Forecasting Financial Volatility Using Asymmetric Normal Mixture GARCH in Risk Management: Evidence from the Greek Stock Market

by

Anastassios A. Drakos¹, Panayiotis J. Arsenos² and Leonidas P. Zarangas³*

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Abstract
In this paper, the return volatility of stocks traded in the Athens Stock Exchange is estimated by different GARCH models. The research is especially interested in the predictive performance of Asymmetric Normal Mixture GARCH (NMAGARCH) based on Kupiec test for the FTSE/ASE 20 Index. The empirical results show that the NMAGARCH perform better based on 99% confidence intervals out-of-sample forecasting while GARCH with normal and student-t distribution perform better based on 95% confidence intervals out-of-sample forecasting Kupiec test. These results show that none of the models including NMAGARCH outperforms other models in all cases as trading position or confidence intervals and these results shows that volatility models should be chosen according to confidence interval and trading positions. In addition, NMAGARCH increases predictive performance for higher confidence internal as Basel requires.

Key Words: Value-at-Risk, GARCH, Asymmetric Normal Mixture GARCH, Kupiec Test

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¹ Department of Business Administration, Athens University of Economics and Business, 76 Patission Street, GR-74100, Athens, Greece.
² Department of Business Administration, Technological Educational Institute of Ionian Islands, Lixouri, Cephalonia, Greece GR-28200,
³ Department of Finance & Auditing, Technological Educational Institute of Epirus, 210 Leoforos Ioanninon, GR-48100, Preveza, Greece.
*Corresponding author: email: lzaragas@hol.gr.
1. Introduction.

During the past two decades, volatility in financial markets and their models and forecasts have attracted growing attention by academic researchers, policy makers and practitioners. First, volatility receives a great deal of concern from policy makers and financial market participants because it can be used as a measurement of risk, providing an important input for portfolio management, option pricing and market regulation (Poon and Granger, 2003). Second, greater volatility in the stock, bond and foreign exchange markets raises important public policy issues about the stability of financial markets and the impact of volatility on the economy. High volatility beyond a certain threshold will increase the risk of investor losses and raise concerns about the stability of the market and the wider economy. Third, from a theoretical perspective, volatility plays a central role in the pricing of derivative securities. An investor's choice of portfolio is intended to maximize his expected return subject to a risk constraint, or to minimize his risk subject to a return constraint. A good forecast of an asset's price volatility provides a starting point for the assessment of investment risk. To price an option, we need to know the volatility of the underlying asset and according to the Black-Scholes formula, volatility is the only parameter that needs to be estimated. Therefore, option markets can be regarded as a place where people trade volatility. Finally, for the purpose of forecasting return series, forecast confidence intervals may be time varying, so that more accurate intervals can be obtained by modelling volatility of returns.

In recent years, the tremendous growth of trading activity and the trading losses of well known financial institutions have led financial regulators and supervisory committees to quantify the risk.

The importance of financial risk management has significantly increased since the mid-1970s, which saw both the collapse of the fixed exchange rate system and two oil price crises. These major events led to considerable volatility in the capital markets, which together with the proliferation of the derivatives markets, increased trading volumes and technological advances, led to considerable concerns about the effective measurement and management of financial risk. This tendency was further reinforced by a number of financial crisis such as the worldwide stock markets collapse in 1987, the Mexican crisis in 1995, the Asian crisis in 1997, as well as the Orange County,
Barings Bank and Long Term Capital Management cases. Such financial uncertainty has increased the likelihood of financial institutions suffering substantial losses as a result of their exposure to unpredictable market changes. These events have made investors become more cautious in their investment decisions, while it has also led to an increased need for more careful study of price volatility in stock markets. Indeed, recently we observe intensive research by academics, financial institutions and regulators of the banking and financial sectors in order to better understand the operation of capital markets and to develop sophisticated models to analyze market risk.

Market risk is one of the four types of risk that financial institutions can expose themselves to. It is considered the most significant one since it represents potential economic loss caused by the reduction in the market value of a portfolio. The existence of market risk and recent financial disasters have raised the need for the development of practical risk management tools for financial institutions. This need has been reinforced by the Basel Committee of Banking Supervision (1996) which called for the use of internal market risk management to capital requirement by financial institutions such as banks and investment firms.1

Value-at-Risk has become the standard tool used by financial analysts to measure market risk. VaR is defined as a certain amount lost on a portfolio of financial assets with a given probability over a fixed number of days. The confidence level represents ‘extreme market conditions’ with a probability that is usually taken to be 99% or 95%. This implies that only 1% (5%) of the cases will lose more than the reported VaR of a specific portfolio. VaR is widely used because of its simplicity. Essentially the VaR provides a single number that represents market risk and therefore it is easily understood.2

In addition, correctly modeling the variance is important for inference and forecasting. Since most data exhibit volatility across time, the unconditional variance is constant even though the conditional variance during some periods is unusually large. Therefore, the estimation methods that use conditional variances are more appropriate

1 For a detailed analysis see the Basel Committee on Banking Supervision’s (1996), “Amendment to the Capital Accord to Incorporate Market Risks”. Duffie and Pan (1997), Alexander (2005) and Drzik (2005) provide a comprehensive overview of value at risk measures.

2 See also Bank for International Settlements (1988, 1999a,b,c, 2001).
for this type of data, as the heteroscedasticity in the disturbances biases the test statistics, leading to incorrect inferences. In the presence of heteroscedasticity, the estimators themselves are no longer efficient. This point is important in testing economic or finance theory. For example Chen (2003) tested the CAPM model by modeling the heteroscedasticity to improve the estimate of beta. For the purpose of forecasting return series, more accurate intervals can be obtained by modeling volatility of returns.

The development of volatility models has been a sequential exercise. Surveys as in Bollerslev, Chou, and Kroner (1992), Bera and Higgins (1993), Bollerslev, Engle, and Nelson (1994), and Poon and Granger (2002) attest to the variety of issues in volatility research. In the finance literature, among many volatility models, the most successful models are seen as the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models by Bollerslev (1986), who generalizes the seminal idea on ARCH by Engle (1982), and their numerous generalizations that add asymmetries, long memory, or structural breaks. GARCH models are popular due to their ability to capture many of the typical stylized facts of financial time series, such as time-varying volatility, persistence and volatility clustering. Andersen and Bollerslev (1998) find that GARCH models do really provide good volatility forecasts, in particular when a good proxy for the latent volatility, such as the realized volatility, is adopted. However, the various model rankings were shown to be sensitive to the error statistic used to assess the accuracy of the forecasts. McMillan et al. (2000) evaluated the performance of ten alternative models for predicting the UK FTA all share and FTSE100 stock index volatility at monthly, weekly and daily frequencies.

They found that the ranking of different forecasting models is dependent on the series, frequency and evaluation criteria. Instead of statistical evaluation criteria, model performance could be judged by some measures of economic significance. Dacco and Satchell (1999) propose the use of alternative economic loss functions. Brooks and Persand (2003) evaluated how adequately the forecasting models perform in modern risk management setting.

The normal distribution, although widely used for its simplicity, cannot effectively describe the tails of returns. To better account for leptokurtosis, other distributions such as student t distribution, mixed normal distribution, GER distribution and skewed student distribution are used. For example, Angelidis, Benos and Dégianakis
evaluate the performance of an extensive family of ARCH models in five stock indices, using a number of distributional assumptions. They found that leptokurtic distributions are able to produce better one-step-ahead forecasts. Giot and Laurent (2003) modeled value at risk for daily asset returns. They suggest using an APARH model based on the skewed student distribution to fully take into account the fat left and right tails of the returns distribution. In all cases the skewed student t based models performed very well.

This paper seeks to combine volatility forecasting and value at risk in a number of ways. Despite a substantial amount of empirical research on stock-market behavior, most studies have concentrated on the major developed stock markets. There have been only a few comparable research works devoted to investigation into developing stock markets.

In this paper, five main GARCH models are used to estimate the stock market volatility. In addition, each model is applied on the time series with different normality assumptions, mainly normal distribution, Student's-t distribution and skewed Student's-t distribution. In recent research, asymmetric normal mixture GARCH models have been used in volatility modelling. Research by Alexander and Lazar (2003, 2005, 2006) uses GARCH(1,1) models with normal mixture conditional densities having flexible individual variance processes and time-varying conditional higher moments. The importance of using (asymmetric) normal mixture GARCH process lies in the fact that it can captures tails in the financial time series more properly. That is very important for modelling return volatility in the emerging financial markets where asymmetric high volatility observed during financial shocks. The emerging markets are open to internal or external shocks observed due to hot money movements, low trade volume, thin trading and instability. Markov regime switching models are used to capture the effects of the sudden shocks in the emerging markets. The normal mixture GARCH models are similar to Markov switching models and easier for use as it will be explained in the methodology part. This paper tries to estimate the return volatility in the Athens Stock Exchange by using five GARCH models including the normal mixture GARCH models with three different normality distributions. The aim of this research is to examine if the normal mixture GARCH models produce more accurate results and are able to capture shocks as long memory processes.

The organization of this paper is as follows. In Section 2, a literature review on the
predictive performance in return volatility in financial markets is presented. The test results in the literature with different markets and sample periods are compared. In Section 3, the methodologies of the GARCH models and different normality distributions are introduced. The importance is given on the methodology of asymmetric normal mixture GARCH model introduced by Alexander and Lazar (2003, 2005). After presenting the descriptive statistics of the data, empirical tests and Kupiec back-test is implemented. The predictive performances of the fifteen GARCH models in-sample and out-of-sample forecasting results are compared. The paper ends with suggestions for risk management and trading functions for their Value-at-Risk calculations and future financial research conducted in the transitory economics.

2. Literature Review

Early empirical evidence has shown that a high ARCH order should be used to capture the dynamics of the conditional variance. The Generalized ARCH (GARCH) process constructed by Bollerslev (1986) solves the problem in the ARCH model. The GARCH model is based on an infinite ARCH specification and reduces the number of estimated parameters by imposing non-linear restrictions on them. The GARCH models are extended under different motivation and assumption by researchers. The alternative models are EGARCH (Nelson, 1991), GJR (Glosten, Jagannathan, and Runkle, 1993), APARCH (Ding, Granger, and Engle, 1993), IGARCH (Engle and Bollerslev, 1986), FIGARCH (Baillie, Bollerslev, and Mikkelsen, 1996 and Chung, 1999), FIEGARCH (Bollerslev and Mikkelsen, 1996), FIAPARCH (Tse, 1998) and HYGRACH (Davidson, 2001). Ackert and Racine (1999), Darrat and Benkato (2003) and Puttonen (1995) use different GARCH models with different markets and time periods and conclude that the GARCH models are successful to model the volatility in the stock returns. As a long memory process, the normal mixture GARCH model captures shocks effects in the time series is used by Alexander and Lazar (2005, 2006).

The GARCH models are used with different assumptions on normality distributions. Bollerslev and Wooldridge (1992) shows that under the normality assumption, the quasi maximum likelihood estimator is consistent if the conditional mean and the conditional variance are correctly specified. This estimator is, however, inefficient with the degree of inefficiency increasing with the degree of departure from

The importance of skewness is explained in many researches. In a recent study, Christoffersen and Jacobs (2004) show that a simple asymmetric GARCH, that captures the leverage effect, performs best of all GARCH model considered. Bekaert and Wu (2000) and Wu (2001) display the fact that the 'leverage effect' in stocks determines a strong negative correlation between returns and volatility, which is the most important reason for skewness in stock returns. Christoffersen, Heston and Jacobs (2004), Bates (1991) focus on the connection between time-variability in the physical conditional skewness and the empirical characteristics of option implied volatility skews.

The difference between the physical and risk neutral skews is among the recent issues in financial research. Bates (2003) states that the difference between the risk-neutral and observed distributions cannot be explained unless the existence of a time-varying volatility risk premium is considered. Bates (2003) conducts the research based on real-world models with a single volatility component. However, Haas, Mittnik and Paolella (2004) and Alexander and Lazar (2005) show that GARCH models with time-varying volatility provide a better fit to the physical conditional densities than GARCH specifications with only one volatility state. The conditional higher moments endogenously determined are time-varying in those models. Therefore, their implied volatility skews exhibit the features of risk neutral index skews.

Non-normalities in conditional and unconditional returns is higher than that can be captured by GARCH(1,1) models with normally distributed errors. Bollerslev (1987) constructs GARCH(1,1) model with Student-t distribution. Fernandez and Steel (1998) extend the model to the skewed t-distribution. These t-GARCH models have no time-variation in the conditional higher moments. On the other hand, Haas, Mittnik and Paolella (2004) and Alexander and Lazar (2006) in their recent researches conduct GARCH(1,1) models with normal mixture conditional densities. The normal mixture GARCH models are flexible in individual variance processes and have time-varying conditional higher moments. Alexander and Lazar (2006) show that if the
model has more than two variance components, biases in parameter estimates are likely to result, and the estimated conditional skewness and excess kurtosis can be unstable over time. For modelling major exchange rate time-series, they find that the mixture of two GARCH(1,1) components models outperform both symmetric and asymmetric t-GARCH models and normal mixture GARCH(1,1) models with more than two components.

For stock market returns volatility, there are certain discrete time-varying models in the literature based on asymmetric GARCH models. Engle and Ng (1993), Glosten, Jagahannathan, and Runkle (1993) Nelson (1991) show that the models capture only one source of skewness, namely, the leverage effect. Additional structure is needed to capture the empirical observations about the nature of skewness in the risk-neutral equity index skew. This paper deals with the problem by using asymmetric normal mixture GARCH model with reality check.

3. Econometric Methodology

Reliable forecasting of return volatility in the financial markets is crucial for trading, risk management and derivative pricing. Return volatility is affected by time dependent information flows resulting in pronounced temporal volatility clustering. Therefore, financial time series should be parameterized with Autoregressive Conditional Heteroskedastic (ARCH) models modelling a time-turn varying conditional variance as a linear function of past squared residuals and of its past values. In other words, ARCH models are used to forecast conditional variances in that the variance of the dependent variable is modeled as a function of past values of the dependent variable or exogenous variables. ARCH models are constructed by Engle (1982) and generalized as GARCH (Generalized ARCH) by Bollerslev (1986) and Taylor (1986).

Different GARCH models are used to estimate the return volatility of financial instruments. EGARCH (Nelson, 1991), GJR (Glosten, Jagannathan and Runkle; 1993), APARCH (Ding, Granger and Engle; 1993), IGARCH (Engle and Bollerslev; 1986), FIGARCH (Chung, 1999), FIEGARCH (Bollerslev and Mikkelsen, 1996), FIAPARCH (Tse, 1998) and HYGARCH (Davidson, 2001) are the most known extensions and/or revisions of the ARCH model. The researches show that GARCH
models can provide good in-sample parameter estimates and, when the appropriate
volatility measure is used, reliable out-of-sample volatility forecasts. Recently the
asymmetric normal mixture GARCH model has been used to capture asymmetric
volatility in the returns. This paper tests the predictive performance of different
GARCH models with normal, Student's t and skewed Student's t distributions of the
error terms. Following fifteen models are constructed and compared for estimating
return volatility in the Athens Stock Exchange.

A. Models with normally distributed errors
   1. GARCH
   2. GRJ
   3. FIGARCH
   4. HYGARCH
   5. NM-AGARCH

B. Models with symmetric Student's t distributed errors
   1. GARCH
   2. GRJ
   3. FIGARCH
   4. HYGARCH
   5. NM-AGARCH

C. Models with skewed Student's t distributed errors
   1. GARCH
   2. GRJ
   3. FIGARCH
   4. HYGARCH
   5. NM-AGARCH

In a static linear model \( y_i = \alpha + \beta x_i + \varepsilon_i \), the error term \( \varepsilon_i \) is accepted as a random
variable with normal distribution and constant variance denoted in the Eq. 1.

\[
E(\varepsilon_i - 0)^2 = E(\varepsilon_i)^2 = \sigma_\varepsilon^2
\]  

(1)

Engle (1982) constructs Autoregressive Conditional Heteroscedasticity (ARCH)
model to explicit the time-varying variance.
\[ \sigma_t^2 = \omega + \sum_{i=1}^{n} \alpha_i \varepsilon_i^2 = \omega + \alpha(L)\varepsilon_i^2 \] (2)

In the Eq. 2, \( \sigma_t \) is the conditional variance of \( \varepsilon_t \) and varies on time. The model has restriction that the sum of \( \alpha_i > 0 \) and \( \alpha \) should be 1. In order to reach for estimations with negative variance Bollerslev (1986) constructs Generalized ARCH (GARCH). The GARCH Model includes the effects of both the linear variance and conditional variance of the past.

\[ \sigma_t^2 = \alpha_0 + \alpha_1 \sum_{i=2}^{n} \varepsilon_i^2 + \sum_{i=2}^{n} \sigma_{i-1}^2 \] (3)

The volatility in the returns increases more than the expected with the negative information if there is asymmetry in the time series. The first GARCH model capturing the asymmetry in the volatility is Exponential GARCH constructed by Nelson (1991).

\[ \ln(\sigma_t) = \delta + (1 + \alpha_i) f(u_{t-1} / \sigma_t^{1/2}) + \beta_i \ln \sigma_{t-1} \] (4)

\[ f(u_{t-1} / \sigma_t^{1/2}) = \theta u_{t-1} + \gamma |u_{t-1} / \sigma_t^{1/2}| - E[u_{t-1} / \sigma_t^{1/2}] \] (5)

In the model, the parameters are positive since the logarithmic values of the conditional variance are employed. Eq. 5 adds the asymmetric characteristic in the model. While "\( \theta \)" determines the sign of the error term affecting the conditional variance "\( \gamma \)" states the size effect. If there is asymmetry in the time series, \( \theta \) should be less than zero.

Gloslen, Jagannathan and Runkle (1993), and Zakoian(1994) state that asymmetry in the return volatility can be modeled by adding a dummy variable into GARCH model. GJR (Threshold GARCH) model is shown on Eq. 6.

\[ \sigma_t = \alpha_0 + \alpha_i u_{t-1}^2 + \gamma_i u_{t-1}^2 I_{t-1} + \beta_i \sigma_{t-1} \] (6)

In the model, if \( u_{t-1} \) higher than zero, \( I_{t-1} \) is equal to 1, otherwise, equal to zero. ARCH parameters in the conditional variance vary between \( \alpha_i + \gamma_i \) and \( \alpha_i \) in accordance with the sign of the error term. The positive news have an effect on the \( \alpha_i \) while the negative news on \( \alpha_i \) and \( \gamma_i \). If \( \gamma_i \) is higher than 1, it is accepted that there is asymmetry effect while on the other hand, if \( \gamma_i \) is equal to zero, the news
impact curve is symmetric. ARCH, GARCH and asymmetric GARCH models do not take into consideration the stationary of the conditional variance in the error terms. In the GARCH (1,1) model, if $\alpha_i + \beta_i < 1; u_i$ is static. The stationary of the conditional variance depends also Alpha and Beta parameters. In the GARCH(p,q) model, if $\alpha_{1-p} + \beta_{1-q} < 1$ in case of a shock, its effect changes the conditional variance in time known as decay factor. When $\alpha_{1-p} + \beta_{1-q} = 1$, the conditional variance behaves like a unit root process and enables the shock effect to change the conditional variance. Therefore, GARCH (p,q) model has the restriction of $\alpha_{1-p} + \beta_{1-q} < 1$ (Harris and Sollis, 2003).

In time series with high frequency, the sum of the Alpha and Beta parameters for the conditional variance estimated by GARCH (p,q) model is near or equal to 1 meaning that the volatility effects of the last observations in dataset increase. The same situation is valid for mean equation, as well. When sum of all AR and MA parameters is equal to 1, ARIMA process is expected (Laurent and Peters, 2002). The GARCH (p,q) process can be modeled as an ARMA process and written as in the Eq. 7 by using the lag operator.

$$[1-\alpha(L) - \beta(L)]e_i^2 = \omega + [1-\beta(L)](e_i^2 - \sigma^2_i)$$ (7)

The function $[1-\alpha(L) - \beta(L)]$ has a unit root, the sum of Alpha and Beta parameters is 1 and gives Integrated GARCH model of Engle and Bollerslev (1986). IGARCH model is denoted in the Eq. 8 (Laurent and Peters 2001).

$$\phi(L)(1-L)e_i^2 = \omega + [1-\beta(L)](e_i^2 - \sigma_i^2)$$ (8)

When the IGARCH process is modeled as a conditional variance of the squared error terms, it can be written in GARCH formulation as in Eq. 9.

$$\sigma_i^2 = \frac{\omega}{1-\beta(L)} + [1-\phi(L)(1-L)[1-\beta(L)]^{-1}]e_{i-1}^2$$ (9)

In time series, if the fractional difference of $y_i$ has a static process, $y_i$ is in the fractional integration. In the $(1-L)^d = y_i = e_i$ equation, if $d$ equals to 0, $y_i$ is static.
and its autocorrelations are zero. On the other hand, if $d$ is 1, $y_t$ has unit root with zero frequency. In case of $0 < d < 1$, the autocorrelations of $y_t$ slowly reaches zero. For this reason, the fractionally integrated models are seen as the models including long memory (Harris and Sollis, 2003).

Baillie, Bollerslev and Mikkelsen (1996) constructed Fractionally Integrated GARCH (FIGARCH) model by replacing the lag operator with $(1-L)^d$ in the IGARCH model. FIGARCH-BBM is represented in the Eq. 10.

$$
\phi(L)(1-L)^d (\varepsilon_t^2 - \sigma_t^2) = [1 - \beta(L)](\varepsilon_t^2 - \sigma_{t-1}^2) \varepsilon_t^2
$$

(10)

The conditional variance in the FIGARCH (BBM) model is can be written as

$$
\sigma_t^2 = \omega [1 - \beta(L)]^{-1} + [1 - [1 - \beta(L)]^{-1} \phi(L)(1-L)^d] \varepsilon_t^2
$$

(11)

where

$$
\omega = [1 - \beta(L)]^{-1}, \lambda(L) = [1 - [1 - \beta(L)]^{-1} \phi(L)(1-L)^d] \varepsilon_t^2, 0 < d < 1, \text{ and } \sigma_t^2 = \omega + \lambda(L)
$$

Chung (1999) modifies the FIGARCH (BBM) model as it is in the Eq. 12 since $\omega$ has theoretical problem and difficulties in the modelling in the practice.

$$
\sigma_t^2 = \sigma_{t-1}^2 + \{[1 - \beta(L)]^{-1} \phi(L)(1-L)^d)[\varepsilon_t^2 - \sigma_{t-1}^2]
$$

(12)

In this article, FIGARCH model suggested by Chung (1999) is tested.

Another integrated model developed by Davidson (2002) as a special version of FIGARCH is Hyperbolic GARCH. Davidson (2002) uses near epoch dependency in order to reach long-term memory. HYGARCH model can be written as

$$
\sigma_t^2 = \omega [1 - \beta(L)]^{-1} + [1 - [1 - \beta(L)]^{-1} \phi(L)[1+\alpha(1-L)^d]]
$$

(13)

Recently, normal mixture GARCH (NM-GARCH) models have been started to be used in detecting the shocks and long-term memory in the returns of the financial instruments. According to Alexander and Lazar (2005), NM-GARCH model can be seen as the Markov switching GARCH model in a restricted form where the transition probabilities are independent of the past state. They argue that the NM-GARCH models are easier to estimate than the Markov switching model constructed by Hamilton and Susmel (1994). What is more, in the NM-GARCH models, the
individual variances are only tied with each other through their dependence on the error term. The methodologies of the NM GARCH models are constructed and formulized by Alexander and Lazar (2005).

The asymmetric normal mixture GARCH model has one equation for the mean and K conditional variance components representing different market conditions. For simplicity the conditional mean equation is written \( y_t = \varepsilon_t \). It contains no explanatory variables as these can be estimated separately. The error term \( \varepsilon_t \) is assumed to have a conditional normal mixture density with zero mean, which is a weighted average of \( K \) normal density functions with different means and variances. We write:

\[
\varepsilon_t / I_{t-1} \sim \text{NM}(p_1, \ldots, p_K, \mu_1, \ldots, \mu_k, \sigma_{i1}^2, \ldots, \sigma_{ik}^2), \sum_{i=1}^{K} p_i = 1, \sum_{i=1}^{K} p_i \mu_i = 1
\]  

From Eq. 14, the conditional density of the error term is derived as

\[
\eta(\varepsilon_t) = \sum_{i=1}^{K} p_i \phi_i(\varepsilon_t)
\]  

where \( \phi \) is normal density functions with different constant means \( \mu_i \) and different time varying variances \( \sigma_{ii}^2 \) for \( i = 1, \ldots, K \).

The conditional variance behaviour is described by K variance components - and these characterize, according to one interpretation, different market circumstances. These variances can follow any GARCH process but for the purpose of this paper we assume there are three possibilities. In addition to the GARCH(1,1) processes studied in Haas et al (2004) and Alexander and Lazar (2005) we consider two types of asymmetric processes:

The NM-GARCH is given by:
\[
\sigma_i^2 = \alpha_0 + \alpha_i \varepsilon_{i-1}^2 + \beta_i \sigma_{i-1}^2, \text{ for } i = 1, \ldots, K
\]  

(16)

NM-AGARCH based on the Engle and Ng, (1993) model is:

\[
\sigma_i^2 = \alpha_0 + \alpha_i (\varepsilon_{i-1}^2 - \lambda_i) + \beta_i \sigma_{i-1}^2, \text{ for } i = 1, \ldots, K
\]  

(17)

NM-GJR GARCH based on Glosten et al, (1993) is given by:

\[
\sigma_i^2 = \alpha_0 + \alpha_i \varepsilon_{i-1}^2 - \lambda_i d_i \varepsilon_{i-1}^2 + \beta_i \sigma_{i-1}^2, \text{ for } i = 1, \ldots, K
\]  

(18)

where \( d_i = 1 \) if \( \varepsilon_i < 0 \) and 0 otherwise.

In all cases, the overall conditional variance is

\[
\sigma_t^2 = \sum_{i=1}^{K} p_i \sigma_i^2 + \sum_{i=1}^{K} p_i \mu_i^2
\]  

(19)

When \( K > 1 \), the existence of second, third and fourth moments are assured by imposing less stringent conditions than in the single component (K=1) models. For instance, Alexander and Lazar (2005) show that \( \alpha_i + \beta_i < 1 \) is not required and Haas et al (2004) as well found that \( \alpha > 1 \) can happen on the second and higher variance components.

For asymmetric NM-GARCH models, the conditions for the non-negativity of variance and the finiteness of the third moment are represented in the Eq. 20.

\[
0 < p_i < 1, i = 1, \ldots, K, \sum_{i=1}^{K} p_i < 1, 0 < \alpha_i, 0 \leq \beta_i < 1
\]  

(20)

In the NM-GARCH Model, we should have:

\[
m = \sum_{i=1}^{K} p_i \mu_i^2 + \sum_{i=1}^{K} \frac{p_i \omega_i}{(1 - \beta_i)} > 0, n = \sum_{i=1}^{K} \frac{p_i (1 - \alpha_i - \beta_i)}{(1 - \beta_i)} > 0
\]  

(21)

and \( \omega_i + \alpha_i \frac{m}{n} > 0 \)

For the NM-AGARCH model we require that:

\[
m = \sum_{i=1}^{K} p_i \mu_i^2 + \sum_{i=1}^{K} \frac{p_i (\omega_i + \alpha_i \lambda_i^2)}{(1 - \beta_i)} > 0, n = \sum_{i=1}^{K} \frac{p_i (1 - \alpha_i - \beta_i)}{(1 - \beta_i)} > 0
\]  

(22)

and \( \omega_i + \alpha_i (\frac{m}{n} + \lambda_i^2) > 0 \)

and for the NM-GJR GARCH Model, we should have.
According to Alexander and Lazar (2005), there are two distinct sources of asymmetry in the model:

**Persistent asymmetry.** This arises in all three models, when the conditional returns density is a mixture of normal density components having different means; it is generated by the difference between the expected returns under different market conditions. However, only the NM-AGARCH and NM-GRJ GARCH models have **Dynamic Asymmetry** emerging when the $\lambda_i$ parameters in the component variance processes capture time-varying short-term asymmetries arising from the leverage effect. If $\lambda_i$ is positive, the conditional variance is higher following a negative unexpected return at time $t-1$ than following a positive unexpected return. In equity markets, since "bad news" corresponds to a negative unexpected return, positive $\lambda_i$ should be expected. On the other hand, negative leverage coefficients may be estimated from commodity returns.

Taken together, these two sources of skewness in the physical conditional returns density offer a much richer structure for capturing the shape of equity index skews than is given by traditional GARCH models. Not only are conditional higher moments time-varying, the unconditional skewness and excess kurtosis are both non-zero. But, since we have conditional normality for each component, the conditional skewness for each component is zero, and thus the unconditional skewness in each component is zero. Hence the unconditional skewness in the overall index returns density stems only from the 'persistent' asymmetry, i.e. from the different means in the components of the normal mixture conditional density.

Normal mixture GARCH can be viewed as a restricted form of the Markov switching GARCH model where the transition probabilities are independent of the past state. These models are considerably easier to estimate than the class of Markov switching GARCH models introduced by Hamilton and Susmel (1994) even with the restrictions and improvements introduced by Cai (1994), among others. The difficulty with
estimating most Markov Switching models lies in the co-dependencies of the state variances. However, normal mixture models have a very straightforward relationship between the states (the individual variances are only tied with each other through their dependence on the error term). Also, the transition probabilities are not historical state-dependent:

One of the assumptions in linear equation is to estimate the variance with normal distribution. The log-likelihood function of the standard normal distribution is given by Eq. 24 (Peters, 2001) where \( T \) is the number of observations. In normal distribution, skewness and kurtosis take the value of (0, 3).

\[
L_z = -\frac{1}{2} \sum_{t=1}^{T} \left[ \ln(2\pi) + \ln(\sigma_t^2) + z_t^2 \right] \quad (24)
\]

Starting with Bollerslev (1987) and Hsieh (1989), Baillie and Bollerslev (1989) and Palm and Vlaar (1997) show that fat-tailed distributions like Student-t perform better to capture higher observed kurtosis. The log-likelihood function of the Student-t distribution is given by Eq. 25. Like normal distribution, Student-t distribution is also a symmetric.

\[
L^{\text{dist}}(\theta) = T \left\{ \ln \Gamma\left(\frac{\nu+1}{2}\right) - \ln \Gamma\left(\frac{\nu}{2}\right) - \frac{1}{2} \ln[\pi(\nu-2)] \right\}
\]

\[
-\frac{1}{2} \sum (\ln(h_t) + (1+\nu) \ln(1 + \frac{\varepsilon_t^2}{\nu-2}))
\]

The main drawback of these two distributions is that although student-t may account for fat tails, they are symmetric. Recently, Lambert and Laurent (2001) applied skewed student-t distribution that is proposed by Fernandez and Steel (1998) in Value at risk estimation (Peters, 2001).

The main advantage of this density is that it considers both asymmetry and fat-tailedness. If \( \Gamma(.) \) denotes the gamma function in the log-likelihood of a standardized skewed student-t is given by Eq. 26 (Peters, 2001).

\[
L^{\text{skewed-dist}}(\theta) = T \left\{ \ln \Gamma\left(\frac{n+1}{2}\right) - \ln \Gamma\left(\frac{n}{2}\right) - 0.5 \ln[\pi(n-2)] + \ln\left(\frac{2}{\xi + \frac{1}{\xi}}\right) + \ln(s) \right\}
\]

\[
-0.5 \sum_{t=1}^{T} \left\{ \ln \sigma_t^2 + (1+\eta) \ln \left[ 1 + \frac{(s\varepsilon + m)^2}{\eta - 2 - \xi^{-2t}} \right] \right\}
\]

\[
(26)
\]
Forecasting ability of Garch Models has been determined by squared daily returns, RMSE or absolute failure rate that is offered by Basle Committee on Banking Supervision (1996a, 1996b). The Basel back-testing is based on recording daily exceptions as comparing one year of Profit &Loss to a %99 one tail confidence 1 day value at risk with an exception whenever Profit &Loss< value at risk. Since Basel back-testing procedure do not consider failure rate in shock positions we do not test models with this test. In order to compare asymmetric mixture Garch and other Garch models we use the widely used back testing procedure, Kupiec test.

In Kupiec’s test, we define \( f \) as the ratio of the number of observations exceeding \( \text{Var}(x) \) to the number of total observation (T) and pre-specified VaR level as a (Tang and Shieh, 2006). The statistic of Kupiec LR test is given by Eq. 27 (Kupiec, 1995). LR is distributed as chi-square distribution.

\[
LR = 2 \left\{ \log[f^+(1 - f)^{T-x}] - \log[\alpha^+(1 - \alpha)^{T-x}] \right\}
\]

(27)

The VaRs of \( \alpha \) quantile for long and short trading position are computed as in Equation 28, 29 and 30 for normal, student-t and skewed student-t respectively(Tang and Shieh, 2006).

\[
\text{VaR}_{\text{long}} = \hat{\mu}_t - z_\alpha \hat{\sigma}_t, \quad \text{VaR}_{\text{short}} = \hat{\mu}_t + z_\alpha \hat{\sigma}_t
\]

(28)

\[
\text{VaR}_{\text{long}} = \hat{\mu}_t - st_{\alpha,\nu} \hat{\sigma}_t, \quad \text{VaR}_{\text{short}} = \hat{\mu}_t + st_{\alpha,\nu} \hat{\sigma}_t
\]

(29)

\[
\text{VaR}_{\text{long}} = \hat{\mu}_t - skst_{\alpha,\nu,\xi} \hat{\sigma}_t, \quad \text{VaR}_{\text{short}} = \hat{\mu}_t + skst_{\alpha,\nu,\xi} \hat{\sigma}_t
\]

(30)

Where \( z_\alpha \), \( st_{\alpha,\nu} \) and \( skst_{\alpha,\nu,\xi} \) are left or right tail quantile at \( \alpha\% \) for normal, student-t and skewed student-t distributions respectively.

We estimate VaR with \( \alpha = 0.01 \) and \( \alpha = 0.05 \) confidence interval and backtest VaR models with Kupiec in-sample and out-of-sample forecasting test. We chose %99 confidence interval as Basel II requires %99 confidence interval and %95 confidence interval. to compare VaR results with different confidence interval level.
4. Data and Empirical Results

Data and Summary Statistics

The data set obtained from Datastream includes daily closings of the FTSE/ASE20, a joint venture between FTSE and the ASE which is a capitalisation weighted index, consisting of the top 20 companies by market capitalisation (mainly banking sector and telecommunications), and the sample period is January 1998 to May 2007 for a total of 2555 observations. The estimation process is run using 10 years of data (1998-2007) while the remaining 5 year (252*5 days) is used for out-of-sample forecasting. Since there is strong evidence of no unit root in the logarithmic first difference of the daily prices according to the GLS augmented Dickey-Fuller test (DF-GLS, by Elliott et.al. and the GLS versions of the modified Phillips-Perron (1988) tests (MZGLS and MZGLSGLS) by Ng and Perron for the null hypothesis of a unit root against the alternative of stationarity (see Table 1) we will focus on the continuously compounded percentage index return per day \( R_t = 100 \times (\log P_t - \log P_{t-1}) \), which corresponds to the approximate percentage nominal return on the index obtained from time \( t \) to \( t-1 \).

Summary statistics for the return series and for the whole period are reported in Table 1. We clearly observe that the mean-return over the whole sample period is positive, on average about 0.6%, the return series are positively skewed whereas the large returns (either positive or negative) lead to a large degree of kurtosis. Furthermore, the Lung-Box \( Q^2 \) statistics for all returns series are statistically significant, providing evidence of strong second-moment dependencies (conditional heteroskedasticity) in the distribution of the stock price changes.
TABLE 1: Descriptive Statistics and Unit Root Tests

<table>
<thead>
<tr>
<th>Statistics</th>
<th>R_FTSE20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample Period</td>
<td>02/01/1998-10/05/2007</td>
</tr>
<tr>
<td>Observation</td>
<td>2555</td>
</tr>
<tr>
<td>Mean</td>
<td>0.000596</td>
</tr>
<tr>
<td>Annual Std.Dev.</td>
<td>0.27596</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.020629</td>
</tr>
<tr>
<td>Excess Kurtosis</td>
<td>3.399204</td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.096048</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.086806</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>1229.8(0.000)</td>
</tr>
<tr>
<td>$Q^2(10)$</td>
<td>639.634(0.000)</td>
</tr>
<tr>
<td>DF-GLS</td>
<td>-4.38772</td>
</tr>
<tr>
<td>$MZ_{GLS}^\alpha$</td>
<td>-18.2810</td>
</tr>
<tr>
<td>$MZ_{t_{GLS}}$</td>
<td>-3.01951</td>
</tr>
</tbody>
</table>

Note: The number in parentheses are $P$-value of corresponding tests. All values are computed using EViews 6. $Q^2(10)$ is the Ljung-Box $Q$ -statistic of order 10 on the squared series. DF-GLS: Dickey-Fuller GLS(ERS) unit root test and $MZ_{GLS}^\alpha$, $MZ_{t_{GLS}}$, Ng-Perron unit root test.

Financial returns do usually, or mostly, not follow the pattern of a normal distribution; the statement of being independently and identically distributed (iid) can in that not be assumed. Facts that on the other hand can be applied to financial returns data contain first, volatility clustering, second, excess kurtosis and fat tails and third, a mild skewness(Alexander, 2004). From the summary statistics the excess kurtosis can directly be seen, and also some degree of skewness. To extend this analysis Figure 1 provides descriptive graphs (level of price series, daily returns, density of the daily returns vs. normal and QQ-plots against the normal distribution) for daily returns series. The density graphs and the QQ-plots against the normal distribution show that all the distributions of returns exhibit fat tails. Furthermore, the QQ-plots imply that there is an asymmetry in the fat tails. An additional result of these graphical expositions show that the return series exhibit volatility clustering, which means that
there are periods of large absolute changes tend to cluster together followed by

Figure 1: FTSE20/ASE stock index in level, daily returns, daily returns density (versus normal) and QQ-plot against the normal distribution. The time period is 02/01/1998 - 10/05/2007.

periods of relatively small absolute changes.

Empirical Results

Given these salient features of the daily returns for the FTSE/ASE20 we now move to perform the VaR analysis based on the Asymmetric Normal Mixture Garch and the other chosen models as described in section 3.3The results for the(approximate maximum likelihood) estimation of the GARCH,GJR,FIGARCH and HYGARCH models with the normal, student-t and skewed studen-t distributions are reported in Tables 2 and 3. The calculated Ljung-Box $Q^2$-statistic is not significant and this implies that the skewed Student models are successful in taking into account the

3 All computations were performed with G@RCH 5.0 procedure on Ox package (see Laurent and Peters,2002). We also use the Ox programming language for the estimation of the Asymmetric Normal Mixture Garch(see Alexander and Lazar,2005,2006) and the parameters are estimated using the Quasi Maximum likelihood method (Bollerslev nad Wooldrige,1992) and the BFGS Quasi-Newton optimization algorithm. Our thanks goes to Atilla Çifter for his help in programming the necessary codes.
conditional heteroskedasticity exhibited by the data. Student parameters \( (v) \) are statistically significant for all the GARCH models and thus shows that time series is fat tailed.

For the skewed student-t distribution, the asymmetric parameters \( (\xi) \) are negative and statistically significant for all Garch models. This is an indication that the density distribution of the FTSE/ASE20 is skewed to the left. Estimated long memory parameter \( d \) for the FIGARCH model and the hyperbolic parameter \( \ln(\alpha) \) for the HYGARCH model are statistically significant (see Table 3).

### Table 2: Estimation Results from GARCH(1,1) and GJR(1,1)

<table>
<thead>
<tr>
<th></th>
<th>Garch</th>
<th>Garch-t</th>
<th>Garch-Skew</th>
<th>GJR</th>
<th>GJR-t</th>
<th>GJR-Skew</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega )</td>
<td>0.04936</td>
<td>0.0706</td>
<td>0.0708</td>
<td>0.04801</td>
<td>0.0737</td>
<td>0.0734</td>
</tr>
<tr>
<td>(5.094)</td>
<td>(4.018)</td>
<td>(4.016)</td>
<td>(5.068)</td>
<td>(4.208)</td>
<td>(4.198)</td>
<td></td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.1163</td>
<td>0.1241</td>
<td>0.1253</td>
<td>0.0821</td>
<td>0.0906</td>
<td>0.0914</td>
</tr>
<tr>
<td>(13.03)</td>
<td>(7.867)</td>
<td>(7.884)</td>
<td>(8.971)</td>
<td>(5.767)</td>
<td>(5.780)</td>
<td></td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>0.8732</td>
<td>0.8571</td>
<td>0.8564</td>
<td>0.8725</td>
<td>0.8508</td>
<td>0.8511</td>
</tr>
<tr>
<td>(100.4)</td>
<td>(53.74)</td>
<td>(53.60)</td>
<td>(100.4)</td>
<td>(53.61)</td>
<td>(53.63)</td>
<td></td>
</tr>
<tr>
<td>( \nu - \text{Student } - t )</td>
<td>6.4889</td>
<td>6.703</td>
<td></td>
<td>(7.997)</td>
<td>(7.721)</td>
<td></td>
</tr>
<tr>
<td>( \xi - \text{Skewness} )</td>
<td>-0.0399</td>
<td>-0.0341</td>
<td></td>
<td>(-2.123)</td>
<td>(-1.5196)</td>
<td></td>
</tr>
<tr>
<td>( \nu - \text{Skewness} )</td>
<td>6.4786</td>
<td>6.688266</td>
<td></td>
<td>(7.999)</td>
<td>(7.722)</td>
<td></td>
</tr>
<tr>
<td>( \gamma_1 - GJR )</td>
<td>0.0769</td>
<td>0.0847</td>
<td>0.0829</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volatility</td>
<td>0.0217201</td>
<td>0.0194261</td>
<td>0.01966775</td>
<td>0.0102672</td>
<td>0.0112142</td>
<td>0.0112976</td>
</tr>
<tr>
<td>LogLike</td>
<td>6999.8504</td>
<td>7057.4518</td>
<td>7058.1301</td>
<td>7013.0935</td>
<td>7065.7830</td>
<td>7065.9273</td>
</tr>
<tr>
<td>AIC</td>
<td>-5.51367</td>
<td>-5.55828</td>
<td>-5.55802</td>
<td>-5.52332</td>
<td>-5.56405</td>
<td>-5.56338</td>
</tr>
<tr>
<td>( Q^2(10) )</td>
<td>19.9797</td>
<td>18.9134</td>
<td>18.2616</td>
<td>15.6626</td>
<td>13.8074</td>
<td>13.8983</td>
</tr>
<tr>
<td></td>
<td>[0.0104]</td>
<td>[0.0183]</td>
<td>[0.0193]</td>
<td>[0.0475]</td>
<td>[0.0869]</td>
<td>[0.0844]</td>
</tr>
</tbody>
</table>

Notes: Estimation results for the validity specification of the GARCH modes. \( Q^2(10) \) is the Ljung-Box \( Q \)-statistic of order 10 on the squared series (p-values in brackets)

As reported in Table 4, parameters \( \omega, \alpha, \beta_1 \) and the normal mixture \( \gamma \) (Gamma) parameter are statistically significant for all of the Asymmetric Normal Mixture
Models.

**TABLE 3: Estimation Results from FIGARCH(1,d,1) and HYGARCH(1,d,1)**

<table>
<thead>
<tr>
<th></th>
<th>Figarch Chung</th>
<th>Figarch Chung-t</th>
<th>Figarch Chung-Skew</th>
<th>Hygarch</th>
<th>Hygarch -t</th>
<th>Hygarch -Skew</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega$</td>
<td>4.0714</td>
<td>4.9295</td>
<td>5.154</td>
<td>0.3072</td>
<td>0.3510</td>
<td>0.3469</td>
</tr>
<tr>
<td></td>
<td>(3.411)</td>
<td>(2.142)</td>
<td>(2.149)</td>
<td>(3.550)</td>
<td>(2.770)</td>
<td>(2.764)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.2687</td>
<td>0.2523</td>
<td>0.2519</td>
<td>-0.2678</td>
<td>-0.1719</td>
<td>-0.1617</td>
</tr>
<tr>
<td></td>
<td>(3.648)</td>
<td>(2.179)</td>
<td>(2.024)</td>
<td>(-1.768)</td>
<td>(-0.7704)</td>
<td>(-0.731)</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.5143</td>
<td>0.4260</td>
<td>0.4213</td>
<td>-0.1149</td>
<td>-0.0054</td>
<td>0.0072</td>
</tr>
<tr>
<td></td>
<td>(6.993)</td>
<td>(3.437)</td>
<td>(3.477)</td>
<td>(-0.667)</td>
<td>(-0.0208)</td>
<td>(0.0278)</td>
</tr>
<tr>
<td>$\nu - Student - t$</td>
<td>6.7040</td>
<td></td>
<td></td>
<td>7.1127</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(7.568)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\xi - Skewness$</td>
<td></td>
<td>-0.0274</td>
<td></td>
<td>-0.02729</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-1.001)</td>
<td></td>
<td>(-1.007)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu - Skewness$</td>
<td></td>
<td>6.6815</td>
<td></td>
<td>7.1011</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(7.592)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d$ Figarch</td>
<td>0.3974</td>
<td>0.4008</td>
<td>0.4044</td>
<td>0.1579</td>
<td>0.2122</td>
<td>0.2138</td>
</tr>
<tr>
<td></td>
<td>(10.45)</td>
<td>(7.693)</td>
<td>(7.757)</td>
<td>(2.935)</td>
<td>(2.316)</td>
<td>(2.356)</td>
</tr>
<tr>
<td>$Hygarch \ln(\alpha)$</td>
<td></td>
<td>0.4901</td>
<td></td>
<td>0.2840</td>
<td>0.2749</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2.221)</td>
<td></td>
<td>(1.189)</td>
<td>(1.187)</td>
<td></td>
</tr>
<tr>
<td>Volatility</td>
<td>0.0298293</td>
<td>0.0244554</td>
<td>0.0251226</td>
<td>0.0047133</td>
<td>0.005460</td>
<td>0.00548113</td>
</tr>
<tr>
<td>LogLike</td>
<td>7028.8083</td>
<td>7074.1989</td>
<td>7074.7354</td>
<td>7025.7463</td>
<td>7071.8861</td>
<td>7072.4357</td>
</tr>
<tr>
<td>AIC</td>
<td>-5.5265</td>
<td>-5.57068</td>
<td>-5.57032</td>
<td>-5.5325</td>
<td>-5.56807</td>
<td>-5.56772</td>
</tr>
<tr>
<td>$Q^2(10)$</td>
<td>4.28644</td>
<td>4.29997</td>
<td>4.38645</td>
<td>2.44310</td>
<td>3.11411</td>
<td>3.18131</td>
</tr>
<tr>
<td></td>
<td>[0.8304]</td>
<td>[0.8291]</td>
<td>[0.8207]</td>
<td>[0.9643]</td>
<td>[0.9270]</td>
<td>[0.9225]</td>
</tr>
</tbody>
</table>

Notes: Estimation results for the validity specification of the GARCH modes. $Q^2(10)$ is the Ljung-Box $Q$-statistic of order 10 on the squared series (p-values in brackets)

Besides student-t and skewed student-t parameters $\nu$ - Student t, $\xi$ - Skewness and $\nu$ - Skewness are also statistically significant. These results shows that Asymmetric Normal Mixture Garch models may perform better – a hypothesis that can be tested
with back-testing procedures.

<table>
<thead>
<tr>
<th></th>
<th>NM-AGARCH</th>
<th>NM-AGARCH -t</th>
<th>NM-AGARCH -Skew</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega )</td>
<td>0.04167</td>
<td>0.065347</td>
<td>0.065465</td>
</tr>
<tr>
<td></td>
<td>(9.008)</td>
<td>(3.573)</td>
<td>(3.592)</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.120339</td>
<td>0.13874</td>
<td>0.130770</td>
</tr>
<tr>
<td></td>
<td>(12.81)</td>
<td>(8.103)</td>
<td>(8.090)</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>0.867620</td>
<td>0.847743</td>
<td>0.848117</td>
</tr>
<tr>
<td></td>
<td>(97.92)</td>
<td>(52.95)</td>
<td>(52.95)</td>
</tr>
<tr>
<td>( \nu - \text{Student -} t )</td>
<td>6.753675</td>
<td>(7.727)</td>
<td></td>
</tr>
<tr>
<td>( \xi - \text{Skewness} )</td>
<td>-0.030200</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-1.3760)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \nu - \text{Skewness} )</td>
<td>6.742793</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(7.725)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \gamma - \text{Normal Mixture} )</td>
<td>0.003736</td>
<td>0.003704</td>
<td>0.003640</td>
</tr>
<tr>
<td></td>
<td>(6.302)</td>
<td>(3.927)</td>
<td>(3.825)</td>
</tr>
<tr>
<td>Volatility</td>
<td>0.0220358</td>
<td>0.0197378</td>
<td>0.0198021</td>
</tr>
<tr>
<td>LogLike</td>
<td>7013.9332</td>
<td>7066.3075</td>
<td>7066.3826</td>
</tr>
<tr>
<td>AIC</td>
<td>-5.52398</td>
<td>-5.56447</td>
<td>-5.56374</td>
</tr>
<tr>
<td>( Q^2(10) )</td>
<td>15.3885</td>
<td>13.0074</td>
<td>13.0034</td>
</tr>
<tr>
<td></td>
<td>[0.0520]</td>
<td>[0.1116]</td>
<td>[0.1117]</td>
</tr>
</tbody>
</table>

Notes: Estimation results for the validity specification of the GARCH modes. \( Q^2(10) \) is the Ljung-Box \( Q \)-statistic of order 10 on the squared series (p-values in brackets)

Previous papers have used a variety of statistics to evaluate and compare forecast errors. This paper considers results from the Root Mean Squared Errors (RMSE), the Mean Squared Errors (MSE), information criteria test and the Nyblom test (Nyblom, 1994). Nyblom tests statistics shows that all of the models' parameters are stable (see Table 5).
<table>
<thead>
<tr>
<th>Method</th>
<th>MSE</th>
<th>RMSE</th>
<th>Akaike</th>
<th>$Q^2(10)$ **</th>
<th>Nyblom test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Garch-Normal</td>
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<tr>
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<td>15.3885</td>
<td>2.37792</td>
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<td>NMAGARCH-t</td>
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</tr>
</tbody>
</table>

* 1 day ahead out-of-sample forecasting based on 252 days evaluation.

**$Q$-Statistics on Squared Standardizes Residuals

However, RMSE or MSE may not be adequate backtesting test as these tests do not consider tail probability and overshooting effects. This can be seen in Figure 2 as
RMSE is maximum for NMGARCH models where Akaike criteria tests are not maximum for NMGARCH models. In this paper we employ relevant VaR evaluation criteria. In order to back-test the VaR results, we use Kupiec's (1995) LR test for long and short trading positions. This test attempts to determine whether the observed frequency of exceptions is consistent with the frequency of expected exceptions according to the VaR model and the chosen confidence interval.

![Figure 2: RMSE and Akaike Values](image)

We compared VaR models with Kupiec test for long and short trading positions. We define a failure rate for long trading position as percentage of negative returns smaller than one-step ahead VaR for long position (left tail of the density distribution of the returns) and a failure rate as the percentage of positive returns larger than one-step ahead VaR for short position (right tail of the density distribution of the returns).

The empirical results based on Kupiec in-sample forecasting test are summarized in Table 6 and Figure 3. The table contains Kupiec failure rates for short and long position VaR.
### TABLE 6: In Sample Forecasting Kupiec Test

<table>
<thead>
<tr>
<th>Model</th>
<th>Failure Rate</th>
<th>Kupiec LR</th>
<th>p-value</th>
<th>Failure Rate</th>
<th>Kupiec LR</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Garch-Normal</td>
<td>0.95114</td>
<td>0.07027</td>
<td>0.79094</td>
<td>0.047281</td>
<td>0.40190</td>
<td>0.52611</td>
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<tr>
<td>Garch-t</td>
<td>0.94720</td>
<td>0.41096</td>
<td>0.52148</td>
<td>0.051615</td>
<td>0.13804</td>
<td>0.71024</td>
</tr>
<tr>
<td>Garch-Skew</td>
<td>0.94602</td>
<td>0.82571</td>
<td>0.36352</td>
<td>0.048463</td>
<td>0.12741</td>
<td>0.72113</td>
</tr>
<tr>
<td>GJR-Normal</td>
<td>0.95193</td>
<td>0.20164</td>
<td>0.65340</td>
<td>0.044917</td>
<td>1.4271</td>
<td>0.23224</td>
</tr>
<tr>
<td>GJR-t</td>
<td>0.94720</td>
<td>0.41096</td>
<td>0.52148</td>
<td>0.051615</td>
<td>0.13804</td>
<td>0.71024</td>
</tr>
<tr>
<td>GJR-Skew</td>
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<td>0.52148</td>
<td>0.046493</td>
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<tr>
<td>Figarch-Normal</td>
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<td>0.19742</td>
<td>0.044129</td>
<td>1.9142</td>
<td>0.16650</td>
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<td>0.93460</td>
<td>0.04523</td>
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<td>0.19742</td>
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<tr>
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<td>0.86229</td>
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<td>0.23224</td>
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<td>0.31853</td>
<td>0.047675</td>
<td>0.29309</td>
<td>0.58825</td>
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</table>

Note: Number of forecasts 15 days

<table>
<thead>
<tr>
<th>Model</th>
<th>Failure Rate</th>
<th>Kupiec LR</th>
<th>p-value</th>
<th>Failure Rate</th>
<th>Kupiec LR</th>
<th>p-value</th>
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</thead>
<tbody>
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<td>Garch-Normal</td>
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<td>0.14623</td>
<td>0.013002</td>
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<td>0.23276</td>
<td>0.62948</td>
<td>0.010244</td>
<td>0.015177</td>
<td>0.90195</td>
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<td>Garch-Skew</td>
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<td>0.62948</td>
<td>0.010244</td>
<td>0.015177</td>
<td>0.90195</td>
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<td>0.14623</td>
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<td>0.10208</td>
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<tr>
<td>GJR-t</td>
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<td>0.23276</td>
<td>0.62948</td>
<td>0.010244</td>
<td>0.015177</td>
<td>0.90195</td>
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<tr>
<td>GJR-Skew</td>
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<td>0.10232</td>
<td>0.74907</td>
<td>0.009850</td>
<td>1.0057756</td>
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<tr>
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<td>Hygarch-Normal</td>
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<td>0.009062</td>
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<td>0.62948</td>
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<tr>
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<td>0.27854</td>
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<td>0.0057756</td>
<td>0.93942</td>
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</table>

Note: Number of forecasts 15 days
The number of days for the in-sample-forecasting is 15 and confidence interval is chosen as with $\alpha = 0.01$ and $\alpha = 0.05$. Table 6 can be read as follows. If the model is estimated accurately, it should explain the actual observations very well. The failure rate should be equal to the pre-specified VaR level, and Kupiec LR test would not reject its null hypothesis as failure rate equals to $a$ (Tang and Shieh, 2006).

The empirical results of Kupiec in-sample forecasting test shows that NMAGARCH with Gaussian distribution for short position and Figarch(1,d,1) with skewed student-t distribution performs better for $\alpha = 0.05$ where NMAGARCH with student-t for short position and GRJ with student-t and Hygarch with skewed student-t distribution for long position performs better for $\alpha = 0.01$. These results show that none of the model outperforms other models based on Kupiec in-sample forecasting. Since in-sample forecasting estimates VaR knowing only the past performance, out-of-sample forecasting is more consistent.
### TABLE 7: Out-of-Sample Forecasting Kupiec Test

<table>
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<tr>
<th></th>
<th>Out-of-Sample Forecasting %95 Confidence Interval</th>
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<th>Out-of-Sample Forecasting %99 Confidence Interval</th>
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<td>VaR for Long Position</td>
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<td>VaR for Short Position</td>
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<td></td>
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<td>Kupiec LR</td>
<td>p-value</td>
<td>Failure Rate</td>
<td>Kupiec LR</td>
<td>p-value</td>
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<td>0.018254</td>
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<td>0.023810</td>
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<tr>
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<td>0.16667</td>
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<td>0.0000</td>
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<tr>
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<td>0.0000</td>
</tr>
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</table>

Note: Number of forecasts: 252*5 days and 1 day ahead
Note: Number of forecasts: 252*5 days and 1 day ahead

![Out-of-Sample Kupiec Test p-value](image)

**Figure 4: Out-of-Sample Kupiec Test p-value**

Our out-of-sample forecast evaluation uses one step ahead prediction for 252*5 days forecast sample. Out of sample VaR results for long and short trading positions are reported in Table 7 and Figure 4. The empirical results of Kupiec out-of-sample forecasting test shows that Figarch(1,d,1) with skewed student-t distribution and Hygarch(1,d,1) with skewed student-t distribution for short position and Hygarch(1,d,1) with student-t distribution performs better for $\alpha = 0.05$ where NMAGARCH with Gaussian distribution for short position and GRJ with Gaussian distribution and NMAGARCH with Gaussian distribution for long position performs better for $\alpha = 0.05$. Based on Kupiec in-sample and out-of-sample forecasting the empirical evidence is in favor of the Figarch with skewed student-t, Hygarch with skewed student-t, GRJ with Gaussian, GRJ with student-t and NMAGARCH with Gaussian distribution.

The above results show that the volatility model should be chosen in accordance with confidence interval and trading positions. However, NMAGARCH model has better predictive performance for higher confidence interval. The Basel II Accord
requires accurate volatility model, which is statistically significant at 99% confidence level.

Figure 6 shows out-of-sample estimation for GARCH and NM-AGARCH with Gaussian and skewed student-t distribution. NM-AGARCH captures fat-tailed behavior of the data (shocks) better than GARCH.

5. Conclusion

Though volatility in stock returns provides opportunity in earning profit for traders, it is a threat for risk managers in balancing risk-return relationship. In emerging markets, return volatility is relatively high due to low market volume, unstable political and economic conditions, and hot money from international investment portfolios. High volatility and nonlinear returns in stock prices require advanced volatility measurement models based on non-normal distribution of returns. They should catch the fat tails and regime switches, which are not easy to be estimated and modeled with static econometric models.

In this paper, the return volatility of stocks traded in the Athens Stock Exchange is
estimated by different GARCH models. The research is especially interested in the predictive performance of Asymmetric Normal Mixture Garch (NMAGARCH) based on Kupiec test for the FTSE/ASE20 Index. In this respect, this article includes the first research employing the NMAGARCH model in Greek equity markets. What is more, it has original contribution to the finance literature by conducting reality check of the NMAGARCH model with comparing the classical GARCH models.

By examining fifteen GARCH models with alternative return distribution assumptions, the paper shows that the NMAGARCH perform better based on the 99 % confidence interval out-of-sample forecasting Kupiec test. On the other hand, Figarch with skewed student-t, Hygarch with skewed student-t, GRJ with normal, GRJ with student-t and NMAGARCH with Gaussian distribution perform better based on 95 % confidence interval out-of-sample forecasting kupiec test. The empirical evidence has a crucial concluding remark in prediction of stock market volatility. The results show that volatility model should be chosen in accordance with confidence interval and trading positions. However, NMAGARCH model has better predictive performance for higher confidence interval. The Basel II Accord requires accurate volatility model, which is statistically significant at 99 % confidence level. The paper show that for accurate internal volatility models being proper for the Basel II Accord, advanced models based on financial computing should be constructed by examining the nature of the markets under investigation.
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