regime variable governed by a two-state, first-order Markov Chain. The panels of **Intercepts** and **Autoregressive Coefficients** report the estimates of $\mathbf{\Phi}_{\circ}$ and $\mathbf{\Phi}_1$. The diagonals of the **Volatility and Correlation** panel are the volatility estimates and the off-diagonals are the correlation ones. The sample period is 1953:1-2007:12. Standard errors are in parentheses.

	$Growth_t$	**$Value_t$**	**$Bond_t$**	**Term $Spread_t$**	**$Ln(D/P)_t$**
Intercepts					
Regime 1	0.0119	0.0142	-0.0012	0.0000	-0.0019
	(0.0154)	(0.0146)	(0.0025)	(0.0001)	(0.0062)
Regime 2	0.0100	0.0569	0.0177	0.0010	-0.0815
	(0.0675)	(0.0657)	(0.0150)	(0.0006)	(0.0345)
Autoregressive coefficients					
			Regime 1		
$Growth_{t-1}$	0.0423	-0.0087	-0.0254	-0.0003	-0.0196
	(0.0563)	(0.0533)	(0.0090)	(0.0003)	(0.0226)
$Value_{t-1}$	-0.1003	0.0586	-0.0038	0.0004	0.0374
	(0.0592)	(0.0564)	(0.0095)	(0.0003)	(0.0238)
$Bond_{t-1}$	0.9136	0.8715	0.3871	-0.0102	-0.4365
	(0.2240)	(0.2138)	(0.0362)	(0.0012)	(0.0901)
Term $Spread_{t-1}$	1.3362	2.1163	1.3293	0.9860	-0.6858
	(1.7949)	(1.6911)	(0.2910)	(0.0096)	(0.7187)
$Ln(D/P)_{t-1}$	0.0057	0.0069	-0.0009	0.0000	0.9988
	(0.0018)	(0.0094)	(0.0016)	(0.0001)	(0.0040)
			Regime 2		
$Growth_{t-1}$	0.2979	-0.0651	-0.0430	-0.0002	-0.1389
	(0.2221)	(0.2135)	(0.0489)	(0.0020)	(0.1052)
$Value_{t-1}$	-0.3861	-0.2873	-0.0659	-0.0019	0.0453
	(0.2185)	(0.2181)	(0.0500)	(0.0022)	(0.1067)
$Bond_{t-1}$	-0.0327	0.9126	0.3965	0.0360	-0.7466
	(0.7777)	(0.7631)	(0.1697)	(0.0073)	(0.3579)
Term $Spread_{t-1}$	2.4317	6.1492	0.4640	0.9106	-1.2290
	(7.4603)	(7.2594)	(1.7176)	(0.0695)	(3.4869)
$Ln(D/P)_{t-1}$	-0.0190	0.0213	0.0114	0.0007	0.9355
	(0.0466)	(0.0449)	(0.0103)	(0.0004)	(0.0241)
Volatility and Correlations					
			Regime 1		
$Growth_t$	0.0391				
	(0.0011)				
$Value_t$	0.7903	0.0372			
	(0.0003)	(0.0000)			
$Bond_t$	0.0669	0.0954	0.0062		
	(0.0003)	(0.0000)	(0.0002)		
Term $Spread_t$	-0.0138	0.0030	0.0088	0.0002	
	(0.0000)	(0.0002)	(0.0002)	(0.0066)	
$Ln(D/P)_t$	-0.9264	-0.8620	-0.0845	-0.0194	0.0158
	(0.0059)	(0.0020)	(0.0001)	(0.0043)	(0.0014)
			Regime 2		
$Growth_t$	0.0652				
	(0.0013)				
$Value_t$	0.4451	0.0660			
	(0.0005)	(0.0007)			
$Bond_t$	0.2152	0.2045	0.0149		
	(0.0002)	(0.0000)	(0.0002)		
Term $Spread_t$	0.1723	0.1117	-0.1754	0.0006	
	(0.0087)	(0.0020)	(0.0001)	(0.0043)	
$Ln(D/P)_t$	0.0363	-0.1747	0.1361	-0.0472	0.0316
	(0.0001)	(0.0043)	(0.0001)	(0.0044)	(0.0031)
	Regime 1	**Regime 2**			
Regime 1	0.9446	0.0554			
	-	(0.0101)			
Regime 2	0.5978	0.4022			
	(0.1185)	-			

Table 3:

Empirical Asset Allocations for Three Investment Opportunity Sets

This table reports the results of the mean optimal portfolio weights that average the whole sample period and two subsample periods sorted by regimes. It covers seven belief formation algorithms and three investment opportunity sets. The parameter values of the investment opporunity set are given in Table 2A, 2B and 2C and the preference parameters are $\psi = 0.99$, $\beta = 0.92$, $\gamma = 5$, $\omega = 0.9$.

	Bayesian	Representative-Biased	Over-Confident	Optimistic	Pessimistic	Reversal	Momentum
			Set A				
Stock	0.974	0.800	0.892	1.697	-0.342	0.171	0.769
Bond	6.837	6.056	6.064	10.641	2.530	3.869	6.033
Cash	-6.810	-5.855	-5.956	-11.338	-1.189	-3.040	-5.802
Regime 1:Bullish Regime							
Stock	1.033	0.848	0.919	1.675	-0.350	0.062	0.857
Bond	7.010	6.159	6.129	10.483	2.411	3.315	6.275
Cash	-7.043	-6.007	-6.048	-11.159	-1.061	-2.377	-6.132
Regime 2:Bearish Regime							
Stock	0.685	0.562	0.763	1.802	-0.300	0.704	0.341
Bond	5.988	5.551	5.745	11.413	3.109	6.575	4.854
Cash	-5.673	-5.113	-5.508	-12.214	-1.809	-6.278	-4.195
			Set B				
Small Caps	0.557	0.569	0.605	0.470	1.000	0.989	0.630
Large Caps	0.038	0.035	-0.016	0.134	-0.149	-0.160	-0.015
Bond	3.257	3.112	2.962	4.172	-0.247	0.114	2.640
Cash	-2.852	-2.717	-2.551	-3.776	0.396	0.058	-2.255
Regime 1:Bullish Regime							
Small Caps	0.546	0.558	0.584	0.427	1.028	1.000	0.622
Large Caps	0.049	0.046	0.014	0.196	-0.152	-0.138	-0.006
Bond	3.585	3.424	3.207	4.439	-0.231	-0.020	2.946
Cash	-3.180	-3.028	-2.804	-4.062	0.355	0.158	-2.562
Regime 2:Bearish Regime							
Small Caps	0.669	0.685	0.824	0.900	0.721	0.876	0.707
Large Caps	-0.066	-0.071	-0.316	-0.489	-0.119	-0.383	-0.102
Bond	-0.027	-0.009	0.512	1.505	-0.409	1.449	-0.409
Cash	0.424	0.394	-0.021	-0.915	0.807	-0.942	0.805
			Set C				
Growth	0.259	0.292	0.263	0.046	0.653	0.590	0.374
Value	0.525	0.511	0.535	0.626	0.638	0.640	0.502
Bond	2.792	2.646	2.549	3.884	-1.130	-0.718	1.986
Cash	-2.577	-2.450	-2.347	-3.556	0.839	0.488	-1.862
Regime 1:Bullish Regime							
Growth	0.228	0.264	0.261	0.056	0.643	0.621	0.341
Value	0.537	0.521	0.532	0.614	0.655	0.641	0.509
Bond	3.155	2.991	2.836	4.206	-1.045	-0.789	2.336
Cash	-2.920	-2.776	-2.628	-3.876	0.747	0.527	-2.187
Regime 2:Bearish Regime							
Growth	0.617	0.628	0.283	-0.069	0.769	0.230	0.758
Value	0.386	0.398	0.576	0.757	0.434	0.631	0.415
Bond	-1.440	-1.389	-0.800	0.132	-2.122	0.112	-2.096
Cash	1.437	1.362	0.941	0.180	1.919	0.028	1.923

Table 4A:
Comaprison of the Empirical Performances for Investors with Different Belief Formation Algorithms I

This table reports the resuls of the empirical performance of the portfolio allocations associated various belief formation algorithms. Panel A represents the metrics associated with the sample averge of portfolio returns and the Graham and Harvey's measures. Panel B reports the intercept estimates of the Fama-French four-factors model and the Newey-West standard errors are in the parentheses. The sample period is 1953:1-2007:12. **Set A (or B, C)** indicates the investment opportunity set. Boldfaced numbers in Panel B indicates the corresponding numbers that are significant at 95% significant level.

Panel A: Summary Statistics of the Portfolio Returns and GH Metrics

	Bayesian	Representative -Biased	Over-Confident	Optimistic	Pessimistic	Reversal	Momentum
In-Sample							
			Set A				
$\overline{r^t_{p,t}}$	0.0756	0.0700	0.0741	0.1111	0.0429	0.0690	0.0671
$\overline{r^t_{p,t}} - \overline{r^B_{p,t}}$		-0.0056	-0.0015	0.0355	-0.0327	-0.0066	-0.0085
GH1	0.0591	0.0554	0.0553	0.0773	0.0334	0.0440	0.0536
GH SCORE	0.0760	0.0732	0.0692	0.0882	0.0498	0.0524	0.0724
			Set B				
$\overline{r^t_{p,t}}$	0.0533	0.0500	0.0569	0.0819	0.0211	0.0399	0.0426
$\overline{r^t_{p,t}} - \overline{r^B_{p,t}}$		-0.0033	0.0036	0.0286	-0.0322	-0.0134	-0.0107
GH1	0.0434	0.0410	0.0456	0.0650	0.0146	0.0278	0.0347
GH SCORE	0.0642	0.0624	0.0647	0.0831	0.0254	0.0386	0.0554
			Set C				
$\overline{r^t_{p,t}}$	0.0537	0.0507	0.0568	0.0779	0.0209	0.0356	0.0421
$\overline{r^t_{p,t}} - \overline{r^B_{p,t}}$		-0.0030	0.0030	0.0241	-0.0328	-0.0182	-0.0116
GH1	0.0442	0.0418	0.0454	0.0618	0.0140	0.0237	0.0346
GH SCORE	0.0662	0.0640	0.0642	0.0800	0.0236	0.0330	0.0561
Out-of-Sample							
			Set A				
$\overline{r^t_{p,t}}$	0.0860	0.0814	0.0797	0.1025	0.0348	0.0345	0.0710
$\overline{r^t_{p,t}} - \overline{r^B_{p,t}}$		-0.0046	-0.0063	0.0166	-0.0512	-0.0515	-0.0150
GH1	0.0589	0.0542	0.0560	0.0746	0.0174	0.0213	0.0544
GH SCORE	0.0753	0.0713	0.0697	0.0848	0.0257	0.0252	0.0730
			Set B				
$\overline{r^t_{p,t}}$	0.0989	0.0942	0.0892	0.1373	0.0159	0.0245	0.0817
$\overline{r^t_{p,t}} - \overline{r^B_{p,t}}$		-0.0047	-0.0098	0.0383	-0.0830	-0.0745	-0.0172
GH1	0.0403	0.0389	0.0338	0.0507	0.0035	0.0013	0.0337
GH SCORE	0.0590	0.0587	0.0475	0.0645	0.0060	0.0019	0.0533
			Set C				
$\overline{r^t_{p,t}}$	0.0895	0.0862	0.0839	0.1179	0.0282	0.0343	0.0751
$\overline{r^t_{p,t}} - \overline{r^B_{p,t}}$		-0.0033	-0.0056	0.0284	-0.0613	-0.0552	-0.0144
GH1	0.0364	0.0354	0.0316	0.0430	0.0088	0.0066	0.0312
GH SCORE	0.0541	0.0537	0.0444	0.0553	0.0147	0.0092	0.0501

Panel B: Intercept Estimates of $\widehat{\alpha_F}$

	Representative-Biased	Over-Confident	Optimistic	Pessimistic	Reversal	Momentum
In-Sample						
Set A	**-0.0054**	-0.0011	**0.0357**	**-0.0323**	-0.0046	**-0.0083**
	(0.0013)	(0.0039)	(0.0118)	(0.0050)	(0.0099)	(0.0024)
Set B	**-0.0032**	0.0039	**0.0283**	**-0.0314**	**-0.0121**	**-0.0104**
	(0.0007)	(0.0022)	(0.0054)	(0.0034)	(0.0045)	(0.0017)
Set C	**-0.0030**	0.0036	**0.0245**	**-0.0324**	**-0.0168**	**-0.0113**
	(0.0005)	(0.0019)	(0.0048)	(0.0031)	(0.0038)	(0.0017)
Out-of-Sample						
Set A	**-0.0027**	**-0.0081**	**0.0131**	**-0.0429**	**-0.0457**	**-0.0129**
	(0.0013)	(0.0037)	(0.0061)	(0.0119)	(0.0111)	(0.0033)
Set B	**-0.0031**	**-0.0113**	**0.0357**	**-0.0779**	**-0.0725**	**-0.0159**
	(0.0010)	(0.0036)	(0.0071)	(0.0134)	(0.0125)	(0.0031)
Set C	**-0.0020**	**-0.0063**	**0.0259**	**-0.0564**	**-0.0523**	**-0.0130**
	(0.0008)	(0.0022)	(0.0054)	(0.0075)	(0.0069)	(0.0021)

Table 4B:

Comaprison of the Empirical Performances for Investors with Different Belief Formation Algorithms II

This table reports the resuls of the empirical performance of the portfolio allocations associated various belief formation algorithms. Panel A represents the metrics associated with the sample averge of long-run portfolio returns. Panel B reports the intercept estimates of the long-run return regression model and the Newey-West standard errors are in the parentheses. The sample period is 1953:1-2007:12. **Set A (or B, C)** indicates the investment opportunity set. Boldfaced numbers in Panel B indicates the corresponding numbers that are significant at 95% significant level.

Panel A:Sample Averages of the Long-Run Returns $\overline{LR^{i}_{t}}(\tau)$

	Bayesian	Representative -Biased	Over-Confident	Optimistic	Pessimistic	Reversal	Momentum
			τ =12M				
Set A	0.9142	0.8468	0.8964	1.3453	0.5192	0.8378	0.8105
Set B	0.6453	0.6051	0.6893	0.9926	0.2600	0.4888	0.5157
Set C	0.6507	0.6142	0.6881	0.9432	0.2575	0.4357	0.5105
			τ =36M				
Set A	2.7800	2.5751	2.7244	4.1015	1.5886	2.5680	2.4604
Set B	1.9637	1.8397	2.1044	3.0376	0.7848	1.4973	1.5642
Set C	1.9727	1.8617	2.0904	2.8706	0.7735	1.3259	1.5440
			τ =60M				
Set A	4.6884	4.3408	4.5942	6.9244	2.6979	4.3594	4.1483
Set B	3.3091	3.0955	3.5617	5.1510	1.3070	2.5378	2.6261
Set C	3.3211	3.1318	3.5283	4.8499	1.2899	2.2394	2.5926

Panel B: the intercept estimates $\widehat{\alpha_L}$

	Representative -Biased	Over-Confident	Optimistic	Pessimistic	Reversal	Momentum
		τ =12M				
Set A	**-0.0545**	-0.0075	**0.3868**	**-0.3235**	-0.0114	**-0.0913**
	(0.0131)	(0.0398)	(0.1318)	(0.0477)	(0.1081)	(0.0253)
Set B	**-0.0411**	**0.0776**	**0.3913**	**-0.3938**	**-0.1252**	**-0.1381**
	(0.0057)	(0.0251)	(0.0654)	(0.0360)	(0.0483)	(0.0185)
Set C	**-0.0398**	**0.0645**	**0.3224**	**-0.4218**	**-0.2022**	**-0.1459**
	(0.0039)	(0.0241)	(0.0595)	(0.0335)	(0.0404)	(0.0195)
		τ =36M				
Set A	**-0.1396**	0.0966	**1.1451**	**-0.8072**	0.2736	**-0.2534**
	(0.0454)	(0.1297)	(0.5109)	(0.1833)	(0.4027)	(0.0971)
Set B	**-0.1710**	**0.3592**	**1.4295**	**-1.3002**	-0.3064	**-0.4884**
	(0.0282)	(0.0888)	(0.2474)	(0.1474)	(0.2052)	(0.0714)
Set C	**-0.1300**	**0.2760**	**1.0327**	**-1.2230**	**-0.4935**	**-0.4536**
	(0.0111)	(0.0732)	(0.2158)	(0.1062)	(0.1579)	(0.0702)
		τ =60M				
Set A	**-0.2079**	-0.0816	0.5344	**-0.7971**	-0.1597	-0.1500
	(0.0941)	(0.1670)	(0.6891)	(0.3101)	(0.5108)	(0.1572)
Set B	**-0.3171**	**0.4445**	**1.8898**	**-2.0602**	**-0.8849**	**-0.7079**
	(0.0544)	(0.1084)	(0.2829)	(0.2932)	(0.3213)	(0.1079)
Set C	**-0.1991**	**0.3206**	**1.1926**	**-1.7832**	**-0.9812**	**-0.5710**
	(0.0283)	(0.0798)	(0.2585)	(0.2079)	(0.2507)	(0.0978)

Table 5A:

Empirical Performances for Investors with Different Belief Formation Algorithms during 70's and 90's

This table reports the resuls of the empirical performance of the portfolio allocations associated various belief formation algorithms. Panel A represents the metrics associated with the sample averge of portfolio returns and the Graham and Harvey's measures. Panel B reports the intercept estimates of the Fama-French four-factors model and the Newey-West standard errors are in the parentheses. The sample period is 1953:1-2007:12. **Set A (or B, C)** indicates the investment opportunity set. Boldfaced numbers in Panel B indicates the corresponding numbers that are significant at 95% significant level.

Panel A: Subsample Period I (1970:1--1979:12)

	Bayesian	Representative-Biased	Over-Confident	Optimistic	Pessimistic	Reversal	Momentum
$\overline{r^t_{p,t}}$							
Set A	0.057	0.054	0.066	0.083	0.046	0.073	0.051
Set B	0.049	0.043	0.061	0.091	0.011	0.042	0.034
Set C	0.052	0.048	0.063	0.087	0.019	0.044	0.038
GH SCORE							
Set A	0.051	0.050	0.050	0.049	0.053	0.046	0.052
Set B	0.056	0.051	0.061	0.087	0.006	0.032	0.041
Set C	0.061	0.056	0.063	0.082	0.016	0.035	0.046
Intercept Estimates of $\widehat{\alpha_F}$							
		Representative-Biased	**Over-Confident**	**Optimistic**	**Pessimistic**	**Reversal**	**Momentum**
Set A		-0.002	0.008	0.021	-0.008	0.015	-0.004
		(0.002)	(0.009)	(0.021)	(0.008)	(0.016)	(0.004)
Set B		**-0.006**	0.011	**0.040**	**-0.036**	-0.008	**-0.015**
		(0.003)	(0.006)	(0.012)	(0.007)	(0.010)	(0.004)
Set C		**-0.003**	0.010	**0.032**	**-0.030**	-0.007	**-0.013**
		(0.001)	(0.005)	(0.011)	(0.005)	(0.007)	(0.003)

Panel B: Subsample Period II (1990:1--1999:12)

	Bayesian	Representative-Biased	Over-Confident	Optimistic	Pessimistic	Reversal	Momentum
$\overline{r^t_{p,t}}$							
Set A	0.085	0.076	0.079	0.102	0.029	0.031	0.082
Set B	0.054	0.052	0.054	0.070	0.021	0.024	0.047
Set C	0.053	0.052	0.053	0.062	0.026	0.027	0.047
GH SCORE							
Set A	0.097	0.090	0.088	0.104	0.035	0.038	0.094
Set B	0.078	0.077	0.079	0.093	0.026	0.030	0.070
Set C	0.080	0.079	0.079	0.087	0.037	0.039	0.073
Intercept Estimates of $\widehat{\alpha_F}$							
		Representative-Biased	**Over-Confident**	**Optimistic**	**Pessimistic**	**Reversal**	**Momentum**
Set A		**-0.010**	-0.007	**0.017**	**-0.059**	**-0.055**	-0.003
		(0.002)	(0.005)	(0.007)	(0.009)	(0.010)	(0.002)
Set B		**-0.001**	0.000	**0.015**	**-0.030**	**-0.028**	**-0.006**
		(0.000)	(0.001)	(0.002)	(0.005)	(0.005)	(0.001)
Set C		**-0.002**	0.000	**0.008**	**-0.024**	**-0.022**	**-0.006**
		(0.001)	(0.000)	(0.002)	(0.005)	(0.005)	(0.001)

Table 5B:

Empirical Performances for Investors with Different Belief Formation Algorithms during 1953:1-1979:12 and 1980:1-2007:12

This table reports the resuls of the empirical performance of the portfolio allocations associated various belief formation algorithms. Panel A represents the metrics associated with the sample averge of portfolio returns and the Graham and Harvey's measures. Panel B reports the intercept estimates of the Fama-French four-factors model and the Newey-West standard errors are in the parentheses. The sample period is 1953:1-2007:12. **Set A (or B, C)** indicates the investment opportunity set. Boldfaced numbers in Panel B indicates the corresponding numbers that are significant at 95% significant level.

Panel C: Subsample Period III (1953:1--1979:12)

	Bayesian	Representative-Biased	Over-Confident	Optimistic	Pessimistic	Reversal	Momentum
$\overline{r^t_{p,t}}$							
Set A	0.035	0.035	0.038	0.044	0.029	0.038	0.034
Set B	0.035	0.032	0.040	0.055	0.016	0.029	0.028
Set C	0.035	0.033	0.040	0.053	0.013	0.024	0.027
GH SCORE							
Set A	0.033	0.034	0.028	0.022	0.037	0.022	0.035
Set B	0.044	0.043	0.044	0.054	0.018	0.026	0.038
Set C	0.045	0.042	0.043	0.053	0.010	0.018	0.037

Intercept Estimates of $\widehat{\alpha_F}$

	Representative-Biased	Over-Confident	Optimistic	Pessimistic	Reversal	Momentum
Set A	0.000	0.001	0.004	-0.005	0.001	-0.001
	(0.001)	(0.003)	(0.007)	(0.004)	(0.005)	(0.001)
Set B	**-0.003**	0.004	**0.018**	**-0.019**	-0.007	**-0.007**
	(0.001)	(0.002)	(0.005)	(0.004)	(0.004)	(0.002)
Set C	**-0.002**	0.004	**0.016**	**-0.021**	**-0.012**	**-0.007**
	(0.000)	(0.002)	(0.004)	(0.004)	(0.004)	(0.001)

Panel D: Subsample Period IV (1980:1--2007:12)

	Bayesian	Representative-Biased	Over-Confident	Optimistic	Pessimistic	Reversal	Momentum
$\overline{r^t_{p,t}}$							
Set A	0.114	0.104	0.109	0.176	0.057	0.099	0.099
Set B	0.071	0.067	0.074	0.108	0.026	0.050	0.057
Set C	0.071	0.068	0.073	0.102	0.028	0.047	0.056
GH SCORE							
Set A	0.119	0.112	0.110	0.154	0.064	0.082	0.109
Set B	0.085	0.083	0.085	0.112	0.032	0.051	0.073
Set C	0.088	0.086	0.085	0.107	0.037	0.048	0.075

Intercept Estimates of $\widehat{\alpha_F}$

	Representative-Biased	Over-Confident	Optimistic	Pessimistic	Reversal	Momentum
Set A	**-0.010**	-0.004	**0.064**	**-0.058**	-0.011	-0.015
	(0.002)	(0.006)	(0.018)	(0.007)	(0.016)	(0.004)
Set B	**-0.004**	0.004	**0.037**	**-0.043**	**-0.017**	**-0.014**
	(0.001)	(0.003)	(0.008)	(0.005)	(0.007)	(0.002)
Set C	**-0.004**	0.003	**0.032**	**-0.043**	**-0.021**	**-0.015**
	(0.001)	(0.002)	(0.007)	(0.004)	(0.006)	(0.002)

Table 6:

Comaprison of the Empirical Performances with/without Predictability Between the Bayesian and Optimistic Investors

This table reports the results of the empirical performance of the portfolio allocations associated various belief formation algorithms. Panel A represents the sample averge of the single-period and long-run portfolio returns, $\bar{r}_{p,t}^{o(orB)}$ and LR$(\tau = 60M)^{o(orB)}$. Panel B reports the intercept estimates of the regression models, (23) and (25), and the Newey-West standard errors are in the parentheses. The sample period is 1953:1-2007:12. Boldfaced numbers in Panel B indicates the corresponding numbers that are significant at 95% significant level.

Panel A:

	Bayesian		Optimistic	
	$\overline{r_{p,t}^{B}}$	$\overline{LR_t^B}(\tau = 60M)$	$\overline{r_{p,t}^{O}}$	$\overline{LR_t^O}(\tau = 60M)$
		Set A		
MS	0.0036	0.2069	-0.0070	-0.0132
MSVAR	0.0756	4.6884	0.1111	6.9244
		Set B		
MS	0.0039	0.2799	0.0007	0.0796
MSVAR	0.0533	3.3091	0.0819	5.1510
		Set C		
MS	0.0002	0.0365	-0.0008	-0.0177
MSVAR	0.0537	3.3211	0.0779	4.8499

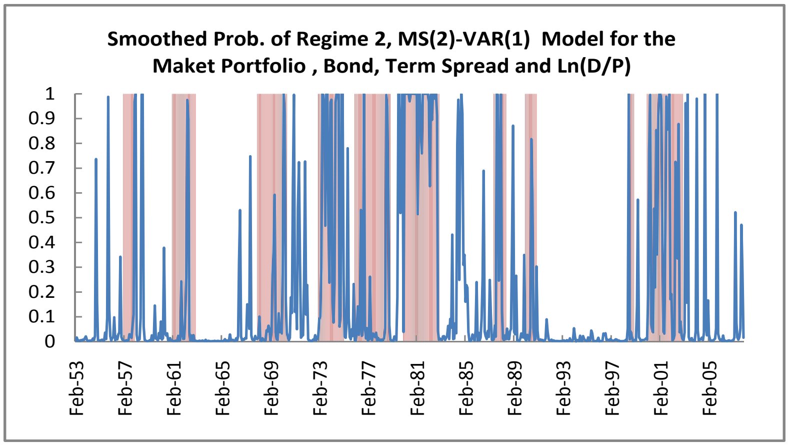
Panel B:

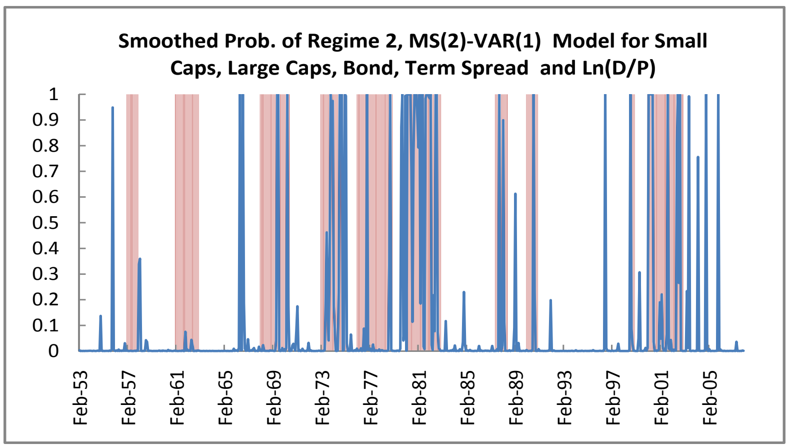
Intercept estimates $\widehat{\alpha_F}$	Set A	Set B	Set C
MS	**-0.0034**	**-0.0030**	-0.0014
	(0.0012)	(0.0009)	(0.0010)
MSVAR	**0.0357**	**0.0283**	**0.0245**
	(0.0118)	(0.0054)	(0.0048)
Intercept estimates $\widehat{\alpha_L}$			
MS	**-0.4673**	**-0.3582**	**-0.2761**
	(0.0603)	(0.0523)	(0.0532)
MSVAR	0.5344	**1.8898**	**1.1926**
	(0.6891)	(0.2829)	(0.2585)

Figure 1:

Smoothed Probabilities of Regime 2 for Three Investment Opportunity Sets

The graphs plot the smoothed probabilities of regime 2 for three investment opportunity sets. The sample period is 1953:1-2007:12. The red bars indicate the bear market periods.





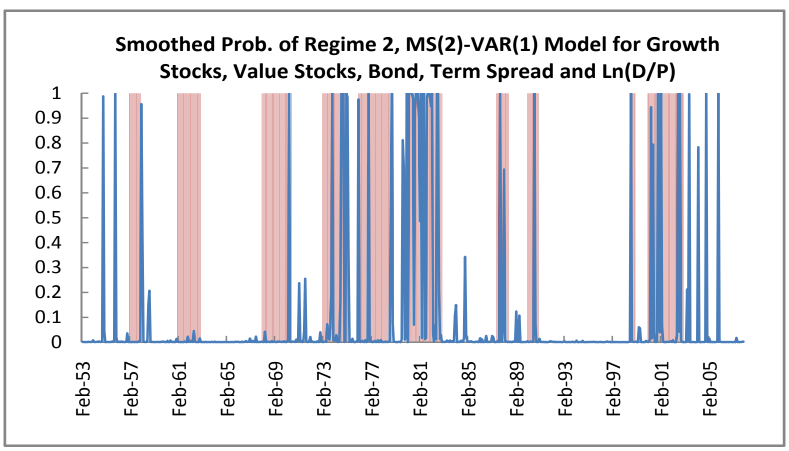


Figure 2A:

The Investor's Beliefs and the Demands for Stocks and Bond

This figure show that the demand curves for stocks and bonds with respect to the investor's beliefs. The Vertical (horizontal) axis is the demands(beliefs $\pi_{t|t}(s_t = 1)$). The parameter values of the investment opporunity set are given in Table 2A and the preference parameters are $\psi = 0.99$, $\beta = 0.92$, $\gamma = 5, 10$.

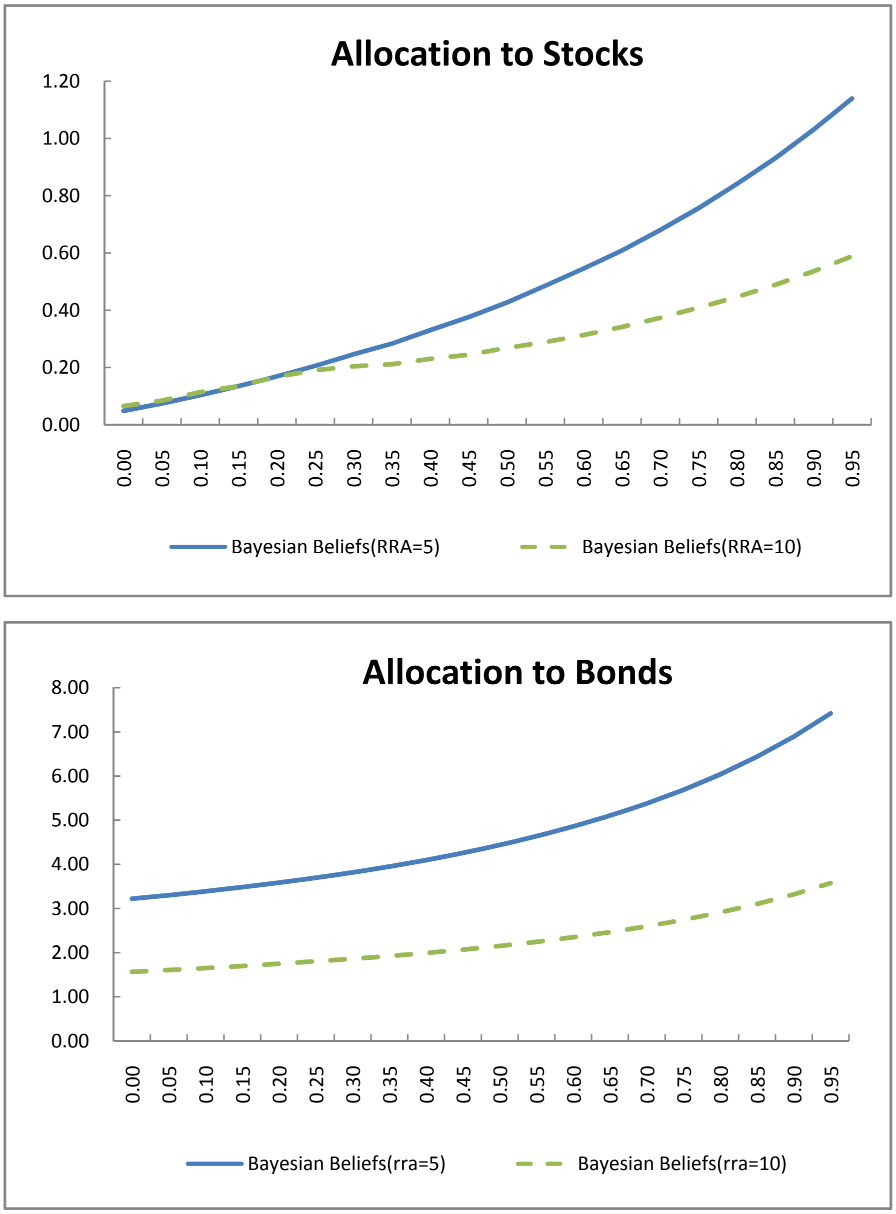


Figure 2B:

Stock Characteristics and the Portfolio Allocations

This figure plots the portfolio allocations under the investment opportunity set B and C. The Vertical (horizontal) axis is the demands(beliefs $\pi_{t|t}(s_t = 1)$). The parameter values of the investment opporunity set are given in Table 2B and 2C and the preference parameters are $\psi = 0.99$, $\beta = 0.92$, $\gamma = 5$.

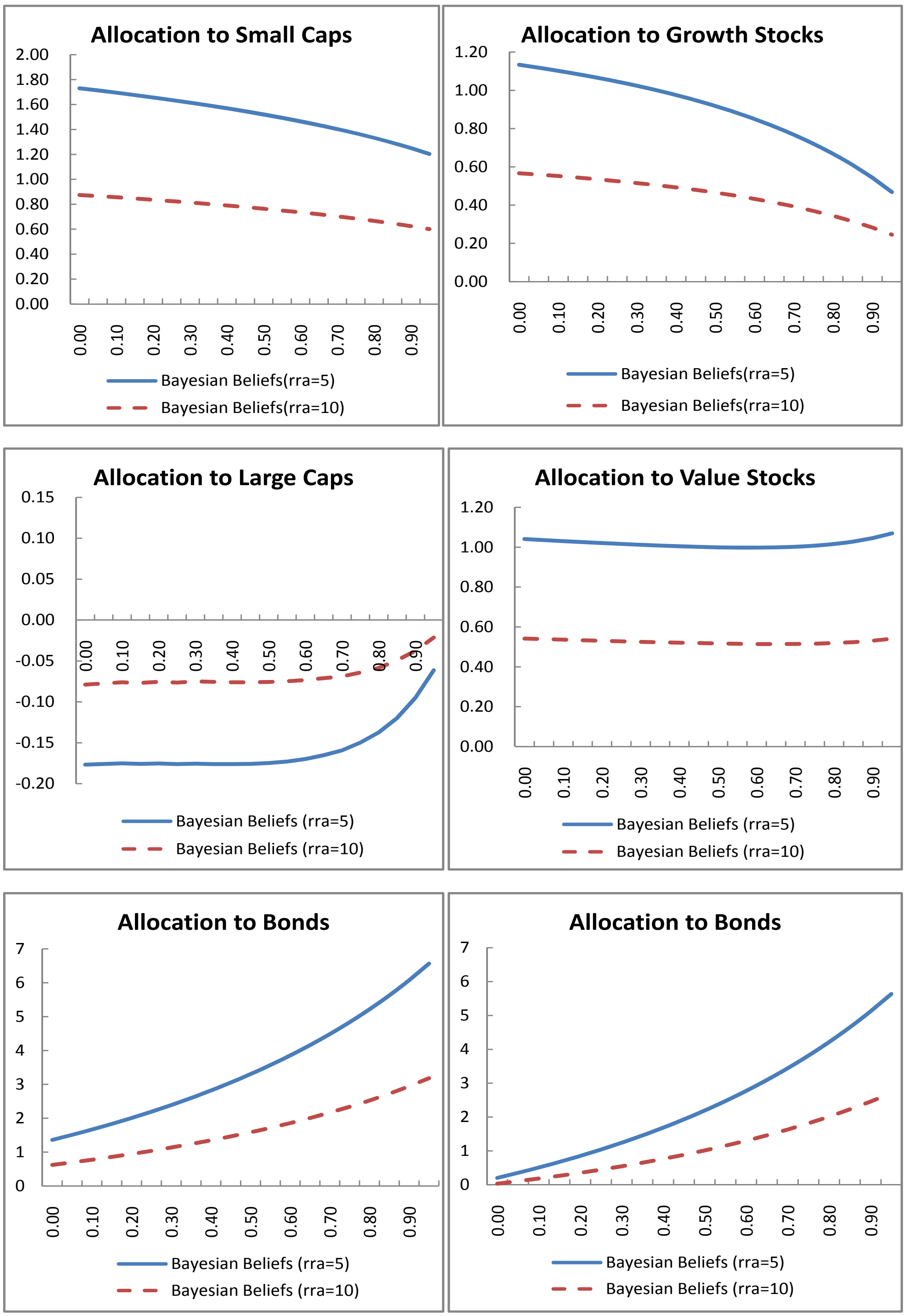


Figure 3:

Predictability from the Term Spread and the log Dividend-Price Ratio

The figure plots the demands for stocks and bonds with respect to the investor's beliefs with different term spread (log dividend-price ratio) values. The Vertical (horizontal) axis is the demands(beliefs $\pi_{t|t}(s_t = 1)$). The parameter values of the investment opporunity set are given in Table 2A and the preference parameters are $\psi = 0.99$, $\beta = 0.92$, $\gamma = 5$.