Loss Aversion in Prospect Theory

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Abstract

Loss aversion, an important element in prospect theory, explains many types of psychological behavior. In this paper, we formally define an index of loss aversion as a function of the location, to reflect gain/loss comparisons. This index, different from the one defined by Kobberling and Wakker (2005), can be used to fully describe the characteristics of loss aversion without making strong assumption on the utility function. Distinctions of our definition from the previous ones are discussed in details. We also attempt to fit the special classes of utility functions into our definition. Indices of different decision makers' loss aversion can be compared through Yaari's acceptance sets or asset demand in investment strategy. Generally speaking, the more loss averse the agent is, the smaller the acceptance set is and the less he will invest in risky asset.

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Introduction

Most economics models in finance and economics have traditionally assumed implicitly or explicitly von Neumann and Morgenstern (1953) expected utility (EU) maximization. However, from the late 1970's, dissatisfaction with EU theory begins to emerge on a number of fronts: experimental economists begin to disclose a lot of evidence that subjects systematically violate the axioms of EU theory in a variety of settings. Actually, many different versions of non-EU behaviors are motivated by two simple-thought experiments, the famous Allais (1953) paradox and Ellsberg (1961) paradox. Allais puzzle refers to the problem of comparing participants' choices in two different experiments, each of which consists of a choice between two gambles. In the first experiment, you are faced with 100% of winning \$1 million or 89% of winning \$1 million, 10% of winning \$5 million and another 1% of receiving nothing. In the second experiment, you can choose either 11% of winning \$1 million and 89% of nothing or 10% of winning \$5 million and 90% of nothing. The result is that a majority of participants choose first gamble in experiment I and second in experiment II. However this result is inconsistent under the EU setting. Ellsberg's paradox also involves a comparison of participants' choices in two different experiments, each of which consists of a choice between two gambles. Suppose you have an urn containing 30 red balls and 60 other balls that are either black or yellow. You do not know the distribution, but the total number is fixed 60. In experiment I, you are faced with A: receiving \$100 if you draw a red ball and B: receiving \$100 if you draw a black ball. In experiment II, choice A gives you \$100 if you draw a red or yellow ball and choice B gives you \$100 if you draw a black or yellow ball. The result shows that most of the participants choose A in the first experiment and B in the second, which is also in contradiction with the EU framework.

Microeconomists and some decision analysts feel that EU theory places too much constraints on the modeling of rational behavior. This urges the development of alternative tractable theories for making decisions. Popular choices include Tversky and Kahneman's (1979) prospect theory and Quiggin's (1981) rank-dependent theory. Prospect theory is recognized as a descriptive model of choice under uncertainty. It is different from EU theory in two main aspects:

1. The determinant of utility is not final wealth, but rather gains and losses relative to a reference point. The value function is normally concave for gains and convex for losses, a phenomenon called the "reflection effect". The value function is generally steeper for losses than it is for gains.

2. Individuals evaluate uncertain prospects with "decision weights" which are distorted versions of probabilities. In particular, they overweight small probabilities and underweighted moderate and high probabilities.

Quiggin's approach, which is an axiomatically sound way to represent preferences, distorts cumulative probabilities rather than original probabilities. This automatically avoids the violation of first stochastic dominance.[1]

Later until 1992, Tversky and Kahneman's improved version of prospect theory succeeds in replacing the distorted probabilities in the original theory with distorted cumulative probabilities. It is both rank and sigh dependent for gains and losses. As commented by Kobberling and Wakker (2005), it combines the mathematical elegance of Quiggin's theory with the empirical realism of Kahneman and Tversky's original prospect theory.

[1] Suppose two prospects $P = (p_1, x_1; \cdots; p_n, x_n)$ and $Q = (q_1, y_1; \cdots; q_m, y_m)$ (The definition of this short writing is introduced later.) First mix $\{x_i\}$ and $\{y_j\}$ and rank them as $\{z_k\}$, then rewrite P and Q with possible outcomes $\{z_k\}$.

$$\begin{aligned} PT(P) &= u(z_1)w(p_1^z) + \sum_{k=2}^{m+n} u(z_k)(w(\sum_1^k p_i^z) - w(\sum_1^{k-1} p_i^z)) \\ &= \sum_{k=1}^{m+n-1} (u(z_k) - u(z_{k+1}))w(\sum_1^k p_i^z) + u(z_{m+n})w(\sum_1^{m+n} p_i^z) \end{aligned}$$

is apparently greater than $PT(Q)$ if P is first stochastic dominant over Q. Note: p_i^z denotes the probability weight assigned for prospect P among outcomes z_i.

Loss aversion, which emerges as an important concept in prospect theory, reflects the phenomenon that individuals are more sensitive to losses than gains. This pattern implies that the utility function is steeper for losses than it is for gains. This theory has been widely used in many applications. Benartzi and Thaler (1995) discuss equity puzzle and conclude if loss aversion is taken into account the risk premium can be more substantial than when it is not considered. This helps explain the empirical fact that stocks have outperformed bonds over the last century by a surprisingly large margin. Thaler (1980) discusses endowment effect and concludes that the disparity between willingness-to-pay and willingness-to-accept can be explained since removing a good from the endowment creates a loss while adding the same good (to an endowment without it) generates a gain. Therefore, people value more for those owned by themselves and less for those belonging to others. The status quo bias is noticed by Samuelson and Zeckhauser (1988) that most real decision-makers, unlike those of economics texts, prefer to maintain their current or previous decisions. Their significant bias towards status quo alternatives is explained since the potential loss from changes will cause more pains than the euphoria brought by potential gains. Barberis, Huang and Thaler (2006) discuss stock market non-participation phenomenon that even though the stock market has a high mean return and a low correlation with other household risks, many households have historically been reluctant to allocate any money to it. They suggest that preferences that combine loss aversion with narrow framing will have an easier time explaining so.

Although referred to as an important element to help explain phenomena, only a few studies formally discuss this concept. Even within the relatively small amount of literature, there exists difference regarding the definitions. Wakker and Tversky (1993) explain loss aversion as value function is steeper for losses than for gains and clarify this concept in their own settings. Neilson (2002) provides another definition for loss aversion that a chord connecting the origin to $f(z)$ for any $z < 0$ is steeper than a chord connecting the origin to $f(y)$ for any $y > 0$. This can be interpreted as the maximal average utility from any gain is less than the minimal average disutility from any loss. Stronger definition for loss aversion, adopted by Bowman et al. (1999), Breiter et al.(2001) and Neilson (2002), states that the marginal utility of a gain is less than the marginal disutility from a loss. This actually restricts the function to be steeper at every loss than it is at every gain.

Kobberling and Wakker (2005) simply define loss aversion through the characteristics of the utility function around the kink. They first bring in the concept of index of loss aversion and then equalize loss aversion as the requirement that the index is greater than 1. Actually only when the strong assumption of the utility function is made can the feature around the kink represent the whole domain. Hence the acceptance of their definition hinges much on the assumption of the utility function. Motivated by their work and aiming to broaden the applicability, we redefine the index of loss aversion as a function, reflecting the comparisons of the steepness between symmetric locations. Similar to their conclusion, loss aversion holds only when the index of loss aversion is always greater than a constant. We hope this way of definition will make the concept more natural and tractable, hence eliminating the dispute with respect to this concept. Distinctions and connections between our and other existing definitions are also provided. Our definition makes possible the comparison of loss aversion between different individuals, subject to certain assumptions.

The paper is structured as follows. First we provide the background framework for this conception. Tversky and Kahneman's latest version of prospect theory has been chosen. The next section defines loss aversion and compares different existing definitions for this concept. Implications for the regular forms of utility functions are discussed in section 3. Section 4 and 5 try to elaborate our definition from Yaari's sense and investment strategy. Conclusion follows in the last section.

1 Cumulative Prospect Theory

Outcomes are monetary. The reference point is the status quo. We may assume 0 as the reference point through rescaling the outcomes. A prospect, denoted by $P = (p_1, x_1; \cdots; p_n, x_n)$, assigns probability p_i for any possible outcome x_i, where $p_i > 0$ and $\sum_{i=1}^{n} p_i = 1$. The outcomes are also ordered $x_1 \geq x_2 \geq \cdots x_k \geq 0 > x_{k+1} \geq \cdots \geq x_n$. The preferences of an agent over some prospects are denoted by $\succeq$, with indifference by $\sim$.

Upon these notations, EU holds if the prospect is evaluated according to:

$$EU(P) = \sum_{i=1}^{n} p_i U(x_i)$$

where U is the utility function.

However, for prospect theory to hold, the prospect P would be evaluated respectively for gain part and loss part. Formally, an evaluation function PT represents preference, where PT is defined as follows:

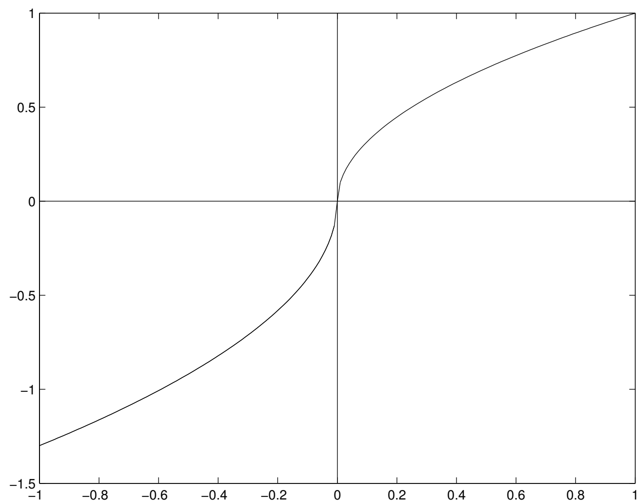
1. There exists a utility function $U : \Re \rightarrow \Re$, which is continuous and strictly increasing with $U(0) = 0$.

2. There exist weighting functions $w^{+(-)} : [0, 1] \rightarrow [0, 1]$, which are continuous and strictly increasing with $w^{+(-)}(0) = 0$ and $w^{+(-)}(1) = 1$.

3. $PT(p_1, x_1; \cdots; p_n, x_n) = \sum_{i=1}^{n} \pi_i U(x_i)$, with $\pi_i = w^+(p_1 + \cdots + p_i) - w^+(p_1 + \cdots + p_{i-1})$ for $i \leq k$ and $\pi_i = w^-(p_i + \cdots + p_n) - w^-(p_{i+1} + \cdots + p_n)$ for $i > k$. In particular $\pi_1 = w^+(p_1)$ if $k \geq 1$ and $\pi_n = w^-(p_n)$ if $k < n$.

For most of the cases, we assume the utility is concave for gains and convex for losses. Although in some literatures it is suggested that the utility function is concave for losses, most studies agree on that the marginal utility should also be decreasing for losses. The sketch of the utility is provided below:

Figure 1: Utility Function in Cumulative Prospect Theory



For the special case when $w^+(p) = 1 - w^-(1-p)$ for any $0 \le p \le 1$, the cumulative prospect theory will degenerate into rank-dependent theory if the utility is not defined over changes in wealth. Tversky and Kahneman choose the expressions for the weighting functions $w^+(p) = \frac{p^\gamma}{(p^\gamma+(1-p)^\gamma)^{1/\gamma}}$ and $w^-(p) = \frac{p^\delta}{(p^\delta+(1-p)^\delta)^{1/\delta}}$. The estimated parameters values are $\gamma = 0.61$ and $\delta = 0.69$. The resulting weighting functions exhibit the following characteristics:

1. regressive: intersecting the diagonal from above which means $w(p) > p$ first and then $w(p) < p$.

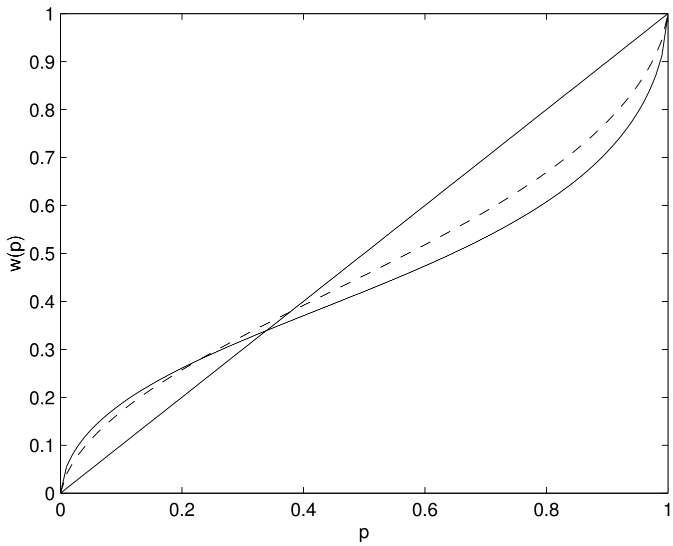
2. asymmetric: with fixed point around 1/3,

3. s-shaped: concave on an initial interval and convex beyond that.

Later, Prelec (1998) suggests an alternative form for the weighting function $w(p) = \exp(-(-\ln p)^\alpha)$, $0 < \alpha < 1$. His construction stems from the subproportional requirement for common-ratio effect.[2] He points out the previous

[2]Common-ratio effect in Tversky and Kahneman (1979): if (x, p) is equivalent to (y, pq) then (x, pr) is not preferred to (y, pqr), $0 < p, q, r \le 1$. It holds if and only if $\ln w(p)$ is a convex function of $\ln p$.

Figure 2: Weighting Functions in Cumulative Prospect Theory



expressions for the weighting functions fail to satisfy this requirement for small probabilities.

We also have a continuous version for cumulative prospect theory:

$$PT(P) = \int_{-\infty}^{0} U(x) \frac{d}{dx}(w^{-}(F(x)))dx + \int_{0}^{+\infty} U(x) \frac{d}{dx}(-w^{+}(1 - F(x)))dx$$

where $F(x) = \int_{-\infty}^{x} dp$ is the cumulative probability.

2 Concept of Loss Aversion

Loss aversion, which is different from risk aversion and actually a component of that in most cases, implies there must be some (potential) losses associated. Therefore, it is expected to reflect the comparisons between two sides of the status quo. In the original prospect theory, Tversky and Kahneman describe loss aversion in the language of preference that individuals dislike symmetric $50 - 50$ bets and the averseness of symmetric fair bets generally increases with the size of the stake. Some others, equalize the definition of loss aversion as the requirement that utility function being steeper for losses than for gains. We first clarify the word "steeper" appeared in their definition, for which there are generally two interpretations.

Definition 1 *A utility function is said to be steeper*[3] *for losses than for gains if* $U(x) - U(y) < U(-y) - U(-x)$ *for all* $x > y \geq 0$.

An alternative expression is:

Definition 2 *A utility function is said to be steeper for losses than for gains if* $U'(x) < U'(-x)$ *for all* $x > 0$.

Seemly, these two definitions are not exactly the same. We will clarify the relationship between them:

1. If $U'(x) < U'(-x)$, then $\int_y^x U'(t)dt < \int_{-x}^{-y} U'(t)dt$ for any $x > y \geq 0$. Hence we have $U(x) - U(y) < U(-y) - U(-x)$.

2. If $U(x)-U(y) < U(-y)-U(-x)$ for all $x > y \geq 0$, then $\lim_{y \to x} \frac{U(x)-U(y)}{x-y} \leq \lim_{(-y) \to (-x)} \frac{U(-y)-U(-x)}{-y-(-x)}$. Hence $U'(x) \leq U'(-x)$. Actually we have $U'(x) < U(-x)$ almost surely for $x > 0$. If not, we integral the inequality for a small area with non-zero length and will contradict with $U(x) - U(y) < U(-y) - U(-x)$.

The second definition therefore is a little bit stronger than the first. Actually for general cases, these two criteria are equivalent.[4] Counter-examples only exist within the realm of mathematicians.

The definition of loss aversion by Tversky and Kahneman in the language of preference is presented below:

Definition 3 *Loss aversion holds if the prospect* $(0.5, y; 0.5, -y)$ *is always preferred to* $(0.5, x; 0.5, -x)$ *for all* $x > y \geq 0$.

[3]Here steeper actually refers to strictly steeper

[4]Hereafter, we simply take the second definition for "steeper".

This statement, which can be decomposed into two aspects of the requirements, is initially formalized by Schmidt and Zank (2005). In the original prospect theory, it has been proved that this definition is equivalent to the fact that the utility function is steeper for losses than for gains,[5] such that we have a kink at the reference point. In the framework of cumulative prospect theory, the conclusion will change as well.

Theorem 1 *In cumulative prospect theory, loss aversion is satisfied if and only if for all $x > 0$ it holds that*

$$\frac{U'(-x)}{U'(x)} > \frac{w^+(0.5)}{w^-(0.5)}$$

We start from the comparisons between two prospects $P = (0.5, x; 0.5, -x)$ and $Q = (0.5, y; 0.5, -y)$ with $x > y \geq 0$. According to its definition in preference language $PT(P) < PT(Q)$, we have $U(x)w^+(0.5) + U(-x)w^-(0.5) < U(y)w^+(0.5) + U(-y)w^-(0.5)$, i.e. $\frac{U(-y)-U(-x)}{U(x)-U(y)} > \frac{w^+(0.5)}{w^-(0.5)}$. This could further be simplified as $\frac{U'(-x)}{U'(x)} > \frac{w^+(0.5)}{w^-(0.5)}$ due to the equivalence of $\frac{U(-y)-U(-x)}{U(x)-U(y)}$ and $\frac{U'(-x)}{U'(x)}$. The advantage of this simplification is that now we have an unary criterion for this concept, which is much easier to check. According to the estimates by Tversky and Kahneman (1992), we have $\frac{w^+(0.5)}{w^-(0.5)} = 0.927$.[6] This is striking because we actually do not need to impose that $U(x)$ is steeper for losses than for gains to guarantee loss aversion in cumulative prospect theory setting.

Definition 4 $\lambda(x) = \frac{U'(-x)}{U'(x)}$ *(where $x > 0$) is defined as the index of loss aversion.*

The index of loss aversion, basically a function, is defined as a ratio of the steepness between two sides. It measures the comparison between symmetric locations $-x$ and x. Based on this, the above theorem can be rewritten as:

[5]We do not allow different weighting functions in original prospect theory.

[6]In the figure for the weighting functions, we notice the real line is below the dashed line at $p = 0.5$.

Proposition 1 *In cumulative prospect theory, loss aversion is satisfied if and only if for all $x > 0$ it holds that*

$$\lambda(x) > \frac{w^+(0.5)}{w^-(0.5)}$$

In cumulative prospect theory, loss aversion is embodied in the interaction between utility functions and weighting functions. One of the purposes of allowing two different weighting functions is to allow more degrees of flexibility in the model, hence better capturing the data. In Tversky and Kahneman (1992), they assume specific functional forms for utilities and probability weightings, and estimate the parameters according to their survey results. If the weighting functions for gains and losses were the same or at least they valued the same for 0.5, we would have a more neat result.

Proposition 2 *In cumulative prospect theory with the same weighting function, loss aversion is satisfied if and only if for all $x > 0$ it holds that*

$$\lambda(x) > 1$$

This conclusion just meets someone's expectation. What's more important? In these two propositions, we succeed in equalizing loss aversion as a condition on $\lambda(x)$. This partly explains why we call it index of loss aversion. Take an analogy in EU theory, risk averse is valid only when absolute risk aversion is uniformly greater than 0.

An alternative definition of the index of loss aversion is raised by Kobberling and Wakker(2005). They suggest the observed utility U is a composition of a loss aversion index $\lambda > 0$, reflecting the different processing of gains and losses, and the basic utility u.[7] Formally,

$$U(x) = \begin{cases} u(x) & \text{if } x \geq 0 \\ \lambda u(x) & \text{if } x < 0 \end{cases}$$

[7] u is hypothetic and not necessarily to be an odd function.

In their supposition, this basic utility function u is differentiable everywhere, including the reference point. Based on the setting, the index of loss aversion can be calculated as $\lambda = -\frac{U(x_2)}{U(x_1)}$ where $u(x_1) = -u(x_2)$ for any $x_2 < 0 < x_1$. Later in their formal definition, they extract the index of loss aversion as $\lambda = \frac{U'_\uparrow(0)}{U'_\downarrow(0)}$. However, they do not provide the intermedial steps for this definition. We try filling this gap through offering an intuitive derivation:

$$\lim_{\epsilon \to 0^+} -u(-\epsilon) = \lim_{\epsilon \to 0^+} u(\epsilon)$$

where $\epsilon > 0$,

$$\begin{aligned}
\lambda &= \lim_{\epsilon \to 0^+} -\frac{U(-\epsilon)}{U(\epsilon)} \\
&= \lim_{\epsilon \to 0^+} \frac{(U(-\epsilon)-U(0))/(-\epsilon-0)}{(U(\epsilon)-U(0))/(\epsilon-0)} \\
&= \lim_{\epsilon_1 \to 0^+, \epsilon_2 \to 0^+} \frac{U'(-\epsilon_1)}{U'(\epsilon_2)} \\
&= \frac{U'_\uparrow(0)}{U'_\downarrow(0)}
\end{aligned}$$

They define that loss aversion is achieved once $\lambda > 1$. Their conclusion has nothing with the weighting functions, therefore can not explain the effect of weighting functions for loss aversion. Besides, loss aversion is supposed to be a holistic phenomenon rather than a local one around the kink. Their so-called "loss aversion" is defined around the origin, therefore only implies local loss aversion. Their definition is meaningful only when strong assumption about the utility functions is assumed, which guarantees the property around 0 can represent that of the whole domain.

Besides the above referred definition for loss aversion through the index, Neilson (2002) characterizes a condition for loss aversion, requiring that $U(x)/x > U(y)/y$ for all $x < 0 < y$. Bowman et al. (1999) and Breiter et al. (2001) impose a stronger condition that $U'(x) > U'(y)$ for all $x < 0 < y$. We plan to compare these conditions listed below:

1. $U(x)/x > U(y)/y$ for all $x < 0 < y$.

2. $U'(x) > U'(y)$ for all $x < 0 < y$.

3. $U'(x) > U'(-x)$ for all $x < 0$.

4. $-U(x) > U(-x)$ for all $x < 0$.

As no weighting functions get involved, we may restrict our comparisons within original prospect theory framework:

(I): $2 \Longrightarrow 3$. This is evident as long as we substitute $-x$ for y in 2.

(II): $2 \Longrightarrow 1$.

$$\frac{U(x)}{x} = \frac{U(x) - U(0)}{x - 0} = U'(\xi)$$

where $\xi \in (x, 0)$. Similarly

$$\frac{U(y)}{y} = \frac{U(y) - U(0)}{y - 0} = U'(\zeta)$$

where $\zeta \in (0, y)$. According to $U'(x) > U'(y)$ for all $x < 0 < y$, $U'(\xi) > U'(\zeta)$. This implies $U(x)/x > U(y)/y$.

(III): $1 \Longrightarrow 4$. This is evident as long as we substitute $-x$ for y in 1.

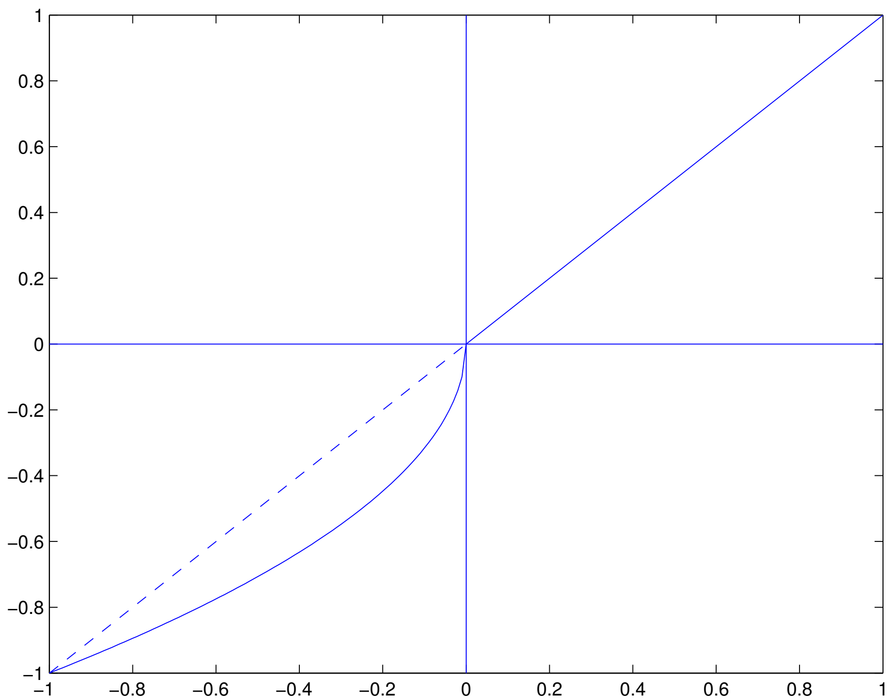
(IV): $3 \Longrightarrow 4$.

$$\begin{aligned}
& \textstyle\int_x^0 (U'(t) - U'(-t))dt > 0 \\
\Longrightarrow \quad & \textstyle\int_x^0 U'(t)dt > \int_x^0 U'(-t)dt \\
\Longrightarrow \quad & \textstyle\int_x^0 U'(t)dt > \int_{-x}^0 U'(s)d(-s) \\
\Longrightarrow \quad & \textstyle\int_x^0 U'(t)dt > \int_0^{-x} U'(s)ds \\
\Longrightarrow \quad & U(0) - U(x) > U(-x) - U(0) \\
\Longrightarrow \quad & -U(x) > U(-x) \text{ for all } x < 0
\end{aligned}$$

(V): However, 1 and 3 are not comparable. Counter examples which satisfy one and violate the other can be easily found (in the following figures).

In figure 3, the average utility for losses is everywhere greater than that for gains. But the slope of the utility for losses varies a lot, sometimes steep and sometimes flat. In figure 4, the marginal utility for losses is greater than that of the symmetric point for gains. However, if we take the dashed line as a standard, the average utility for gains sometimes exceeds that for losses.

Figure 3: An Example Which Satisfies Definition 1 but Violates Definition 3



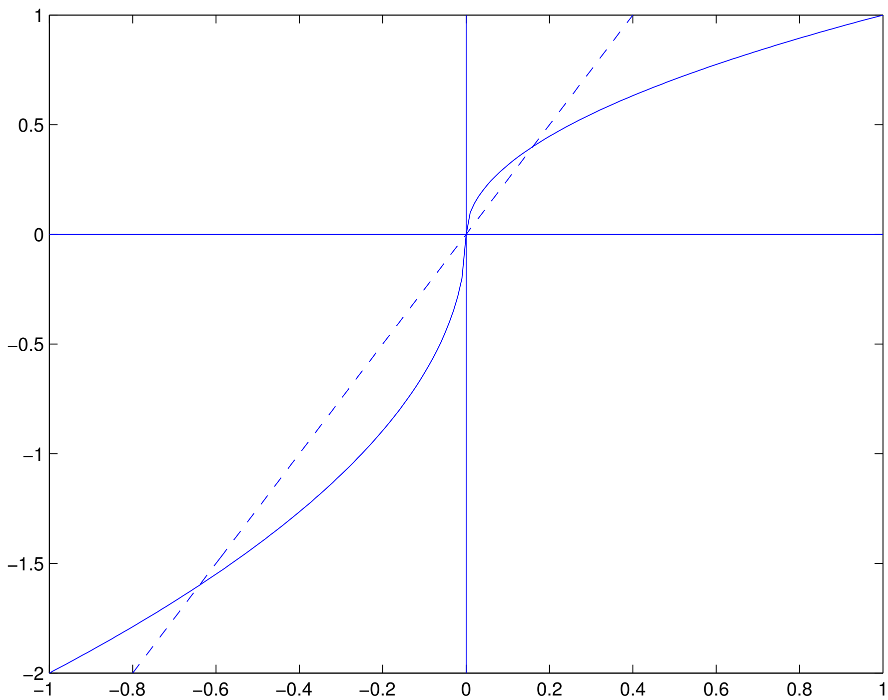
To summarize, 2 is the strongest definition and 4 is the weakest. 1 and 3 are moderate definitions and can not be compared. Neilson (2002) proves if utility function is s-shaped, definition 1 and 2 are equivalent. Actually, we think definition 2 is a little bit too strong because euphoria from gains and pains for losses are both marginal decreasing. The first dollar gain will probably bring you more pleasure than the additional pain caused by the n^{th} dollar loss.

Schmidt and Zank (2005) raise strong loss aversion in comparison with loss aversion and suggest strong loss aversion may be the most appropriate definition of loss aversion.

Definition 5 *Strong loss aversion holds if the prospect*

$$(p_1, z_1; \cdots; p_{i-1}, z_{i-1}; \alpha, y; p_{i+1}, z_{i+1}; \cdots; p_{j-1}, z_{j-1}; \alpha, -y; p_{j+1}, z_{j+1}; \cdots; p_n, z_n)$$

Figure 4: An Example Which Satisfies Definition 3 but Violates Definition 1



is always preferred to

$(p_1, z_1; \cdots; p_{i-1}, z_{i-1}; \alpha, x; p_{i+1}, z_{i+1}; \cdots; p_{j-1}, z_{j-1}; \alpha, -x; p_{j+1}, z_{j+1}; \cdots; p_n, z_n)$

for all $x > y \geq 0$ *and* $0 < \alpha \leq 0.5$.

The prospects here differ from the above ones in that they no longer require the middle payoff to be equal to the status quo, and can be any common outcome. Strong loss aversion means among two lotteries for which one can win or lose a given amount with equal probability, that lottery will be preferred for which this amount is smaller. This is apparently a model-independent behavior concept, for which loss aversion is a special case when $\alpha = 0.5$.

Theorem 2 *In cumulative prospect theory, strong loss aversion is satisfied if* *and only if for all $x > 0$ and $p, q \geq 0$ that $p + q \leq 1$ it holds that*

$$\lambda(x) > \frac{(w^+)'(p)}{(w^-)'(q)}$$

We start from comparing two prospects and get the equivalent condition that for all $x > y \geq 0$, all $0 < \alpha \leq 0.5$ and all $\gamma, \delta \geq 0$ with $1 - 2\alpha \geq \gamma + \delta$ it holds that

$$\frac{U(-y) - U(-x)}{U(x) - U(y)} > \frac{w^+(\gamma + \alpha) - w^+(\gamma)}{w^-(\delta + \alpha) - w^-(\delta)}$$

The left hand side of this inequality can be simplified as $\frac{U'(-x)}{U(x)}$, and the right hand side can be summarized as $\frac{(w^+)'(p)}{(w^-)'(q)}$ where p, q need to satisfy some conditions. Therefore, strong loss aversion means

$$\lambda(x) > \frac{(w^+)'(p)}{(w^-)'(q)}$$

for all $x > 0$, $0 \leq p, q$ and $p + q \leq 1$. It is fansinating that strong loss aversion can also be defined through the index of loss aversion.

If strong loss aversion holds under cumulative prospect theory, for the special case $0 < p = q < 0.5$, we have

$$(w^+)'(p) < (w^-)'(p) * \lambda(x)$$

Do the integration from 0 to 0.5,

$$\int_0^{0.5} (w^+)'(p)dp < \int_0^{0.5} (w^-)'(p)dp * \lambda(x)$$

i.e.

$$w^+(0.5) < w^-(0.5) * \lambda(x)$$

This is exactly the condition for loss aversion to hold in cumulative prospect theory. Someone dispute that loss aversion should be replaced by strong loss aversion since strong loss aversion is more generous and has been partly verified by experimental study (Brooks and Zank (2004)).

3 Implications for the Regular Form of Utilities

We are interested in the cases with regular form of utilities. In academy, a certain number of researchers agree on constant relative risk aversion (CRRA hereafter). Referring to their suggestion, we assume

$$U(x) = \begin{cases} x^r & \text{if } x \geq 0 \\ -s(-x)^t & \text{if } x < 0 \end{cases}$$

where $0 < r, t < 1$. Actually this form of utility function is also taken by Tversky and Kahneman (1992). For $x > 0$

$$\lambda(x) = \frac{U'(-x)}{U'(x)} = \frac{st}{r} x^{t-r}$$

Disregarding the factor of weighting functions, loss aversion holds only when $\lambda(x) > 1$ for all $x > 0$. This actually requires $t = r$ [8] and $s > 1$.[9]

Another common preference assumption is constant absolute risk aversion (CARA hereafter),

$$U(x) = \begin{cases} \frac{1-e^{-\mu x}}{\mu} & \text{if } x \geq 0 \\ s(\frac{e^{\nu x}-1}{\nu}) & \text{if } x < 0 \end{cases}$$

where $\mu, \nu > 0$. Then for $x > 0$

$$\lambda(x) = \frac{U'(-x)}{U'(x)} = se^{(\mu-\nu)x}$$

Not considering the distinction of weighting functions, loss aversion is achieved as long as $\mu \geq \nu$ [10] and $s > 1$.

To step further, we will introduce a general group: HARA utility

$$U(x) = \begin{cases} \frac{(x+a)^{1-\gamma}}{1-\gamma} - \frac{a^{1-\gamma}}{1-\gamma} & \text{if } x \geq 0 \\ s(\frac{b^{1-\beta}}{1-\beta} - \frac{(-x+b)^{1-\beta}}{1-\beta}) & \text{if } x < 0 \end{cases}$$

[8] If $t > r$, as long as x is small enough, $\lambda(x) < 1$; if $t < r$, as long as x is large enough, $\lambda(x) < 1$.

[9] if weighting functions are considered, we need $s > \frac{w^+(0.5)}{w^-(0.5)}$. This applies in the following two cases as well.

[10] when $\mu < \nu$, we always have sufficient large x to make $\lambda(x) < 1$ no matter how large s is.

where $a, b > 0$ and $0 < \gamma, \beta < 1$. Then for $x > 0$

$$\lambda(x) = \frac{U'(-x)}{U'(x)} = s\frac{(x+a)^{\gamma}}{(x+b)^{\beta}}$$

This is composed of three different situations:

1. First case is when $a = b$: we need $s\frac{(x+a)^{\gamma}}{(x+b)^{\beta}} = s(x+a)^{\gamma-\beta} > 1$. Obviously $\gamma \geq \beta$ is necessary. Based on this we further have $s > a^{\beta-\gamma}$.

2. Second case is when $a > b$: to guarantee $\lim_{x\to\infty} \lambda(x) > 1$, we need $\gamma \geq \beta$. We further consider the monotony of $\frac{(x+a)^{\gamma}}{(x+b)^{\beta}}$. If $\gamma b - \beta a \geq 0$, it is always increasing in the domain $x > 0$. We only need $s > \frac{b^{\beta}}{a^{\gamma}}$ to ensure loss aversion. Else if $\gamma b - \beta a < 0$, $\frac{(x+a)^{\gamma}}{(x+b)^{\beta}}$ is first decreasing and then increasing in the area $x > 0$ with the minimum value achieved at $x_0 = \frac{\beta a - \gamma b}{\gamma - \beta}$. As long as we ensure $s > \frac{(x_0+b)^{\beta}}{(x_0+a)^{\gamma}}$ loss aversion is valid.

3. The last case is when $a < b$: the necessary condition is $\gamma \geq \beta$. We simply know $\frac{(x+a)^{\gamma}}{(x+b)^{\beta}}$ is always increasing in the positive domain. Therefore loss aversion can be ensured as long as $s > \frac{b^{\beta}}{a^{\gamma}}$.

The discussion about the compatibility of the regular form of utilities with our definition of loss aversion in prospect theory will hopefully shed some light on the future empirical work in this area. If you are based on our framework and assume regular form of utilities, some basic constraints about the parameters have to be satisfied.

4 Comparative Loss Aversion in Yaari's Sense

Another advantage that we define the index of loss aversion is we can easily compare different individuals' loss aversion. We use Yaari's acceptance sets (1969) as a supplementary tool to help explain comparative loss aversion. Different from Kobberling and Wakker's definition where index of loss aversion is only a number, here the index is a function of the location. Unless we make some assumptions for the utility functions and the weighting functions, it is impossible to use the acceptance sets to compare them.

Assume two agent, 1 and 2, whose preferences over L, denoted by $\succeq_1$ and $\succeq_2$, can be modeled by prospect theory with utility functions U_1 and U_2, loss aversion indices λ_1 and λ_2 and weighting functions w_1^+, w_1^-, w_2^+, w_2^-, respectively. We assume the reference point of each agent by 0, where this can refer to different absolute levels of wealth for the two agents.

L^+ denotes the set of prospects with no loss result, and L^- denotes the opposite. A prospect is said to be mixed if is neither contained in L^+ nor in L^-, so that it yields both gains and losses with positive probability. We are interested in mixed prospects because we always believe loss aversion should be a concept describing the comparison between two sides of the reference point. For an outcome x, we define $A_1(x) = \{P \in L | P \succeq_1 x\}$ to be agent 1's acceptance set, i.e. the set of prospects that agent 1 prefers to a fixed outcome x. The gain acceptance set $A_1^+(x) = A_1(x) \cap L^+$ restricts the set to prospects with no possible losses. Analogically the loss acceptance set is defined as $A_1^-(x) = A_1(x) \cap L^-$. Apparently for $x > 0$, $A_1^+(-x) = L^+$ and $A_1^-(x) = \emptyset$. Agent 2's $A_2^+(x)$, $A_2^-(x)$ and $A_2(x)$ could be similarly defined.

Theorem 3 *Assume that the preferences of agents 1 and 2 can be modeled through cumulative prospect theory and* $W_1^- = W_2^-$*, then the following statement (i) will imply (ii).*

(i) The following three conditions hold:

(a) $W_1^+ = W_2^+$*;*

(b) $U_1(x) = \sigma U_2(x)$ *for all* $x > 0$*;*

(c) $\lambda_1(x) \leq \lambda_2(x)$ *for all* $x > 0$*.*

(ii) The following two conditions hold:

(a) $A_1^+(x) = A_2^+(x)$ *for all* $x > 0$*;*

(b) $A_1(x) \supset A_2(x)$ *for all* $x > 0$*.*

We introduce these two groups of conditions first. Group (i) consists of three items, all of which are from the view of prospect theory. The first condition means the weighting functions for gains coincide across individuals. The second implies there only exists a multiple difference for the gain part utilities. The last one says the index of loss aversion for agent 1 does not exceed that for agent 2. The conditions in group (ii) are written in the language of acceptance sets. The first item means the gain acceptance sets are coincident across two agents.[11] The other one requires the acceptance set of agent 1 w.r.t. a positive outcome is greater than that of agent 2. We should not neglect the premise ahead that weighting functions for losses are assumed the same across two agents. In Kobberling and Wakker (2005), risk attitude is decomposed into three distinct components: basic utility , probability weighting and loss aversion. That's why they restrict the basic utilities and probability weightings to be the same for both agents. In our definition, risk attitude is composed of utility function for gains, probability weighting and index of loss aversion. Statement (i) exactly reflects this decomposition. Proof is provided as follows:

(i)→(ii): Let (i.a-c) hold. We have to show that (ii.a-b) hold, of which only (ii.b) is explained, because (ii.a) is trivial. Assume $x > 0$ and $P \in A_2(x)$, i.e. $PT_2(P) \geq U_2(x)$. Suppose $P = (p_1, x_1; \cdots; p_n, x_n)$ with $x_1 \geq \cdots \geq x_k \geq 0 > x_{k+1} \geq \cdots \geq x_n$ where $1 \leq k \leq n$. Define $P^+ = (p_1, x_1; \cdots; p_k, x_k; 1 - p_1 - \cdots - p_k, 0)$ and $P^- = (1 - p_{k+1} - \cdots - p_n, 0; p_{k+1}, x_{k+1}; \cdots; p_n, x_n)$. We may, and will, assume $\sigma = 1$ which does not affect the result. $\lambda_2(x) \geq \lambda_1(x)$ for $x > 0$, i.e. $U_2'(-x) \geq U_1'(-x)$. We further have $U_2(-x) \leq U_1(-x)$. $PT_1(P) = PT_1(P^+) + PT_1(P^-) \geq PT_2(P^+) + PT_2(P^-) = PT_2(P) \geq U_2(x) = U_1(x)$. Hence $P \in A_1(x)$ and therefore $A_2(x) \subset A_1(x)$.

However, we could not infer (i) from (ii). According to the standard unique-

[11] $A_1^+(x) = A_2^+(x) = L^+$ for all $x < 0$ naturally holds.

ness by Wakker (1994), we could have (i.a) and $U_2(x) = \alpha U_1(x) + \beta$ for $x > 0$. $U_1(0) = U_2(0) = 0$ implies $\beta = 0$. We could assume $\alpha = 1$, i.e. $\sigma = 1$. Through constructing the prospect $P = (p, y; 1-p, -x)$ where $x > 0$, $y > 0$ and y is flexible, we could always have $PT_2(P) = U_2(t)$ where $t > 0$. According to (ii.b), $PT_1(P) \geq U_1(t)$, which implies $-U_2(-x) \geq -U_1(-x)$. However this is a weaker condition than $\lambda_2(x) \geq \lambda_1(x)$ for $x > 0$.

If we replace (i.c) by $-U_2(-x)/U_2(x) \geq -U_1(-x)/U_1(x)$ for all $x > 0$, we can say (i) and (ii) are equivalent. However, we reserve our agreement that $-U(-x)/U(x)$[12] can be used as an index for loss aversion. Although this definition satisfies individuals dislike symmetric 50-50 bets, it can not ensure averseness of symmetric fair bets increases with the size of the stake.

In the above theorem, we did not compare the situations for $x < 0$. The feasibility of comparison for $x < 0$ hinges on the assumption that utilities are coincident in the domain $x < 0$. The conjugated version of the above theorem is provided below:

Theorem 4 *Assume that the preferences of agents 1 and 2 can be modeled through cumulative prospect theory and $W_1^+ = W_2^+$, then the following statement (i) will imply (ii).*

(i) The following three conditions hold:

(a) $W_1^- = W_2^-$;

(b) $U_1(-x) = \sigma U_2(-x)$ for all $x > 0$;

(c) $\lambda_1(x) \leq \lambda_2(x)$ for all $x > 0$.

(ii) The following two conditions hold:

(a) $A_1^-(-x) = A_2^-(-x)$ for all $x > 0$;

(b) $A_1(-x) \supset A_2(-x)$ for all $x > 0$.

These two results can be summarized as that the acceptance set of one agent

[12]This is actually the definition 4 above for loss aversion.

who is comparatively more loss averse is included in the acceptance set of the other. As we elaborate earlier loss aversion is a property reflecting the difference of referent point's two sides, only mixed prospects can embody this difference. Initially, we expect to find an equivalent condition for comparative loss aversion. However, the above results fail to do so.

Kobberling and Wakker (2005) also use Yaari's definition to compare loss aversion. They get equivalent conditions because their definition for loss aversion is only a number and they assume homogeneous basic utility across the whole domain. Whereas the advantage of our theories lies in that we do not impose identical acceptance sets over the pure gain and pure loss domain: we leave one-side flexibility to the individuals. As they summarize, in EU theory, Yaari's condition characterizes concave transformations of utility while their restriction characterizes a special concave transformation of the utility: with a kink at zero and linear anywhere else. In our theories, we assume linear transformation for only one side, a kink at zero, and irregular transformation which has to meet some conditions for the other side. Compared to Kobberling and Wakker's definition, ours provides a more general framework.

If we play one trick in Theorem 3, we can eventually get equivalent conditions as well. Suppose for some specific utilities, we can conclude $U_2'(-x) \geq U_1'(-x)$ from $-U_2(-x) \geq -U_1(-x)$ where $x > 0$. This means $\lambda_1(x) \leq \lambda_2(x)$ is acting as $-U_2(-x)/U_2(x) \geq -U_1(-x)/U_1(x)$ where $x > 0$. Then we will have a new version for theorem 3.

Theorem 5 *Assume that the preferences of agents 1 and 2 can be modeled through cumulative prospect theory with CRRA utilities, and* $W_1^- = W_2^-$*, then the following statements (i) and (ii) are equivalent.*

(i) The following three conditions hold:

(a) $W_1^+ = W_2^+$*;*

(b) $U_1(x) = \sigma U_2(x)$ *for all* $x > 0$*;*

(c) $\lambda_1(x) \le \lambda_2(x)$ *for all* $x > 0$.

(ii) The following two conditions hold:

(a) $A_1^+(x) = A_2^+(x)$ *for all* $x > 0$;

(b) $A_1(x) \supset A_2(x)$ *for all* $x > 0$.

Take CRRA case as an example: we start from $s_1 x^{t_1} \le s_2 x^{t_2}$ for all $x > 0$. Cases for large x guarantee that $t_1 \le t_2$. Cases for small x ensure that $t_1 \ge t_2$. Therefore $t_1 = t_2$. Based on that, we further conclude $s_1 \le s_2$. Therefore, $s_1 t_1 x^{t_1 - 1} \le s_2 t_2 x^{t_2 - 1}$ for all $x > 0$, which is exactly $\lambda_1(x) \le \lambda_2(x)$ for all $x > 0$. We say condition (i.c) equals to the condition $-s_1(-x)^{t_1} \ge -s_2(-x)^{t_2}$ for all $x < 0$, which combined with (i.a) and (i.b) are proved to be equivalent to (ii).

However, for CARA case: $s_1 \frac{e^{\gamma_1 x} - 1}{\gamma_1} \ge s_2 \frac{e^{\gamma_2 x} - 1}{\gamma_2}$ for all $x < 0$. Cases for small $-x$ ensure $s_1 \le s_2$. Cases for big $-x$ guarantee that $\frac{\gamma_1}{\gamma_2} \ge \frac{s_1}{s_2}$. However, these two conditions are not sufficient to guarantee $\lambda_1(x) \le \lambda_2(x)$ for all $x > 0$. If we let $s_1 = 0.8, s_2 = 0.9, \gamma_1 = 0.45, \gamma_2 = 0.5$, it satisfies $s_1 \frac{e^{\gamma_1 x} - 1}{\gamma_1} \ge s_2 \frac{e^{\gamma_2 x} - 1}{\gamma_2}$ for all $x < 0$ but violates $s_1 e^{\gamma_1 x} \le s_2 e^{\gamma_2 x}$ for some $x < 0$. Therefore $U_2'(-x) \ge U_1'(-x)$ and $-U_2(-x) \ge -U_1(-x)$ where $x > 0$ are not equivalent in CARA case.

Actually, it's quite common that these two conditions are not equivalent. In the particular CRRA case, the index of loss aversion is actually a number, which makes possible the equivalence of two conditions. A conjugated version of necessary and sufficient condition, where utilities functions are coincident in $x < 0$, is presented below.

Theorem 6 *Assume that the preferences of agents 1 and 2 can be modeled through cumulative prospect theory with CRRA utilities, and* $W_1^+ = W_2^+$, *then the following statements (i) and (ii) are equivalent.*

(i) The following three conditions hold:

(a) $W_1^- = W_2^-$;

(b) $U_1(-x) = \sigma U_2(-x)$ *for all* $x > 0$*;*

(c) $\lambda_1(x) \leq \lambda_2(x)$ *for all* $x > 0$*.*

(ii) The following two conditions hold:

(a) $A_1^-(-x) = A_2^-(-x)$ *for all* $x > 0$*;*

(b) $A_1(-x) \supset A_2(-x)$ *for all* $x > 0$*.*

5 Comparative Loss Aversion in Asset Demand

This section is specially in comparison with the concept comparative risk aversion in EU theory. As demonstrated by Pratt (1964), more risk aversion implies more demand for asset in a simple investment strategy problem in which an agent is supposed to have initial wealth w, and can invest in only two assets: risk-free asset with zero return and risky asset with excess return $\tilde{x}$ where $E(\tilde{x}) \geq 0$. In original EU framework, the maximum problem is

$$\max_a E(u(w + a\tilde{x}))$$

However in prospect theory, because only gains and losses matter, the new maximum problem is

$$\max_a E(u(a\tilde{x}))$$

In original EU framework, the problem is easy to handle due to global concavity of u, which implies the solution to the first order condition is the optimal investment level in risky asset. However, in prospect theory, u is assumed to be concave for gains but convex for losses. That's why we have to make the following assumption:

Assumption 1 *When* $E(\tilde{x}) \geq 0$*,* $E(\tilde{x}u'(a\tilde{x}))$ *is strictly decreasing in* a*.*

This assumption which indicates the first order derivative is decreasing can ensure the solution to the first order condition is the optimal one. Theorem for comparative loss aversion in asset demand is presented below:

Theorem 7 *Assume that the preferences of agents 1 and 2 can be modeled through cumulative prospect theory with* $W_1 = W_2$ *and* $U_1(x) = \sigma U_2(x)$ *for all* $x > 0$*, and the above assumption is valid for both agents, then the following statements (i) and (ii) are equivalent.*
(i) $\lambda_1(x) \leq \lambda_2(x)$ *for all* $x > 0$.
(ii) $a_1 \geq a_2$

Suppose a_2 is the optimal investment in risky asset for agent 2. It means $E(\tilde{x}u_2'(a_2\tilde{x})) = 0$.

$$
\begin{aligned}
& E(\tilde{x}u_1'(a_2\tilde{x})) \\
= \;& E(\tilde{x}u_1'(a_2\tilde{x})|\tilde{x} > 0)p(\tilde{x} > 0) + E(\tilde{x}u_1'(a_2\tilde{x})|\tilde{x} < 0)p(\tilde{x} < 0) \\
= \;& \sigma E(\tilde{x}u_2'(a_2\tilde{x})|\tilde{x} > 0)p(\tilde{x} > 0) + E(\tilde{x}\lambda_1(-a_2\tilde{x})u_1'(-a_2\tilde{x})|\tilde{x} < 0)p(\tilde{x} < 0) \\
\geq \;& \sigma E(\tilde{x}u_2'(a_2\tilde{x})|\tilde{x} > 0)p(\tilde{x} > 0) + E(\tilde{x}\lambda_2(-a_2\tilde{x})u_1'(-a_2\tilde{x})|\tilde{x} < 0)p(\tilde{x} < 0) \\
= \;& \sigma E(\tilde{x}u_2'(a_2\tilde{x})|\tilde{x} > 0)p(\tilde{x} > 0) + \sigma E(\tilde{x}\lambda_2(-a_2\tilde{x})u_2'(-a_2\tilde{x})|\tilde{x} < 0)p(\tilde{x} < 0) \\
= \;& \sigma(E(\tilde{x}u_2'(a_2\tilde{x}))) \\
= \;& 0
\end{aligned}
$$

Therefore we have $a_2 \leq a_1$. If $\lambda_1(x) > \lambda_2(x)$ for some $x > 0$, due to the continuity, we will have an interval of x satisfying $\lambda_1(x) > \lambda_2(x)$. Choose a binary distribution of $\tilde{x}$ and let $a_i\tilde{x}$ lies in this interval, we can easily have $a_1 < a_2$. This actually means we can infer (i) from (ii).

6 Conclusion

Stressing our view that loss aversion should be a concept reflecting gain/loss comparisons, we define the index of loss aversion as a function of the location. Even though the introduction of cumulative prospect theory would make the equivalent condition a little more complicated, it is still tractable if we start from Tversky and Kahneman's model-free preference definition. We equalize loss aversion by the requirement that index of loss aversion is always greater than a constant. Different versions of the definition are compared under the framework of original prospect theory so that we can see the differences and connections. We typically compare our definition of loss aversion index with that of Kobberling and Wakker's à la Yaari. The failure to obtain equivalent condition is discussed. Tricks have been applied to get necessary and sufficient conditions. Comparative loss aversion in asset demand is specially discussed in contrary to comparative risk aversion in EU theory. All the conclusions are within our expectation. We also attempt to relate our definition to several well-know groups of utilities and provide some useful tips for the future research.

In the area of empirical research, loss aversion, as an important aspect of prospect theory, begins to show its power. Especially combined with "mental accounting" and "narrow framing", it offers some alternative explanations for some empirical phenomena. What's more important, it relights our hope for solving some puzzles which have confused us for a long time. Some practitioners start bringing this weapon into finance market, such as stock market (Barberis and Huang in 2001), and make some breakthrough. We expect the explosive resolution of its application in the near future.

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