A Model For Time Varying Betas

by

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Abstract

This paper draws attention to the fact that under standard assumptions the time varying betas model cannot capture the dynamics in beta. Using the multivariate normal as a model for the joint distribution of returns on the market portfolio and predetermined information variables, it is shown how to capture skewness and kurtosis in the unconditional distributions of asset returns. It is also shown that the predetermined information variables have the potential to account for the time series properties of returns, including heterogeneity of variance. The model may be extended empirically by using different distributions for the residual returns. It may be extended theoretically by considering other members of the elliptically symmetric class of distributions. An empirical study applies the model to returns on European bond funds. An analysis of the residuals from fitting several versions of the time varying betas models shows that such models are able both to capture the dynamics of alpha and beta and account for other features of the time series of returns for a significant number of European bond funds.

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1. Introduction

The aim of this paper is to present some new theoretical and empirical insights into a model that is widely used in financial economics for modelling the returns on risky financial assets. Specifically, the paper is concerned with extensions to the market model in which, in the usual notation, the parameters α and β are time varying. In the case under consideration, the dynamics of α and β are modelled by positing linear regression models in which the unobserved values of these two parameters are related to economic conditions by incorporating predetermined information variables. This model, which is described in detail in section 2 of this paper, is both well known and widely used. Indeed, it may be regarded as a standard technique in asset pricing and portfolio theory as it is described in standard textbooks such as Elton et al. (2003, p. 150). The method itself is attributed to Beaver et al. (1970) and was popularised by Barr Rosenberg and his co-workers, for example Rosenberg and McKibben (1973) or Rosenberg and James (1976). This paper is motivated by the increasing application of this model to asset pricing studies (for example Jagannathan and Wang, 1996) and specifically to mutual fund performance studies (for example Ferson and Schadt, 1996, Christopherson et al., 1998, Silva et al., 2003).

The theoretical contribution of this paper is twofold. First, for the case when standard econometric assumptions hold, it is shown that the model is unable to describe the dynamics in β. Secondly, the paper describes the implications for modelling time varying α and β when the standard econometric assumptions do not hold. The first theoretical contribution therefore offers an explanation for those applications where the model has not provided evidence of time variation in α or β. More interestingly, for those applications which have been successful, the other contribution provides new insights into the unconditional distribution of asset returns. As is shown in section 4, these distributions may include both skewness and kurtosis and therefore offer new possibilities for portfolio selection as well as for modelling returns. It is also shown in section 4 that the time varying betas model offers the possibility of capturing both serial-correlation and heterogeneity in the variance of asset returns. A correctly specified time-varying betas model therefore offers the possibility of accounting for the empirical properties that are often observed in the time series of returns on financial assets, even though the model is estimated using OLS or similar methods.

The structure of this paper is as follows. Section 2 describes the model under consideration and presents a short review of relevant literature. Section 3 explains why the model fails under multivariate normality and related elliptically symmetric distributions. Section 4 presents a new model. This is based on the assumption that the joint probability distribution of returns on the market portfolio and the information variables is multivariate normal. As is shown, this leads to a model for the unconditional distribution of returns on an asset which can exhibit both skewness and kurtosis. It is also shown how the model may account for heterogeneity in variance and for other time series effects. Section 5 describes a short empirical study of returns on European bond funds. As the results of the study show, a time varying betas model is often able to account for the time series behaviour of a majority of fund returns, leaving estimated residuals that satisfy the usual OLS assumptions. Section 6 concludes. There are two appendices with technical results. Notation is that in standard use. In keeping with increasingly common practice, only the main results are presented. Further detail is available from the authors on request.

2. The Model and its Properties

The model for time varying betas in this paper has two components. The first is the market model in which returns on individual assets are related to returns on a proxy to the market portfolio. In the second component, linear models, in which the unobserved parameters are related to predetermined information variables, represent the dynamics in the two parameters of the market model. The first component of the model is:

$$R_t = \alpha_t + \beta_t R_{mt} + \omega_t . \tag{1.}$$

The notations R_t and R_{mt} are used indifferently to denote either total returns or excess returns on an asset and the market proxy respectively for the time period ending at time t. Excess return is defined in the usual way as total return minus the risk free return over the period. To avoid unnecessary notation, the subscript *i* to denote asset *i* is omitted except in cases where it is explicitly required. The time series of unobserved residual returns $\{\omega_t\}$ are assumed to be IID with zero mean and constant variance. In the remainder of the paper, the model defined by equation (1.) is referred to as the single index or SI model.

The second component of the model describes the dynamics of α and β using two linear models:

$$\begin{aligned} \alpha_t &= \phi_0 + \phi_1^T X_t + \eta_t , \\ \beta_t &= \gamma_0 + \gamma_1^T X_t + \varsigma_t . \end{aligned} \tag{2.}$$

In this pair of equations, X_t is a vector of lagged information variables (that is, it contains variables which are known at the start of period t), ϕ_1 γ_1 are the corresponding vectors of parameters, ϕ_0 and γ_0 are scalars. The residuals η_t and ζ_t are unobserved. It is assumed that values of the 2-vector (η_t, ζ_t) are IID and that it is distributed independently of ω_t.

Substitution of equation (2.) into equation (1.) gives the model:

$$R_t = \phi_0 + \phi_1^T X_t + \gamma_0 R_{mt} + \gamma_1^T X_t R_{mt} + \varepsilon_t ; \tag{3.}$$

where, by definition, the residual return ε_t is given by:

$$\varepsilon_t = \omega_t + \eta_t + R_{mt}\varsigma_t . \tag{4.}$$

This model is used by Jagannathan and Wang (1996) in asset pricing and by Ferson and Schadt (1996) in a conditional performance evaluation framework. Both papers focus on the dynamics in beta. The dynamics of alpha are considered by Christopherson et al. (1998) and by Bernhardt and Jung (1979), who are concerned with statistical desirability of the inclusion of the intercept term. Ferson and Schadt (1996) further extend the model by incorporating a quadratic term in R_{mt} thus obtaining a conditional version of the Treynor and Mazuy (1966) timing model:

$$R_t = \phi_0 + \phi_1^T X_t + \gamma_0 R_{mt} + \gamma_1^T X_t R_{mt} + \gamma_2 R_{mt}^2 + \varepsilon_t; \tag{5.}$$

where γ_2 is a scalar. Given the specification above, the error terms $\{\varepsilon_t\}$ have zero mean but a variance that is time varying through its dependence on R_{mt}. Denoting the variance of the term $(\omega_t + \eta_t)$ by σ_ω^2, the variance of ε_t is:

$$V[\varepsilon_t] = \sigma_\omega^2 + R_{mt}^2 \sigma_\varsigma^2. \tag{6.}$$

Generally, this complication is ignored and the models defined at equations (3.) or (5.) are estimated using OLS.

A special case arises if all the elements of the parameter vector ϕ_1 are equal to zero. In this case, the alpha coefficient in equation (2.) is constant and only the beta is time varying. In the remainder of the paper the model defined by (3.) or (5.) together with the restriction that alpha is constant is referred to as the conditional single index or CSI model. The more general case, in which alpha may be time varying is referred to as the ACSI model.

The non-linearity that arises because of the presence of the vector $X_t R_{mt}$ is ignored since, by specification, these terms are given. The model defined at equations (3.), (5.) and, if used, (6.) is applied to individual assets. If the vector X_t contains common factors, for example the change in interest rates, then it also applies to a portfolio of the assets. In this case the parameters ϕ_0, γ_0, ϕ_1, γ_1 and, if used, γ_2 will in general vary with each asset; giving parameters ϕ_{0i} and so on. Furthermore, if w_i denotes the weight of asset i in the market proxy, the model parameters must satisfy the following restrictions:

$$\begin{aligned} &\sum_{i=1}^{n} w_i \phi_0 = 0, \sum_{i=1}^{n} w_i \gamma_0 = 1, \\ &\sum_{i=1}^{n} w_i \phi_1 = \mathbf{0}, \sum_{i=1}^{n} w_i \gamma_1 = \mathbf{0}; \end{aligned} \tag{7.}$$

where in the second pair of equations **0** is a vector of zeros. It is also the case the error terms $\{\varepsilon_t\}$ must satisfy a linear restriction of the form:

$$\sum_{i=1}^{n} w_i \varepsilon_{it} = 0 .$$

The implication of this is that the errors are not cross-sectionally independent and that their variance-covariance matrix (henceforth VC matrix) has rank at most n-1. Adcock and Clark (1999) argue that this may be ignored when the market portfolio is well diversified. This is indeed common practice. However, it may be noted that this assumption may result in some loss of estimation efficiency in markets for which the index has only a small number of assets.

The SI model at equation (1.) may be replaced by a model in which there is more than one factor or explanatory variable. This may be motivated by Ross' arbitrage pricing theory, Ross (1976), or by empirical considerations. In such models, the scalar coefficient β_t is replaced by a vector, $\boldsymbol{\beta}_t$ say. Such models are referred to as multi-index or MI models. Time variation in the elements of $\boldsymbol{\beta}_t$ may be captured by positing linear models which are similar to the second component of equation of (2.). This results in models that are similar in structure to equations (3.) and (5.); containing linear, cross-product and quadratic terms. Such models are referred to as CMI models, when alpha is constant, and ACMI models, when alpha is time varying.

In the interests of brevity, the detailed equations for (A)CMI models are omitted. To simplify exposition, the technical results described in section 4 are based on the (A)CSI model. Extension to the multi-index case is straightforward, although the notation required for the theoretical results is cumbersome. However, empirical work is straightforward and the study described in section 5 covers both SI and MI models.

3. Elliptically Symmetric Distributions

The variables R_{mt} and the elements X_{jt} of the vector X_t are also random variables. The justification for the use of OLS is that conditional on values of R_{mt} and X_t the probability distribution of R_t is normal and that the expected value of the right hand side of equation (3.) exactly describes the conditional mean of R_t. If it may be assumed that the variables R_t, R_{mt} and X_t have a joint multivariate normal distribution, then the model at equation (3.) or (5.) are mis-specified. This is because the conditional distribution of R_t given R_{mt} and X_t has a mean that is strictly linear in R_{mt} and X_t. The details of this are in appendix A. This is a standard result in the theory of the multivariate normal distribution. See, for example, Anderson (1958, p. 29) for further details.

In the case of multivariate normality, using OLS to estimate the parameters of (3.) must give estimated values of the elements of the vector γ_1 that are not significantly different from zero. If such a model is estimated and the null hypothesis H_0: $\gamma_1 = 0$ is not rejected (against any suitable alternative), the implication is that the dynamics of β, although not necessarily those of α, must be captured another way. It may also be conjectured that failure to reject H_0 is also a test of the joint multivariate normality of R_t, R_{mt} and X_t, although this idea requires development which is beyond the scope of this paper. The same comments apply to the model at (5.); the additional null hypothesis H_0: $\gamma_2 = 0$ would not be rejected either.

These comments apply, with relatively minor modifications, if the joint multivariate probability distribution of R_t, R_{mt} and X_t comes from any member of the elliptically symmetric class. This includes the multivariate Student distribution, which is well established in finance (for example Chamberlain, 1983, Ingersoll, 1987, or Zhou, 1993), and the multivariate Laplace distribution (see Fang et al., 1990, p. 92). Under these distributions, the conditional mean of R_t given values of R_{mt} and X_t is unchanged. The residual variance, however, will in general be a function of R_{mt} and X_t, depending on the joint distribution of these variables. Thus, it is possible to accommodate kurtosis in asset returns but to remain in the position where time variation in β cannot be captured by a model of the type defined at equations (3.), (4.) and (5.). Further technical details of the results for the multivariate Student distribution are available on request.

4. Model for Non-normal Asset Returns

This section presents a model for the case when the unconditional distribution of asset returns is non-normal. The development described is based on the model at equation (3.). As will be shown, the extended model of Ferson and Schadt (1996), which has a quadratic term in R_{mt}, is easily incorporated in this framework. As noted above, the results below may be extended to the case where the SI model at (2.) is replaced by a MI model, albeit at the price of some notational complexity.

When the null hypothesis H_0: $\gamma_1 = 0$ is rejected (against any suitable alternative), the implication is that the unconditional distribution of R_t will not be normal (or from an appropriate elliptically symmetric distribution). Depending on the joint probability distribution of R_{mt} and X_t, the unconditional distribution of R_t will exhibit both skewness and kurtosis. As noted in the introduction, the recent study in Silva et al. (2003) applies the time varying betas model to returns on European bond funds. As their results demonstrate, there are some funds for which there is no time variation in the estimated betas and which therefore leads to the inference that the returns on such funds are normally distributed. There are other funds, however, which show statistically significant time variation, thus leading to the inference that the unconditional distribution of their returns is not normal.

Given the above, the conditional distribution of R_t however is normal. In the usual notation:

$$R_t \mid r_{mt}, x_t \sim N(\phi_0 + \phi_1^T x_t + \gamma_0 r_{mt} + \gamma_1^T x_t r_{mt}, \sigma_\omega^2 + r_{mt}^2 \sigma_\varsigma^2) \qquad (8.)$$

In the rest of this paper it is assumed that the joint distribution of R_{mt} and X_t is multivariate normal. In this case the market model, which is taken to be the conditional distribution of R_t given R_{mt}, is normal but with mean and variance that are functions of R_{mt}. For this case, the unconditional moments of R_t will, depending on the values of the parameters in the model defined at equation (8.), exhibit skewness and kurtosis. This approach, namely to make an exogenous assumption about the distribution of R_{mt} and X_t, is similar to that adopted in Pedersen and Satchell (2000).

The result is derived as follows using the result in appendix A. When R_{mt} and X_t have a joint multivariate normal distribution, the conditional distribution of X_t given R_{mt} is also multivariate normal. If the mean vector and VC matrix of R_{mt} and X_t are denoted by:

$$\begin{bmatrix} \mu_m \\ \mu_X \end{bmatrix}, \begin{bmatrix} \sigma_m^2 & \sigma_{mX}^T \\ \sigma_{mX} & \Sigma_{XX} \end{bmatrix},$$

respectively, the conditional mean vector and VC matrix of X_t given that $R_{mt} = r_{mt}$ are:

$$\mu_{X|r_m} = \mu_X + \lambda_m (r_{mt} - \mu_m), \Sigma_{X|r_m} = \Sigma_{XX} - \sigma_m^2 \lambda_m \lambda_m^T \,, \tag{9.}$$

where the vector λ_m is defined as:

$$\lambda_m = \frac{\sigma_{mX}}{\sigma_m^2} \,. \tag{10.}$$

Since only the mean of the conditional distributions defined at equations (8.) and (9.) depends on r_{mt}, the conditional distribution of the return on an asset R_t given market return r_{mt} is also normal. It is convenient to define $\tilde{r}_{mt}$ as:

$$\tilde{r}_{mt} = r_{mt} - \mu_m \,.$$

It is shown in appendix B that conditional expected value and conditional variance are in general quadratic functions of $\tilde{r}_{mt}$:

$$E[R_t \mid r_{mt}] = \psi_0 + \psi_1 \tilde{r}_{mt} + \psi_2 \tilde{r}_{mt}^2 \,, \tag{11.}$$

and:

$$V[R_t \mid r_{mt}] = \Lambda_0 + 2\Lambda_1 \tilde{r}_{mt} + \Lambda_2 \tilde{r}_{mt}^2 \,, \tag{12.}$$

respectively. The constants ψ_j and Λ_j, j = 0,1,2 are also defined in appendix B. The equation for the conditional mean is essentially the market-timing model of Treynor and Mazuy (1966) or the model proposed in Harvey and Siddique (2000) to capture skewness. Comparison of (11.) with (5.) shows that it is straightforward to include the quadratic term in R_{mt}. This only requires modification of the model parameters ψ_0 and ψ_2 in (11.).

Using exactly the same approach, it is straightforward to derive the conditional expected value and variance of R_t for the case when the conditioning variable is the vector X_t. The equations are omitted in the interests of brevity. Recalling that, by specification, the elements of X_t are known at the start of the time period, this representation is able to account for heterogeneity in both the conditional mean and variance of returns and thus has the capability of playing a role similar to that of a member of the ARCH family of models. Depending on the properties of X_t, it also offers the possibility of accounting for serial correlation.

Using the above notation, the unconditional mean and variance of R_t are, respectively:

$$E[R_t] = \psi_0 + \psi_2\sigma_m^2, V[R_t] = \Lambda_0 + \left(\Lambda_2 + \psi_1^2\right)\sigma_m^2 + 2\psi_2^2\sigma_m^4$$

To compute higher moments, it is simpler to use the moment generating function (MGF) of R_t. The conditional moment generating function (MGF) of R_t given r_{mt} is:

$$M_{R_t|r_{mt}}(s) = \exp\left[s\left(\psi_0 + \psi_1\tilde{r}_{mt} + \psi_2\tilde{r}_{mt}^2\right) + \frac{1}{2}s^2\left(\Lambda_0 + 2\Lambda_1\tilde{r}_{mt} + \Lambda_2\tilde{r}_{mt}^2\right)\right]$$

Integrating over the distribution of r_{mt} gives the unconditional MGF of R_t. Taking logs gives the cumulant generating function of R_t. This is:

$$K_R(s) = \psi_0 s + \frac{1}{2}\Lambda_0 s^2 - \frac{1}{2}\left[ln\left\{1 - \sigma_m^2\left(\Lambda_2 s^2 + 2\psi_2 s\right)\right\} + \frac{\sigma_m^2\left\{\psi_1 s + \Lambda_1 s^2\right\}^2}{\left\{1 - \sigma_m^2\left(\Lambda_2 s^2 + 2\psi_2 s\right)\right\}}\right]$$

The Skewness of R_t is:

$$SK(R_t) = 2\sigma_m^2\psi_2^3 + 3!\sigma_m^2\left\{\psi_1\Lambda_1 + \sigma_m^2\psi_2\left(\Lambda_2 + \psi_1^2\right)\right\}$$

The equation for kurtosis is a more complex function of the parameters and so is omitted.

It is clear from equations (11.) and (12.) that, even for the case where it is assumed that R_{mt} and X_t have a joint multivariate normal distribution, the exact closed form expression for the unconditional distribution of R_t is complicated[1]. Estimation of the parameters of the model at (11.) and (12.) is however straightforward. If the terms involving $\Lambda_{1,2}$ are ignored, the other model parameters may be estimated by OLS. Otherwise they may be estimated using weighted least squares iteratively.

To summarise: the time varying betas model may be used in conjunction with standard assumptions about the distribution of the return on the market portfolio and explanatory factors to generate a model which will capture skewness and kurtosis in returns, as well as time series properties of returns. The model, which is defined at equations (3.) or (5.), may nonetheless be estimated using standard

[1] Under the assumptions made in this section, the unconditional distribution of R_t is that of a non-central quadratic form in normal variables, see for example Johnson and Kotz (1970, Ch. 29).

methods. As shown at equations (11.) and (12.), the market-timing model of Treynor and Mazuy (1966) arises as a by-product. From an empirical perspective it is easy to generalise the models described in this section, for example by assuming that the residuals follow a different distribution from the normal, or by specifying a different functional form, for the joint distribution of R_{mt} and X_t.

5. Empirical Study

As described in the introduction, this paper is motivated by the increasing application of models with time varying alphas and betas to fund performance evaluation. The implication of the results reported in section 4 is that a correctly specified model will result in estimated residuals that satisfy the standard diagnostic tests used in time series regression. Similarly, a model that is incorrectly specified will result in estimated residuals that fail to satisfy the standard assumptions. As also noted in the introduction, a set of estimated residuals that satisfies the OLS assumptions implies that the predetermined information variables and precise functional form of the time varying betas model has the ability to capture the properties of the variation in returns. The overall aim of the empirical study that follows is therefore to examine the residuals from the models for bond fund performance reported in Silva et al. (2003) and to establish to what extent these models are able to capture non-normality in the unconditional distributions of bond fund returns.

5.1 The data

The same database is used as in Silva et al. (2003). This consists of 638 bond funds from Italy (58), Spain (157), France (266), Germany (90), UK (45) and Portugal (22). These markets represented around 76.5% of the European bond fund market (excluding Luxembourg), as of December 2000.

Monthly data, in local currency, was obtained from Datastream, Micropal and APFIN[2] for bond funds investing mainly in the domestic market and/or in the European market over the period February 1994 to December 2000[3].

All fund returns are monthly continuously compounded returns, with income distributions reinvested, and in local currency. These returns are net of management expenses but not of load charges. In order to obtain excess returns, the risk free rate, proxied by the 3-month Interbank offered rate, is subtracted from this return. The analysis reported in Silva et al. (2003) is based on consideration of the 638 individual bond funds and on equally weighted portfolios formed from the individual funds for each country.

Table 1 presents the summary statistics and main characteristics of the European bond fund sample, based on the equally weighted portfolios described above. The average size and management fees as well as the average of monthly excess returns, its standard deviation, skewness, kurtosis, and the probability of the Bera

[2] APFIN is the Portuguese association of mutual fund companies.

[3] For Portugal a shorter period (January 1995-December 2000) is considered due to the availability of the index used as benchmark

Jarque test are shown. The average bond fund size is highest in Italy and much lower in the other countries. Management fees range from approximately 0.5% annually (Portugal and Germany) to approximately 1.4% (Spain). For most countries, bond funds present negative mean excess returns. The few cases presenting positive mean excess returns (Germany and France) did not turn out to be statistically significant. Volatility of monthly excess returns is highest for UK bond funds. According to the Bera Jarque test statistic, the hypothesis of a normal distribution is rejected (at the 5% level) for the equally weighted portfolio of bond funds for UK, Spain and Italy.

Insert Table 1 here

Although not reported in this paper, the summary statistics for each individual fund were also computed. Relatively to the distribution of returns, the results have shown that for a large number of funds we do not reject (at the 5% level) the hypothesis of a normal distribution (372 funds, which represent 58% of our sample). This is particularly evident for German and French bond funds, while in the other countries the rejection of this hypothesis is more frequent. This is consistent with the results obtained for the equally weighted portfolio of funds. It is commonly advocated that, given the dynamics of the term structure of interest rates and their finite life, bonds often exhibit non-normal and auto-correlated returns. However, this does not seem to be an obvious problem for the sample used in the study.

Both unconditional and conditional models (as described in equations 1., 3. and 5.) are used as the return generating process. For both cases, SI and MI versions are considered. The Salomon Smith Barney WGBI all maturities for each country is used as the benchmark for the SI model. For the MI model, in addition to this market index, the excess return on a stock market index (the MSCI stock index for each country)[4] and a default spread (calculated as the difference between the MSCI Euro Credit Index BBB rated and the MSCI Euro Credit Index AAA rated)[5] are included. The correlations between these indices in each country are low, suggesting that multicollinearity should not be a problem. Table 2 presents the summary statistics on these market indices.

Insert Table 2 here

For the series corresponding to the Default spread (Def), the hypothesis of a normal distribution is rejected. In the case of UK and Spain, normality is also rejected for the other two market indices. Furthermore, the first-order auto-correlation coefficients are relatively small.

[4] The excess return on the stock index is included as it can be viewed as a measure of expectations about general economic conditions (see Elton et al., 1995, Cornell and Green, 1991) and also because some of the funds can hold a small percentage of stocks.
[5] The default spread is a measure of the default risk that may affect corporate bond returns. As we do not have information on local spreads (for most European countries the corporate bond market is still a market with a low degree of liquidity) we used a spread for the aggregate Euro zone.

The variables used as conditioning information were the term spread, inverse relative wealth (IRW) and a dummy variable for the month of January (jd). Previous research has motivated the choice of these variables (Ilmanen, 1995, Silva et al., 2003). The term spread is the difference between the yield on a long-term government bond and a short-term bond rate (or the 3-month Interbank offered rate). The yield on a 10-year Government bond (or approximately), obtained from the Central Banks[6], is used as the long-term bond yield. IRW is the ratio between the exponentially weighted average of past real wealth and current wealth. The MSCI stock indices for each country (obtained from Datastream) deflated by the Consumer Price Index (obtained from the International Monetary Fund) were used to measure real wealth. All these variables are stochastically detrended (by subtracting a 12-month moving average)[7] and mean zero variables. The summary statistics on these variables are presented in table 3.

Insert Table 3 here

Similarly to what were observed for the market indices excess returns, there is evidence that the predetermined information variables do not always follow a normal distribution. Furthermore, although the series have been stochastically detrended in order to reduce the problem of persistent series, they still exhibit quite high first-order auto-correlation coefficients (although slightly lower than the ones obtained considering the original series of the variables).

The two conditional models are estimated with the alpha restricted to be constant and with alpha allowed to be time varying.

5.2 Results

The models are estimated both in aggregate terms, considering the equally weighted portfolio of funds for each country, and for each individual fund. Table 4 summarises the results of bond fund performance on the basis of unconditional models (both SI and MI benchmarks). The analysis of this table shows that, when the unconditional SI model is used, the equally weighted portfolios of funds, in general, present negative alphas, being more significant in Italy, Spain and Portugal. For the UK and Germany, several funds have positive alphas, although not statistically significant. In the case of Germany only 8 out of 90 funds present statistically significant negative alphas, at the 5% level.

Insert Table 4 here

[6] This is the most commonly used maturity for representing a long-term bond yield. For Portugal we used the yield on Treasury bonds with remaining maturity between 108 and 126 months; for Spain, the yield on a 10-year government bond; for Italy, the yield on the 10-year BTP (Buoni Poliennali del Tesoro); for France, the "taux de l`emprunt phare a 10 ans"; for Germany, the yield on listed Federal securities with a residual maturity of 9-10 years (only bonds eligible as underlying instruments for futures contracts are included) and for UK, the yield on a 10-year Government bond.

[7] This procedure was used in order to reduce the problem of spurious regression, a problem that may be found when persistent regressors are used (see Ferson et al., 2003a, 2003b).

It can also be observed that the index used, the Salomon Smith Barney WGBI for all maturities for each country, in general explains a large percentage of bond fund excess returns.[8]

In relation to the unconditional MI model, the inclusion of the two additional factors adds some explanatory power. The stock index is an important factor, mainly for Germany and UK. The default spread is also significant, particularly for Germany and France. For a large number of funds (at the 5% level) the null hypothesis that the coefficients of the two factors are equal to zero is rejected. Furthermore, the alphas of the funds, in general, decrease comparatively to those of the unconditional SI model. This outcome is consistent with the findings of Elton et al. (1993) in relation to the Ippolito (1989) study and with other studies on stock fund performance and reinforces the argument that SI models might overestimate fund performance.

Estimated values of the conditional models (as described in equation 3.) are computed both for the case where alpha is restricted to be constant (CSI and CMI) and for the case where alpha is time-varying (ACSI and ACMI).

Table 5 presents the estimates for the CSI and CMI models. In general, the sign of the estimated alphas does not change much when time-varying betas are allowed. The inclusion of the predetermined information variables seems to add explanatory power to the model. Although the R^2 (adj.) of the portfolios of funds remain similar, or even decreases slightly, at the individual fund level and for both models (SI and MI), the hypothesis that the additional coefficients are jointly equal to zero (at the 5% level) is rejected for a large number of funds. For the CMI model this is observed for 446 bond funds, representing approximately 70% of the funds (39 German funds, 177 French funds, 35 UK funds, 133 Spanish funds, 47 Italian funds and 15 Portuguese funds). In the case of the CSI model, the null hypothesis is rejected for 260 funds (41% of the funds).

Insert table 5 here

Estimates of ACSI and ACMI, presented in Table 6, show that when alphas are not restricted to be constant, estimates of fund performance are similar. The hypothesis that the additional coefficients of time-varying betas and time-varying alphas are jointly equal to zero (at the 5% level) is rejected for a large number of funds. For the ACMI this is observed for 529 bond funds, representing approximately 83% of the funds. In the case of the ACSI, the null hypothesis is rejected for 338 funds (53% of the funds).

Insert table 6 here

[8] This is particularly the case of categories of funds that include mainly Government bonds. Although not reported in this paper, the bond funds in each country are grouped into different categories according to the type of bonds held by the funds (for more detail see Silva et al., 2003).

These results can be interpreted as evidence of time-varying betas and alphas, which seem to be stronger for the ACMI model. Table 7 shows, in more detail, the estimates for the slope coefficients of the conditional beta function.

Insert Table 7 here

The analysis of the sign of the estimates for the conditional alpha function, resulting from equation (3.) provides insights relatively to the issue of time-varying alphas. The results (not reported in this paper) show that there is a relationship between the predetermined information variables and bond fund performance for both the ACSI and ACMI models. In the latter case, for 348 funds (representing approximately 55% of the sample) the null hypothesis that the coefficients on the lagged information variables are jointly equal to zero is rejected.

5.4 The Treynor and Mazuy Model

Previous studies that have applied the Treynor and Mazuy (1966) model in an unconditional framework have found more evidence of negative than positive timing ability. Considering the analysis of naïve strategies, Ferson and Schadt (1996) concluded that this model is not well-specified and that its conditional version can control for this misspecification. Their results show that the evidence of negative timing disappears when a conditional timing model is used. Other studies (e.g.: Becker et al., 1999, Sawicki and Ong, 2000, Gallagher and Jarnecic, 2002) also document this phenomenon.

This issue is explored by applying three versions of the Treynor and Mazuy model: the original model (TM), the Ferson and Schadt conditional timing model (CTM) and the extended timing model considering time-varying alphas (ACTM). The results are presented in table 8.

Insert Table 8 here

As can be observed, the results are very similar whatever timing model is used. Unlike Ferson and Schadt (1996), introducing the conditioning information variables does not impact estimates of timing. Timing coefficients for the equally weighted portfolios of funds are, in general, negative for all European countries. The exception is for Portuguese and Spanish bond funds (in the latter case this is only observed in the context of the extended conditional timing model). Considering the individual fund estimates, only a very small number of bond funds present statistically significant timing coefficients. It should be noted, however, that the number of statistically negative coefficients is higher for Italy and to a less extent for Germany and UK. For Spanish and French bond funds the number of statistically significant positive and negative timing coefficients is balanced. The only market in which there is more evidence of positive than negative timing is Portugal.

5.4 Analysis of the Residuals

The implication of the results summarised above, is that there is evidence of time variation in both alpha and beta for many bond funds. The unconditional distribution of returns of such funds may therefore be expected to exhibit skewness or kurtosis or both. As noted in section 4, if the models defined at (3.) and (5.) accurately describe the dynamics in alpha and beta, then it may be expected that the estimated residuals will satisfy the standard diagnostic tests.

This section therefore reports an analysis of the fitted residuals for all the 638 funds as shown in table 9. For each fund, the 9 models (SI, MI, CSI, CMI, ACSI, ACMI and the three versions of the Treynor and Mazuy model) are considered. For each of the time series of fitted residuals, two standard diagnostic test statistics are computed. These are the Bera Jarque test of normality and the Ljung-Box test for auto-correlation. The Bera-Jarque test is disaggregated into its two components, each of which has a Chi-squared distribution with one degree of freedom under the null hypothesis that the residuals are IID normal.

In order to test the heterogeneity of variance, a GARCH(1,1) model is fitted to the residuals. In this paper, the GARCH(1,1) model is fitted subject to the restriction that all parameters in the conditional variance equation are non-negative. The null hypothesis that the residuals are IID normal with constant variance is then tested using a likelihood ratio test with two degrees of freedom.

Insert Table 9 here

The first panel of table 9 shows the percentage of funds in each country for which the skewness component of the Bera Jarque test is significant at the 1% level. The first column reports the results for the SI model, with corresponding results for the other eight models being shown in the subsequent columns. As the results for the SI column indicate, skewness is not a very common phenomenon in the estimated residuals. Furthermore, as the results corresponding to all six conditional models show, the predetermined information variables that are used to model time variation in alpha and beta also account for much for the residual skewness in the SI model. In particular, the conditional models with time varying alphas account for skewness in better than two out of three funds in five out of six countries. The first panel of the tables also shows that the percentages recorded for the three versions of Treynor-Mazuy market timing model are similar to the corresponding models from which the quadratic term in R_{mt} is omitted. Indeed, this applies to each panel in table 9, leading to the conclusion that, for this data set at least, the inclusion of a quadratic term does not improve a model's explanatory power.

Panel (ii) of table 9 shows the same results for the kurtosis component of the Bera Jarque test. The results in panel (ii) indicate that the conditional models are less successful at removing the effects of fat tails. The implication is that the parameter estimates for some funds are unbiased but may not be BLUE estimators. Reconsideration of some of the conditional models using a different probability distribution for the residuals is a topic for future work. Panel (iii) reports the percentage of funds for which the overall Bera Jarque test is significant. This panel confirms the findings reported in section 5.1, namely that there is a significant number of funds for which non-normality is not an issue.

Panel (iv) shows the results for the Ljung-box test of auto-correlation. It is clear from this panel that auto-correlated residuals only affect a minority of funds, regardless of the model that is used.

Panel (v) shows the percentages for the likelihood ratio test for a GARCH(1,1) model. The SI column shows that there are two countries, Italy and Spain, for which the SI model leaves time varying residual variances in a majority of funds. The ACSI and ACMI models in particular substantially reduce the incidence of GARCH(1,1) effects in the other four markets. Indeed, it is only in Italy that GARCH effects persist for a substantial number of funds after fitting the ACMI model. For a small number of funds, the estimated parameters of the GARCH(1,1) model indicated that the unconditional variance does not exist. The details of this analysis are omitted, but investigation of the effects of this is also a topic for further research.

6. Conclusions

This paper draws attention to the fact that under standard assumptions the time varying betas model cannot capture the dynamics in beta. Using the multivariate normal as a model for the joint distribution of returns on the market portfolio and the predetermined information variables, it is shown how to capture skewness and kurtosis in the unconditional distributions of asset returns. It is also shown that the predetermined information variables have the potential to account for the time series properties of returns, including heterogeneity of variance. The model may be extended empirically by using different distributions for the residual returns. It may be extended theoretically by considering other members of the elliptically symmetric class of distributions.

The empirical study applies the model to returns on European bond funds. An analysis of the residuals from fitting several versions of the time varying betas models shows that such models are able both to capture the dynamics of alpha and beta and account for other features of the time series of returns for a significant number of European bond funds. There are, however, a number of funds for which further refinements of the model specifications are required.

Appendix A - The multivariate normal distribution

Consider an (n+p+1) vector Y which is partitioned as:

$$Y = \begin{bmatrix} R \\ \tilde{X} \end{bmatrix},$$

where R is an n vector which denotes the return on all assets and $\tilde{X}$ is a vector of length p+1 defined as:

$$\tilde{X} = \begin{bmatrix} R_m \\ X \end{bmatrix}.$$

The time subscript is omitted. The vector of expected values and the variance-covariance matrix are similarly partitioned as:

$$\mu_Y = \begin{bmatrix} \mu_R \\ \mu_{\tilde{X}} \end{bmatrix},$$

and

$$\Sigma = \begin{bmatrix} \Sigma_{RR} & \Sigma_{R\tilde{X}} \\ \Sigma_{R\tilde{X}}^T & \Sigma_{\tilde{X}\tilde{X}} \end{bmatrix}.$$

It is assumed that Σ, Σ_{RR} and $\Sigma_{\tilde{X}\tilde{X}}$ are all of full rank. If the vector Y has a multivariate normal distribution, the conditional distribution of R given that $\tilde{X} = \tilde{x}$ is also multivariate normal with expected value:

$$\mu_{R|\tilde{X}} = \mu_R + \Sigma_{R\tilde{X}}\Sigma_{\tilde{X}\tilde{X}}^{-1}\left(\tilde{x} - \mu_{\tilde{X}}\right) = \mu_R + B\left(\tilde{x} - \mu_{\tilde{X}}\right), \text{ say,}$$

and variance-covariance matrix:

$$\Sigma_{R|\tilde{X}} = \Sigma_{RR} - \Sigma_{R\tilde{X}}\Sigma_{\tilde{X}\tilde{X}}^{-1}\Sigma_{R\tilde{X}}^T = \Sigma_{RR} - B\Sigma_{\tilde{X}\tilde{X}}B^T$$

For further details, see Anderson (1958, p. 29).

Appendix B – conditional distribution of R_t given r_{mt}

On applying the results in appendix A and using the notation of section 4, the conditional expected value of R_t given r_{mt} is:

$$E\left[R_t \mid r_{mt}\right] = \phi_0 + \gamma_0 r_{mt} + \left(\phi_1^T + \gamma_1^T r_{mt}\right)\mu_{X|r_m}.$$

This may be written as:

$$E\left[R_t \mid r_{mt}\right] = \psi_0 + \psi_1 \tilde{r}_{mt} + \psi_2 \tilde{r}_{mt}^2,$$

where the constants $\psi_{0,1,2}$ are defined as:

$$\psi_0 = \phi_0 + \gamma_0\mu_m + \left(\phi_1^T + \mu_m\gamma_1^T\right)\mu_X$$

$$\psi_1 = \gamma_0 + \gamma_1^T\mu_X + \left(\phi_1^T + \mu_m\gamma_1^T\right)\lambda_m.$$

$$\psi_2 = \gamma_1^T\lambda_m$$

The conditional variance is

$$V\left[R_t \mid r_{mt}\right] = \sigma_\omega^2 + r_{mt}^2\sigma_\varsigma^2 + \left(\phi_1 + r_{mt}\gamma_1\right)^T \Sigma_{X|r_m}\left(\phi_1 + r_{mt}\gamma_1\right)$$

This may be written as:

$$V\left[R_t \mid r_{mt}\right] = \Lambda_0 + 2\Lambda_1 \tilde{r}_{mt_\varsigma}^{\;2} + \Lambda_2 \tilde{r}_{mt}^2,$$

where the constants $\Lambda_{0,1,2}$ are defined as:

$$\Lambda_0 = \sigma_\omega^2 + \left(\phi_1 + \mu_m\gamma_1\right)^T \Sigma_{X|r_m}\left(\phi_1 + \mu_m\gamma_1\right)$$

$$\Lambda_1 = \left(\phi_1 + \mu_m\gamma_1\right)^T \Sigma_{X|r_m}\gamma_1$$

$$\Lambda_2 = \sigma_\varsigma^2 + \gamma_1^T \Sigma_{X|r_m}\gamma_1$$

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Table 1 – Summary statistics for equally weighted portfolios of bond funds

Summary statistics based on equally weighted portfolios of all bond funds are presented for each country. Average size in million Euros and management fees in annual percentage of assets invested as of 31/12/00. Mean excess returns (considering monthly continuously compounded returns), standard deviation, skewness, kurtosis and the probability of the Bera Jarque test are reported for the period February 1994 to December 2000 (January 2005 to December 2000 in the case of Portugal).

	Nº. of Funds	**Average Size** (Millions Euro)	**Management Fees** (annual %)	**Mean Excess Return** (Monthly %)	**Standard Deviation**	**Skewness**	**Kurtosis**	**JB prob.**
Germany	90	284	0.46	0.115	0.831	-0.506	2.540	0.118
France (1)	266	128	1.01	0.030	0.783	-0.375	2.452	0.224
UK	45	236	0.95	-0.019	1.437	-0.738	3.977	0.004
Spain	157	96	1.39	-0.101 **	0.454	-0.421	4.059	0.042
Italy (2)	58	990	0.98	-0.129 **	0.586	-0.916	4.749	0.000
Portugal	22	159	0.53	-0.074 ***	0.194	-0.214	2.301	0.365
All Sample	638							

*** Statistically significant at 1%   ** Statistically significant at 5%   * Statistically significant at 10%

(1) The average size for France includes mainly French SICAVs as we could not obtain the information on the majority of the French FCPs that compose our sample.

(2) The management fees are average fees for the categories of Italian funds as reported by Assogestioni, the Italian association of mutual fund companies.

Table 2 – Summary statistics on the market indices

Summary statistics on the market indices for the period February 1994 to December 2000 (January 2005 to December 2000 in the case of Portugal) are reported. Bindex refers to the Salomon Smith Barney WGBI for all maturities, Sindex refers to the MSCI stock index and Def correspond to the default spread (measured by the difference between the monthly continuously compounded excess return on the MSCI Euro Credit Index BBB rated and the monthly continuously compounded excess return on the MSCI Euro Credit Index AAA rated). The reported statistics are mean excess returns (considering monthly continuously compounded returns), standard deviation, skewness, kurtosis, the probability of the Bera Jarque test and the first-order auto-correlation coefficient (AC1).

	Mean	Standard Deviation	Skewness	Kurtosis	BJ prob.	AC1
Germany						
Bindex	0.176	0.930	-0.497	2.779	0.167	0.119
Sindex	0.943	5.800	-0.507	4.157	0.017	-0.077
Def	0.071	0.495	-0.546	5.071	0.000	0.131
UK						
Bindex	0.122	1.504	-0.697	3.649	0.017	0.080
Sindex	0.360	3.781	-0.728	3.602	0.014	-0.034
Def	0.071	0.495	-0.546	5.071	0.000	0.131
France						
Bindex	0.162	1.155	-0.377	2.426	0.211	0.149
Sindex	0.963	5.482	-0.407	2.851	0.306	0.014
Def	0.071	0.495	-0.546	5.071	0.000	0.131
Spain						
Bindex	0.203	1.224	-0.469	3.922	0.050	0.179
Sindex	0.967	6.364	-0.755	5.551	0.000	0.093
Def	0.071	0.495	-0.546	5.071	0.000	0.131
Italy						
Bindex	0.192	1.191	-0.084	2.991	0.952	0.183
Sindex	0.753	6.804	0.322	2.713	0.423	-0.117
Def	0.071	0.495	-0.546	5.071	0.000	0.131
Portugal						
Bindex	0.368	0.910	-0.370	3.378	0.356	0.282
Sindex	0.982	6.093	-0.133	3.409	0.700	0.063
Def	0.066	0.481	-0.918	5.551	0.000	0.131

Table 3 – Summary statistics on the predetermined information variables

Summary statistics on the predetermined information variables for the period January 1994 to November 2000 (December 1994 to November 2000 in the case of Portugal) are reported. Term Spread is the difference between the yield on a long-term government bond and a short-term bond rate (or the 3-month Interbank offered rate). IRW is the ratio between the exponentially weighted average of past real wealth and current wealth. These variables are stochastically detrended (by subtracting a 12-month moving average) and mean zero variables. The reported statistics are mean, standard deviation, skewness, kurtosis, the probability of the Bera Jarque test and the first-order auto-correlation coefficient (AC1).

	Mean	Standard Deviation	Skewness	Kurtosis	BJ prob.	AC1
Germany						
Term Spread	0.000	0.801	0.414	2.417	0.170	0.937
IRW	0.000	0.079	0.154	2.976	0.847	0.724
UK						
Term Spread	0.000	0.926	0.585	2.504	0.061	0.929
IRW	0.000	0.053	0.739	3.379	0.018	0.711
France						
Term Spread	0.000	1.019	0.094	2.450	0.557	0.915
IRW	0.000	0.074	0.649	2.973	0.054	0.720
Spain						
Term Spread	0.000	1.082	0.739	3.111	0.022	0.933
IRW	0.000	0.093	0.478	2.883	0.201	0.759
Italy						
Term Spread	0.000	0.905	0.451	2.215	0.084	0.910
IRW	0.000	0.090	0.057	2.596	0.739	0.752
Portugal						
Term Spread	0.000	0.723	0.595	2.957	0.065	0.873
IRW	0.000	0.653	0.537	3.101	0.174	0.838

Table 4 – Estimates for the unconditional single (SI) and multiple index (MI) models

For each country, an equally-weighted portfolio of all bond funds is formed. This table shows the results for both the SI, with the Salomon Smith Barney WGBI for all maturities as the benchmark index, and the MI model. In addition to the monthly continuously compounded excess return on the WGBI for all maturities (Bindex), we consider two more factors: the monthly continuously compounded excess return on the MSCI stock index (Sindex) and the difference between the monthly continuously compounded excess return on the MSCI Euro Credit Index BBB rated and the monthly continuously compounded excess return on the MSCI Euro Credit Index AAA rated (Def). The estimates for the alphas (in percentage) and the regression coefficients with their significance based on heteroscedasticity and auto-correlation adjusted errors (following Newey and West, 1987), and also the R^2 (adj.) for each of the equally-weighted portfolios of funds are presented. The number of individual funds presenting statistically significant positive, not different from zero and negative alphas, at the 5% level, is also reported (N +/0/-). The W(p-val) is the probability value for the Chi-square statistic of the Wald test for the restriction that the coefficients for the additional factors are jointly equal to zero. The number of funds for which we reject that hypothesis, at the 5% level, are reported in brackets.

	Nº of	Unconditional SI			Unconditional MI					
	Funds	α	β	R^2(adj.)	α	Bindex	Sindex	Def	R^2(adj.)	W(p-val)
Germany	90	-0.026	0.799 ***	79.7%	-0.073 **	0.809 ***	0.033 ***	0.202 ***	87.4%	0.000 [70]
N +/0/-		0/82/8			0/65/25					
France	266	-0.077 ***	0.660 ***	94.6%	-0.088 ***	0.657 ***	0.004	0.110 ***	95.1%	0.000 [126]
N +/0/-		0/124/142			0/96/170					
UK	45	-0.127 **	0.886 ***	85.9%	-0.159 ***	0.836 ***	0.076 ***	0.151	89.9%	0.000 [12]
N +/0/-		0/23/22			0/22/23					
Spain	157	-0.173 ***	0.356 ***	92.2%	-0.179 ***	0.348 ***	0.004 **	0.051 **	92.8%	0.000 [47]
N +/0/-		0/17/140			0/10/147					
Italy	58	-0.218 ***	0.465 ***	89.0%	-0.217 ***	0.453 ***	0.006	-0.036	89.2%	0.113 [12]
N +/0/-		0/4/54			0/3/55					
Portugal	22	-0.143 ***	0.186 ***	75.9%	-0.144 ***	0.187 ***	0.002	-0.014	75.6%	0.429 [5]
N +/0/-		0/0/22			0/0/22					
All Sample	638									[272]
N +/0/-		0/250/388			0/196/442					

*** Statistically significant at the 1% level ** Statistically significant at the 5% level * Statistically significant at the 10% level.

Table 5 – Estimates for the conditional single (CSI) and multiple index (CMI) models when alpha is constant

For each country, an equally-weighted portfolio of all bond funds is formed. This table shows the results for both the CSI and CMI models. The predetermined information variables are the term spread (term), the IRW and a dummy variable for the month of January (jd). Term is the difference between the yield on a long-term government bond and a short-term bond rate (or the 3-month Interbank offered rate). IRW is the ratio between the exponentially weighted average of past real wealth and current wealth. These variables are stochastically detrended (by subtracting a 12-month moving average) and mean zero variables. The estimates for alpha (in percentage) and for the average conditional beta(s) γ_0 (for the CSI model) and γ_{0b}, γ_{0s}, γ_{0Def} (for the CMI model) and also the R^2(adj.) for each of the equally-weighted portfolios of funds are presented. The statistical significance of the estimates is based on heteroscedasticity and auto-correlation adjusted errors (following Newey and West, 1987). The number of individual funds presenting statistically significant positive, not different from zero and negative alphas, at the 5% level, is also reported (N +/0/-). The W(p-val) is the probability value for the Chi-square statistic of the Wald test for the restriction that the coefficients on the additional variables (the cross products between the factors and the predetermined information variables) are jointly equal to zero. The number of funds for which we reject that hypothesis, at the 5% level, are reported in brackets.

	Nº of Funds	CSI				CMI					
		α	β_0	R^2(adj.)	W(p-val)	α	BIndex	SIndex	Def	R^2(adj.)	W(p-val)
Germany	90	-0.008	0.781 ***	79.3%	0.324 [17]	-0.065	0.812 ***	0.033 ***	0.200 **	86.3%	0.263 [39]
N +/0/- alphas		2/82/6				0/76/14					
France	266	-0.080 ***	0.659 ***	95.2%	0.002 [122]	-0.086 ***	0.662 ***	0.003	0.098 ***	95.6%	0.087 [177]
N +/0/- alphas		0/121/145				0/113/153					
UK	45	-0.081	0.869 ***	88.9%	0.000 [32]	-0.120 **	0.820 ***	0.069 ***	0.147	91.2%	0.000 [35]
N +/0/- alphas		0/26/19				0/28/17					
Spain	157	-0.175 ***	0.352 ***	92.6%	0.016 [64]	-0.182 ***	0.352 ***	0.003	0.055 **	92.9%	0.000 [133]
N +/0/- alphas		0/9/148				0/7/150					
Italy	58	-0.210 ***	0.465 ***	89.1%	0.419 [12]	-0.204 ***	0.445 ***	0.005	0.015	89.5%	0.000 [47]
N +/0/- alphas		0/3/55				0/2/56					
Portugal	22	-0.144 ***	0.181 ***	78.7%	0.000 [13]	-0.143 ***	0.185 ***	0.001	-0.005	76.8%	0.001 [15]
N +/0/- alphas		0/1/21				0/1/21					
All Sample	638				[260]						[446]
N +/0/- alphas		2/242/394				0/227/411					

*** Statistically significant at the 1% level ** Statistically significant at the 5% level * Statistically significant at the 10% level.

Table 6 – Estimates for the conditional single (ACSI) and multiple index (ACMI) models

For each country, an equally-weighted portfolio of all bond funds is formed. This table shows the results for both the ACSI and ACMI models. The predetermined information variables are the term spread (term), the IRW and a dummy variable for the month of January (jd). Term is the difference between the yield on a long-term government bond and a short-term bond rate (or the 3-month Interbank offered rate). IRW is the ratio between the exponentially weighted average of past real wealth and current wealth. All these variables are stochastically detrended (by subtracting a 12-month moving average) and mean zero variables. The estimates for the average conditional alpha ϕ_0 (in percentage) and for the average conditional beta(s) γ_0 (for the conditional single-index model) and γ_{0b}, γ_{0s}, γ_{0Def} (for the conditional multi-index model) and also the R^2(adj.) for each of the equally-weighted portfolios of funds are presented. The statistical significance of the estimates is based on heteroscedasticity and auto-correlation adjusted errors (following Newey and West, 1987). The number of individual funds presenting statistically significant positive, not different from zero and negative alphas, at the 5% level, is also reported (N +/0/-). The W(p-val) is the probability value for the Chi-square statistic of the Wald test for the restriction that the coefficients on the additional variables (the cross products between the factors and the predetermined information variables) are jointly equal to zero. The number of funds for which we reject that hypothesis, at the 5% level, are reported in brackets.

	Nº of Funds	ACSI				ACMI					
		Φ_0	γ_0	R^2(adj.)	W(p-val)	Φ_0	γ_{0b}	γ_{0s}	γ_{0Def}	R^2(adj.)	W(p-val)
Germany	90	0.003	0.756 ***	80.0%	0.397 [23]	-0.057	0.789 ***	0.033 ***	0.223 ***	88.1%	0.000 [72]
N +/0/- alphas		3/82/5				0/76/14					
France	266	-0.074 ***	0.647 ***	95.4%	0.000 184]	-0.078 ***	0.642 ***	0.003	0.115 ***	96.0%	0.000 [223]
N +/0/- alphas		0/134/132				0/132/134					
UK	45	-0.069	0.868 ***	89.3%	0.000 [33]	-0.135 **	0.824 ***	0.080 ***	0.157	92.1%	0.000 [41]
N +/0/- alphas		0/27/18				0/27/18					
Spain	157	-0.172 ***	0.332 ***	93.9%	0.000 [75]	-0.178 ***	0.331 ***	0.003	0.056 **	94.1%	0.000 [131]
N +/0/- alphas		0/8/149				0/6/151					
Italy	58	-0.207 ***	0.466 ***	88.8%	0.130 [14]	-0.203 ***	0.447 ***	0.005	0.014	89.1%	0.000 [46]
N +/0/- alphas		0/3/55				0/2/56					
Portugal	22	-0.139 ***	0.176 ***	77.5%	0.003 [9]	-0.143 ***	0.185 ***	0.001	-0.005	76.8%	0.001 [16]
N +/0/- alphas		0/1/21				0/1/21					
All Sample	638				[338]						[529]
N +/0/- alphas		3/255/380				0/244/394					

*** Statistically significant at the 1% level   ** Statistically significant at the 5% level   * Statistically significant at the 10% level.

Table 7 – Estimates of conditional betas

This table presents the coefficients' estimates for the conditional beta function for the equally-weighted portfolios of funds and for both the SI and MI models. The predetermined variables Term, IRW and Jd are as defined in table 5. The conditional beta for the Bindex is designated by γ_{1b}, γ_{1s} identifies the conditional beta for Sindex and γ_{1Def} the conditional beta for the Def. Bindex, Sindex and Def as defined in table 2. The number of funds with positive (N+) or negative (N-) coefficients with respect to the lagged information variables are also reported, with the number of those which are statistically significant, at the 5% level, reported in brackets.

	Nº of Funds	ACSI			ACMI								
		$\gamma_{1,\,term}$	$\gamma_{1,irw}$	$\gamma_{1,jd}$	$\gamma_{1b,\,term}$	$\gamma_{1b,irw}$	$\gamma_{1b,jd}$	$\gamma_{1s,\,term}$	$\gamma_{1s,irw}$	$\gamma_{1s,jd}$	$\gamma_{1def,\,term}$	$\gamma_{1def,irw}$	$\gamma_{1def,jd}$
Germany	90	0.077	-0.269	0.101	0.040	-0.187	0.058	-0.003	0.011	0.003	0.102	-0.384	0.557 ***
N+		63	50	64	57	51	58	35	38	46	78	24	71
		[15]	[3]	[9]	[14]	[4]	[26]	[1]	[3]	[7]	[1]	[0]	[51]
N-		27	40	26	33	39	32	55	52	44	12	66	19
		[1]	[4]	[6]	[3]	[3]	[9]	[0]	[1]	[2]	[0]	[0]	[4]
France	266	0.000	-0.735 **	0.161 ***	0.014	-0.636 *	0.127 **	-0.004	-0.099 *	-0.003	0.028	-0.057	0.110
N+		125	40	230	154	43	206	85	64	114	198	109	173
		[26]	[0]	[124]	[33]	[1]	[60]	[4]	[7]	[14]	[21]	[4]	[35]
N-		141	226	36	112	223	60	181	202	152	68	157	93
		[22]	[79]	[5]	[10]	[62]	[11]	[9]	[52]	[33]	[2]	[4]	[17]
UK	45	0.027	-3.047 ***	0.203	-0.033	-1.830 ***	0.073	0.022 *	-0.113	-0.023	0.167	0.976	0.629 ***
N+		21	2	31	18	7	28	34	14	16	32	27	37
		[3]	[0]	[8]	[2]	[0]	[5]	[9]	[4]	[1]	[3]	[1]	[10]
N-		24	43	14	27	38	17	11	31	29	13	18	8
		[6]	[34]	[1]	[6]	[19]	[3]	[1]	[8]	[3]	[1]	[1]	[0]
Spain	157	0.002	-0.373 ***	0.029	0.005	-0.347 **	-0.171	-0.001	0.009	0.027 ***	0.035 *	-0.228	0.196 *
N+		84	57	89	82	56	57	57	88	105	116	67	113
		[21]	[5]	[26]	[31]	[6]	[9]	[1]	[25]	[38]	[22]	[6]	[38]
N-		73	100	68	75	101	100	100	69	52	41	90	44
		[28]	[32]	[10]	[21]	[40]	[36]	[8]	[22]	[9]	[2]	[13]	[9]
Italy	58	0.029	-0.459	0.047	0.022	-0.542	0.090 *	0.006	0.019	-0.007	0.085	1.356 **	-0.043
N+		42	11	40	41	7	47	48	42	16	47	52	29
		[8]	[0]	[8]	[6]	[0]	[12]	[4]	[3]	[4]	[3]	[6]	[6]
N-		16	47	18	17	51	11	10	16	42	11	6	29
		[1]	[5]	[1]	[2]	[7]	[1]	[1]	[0]	[13]	[0]	[0]	[3]
Portugal	22	-0.002	-0.191	0.092 ***	0.009	0.137	0.079 **	-0.001	-0.022 **	-0.001	-0.020	0.237	0.008
N+		13	7	14	15	8	10	7	2	14	10	16	16
		[2]	[4]	[4]	[2]	[2]	[4]	[1]	[0]	[5]	[1]	[1]	[5]
N-		9	15	8	7	14	12	15	20	8	12	6	6
		[0]	[5]	[0]	[0]	[7]	[2]	[1]	[3]	[2]	[1]	[0]	[2]
All Sample	638												
N+		348	167	468	367	172	406	266	248	311	481	295	439
		[75]	[12]	[179]	[88]	[13]	[116]	[20]	[42]	[69]	[51]	[18]	[145]
N-		290	471	170	271	466	232	372	390	327	157	343	199
		[58]	[159]	[23]	[42]	[138]	[62]	[20]	[86]	[29]	[6]	[18]	[35]

*** Statistically significant at the 1% level   ** Statistically significant at the 5% level   * Statistically significant at the 10% level.

Table 8 - Estimates of Timing

For each country, an equally-weighted portfolio of all bond funds is formed. This table shows estimates of the Treynor and Mazuy (1966) timing model, its conditional version as in Ferson and Schadt (1996) and the extended timing model considering time-varying alphas. The predetermined information variables are the term spread (term), the IRW and a dummy variable for the month of January (jd). Term is the difference between the yield on a long-term government bond and a short-term bond rate (or the 3-month Interbank offered rate). IRW is the ratio between the exponentially weighted average of past real wealth and current wealth. All these variables are stochastically detrended (by subtracting a 12-month moving average) and mean zero variables. The estimates for alphas (α and Φ_0), for betas (β and γ_0) and for timing coefficients (γ_2) as well as the R^2(adj.) for each of the equally-weighted portfolios of funds are presented. The statistical significance of the estimates is based on heteroscedasticity and auto-correlation adjusted errors (following Newey and West, 1987). The number of funds presenting statistically significant positive or negative timing coefficients (γ_2) is reported in brackets (at the 5% level). The number of individual funds presenting statistically significant positive, not different from zero and negative alphas, at the 5% level, is also reported (N +/0/-).

	Nº of	Treynor & Mazuy Model				Conditional Treynor & Mazuy				Conditional Treynor&Mazuy (time-varying alphas)			
	Funds	α	β	γ_2	R^2(adj.)	α	β_0	γ_2	R^2(adj.)	Φ_0	γ_0	γ_2	R^2(adj.)
Germany	90	0.006	0.795 ***	-0.035 [0/3]	79.6%	0.012	0.780 ***	-0.025 [1/2]	79.2%	0.012	0.756 ***	-0.011 [1/1]	79.7%
N +/0/-		0/87/3				0/87/3				0/88/2			
France	266	-0.072 ***	0.659 ***	-0.004 [12/12]	94.5%	-0.069 ***	0.659 ***	-0.009 [6/14]	95.2%	-0.070 ***	0.647 ***	-0.003 [6/7]	95.3%
N +/0/-		0/143/123				0/150/116				0/158/108			
UK	45	-0.065	0.865 ***	-0.026 [1/6]	86.1%	-0.070	0.865 ***	-0.006 [4/9]	88.8%	-0.048	0.860 ***	-0.012 [3/10]	89.2%
N +/0/-		0/30/15				0/30/15				0/32/13			
Spain	157	-0.172 ***	0.356 ***	-0.001 [27/26]	92.1%	-0.174 ***	0.352 ***	-0.001 [9/30]	92.5%	-0.177 ***	0.328 ***	0.006 [26/24]	93.9%
N +/0/-		0/8/149				0/8/149				0/8/149			
Italy	58	-0.161 ***	0.476 ***	-0.041 *** [0/29]	90.9%	-0.163 ***	0.475 ***	-0.040 *** [2/28]	90.7%	-0.157 ***	0.479 ***	-0.042 *** [1/30]	90.5%
N +/0/-		1/3/54				1/3/54				1/3/54			
Portugal	22	-0.143 ***	0.186 ***	0.000 [4/1]	75.6%	-0.148 ***	0.178 ***	0.005 [3/0]	78.5%	-0.143 ***	0.173 ***	0.006 [4/0]	79.2%
N +/0/-		0/1/21				0/1/21				0/1/21			
All Sample	638												
N +/0/-				44/517/77				25/530/83				41/525/72	

*** Statistically significant at the 1% level ** Statistically significant at the 5% level * Statistically significant at the 10% level.

Table 9 - Analysis of the Estimated Residuals for 638 European Bond Funds

The table entries are the percentage of bond funds for which the diagnostic tests in each of panels (i) through (v) are significant at the 1% level. This is done for each of the six European Countries covered and for each of the nine models. The abbreviations used to denote the models (SI etc.) are as defined in section 2. In panel (i) the numbers in parentheses for each country indicate the number of bond funds in the study for that country. The table entries are shown correct to the nearest whole percentage.

Country	Model SI	MI	CSI	CMI	ACSI	ACMI	TM	CTM	ACTM
(i) Skewness component of Bera Jarque Test									
France(266)	29	33	21	22	18	17	28	20	16
Germany(90)	47	39	39	36	30	26	40	38	27
Italy(58)	48	50	40	40	45	38	31	24	24
Portugal(22)	27	27	27	23	32	32	27	27	32
Spain(157)	48	45	37	41	31	27	43	33	27
UK(45)	20	18	18	18	18	18	13	16	16
(ii) Kurtosis component of Bera Jarque Test									
France	52	55	38	29	35	27	53	37	35
Germany	41	40	37	40	31	29	39	34	30
Italy	79	76	69	67	81	74	74	72	72
Portugal	55	55	55	50	59	50	59	50	45
Spain	66	64	62	61	55	44	62	59	52
UK	44	44	42	29	44	29	42	36	38
(iii) Bera Jarque Test									
France	52	55	38	33	32	30	54	38	33
Germany	51	50	48	47	37	36	44	44	37
Italy	76	74	72	67	79	74	74	71	72
Portugal	55	55	55	55	59	50	64	50	50
Spain	69	66	64	64	56	45	63	62	53
UK	49	42	42	33	44	33	42	33	36
(iv) Ljung Box test for serial correlation									
France	3	6	4	10	3	10	3	3	3
Germany	10	12	9	11	8	7	10	9	6
Italy	3	2	2	5	0	3	2	3	0
Portugal	18	18	18	18	14	14	18	23	14
Spain	9	8	9	9	8	6	10	11	9
UK	7	7	16	13	13	9	9	13	16
(v) Likelihood ratio test for GARCH effects									
France	40	36	29	20	27	20	39	28	26
Germany	20	18	17	17	12	10	17	11	9
Italy	71	71	60	50	53	50	64	57	50
Portugal	45	45	36	18	23	9	45	41	27
Spain	61	58	47	41	38	32	54	44	38
UK	16	11	20	9	18	11	18	20	20