An alternative model to forecast default based on Black-Scholes-Merton model and a liquidity proxy

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Abstract

Building upon the theoretical Black-Scholes-Merton model, we develop an alternative model to forecast default. Without solving the required nonlinear equations, we deviate from Bharath and Shumway (2008) approach by estimating volatility in a simpler manner. Similar to Charitou and Trigeorgis (2006), we consider the probability of intermediate involuntary default before debt-maturity which we capture via a liquidity proxy. Finally, we use a weighted average life of total debt as time-to-option-maturity. Cox proportional hazard models and several approaches that test the model predictive ability suggest that our alternatives indicate higher sufficient statistic and ability to forecast default.

JEL codes: G33, G3, G0, M4

Key words: bankruptcy, option-pricing theory, intermediate default

1. Introduction

This study focuses on developing a simple and rational model to forecast default that has its basic intuition behind the standard option-pricing theory. Merton (1974, 1977) investigates bankruptcy based on the Black and Scholes (1973) option-pricing theory, the known as the Black-Scholes-Merton model [hereafter BSM-model]. According to the BSM-model, the equity of a levered firm can be viewed as a call option on the value of the firm assets *V*. When *V* falls below the face value of liabilities (B), $(V < B)$, the call option is left unexercised and the bankrupt firm is turned into its debt-holders. The firm voluntary defaults, since equity holders equity-holders (the residual claimants of firm value) have no positive pay-off to exercise the option $\{E = \max(V - B, 0)\}$.

The BSM-model assumes the option time-to-maturity equals the time-to-debt maturity. Hence, it ignores the possibility of involuntary default before maturity. Moody's-KMV model uses option-pricing theory on a discrete hazard model and recognizes that default may be triggered by firm inability to meet any scheduled payment before maturity [hereafter KMV-Merton]. They follow the algorithms of BSM-model to estimate firm value (V) and its volatility (σ_V) and account for the probability of intermediate default by adjusting the default boundary downward at a maturity based on their proprietary database and experience to (B=current liabilities + 0.5*long-term liabilities). They primarily focus on a distance-to-default measure, defined as the difference between the firm value and its default point (debt amount due), divided by the firm volatility. It defines the distance of firm value (V) from the default point (B), measured in units of firm standard deviation (σ_V). More specifically, how many standard deviations (the equity-holder call option is in-the-money) it takes for firm value to move down before it can trigger bankruptcy filing.

Bharath and Shumway (2008) [hereafter BS] follow KMV-Merton distance-to-default measure, without following the algorithms specified by the Merton distance-to-default (they do not solve the simultaneous nonlinear equations required by BSM-model, in order to assess the firm value and its volatility). Contrary, their "naive" probability is estimated by using market observable variables. Firm value is defined as the sum of firm value and face value of debt, whereas volatility is estimated on historic firm returns. They argue the BSM-model is not a sufficient statistic for forecasting bankruptcy. Their default measure appears to produce more sufficient statistic. Accepting this approach, the methodology for estimating volatility to apply the BSM-model should not be important.

Hence, our first hypothesis examines whether a simpler approach would still provide similar or more sufficient statistic. We calculate the value volatility following the BS approach $(\sigma_{V[BS]})$ and estimating the option variables using items directly observed by the market. We also apply a simpler approach $(\sigma_{V[DLCT]})$ again without solving the equations implied by the BSM-model.

However, instead of using the KMV distance-to-default, we use a more direct proxy that captures the information provided by the KMV-measure. Similar to Charitou and Trigeorgis (2006), we extend the theoretical BSM-model into a European compound call option (Geske, 1979) that triggers default when firm value falls below the default boundary. Such case would imply that the firm has insufficient cash flows to meet its intermediate interest and debt repayment instalments. Thus, it will involuntary default at a time before debt maturity. To capture the intermediate default probability, we use a cash-flow coverage ratio that covers firm inability to meet its intermediate interest and debt payments (we maintain the original default boundary, B=current + long term debt).

Existing studies argue that adding market variables along with accounting variables helps in improving forecast accuracy (Shumway, 2001; Hillegeist et al., 2004). Hence, the liquidity proxy derives from accounting measures and is included into the BSM-model as an option variable. On that basis, our second hypothesis examines the ability of our extended measure to forecast default. It captures the possibility of earlier default separately (via a transformation of the cash flow coverage), without solving the simultaneous equations of the BSM-model.

Finally, it is common for researchers to use a forecasting horizon of 1 year *(T=1)*. Nonetheless, this is a simplistic assumption as any default boundary used has time-to-maturity more than one year, which restrict the BSM-model assumptions (Hillegeist et al., 2004). According to this, our third hypothesis examines whether the estimation method of the time-to-option maturity has significant impact on the probability of default as employed by KMV-model. Beyond the *T=1*, we follow the approach of Charitou and Trigeorgis (2006) and use an additional alternative of *T* that equals the weighted average life of total debt.

We apply Cox proportional hazard models in a sample of 7,833 U.S. firms (1,269 firms that filed for bankruptcy and 6,564 available healthy firms). We first estimate their goodness of fit Cox partial -2LogLikelihood and rank the competitive models fit according to their AIC information criterion (Akaike, 1974). To examine their predictive ability and test if our extended measure has more or less predictive ability, we examine the area under the ROC curve (Vassalou and Xing, 2004; Agarwal and Taffler, 2008) and their predictive ability deriving from out-of-sample tests (Shumway, 2001; Bharath and Shumway, 2008). Our findings suggest that our methodology provides sufficient statistic which is similar or higher than that provided by BS and therefore, it slightly outperforms the information provided by the KMV-Merton model.

The remainder of the paper is organized as follows. Section two describes the theoretical framework of the standard option pricing theory of Black and Scholes (1973) and Merton (1974, 1977), the KMV-Merton model and the methodology employed by Bharath and Shumway (2008). It also explains the extended compound call option for intermediate default and liquidity proxy (Geske, 1979). Section three describes our research design, whereas section four discusses our empirical findings. The last section concludes.

2. Theoretical Framework

2.1. The BSM Option-Pricing Model of Business Default

Firm value V_t is assumed to follow a Geometric Brownian Motion $dV_t / V_t = (\alpha - D)dt + \sigma_V dz$, where α represents the total expected rate of return on firm value, D the total payout (% of V), σ_V the standard deviation of the firm value returns and dz an increment of a standard Wiener process. Merton (1974, 1977) suggests that any claim whose value is contingent on a traded asset with value *V*, having a payout *D* and time to maturity *T* must satisfy the fundamental partial differential equation (p.d.e.):

$$\sigma^2 V^2 F_W + (r - D)VF_V - F_T - \gamma F + d = 0 \tag{1}$$

where d is the payout from the firm to the particular contingent claim *F*. Solution to the p.d.e. is given by the Black-Scholes formula for a European call option, on a dividend-paying asset:

$$E(V,T) = Ve^{-DT} N(d_1) - Be^{-rT} N(d_2) \tag{2}$$

where $$d_1 = \frac{\ln(V/B) + (r - D + 0.5\sigma_V^2)T}{\sigma_V \sqrt{T}} \tag{3}$$

$$d_2 = d_1 - \sigma_V \sqrt{T} = \frac{\ln(V/B) + (r - D - 0.5\sigma_V^2)T}{\sigma_V \sqrt{T}} \tag{4}$$

where E = European call option (firm equity), B = face value (principal) of the debt, V = value of firm assets, D = total constant payout yield, σ_V = standard deviation of firm value changes (returns in V), T = time to debt maturity, r = risk-free interest rate, $N(d)$ = cumulative standard normal distribution function (from -∞ to d).

The term $VN(d)$ is the discounted expected value of the firm if it is solvent. Be^{-rT} is the present value of the principal debt *B*, and $N(d_2)$ is the risk-neutral probability the firm to be solvent at maturity *T*, $\Pr ob.(V_T < B)$. The risk-neutral probability of default at the debt maturity is given by: $\Pr.Default = \Pr ob(V_t < B) = 1 - N(-d2) = N(-d_2) = N\{-\frac{\ln(V/B) + (r - D - 0.5\sigma_V^2)T}{\sigma_V \sqrt{T}}\}$ (5)

The probability is driven by the five primary option-pricing variables as described in BSM-model formula (equation 3): $N(-d_2) = f\{\ln(V), \ln(B), \sigma_V, T, (r-D)\}$.

Most applications of BSM-model consider the value of the European option as a function of the four variables that are easily observable by the market (*V, B, T* and *r-D*) and, one variable that can be estimated (σ_V). However, in the Merton model, the value of the option is observed directly from the market as the firm equity *E*, while the value of firm value *V* and its volatility σ_V should be inferred. However, under the Merton assumptions, *E* is a function of the firm value and time, which follows Ito's lemma: $\sigma_E = (\frac{V}{E})\frac{\partial E}{\partial V}\sigma_V$. As in the BSM-model $\frac{\partial E}{\partial V} = N(d_1)$, the Merton model assumes the volatilities of the firm and equity are related by:

$$\sigma_E = (\frac{V}{E})N(d_1)\sigma_V \tag{6}$$

The equity volatility can be estimated by historical stock returns (observable at the marketplace similar to the rest four variables). Thus, equations (2) and (6) should be solved simultaneously, providing numerical values for all variables required to assess the distance-to-default (d_2) and default probability $(N(-d_2))$.

2.2. KMV-Merton Model

The method employed by Moody's corporation, the known KMV-Merton model, builds upon the BSM-model to allow for various classes and maturities of debt. The value of the

European option is directly observable by the market (the equity, E) while the underlying asset value *(V)* and its volatility *(σ_V)* are inferred. The KMV-Merton model estimates the σ_E from historical stock returns or option implied volatility data. It chooses a forecasting time horizon and measure the relative default boundary B_{KMV} (current + 0.5*long-term liabilities). After having values of the marketable variables they simultaneously solve equations (2) and (6) to estimate V and σ_V. Once this numerical solution is obtained, they calculate their distance-to-default measure *(DD)*:

$$DD = \frac{\ln(V/B_{KMV}) + (\mu - 0.5\sigma_V^2)T}{\sigma_V \sqrt{T}} \qquad (7)$$

Based on the BSM-model, the probability of default equals to $N(-DD)$, and thus to $N(-DD) = N\{-\frac{\ln(V/B) + (\mu - 0.5\sigma_V^2)T}{\sigma_V \sqrt{T}}\}$. However, the KMV-Merton model does not use the cumulative normal distribution to convert the *DD* to default probabilities. Moody's KMV uses its large historical database. It estimates the empirical distribution of *DD*, based on which it calculates the default probabilities.

Thus, exact replications of the KMV-Merton model are not feasible. Not only because of their probability of default, but also because of the likelihood that Moody's KMV makes adjustments to the accounting information used to calculate their default boundary due to the several modelling choices they have. Still, recent academic articles examining default such as Vassalou and Xing (2004), Duffie et al. (2007), Cambel et al. (2008) adopt the arbitrary default boundary of KMV-Merton model. We do not examine these latest models, as they are beyond the scope of this article.

2.3. Bharath and Shumway (2008) measure [BS]

BS develop a simple predictor for distance-to-default without solving the nonlinear equations required by the BSM-model. They suggest that all option variables should be observable from the marketplace, assuming the market is efficient and well informed. Thus, firm value is defined as the summation of the firm equity and debt, $V = E + B$. Equity *E* is defined as the shares outstanding multiplied by the market price, whereas *B* is the KMV-Merton default boundary (B_{KMV}) . After estimating *V,* they approximate the volatility of each firm debt as $\sigma_B = 0.05 + 0.25\sigma_E$, and the firm volatility as the weighted average volatility of *E* and B_{KMV}:

$$\sigma_{V[BS]} = \frac{E}{E+B}\sigma_E + \frac{B_{KMV}}{E+B}\sigma_B \tag{8}$$

They set the expected firm return equal to the firm stock return over the previous year and calculate their *DD* equal to:

$$DD_{BS} = \frac{\ln(V/B_{KMV}) + (\mu - 0.5\sigma^2_{V[BS]})T}{\sigma_V\sqrt{T}} \tag{9}$$

and the probability of default to

$$N(-DD_{BS}) = N\{-\frac{\ln(V/B) + (\mu - 0.5\sigma_V^2)T}{\sigma_V\sqrt{T}}\} \tag{10}$$

Similar to KMV-Merton model, they use a time to debt maturity 1 year *(T=1)*.

2.4. Extension of BSM -Model for Intermediate Default

The BSM-model above assumes that the firm debt mature in *T* periods, while in real world firms have obligations at intermediate times $(T' < T)$. Consider *"I"* the firm intermediate obligations. *I* represents interest and debt repayment instalments payable at intermediate times before debt maturity. Within this context, the option can be treated as compound option where each payment *I* constitutes the exercise price that must be paid in order to continue to the next stage. If at intermediate time T' the firm value $V_{T'}$ is lower than its interest and debt payment *I* $(V_{T'} < I)$, equity-holders will involuntary default. The payment *I* due at intermediate time T' $(T' < T)$, gives equity-holders the option to continue with the option to acquire the firm at debt maturity *T*. Thus, the value of equity can be seen as a call-on-a-call or a compound call option. Geske (1979) derived a model to value compound options:

$$E(V,T) = VN(d_1', d_1; \rho) - Be^{-rT'} N(d_2', d_2; \rho) - Ie^{-r\tau} N(d_2') \tag{11}$$

where $d'_1(V', \tau) = \{\frac{\ln V/V' + (r - D + 0.5\sigma_V^2)T'}{\sigma_V \sqrt{T'}}\}$ (12)

$$d_2' \equiv d_2'(V', T') = d_1' - \sigma\sqrt{T'} = \{\frac{\ln V/V' + (r - D - 0.5\sigma_V^2)T'}{\sigma_V \sqrt{T'}}\} \tag{13}$$

d_1 and d_2 are as earlier (equations 3 and 4, respectively)

Notation:

- N(d) = univariate cumulative standard normal distribution function (from $-\infty$ to d)
- N(a, b; ρ) = bivariate cumulative standard normal distribution function with upper integral limits a and b and correlation coefficient *ρ*, where $\rho = \sqrt{T'/T}$. The bivariate cumulative normal distribution $N(-d'_2, -d_2; \rho)$ represents the probability that equity-holders exercise their call option by paying off the principal B at the maturity, given that they previously decide to keep alive their option to continue.
- V' is the cut-off firm value *V* at the intermediate time *T′* when payment *I* comes due.
- The volatility parameter σ_V is not constant, but depends on the value of the firm.

In this option-based formulation, equity-holders may default not only at the debt maturity T $(V_T < B)$, but also at an intermediate time *T′*, just before the payment *I* comes due. That is the case of the firm value at time *T′* to fall below its cut-off option value $V'(V_{T'} < V')$. This default probability at an intermediate time *T′* is given by:

$$\Pr ob.default_{(T')} = \Pr ob(V_{T'} < V') = \Pr ob\{E(V', T') < I\} = 1 - N(d_2') = N(-d_2') \tag{14}$$

where d_2' is estimated as in equation (13).

However, even when the firm is profitable and equity is valuable, $E(V',T') > I$ and $V_T > B$, default may additionally be triggered when firm has insufficient cash flows to pay for the next *I*, $(CF_{T'} < I)$.

3. Research design

3.1. Liquidity proxy and distance-to-default measure

Within the context of intermediate default, Charitou and Trigeorgis (2006) develop a distance to default measure which includes a cash-flow variable as an option variable. In a similar vein, we use a variation of the liquidity variable to capture firm ability to generate cash to cover its interest expense and debt repayment obligations, at a given time before debt maturity. The cash flow coverage-ratio is defined as:

$$CFC = \frac{CashFlowFromOperations_t + Cash \,\&\, CashEquivalents_{t-1}}{InterestExpense_t + (\frac{Dept\,\mathrm{Re}\,payment_t + \mathrm{Pr}\,ef.Dividends_t}{1 - TaxRate})} \quad (15)$$

The numerator represents the available cash-flow, whilst the denominator represents the cash obligations, over an intermediate time period T'.[1] Therefore, if the firm has sufficient cash flows to pay its upcoming debts, *CFC* will be higher than 1 *(CFC>1)*. Otherwise, if *CFC<1*, its upcoming debts are higher than its cash flows and thus, its probability of default should be high. Hence, negative relation of the *CFC* with the default probability is expected.

Assuming the cash flow from operations *(CFO)* is a constant proportion of the firm value at time T' $(CFO = cV_{T'})$, it will trigger involuntary early default if the intermediate payment *I* is higher than the *CFO*+ *Cash & CashEquivalents (cash).* Thus, $I > (cV_{T'} + cash)$, whilst the cumulative intermediate probability of default should be:

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[1] Charitou and Trigeorgis (2006) subtract the *Cash & Cash Equivalents* from the denominator, arguing that some cash & cash equivalents might already been in place when the intermediate payment for interest or debt repayment come due. They also waive preference dividends and tax payments, as they can be deferred without triggering bankruptcy. As their ratio might not be well defined for negative values, we include the available cash and payments due in the numerator and denominator, respectively.

$$\Pr ob.default = \Pr ob((I - cash) > cT_{T'}) = \Pr ob(CFC < 1) = N(-d_2'') \quad (16)$$

where $d_2'' = \dfrac{\ln(CFC) + (r - D - 0.5\sigma_V^2)T'}{\sigma_V \sqrt{T'}}$.

Instead of using the risk neutral probability to default *(r-D)*, we use an empirical model with an option variable the difference between the firm value return R_V and the firm payout yield *D* (the coupon interest payment plus dividends at fiscal year end). However, as R_V is sometimes negative or lower than riskless rate *r*, firms are assumed to obtain the maximum return between R_V and *r* $\{\mu = \max(R, r)\}$, having a *DD*:

$$d_2'' = \frac{\ln(CFC) + (\mu - D - 0.5\sigma_V^2)T'}{\sigma_V \sqrt{T'}} \quad (17)$$

Thus, our default probability is $N(-d_2'') = N(-\dfrac{\ln(CFC) + (\mu - D - 0.5\sigma_V^2)T'}{\sigma_V \sqrt{T'}})$ (18)

Similar to *CFC*, *(μ - D)* is also negatively related to the default probability. As σ_V is the annually standard deviation of firm return (% of V), is proxy for risk, it has positive relationship with the option to default; the greater the σ_V the greater the default option value. *T* is the time to debt maturity, whereas *T′* is the time to the next intermediate *I.* All else equal, the longer the maturity the greater the default option value.

Considering the components of *DD* and $N(-DD)$ of the KMV-model (equation 8), our default variable (equation 18) incorporates the information content provided by the KMV-Merton model. As we also account for the dividend payments (contrary to KMV-Merton model) and capture the probability of intermediate default via the CFC, our *DD* measure should improve the KMV-Merton model as well as the BS.[2]

3.2. Data and Option variable calculations

[2] Further methodological differences of the two models are described in the following section.

The dataset used consists of a sample of 1269 U.S. industrial firms that filed for bankruptcy during the 1983–2001 periods and have data available in the Compustat and CRSP databases. We require the firms to be identified in the Wall Street Journal or in the Internet Bankruptcy Library as having filed for bankruptcy. We also use a sample of 6564 available healthy firms, resulting to a total sample of 7833 firms. To estimate option variables, we follow the BS approach and do not apply the algorithms required by the BSM-model. All items used are directly collected from the marker.

First, the market firm value *(V)* is set equal to *V*= *E* + *B*. *E* represents firm equity, defined as the number of shares outstanding multiplied with their market price (Compustat items #*A2*5 and #A24, respectively). Regarding the *B*, we maintain the original default boundary which equals the face value of total debt (book value of total liabilities #*A181*).

The standard deviation on firm asset value is first estimated as suggested by BS, $\sigma_{V[BS]}$. As the debt volatility is a function of the equity return volatility $(\sigma_B = 0.05 + 0.25\sigma_E)$, we estimate the monthly return on equity, adjusted for dividend payments: $R_{E,t} = \ln\{\frac{E_t + DV_t}{E_{t-1}}\}$, where DV_t is the cash dividends (#*A127*). *E* is the monthly firm equity as estimated earlier. Using a window of 60 months, we calculate σ_E and then assess the *BS* volatility (equation 9).

We then calculate our simpler alternative $\sigma_{V[DLCT]}$, by estimating the annually standard deviation derived by the monthly firm return (of V). We do not require the estimations of σ_E and σ_B, but the volatility of the monthly return on *V*. Thus, the first step is to calculate the monthly *V*. As V=E+B, we estimate the equity *(E)* and debt *(B)* on monthly basis. *E* is easily observable, whereas the monthly *B* is calculated by transforming the quarterly long-term debt (#*Q54*) into monthly value. For the transformation we use an averaging method based on the two surrounding months to estimate the two missing months.

Once the monthly V is calculated, the return on its value is defined as: $R_V = \ln\{\frac{V_t + D_t}{V_{t-1}}\}$, where D is the constant payout yield, set equal to $D = \frac{DV_t + XINT_t}{V_{t-1}}$. DV_t is the cash dividends (*#A127*) and $XINT_t$ is the Interest Expense (*#A15*). As mentioned, R_V is sometimes negative or lower than the riskless rate. Thus, we use the maximum return between actual and riskless return *r*, $\{\mu = \max(R, r)\}$. *r* is the 3-month US Treasury-bill rate. We then estimate the volatility of the monthly R_V $(\sigma_{V[DLCT]})$, having a 60-month return window. If $\sigma_{V[DLCT]}$ incorporates sufficient information similar to $\sigma_{V[BS]}$, the two variables are expected to be highly correlated. This would support our first hypothesis.

Having estimated the necessary items for four of the required option variables $\{V, B, (m-D), \sigma_V\}$, *T* is set equal to one (T=1) when we test the default probability in a year, as Helligeist et al (2004) explicitly do. However, we also follow Charitour and Trigeorgis (2006) and set the time to option maturity equal to the duration of weighted average life of debt maturity: $T_{wa} = \frac{\sum_{t=1}^{20} PV(DDt) * t}{\sum_{t=1}^{20} PV(DDt)}$, where $PV(DDt)$ is the present value of debt due in the year *t*, representing the present value of debt due in each year for the period 1983 - 2002.[3] Our implementation of the duration concept involves an approximation, repeated for all years tested resulting to an estimation of the average life of debts, for each

[3] For t = 1 to 5, DDt (debt due in year 1…5) was obtained from relative Compustant data items. *DD1*-debt due in one year (item *#A44*), *DD2*-debt due in two years (item *#A91*), *DD3*-debt due in three years (item *#A92*), *DD4*-debt due in four years (item *#A93*), *DD5*-debt due in five years (item *#A94*). For t > 5 an approximation was used by taking account of the total long-term debt (*DLTT #49*). The model implies that *T* should be the maturity date for all firm debts *B*. However, it is not possible to calculate *T* for some liabilities. As an example, current operating liabilities typically turn over, which makes it impossible to determine the maturity date for longer-term operating liabilities such as deferred income taxes. To estimate the cumulative debt from year 6 and forward, we first subtract the sum of *DD2* to *DD5* from the total long-term debt (*DLTT*). We then determine the average annual debt for the first five-years (debt *DD2* to *DD5*) and ultimately apportion debt to the remaining years (to *DD6* up to *DD20* for the 20 years tested) until the cumulative debt is exhausted up to year 20.

year and each firm. The procedure leads in a realistic approximation of the probability to default in a time equal to the average debt life.

Finally, regarding our liquidity proxy CFC (equation 15), Cash flow from operations are proxied by the item *#A98,* whereas cash & cash equivalents by the *#A1*. Tax Rate equals the total income taxes (item *#A16*) multiplied by 100 and divided by the pre-tax income (item *#A170*). We use the item #A19 for the Preference Dividends, *#A15* for Interest Expense and the debt due in one year (DD1 item *#A44*) for the debt repayment.

Once calculation of all option variables is completed, we estimate the default variables according to BS approach $-d2$ (equation 10 but adjusting for dividend payments and using the original default boundary B) and according to our extended compound-call option model $-d2''$ (equation 17). Both default variables are estimated using our $\sigma_{V[DLCT]}$ and BS $\sigma_{V[BS]}$, and also using T=1 (T_1) and $T = T_{wa}$. To avoid heavy effect of outliers, we winsorize all observations at the 1st and 99th percentiles.

3.3. Empirical models and predictions

Similar to various researchers we test our default variables running Cox proportional hazard models.[4] In Cox hazard models with time-varying covariates, the dependent variable is the time spent by a firm into the healthy group; the time-to-default *(T2D)*. Bankrupt firms prior the filing year are included in the healthy sample, however when they file for bankruptcy they leave the healthy sample and join the bankrupt one. For the following years they are removed from the samples (censored).

Proportional hazard models assume the default probability, known as the hazard rate $\lambda(t)$ increases linearly with time, conditional on the covariates in the model:

[4] Among others see Shumway (2001), Chava and Jarrow (2004), Agarwal and Taffler (2008), Bharath and Shumway (2008); Duffie et al. (2007).

$$\lambda(t) = \phi(t)[\exp(x(t')b)] \qquad (19)$$

where $\phi(t)$ is the baseline hazard rate. It depends on time only, without being affected by the time-covariates. $\exp(x(t')b)$ allows for the time-to-default to vary according to the firm covariates $x(t)$.[5]

To examine our hypotheses we run two sets of empirical models with explanatory variables the default measures. The two regression sets defer only in the volatility calculation. The first includes our $\sigma_{V[DLCT]}$ (*DLCT* models), whilst the second replicates the same models but includes the $\sigma_{V[BS]}$ (*BS* models). To account for the possibility that the overall health of the economy drives our measures, each model includes an annual bankruptcy rate $(AnRate)$. It is the number of the corporate bankruptcies divided by the total number of the traded firms over the previous year. It is hence expected to indicate high fluctuation along recession and expansion periods.

	Set 1: *DLCT* models	Set 2: *BS* models
Model 1:	$T2D = f(-d2^{DLCT}_{Twa}, AnRate)$	$T2D = f(-d2^{BS}_{Twa}, AnRate)$
Model 2:	$T2D = f(-d2^{DLCT}_{T1}, AnRate)$	$T2D = f(-d2^{BS}_{T1}, AnRate)$
Model 3:	$T2D = f(-d2''^{DLCT}_{Twa/2}, AnRate)$	$T2D = f(-d2''^{BS}_{Twa/2}, AnRate)$
Model 4:	$T2D = f(-d2''^{DLCT}_{T1/2}, AnRate)$	$T2D = f(-d2''^{BS}_{T1/2}, AnRate)$

The default variables assessed using the our volatility are notated as $-d2^{DLCT}$ or $-d2''^{DLCT}$, whereas those estimated following the BS as $-d2^{BS}$ or $-d2''^{BS}$. The notation used is consistent with previously. The default variable $-d2$ is assessed based on BSM-model modified by BS (equation 10 but adjusted for dividend payments and using the original default boundary *B*) and $-d2''$ based on our extended option formula via the *CFC* (equation

[5] For further information on the properties of Cox proportional regression models see Cox and Oakes (1984), Lin and Wei (1989), White (1989).

17). To distinguish when the T=wa we use the index $_{Twa}$, whereas $_{T_1}$ tests the default probability in a year. When we allow for one intermediate default before debt maturity $(-d2'')$, *T* is divided by 2, which implicitly assumes the default time will be about half of the *T*.

The two sets of hazard models examine the predictive ability of $\sigma_{V[DLCT]}$ in comparison $\sigma_{V[BS]}$ and hence, hypothesis one. Both sets are expected to have sufficient predictive ability, suggesting that the BSM-model is not sensitive to the manner firm value and its volatility are calculated. Comparing the two sets, we compare which approach captures the highest sufficient statistic (*DLCT* or *BS*). Similarly, models 3 and 4 of both sets examine the predictive ability of our extended option for intermediate default $(-d2'')$, consistent with our second hypothesis. Models 1 and 3 are in line with our third hypothesis, examining whether the estimation method of the time to option maturity has significant impact on the probability of default as employed by KMV-model.

To examine the models fit and rank the models according to their forecasting ability, we use four different tests. We first examine the Cox partial likelihood, where, the closer the log likelihood to zero, the better the model fit. The likelihood ratio (-2LL) tests the null hypothesis that covariate coefficients are not different from zero. If -2LL test is significant, the null hypothesis is rejected suggesting the covariates are contributing to explanation of the dependent variable (beyond the baseline hazard). Hence, we expect significant goodness of fit test in all tested models.

Second, the AIC information criterion is used (Akaike, 1974, 1981; Bozdogan, 1987). Unlike the Likelihood Ratio Test, AIC criterion is not test in the sense of hypothesis testing, but tool for model selection among non-nested models. It offers a relative measure of the information lost when a given model is used to describe the tradeoff between bias and variance in model construction. Therefore, competing models are ranked with the one having the lowest AIC being the best. We expect to report the lowest AIC value when we estimate volatility based on our approach, account for intermediate default via our *CFC* coverage ratio and use the time-to-option maturity equals the weighted average duration of debt life. In other words, we expect the 3rd *DLCT* model to indicate the highest predictive ability.

Third, we use a Receiver or Relative Operating Characteristic Curve (ROC curve) analysis to examine the predictive ability of the models. The ROC curve plots graphically the fraction of true positive versus the false positive rate, comparing the two characteristics/samples. The accuracy of the test depends on how well the test separates the group being tested into bankrupt and healthy, measured by the area under the ROC curve. ROC curve analysis is a tool widely used to select possibly optimal models and to discard suboptimal ones.[6] This approach allows model comparisons and rankings based on the area the ROC curve covers. ROC curve ratio indicates the area covered by the model's average function divided by that of a "perfect" model. ROC curve ratio equal to 1 indicates a model with "perfect" predictive ability, whereas 0.5 represents a worthless test. We therefore expect a close to 1 ROC curve for the models, especially for our 3rd model.

Ultimately, to robust our results we assess the out-of-sample forecast ability of all models (Shumway, 2001; BS; Duffie et al., 2007). Once we estimate the coefficients of each hazard model, firms are sorted into probability deciles. Probability deciles represent the bankruptcies occurred, documenting the forecasted default percentages. Thus, firms are sorted into default percentages. The top decile should predict the highest percentage of bankruptcies. To calculate the actual probability that firms in the top decile will default in the next period, we should divide the frequency of bankruptcies in that decile with the number of bankrupt firms in the model. With this approach we have the ability to rank firms into

[6] See Vassalou and Xing (2004), Agarwal and Taffler (2008), Crook (2004), Crook et al. (2007), Stein (2005), Blochlinger and Leippold (2006).

probability deciles without estimating the actual probabilities of bankruptcy. If models have misspecifications, out of sample results are not affected.

Table 1 about here

4. Empirical Results

4.1. Summary and descriptive statistics

Table 1 provides summary statistics for the bankruptcy rate within the years tested. During 1983 and 2001, the fluctuation between the worst and the best economic years is noticeable: 1996 seem to be an expansion period which indicates the lowest annual bankrupt rate of 0.95%. Contrary, during 2000 that the economic conditions were heading towards recession, the bankrupt rate (3.05%) is the highest for the 18 year-period.

Table 2 about here

Table 2 reports correlation coefficients for the default variables $-d2$ and the volatility estimations $\sigma_{V[DLCT]}$ and $\sigma_{V[BS]}$. Based on the first hypothesis, if our new volatility approach provides similar information as previous models (e.g. KMV, BS), *DLCT* and *BS* models should be highly correlated. In line with our expectations, the two standard deviations have a correlation of 0.509***, whereas the *DLCT* and *BS* default variables ($-d2^{DLCT}$,$-d2^{BS}$, respectively) are correlated by at least 0.724***.

The correlations change depending on: First, whether we compare *DLCT* and *BS* variables based on the BSM-models $(-d2)$ and use time-to-maturity T=1 $(-d2_{T1})$ or T=wa $(-d2_{Twa})$. Second, whether we compare *DLCT* and *BS* variables based on the extended variables for intermediate default $(-d2'')$ again using $(-d2''_{T1})$ or $(-d2''_{Twa})$. According to the table, *DLCT* and *BS* variables using our extended variable and T=1 indicate the highest correlation of 0.834*** $(-d2''^{DLCT}_{T1}, -d2''^{BS}_{T1})$. The high correlations suggest the default variables using our $\sigma_{V[DLCT]}$ or $\sigma_{V[BS]}$ capture similar information.

Table 3 about here

Table 3 provides signals regarding the accuracy of our methodology. It presents descriptive statistics of the *DLCT* and *BS* default variables, for the healthy and bankrupt firms $(-d2_{Twa}, -d2_{T1}, d2''_{Twa/2}, d2''_{T1/2})$. Panel A refers to the *DLCT* measures and panel B to *BS*. Regardless the volatility estimation method, the default variable (the firm distance-to-default to file for bankruptcy at *T* years) is expected to be higher for bankrupt rather than healthy firms.

Indeed, the means and medians of all default proxies are significantly higher (at 1%) for the bankrupt firms compared to the corresponding variables of healthy firms. *DLCT* and *BS* measure differences between healthy and bankrupt firms are still significant when T=wa (not only with T=1) and allow for intermediate default using the extended variable via our *CFC* ratio $(-d2'')$. These are in line with the expectations for our three hypotheses that our alternatives provide similar information as previous measures.

4.2. Cox proportional hazard models and AIC information criterion

Table 4 documents the coefficients of the Cox hazard models. Panel A refers to the *DLCT* models. The annual rate and the default variable are positive and significant at the 1% level, in all models. This is consistent with our expectations as the bankrupt variables $-d2$ and the annual bankrupt rates should be positively related to the bankrupt probability. Models 3-4 that triggers intermediate default via our *CFC* ratio report the lowesr

-2LL. This implies better goodness of fit test, especially for the 3rd model. Also, comparing models 1 with 2, and 3 with 4, when T=wa, models have lower -2LL.

AIC criterion ranks competing models with the one having the lowest AIC being the best. Model 3 which incorporates both, T=wa $(-d2''_{Twa/2})$ and accounts for intermediate default seems to provide the most sufficient statistic of the *DLCT* models. $DLCT_3$ model appears to be superior among the *DLCT* models, followed by the $DLCT_4$, $DLCT_2$ and $DLCT_1$. Hence, models that trigger for intermediate default indicate higher goodness of fit.

Panel B documents the *BS* models, which indicate similar results in the sense that models containing our *CFC* to capture the probability for intermediate default indicate lower -2LL. The AIC information criterion suggest that when T=wa, *BS* models perform less well than when T=1. The BS_4 model documents the highest goodness of fit among the *BS* models, followed by the $BS_{3,}$, BS_2 and BS_1. Overall, all models (*DLCT* and *BS*) indicate significant sufficient statistic, consistent with our hypotheses.

Ranking the models according to the AIC information criterion, when we extend the BSM-model to European compound call option and allow for intermediate default, it exhibits the *highest* statistic information in comparison with models that do not trigger intermediate involuntary default. Whether the time-to-debt-maturity has higher impact on the forecasting model, we find some supportive evidence. T=wa seems to have *higher* impact in the default probability information only in *DLCT* models and when *CFC* is included into the model.

Table 4 about here

4.3. ROC curve predictive ability

According to the ROC curve analysis, a rough guide for classifying the accuracy of a diagnostic test is the traditional academic point system: 0.90-1= excellent, 0.80-0.90 = good, 0.70-0.80 = fair, 0.60-0.70 = poor, 0.50-0.60 = fail. Based on the area coved by the ROC curve, models 1, 3 and 4 indicate "fair" performance (0.77***) whereas model 2 appear to have "good" performance (0.809***). Therefore, the *DLCT* models indicate ability to predictive bankruptcy, with the 2nd model being the best.

Regarding the *BS* models, models 2,3,4 indicate "fair" predictive ability (0.72***-0.75***) but model 1 indicates "good" ability (0.689***). This predictive ability is worse than *DLCT* models, consistent with our first hypothesis. Our volatility approach $(\sigma_{v[DLCT]})$ provides incremental information and is better suited for bankruptcy prediction compared to the *BS* approach $(\sigma_{v[BS]})$. BS report that by applying their methodology to KMV-Merton model result to a much simpler and easier to implement model, with no less sufficient statistic. Our methodology appears to incorporate more information than *BS* and the KMV-Merton model. As our methodology marginally outperforms *BS*, our model marginally outperforms KMV-Merton model as well.

Comparing all models (*DLCT* and *BS*), they can be ranked from the highest to worse forecasting ability as: $DLCT_2$, $DLCT_3$, $DLCT_4$, $DLCT_1$, BS_2, BS_4, BS_1, BS_3. Although all models have significant predictive ability with the *DLCT* to outperform *BS*, the model ranking cannot clearly suggest whether our extended formula to count for intermediate default has higher predictive ability than the original BSM-model. The models that extend the BSM-model to European compound call option and trigger intermediate default via our *CFC* coverage ratio $(-d2'')$ do not necessarily exhibit more sufficient statistic compare to the corresponding boundary default of KMV-Merton approach $(-d2)$. The same holds when comparing the models the impact of the time-to-maturity estimation method (*T=1* vs. *T=w*a).

Table 5 about here

4.4. Out-of-sample forecasts

To robust our results regarding the model rankings, we assess the out-of-sample forecast ability of all models, presented in table 5. With this approach we have the ability to rank firms into probability deciles without estimating the actual probabilities of bankruptcy. The top probability deciles should predict the highest percentage of bankruptcies. If models have misspecifications, out of sample results are not affected. Panel A demonstrates the out-of-sample results for *DLCT* models, whereas panel B demonstrates the out-of-sample results for *BS* models.

Out of sample assessments are similar to our previous findings. All models seem able to forecast the bankruptcy percentages by at least 35% at the top quintile. Specifically, they could be ranked from the highest to worse forecasting ability as: $DLCT_3$, $DLCT_2$, $DLCT_4$, BS_4, BS_3, $DLCT_1$, BS_2, BS_1.

$DLCT_3$ that employs our extended formula to capture for the intermediate default seems able to classify most of the bankrupt firms in the top deciles. In the highest probability deciles, it forecasts 56.08% of the bankrupt firms at the beginning of the period in which they default. $DLCT_2$ predicts 55.7%, followed by $DLCT_4$ that predicts 52.97% and by BS_4 that appears able to forecast 52.39% of the bankruptcies in their top deciles. The rest models indicate lower predictive ability with the BS_1 being the worse as it predicts only 35.09% of the bankrupt firms in its top deciles. Consistent with our previous findings, *BS* models indicate lower predictive ability.

5. Conclusions

Our main research objective is to provide a more effective and easier to practice bankruptcy prediction model which builds upon the option pricing theory. Like BS, we do not solve the simultaneous nonlinear equations required by the Black-Sholes-Merton model. We estimate volatility in a simpler manner. Our first hypothesis examines whether our volatility calculation captures sufficient information. Such findings would be consistent with the BS argument that following the algorithms required by the BSM-model to estimate the option volatility is not significant.

Second, we extend the theoretical BSM-model into a conceptual European compound call option, counting for the probability of an intermediate default. That is the case of a firm to have insufficient cash flows to meet any of its intermediate scheduled obligations before debt maturity. We account for this possibility by including a liquidity coverage ratio, examining whether the extended option provides sufficient statistic to predict bankruptcy. Finally, our third hypothesis examines whether the estimation method of the time-to-maturity option variable affects the default probability. Previous studies use time-to-maturity equal to one (Hilligeist et al., 2002; KMV-Merton; BS), whereas the debt maturity is beyond far from one. Following Charitou and Trigeorgis (2006), we also use a weighted average life of total liabilities as time to option maturity.

Our results are in line with our expectations. After comparing our methodology and results with the previous models, our extended formula appears to be a valuable tool in forecasting default. It is more realistic and easy to implement, whilst it provides sufficient statistic and significant predictive ability.

Consistent with the BS, the volatility estimation following the algorithms of the BSM-model is not significant to the distance to default measurement. BS also report that by applying their methodology to KMV-Merton model result to a model with no less sufficient statistic. Our measures using a new approach for the volatility estimation appears to outperform BS, consistent with our first hypothesis. Hence, our methodology should exhibit higher statistic information than the one provided by the KMV-Merton model.

As far as our second hypothesis and third hypotheses are concerned, the measures still indicate significant ability to forecast default. Nonetheless, the model ranking cannot clearly suggest whether our extended formula to count for intermediate default has *higher* predictive ability than the original BSM-model. The same holds when comparing the models the impact of the time-to-maturity estimation method (*T=1* vs. *T=w*a). These ambiguities are likely to be related to the fact that the default boundary used to estimate the default measures (total liabilities or cash flow coverage) has specific time-to-maturity which differs from the option forecasting horizon (*T=1* or *T=w*a). This inconsistency however, is one of the limitations of the BSM-model.

Overall, our alternative measures, our deviated simple volatility estimation, provide sufficient statistic and ability to forecast default that appears to outperforms prior models.

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Table 1: Summary statistics – bankruptcies by year

The table provides summary statistic of our sample. Annual bankruptcy rate is the number of the corporate bankruptcies divided by the total number of the traded firms over the previous year. Observations for a firm after it has file for bankruptcy are eliminated. The bankrupt rate represents the overall health of the economy. During recession periods the annual bankrupt rate is relatively, whereas it is relatively low rates during expansion periods.

Year	Bankruptcies	Traded Co	An.Rate	An.R%
1983	6	1772	0.0034	0.34%
1984	13	1906	0.0068	0.68%
1985	27	2030	0.0133	1.33%
1986	22	2250	0.0098	0.98%
1987	21	2436	0.0086	0.86%
1988	41	2510	0.0163	1.63%
1989	67	2580	0.0260	2.60%
1990	73	2672	0.0273	2.73%
1991	72	2834	0.0254	2.54%
1992	59	3056	0.0193	1.93%
1993	38	3376	0.0113	1.13%
1994	47	3696	0.0127	1.27%
1995	53	4454	0.0119	1.19%
1996	48	5070	0.0095	0.95%
1997	67	5496	0.0122	1.22%
1998	103	5949	0.0173	1.73%
1999	116	6416	0.0181	1.81%
2000	208	6815	0.0305	3.05%
2001	166	6863	0.0242	2.42%

Table 2: Correlation Matrix for the default variables

The table presents the correlation coefficients between the default variables $-d2$ and correlations between the volatility σ_V estimated based on our approach $\sigma_{V[DLCT]}$ and based on Bharath and Shamway (2008) $\sigma_{V[BS]}$. The default variables $-d2$ are assessed based the description of BSM-model 6 but adjusted for dividend payments and $-d2''$ based the description of our extended option formula via our liquidity proxy. The default variables assessed using $\sigma_{V[DLCT]}$ are notated as $-d2^{DLCT}$ or $-d2''^{DLCT}$, whereas those estimated following the Bharath and Shamway (2008) as $-d2^{BS}$ or $-d2''^{BS}$. The index $_{Twa}$ denotes that time to maturity is set equal to the weighted average duration life of debt (equation 21), whereas T_1 tests the default probability in a year. The time to maturity T_{wa} and T_1 are divided by 2 when we allow for one intermediate default before debt maturity $(-d2'')$. Thus, we implicitly assume the default time will be about half of the *T*.

	$-d2^{DLCT}_{Twa}$	$-d2^{BS}_{Twa}$	$-d2^{DLCT}_{T1}$	$-d2^{BS}_{T1}$	$-d2''^{DLCT}_{Twa/2}$	$-d2''^{BS}_{Twa/2}$	$-d2''^{DLCT}_{T1/2}$	$-d2''^{BS}_{T1/2}$	$\sigma_{V[DLCT]}$	$\sigma_{V[BS]}$
$-d2^{DLCT}_{Twa}$	1 .									
$-d2^{BS}_{Twa}$	**0.724** **(0.000)*** **	1 .								
$-d2^{DLCT}_{T1}$	0.052 (0.000)***	0.037 (0.000)***	1 .							
$-d2^{BS}_{T1}$	0.047 (0.000)***	0.028 (0.000)***	**0.760** **(0.000)*** **	1 .						
$-d2''^{DLCT}_{Twa/2}$	0.012 (0.138)	0.013 (0.102)	0.019 (0.019)**	0.002 (0.795)	1 .					
$-d2''^{BS}_{Twa/2}$	0.018 (0.027)**	0.017 (0.032)**	0.011 (0.159)	0.000 (0.967)	**0.819** **(0.000)*** **	1 .				
$-d2''^{DLCT}_{T1/2}$	0.040 (0.000)***	0.037 (0.000)***	0.034 (0.000)***	0.031 (0.000)***	0.022 (0.007)***	0.020 (0.014)**	1 .			
$-d2''^{BS}_{T1/2}$	0.040 (0.000)***	0.036 (0.000)***	0.035 (0.000)***	0.029 (0.000)***	0.022 (0.007)***	0.025 (0.002)***	**0.832** **(0.000)*** **	1 .		
$\sigma_{V[DLCT]}$	0.055 (0.000)***	0.052 (0.000)***	0.033 (0.000)***	0.027 (0.000)***	0.019 (0.017)***	0.018 (0.026)**	0.032 (0.000)***	0.026 (0.001)***	1 .	
$\sigma_{V[BS]}$	0.034 (0.000)***	0.037 (0.000)***	0.009 (0.234)	0.012 (0.102)	0.006 (0.424)	0.014 (0.074)*	0.008 (0.311)	0.003 (0.672)	**0.509** **(0.000)*** **	1 .

*, **, *** indicate the level of significance at 10%, 5% and 1%, respectively

Table 3: Descriptive Statistics of default measures

The table presents descriptive statistics for the bankruptcy variables. The default variables $-d2$ are assessed based the description of BSM-model adjusted for dividend payments and $-d2''$ based the description of our extended option formula via the liquidity proxy to allow for intermediate default before debt maturity. The default variables assessed using our volatility $\sigma_{V[DLCT]}$ are notated as $-d2^{DLCT}$ or $-d2''^{DLCT}$, whereas those estimated following the Bharath and Shamway (2008) as $-d2^{BS}$ or $-d2''^{BS}$. The index $_{Twa}$ denotes that time to maturity is set equal to the weighted average duration life of debt, whereas T_1 tests the default probability in a year. When we allow for one intermediate default before debt maturity $(-d2'')$, we implicitly assume the default time will be about half of the T and divide by 2. Paired t-test and Wilcoxon test are tests of significance for mean and median differences between healthy and bankrupt firms; p-values in parenthesis.

Model		Firms	N	Mean	Mean Difference	paired t-test p-value	Median	Median Difference	paired non-par p-value
Panel A: Descriptive statistics for our default varialbes									
1	$-d2^{DLCT}_{Twa}$	Healthy	19119	-1.75761			-1.2714		
		Bankrupt t=0	594	0.18687	-1.944	(0.000)***	0.32302	-1.594	(0.000)***
		Healthy	19119	-1.75761			-1.2714		
		Bankrupt t<0	2452	-0.91733	-0.840	(0.000)***	-0.5821	-0.689	(0.000)***
2	$-d2^{DLCT}_{T1}$	Healthy	24026	-4.113			-3.2021		
		Bankrupt t=0	675	-0.68665	-3.426	(0.000)***	-0.2369	-2.965	(0.000)***
		Healthy	24026	-4.113			-3.2021		
		Bankrupt t<0	2738	-2.58205	-1.531	(0.000)***	-1.9611	-1.241	(0.000)***
3	$-d2''^{DLCT}_{Twa/2}$	Healthy	17475	-3.13358			-2.5425		
		Bankrupt t=0	362	1.63343	-4.767	(0.000)***	0.8096	-3.352	(0.000)***
		Healthy	17475	-3.13358			-2.5425		
		Bankrupt t<0	1736	-0.90964	-2.224	(0.000)***	-0.8026	-1.740	(0.000)***
4	$-d2''^{DLCT}_{T1/2}$	Healthy	19614	-7.51246			-6.1258		
		Bankrupt t=0	404	1.28272	-8.795	(0.000)***	0.24457	-6.370	(0.000)***
		Healthy	19614	-7.51246			-6.1258		
		Bankrupt t<0	1856	-3.06977	-4.443	(0.000)***	-2.8395	-3.286	(0.000)***
Panel B:Descriptive statistics for BS default variables									
1	$-d2^{BS}_{Twa}$	Healthy	19518	-0.77926			-0.7173		
		Bankrupt t=0	587	0.13391	-0.913	(0.000)***	0.27655	-0.994	(0.000)***
		Healthy	19518	-0.77926			-0.7173		
		Bankrupt t<0	2444	-0.702	-0.077	(0.181)	-0.364	-0.353	(0.000)***
2	$-d2^{BS}_{T1}$	Healthy	24613	-3.49092			-2.3403		
		Bankrupt t=0	668	-0.55464	-2.936	(0.000)***	-0.2777	-2.063	(0.000)***
		Healthy	24613	-3.49092			-2.3403		
		Bankrupt t<0	2728	-2.14663	-1.344	(0.000)***	-1.5785	-0.762	(0.000)***
3	$-d2''^{BS}_{Twa/2}$	Healthy	16449	-2.11856			-1.6971		
		Bankrupt t=0	355	1.32705	-3.446	(0.000)***	0.7692	-2.466	(0.000)***
		Healthy	16449	-2.11856			-1.6971		
		Bankrupt t<0	1728	-0.67559	-1.443	(0.000)***	-0.6065	-1.091	(0.000)***
4	$-d2''^{BS}_{T1/2}$	Healthy	18433	-6.54178			-4.4555		
		Bankrupt t=0	397	0.75821	-7.300	(0.000)***	0.2356	-4.691	(0.000)***
		Healthy	18433	-6.54178			-4.4555		
		Bankrupt t<0	1845	-2.55677	-3.985	(0.000)***	-2.4321	-2.023	(0.000)***

*, **, *** indicate the level of significance at 10%, 5% and 1%, respectively.

Table 4: Cox proportional hazard models with DLCT and BS default measures

The table presents coefficients of the hazard models with depended the time-to-default. It documents the hazard rates of the annual rate as described in table 1 and the default probabilities: $-d2$ are assessed based the description of BSM-model but adjusted for dividend payments, whereas $-d2''$ are based on the description of our extended option formula via our liquidity proxy which allows for intermediate default. The default variables use the $\sigma_{V[DLCT]}$ are notated as $-d2^{DLCT}$ or $-d2''^{DLCT}$, whereas those use the $\sigma_{V[BS]}$ are notated as $-d2^{BS}$ or $-d2''^{BS}$. The index $_{Twa}$ denotes that time to maturity is set equal to the weighted average duration life of debt, whereas T_1 tests the default probability in a year. The time to maturity is divided by 2 when we allow for one intermediate default before debt maturity $(-d2'')$. Thus, we implicitly assume the default time will be about half of the *T*. Each model includes a binary variable that takes 1 for bankrupt firms at the bankrupt year and zero when firms are included in the healthy sample. AIC test constitutes test for goodness of model fit among several competitive models. ROC curve ratio indicates the area covered by the model's average function divided by that of a "perfect" model. ROC curve ratio equal to 1 indicates a model with "perfect" predictive ability.

Panel A: DLCT models

Model	An.Rate	$-d2^{DLCT}_{Twa}$	$-d2^{DLCT}_{T1}$	$-d2''^{DLCT}_{Twa/2}$	$-d2''^{DLCT}_{T1/2}$	Obs	Cox partial -2LL	AIC	ROC curve ratio
1	86.613 (0.000)***	0.316 (0.000)***				3046	9119.065 (0.000)***	0.250	0.769 (0.000)***
2	64.288 (0.019)**		0.434 (0.000)***			3413	10423.858 (0.000)***	0.233	0.809 (0.000)***
3	102.415 (0.000)***			0.072 (0.000)***		2098	5285.930 (0.000)***	0.171	0.772 (0.000)***
4	100.914 (0.000)***				0.033 (0.000)***	2260	5984.512 (0.000)***	0.173	0.761 (0.000)***

Panel B: BS models

Model	An.Rate	$-d2^{BS}_{Twa}$	$-d2^{BS}_{T1}$	$-d2''^{BS}_{Twa/2}$	$-d2''^{BS}_{T1/2}$	Obs	Cox partial -2LL	AIC	ROC curve ratio
1	91.152 (0.000)***	0.300 (0.000)***				3031	9034.651 (0.000)***	0.254	0.686 (0.000)***
2	67.572 (0.001)***		0.511 (0.000)***			3396	10305.715 (0.000)***	0.233	0.746 (0.000)***
3	103.824 (0.000)***			0.066 (0.000)***		2083	5185.686 (0.000)***	0.193	0.716 (0.000)***
4	103.236 (0.000)***				0.030 (0.000)***	2242	5880.330 (0.000)***	0.186	0.722 (0.000)***

*, **, *** indicate the level of significance at 10%, 5% and 1%, respectively.

Table 6: Out of sample forecasts

The table reports the out of sample forecasting ability of the hazard models. Panel A examines the accuracy of our DLCT models, whereas panel B examines the accuracy of BS models (panels A and B of table 4, respectively). DLCT and BS bankruptcy variables differ only in volatility estimation: *DLCT* include the volatility $\sigma_{V[DLCT]}$, whilst BS but includes the $\sigma_{V[BS]}$. For the out of sample estimations, firms are sorted into deciles on each forecasting model. The resulting coefficients of each hazard model are used to estimate the predicted time-to-default. The frequency column indicates the bankruptcies occurred within the specific probability deciles whereas the forecast column indicates the percentage of default within the same deciles. The top probability quintiles should predict the highest percentage of bankruptcies. With this approach we are able to rank firms into probability deciles without estimating the actual default probabilities, therefore, if the models have misspecifications, out of sample results are not affected. To calculate the actual default probabilities, we divide the default frequency by the model observations (Default Prob. Column).

Model:	1			2			3			4		
Deciles	Frequen.	Forecast	Default.Prob.	Frequen.	Forecast	Default.Prob.	Frequen.	Forecast	Default.Prob.	Frequen.	Forecast	Default.Prob.
Panel A: DLCT models												
1-2	261	43.94%	8.57%	376	55.70%	11.02%	203	56.08%	9.68%	214	52.97%	10.39%
3-4	181	30.47%	5.94%	214	31.70%	6.27%	58	16.02%	2.76%	69	17.08%	3.35%
5-6	105	17.68%	3.45%	65	9.63%	1.90%	53	14.64%	2.53%	62	15.35%	3.01%
7-8	42	7.07%	1.38%	14	2.07%	0.41%	37	10.22%	1.76%	40	9.90%	1.94%
9-10	5	0.84%	0.16%	6	0.89%	0.18%	11	3.04%	0.52%	19	4.70%	0.92%
Defaults:	594			675			362			404		
Panel B: BS models												
1-2	206	35.09%	6.80%	293	43.86%	8.63%	173	48.73%	8.31%	208	52.39%	9.28%
3-4	191	32.54%	6.30%	239	35.78%	7.04%	70	19.72%	3.36%	82	20.65%	3.66%
5-6	110	20.10%	3.63%	110	16.47%	3.24%	55	15.49%	2.64%	56	14.11%	2.50%
7-8	23	11.58%	0.76%	23	3.44%	0.68%	42	11.83%	2.02%	40	10.08%	1.78%
9-10	3	0.68%	0.10%	3	0.45%	0.09%	15	4.23%	0.72%	11	2.77%	0.49%
Defaults:	587			668			355			397		