Resolution of Financial Distress under Chapter 11: A Dynamic Game Approach (Preliminary version)

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Abstract

This paper examines the impact of bargaining under the U.S. bankruptcy code (Chapter 11) on the valuation of senior (unsecured) and subordinated corporate debt within a structural model of default. We model the strategic interaction between claimants in Chapter 11 as a multiple-stage costly bargaining process, and solve it in a game theory setting. In a structural framework, and using a dynamic programming approach, we solve for the equilibrium strategies of the various claimants. We then measure the impact of the bankruptcy judge's strategy and behavior on these strategies and on the yield spreads of restructured corporate senior and subordinated debt.

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1 Introduction

The beginning of the 21^{th} century was marked by the most spectacular financial bankruptcies in the last decades and raised the awareness of both academic and professional communities of the importance of credit risk events for the valuation of corporate risky debt. In the U.S., Chapter 11 of the bankruptcy Code, thereafter called the Code, presents an alternative to the liquidation of a bankrupt firm, by defining a judicial context in which the firm can reorganize its activities in order to emerge as a viable entity. For example, Enron (31,237.00 $MM liabilities) and WorldCom (45,984.00 $MM liabilities),[1] considered as the most important recent defaults, did use this provision of the Code in their attempt to survive and avoid costly liquidation. The aim of this paper is to account for the characteristics of Chapter 11 negotiation in the evaluation of corporate debt and the estimation of post-default credit spread levels.

This paper models the strategic interaction between claimants in Chapter 11 as a multiple-stage bargaining process, and solves it in a game theory setting. Our paper adds to earlier literature by modeling a complex and realistic negotiation process, incorporating different features of Chapter 11 negotiation described in the Code, and by considering two classes of creditors with different seniorities. Thus, the claimants are allowed to sequentially propose reorganization plans, one of which can be confirmed by the bankruptcy judge if all claimants agree on its implementation. Moreover, according to Chapter 11's cram-down provision, the judge can impose a reorganization plan, even if it was rejected by one of the claimants during formal negotiation, allowing him to stop lengthy and costly negotiation. We extend the definition of cram-down by allowing the bankruptcy judge to impose his own *cram-down plan*. We also account for the fact that the bankruptcy judge can stop negotiation by converting Chapter 11 reorganization into Chapter 7 liquidation.

Our results are as follows: We characterize the conditions under which are obtained various outcomes of the negotiation process under Chapter 11 (liquidation and reorganization). We also relate these conditions to the length of the negotiation process. We assess the impact of the perceived behavior of the bankruptcy judge (equity-favoring versus creditor-favoring) and his strategy on these results. Finally, we investigate the impact of Chapter 11 negotiation provision on the valuation of corporate risky debt and post-default credit spreads.

The remainder of the paper is organized as follows: After a brief review of the related literature in

[1] Source: Altman and Hotchkiss (2006).

Section 2, we introduce our main assumptions on the game-theoretic framework and the negotiation game under Chapter 11 in Section 3. Then, in Section 4, we derive the capital structure of the firm using a structural approach which allows us to specify the optimal reorganization plans in Section 5. In Section 6, we present the dynamic programming algorithm used to solve the negotiation game. Then, in section 7, we present the numerical results and we conclude in Section 8.

2 Related literature

Our paper is related to the literature that focuses on the resolution of financial distress in the context of structural models of default. This approach assumes that credit events are predictable and triggered by the movement of the firm's value relative to some barrier, and mainly relies on the option pricing approach of Black and Scholes (1973). Merton (1974) is the first paper that considers the firm's assets as contingent claims and prices equities as a call option on the value of the firm. He assumes that default occurs if, at maturity, the value of the assets of the firm do not cover the face value of the debt. Black and Cox (1976) relax this assumption by allowing default to occur when the unlevered firm's value reaches a lower barrier defined by a safety covenant. A crucial aspect of structural models of default if the definition of the default barrier, which can take different forms. It can be defined by an exogenous constant bond covenant as in Black and Cox (1976) and Leland (1994), or by an exogenous time-varying boundary as in Collin-Dufresne and Goldstein (2001). It can also be endogenous, reflecting the limited liability of equityholders (as in Leland (1994), Fan and Sundaresan (2000) and François and Morellec (2004)).

The issue of default is largely covered in the literature, that considers mainly three cases: In the first one, default leads to an immediate liquidation of the firm under Chapter 7 of the Code (see Merton (1974), Black and Cox (1976), Brennan and Schwartz (1984), Collin-Dufresne and Goldstein (2001)). In the second case, the firm enters out-of-court or private negotiation. Thus, in a binomial model, Anderson and Sundaresan (1996) model negotiation as a the take-it-or-leave-it offer made by the manager to creditors. Anderson et al. (1996) generalize the preceding model to continuous time. Mella-Barral and Perraudin (1997) and Mella-Barral (1999) study, in a continuous time private negotiation, the impact of bargaining power on the bilateral offer made by equityholders or creditors. Hege and Mella-Barral (2000) generalize Mella-Barral (1999) by introducing the tax advantage of debt and allowing for multiple negotiations.

The third case considers public negotiation, mainly under Chapter 11 of the U.S. bankruptcy code. The empirical literature on Chapter 11 is extensive, and its description is beyond the scope of this paper. Nevertheless, some empirical papers that highlight the features used in this paper are worth noticing here. On the importance of the bankruptcy judges, Bris et al. (2006) and Chang and Schoar (2006) show that the judges differ on their behavior towards claimants, where some of them are debtor-favoring and others creditor-favoring. Chang and Schoar (2006) show that this judge's specificity has an impact on post-bankruptcy credit-ratings and rates of re-filing in Chapter 11. On the other hand, some empirical papers examine the duration of Chapter 11 negotiation. Carapeto (2005), for example, documents that two-thirds of Chapter 11 firms require more than one reorganization plan before an agreement can be attained. Empirical studies indicate that the average duration of negotiation is somewhat over two years; Observed durations range from essentially zero (Franks and Torous (1989) report one case that lasted only 37 days) to more than seven years (Warner (1977)). The outcome of Chapter 11 reorganization is also widely discussed in the literature. For example, in their study of a sample of 1770 public companies that filed for Chapter 11 between 1979 and 2002, Hotchkiss and Mooradian (2004) are able to determine some resolution of the case by June 2004, for 79% of the firms studied. They find that almost 21.5% of the firms are liquidated under Chapter 7, while the remaining emerge as publicly or non publicly-registered companies, or merge with another operating company. However, Chapter 11 is not always a successful reorganization, even if the firm emerges as an entire entity. In fact, Altman and Hotchkiss (2006) show that, between 1984 and 2004, some firms that already filed and emerged from Chapter 11 once, filed for a second (Chapter 22), and even a third (Chapter 33) time in Chapter 11. The reported numbers show that these events are rather rare. For example, in 2003, Altman reports only 17 Chapter 22 cases, and 1 Chapter 33 case over 9,404 Chapter 11 cases.

The bankruptcy costs play an important role in the reorganization under Chapter 11, especially for small firms. In fact, these costs can be so high that they can push the liquidation of the firm. The bankruptcy costs can be classified into two categories: the direct and indirect bankruptcy costs. The first category represents out-of-pocket expenses such as legal and administrative fees, including the costs of lawyers, accountants, trustee and other professionals involved in the reorganization process. The indirect costs expresses the opportunity costs such as *the lost sales and profits caused by customers choosing not to deal with a firm that may enter bankruptcy.*[2] Bris et al. (2006) approximate the indirect bankruptcy by the reported asset value changes during bankruptcy and

[2] See Altman and Hotchkiss (2006), p. 93-94.

the time spent in reorganization. With respect to Chapter 11 reorganizations, the literature offers a wide range of values and estimation benchmark of the direct bankruptcy costs. In fact, these costs can be estimated as a percentage of firm value prior to bankruptcy (Warner (1977), Altman (1984), Weiss (1990)), total liquidating value of assets (Ang, Chua and McConnell (1982)), pre-bankruptcy assets (Bris et al. (2006)), book value of assets (Gilson, John and Lang (1990)), assets at beginning of case (Luben (2000), LoPucki and Doherty (2004)). Direct bankruptcy costs values are very heterogenous, and range from an average of 1.4% of assets at beginning of Chapter 11 case (LoPucki and Doherty (2004)) to 16.9% of pre-bankruptcy assets (Bris et al. (2006))

While significant progress has been done in empirical exploration of Chapter 11, little has been done regarding rigorous theoretical modeling of negotiation. Brown (1984) proposes a description of the agenda rules, and the cram-down rules that determine the outcome of a public negotiation. Theoretical structural models on public negotiation include the works of Fan and Sundaresan (2000), François and Morellec (2004), and Ericsson and Renault (2006). In these models, the outcome is modeled as a that of Nash bargaining game between equityholders and one class of creditors, that share the emerged firm according to their bargaining powers. A crucial issue, only recently addressed in the literature, is that negotiation involves more than one class of creditors. For example, Hackbarth, Hennessy and Leland (2006) examine a firm financed with a mix of bank and market debt, where only the bank has the ability to renegotiate. Breccia (2004), who extends Mella-Barral and Perraudin (1997) technique, studies two-stage sequential restructuring of two debt classes, senior unsecured and subordinated debt. Taking into account impairment strategy allows the negotiation to occur, in the first round, between equityholders and only one class of creditors. However, if the game reaches the second stage, all the claimants are involved in negotiation which is modeled as a Nash Bargaining Game, as in Fan and Sundaresan (2000). In contrast, in this paper, we rather model the negotiation process as a non-cooperative game.

3 The negotiation process under Chapter 11

We will be modeling a negotiation process which ultimately consists in sharing some value between three players. For that reason, it is convenient to define the set $X \in \mathcal{R}^3$ of sharing vectors (x_1, x_2, x_3) such that:

$$0 \leq x_i \leq 1 \tag{1}$$

and

$$\sum_{i \in \{1,2,3\}} x_i = 1. \tag{2}$$

3.1 Framework of the dynamic negotiation game

When the firm starts the reorganization under Chapter 11, we assume that it is managed by the same board of directors as prior to filing. Then, we can characterize the debtor as being in possession of its own affairs, and as stipulated in the Code, label him debtor-in-possession or DIP. In our paper, we assume that during the negotiation process under Chapter 11 the interests of equityholders (denoted by e) are represented by the DIP. Moreover, abstracting from hold-out problems among creditor's classes, we assume that each class is represented by a single creditor, and we denote the representative senior creditor by s and junior creditor by j. During the negotiation process, each claimant has the opportunity to propose a reorganization plan, and each remaining claimant can approve it or reject it.[3] We model the negotiation between the claimants as a mix of sequential and simultaneous non-cooperative games, which are played in successive bargaining rounds, which are assumed of equal length d for simplicity.[4]

In this paper, we allow the bankruptcy judge to have an important role in the negotiation process. In fact, he has the opportunity to decide on the maximum number of negotiation rounds, and to interfere at each round k with a probability q_k. This probability reflects the impatience of the judge at each bargaining round, which increases with the length of the negotiation process.[5]

The dynamic game is as follows (see Figure 1): As described in the Code, we assume that the DIP has the privilege to propose the first reorganization plan $p \in X$ at time $t = d$, where four scenarios are possible: In the first one, DIP's plan is implemented if the creditors unanimously

[3] Under the Code, the requirements for the approval are two-thirds in value and a majority in number for each class of creditors, and two-thirds in value for shareholders.

[4] Different bargaining rounds' lengths can be accomodated easily, without changing the results qualitatively.

[5] Bris et al. (2006) show that the identity of the bankruptcy judge matters in the determination of the total duration of the reorganization.

accept it, or if the creditors make opposite decisions and the judge interferes and decides to impose (or cram-down) this plan. In the second scenario, Judge's plan $\gamma \in X$ is implemented if the creditors do not agree to accept DIP's plan, and the judge interferes and decides to impose his own plan. In the third scenario, the firm is liquidated if the creditors reject unanimously the DIP's plan and the judge interferes to stop the process thus converting Chapter 11 negotiation into Chapter 7 liquidation.[6] In the last scenario, the game continues to the next bargaining round if the creditors do not agree to implement the plan, and the judge does not interfere in the bargaining process. We assume that, at each round, the judge interferes with a probability q_k, which is increasing with k, and that the judge decides between imposing either the last plan proposed, or his own arbitrary plan with a probability z, known by all players.

If a second round of negotiation is reached at $t = 2d$, then a new plan $p \in X$ is proposed by the subordinated creditor. Again, a simultaneous game is played where DIP and senior creditor decide independently on accepting or rejecting the plan, and where the possible outcomes are the same as in the first round. If the game moves to the third round, then the senior creditor proposes a new plan at $t = 3d$, and so on. If again the remaining players decide to continue the negotiation, then creditors alternate in proposing a new reorganization plan until the firm is liquidated or reorganized at $t^* = k^*d$, $k^* \leq K + 1$, or the players still do not agree on a plan at the last bargaining round $K + 1$, with $q_{K+1} = 1$.

3.2 Resolution of an auxiliary static negotiation game

Before solving for the entire negotiation process, we start by defining an auxiliary static game, which is played as follows (see Figure 2): One of the players (or claimants), labeled Leader and denoted l, proposes a plan $p \in X$ to the other two players, labeled Follower 1 (denoted f_1) and Follower 2 (denoted f_2). Followers 1 and 2 then decide whether to accept (A) or reject (R) the plan. As indicated in Table 1, the outcomes of the game depend on the pair of binary decisions made by Followers 1 and 2, on the proposition p by Leader, on the continuation share θ and the cram-down share offered by the judge γ, where p, θ and $\gamma \in X$. We assume that ω^I is a function of $x \in X$, and ω^C and ω^L are arbitrary positive constants, corresponding respectively to the value to be shared if a plan p is Implemented by the Leader or Imposed, if the game Continues, and if the

[6] To liquidate the firm, the bankruptcy judge can argue one of the causes described in section 1112(b)(4) of the Bankruptcy Code. For example, *showing that there is substantial or continuing loss to the estate and the absence of a reasonable likelihood of rehabilitation.* Other causes can include some negotiation technicalities.

firm is Liquidated. Finally, q and z are non negative parameters $\in [0,1]$ that represent respectively the judge's impatience and the probability that the judge implements his own plan.

Table 1: Normal-form representation of the static game

		Follower 2	
		A	R
Follower 1	A	$p\omega^I(p)$	$q\left[z\gamma\omega^I(\gamma)+(1-z)\,p\omega^I(p)\right]+(1-q)\,\theta\omega^C$
	R	$q\left[z\gamma\omega^I(\gamma)+(1-z)\,p\omega^I(p)\right]+(1-q)\,\theta\omega^C$	$q\left(L\omega^L\right)+(1-q)\,\theta\omega^C$

We now investigate the three-player static game, where the Leader proposes the sharing vector p, taking into account the reactions of the two followers, first computing the best strategy for the Leader for all possible outcomes of the game:

Leader proposes a plan and outcome is (R,R) In this case, Leader's outcome is

$$O_l^{RR}=q\left(L_l\omega^L\right)+(1-q)\left(\theta_l\omega^C\right) \tag{3}$$

Leader proposes a plan expecting it to be (A,A) In this case, the best Leader can achieve is

$$O_l^{AA}=\max_p\left\{p_l\omega^I(p):p_i\omega^I(p)\geq b_i,\ i=f_1,f_2\right\} \tag{4}$$

where

$$b_i=\frac{qz\left(\gamma_i\omega^I(\gamma)\right)+(1-q)\left(\theta_i\omega^C\right)}{1-q(1-z)},\ i=f_1,f_2 \tag{5}$$

For the particular case where $q=1$ and $z=0$, the solution to the optimization problem (4) is $p_l=1$ and $p_i=0$, $i=f_1,f_2$.

Leader proposes a plan expecting it (A,R) Again, the best the Leader can achieve is

$$O_l^{AR}=\max_p\left\{\begin{array}{c} q\left[z\left(\gamma_l\omega^I(\gamma)\right)+(1-z)\left(p_l\omega^I(p)\right)\right]+(1-q)\,\theta_l\omega^C: \\ p_{f_1}\omega^I(p)\geq b_{f_1}\text{ and }p_{f_2}\omega^I(p)\leq b_{f_2}\end{array}\right\} \tag{6}$$

where

$$b_{f_1}=\frac{L_{f_1}\omega^L-z\left(\gamma_{f_1}\omega^I(\gamma)\right)}{(1-z)} \tag{7}$$

and

$$b_{f_2}=\frac{(1-q)\left(\theta_{f_2}\omega^C\right)+qz\left(\gamma_{f_2}\omega^I(\gamma)\right)}{(1-q(1-z))} \tag{8}$$

Again, for the particular case where $q = 1$ and $z = 0$, the solution of the optimization problem (6) is $b_{f_1} = L_1\omega^L$, and $\forall\ b_{f_2}$. Moreover, if $z = 1$, $\forall\ q \in [0,1]$, then the the solution of the optimization problem (6) is $p_l = 1$ and $p_i = 0$, $i = f_1, f_2$ under the condition that $\gamma_{f_1}\omega^I(\gamma) \geq L_{f_1}\omega^L$. Finally, if $z = 0$, $\forall\ q \in [0,1]$, then $b_{f_2} = L_{f_2}\omega^L$.

Leader proposes a plan expecting it (R,A) The solution is obtained as in the preceding case, by reversing the identities of the followers.

3.3 Resolution of the dynamic negotiation game

At the k^{th} negotiation round between claimants, as in the single round game described in section 3.2, the Leader proposes p^k to Followers 1 and 2. Thus, a strategy for Leader at round $k = \frac{t}{d}$ is a mapping $\varkappa^k$ from $(v,t) \in \Re^2$ to X

$$\varkappa^k(v,t) = p^k, \tag{9}$$

where the assets value of the firm denoted v is a state variable of the dynamic game, and is assumed to evolve stochastically and be observable by all the players at any time $0 < t < (K+1)\,d$.

A strategy for Player $i \in \{f_1, f_2\}$ at stage k is a mapping μ_i^k from $(p,v,t) \in X \times \Re^2$ to $\{A, R\}$

$$\mu_i^k(p,v,t) = \phi_i^k. \tag{10}$$

We will use the auxiliary one-stage game described in section 3.2 to solve for the dynamic bargaining game under Chapter 11 described in 3.1 by backward recursion.

4 Firm value and capital structure

We define an economy in which, under the risk neutral measure, the value of the assets follows a log-normal process:

$$dV_t = (r - \delta)\,V_t dt + \sigma V_t dW_t \tag{11}$$

where r denotes the risk-free rate, σ the volatility of the assets, and W_t is a $Q-$Brownian motion defined on a probability space $(\Omega, \mathbb{F}, \mathcal{F}, Q)$. The payout δV_t represents the cash flows generated by the assets at time t, at a payout rate δ, which is used to pay coupons and dividends. During Chapter 11 reorganization, we assume that all claimants payments are suspended, and the payout cash is used to cover the bankruptcy costs.

4.1 Capital structure of the reorganized firm

The firm is owned by the equityholders and is financed initially by a mix of perpetual senior (unsecured) and subordinated debt, receiving constant and continuous coupons (respectively c_s and c_j), until the firm is in default and starts reorganization under Chapter 11. Thus, the total value of the firm, denoted $\omega_t(v)$, when $V_t = v$, can be expressed as

$$\omega_t(v) = S_t(v) + J_t(v) + E_t(v) \tag{12}$$

where $S_t(v)$ denotes the value of senior debt, $J_t(v)$ the value of junior debt and $E_t(v)$ the total value of equity.

If the firm emerges from financial distress at some time t^*, then the reorganized firm continues operations by distributing new continuous reorganized coupons to the senior and junior creditors (respectively c_s^* and c_j^*). We assume that further financial distress leads to the liquidation of the firm when the value of the assets hits the default barrier B at T

$$T = \inf\{t \geq t^* : V_t = B\}, \tag{13}$$

where B represents the default barrier after reorganization, that leads to the liquidation of the firm. Following Leland (1994), we assume that default is decided by equityholders and default barrier B is defined so as to maximize firm value given limited liability of equityholders. At default time T, the firm is liquidated following Absolute Priority Rule (APR) and assumes proportional liquidation costs α. The expectation $\mathbb{E}^Q(\cdot|t, V_t = v)$ will be denoted $\mathbb{E}_{vt}(\cdot)$. The value of the reorganized firm, for $t^* \leq t \leq T$, is given by:

$$\omega_{t^*}(v, c^*) = \mathbb{E}_{vt^*}\left[\int_{t^*}^{T} (\tau c^* + \delta V_t)\, e^{-r(t-t^*)} dt\right] - \alpha \mathbb{E}_{vt^*}\left[e^{-r(T-t^*)} B\right] \tag{14}$$

where τ represents the tax rate, α the proportional liquidation costs and, $c^* =: c_s^* + c_j^*$ the total reorganized coupons (senior and junior creditors), distributed when the firm emerges from Chapter 11 at t^*. Then, the value of the firm after emergence, which is the object of bargaining between claim holders, is endogenous and is given by

$$\omega_{t^*}(v, c^*) = v + \frac{\tau c^*}{r}\left(1 - \left(\frac{B}{v}\right)^{\lambda/(1-\lambda)}\right) - \alpha B \left(\frac{B}{v}\right)^{\lambda/(1-\lambda)} \tag{15}$$

where $\lambda = \frac{b+\eta}{b+\eta+\sigma}$, $b = \frac{\left(r-\delta-\frac{\sigma^2}{2}\right)}{\sigma}$ and $\eta = \sqrt{2r + b^2}$.[7] The first term in (15) is the value of the

[7] See Appendix A.

assets, the second is the value of the tax benefits of the operating firm, and the third term is the outcome of a default event leading to immediate liquidation of the firm. The senior debt is given by

$$S_{t^*}(v,c^*) = \frac{c_s^*}{r}\left(1-\left(\frac{B}{v}\right)^{\lambda/(1-\lambda)}\right) + \min\left[(1-\alpha)\,B,\frac{c_s^*}{r}\right]\left(\frac{B}{v}\right)^{\lambda/(1-\lambda)} \tag{16}$$

and the junior debt is given by

$$J_{t^*}(v,c^*) = \frac{c_j^*}{r}\left(1-\left(\frac{B}{v}\right)^{\lambda/(1-\lambda)}\right) + \min\left[\max\left[(1-\alpha)\,B-\frac{c_s^*}{r},0\right],\frac{c_j^*}{r}\right]\left(\frac{B}{v}\right)^{\lambda/(1-\lambda)} \tag{17}$$

Following Leland (1994), we define the liquidation barrier as

$$B = \frac{(1-\tau)\,\lambda}{r}c^*$$

where it is apparent that the reorganized coupon c^* has a direct impact on the liquidation barrier, and consequently on the value of the reorganized firm.

For a given v and c^*, we denote

$$y\,(v,c^*) =: \frac{B}{v} = \frac{\lambda\,(1-\tau)}{rv}c^* \tag{18}$$

such that $0 < y < 1$. For a given v, it is apparent that the choice of the reorganized coupon is equivalent to the choice of the value of y, where $y^{\lambda/(1-\lambda)}$ expresses the probability of default. To simplify notation, the symbol y is used to represent $y\,(.)$ when no confusion arises. Under these assumptions, the total value of the firm and the payoffs of the claimants at $t^* \leq t \leq T$, are given by:

$$\omega_t\,(v,y) = \frac{v}{\lambda}\left(\lambda + \frac{\tau}{1-\tau}y - \left(\frac{\tau}{1-\tau}+\lambda\alpha\right)y^{1/(1-\lambda)}\right) \tag{19}$$

$$S_t(v,y) + J_t(v,y) = \frac{v}{\lambda}\left(\frac{1}{1-\tau}y - \left(\frac{1}{1-\tau}-\lambda\,(1-\alpha)\right)y^{1/(1-\lambda)}\right) \tag{20}$$

$$E_t(v,y) = \frac{v}{\lambda}\left(\lambda - y + (1-\lambda)\,y^{1/(1-\lambda)}\right) \tag{21}$$

where it is also apparent that the variable y can be identified with a sharing vector in X for a given v.

4.2 The bankruptcy costs

While the bankruptcy costs are largely described and measured in the literature, they are surprisingly not incorporated in the modeling of the negotiation under Chapter 11. One exception is the work of François and Morellec (2004) who consider continuous bankruptcy costs, proportional to the remaining asset values, paid while the firm is reorganizing its activity. LoPucki and Doherty (2004) find that the amount of legal fees depend primarily on the firm's size, measured by assets, and the total time elapsed from filing to confirming reorganization plan.

In this paper, we assume that the negotiation is costly, and as in LoPucki and Doherty (2004), proportional to the assets and the time spent in the negotiation. We assume that a proportion of the bankruptcy costs is covered by the firm through the payout cash, δv. The remaining costs are paid by the claimants of the distressed firm.[8]

If the firm emerges from Chapter 11 at t^*, and if we assume that $0 < \delta < \varphi$,[9] the bankruptcy costs to be born by the claimants are given by:

$$C\left(v, t^*\right) = \left(\varphi - \delta\right) t^* v + \mathcal{K} \tag{22}$$

where v represents end-of-bankruptcy declared assets values, φ the proportional bankruptcy costs, δ the payout rate and $t^* = dk^*$ the time spent in the negotiation. $\mathcal{K}$ represents the constant bankruptcy costs such that the filing fees paid at the entry in Chapter 11 negotiation.

The value to be shared by the claimants when the assets value is v and a sharing plan corresponding to y is implemented, is expressed from (19) as follows:

$$\omega^I_{t^*}\left(v, y\right) \equiv \omega_{t^*}\left(v, y\right) - C\left(v, t^*\right) \tag{23}$$

where $\omega_{t^*}\left(v, y\right)$ is the total value of the reorganized firm as defined in (19).

Bris et al. (2006) point out that Chapter 11 reorganizations have two identifiable reimbursable cost components: debtor expenses and unsecured creditors' committee expenses. According to Bris et al. (2005), which is the first paper that examines the optimal allocation of bankruptcy costs, the current Code does not authorize the court to compensate for the senior creditors expenses, but sometimes reimburses junior creditors. In this paper, we assume that the court does not reimburse junior creditors. Moreover, given the limited liability of debtors, we assume that their expenses are totally reimbursed by the court. Finally, we assume that the allocation of bankruptcy costs among the creditors is independent of the plan p proposed by the Leader, and follows the same rule as their initial contractual engagements.[10]

[8] Bris and al. (2005) document that the expenses are submitted and are almost always approved by the bankruptcy court.

[9] If $\delta > \varphi$, then the DIP receives $\left(\delta - \varphi\right) v$ as a compensation for running the firm during Chapter 11 reorganization. This case is not treated is this paper but could be easily incorporated in the analysis.

[10] Each creditor assume a part of the costs proportional to their contractual coupons.

As a consequence, the relative shares of debt and equity at v, for a given plan y, and accounting for bankruptcy costs, are given respectively by:

$$S^I_{t^*}(v,y) \equiv S_{t^*}(v,y) - \frac{c_s}{c_s+c_j} C(v,t^*) \tag{24}$$

and

$$J^I_{t^*}(v,y) \equiv J_{t^*}(v,y) - \frac{c_j}{c_s+c_j} C(v,t^*)$$

where $S_{t^*}(v,y)$ and $J_{t^*}(v,y)$ represent respectively the senior and junior share in the reorganized firm, as defined in (20), and

$$E^I_{t^*}(v,y) == E_{t^*}(v,y) \tag{25}$$

where $E_{t^*}(v,y)$ is the equityholders' share as defined in (21).

If however the firm does not emerge from Chapter 11 but is liquidated, the presence of bankruptcy costs modifies the absolute priority rule, since these costs are deemed in priority. We assume that the bankruptcy costs are paid at the emergence from Chapter 11, either through liquidation or reorganization. The senior, junior and equityholders liquidation payoffs are then given respectively by

$$\begin{aligned} L_s\omega^L &= L_s(v,c_s,t^*) \\ &= \min\left[(1-\alpha)v - C(v,t^*), \frac{c_s}{r}\right], \end{aligned} \tag{26}$$

$$\begin{aligned} L_j\omega^L &= L_j(v,c_j,t^*) \\ &= \min\left[\max\left[(1-\alpha)v - C(v,t^*) - \frac{c_s}{r}, 0\right], \frac{c_j}{r}\right] \end{aligned} \tag{27}$$

and

$$\begin{aligned} L_e\omega^L &= L_e(v,c,t^*) \\ &= \max\left[(1-\alpha)v - C(v,t^*) - \frac{c}{r}; 0\right]. \end{aligned} \tag{28}$$

5 Optimal reorganization plans

As seen in the preceding section, for a given value of the assets, the value of the reorganized firm depends on the relative share of the equity and debt, represented by the dependence on y. In this section, we solve, for a given value of the assets v and time elapsed $t = kd$, the optimization problem of Leader as described by $(4) - (8)$, that is solving for p^*

$$\max_p p_l \omega^I(p) \tag{29}$$

$$s.t. \qquad p_i \omega^I(p) \geq b_i,\ i = \{f_1,\ f_2\} \tag{30}$$

where b_i is an arbitrary constant, representing what Follower i can achieve otherwise. If the Leader's strategy is to propose a plan expecting it to be accepted by both followers, i.e. (A,A), then b_i, $i = f_1$, f_2 is defined in (5). If however the Leader's strategy is to propose a plan and expecting it to be accepted by Follower 1 and rejected by Follower 2, i.e. (A,R), then b_{f_1} is defined in (7) and b_{f_2} is defined in (8). Finally, if the Leader's strategy is to propose a plan and expecting it to be rejected by Follower 1 and accepted by Follower 2, i.e. (R,A), then b_{f_1} and b_{f_2} are defined as in the (A,R) case by reversing the identities of the followers.. According to the identity of the Leader, we consider two cases.

5.1 Debtor-in-possession' plan

The DIP has the opportunity to propose the first reorganization plan at $t = d$. Then, the Leader is the DIP who maximizes

$$\begin{aligned} p_e \omega^I(p) &= E^I(v, y) \\ &= \frac{v}{\lambda}\left(\lambda - y + (1-\lambda)\, y^{1/(1-\lambda)}\right) \end{aligned} \tag{31}$$

by deciding on the reorganized coupons through $y = \frac{(1-\tau)\lambda}{rv} c^*$, as defined in (18). Differentiating (31) with respect to y yields

$$\frac{d}{dy}\left(p_e \omega^I(v, y)\right) = -\frac{v}{\lambda}\left(1 - y^{\lambda/(1-\lambda)}\right) < 0.$$

The objective function is decreasing in $y \in [0, 1]$. Solution of problem (29) under conditions (30) is therefore obtained by setting senior and junior creditors as the followers of this bargaining round, where $p_i \omega^I(p) = b_i$, $i = s, j$; that is, solving for y_e the following:

$$S^I(v, y) + J^I(v, y) = b_s + b_j \tag{32}$$

and from which the value of the reorganized firm and the payoff of each player can then be obtained using $(23) - (25)$.[11]

[11] For the resolution of equation (32), see Appendix B.1.

5.2 Creditor's plan

Except for the first bargaining round, creditors alternate in proposing reorganization plan for subsequent rounds. If Leader is one of the creditors, $l = s, j$; we index the other creditor by f and denote O_f his total share. Leader maximizes his share in the reorganized firm

$$p_l \omega^I (p) = S^I (v, y) + J^I (v, y) - O_f$$

by deciding on y and on the relative share of the follower creditor O_f. For a given y, it is apparent that the objective function is decreasing in O_f, so that the optimal solution is such that

$$O_f = b_f \ , \tag{33}$$

and Leader maximizes

$$\frac{v}{\lambda}\left(\frac{1}{1-\tau}y - \left(\frac{1}{1-\tau} - \lambda\left(1-\alpha\right)\right) y^{1/(1-\lambda)}\right) - C(v,t) - b_f \tag{34}$$

This is a concave function admitting a maximum at

$$y_d^* = \left(\frac{1-\lambda}{1-\lambda\left(1-\tau\right)\left(1-\alpha\right)}\right)^{(1-\lambda)/\lambda} . \tag{35}$$

There are two possibilities for the optimal plan, depending on the fact if y_d^* satisfies the constraint (30) or not:

The DIP gets more than otherwise payoff (b_e) In this case

$$E^I (v, y_d^*) > b_e$$

and the optimal plan proposed by the Leader is y_d^*. The share of the Leader creditor is

$$p_l^* \omega^I (p^*) = S^I (v, y_d^*) + J^I (v, y_d^*) - b_f,$$

the share of the DIP is

$$p_e^* \omega^I (p^*) = E^I (v, y_d^*)$$

and the share of the Follower creditor is

$$p_f^* \omega^I (p^*) = b_f.$$

The DIP gets otherwise payoff (b_e) In this case, solution of problem (34) is obtained by setting the shares of the DIP and the other creditor to their lower bound, such that

$$p_e^* \omega^I(p^*) = b_e \tag{36}$$

and

$$p_f^* \omega^I(p^*) = b_f. \tag{37}$$

The optimal plan for the Leader then solves for (36), which is rewritten as follows:

$$y_l^* - (1-\lambda)(y_l^*)^{1/(1-\lambda)} = \lambda\left(1 - \frac{b_e}{v}\right) \tag{38}$$

and which has a unique solution in $[0,1]$ if $0 < b_e < v$ (and no solution otherwise) from which the value of the firm and the shares of all claimants can be found using (23) – (25).[12]

5.3 The bankruptcy judge's plan

Recent empirical papers (e.g. Chang and Schoar (2006), Bris et al. (2006)) report on the impact of the bankruptcy judge's strategies and behaviors on the outcome of the game. In order to incorporate this feature, we the judge not only to cram-down the Leader's plan, but also to impose his own reorganization plan: in our model, when the judge interferes in the negotiation process, he has the opportunity to end the negotiation process by imposing the last reorganization plan that was rejected by the players, or to impose his own reorganization plan.

The definition of the judge's plan we use is consistent with the Code, which stipulates that the cram-down plan must be *fair and equitable*. To ensure the fairness condition, the judge offers to each player at least the liquidation payoff, and distributes the residual value among the claimants according to the same sharing vector β in X. Therefore, we can write, for a given v, each claimant's payoffs as following[13]

$$\gamma_i \omega_{t^*}^I(\gamma) == \beta_i \underbrace{\left(\omega_{t^*}^I(\gamma) - \sum_i L_i(v,c,t^*)\right)}_{\text{Residual value}} + L_i(v,c,t^*), \quad i \in \{s,j,e\} \tag{39}$$

where c represents the total contractual coupon, t^* the emergence time from Chapter 11, and $L_i(v,y)$, $i = s,j$ are defined as in (26) – (27) thus including the bankruptcy costs. In this case,

[12] For the resolution of equation (38), see Appendix B.2.
[13] We assume that claimants have common knowledge on the judge's expected strategy and β.

$\beta \in X$ measures the proportion of the benefit of cooperation that each player gets in cram-down, and which is decided by the judge.[14] For a given v, the value of y is obtained by solving for a given v and t^*

$$E^I_{t^*}(v, y) == L_e(v, t) + \beta_e \left(\omega^I_{t^*}(v, y) - \omega^L_{t^*}(v, t)\right) \tag{40}$$

[which has a unique solution in $[0, 1]$ if $L_e(v, t) < (1 - \beta_e \alpha) v$, from which the payoffs of all claimants can be obtained as a function of v using $(23) - (25)$.[15]

6 Numerical implementation

We propose a numerical procedure that combines a dynamic programming (DP) with the finite elements technique to price the optimal capital structure of a distressed firm and define the equilibrium strategy.

6.1 Dynamic programming

For each strategic player or claimant, the payoff at the end of the negotiation process at $t = (K + 1) d$ is given by the solution to the static game described in table 1, where $q = 1$.

The players' holding values $h^h_{i,t}(v)$ at time $t = kd$ and asset's value $v = V_t$, for $k = \{1, .., K\}$, can be written as

$$h^h_{i,t}(v) = \mathbb{E}_{(v,t)}\left[h_{i,t+d}(V_{t+d}) e^{-rd}\right], \text{ for } i = \{s, j, e\}, \tag{41}$$

Moreover, the continuation firm's value at any time $t = kd$, denoted by ω^C, is defined as

$$\omega^C_t(v) = \sum_{i=\{s,j,e\}} h^h_{i,t}(v). \tag{42}$$

6.2 Piecewise linear approximation

Let $a_0 < a_1 < ... < a_m < .. < a_M < a_{M+1}$ be a set of points on the space of asset values, where $a_0 = 0$ and $a_{M+1} \to +\infty$. For each dynamic programming value function of this model, the

[14] In fact, this plan can be assimilated to the solution of a Nash bargaining game, where β represents the vector of the different claimant's bargaining powers, and the liquidation payoff as the threat in case of not cooperating.

[15] For the resolution of equation (40), see Appendix B.3.

piecewise-linear interpolation between the evaluation points a_m at $t = kd$ has the form

$$\begin{aligned}
\widehat{h}_{i,t}(v) &= \begin{cases} 0 & \text{for } v < a_0 \\ \theta^t_{i,m} + \psi^t_{i,m} v & \text{for } a_m \le v \le a_{m+1},\ m = 0, ..., M \end{cases} \\
&= \sum_{m=0}^{M} \left(\theta^t_{i,m} + \psi^t_{i,m} v\right) \mathbf{1}_{(a_m \le v < a_{m+1})}
\end{aligned}$$

then, solving for these equations, we get

$$\begin{array}{c}
\theta^t_{i,m} = \frac{\widehat{h}_{i,t}(a_m).a_{m+1} - \widehat{h}_{i,t}(a_m).a_m}{a_{m+1} - a_m} \\
\text{and} \\
\psi^t_{i,m} = \frac{\widehat{h}_{i,t}(a_{m+1}) - \widehat{h}_{i,t}(a_m)}{a_{m+1} - a_m}
\end{array} \tag{43}$$

for $m = 1, ..., M$, and we add the following restrictions: $\theta^t_{i,m+1} = \theta^t_{i,m}$ and $\psi^t_{i,m+1} = \psi^t_{i,m}$ for $m = \{0, M\}$.

Using the interpolated value function $\widehat{h}_{i,((K+1)d)}(v)$, into the expression (41), yields for any $k = \{0, ..., K\}$ and $t = kd$

$$\begin{aligned}
\widehat{h}^h_{i,t}(a_n) &= \mathbb{E}_{a_n,t}\left[\widehat{h}_{i,t}(V_{t+d}) e^{-rd}\right] \\
&= \mathbb{E}_{a_n,t}\left[\sum_{m=0}^{M} \left(\theta^{t+d}_{i,m} + \psi^{t+d}_{i,m} V_{t+d}\right) \mathbf{1}_{(a_m \le V_{t+d} < a_{m+1})}\right] \\
&= \sum_{m=0}^{M} \left(\theta^{t+d}_{i} A_{n,m} + \psi^{t+d}_{i} B_{n,m}\right) \qquad (44)
\end{aligned}$$

where $A_{n,m}$ and $B_{n,m}$ are transition matrices from state a_n to state a_m

$$A_{n,m} = \mathbb{E}_{(a_n,t)}\left[\mathbf{1}_{(a_m \le V_{t+d} < a_{m+1})} e^{-rd}\right]$$

and

$$B_{n,m} = \mathbb{E}_{(a_n,t)}\left[V_{t_{k+1}} \mathbf{1}_{(a_m \le V_{t+d} < a_{m+1})} e^{-rd}\right].$$

After computations[16]

$$A_{n,m} = \left[\Phi(x_{n,m+1}) - \Phi(x_{n,m})\right] e^{-rd} \tag{45}$$

and

$$B_{n,m} = a_n \left[\Phi\left(x_{n,m+1} - \sigma\sqrt{d}\right) - \Phi\left(x_{n,m} - \sigma\sqrt{d}\right)\right] e^{-\delta d} \tag{46}$$

where

$$\begin{cases}
x_{n,m+1} = \frac{\log\left(\frac{a_{m+1}}{a_n}\right) - \left(r - \delta - \sigma^2/2\right)d}{\sigma\sqrt{d}} \\
\text{and} \\
x_{n,m} = \frac{\log\left(\frac{a_m}{a_n}\right) - \left(r - \delta - \sigma^2/2\right)d}{\sigma\sqrt{d}}
\end{cases}$$

[16] See Appendix C.

6.3 Algorithm

We specify the procedure to be implemented in order to define the equilibrium strategy at each bargaining round k and compute the corresponding claimant's payoffs:

1. For $k = K + 1$ to 0,
2. Compute the judge's impatience $q_k = \frac{k}{K+1}$,
3. Compute the liquidation payoffs $L_i(a_m, kd)$ from (26)-(28), for $m = 1, ..., M$ and $i = \{s, j, e\}$,
4. Compute the holding payoffs $\widehat{h}^h_{i,kd}$ by (44), for $i = \{s, j, e\}$,
5. Compute the continuation firm's value $\omega^C_{kd}(a_m)$ by (42), for $m = 1, ..., M$,
6. Solve for the optimal reorganization plan, as described in Section (5). We then obtain reorganized firm's value $\omega^I(v)$, for $m = 1, ..., M$,
7. Identify the optimal decision of the Leader, whether he proposes a plan to be accepted by at least one follower or unanimously rejected by the followers.
8. Compute $h_{i,kd}(a_m)$ corresponding to the equilibrium strategy.

7 Numerical analysis

For our numerical illustration, we use the following parameters for the dynamics of the assets value: the risk-free interest rate $r = 8\%$, the payout ratio $\delta = 6\%$.[17] The volatility $\sigma \in [20\%, 80\%]$ is an input in the model, and reflects the depth of financial distress. To compute the value of the reorganized firm, we set the tax rate $\tau = 15\%$,[18] the liquidation costs $\alpha = 8.1\%$ and the direct bankruptcy costs $\varphi = 16.9\%$.

The Code specifies that the minimum length of the first bargaining rounds is 120 days, and that it can generally be extended. For simplicity, we assume that bargaining rounds are of equal length, such that $d = 6$ months. At the beginning of Chapter 11, we assume that judge decides on the maximum length of Chapter 11 reorganization, i.e. the total number of bargaining rounds K and the length of each round d. Moreover, We assume that the impatience of the judge increases with time, such that $q_k = \frac{k}{K+1}$, where $K + 1$ represents the maximum number of reorganization rounds decided by the bankruptcy judge at the beginning of the reorganization under Chapter 11. We assume that these features are known by all the claimants at the beginning of Chapter 11's reorganization.

7.1 The bankruptcy judge's plan

In this section, we study the particular features of the fair cram-down plan described in section (5.3), and the impact of the judge's strategy on the different claimants shares.[19] Bris et al. (2006) find that APR violations are strongly judge specific. For example, when comparing APR violations among Arizona and New York court, they find that Arizona judges violate APR more than New York judges, which is explained in part by the judge's identity.

Unlike previous studies in the literature, we focus on the importance of the judge's strategy and its impact on debtor's choice of a supervising court. In fact, we assume that bankruptcy petitions are filed "voluntarily" by debtors.[20] Moreover, debtors in large corporate bankruptcy have generally the choice of the district in which to file. Weiss (1990) reports that evidence supports that debtors file in the district they think will be most favorable to them. As in Bris et al. (2006) and Chang

[17] Huang and Huang (2003) compute r as an average over the period $1985 - 1995$ and δ as the average of dividends and coupons over the period $1973 - 1998$.

[18] See Graham (2003)

[19] In this study, we are not looking for the motivations beyond judge's behavior, but rather to its impact on the equilibrium strategies and the value of claims in the reorganized firm.

[20] Another way is an "unvoluntary" filing by creditors, which is not discussed in this paper.

and Schoar (2006), we assume that judges are regrouped into two distinct classes: equity-favoring and creditor-favoring judges.

Under the fair cram-down plan defined in section 5.3, the judge does not decide on the different shares γ directly, but rather on the allocation of the benefit of cooperation proportional to the allocation of benefit of cooperation among the claimants, where each claimant's share is expressed as in (39). We assume that equityholders are favored over creditors (senior and junior) if $\beta_e > 0.5$. Moreover, senior creditor is favored over junior creditor if $\beta_s > \frac{1-\beta_e}{2}$. In case of a fair cram-down strategy, claims are bounded by the liquidation payoffs, that respect APR. The minimum share allocated by the judge to different claimants, at $t = (K+1)\,d$, and $v = V_t$, which is defined as as

$$(\gamma_i)_{\min} \geq \frac{L_i(v, y)}{\omega^I(\gamma)} \text{ for } i \in \{s, j, e\} \tag{47}$$

and is independent on the judge's behavior (equity-favouring versus creditor-favouring). Figure 3 shows the minimum cram-down shares allocated to claimants at round $k = 1$. When the asset values are low (region 1) ($(1-\alpha)v - C\,(v, d) < \frac{c_s}{r}$), and junior creditor and equityholders does not expect to recover in case of liquidation ($L_j\,(v, t) = L_e\,(v, t) = 0$), then the senior creditor collects the greater part in the reorganized firm. However, when the asset value is increasing (region 2) ($\frac{c_s}{r} < (1-\alpha)v - C\,(v, d) < \frac{c_s+c_j}{r}$), the junior creditor starts collecting a part in the reorganized firm. Finally, when the threat of liquidation is vanished (region 3) ($\frac{c_s+c_j}{r} < (1-\alpha)v - C\,(v, d)$), then equityholders shares increase at the expense of creditors shares and the APR between equityholders and creditors is violated.

Figure 4 illustrates the impact of judge's behavior on cram-down payoffs, by showing the relationship between claims values for varying equityholders shares allocated by the judge. Note that at low levels of β_e, the APR in the sharing of the benefit of cooperation is strictly observed between the claimants in the restructured firm. However, as β_e increases, equityholders are gaining shares in the restructured firm, and the APR is no more respected, between equityholders and junior creditor first on one hand, and then between equityholders and creditors on the other hand.

7.2 Equilibrium strategies and post-bankruptcy credit spreads

As pointed out by Bris et al. (2006), one important characteristic of the supervising bankruptcy court is the behavioral difference among the bankruptcy judges. They empirically document three factors that statistically differentiates the judges: the fraction paid out to creditors in case of cram-down[21], the duration of Chapter 11 negotiation[22] and the violation of APR[23]. We measure the sensitivity of the equilibrium strategies, for a given negotiation length, to the judge's strategies and behaviors. Figure 5 show that the equilibrium strategies depend on the following factors:[24]

1. The identity of the claimant who is proposing the reorganization plan: As explained and motivated in section 3.1, the debtor-in-possession, who represents the equityholders during the negotiation, proposes the first plan (at $k = 1$); the junior creditor proposes the second plan (at $k = 2$), and the senior creditor proposes the third plan (at $k = 3$). For a given judge's strategy z and behavior β, we find that the equilibrium strategy depend on the leader's identity. For example, if we consider the middle panel ($z = 0.5$) and we compare the cases where the judge is equity-favouring, we notice that the senior accepts the DIP's plan at $k = 1$ for some value of assets ($V > 23.17$), whether he always rejects junior's plan at $k = 2$.

2. The bankruptcy judge's behavior (β) which is reflected by his favoritism towards one class of claimants. If we take the example in the middle panel where $k = 1$, $z = 0.5$ and $q = 1/3$, we find that when the judge becomes more creditor-favouring, the propensity of the senior creditor to accept the DIP's plan increases.

3. The bankruptcy judge's strategy (z), i.e. whether he imposes the leader's rejected plan or his own reorganization plan. Except for the case where $k = 2$ and the junior proposes the plan, the bankruptcy judge's strategy has an impact on the equilibrium strategy.

Then, depending on the judge's strategy and behavior, Chapter 11 reorganization requires more than one plan of reorganization before a reorganization is reached. Carapeto (2005) find that about two-thirds of Chapter 11 firms in a sample of 144 firms that reorganized successfully over the period January 1986 to December 1997 needed more than one reorganization plan. She explains the multiplicity of the plans by a possible imperfect and incomplete information that arises between players during the negotiation. In our paper, we explain multiple-plan reorganization by the uncertainty about the judge's intervention in the negotiation process. If the judge does not intervene in the

[21] In our paper, we consider that this fraction is represented by the shares γ_s and γ_j offered by the judge when he imposes his own reorganization plan. From (39), this fraction depends mainly on the vector β.

[22] Represented by the length of negotiation, dk, $0 < k \leq K + 1$.

[23] By defining the bankruptcy judge's plan as *fair and equitable*, we ruled out the possibility of APR violation.

[24] Bris et al. (2006) show that Chapter 11 reorganization takes about two years to resolve. In figure 5, we assume that the maximum length is 1.5 years.

negotiation process at any round, i.e. $q_k = 0$, $0 < k < K$, we can show that the firm is always reorganized at the first round.

We also study the impact of renegotiation on the credit spreads of senior and junior subordinated corporate debt. Intuitively, compared with a pure liquidation case[25], we expect a decrease in credit spreads due to the opportunity to renegotiate. For any promised reorganized coupons c^* at reorganization time $t = t^*$, post-bankruptcy credit spreads on corporate debt in the reorganized firm are defined by

$$Y(v, c^*) = \frac{c^*}{S_t(v, y) + J_t(v, y)} - r \tag{48}$$

where the reorganized coupon c^* is obtained from (18) as follows

$$c^* = \left(\frac{r}{\lambda(1-\tau)} v\right) y(c^*, v). \tag{49}$$

In order to define the senior and junior reorganized coupons, c_s^* and c_j^*, we have to solve

$$x_s \omega(x_s) = \frac{c_s^*}{r}\left(1 - y^{\lambda/(1-\lambda)}\right) + \min\left[(1-\alpha) B, \frac{c_s^*}{r}\right] y^{\lambda/(1-\lambda)},\ x_s = \{p_s, \gamma_s\} \tag{50}$$

and

$$c_j^* = c^* - c_s^*, \tag{51}$$

where $x_s \in X$ represents the share of the senior creditor in the reorganized firm, obtained when the Leader implements his reorganized plan p, or where the judge imposes his own plan γ. Moreover, the endogenous liquidation barrier B is given by

$$B = yv. \tag{52}$$

Then, the credit spread for the different classes of debt is defined as

$$Y_s(v, c_s^*) = \frac{c_s^*}{S_t(v, y)} - r \tag{53}$$

and

$$Y_j\left(v, c_j^*\right) = \frac{c_j^*}{J_t(v, y)} - r \tag{54}$$

In Figure (6), we measure the sensitivity of the reorganized coupons to the judge's strategy, and compare the obtained coupons to the contractual ones. We find that, for senior as well as junior reorganized coupons, an equity-favouring judges induces a decreasing in the reorganized

[25] For example, as in Leland (1994).

coupons level. This implies that a pro-debtor bankruptcy judge prevents creditors from extracting the maximum wealth from the reorganized firm. Moreover, we notice that for high levels of assets, and for any bankruptcy judge's strategy, the new debt contract offers to creditors more than the contractual coupons of the distressed firm. The sensitivity of post-default credit spread to the bankruptcy judge's behavior is illustrated in Figure (7). We show that pro-debtor judge's behavior leads to an increase is senior post-default credit level. If we admit that a decrease in credit ratings induces an increase in credit spreads, our results are thus consistent with the empirical findings of Chang and Schoar (2006). When it comes to the junior credit spread, we notice that its evolution depends on the junior's liquidation payoffs[26]. In fact, we delimited three regions in junior credit spread behavior: In the first and second one (from the left), the junior collects less than his contractual coupons, and in the third regions, he recovers more than c_j. In this last region, the junior credit spread, as well as the senior one, increases when the judge becomes more equity-favoring.

In Figure (8), we find that the credit spread level, for senior as well as junior creditors, is an increasing function of the assets' volatility, which is consistent with some very well known stylized facts as well as the subordination feature between senior and subordinated debt. Moreover, for each volatility level, the credit spread level for the subordinated debt is higher than the credit spread level for the senior debt (for example, when $\sigma = 50\%$, the credit spread level for senior debt is around 5%, and the credit spread level for junior debt is around 20%). In Figure (9), we measure the sensitivity of the senior and junior reorganized coupons levels to the initial leverage of the distressed firm, which also expresses the depth of financial distress. We find that reorganized firm leverage is a positive function of distressed firm leverage. Moreover, we find that post-bankruptcy credit spread levels are not sensitive to the initial leverage.

8 Conclusion

The application of game theory to the resolution of financial distress is a growing field in the literature. We extend the existing credit risk literature by proposing a model that rigorously solves a multiple-stage bargaining game where in each stage there is a mix of sequential and simultaneous games. Our model accommodates several features of Chapter 11 and considers two classes of

[26] The reorganized senior debt value, that affects the estimation of credit spreads (see 53), depends on the liquidation payoffs through the bankruptcy judge's plan (see 39)

creditors. As stipulated in the Code, we allow the bankruptcy judge to interfere in the reorganization process to impose the rejected plan, his own reorganization plan, or to convert Chapter 11 reorganization into Chapter 7 liquidation. We formulate the negotiation process as a dynamic programming problem and numerically solve for the optimal capital structure using a piecewise approximation of the reorganized firm value. We also provide a numerical illustration and show that the equilibrium strategies and post-bankruptcy credit spreads depend on the bankruptcy judge's behavior and strategy. Our results open the way for a deeper investigation of the efficiency of the Chapter 11 as described in the Code. This includes for instance asymmetric information problems that can arise during negotiation, and the possibility for bankruptcy judge to extend the exclusivity period at the request of the DIP. These and other extensions are left for future works.

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A Proof of equation (15)

When the firm emerges from financial distress at t^*, its value is given by

$$\omega_{t^*}(v) = \mathbb{E}_{t^*}^{Q}\left[\int_{t^*}^{T}\left(\delta V_t + \tau c^*\right)e^{-r(t-t^*)}dt\right] + \alpha\mathbb{E}_{t^*}^{Q}\left[Be^{-r(T-t^*)}\right]$$

which can be decomposed into three terms that will be calculated separately

$$\omega_{t^*}(v) = \underbrace{\delta\mathbb{E}_{t^*}^{Q}\left[\int_{t^*}^{T} V_t e^{-r(t-t^*)}dt\right]}_{(A.1)} + \underbrace{\tau c^*\mathbb{E}_{t^*}^{Q}\left[\int_{t^*}^{T} e^{-r(t-t^*)}dt\right]}_{(A.2)}$$
$$-\underbrace{\alpha\mathbb{E}_{t^*}^{Q}\left[Be^{-r(T-t^*)}\right]}_{(A.3)}$$

We define the following Radon-Nikodym derivative

$$\frac{dP}{dQ}\mid_{F_t} = e^{-\frac{b}{2}t + b\widetilde{W}_t}$$

where $V_t = V_s e^{\sigma \widetilde{W}_{(t-s)}}, t^* \leq s \leq t$ and $\widetilde{W}_t = W_t + bt$ is a $P-$Brownian motion, where $b = \frac{\left(r-\delta-\frac{\sigma^2}{2}\right)}{\sigma}$. Moreover, We make a change of variables such that $s = t - t^*$, $ds = dt$ and $z = t - T$, $dz = dt$. Then, we can re-rewrite the term $(A.1)$ as follows

$$v\left[\underbrace{\mathbb{E}_{t^*}^{P}\left[\int_{0}^{+\infty} e^{(\sigma+b)\widetilde{W}_s}e^{-\frac{(\eta)^2}{2}s}ds\right]}_{(A.1.1)} - \underbrace{\mathbb{E}_{t^*}^{P}\left[\int_{0}^{+\infty} e^{(\sigma+b)\widetilde{W}_{z+(T-t^*)}}e^{-\frac{(\eta)^2}{2}(z+(T-t^*))}dz\right]}_{(A.1.2)}\right]$$

where $\eta = \sqrt{2r+b^2}$. We will make the calculus thanks to the following lemma (see Karatzas and Shreve [1991, p.272])

Lemma 1 *If $\varphi : R \to R$ is a piecewise continuous function with*

$$\int_{-\infty}^{+\infty} dy\left|\varphi(x+y)\right|e^{-|y|\sqrt{2\alpha}} < \infty, \nabla x \in R$$

for some constant $\alpha > 0$ and $(W_t, t \geq 0)$ is a standard Brownian motion, the resolvent operator of Brownian motion $K_\alpha(\varphi)$ is defined by

$$K_\alpha(\varphi) \triangleq \mathbb{E}\left\{\int_{0}^{+\infty} dt\ e^{-\alpha t}\varphi(W_t)\right\} = \frac{1}{\sqrt{2\alpha}}\int_{-\infty}^{+\infty} dy\varphi(y)e^{-|y|\sqrt{2\alpha}}$$

then, by applying lemma 1, $(A.1.1)$ could be rewritten as

$$\begin{aligned}
& \frac{1}{\eta}\int_{-\infty}^{+\infty} e^{-|y|\eta} e^{(\sigma+b)y} dy \\
= & \frac{1}{\eta}\left[\int_{-\infty}^{0} e^{(\sigma+b+\eta)y} dy + \int_{0}^{+\infty} e^{(\sigma+b-\eta)y} dy\right] \\
= & \frac{1}{\eta}\left[\frac{1}{\sigma+b+\eta} - \frac{1}{\sigma+b-\eta}\right] \\
= & \frac{1}{\delta}
\end{aligned}$$

and the calculations of $(A.1.2)$ are made in the same way, where

$$\begin{aligned}
& \mathbb{E}_{t^*}^{P}\left[e^{bf-|f|\eta}\int_{0}^{+\infty} e^{(\sigma+b)\widetilde{W}_z} e^{-\frac{(\eta)^2}{2}z} dz\right] \\
= & \frac{1}{\delta}\left(\frac{v}{B}\right)^{-\lambda/(1-\lambda)}
\end{aligned}$$

where $f = \frac{1}{\sigma}\log\left(\frac{B}{v}\right)$, $\lambda = \frac{b+\eta}{b+\eta+\sigma}$, $b = \frac{\left(r-\delta-\frac{\sigma^2}{2}\right)}{\sigma}$ and $\eta = \sqrt{2r+b^2}$. Then, term $(A.2)$ is calculated in the same manner as $(A.1)$,

$$\mathbb{E}_{t^*}^{Q}\left[\int_{t^*}^{T} e^{-r(t-t^*)} dt\right] = \frac{1}{r}\left[1-\left(\frac{v}{B}\right)^{-\lambda/(1-\lambda)}\right]$$

and finally, $(A.3)$ is given by

$$\begin{aligned}
& \mathbb{E}_{t^*}^{Q}\left[Be^{-r(T-t^*)}\right] \\
= & B\mathbb{E}_{t^*}^{P}\left[e^{b\widetilde{W}_{(T-t^*)}} e^{-\frac{(\eta)^2}{2}(T-t^*)}\right] \\
= & Be^{bf-|f|\eta} = B\left(\frac{v}{B}\right)^{-\lambda/(1-\lambda)}
\end{aligned}$$

Finally, the value of the firm at the end of the second stage is given by

$$\omega_{t^*}(v) = v + \frac{\tau c^*}{r}\left(1-\left(\frac{B}{v}\right)^{\lambda/(1-\lambda)}\right) - \alpha B\left(\frac{B}{v}\right)^{\lambda/(1-\lambda)}.$$

B Optimal reorganization plans

B.1 Debtor-in-possession optimal reorganization plan

We formulate the equation (32) as follows:

$$\begin{aligned}
f(v, y_e) &= \frac{v}{\lambda}\left(\frac{1}{1-\tau}y - \left(\frac{1}{1-\tau} - \lambda(1-\alpha)\right)y^{1/(1-\lambda)}\right) - C(v, t^*) - (b_s + b_j) \\
&= 0.
\end{aligned}$$

$$\frac{d^2}{dy^2} f(v,y) = -\frac{\lambda}{(1-\lambda)^2}\left(\frac{1}{1-\tau} - \lambda(1-\alpha)\right) y^{(2\lambda-1)/(1-\lambda)} < 0$$

f is a concave function and y_e^* is the solution of

$$\frac{d}{dy} f(v,y) = \frac{1}{1-\tau} - \frac{1}{1-\lambda}\left(\frac{1}{1-\tau} - \lambda(1-\alpha)\right) y^{\lambda/(1-\lambda)} = 0.$$

such that

$$y_e^* = \left(\frac{1-\lambda}{1-\lambda(1-\tau)(1-\alpha)}\right)^{\frac{1-\lambda}{\lambda}} < 1.$$

From the definition of y_e in (18), we know that $0 < y_e < 1$. Moreover

$$f(v,0) = -\left[(b_s + b_j) + C(v,t)\right]$$
and
$$f(v,1) = v(1-\alpha) - \left[(b_s + b_j) + C(v,t)\right]$$

Given that the function $f(v,y)$ is a concave function in y, then

It has a unique root in $[0,1]$ if this function crosses the x-axis one time, i.e. $f(v,0) < 0$ and $f(v,1) > 0$. These conditions are satisfied if

$$0 < (b_s + b_j) + C(v,t) < v(1-\alpha)$$

It has two roots in $[0,1]$ if $f(v,0) < 0$, $f(v,1) < 0$ and $f(v,y_e^*) > 0$. In this case,

$$\left\{\begin{array}{c} v(1-\alpha) < (b_s + b_j) + C(v,t) \\ \text{and} \\ f(v,y_e^*) > 0 \end{array}\right.$$

and the DIP will choose the smallest root.

It has no roots in $[0,1]$ when the function f never crosses the x-axis, which occurs in the following cases:

Case 1: $f(v,0) < 0$, $f(v,1) < 0$ and $f(v,y_e^*) < 0$. These conditionals are expressed as follows:

$$\left\{\begin{array}{c} v(1-\alpha) < (b_s + b_j) + C(v,t) \\ f(v,y_e^*) < 0 \end{array}\right.$$

This case occurs if

$$p_e^* \omega^I(p^*) \frac{v}{\lambda}\left(\lambda - y_d^* + (1-\lambda)(y_d^*)^{1/(1-\lambda)}\right) < b_s + b_j,\ \forall v,t > 0$$

Then, any organization's plan proposed by DIP is rejected by creditors.

Case 2: $f(v,0) > 0$, $f(v,1) > 0$ and $f(v,y_e^*) > 0$. In this case

$$\begin{gathered} (b_s + b_j) + C(v,t) < 0 \\ v(1-\alpha) - [(b_s + b_j) + C(v,t)] > 0 \\ f(v, y_e^*) > 0 \end{gathered}$$

where the second condition is always satisfied in this case. This case occurs if

$$p_e^* \omega^I(p^*) \frac{v}{\lambda} \left(\lambda - y_d^* + (1-\lambda)(y_d^*)^{1/(1-\lambda)} \right) > b_s + b_j,\ \forall v, t > 0$$

Then, any organization's plan proposed by DIP is accepted by creditors. We set $y_e = 0$ and senior and junior creditors get nothing.

B.2 Creditor optimal reorganization plan

Solving for (36) in the general case, where Leader can be senior or junior creditor, leads to

$$g(v,y) = \frac{v}{\lambda} \left(\lambda - y + (1-\lambda) y^{1/(1-\lambda)} \right) - b_e = 0$$

which is a decreasing function in y. In fact,

$$\frac{d^2}{dy^2}(g(v,y)) = \frac{\lambda}{(1-\lambda)} y^{1/(\lambda-1)} > 0$$

and

$$\frac{d}{dy}(g(v,y)) = y^{\lambda/(1-\lambda)} - 1 < 0$$

Moreover,

$$g(v,0) = v - b_e$$

and

$$g(v,1) = -b_e.$$

Given that the function $g(v,y)$ is a convex function in y, then

It has a unique root in $[0,1]$ if this function crosses the x-axis one time, i.e. $g(v,0) > 0$ and $g(v,1) < 0$. These conditions are satisfied if

$$b_e < v$$

It has no roots in $[0,1]$ if this function never crosses the x-axis.

Case 1 $g(v,0) < 0$ and $g(v,1) < 0$. These conditions are satisfied if

$$v < b_e$$

Case 2 $g(v,0) > 0$ and $g(v,1) > 0$. These conditions are never satisfied because $g(v,1) < 0$ always.

B.3 Judge optimal reorganization plan

Equation (40) can be rewritten as

$$\begin{aligned}
& f(v,y) \\
= & \left(1-\lambda+\beta_e\left(\frac{\tau}{1-\tau}+\lambda\alpha\right)\right)y^{1/(1-\lambda)}-\left(1+\beta_e\frac{\tau}{1-\tau}\right)y+\lambda\left(1-\alpha\beta_e-\frac{L_e(v,t)}{v}\right) \\
= & \ 0
\end{aligned}$$

This equation has a unique root, such that $\frac{d}{dy}f(v,y)=0$ leads to

$$y^* = \left(\frac{(1-\lambda)\left(1+\beta_e\frac{\tau}{1-\tau}\right)}{1-\lambda+\beta_e\left(\frac{\tau}{1-\tau}+\lambda\alpha\right)}\right)^{(1-\lambda)/\lambda} > 0$$

and

$$\frac{d^2}{dy^2}f(v,y) < 0$$

then f is a concave function and has a unique root under the following conditions:

$$f(v,0)=\lambda\left(1-\alpha\beta_e-\frac{L_e(v,t)}{v}\right) > 0 \text{ if } L_e(v,t) < (1-\beta_e\alpha)\,v$$

and

$$f(v,1) = -\lambda\frac{L_e(v,t)}{v} < 0.$$

C Computation of transition matrices

At $t = kd$, the first transition term is defined as

$$\begin{aligned}
A^t_{n,m} &= \mathbb{E}_{(a_n,t)}\left[\mathbf{1}_{(a_m \le V_{t+d} < a_{m+1})}|V_t = a_n\right]e^{-rd} \\
&= \mathbb{E}_{(a_n,t)}\left[\mathbf{1}_{\left(a_m \le V_t e^{(r-\delta-\sigma^2/2)d+\sigma Z\sqrt{d}} < a_{m+1}\right)}|V_t = a_n\right]e^{-rd}
\end{aligned}$$

where $Z \sim \mathcal{N}(0,1)$, which implies that

$$A_{n,m}^t = \left[\Phi\left(x_{n,m+1}\right) - \Phi\left(x_{n,m}\right)\right] e^{-rd},$$

where $x_{n,m+i} = \frac{\log\left(\frac{a_{m+i}}{an}\right) - \left(r-\delta-\sigma^2/2\right)d}{\sigma\sqrt{d}}$, for $i = 0, 1$, and Φ is the standard normal cumulative distribution function.

At $t = kd$, the second transition term is defined as

$$\begin{aligned} B_{n,m}^t &= \mathbb{E}_{(a_n,t)}\left[V_{t_{k+1}}\mathbf{1}_{(a_m \le V_{t+d} < a_{m+1})} e^{-rd} | V_t = a_n\right] \\ &= a_n e^{-\left(\delta+\sigma^2/2\right)d}\left[\mathbb{E}_{(a_n,t)}\left[e^{\sigma\sqrt{d}Z}\mathbf{1}_{Z<x_{n,m+1}}\right] - \mathbb{E}_{(a_m,t)}\left[e^{\sigma\sqrt{d}Z}\mathbf{1}_{Z<x_{n,m}}\right]\right] \end{aligned}$$

where

$$\begin{aligned} & \mathbb{E}_{(a_n,t)}\left[e^{\sigma\sqrt{d}Z}\mathbf{1}_{Z<x_{n,m+1}}\right] \\ = \; & \mathbb{E}_{(a_n,t)}\left[\int_{-\infty}^{x_{n,m+1}} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}Z^2+\sigma\sqrt{d}Z} dZ\right] \end{aligned}$$

Now writing

$$\begin{aligned} -\frac{1}{2}Z^2 + \sigma\sqrt{d}Z &= -\frac{1}{2}\left[Z^2 - 2\sigma\sqrt{d}Z + \sigma^2 d\right] + \frac{\sigma^2 d}{2} \\ &= -\frac{1}{2}\left[Z - \sigma\sqrt{d}\right]^2 + \frac{\sigma^2 d}{2} \end{aligned}$$

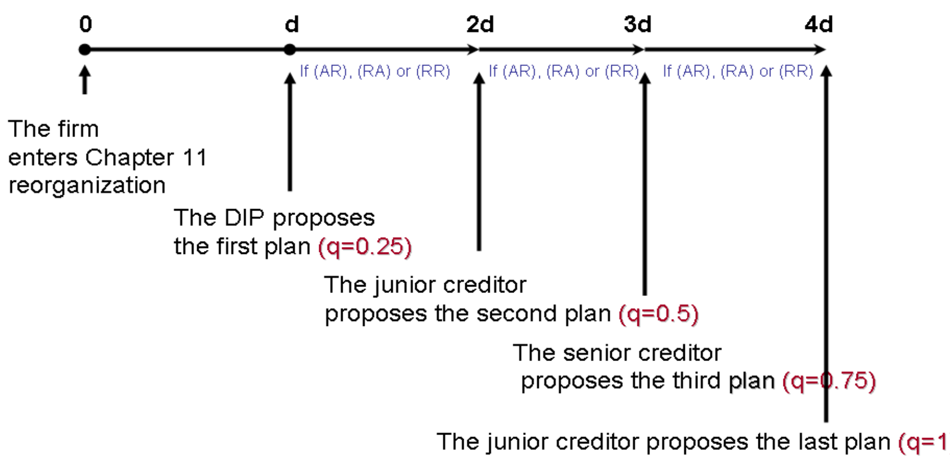
we have

$$\begin{aligned} \int_{-\infty}^{x_{n,m+1}} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}Z^2+\sigma\sqrt{d}Z} dZ &= e^{\frac{\sigma^2 d}{2}} \int_{-\infty}^{x_{n,m+1}} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left[Z-\sigma\sqrt{d}\right]^2} dZ \\ &= e^{\frac{\sigma^2 d}{2}} \int_{-\infty}^{x_{n,m+1}-\sigma\sqrt{d}} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} du \\ &= e^{\frac{\sigma^2 d}{2}} \Phi\left(x_{n,m+1} - \sigma\sqrt{d}\right), \end{aligned}$$

which implies that

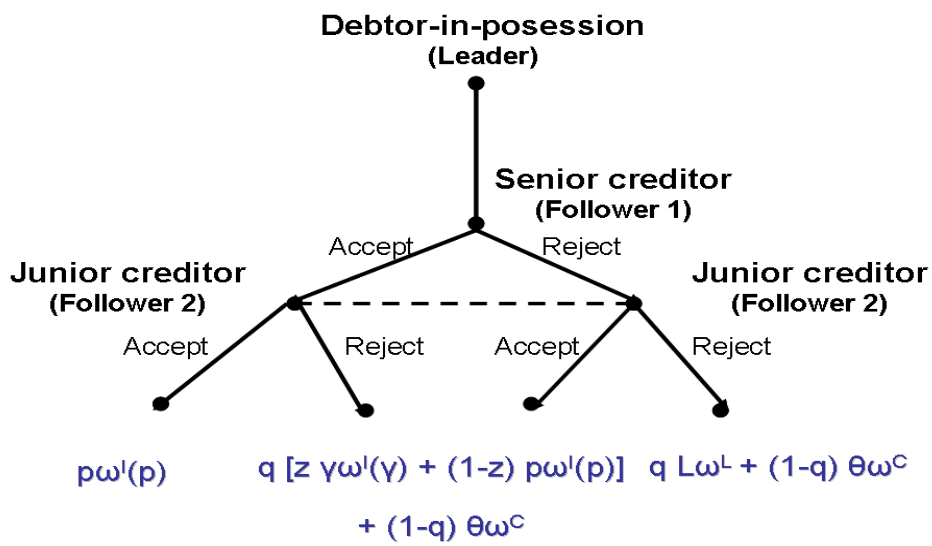
$$B_{n,m}^t = a_n\left[\Phi\left(x_{n,m+1} - \sigma\sqrt{d}\right) - \Phi\left(x_{n,m} - \sigma\sqrt{d}\right)\right] e^{-\delta d}.$$

Figure 1: Timeline of the dynamic negotiation game



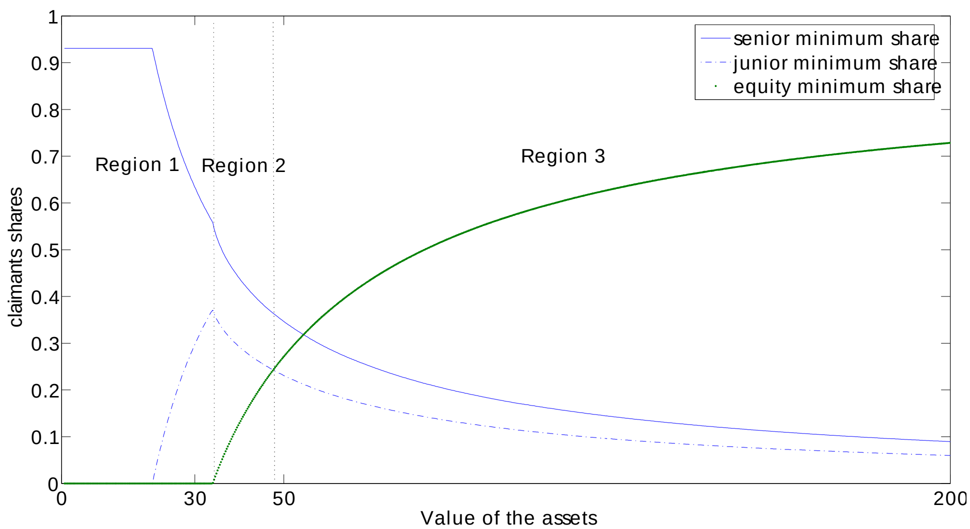
Note.- At the beginning of Chapter 11 negotiation, $t = 0$, the DIP proposes the first reorganization plan (as stipulated in the Code). The DIP spends d units of time from filing in Chapter 11 to submitting a reorganization plan. The Code prescribes an exclusivity period of 120 days, that can be extended at the request of the DIP. In this example, we assume that the negotiation lasts at most 4 bargaining rounds, of equal length d. We also assumes that the probability that the judge interferes in the negotiation, q, is a linear increasing function of the number of rounds k spent into Chapter 11 negotiation.

Figure 2: Extensive-form representation of the first-round game



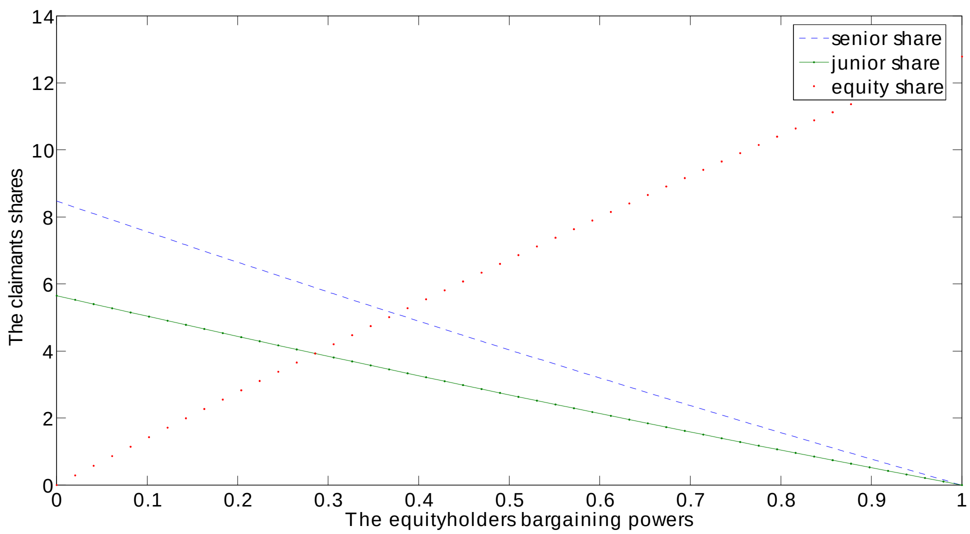
Note.- In the first bargaining round, the DIP (or Leader in this round) proposes the first reorganization plan. The creditors, or followers, decide separately to accept or reject this plan. The output of this game depend on the pair of binary decisions made by followers, i.e. (A,A), (R,R) or ((A,R),(R,A)).

Figure 3: Minimum shares offered by the bankruptcy judge when he imposes his own reorganization plan



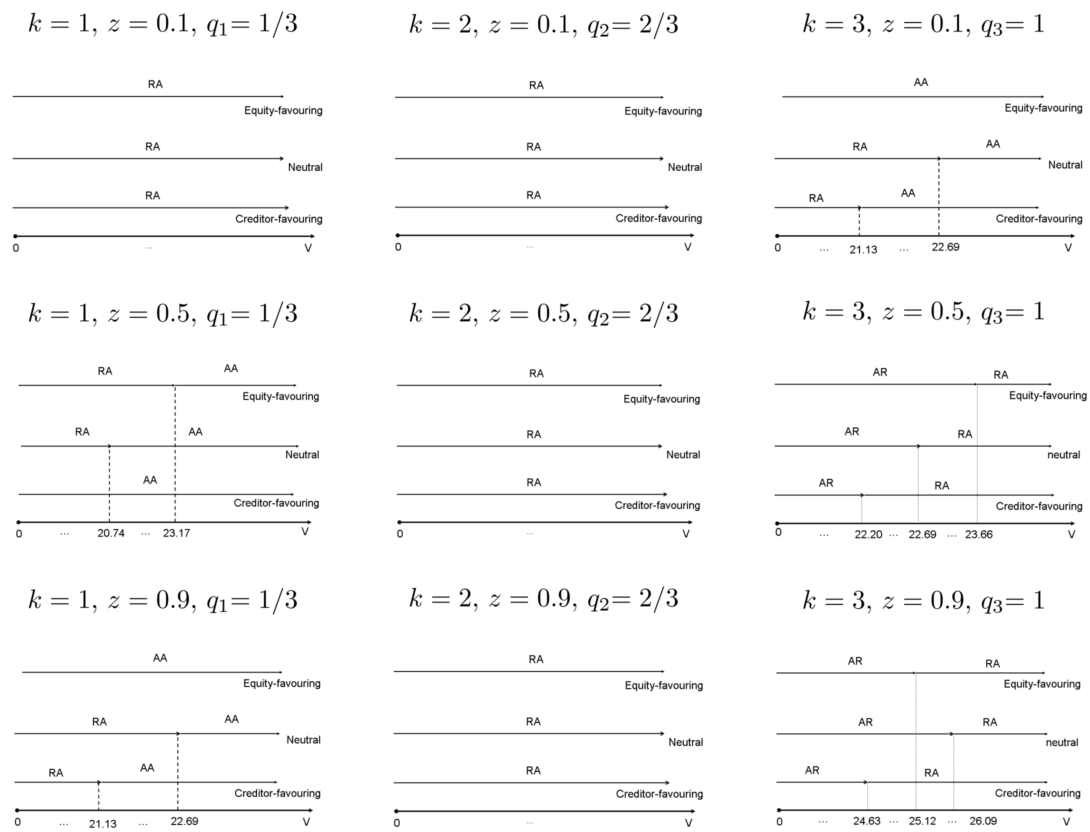
Note.- We compute the minimum cram-down shares offered by the bankruptcy judges when he imposes his organization plan. We use the following values for the parameters: the risk-free interest rate $r = 8\%$, the payout ratio $\delta = 6\%$, the contractual senior coupon level $c_s = 1.4118$, the contractual junior coupon level $c_j = 0.9412$, the tax rate $\tau = 15\%$, the liquidation costs $\alpha = 8.1\%$, the value of assets $V \in [0, 200]$, the proportional bankruptcy costs $\varphi = 16.9\%$ and the constant bankruptcy costs $\mathcal{K} = 0$. We set the volatility σ equal to 50%. The bankruptcy judge is neutral, i.e. $\beta_e = \beta_s + \beta_j = 0.5$. In region 1 (from the left), $0 < v < \frac{c_s}{r(1-\alpha-(\varphi-\delta)d)}$. In region 2, $\frac{c_s}{r(1-\alpha-(\varphi-\delta)d)} < v < \frac{c_s+c_j}{r(1-\alpha-(\varphi-\delta)d)}$. In region 3, $v > \frac{c_s+c_j}{r(1-\alpha-(\varphi-\delta)d)}$.

Figure 4: Sensitivity of the cram-down shares to the bankruptcy judge's behavior



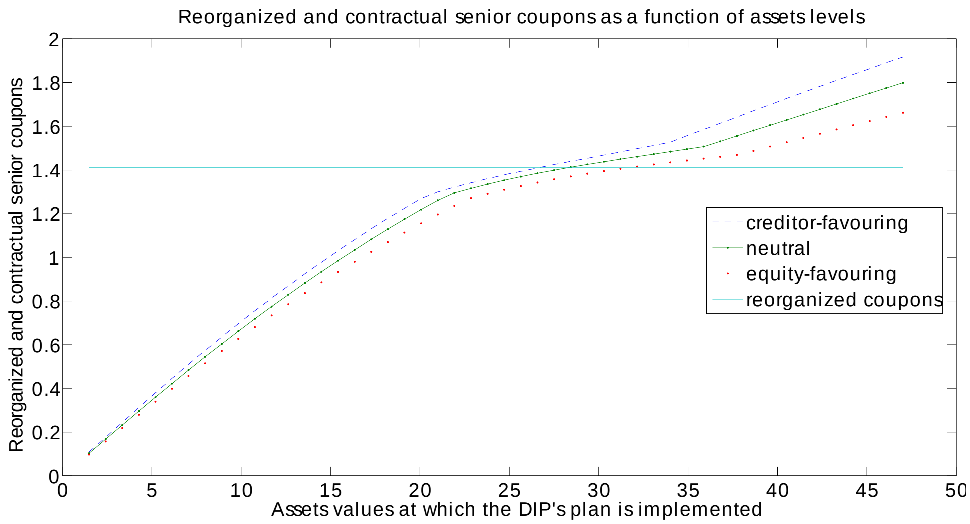
Note.- We compute the claimants shares when the bankruptcy judge imposes his reorganization plan at the first bargaining round, for different bankruptcy judge's behaviors. We use the following values for the parameters: the risk-free interest rate $r = 8\%$, the payout ratio $\delta = 6\%$, the tax rate $\tau = 15\%$, the liquidation costs $\alpha = 8.1\%$, the proportional bankruptcy costs $\varphi = 16.9\%$ and the constant bankruptcy costs $\mathcal{K}= 0$. We set the volatility σ equal to 50%. We assume that the bankruptcy judge allows the game to last for 3 bargaining rounds, i.e. $K + 1 = 3$.

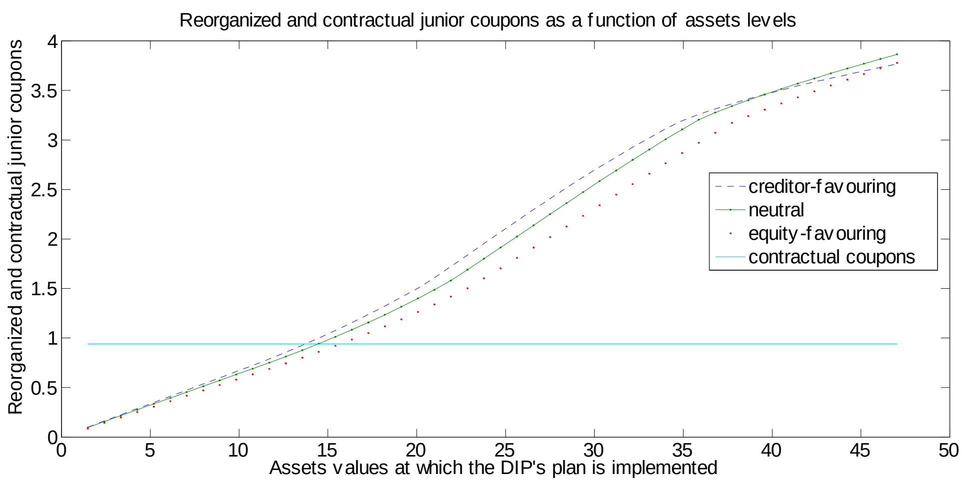
Figure 5: Sensitivity of the equilibrium startegies to the bankruptcy judge's startegy and behavior



Note.- We define the equilibrium strategies of the bargaining game, at the first, second and third round, for different bankruptcy judge's behaviors β and strategies z. All the computations are made for the values of assets $V \in [0, 26]$. We use the following values for the parameters: the risk-free interest rate $r = 8\%$, the payout ratio $\delta = 6\%$, the contractual senior coupon level $c_s = 1.4118$, the contractual junior coupon level $c_j = 0.9412$, the tax rate $\tau = 15\%$, the liquidation costs $\alpha = 8.1\%$, the proportional bankruptcy costs $\varphi = 16.9\%$ and the constant bankruptcy costs $\mathcal{K} = 0$. We set the volatility σ equal to 50%. The probability that the judge interferes is defined by q_k, $k = 1, 2, 3$. We assume that the bankruptcy judge allows the game to last for 5 bargaining rounds, i.e. $K + 1 = 3$. In the top panel, $z = 0.1$, in the middle panel, $z = 0.5$ and in the bottom panel, $z = 0.9$. In each panel, we vary k from 1 (left figure) to 3 (right figure). When the judge is equity-favouring, $\beta_e = 0.8$ and $\beta_s + \beta_j = 0.2$. When the judge is creditor-favouring, $\beta_e = 0.2$ and $\beta_s + \beta_j = 0.8$. Finally, When the judge is neutral, $\beta_e = \beta_s + \beta_j = 0.5$.

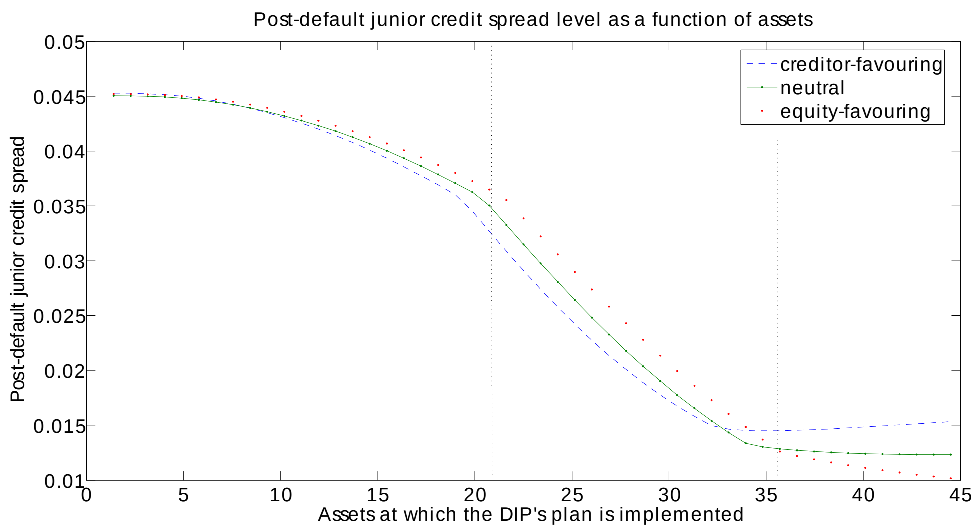
Figure 6: Sensitivity of the senior and junior coupons to the bankruptcy judge's behaviors, when the firm is reorganized at the first bargaining round

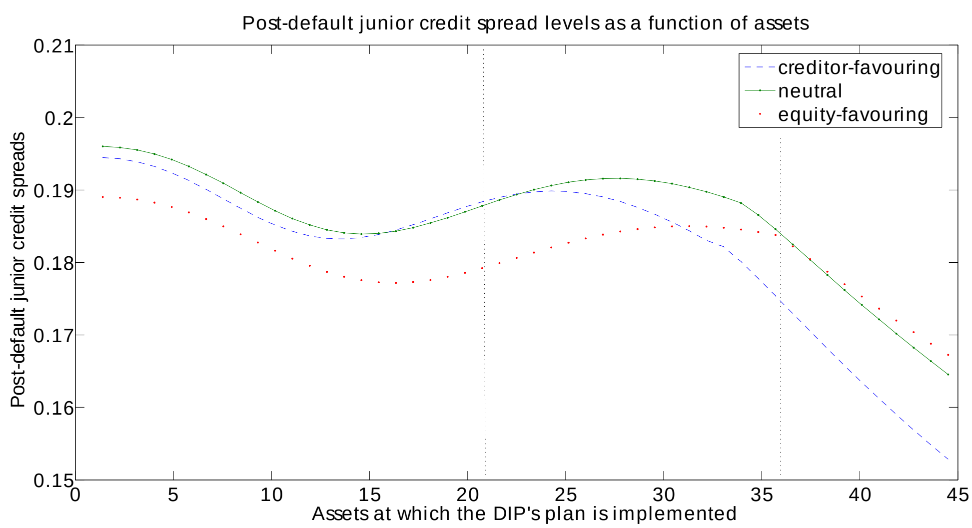




Note.- We compute the senior and junior reorganized coupons, at the first bargaining round, for different bankruptcy judge's behaviors. In this round, the firm is reorganized through unanimous acceptance of the DIP's plan for all the values of assets $V \in [0, 47]$. We use the following values for the parameters: the risk-free interest rate $r = 8\%$, the payout ratio $\delta = 6\%$, the tax rate $\tau = 15\%$, the liquidation costs $\alpha = 8.1\%$, the proportional bankruptcy costs $\varphi = 16.9\%$ and the constant bankruptcy costs $\mathcal{K} = 0$. We set the volatility σ equal to 50%. The probability that the judge interferes is $q_1 = \frac{1}{5}$, and that he imposes his own plan $z = 0.5$. We assume that the bankruptcy judge allows the game to last for 5 bargaining rounds, i.e. $K + 1 = 5$.

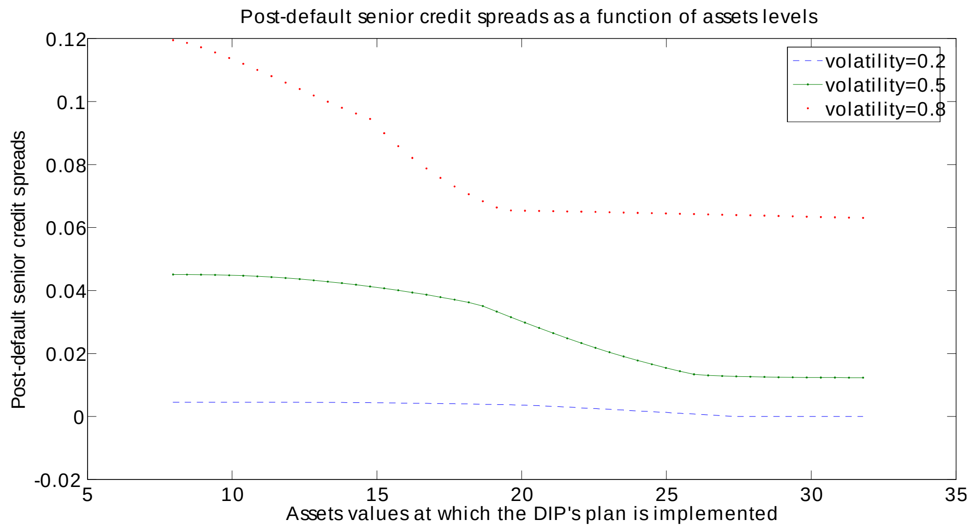
Figure 7: Sensitivity of senior and junior post-default credit spreads to the bankruptcy judge's behaviors, when the firm is reorganized at the first bargaining round

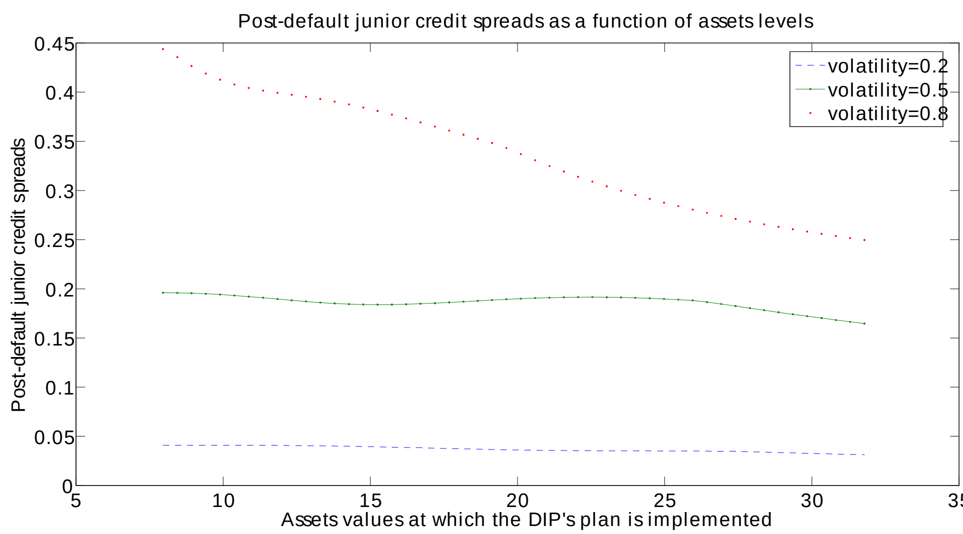




Note.- We compute the senior and junior post-default credit spreads, at the first bargaining round, for different bankruptcy judge's behaviors. In this round, the firm is reorganized through unanimous acceptance of the DIP's plan for all the values of assets $V \in [0, 45]$. We use the following values for the parameters: the risk-free interest rate $r = 8\%$, the payout ratio $\delta = 6\%$, the contractual senior coupon level $c_s = 1.4118$, the contractual junior coupon level $c_j = 0.9412$, the tax rate $\tau = 15\%$, the liquidation costs $\alpha = 8.1\%$, the proportional bankruptcy costs $\varphi = 16.9\%$ and the constant bankruptcy costs $\mathcal{K} = 0$. We set the volatility σ equal to 50%. The probability that the judge interferes is $q_1 = \frac{1}{5}$, and that he imposes his own plan $z = 0.5$. We assume that the bankruptcy judge allows the game to last for 5 bargaining rounds, i.e. $K + 1 = 5$.

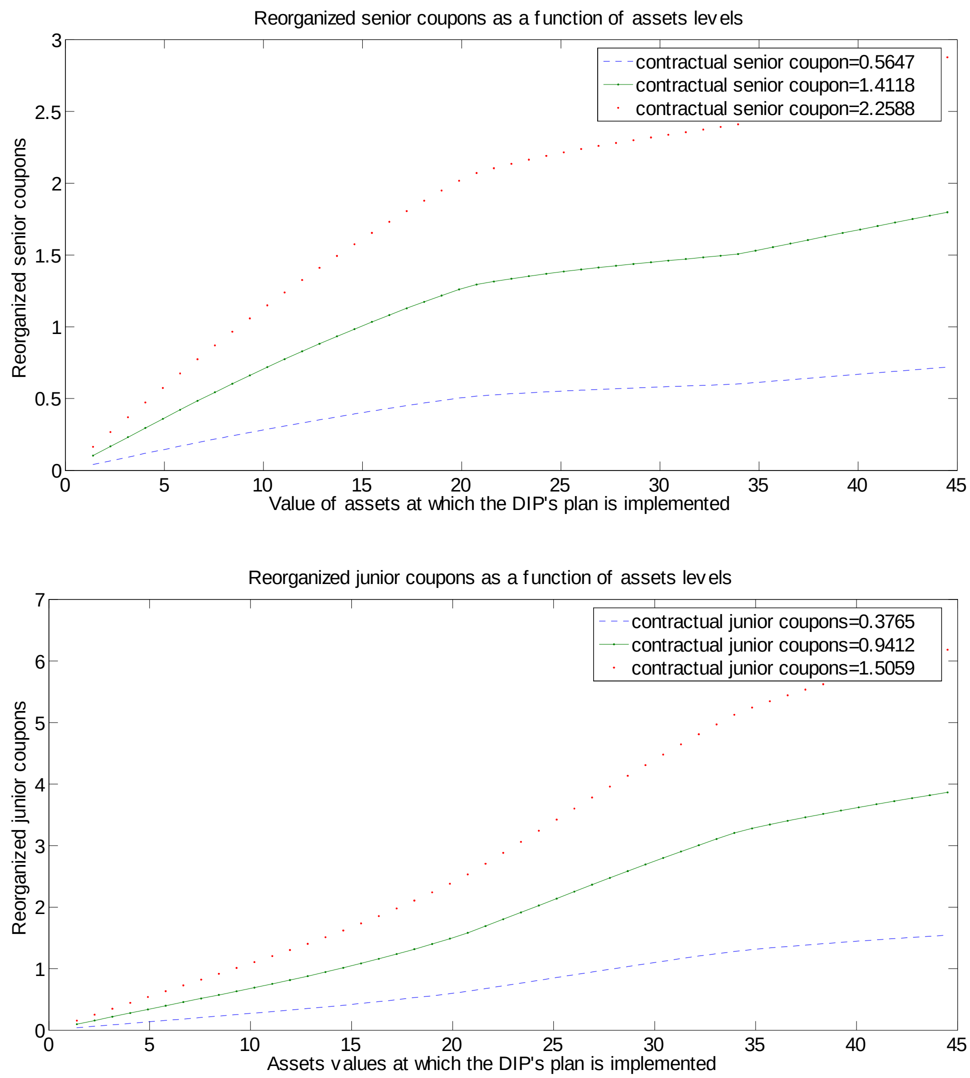
Figure 8: Sensitivity of the senior and junior credit spreads to the volatility level





Note.- We compute the senior and junior post-default credit spreads, at the first bargaining round, for different volatility levels. In this round, the firm is reorganized through unanimous acceptance of the DIP's plan for all the values of assets $V \in [0, 32]$. We use the following values for the parameters: the risk-free interest rate $r = 8\%$, the payout ratio $\delta = 6\%$, the contractual senior coupon level $c_s = 1.4118$, the contractual junior coupon level $c_j = 0.9412$, the tax rate $\tau = 15\%$, the liquidation costs $\alpha = 8.1\%$, the proportional bankruptcy costs $\varphi = 16.9\%$ and the constant bankruptcy costs $\mathcal{K} = 0$. The probability that the judge interferes is $q_1 = \frac{1}{5}$, and that he imposes his own plan $z = 0.5$. The bankruptcy judge's is neutral, i.e. $\beta_e = \beta_s + \beta_j = 0.5$. We assume that the bankruptcy judge allows the game to last for 5 bargaining rounds, i.e. $K + 1 = 5$.

Figure 9: Sensitivity of the senior and junior reorganized coupons to the initial leverage



Note.- We compute the senior and junior reorganized coupons, at the first bargaining round, for different leverage levels. In this round, the firm is reorganized through unanimous acceptance of the DIP's plan for all the values of assets $V \in [0, 45]$. We use the following values for the parameters: the risk-free interest rate $r = 8\%$, the payout ratio $\delta = 6\%$, the tax rate $\tau = 15\%$, the liquidation costs $\alpha = 8.1\%$, the proportional bankruptcy costs $\varphi = 16.9\%$ and the constant bankruptcy costs $\mathcal{K}= 0$. We set the volatility σ equal to 50%. The probability that the judge interferes is $q_1 = \frac{1}{5}$, and that he imposes his own plan $z = 0.5$. The bankruptcy judge's is neutral, i.e. $\beta_e= \beta_s+\beta_j= 0.5$. We assume that the bankruptcy judge allows the game to last for 5 bargaining rounds, i.e. $K + 1 = 5$.