Capital Regulation and Banks’ Financial Decisions*

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JEL Classification Numbers: G21, G28

Keywords: Capital Requirement; Economic Capital; Regulatory Capital; Actual Capital; Procyclicality Effect; Dynamic Programming; Prudential Regulation

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Abstract

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1 Introduction

Banking is by far one of the most regulated industries in the world. Among various regulatory measures, the regulation of bank capital is crucial due to the important role it plays in banks’ soundness and risk-taking behavior, and its influence on the competitiveness of banks.\footnote{See Berger et al. (1995) and Santos (2001) for surveys on the motivations behind capital regulation.} In practice, a key aspect of capital regulation is the calculation of minimum regulatory capital, which is typically based on the credit risk of bank assets. Traditionally, the risk matrix was very simple in the sense that the risk weights were practically flat for different bank loans. More recently, there has been a trend towards adopting a more risk-sensitive capital standard, mainly because the low-sensitivity had arguably led to severe market distortions as banks swapped low-risk assets against high-risk ones with more favorable risk weighting (regulatory arbitrage).

The transition of capital standards has generated the discussion on its implications – pros and cons – for the banking industry. Two major concerns arise. First, it tends to cause substantial changes in the level of bank capital. The study of Edwin and Wilde (2001), which is based on a hypothetical bank portfolio in the U.S. in 1990, suggests that the minimum capital requirement will drop from 8% to 6.8% with the introduction of the risk-based capital rule. Similarly, a study conducted by U.S. banking agencies shows that the new capital rule will cause the minimum required risk-based capital (MRC), the sum of expected and unexpected losses, to drop on average by 12.5%. Across entities, the changes in MRC vary substantially within a range of $[-50\%, 70\%]$, with a median reduction of 24%. These results raise questions on the appropriateness and fairness of the new capital standard. Second, the greater risk-sensitivity in the new capital regime may cause additional volatility in economic activity, sometimes cited as the procyclicality problem.\footnote{See Borio et al. (2001); Jokivuolle and Kauko (2001) for related discussion and Allen and Saunders (2004) for a literature survey.} In particular, capital requirements can increase as the economy falls into recession and fall as the economy enters an expansion. The increase in capital requirements during the downswing may result in a credit crunch and thus worsen already adverse economic conditions. The empirical relevance of this concern was later confirmed in several countries including the United States (Catarineu-Rabell et al., 2005; Jordan et al., 2003; Gordy and Howells, 2006; Kashyap and Stein, 2004), Spain (Corcostegui et al., 2002) and Mexico (Segoviano and Lowe, 2002).

However, there are several major caveats in these studies, which suggest that the above conclusions are debatable. First, these studies focus exclusively on the change in regulatory capital. Nevertheless, in order to understand the impact of regulatory measures, it is more
important to examine the change in banks’ actual capital holdings. A well-known fact is that most banks tend to hold a significant amount of capital above the regulatory requirement in practice, either for efficiency reasons or because the capital cushion is established as a precaution against contingencies (adverse events or regulatory penalties, see Barrios and Blanco, 2003; Elizalde and Repullo, 2006). Taking into account the role of capital buffers may change the above conclusion. Indeed, some researchers (Estrella, 2004b; Heid, 2005; Koopman et al., 2005; Peura and Jokivuolle, 2004) suggest that the existence of capital buffers can potentially mitigate the volatility in total capital. By contrast, the empirical evidence in Germany (Stolz and Wedow, 2005) and Spain (Ayuso et al., 2004) shows that capital buffers are also anti-cyclical, therefore it is not clear whether actual capital holdings are more volatile than regulatory capital, or the vice versa.

Second, these studies adopt passive portfolio assumptions, e.g. banks hold the same portfolio under the two capital regimes or hold portfolios as specified by the minimum capital requirements. This is questionable, given that an important objective of introducing the risk-based capital regime is to avoid excessive risk-taking that may arise under the old (less risk-sensitive) regime. Hence, it is more interesting to investigate how banks adjust their risk-taking behavior in their loan extension process. In other words, an active portfolio strategy is more appropriate for the purpose of this study.

Moreover, some of these impact studies are based on the industry information at a particular point at time. Therefore, the results fail to recognize the dynamic aspect of regulatory implications. The dynamic perspective is crucial in the sense that the underlying credit risk of bank assets can change substantially over the business cycle. As a result, the impacts on banks’ capital holdings (both regulatory and actual capital) and portfolio composition tend to be different at different phases of the business cycle.

This paper seeks to fill these gaps and provide new insights on these important issues. I propose a stochastic dynamic model in which the banks try to maximize shareholder value by making loans. A larger investment yields lower return, but is associated with lower risk due to the diversification benefits. Each bank has to choose its optimal capital structure, facing the tradeoff between one-period investment gains and future franchise value. In equilibrium, the bank’s leverage ratio, lending strategy and failure probability are jointly determined. Moreover, an important contribution of this model is that it allows for heterogeneity in

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3This paper uses three distinct definitions of bank capital. The economic capital refers to banks’ capital holdings in the absence of capital regulation, the regulatory capital refers to the minimum capital requirements based on specific capital rules, and the actual capital (or total capital) refers to banks’ equilibrium capital holdings in an economy with capital regulation.

4Cross-country studies, such as Bikker and Metzemakers (2004) and Kim and Lee (2006), find mixed results on the impact of capital regulation on the dynamics of actual capital holdings.
the banking industry. Interestingly, the implications of the transition of capital regimes are substantially different across banks depending on their size and risk profiles. I consider three economic environments: the baseline economy in which capital regulation is absent, a flat-rate capital regime in which a uniform capital-asset ratio requirement is imposed, and a risk-based capital regime in which the minimum capital requirement is determined by the credit risk of bank assets. The model is then calibrated to the U.S. economy in 1980-2002.

The numerical results show that banks’ financial decisions crucially depend on the state of the business cycle, banks’ initial equity holdings and the form of capital standards. Comparing equilibrium outcomes under different capital regimes sheds new insight on the issues of interest. On the one hand, my results support the view that the transition from a uniform to a risk-sensitive capital rule causes significant changes in minimum capital requirements, and the impacts are substantially different across banks depending on the risk profile of their loan portfolios. Nevertheless, the calibration results suggest that changes in actual capital holdings can be much smaller due to the offsetting effect of capital buffers. On the other hand, there is little evidence that capital regulation causes the procyclicality effect, in that the volatility of bank credits (and economic outputs) over the business cycle is smaller under both capital regimes than the volatility under the baseline economy (without capital regulation). Even between the two capital regimes, the larger volatility of capital charges under the risk-based capital standard does not necessarily lead to amplified swings in banks’ lending activity, because the active portfolio strategy implies that the loan portfolios chosen by banks in equilibrium are often different from those implied by regulatory capital constraints. In fact, for small banks the swing in bank lending can be mitigated with the adoption of a risk-based capital standard.

The model also provides a framework to compare the optimality of the two capital regimes from a regulator’s perspective, assuming that the regulator has to choose between maximizing franchise value and minimizing the negative externality caused by bank failure. Illustrative examples suggest that the risk-based capital rule can outperform the flat-rate capital requirement, in that expected franchise value is higher to achieve the same level of bank safety. The results hold both on the micro level (comparison based on individual banks) and on the macro level (comparison based on the banking system as a whole). The intuition behind this observation is: under the flat-rate capital requirement, the failure to reward sound investments causes a big distortion in the banking industry, limiting the scale of lending for those banks that have access to high-return, low-risk investment opportunities. From a dynamic perspective, such distortion implies that banks have to make a big sacrifice of future franchise value in order to reduce the default risk in the current period. By contrast,
the future franchise value at risk is much smaller under the risk-based capital regulation.

The remainder of the paper is organized as follow. Section 2 outlines the model setup and section 3 discusses the qualitative results. The model is then calibrated to the U.S. economy, allowing us to examine the equilibrium financial decisions under different capital regimes. Section 5 compares between the two capital regimes from a regulator’s perspective. The final section concludes.

2 The model

The benchmark model follows a modified version of the model developed by Cooley and Quadrini (2001). I treat a bank as a special type of firm, whose objective is to maximize the expected discounted value of dividends, $E_0\{\sum_{t=0}^{\infty} \beta^t d_t\}$, where $d_t$ is the dividend payment at time $t$, $\beta$ the discount factor for the bank shareholder, and $E$ the expectation operator. Accordingly, the assumptions on the asset and liability of the firm’s balance sheet will be modified to reflect distinctive features of the banking sector. The modified framework shares much similarity with the one developed by van den Heuvel (2002).

2.1 Loans

In each period, the bank tries to maximize shareholder value by making loans. To maintain analytical tractability, this model abstracts from the determination of lending rates by working directly with a return function of $Y_t = (z_t + \epsilon_t + \mu_t)F(L_t)$, where $Y_t$ is the revenue from making loans (adjusted by defaults), $L_t$ is the size of the loan portfolio, and $F(\cdot)$ satisfies $F(0) = 0$, $F'(\cdot) > 0$ and $F''(\cdot) < 0$. The concavity of $F(\cdot)$ function implies decreasing returns to scale of bank investments, which is standard in the literature and has been supported by empirical findings in Berger et al. (2005), Carter and McNulty (2005) and Cole et al. (2004), among others.

The other three variables, $z_t$, $\epsilon_t$ and $\mu_t$, jointly determine the efficiency of bank lending. In particular, $z_t$ represents the state of the economy that determines the level of investment yields and follows a first-order Markov process (Assumption 1); $\epsilon_t$ is a bank-specific shock that determines the risk of bank assets (Assumption 2); and $\mu_t$ measures the benefit from monitoring as it is explained below (Assumption 3).

Assumption 1 The state of the economy, $z$, takes values in a finite set $Z = \{z_1, z_2, \ldots, z_N\}$ and follows a first-order Markov process with transition probability $P(z_{t+1} | z_t)$.

This partial equilibrium assumption enables me to focus exclusively on banks’ investment decisions. As a tradeoff, the financing strategies of borrowers and the impact of banks’ lending decisions on the real economy, which would be important in a general equilibrium setting, are ignored in this paper.
Assumption 2 The bank-specific shock $\epsilon$ is independently and identically distributed and has a normal density function $N(0, \sigma^2_\epsilon(L))$. The volatility is inversely related to the size of bank assets, i.e. $\frac{d\sigma_\epsilon}{dL} < 0$.

At the beginning of each period, each bank observes $z_t$ but not $\epsilon_t$. In other words, each bank knows the level of expected investment yields in the current period, and is aware that the underlying risk depends on its lending strategy. A larger portfolio tends to have a lower risk for two reasons. First, this is consistent with the decreasing returns to scale assumption as mentioned above, in that high yields are accompanied by high risk. Second, it can be justified by the diversification benefit as proposed in finance theory. For instance, a recent survey by Group of Ten (2001) suggests that consolidation of banks within the United States has led to reductions in risk due to geographic diversification. As we will see later, the negative linkage between bank asset and risk, which is absent in Cooley and Quadrini (2001), turns out to be important in explaining the cross-sectional difference in banks’ capital structure observed in the data.

In addition, there exists another bank-specific drift term, $\mu$, which represents the bank-specific effort in improving its investment returns. Such effort can vary across banks due to differences in managerial skills, market power and monitoring incentive. A simple treatment is to consider this term as the return benefit from the bank’s monitoring effort. By paying a monitoring cost of $\theta(\mu)$ per unit of loan, the bank is able to improve the average investment return by $\mu$. This unit cost increases in $\mu$ at a growing rate, implying that the room for return enhancement tends to become smaller, i.e. a diminishing benefit from monitoring.

Assumption 3 The function $\theta(\mu)$ satisfies $\theta(0) = 0$, $\theta'(.) > 0$ and $\theta''(.) > 0$.

The model abstracts from the determination of lending rates and instead assumes a continuous distribution of net revenues. The continuity can be justified as follows. Suppose that a bank can make many individual loans, with a fixed lending rate for each loan. Since each loan can go bad with positive probability, the effective average return of the whole portfolio tends to be distributed over a large set of real numbers, with the maximum being the highest possible return if no default occurs and the minimum the worst scenario if all loans default.

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6In the literature, the monitoring effort could have two effects. One is to induce entrepreneurs to choose high-productivity projects, and the other to reduce the business risk. Since the revenue function refers to default-adjusted returns, both effects tend to increase the expected revenue.

7The same assumption is adopted by Acharya (2003).
2.2 Deposits and equity

A bank enters each period with an equity equal to \( e_t \). The bank finances its investment by equity capital and bank deposits, the latter being a debt instrument with a fixed return of \( r^d_t \).

Bank deposits are not risk-free, in the sense that the bank may default if its loan investment does not generate enough profits. Therefore a risk premium is charged to compensate for the default risk. This paper abstracts from the supply of bank deposits from the household sector by assuming that the elasticity of the deposit supply equals infinity and:

**Assumption 4** Depositors are risk-neutral and their reservation return is the risk-free rate, \( r^f_t \).

The assumption of perfect access to non-reservable deposits is also adopted in van den Heuvel (2002). The advantage of this assumption is that it allows me to focus on the bank capital channel through which the change in capital regimes affects banks’ lending behavior. On the other hand, it implies that the bank lending channel does not exist in this model, since a necessary condition for the effectiveness of the bank lending channel – shocks to reservable deposits affect the bank’s loan supply (Bernanke and Blinder, 1988; Kashyap and Stein, 1994) – is absent in the model.

However, this paper departs from the counter-intuitive assumption in van den Heuvel (2002) that bank deposits are fully insured at zero cost. Instead, the bank will need to pay a risk premium in line with its likelihood of default. Assuming that the probability of bank default is \( p \) and the expected recovery rate is \( \zeta \), the supply function of bank deposits is \( (D^s_t) \):

\[
D^s_t = \begin{cases} 
0 & \text{if } E(r) < r^f_t \\
[0, \infty) & \text{if } E(r) = r^f_t \\
\infty & \text{if } E(r) > r^f_t 
\end{cases}
\]  

(1)

where \( E(r) \equiv (1 - p)r^d_t + p\zeta r^d_t \) is the expected payoff of bank deposits.

At the end of each period, banks receive revenues on their asset portfolio and pay back deposits. A solvent bank chooses its dividend payment \( (d_t) \), or equivalently, its initial equity holding \( (e_{t+1}) \) in the next period. Noticeably, the amount of dividend is allowed to be negative in this model, i.e., the bank is able to issue new equity if necessary. Nevertheless, issuing new equity is costly. This is consistent with the statement made by practitioners, and can be justified with respect to transaction costs and differential taxation levels for equity. Specifically, the cost of raising new equity is assumed to be linear following Cooley and Quadrini (2001).\(^8\)

\(^8\)The cost of raising new equity is assumed to be constant over time. In practice, it is often more expensive
Assumption 5 The bank pays an issuance cost of $\Lambda(-d_t) = -\lambda d_t$ to raise new equity in the amount of $-d_t$ ($d_t < 0$).

Finally, this model makes the following assumption on the exit of a bank.

Assumption 6 A bank defaults if its net asset value drops below zero. A bank cannot re-enter the market once it exits, i.e. its franchise value is zero thereafter.

Here the bank’s net asset value is defined as the end-of-period total revenue minus the bank’s debt liability, i.e. the principal and interest payments on bank deposits. If the bank’s net asset value drops below zero, its equity capital is negative and it claims bankruptcy. In other words, the renegotiation of bank debt is not allowed in this paper and a default automatically leads to permanent exit.

2.3 Capital regulation

In addition, capital regulation can be introduced by the regulatory authority in the form of a minimum capital requirement. Each bank has to meet this requirement when it makes lending decisions.

Assumption 7 The bank has to meet the minimum capital requirement, $c_{min}$, which depends on the characteristics of bank assets.

Notice that the capital regulation, combined with the equity issuance cost assumption, leads to the deviation from the Modigliani-Miller theorem. Without these assumptions, whether bank loans are financed with deposits or equity is irrelevant. As we will see, the probability of default and the subsequent losses increase banks’ willingness to hold more capital than required.

Owing to the simplified maturity structure of bank assets and liabilities in this paper, the minimum capital requirement is effectively operative only when the bank makes loan decisions. Throughout the paper, I assume that the minimum capital requirement cannot be violated, while acknowledging that in a model with richer maturity structure of bank assets and liabilities the capital requirement might be violated subject to a penalty.

\footnote{Given that a bank’s portfolio is fixed during each period, meeting the minimum capital requirement at the beginning of a period (when the loan is extended) implies that the requirement is not violated throughout this period. Moreover, it is not meaningful to discuss capital charges after the realization of bank-specific shocks, particularly under the risk-based capital regime, because the uncertainty in asset returns is gone.}

for banks to raise fresh capital in economic downturns. Numerical studies show that introducing time-varying cost of equity issuance does not change the results qualitatively. The results are omitted in this paper for space reason.
This paper examines two forms of capital requirements. The first form of capital regulation imposes a flat capital-asset ratio requirement ($\kappa$) of 8% on all banks. Under this capital rule, the level of regulatory capital only depends on the size of bank assets but not their riskiness. By contrast, under the alternative form, the minimum regulatory capital is calculated based on the risk profile of bank assets. In particular, it is determined in a way that the probability of default for the bank does not exceed a threshold level (0.1%). That is, the minimum level of regulatory capital should satisfy:

$$
(z + \epsilon_{0.001} + \mu)F(L) - \theta(\mu)L = (L - e_{\text{min}})r^d
$$

where $\epsilon_{0.001}$ is the 0.1% critical value of the normally distributed variable $\epsilon$ with mean zero and standard deviation of $\sigma_{\epsilon}$. The above equation implies that, if a bank holds the required level of capital, there is 0.1% probability that the bank’s revenue is smaller than the bank’s deposit liability, i.e. the bank will claim insolvent. Because the default risk is very small, the deposit rate should be very close to the risk-free rate. Therefore, the above equation can be written as

$$
e_{\text{min}} \approx -\frac{(z + \mu)F(L) - (r^f + \theta(\mu))L}{r^f} - \frac{\epsilon_{0.001}F(L)}{r^f}
$$

(2)

In Equation 2, the minimum capital requirement consists of two components, the first part referring to the expected loss and the second one the unexpected loss of bank portfolio. The expected loss is jointly determined by the state of the economy and the bank’s choice of monitoring effort.

### 2.4 Timing

At the beginning of each period, there exist a continuum of banks with equity in the amount of $e_{i,t}$ carried over from last period. The state of the economy, $z_t$, is revealed one period in advance, while the bank-specific shock $\epsilon_{i,t}$ is realized at the end of the current period. That is, at the moment of making lending decisions, each bank knows $z_t$ but does not know $\epsilon_{i,t}$. Since there is no interaction between banks’ decisions, in the remaining part the analysis will focus on the optimization problem of an individual bank and the subscript $i$, which

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10Here the calculation is based on the credit VaR (value-at-risk) for the bank portfolio as a whole rather than on the basis of individual loans. Gordy (2003) and Kupiec (2004) prove that, when the default risk is driven by a single common factor and the bank portfolio is fully diversified, the two definitions yield the same level of regulatory capital.

11Implicitly, I assume that banks have an incentive to disclose accurate information concerning their risk and capital adequacy. Estrella (2004a) suggests that it can be achieved via the combination of voluntary disclosure, direct supervision and financial market discipline, which is beyond the scope of this paper.
represents bank-specific variables, will be dropped.

The bank then decides on the volume of bank credit \((L_t)\) and the level of its monitoring effort \((\theta_t)\). To finance the loan decision the bank’s liability includes the equity holding \(e_t\) and bank deposits in the amount of \(L_t - e_t\), which pay an equilibrium deposit rate of \(r^d_t\). The bank’s financial structure has to meet the regulatory capital requirement, if any.

At the end of each period, the bank-specific shock \(\epsilon_t\) is realized and depositors are paid back. Depending on the solvency condition, the bank either files for bankruptcy (if its net asset value drops below zero) or continue operating.

Lastly, at the end of each period the market also observes the state of the economy in the next period, \(z_{t+1}\). The realization of \(z_{t+1}\) is history-dependent following a transition probability \(P(z_{t+1}|z_t)\). Depending on the amount of net profits and the future state of the economy, a solvent bank chooses whether to distribute dividend to its shareholders or to issue new equity \((d_t)\). This choice is reflected on the bank’s initial equity holding for the next period, \(e_{t+1}\).

### 3 The solution to the model

The bank’s problem can be solved using dynamic programming techniques and the backward induction method. At each period, a bank makes three important decisions: choosing the size of its loan portfolio, choosing the monitoring effort at the beginning of the period and deciding on the dividend policy at the end of the period.

I first examine the solvency condition for each bank at the end of each period. For given initial equity \((e_t)\), bank lending \((L_t)\), deposit rate \((r^d_t)\) and monitoring effort \((\theta_t)\), the end-of-period net asset value of the bank is

\[
\pi(L_t, \mu_t, \epsilon_t, r^d_t) = (z_t + \epsilon_t + \mu_t)F(L_t) - \theta(\mu_t)L_t - r^d(L_t - e_t) \tag{3}
\]

There are two possible outcomes. If the bank’s net asset value is positive, i.e. \(\pi_t > 0\), the bank is in a healthy condition and depositors receive the full payment. Otherwise the bank files for bankruptcy and depositors have to accept a smaller payment by sharing bank assets on its balance sheet, which equals \((z_t + \epsilon_t + \mu_t)F(L_t) - \theta(\mu_t)L_t\).

Accordingly, I define the following critical value of the bank-specific shock, \(\bar{\epsilon}\):

\[
\pi(L_t, \mu_t, \bar{\epsilon}_t, r^d_t) = 0 \tag{4}
\]

above which depositors receive the full payment and below which the bank defaults. Since depositors are risk-neutral, in equilibrium their expected payoff should equal the return on
risk-free investments. That is,

\[
\int_\epsilon^\infty r^d_t(L_t - \epsilon_t)\phi(\epsilon)d\epsilon + \int_{-\infty}^\epsilon [(z_t + \epsilon_t + \mu_t)F(L_t) - \theta(\mu_t)L_t] \phi(\epsilon_t)d\epsilon = r^d_t(L_t - \epsilon_t) \tag{5}
\]

where \(\phi(\epsilon)\) is the p.d.f. function of the shock variable \(\epsilon\). Combining Equations (4) and (5), \(\bar{\epsilon}_t\) and \(r^d_t\) can be endogenously determined in the deposit market once \(L_t\) and \(\mu_t\) have been chosen.

\[
\bar{\epsilon}_t = \bar{\epsilon}(L_t, \mu_t) \tag{6}
\]

\[
r^d_t = r^d(L_t, \mu_t) \tag{7}
\]

**Lemma 1** \(\bar{\epsilon}\) increases in bank asset (\(L\)) and decreases in bank equity (\(\epsilon\)).

Proof: based on Equations (4) and (5), it can be easily derived that

\[
\frac{\partial \bar{\epsilon}}{\partial L} = \frac{1}{1 - \Phi(\bar{\epsilon})} \cdot \left\{ \frac{\theta(\mu) + r_f}{F(L)} \cdot (1 - \frac{F'(L)L}{F(L)^2}) + \frac{F'(L)e}{F(L)^2} \right\} > 0
\]

\[
\frac{\partial \bar{\epsilon}}{\partial \epsilon} = -\frac{r_f}{F(L) \cdot (1 - \Phi(\bar{\epsilon}))} < 0
\]

Lemma (1) is quite intuitive. Notice that \(\bar{\epsilon}\) is the threshold value above which a bank can generate positive net profits, hence a higher \(\bar{\epsilon}\) indicates lower profitability and a higher probability of bank failure. An increase in bank assets tends to lower bank profits and increase bank fragility for two reasons. On the one hand, the marginal return of bank assets decreases due to the concavity of the revenue function. On the other hand, the leverage ratio is higher, implying that the default boundary is higher and consequently the default probability is higher. Similarly, lower bank equity increases the probability of bank failure for the leverage reason.

Once \(\bar{\epsilon}_t\) is determined, the net asset value of the bank can be rewritten as

\[
\pi(L_t, \mu_t) = \begin{cases} 
(\epsilon_t - \bar{\epsilon}_t)F(L_t) & \text{if } \epsilon_t \geq \bar{\epsilon}_t \\
0 & \text{if } \epsilon_t < \bar{\epsilon}_t 
\end{cases} \tag{8}
\]

Define \(V(e, z)\) as the value function for a bank with an initial equity of \(e\) and the initial state of the economy \(z\), the bank’s optimization problem can be written as follows:

\[
V(\epsilon_t, z_t) = \max_{(L_t, \mu_t)} \sum_{z_{t+1}} \int_{\bar{\epsilon}_t}^\infty W(\pi(L_t, \mu_t, \epsilon_t, z_{t+1}), z_{t+1}) \cdot P(z_{t+1}|z_t)\phi(\epsilon)d\epsilon \tag{9}
\]
subject to Equations (6), (7), (8) and

\[ L_t \geq e_t \]  
\[ e_t \geq e_{\min}(L) \]  \hspace{1cm} (10)

\[ W(\pi(L_t, \mu_t), z_{t+1}) = \max_{e_{t+1}} \{ d(\pi_t, e_{t+1}) + \beta V(e_{t+1}, z_{t+1}) \} \]  \hspace{1cm} (12)

subject to

\[ d(\pi(L_t, \mu_t), e_{t+1}) = \begin{cases} 
\pi(L_t, \mu_t) - e_{t+1} & \text{if } \pi_t \geq e_{t+1} > 0 \\
[\pi(L_t, \mu_t) - e_{t+1}] (1 + \lambda) & \text{if } 0 \leq \pi_t < e_{t+1} \\
0 & \text{if } \pi_t < 0
\end{cases} \]  \hspace{1cm} (13)

\[ e_{t+1} = 0 \] if \( \pi_t < 0 \)  \hspace{1cm} (14)

By solving the above dynamic problem, the bank maximizes the expected value of its shareholders. The uncertainty in future franchise value comes from both the uncertainty in future state of the economy and the riskiness of bank assets. Franchise value drops to zero when \( \epsilon < \bar{\epsilon}(L_t, \mu_t) \), because in that case the bank loses all its equity capital and is forced to exit the market. Equation (11) is the regulatory capital constraint (if applicable). The bank’s equity holding must be higher than the minimum requirement, which equals \( \kappa \cdot L \) under the flat-rate capital requirement and is endogenously determined under the risk-based capital requirement (Equation 2). The imbedded dynamic program program (Equations 12-14) is the dividend policy to be chosen by the bank at the end of each period, which is equivalent to the choice of \( e_{t+1} \). If the bank chooses to issue new equity, \( d_t \) is negative and the bank pays an additional cost of \( \lambda \) for each unit of fresh capital.

The solution to this problem has the following properties.

**Proposition 1** There exists a unique function \( V^*(e, z) \) that solves the dynamic program (9). The optimal value function is strictly concave in \( e \).

Proof: see Appendix A.

**Proposition 2** (optimal policy functions)

1. The optimal monitoring effort, \( \mu^* \), is a function of \( L \) and independent of \( z \) and \( \epsilon \). It satisfies \( F(L) = \theta'(\mu^*)L \);

2. The optimal dividend policy follows a cutoff rule, i.e. there exists \( \pi_1 \leq \pi_2 \), for which the bank issues new equity \( (d < 0) \) if \( \pi < \pi_1 \), distributes dividend \( (d = \pi - \pi_2) \) if \( \pi > \pi_2 \) and retain all profits \( (d = 0) \) if \( \pi \in [\pi_1, \pi_2] \).
Proof: see Appendix B.

Proposition 2.1 suggests that the level of monitoring effort is chosen to maximize the expected net profit in the current period, and is independent of the uncertainty in asset returns and the bank’s future franchise value. This reduces the number of free parameters by one in the value function iteration procedure and simplifies the algorithm significantly.

The strict concavity of $V^*$ implies that the dividend policy assumes a simple form as specified in Proposition 2.2. Suppose that the feasible values of $e$ are restricted to a set $[e_{\text{min}}, e_{\text{max}}]$. In the embedded dynamic programming problem (13), the marginal cost of bank equity is 1 if the bank does not issue new equity or $1 + \lambda$ if the bank issues new equity. At the same time, the marginal benefit of bank equity is $V'(e_{t+1}|z_t)$. This tradeoff is illustrated in Figure 1. Suppose that $e^1$ (or $e^2$) is the critical value of $e_{t+1}$ at which the marginal benefit equals the marginal cost if new bank equity is issued (or if no new equity is issued), the bank’s dividend policy must follow the cutoff-rule as stated above with $\pi_1 = e^1$ and $\pi_2 = e^2$. This is quite intuitive. If the bank’s net asset value is smaller than $e^1$, the bank has the incentive to raise new equity to the target level $e^1$, because the increase in the bank’s franchise value exceeds the equity issuance cost. By contrast, when $\pi > e^2$, the bank chooses to distribute dividend in the amount of $\pi - e^2$, at which point the marginal decrease in bank value equals the marginal benefit of dividend payment. When the net asset value is in the middle range, the bank prefers to reinvest all profits because it increases the shareholders’ value by more than paying them out as dividend; nevertheless, no extra equity will be raised because the cost is relatively too high.

Notice that the two critical values, $\pi_1$ and $\pi_2$, change with the state of the business cycle but are independent of bank characteristics. In other words, the “optimal” range of equity holdings is uniform for all banks. Of course, the choice of equity holding within this range depends largely on the realization of bank-specific shocks, suggesting that the equilibrium size distribution of bank equity exhibits substantial cross-sectional differences (this issue will be re-examined later).

Going backward, the optimal cutoff rule implies that, when a bank chooses its lending strategy ($L$), it faces the following tradeoffs: First, the choice of bank assets affects the level of expected net asset value. Generally, the concavity of the return function leads to the concavity of expected net asset value (using Equation 3). Therefore, for each bank there exists an optimal loan volume that maximizes its end-of-period expected gains. The bank has the incentive to choose the size of its loan portfolio close to the optimal level. Second,

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12 The first-order condition is too complicated so I leave it out here.
the lending decision also affects the distribution of net asset values. Lower $L$ tends to reduce the probability of bank failure (Lemma 1). Meantime, lower $L$ is also associated with higher volatility of asset returns (following Assumption 2), implying that the bank is more exposed to extreme (good or bad) events. The overall impact is ambiguous. Hence, there is no clear indication whether the volatility concern will cause the bank to increase or reduce the volume of loan extension.

In addition, the lending decision is further complicated by the existence of equity issuance cost, because the change in asset size has an ambiguous effect on the magnitude of this cost. As a result, a closed-form solution is ruled out in the model. The remaining part of the paper tries to employ numerical examples to illustrate the properties of the solution and the equilibrium relationship among bank size, leverages, profits and the probability of failure.

4 Calibration

4.1 Parameterization

The model parameters are calibrated based on the U.S. data in 1981-2002. Table 1 summarizes the input parameters used in the numerical analysis.

- The risk-free rate is fixed at 3 percent, close to the average real risk-free rate (short-term Treasury rate adjusted by inflation) in the United States in the period 1981-2002 (which equals 3.07%).

- The concave return function $F(\cdot)$ is specified as $F(L) = L^\alpha$ with $\alpha = 0.985$. This returns to scale parameter is close to the one adopted by Cooley and Quadrini (2001). The parameter must be strictly less than one, because otherwise the optimal size of the bank goes to infinity. In addition, it is chosen close to one to avoid the large discrepancy in investment returns between large and small banks.

- The state of the economy. For simplicity, I assume that $z$ only takes two possible states – high (H) or low (L) returns. The probability of staying at the same state in the next period is set to be 0.800, which implies that the same state of economy lasts for four

13Based on Equation (8), the two critical net asset values can be transformed into two critical values for bank-specific shocks. Specifically, the threshold value of the bank-specific shock, below which new equity will be issued, is $\epsilon_1 = \bar{\epsilon} + \frac{\bar{\epsilon}}{\delta}$. Higher $L$ increases the first component ($\bar{\epsilon}$) but decreases the second, thus having an ambiguous effect on the probability (and the cost) of new equity issuance.
years on average. The bank-specific shock $\epsilon$ is assumed to be normally distributed with mean zero and standard deviation $\sigma$, which will be discussed later.

The choice of two possible states of the economy, $z_H$ and $z_L$, attempts to match the loan premium observed in the data. I rank the annual loan premium (defined as loan rates minus short-term Treasury rates) during the period 1981-2002, and divide the sample into two subgroups. The average loan premium in the good eleven years is 3.88% and that in the bad eleven years is 2.12%. Hence I set the two shocks as $z_H = 1.07$ and $z_L = 1.052$ (i.e. excess returns of 4% and 2.2%), respectively.

- The determination of $\sigma$, the return volatility, is not straightforward. Following conventional finance theory, I first assume that the return volatility is inversely related to the size of bank assets, i.e. $\sigma(L) = \frac{a_1}{L + a_2}$. The two constant terms $a_1$ and $a_2$ are then chosen so that the Sharpe ratios in equilibrium are within a reasonable range. Rosen (2005) documents that the median Sharpe ratio (the ratio of ROA to the standard deviation of the ROA) for U.S. banks during the period 1977-2003 is about 0.90. In this study, $a_1 = 1.05$ and $a_2 = 15$ so that, in equilibrium, the Sharpe ratios vary between 0.40 and 1.40 with a median of 0.92.

- The cost of monitoring effort is specified as a quadratic form $\theta(\mu) = c_0\mu^2$, where $c_0 = 6.5$. I choose this particular value to offset the decreasing returns to scale effect due to the concavity of the return function, so that in equilibrium the average loan premium matches the target level. Nevertheless, the monitoring cost parameter is not crucial in this study, and varying this parameter has a minor impact on the results.

- The intertemporal discount factor equals 0.95. This is very close to the one adopted by Cooley and Quadrini (2001) and is also consistent with the business cycle literature.

- Following Cooley and Quadrini (2001), the cost of issuing new equity, or the new share’s premium, is set to be 30%. The magnitude of the equity issuance cost is also in line with the size of the tax burden (40%) discussed in van den Heuvel (2002), an alternative way to introduce the differentiation between equity and debt financing.

- Under the flat-rate capital requirement, the minimum capital-asset ratio equals 8%.

The computational procedure of the dynamic programming problem is described in Appendix C. The results presented are based on the discretization using a grid with 240 points for bank equity ($e$), net asset value ($\pi$) and loan volume ($L$).
4.2 A baseline model without capital regulation

The baseline model examines the banks’ financial decisions when there is no capital requirement, which provides a benchmark for judging the impact of different forms of capital regulation. The characteristics of banks’ equilibrium behavior are shown in Figures 2 and 3.

Figure 2 shows the optimal dividend policy adopted by banks at the end of each period. All banks adopt a uniform cut-off rule as described in Proposition 2.2 (top panels). That is, irrespective of banks’ initial equity in current period, their initial equity in the next period will be distributed within the same “optimal” range. However, the size distribution within this range is different across banks, as illustrated in the middle and bottom panels of Figure 2. The panels compare the probability distributions of end-of-period net asset values and next-period initial equities for three hypothetical banks – a large bank \( (e_0 = 2) \), a small bank \( (e_0 = 1) \) and a mid-sized bank \( (e_0 = 1.5) \). The results suggest that large banks are very likely to remain the business leaders in the next period. This is not surprising, because strong capitalization implies that these banks can choose to maximize investment profits without worrying too much about bank safety. By contrast, the growth capacity of small banks is limited due to the concern that high-leveraged capital structure, which is desirable from the perspective of profit maximization, tends to increase the probability of bank failure substantially.

Figure 3 summarizes the characteristics of equilibrium solutions in the baseline economy, including banks’ lending decisions, monitoring efforts, and default probabilities. In each panel, the solid lines refer to equilibrium outcomes in the good state of the economy and the dashed lines represent equilibrium outcomes in the bad state. There are several major observations.

First of all, the upper-left panel indicates that the bank’s franchise value is higher in the good state and it increases with bank size.

Second, the upper-middle and upper-right panels plot the bank’s lending decisions. Generally, bank lending increases during the expansion period and falls during the downturn period. This pro-cyclical lending behavior is a result of maximizing profits when the average return is high. The optimal leverage ratio (the inverse of equity/asset ratios) can be as high as in the range of 15-35 in the good state and as low as 6-10 in the bad state.

Moreover, the optimal leverage ratio differs across banks. As shown in the figure, large and very small banks choose low leverages in equilibrium, and mid-sized banks often choose high leverages.\(^{14}\) This cross-sectional difference is driven by the tradeoff between one-period

\(^{14}\)Figure 2 suggests that the optimal bank size is approximately between 1 and 2 for all banks. Although bank size here does not map directly into the observed data, it is reasonable in this paper to interpret banks
gains and future franchise value. On the one hand, there is an optimal scale of bank lending that maximizes the expected profit of each bank in the current period due to the decreasing returns to scale assumption.\(^\text{15}\) Other things equal, each bank has the incentive to choose this optimal lending scale, implying that the optimal leverage ratio decreases with bank size. On the other hand, high leverage increases the probability of bank failure and reduces the bank’s value in the future. The equilibrium lending decision depends on the relative importance of these two effects. The numerical example suggests, for very small banks, reaching the optimal loan scale that maximizes current period gains leads to very high leverages and high default risk. The future franchise value cost is so high that these banks will choose a prudent lending strategy (i.e. the second effect dominates). As bank equity increases, the stronger capital position implies that banks can maximize current period gains without increasing the probability of default substantially. Hence the optimal leverage ratio increases to reach the balance between the marginal benefit from investment gains and the marginal cost of bank failure. However, at certain point (when the capital position is strong enough) the first effect dominates the safety concern and the optimal leverage ratio decreases with bank size. The combined effects explain the “U-shape” optimal equity/asset ratios as observed in the graph.

Third, the left and middle panels in the bottom row show the risk profile of bank investments and banks’ monitoring efforts. Smaller banks are associated with smaller but riskier portfolio investments, and are willing to devote a higher monitoring effort to reducing the probability of failure. In addition, when the economic conditions are favorable, high productivity induces banks to lower their monitoring efforts because the marginal benefit of higher returns is smaller.

Lastly, the bottom-right panel shows the probability of bank failure during the two stages of the business cycle. Overall, the probability of default is very small for all banks, therefore the difference between default-adjusted deposit rates and the risk-free rate is negligible (not shown here). However, the failure probability differs substantially across banks. In line with the difference in lending decisions, an inverse “U-shape” curve is observed across banks of different sizes. The largest and smallest banks lend more prudently and their default probabilities are very low. By contrast, mid-range bank size is associated with aggressive lending and high insolvency risk.

of size 2 as the largest ones in the industry, banks of size 1 as the small and established banks, and banks of size in between as mid-sized banks. Banks with a size smaller than 1 represent the smallest ones that are still at their infancy stage.

\(^{15}\)Based on Equation 3 and Proposition 2.1, expected profit in the current period equals \(E(\pi | z) = zL^\alpha + \frac{(2\alpha - 1)}{4\alpha} - r'(L - e)\). It is straightforward to show that there is a uniform \(L\) that maximizes the expected profit for each bank.
Interestingly, the probability of bank failure is much higher in the upturn of the business cycle. This supports the view that business risks build up during the expansion period. As the former Federal Reserve Chairman Alan Greenspan noted, “the worst loans are made at the top of the business cycle” (Chicago Bank Structure Conference 2001). Borio and Lowe (2001) and Borio et al. (2001) document the same pattern that systemic risk tends to build up in the booming period. Notice that, however, in this paper the higher systemic risk during upswings is not caused by underestimation of the business risk. Instead, the forward-looking banks are fully aware of the risk involved: they are willing to increase their risk exposure in the pursuit of higher returns. In other words, the one-period expected gain from risk-taking exceeds the franchise value at risk.

The bank failure rates in the simulation are in line with the data (Figure 4). The highest bank failure rate, in the range of 1-2%, matches historical observations in the U.S. before a mandatory capital requirement was applied in 1988. At the other extreme, the lowest bank failure rate is 0.01-0.04%, both in the simulation and in the data.

### 4.3 A flat-rate capital requirement

The rest of the paper introduces capital regulation and examines the impacts on equilibrium outcomes, including banks’ lending behavior, optimal leverage ratios, probability of failure, etc. The capital requirement is assumed to be exogenously imposed for regulatory reasons that are not explored in this model. This subsection focuses on the effect of a flat-rate capital standard, which requires a minimum capital-asset ratio of 8%.

Figure 5 shows the characteristics of equilibrium outcomes under this flat (risk-insensitive) capital rule. Many qualitative aspects of the results in the baseline economy remain valid. For instance, the bank’s franchise value is concave and is strictly increasing in bank size and the state of the economy. In equilibrium, all banks choose a cutoff dividend rule as described in Proposition 2.2. Furthermore, the initial bank equity is important in determining the bank’s lending behavior. Large banks tend to lend more and spend less resources on monitoring (per unit of asset). The optimal equity/asset ratio is “U-shape” distributed across banks, and the probability of default has an inverse “U-shape” distribution.

However, the imposition of the minimum capital requirement does have important effects on the bank’s financial decisions. The counter-cyclicality of the economic capital under the

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16In this paper, it is hard to distinguish between the “build-up” and the “materialization” of business risks because all bank loans mature in one period. The simplified assumption on maturity structure misses the realistic situation that there exists a lag for risks to materialize. Therefore, the readers should not link directly the realization of bank failures with the state of the business cycle when fitting model predictions with the data. Moreover, for general discussion on the impact of the timing assumption, see Crocket (2000).
unregulated economy (see Figure 3) implies that the uniform minimum capital requirement is more likely to be binding in the good state. The binding capital constraint forces banks to lend less aggressively during the expansion period. As a result, the probability of bank failure is much lower than in the baseline economy. By contrast, during the downturn period, the bank’s financial decisions are practically unaffected.\footnote{The bank’s equilibrium financial decisions in bad times differ from the results in the unregulated economy, even if the capital requirement is not binding. This is because banks are forward-looking and the possibility of a binding capital constraint in the future affects their financial decisions in the current period.}

The cross-sectional difference in economic capital under the baseline economy suggests that the effectiveness of the capital requirement is different across banks. In good times, the capital constraint is binding for most banks except for the smallest ones. Accordingly, most banks have to reduce the scale of investment to abide by the regulatory constraint. In bad times, banks are willing to hold capital above the minimum regulatory requirement, and the level of capital buffers reflects the difference between economic capital and regulatory capital. Consistent with the baseline results, large and small banks hold larger capital buffers than the others.

Importantly, contrary to the baseline results, the probability of bank failure is not always higher in the good state than in the bad state. Except for very small banks, banks’ failure rates are lower in the good state, suggesting that the capital requirement plays a powerful disciplinary role in containing excessive risk-taking during economic upswings. By contrast, it has little impact on banks during the period when banks hold strong capital positions and are less fragile.\footnote{The U.S. experience suggests that the flat-rate capital requirement has helped to mitigate the severity of the worst scenario, in that the average default rates in bad years dropped significantly after the introduction of capital regulation (Figure 4). First, I divide the sample into two sub-periods: 1971-1988 and 1991-2002. For each sub-period, I then calculate the average of the half high bank failure rates and that of the half low failure rates. During the first eighteen years, the two averages were 0.67% and 0.04%, respectively. In the twelve years after capital regulation was introduced, the two averages were 0.43% and 0.04%, respectively.}

The prediction of cross-sectional differences in actual capital holdings is partially supported by empirical evidence. Figure 4 shows that, in the U.S. banking industry, the smallest banks typically hold the highest capital/asset ratio. This is consistent with the calibration results. What deviates from the model prediction is, however, that the data observe downward-slope capital ratios, i.e. the largest banks hold the least amount of capital buffers. This could be attributed to a number of factors that are absent in this model. For instance, the largest banks have better access to the inter-bank market, arguably they are considered by market participants to be safer due to the “too-big-to-fail” argument. Hence, they may be able to raise new equity at lower cost. These privileges entice the largest banks into extra lending; hence their capital ratios could be lower than what the model has predicted.
Lastly, the calibration results allow us to examine the procyclicality issue. In this paper, I distinguish between the procyclical movement of bank credit and the procyclicality effect, in that the latter refers to the scenario that the procyclical credit cycle (or the cycle of economic outputs) is amplified with the introduction of capital regulation. From a regulator’s perspective, the amplified credit cycle can lead to excess volatility and cause potential harm to the economy, whereas the cyclical movement of bank credit itself is more likely to be a manifestation of an optimal allocation of economic resources over the business cycle.

I quantify the magnitude of credit cycle for each bank by calculating the ratio of economic outputs (investment yields minus the monitoring cost) in good times vs. in bad times. Furthermore, under specific capital regulation, the cycle is further broken down into two components, which are attributed to the cyclical movements in capital charges and capital buffers, respectively. The comparison of credit cycles between the baseline economy and the one with capital regulation sheds light on whether the procyclicality effect exists, and if it exists, its magnitude and the driving forces behind.

Figure 6 plots the changes in economic outputs, first assuming that banks choose loan portfolios as indicated by the minimum capital requirement (left panels) and then showing the actual outcomes in equilibrium (right panels). Under the flat-rate capital requirement (the dashed lines), economic outputs co-move with the business cycle and this cyclical movement is mainly driven by the anti-cyclical movement in capital buffers.\(^{19}\) Differentiation in capital buffer holdings across banks implies that the magnitude of lending cycles are also different. Those banks that hold low capital buffers in both states (initial equity $\approx 1$) have the smallest credit cycle in the banking industry.

Importantly, the results suggest that the concern about the procyclicality effect is irrelevant under the flat-rate capital requirement, in that the introduction of capital regulation actually causes less volatility in economic activities than in the baseline economy (the solid lines in Figure 6). The mitigation of credit cycles is mainly because capital regulation contains banks’ imprudent lending during upswings.

### 4.4 A risk-based capital requirement

This subsection examines the impact of a risk-based capital rule, which is defined in a way that the maximum failure probability for each bank does not exceed a pre-given threshold value (see Section 2.3). Compared with the flat-rate capital requirement, under the risk-based

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\(^{19}\)This anti-cyclical movement in capital buffers is consistent with the findings in Germany (Stolz and Wedow, 2005) and Spain (Ayuso et al., 2004). Jokipii and Milne (2006) further point out that the fluctuation of capital buffers over the business cycle can vary across country and with the type and size of banks.
capital regime the level of regulatory capital not only varies with the state of the economy but also changes with the risk profile of bank assets, the latter of which is endogenously determined. Specifically, the new capital rule is able to reward banks with high-quality assets by lowering their capital requirements and penalize banks with low-quality assets by doing the opposite.

Figure 7 summarizes the characteristics of equilibrium outcomes under the new capital regime. Overall, banks’ financial decisions exhibit similar patterns as in other economic environments. Larger banks are associated with bigger loan extension, lower returns and lower risk. Equilibrium capital structure exhibits a U-shape across banks, reflecting that the tradeoff between current period gains and future franchise value has different impacts on banks depending on their initial capital positions. In addition, the optimal dividend policy follows a cutoff rule.

It is worth mentioning that the new capital regime appears to be quite efficient in improving the safety of individual banks in that the probability of bank failure is successfully contained below the target level. The disciplinary role of the new capital regime is particularly remarkable for small banks, which are forced by higher capital charges to reduce the scale of bank lending substantially in good times. In other words, the build up of excessive risk-taking by those banks are substantially mitigated.

Based on the simulation results, I am able to re-examine the two interesting issues raised in previous studies, i.e. the change in bank capital and the arise of the procyclicality effect during the transition from flat-rate to risk-based capital regimes. There are two possible answers to the first question. The first answer, consistent with the mainstream of the existing literature, examines the changes in minimum capital requirements. The dotted and dash-dotted lines in the top two panels, and the solid line in the bottom panel of Figure 8, make such comparison. The key message is that the new capital rule benefits large (and less risky) banks at the cost of small (and more risky) banks. In particular, the new capital rule causes the average minimum capital requirement for large banks to be approximately 50% lower than under the flat-rate capital regime, and that for small banks to be 50-60% higher. These findings are roughly in line with previous studies.

However, there is a second answer, which might be more interesting, to the question by investigating the change in actual capital holdings. The solid and dashed lines in the top two panels, and the dashed line in the bottom panel of Figure 8, report the results of this comparison. Overall, the change in total capital is much smaller than the change in regulatory capital, because of the offsetting effect by endogenous adjustments in banks’ capital buffers. And the impacts are less uneven across banks, in that large banks will hold
15-20% less capital and small banks will hold approximately 20% more capital during the transition.

In addition, the uneven impact across banks is not necessarily undesirable. To some extent, it reflects an important motivation behind the transition of capital regimes: to level the de-levered playing field. The flat-rate capital requirement imposes a penalty on taking excessive risk but fails to reward high-quality investments, in that (mid-sized and large) banks are forced to waive high-return, low-risk lending opportunities in good times because of binding capital constraints. By contrast, the risk-based capital regime successfully restores market efficiency by linking capital requirements with asset quality. As the simulation results show, under the risk-based capital regime, the more favorable capital regulation for large (and safer) banks makes their lending portfolios closer to the choices under the baseline economy, and the stricter capital requirement for small (and more risky) banks forces them to cut loan extension substantially.

The answer to the second question, the procyclicality effect, is illustrated in Figure 6. Swings in economic outputs over the business cycle reflect swings in banks’ capital structure and bank credits. There are three main messages from the numerical results. First, under the new capital framework, the cyclical movement in economic activities is mainly driven by the counter-cyclicality of capital charges, and the variation in capital buffers has hardly any impact.\(^{20}\) This is in sharp contrast with the result under the flat-rate capital requirement, in which the cycle is primarily caused by the counter-cyclical movement in capital buffers. To some extent, this difference implies that the risk-based capital rule achieves the goal of aligning regulatory capital with actual capital that a bank is willing to hold. Second, compared between the two capital regimes, it is unclear to judge the aggregate impact on the economic activities. The simulation results show that the cycle for very small banks is smaller under the new capital rule, whereas that for large banks becomes more volatile. The aggregate effect depends on the size distribution of the banking industry for which the capital rule will be applied. Lastly, compared with the results under the baseline economy, the introduction of risk-based capital requirements does not lead to an amplified credit cycle, in that the volatility of bank loans is much smaller for small banks (for which the loan activity is constrained in good times when business risks are likely to build up). Even for

\(^{20}\)The large swing in minimum capital requirements is partly driven by the short horizon of the portfolio risk assessment. For simplicity reasons, this model assumes that bank assets mature in one period (or one year), therefore the risk profile of bank portfolio is a point-at-time measure. This is consistent with the prevalent practice of using 1-year default probability in computing risk-based capital requirements. If risk assessments over a longer time horizon are adopted (eg “through-the-cycle” ratings), the variation in regulatory capital over time tends to be smoother. This requires a richer maturity structure of bank assets in a dynamic setting, which is beyond the scope of this paper.
large (and safe) banks, for which the capital rule is not binding, the cyclical movement of lending activity mimics the lending cycle in the baseline economy and does not cause extra volatility into the economy. Therefore, the cyclical movement in bank lending itself is not equivalent to the existence of a procyclicality effect in this study.

5 Potential gains from the transition of capital regimes

The previous section discusses how the change in capital rules affects banks’ equilibrium financial decisions. Nevertheless, there is no indication whether this change is desirable. For instance, lower leverage and higher bank safety are not always appealing if they imply big sacrifice in lending efficiency. The analysis in this section can shed some light on this important issue.21

Although there is no consensus view on the objective function of a bank regulator, researchers tend to agree that the introduction of capital regulation can be justified on the basis of improving the safety and soundness of the banking sector and to minimize negative externality associated with bank failure. Following the mainstream of the literature (see Kashyap and Stein, 2004; Hellmann et al., 2000, for example), I assume that there are two major components in a regulator’s objective function: franchise value and the probability of bank failure. For obvious reasons, the regulator favors higher franchise value and lower bank fragility.

There are two ways for a regulator to examine the welfare implications of capital regulation, a micro perspective that draws conclusion from the analysis based on individual banks, and a macro perspective that focuses on the banking system as a whole. This is in line with the two distinct views regarding the objective of prudential regulation, known as the micro- vs. macro-prudential perspectives.22 The first view considers the failure of any financial institution to be undesirable, hence the objective of banking regulation is to ensure the safety of individual institutions. By contrast, the macro-view treats the financial system as a whole. Therefore, systemically important banks receive more attention from the regulator than small banks that contribute less to the systemic risk.

Figure 9 illustrates the impacts of the two capital regimes on the tradeoff between efficiency and safety from a micro-perspective, using a very small bank (initial equity $e_0 = 0.5$), a small bank ($e_0 = 1$) and a large bank ($e_0 = 2$) as examples. The horizontal axis represents

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21 The following discussion focuses exclusively on the comparison between the two particular types of capital regimes, within a framework that a regulator cares about franchise value and bank fragility but nothing else. It does not intend to address the issue on optimal design of capital regulation.

22 See Crocket (2000) and Borio (2003) for further discussion. Nevertheless, the distinction between the two strands of views is not always a clear-cut.
the expected probability of bank failure and the vertical line represents expected franchise value, using the fact that the good and bad states are equally likely to occur in the long run. The exercise is implemented by studying the flat-rate capital regime with different minimum capital ratio requirements (\(\kappa = 2\%, 4\%, \cdots, 14\%)\) and the risk-based capital regime with different threshold default probabilities (based on the credit VaR definition as specified in Equation 2).

The results are very interesting. First, a tightening in capital standards (higher uniform capital ratios or lower threshold default probabilities) moves the equilibrium outcome to the lower-left corner, i.e. it reduces the probability of bank failure at the cost of banks’ franchise value. This is quite intuitive. On the other hand, when the capital standard is very loose, the equilibrium outcome converges to the result under the baseline economy. Second, the more interesting finding is that, when achieving the same level of bank safety, the risk-based capital rule yields higher franchise value. This implies that the risk-based capital regulation is more appealing from a regulator’s perspective. At a first glance, this result is not quite intuitive. For instance, comparing between the flat-rate capital requirement with \(\kappa = 8\%\) and the risk-based capital requirement with the threshold default probability at 0.1\%, a small bank actually faces a more restrictive capital requirement under the second regime and is forced to choose lower leverage and lower probability of failure in the current period. Nevertheless, the bank has higher franchise value than the counterpart under the flat-rate capital regime despite the current-period losses.

The answer to this seemingly puzzle lies in the dynamic benefit from the risk-based capital regulation. As we have mentioned earlier, a potential problem with the flat-rate capital rule is that it is not capable of distinguishing between sound and risky loans, therefore it tends to suffocate efficiency at the same time when it reduces bank fragility. This is most obvious for large banks (the bottom panel), which are forced to waive sound investment opportunities to achieve extremely low probability of failure. This conservative lending strategy is no longer appealing in that efficiency losses are too high. From a dynamic perspective, it implies that all banks pay a very high future franchise value cost under the flat-rate capital regime. This future cost tends to overshadow one-period expected gains (if it exists, such as in the above example when small banks choose high default probabilities and higher profits in the current period) and cause expected franchise value to be significantly lower than under a risk-based capital regime.

In addition, Figure 10 illustrates the comparison from a macro-perspective, based on a hypothetical banking sector that consists of 5 large banks \((e_0 = 2)\) and 50 small banks.
Bank-specific shocks are assumed to be independent across banks. The horizontal axis represents the fragility of the whole banking sector, defined as the probability of more than 5% of total bank assets default in the current period. In this example, the threshold is equivalent to the failure of one of the 5 large banks, or the failure of 4 small banks simultaneously. In this simplified banking sector, the comparison result between the two capital regimes is in line with the micro-based results. The risk-based capital requirement achieves a better balance between lending efficiency and safety, in that (expected) aggregate franchise value is higher to reach the same level of banking stability.

It is worth noting that, this macro-perspective example is only for illustrative purpose and the results should not be interpreted as general and conclusive. There are many complicated issues to be dealt with in order to apply this method into practice, which are beyond the scope of this paper. For instance, the results crucially depend on the size distribution of the banking sector and the inter-dependence of asset returns across banks, both of which are not explored in this paper. Nevertheless, the essence of this macro approach is important and worthy of special attention for bank regulators.

6 Concluding remarks

This paper develops a dynamic equilibrium model to examine the changes in banks’ financial decisions during the transition from a flat-rate to a risk-sensitive capital regulation. Several important policy implications arise. First, in the impact study, it is important to take into consideration the potential impacts on banks’ capital buffers and portfolio decisions. I show that the change in actual capital holdings is much smaller than the change in regulatory capital during the transition. Second, the impacts of the capital regime switching differ substantially across banks. The risk-sensitive capital standard leads to a more favorable capital requirement for large (and less risky) banks. Hence, the cyclical movement in lending behavior is more remarkable for these banks. By contrast, small (and more risky) banks are subject to a stricter capital regulation under the new regime. They have to cut loan extension substantially in good times, when risk is most likely to build up without regulatory restrictions. As a result, the lending cycle for these banks is actually less volatile after the transition. Therefore, it is important to examine uneven impacts on different groups of banks. Lastly, the analysis suggests that the transition might be welfare-improving from a regulator’s perspective, in that the risk-based capital regime finds a better balance between

\( e_0 = 1 \).\textsuperscript{23} In equilibrium, the concentration ratio of the top 5 banks is about 30-40%, which is in line with the survey results in the U.S. and other industrial countries (see BIS, 2004, page 131).
safety and efficiency and causes less distortion in loan decisions.

The analysis provides a starting point to examine issues on the design of bank capital regulation and the interaction between the banking sector and the real economy. It is by no means a completed task. One caveat of this model is that the asset side of the bank’s balance sheet is not explicitly modeled, implying that the important feedback effect from the bank’s lending behavior to the real economy is missing in the current analysis. An extension of the model into a general equilibrium framework will make it possible to address this issue. Furthermore, there is no active role of monetary policy in this paper in that the risk-free rate is constant over time. An extension of this model could be used to discuss the impact of monetary policy on the channel through which capital regulation affects the bank’s financial decisions. These issues could be potentially very interesting in the research along this line.
References


Appendix

A Proof of Propositions 1

1. The dynamic program problem is equivalent to:

\[ V(e_t, z_t) = \max_{(\pi_t, \mu_t, \epsilon_{t+1})} \sum_{z_{t+1}} \left\{ \int \phi(\epsilon) d\epsilon \cdot P(z_{t+1}|z_t) \right\} \tag{A.1} \]

subject to Equations (6), (7), (8), (10), (11) and (13).

It suffices to prove the existence of a unique solution \( V^* \) by showing that the \( V(\cdot) \) function satisfies the two Blackwell’s sufficient conditions and therefore is a contraction mapping. To check the two conditions:

- *(Monotonicity)* Take \( V_1 \geq V_2 \), then:

\[
T(V_1) = \max E[d(L^*, \mu^*, e^*) + \beta V_1(e^*, z_{t+1})] \\
\geq \max E[d(L^*, \mu^*, e^*) + \beta V_2(e^*, z_{t+1})] \\
= T(V_2)
\]

- *(Discounting)* For any positive real number \( c > 0 \),

\[
T(V + c) = E\{d(L^*, \mu^*, e^*) + \beta [V(e^*, z_{t+1}) + c]\} \\
= E\{d(L^*, \mu^*, e^*) + \beta V(e^*, z_{t+1})\} + c \\
= T(V) + c
\]

2. The concavity of \( V^*(e) \) function can be proved based on Theorem 4.8 (page 81) in Stokey et al. (1989). For simplicity, I only prove the case in which \( z \) is constant but the proof can be easily extended to the case in which \( z \) is random.

First, it is straightforward that \( d(\pi_t, \epsilon_{t+1}) \) function is continuous and bounded if we can restrict feasible values of \( e \) within the range \([e_{\min}, e_{\max}]\), where \( e_{\min} = 0 \) and \( e_{\max} \) is sufficiently large so that banks’ equity position will never exceed the boundary.

Second, \( d(\cdot) \) function is concave. That is, for any \((\pi_1, e_1'), (\pi_2, e_2')\) and all \( h \in [0, 1] \),

\[ d(\pi_3, e_3') \geq hd(\pi_1, e_1') + (1 - h)d(\pi_2, e_2') \]

where \( \pi_3 \equiv h\pi_1 + (1 - h)\pi_2 \) and \( e_3' \equiv he_1' + (1 - h)e_2' \). This is because

- if \( \pi_1 \geq e_1' \) and \( \pi_2 \geq e_2' \), then \( \pi_3 \geq e_3' \) and \( d(\pi_3, e_3') = hd(\pi_1, e_1') + (1 - h)d(\pi_2, e_2') \).

- similarly, if \( \pi_1 < e_1' \) and \( \pi_2 < e_2' \), then \( \pi_3 < e_3' \) and \( d(\pi_3, e_3') = hd(\pi_1, e_1') + (1 - h)d(\pi_2, e_2') \).
• if $\pi_1 > e'_1$, $\pi_2 < e'_2$, and $\pi_3 > e'_3$, then

\[
d(\pi_3, e'_3) = h(\pi_1 - e'_1) + (1 - h)(\pi_2 - e'_2) > h(\pi_1 - e'_1) + (1 - h)(1 + \lambda)(\pi_2 - e'_2) = hd(\pi_1, e'_1) + (1 - h)d(\pi_2, e'_2)
\]

• if $\pi_1 > e'_1$, $\pi_2 < e'_2$, and $\pi_3 < e'_3$, then

\[
d(\pi_3, e'_3) = h(1 + \lambda)(\pi_1 - e'_1) + (1 - h)(1 + \lambda)(\pi_2 - e'_2) > h(\pi_1 - e'_1) + (1 - h)(1 + \lambda)(\pi_2 - e'_2) = hd(\pi_1, e'_1) + (1 - h)d(\pi_2, e'_2)
\]

Finally, the feasible set of $e'$ is convex if we restrict the feasible value of $e$ within $[e_{\text{min}}, e_{\text{max}}]$.

Based on the above three properties, $V^*(e)$ function is strictly concave in $e$.

### B Proof of Proposition 2

To determine the optimal monitoring effort in the current period, it is worth noticing that future bank value, $V(e_{t+1}, z_{t+1})$, is independent of $\mu_t$. Therefore, the optimal $\mu_t$ should maximize

\[
\int_{\tilde{\epsilon}}^{\epsilon_t} (\pi_t - e_{t+1})(1 + \lambda)\phi(\epsilon)d\epsilon + \int_{\tilde{\epsilon}}^{\epsilon_t} (\pi_t - e_{t+1})\phi(\epsilon)d\epsilon = 0
\]

where $\tilde{\epsilon}$ is the critical value below which new equity needs to be raised. It is determined by $(\tilde{\epsilon} - \bar{\epsilon})F(L) = e_{t+1}$.

The first-order condition of the maximization problem can be rearranged as:

\[
\frac{d\tilde{\epsilon}}{d\mu} [e_{t+1}(1 + \lambda)\phi(\tilde{\epsilon}) - (1 + \lambda)F(L)(\Phi(\tilde{\epsilon}) - \Phi(\bar{\epsilon})) - F(L)(1 - \Phi(\epsilon))] = 0
\]

Since the bank chooses $\mu_t$ at the beginning of period $t$, at which time the choice of $e_{t+1}$ is still uncertain, the above condition is satisfied if and only if $\frac{d\tilde{\epsilon}}{d\mu} = 0$. From Equations (4) and (5), it is straightforward to derive that $\frac{d\tilde{\epsilon}}{d\mu} = \frac{\theta'(\mu)L - F(L)}{(1 - \Phi(\tilde{\epsilon}))F(L)}$. Therefore

\[
\frac{d\tilde{\epsilon}}{d\mu}_{\mu = \mu^*} = 0 \quad \Rightarrow \quad F(L) = \theta'(\mu^*)L.
\]

In addition, given the strict concavity of $V^*$, the dividend policy assumes a cutoff rule for a given initial bank equity. Since this is quite straightforward, I leave out the strict proof here but the intuitions are described in the text.
C Computational procedure

The computational procedure for the dynamic programming is based on value function iteration as introduced in Judd (1998). The value function is solved based on the discretization of the state variable $e_t$, the quasi-state variable $\pi_t$ and control variables $L_t$ and $e_{t+1}$. The main features of the numerical algorithm are:

1. Set the minimum and maximum values of bank equity ($e$) and end-of-period net profit ($\pi$);

2. The range of bank lending ($L$) is set within $[e, e/\kappa]$ under the flat-rate capital requirement, or within $[e, L_{max}]$ under the risk-based capital regime ($L_{max}$ is calculated from Equation (2) by setting $e_{min} = e$).

3. For each pair of $(e, L)$, calculate the corresponding $\bar{\epsilon}$ and $\mu$ based on Equations (6), (7) and Proposition 2.1.

4. For each pair of $(e, L)$, calculate the probability distribution of end-of-period net asset values.

5. Guess initial values of $V(e)$ function for $z = z_H$ and $z = z_L$.

6. Solve for the $W(\pi, z)$ function and the policy function $e'$ for given $\pi$, $e$ and $z$;

7. Based on the $W$ function, solve for the $V(e, z)$ function and the policy function $L$ for given $e$ and $z$; The found values are the new guesses of the $V$ function. The procedure is then restarted from step 6 until converged.
Table 1: **Model parameters for the calibration**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk-free rate</td>
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<tr>
<td>Return to scale parameter</td>
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<td>Economic state</td>
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</tr>
<tr>
<td>$z_H$</td>
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<tr>
<td>$z_L$</td>
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<tr>
<td>Transition matrix</td>
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<tr>
<td>$H \rightarrow H$</td>
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</tr>
<tr>
<td>$L \rightarrow L$</td>
<td>0.800</td>
</tr>
<tr>
<td>Standard deviation of the shock</td>
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</tr>
<tr>
<td>$\sigma_e(L)$</td>
<td>$\frac{1.05}{L+15}$</td>
</tr>
<tr>
<td>Intertemporal discount rate</td>
<td>$\beta$</td>
</tr>
<tr>
<td>Monitoring cost parameter</td>
<td>$c_0$</td>
</tr>
<tr>
<td>New equity premium</td>
<td>$\lambda$</td>
</tr>
<tr>
<td>Minimum capital requirement</td>
<td>$\kappa$</td>
</tr>
</tbody>
</table>
Figure 1: The determination of the bank’s dividend policy
Figure 2: Optimal policy function under the baseline economy without capital regulation

**Note:** The top panels show the optimal dividend policy under the baseline economy, i.e. the determination of next period’s equity ($e_{t+1}$) for given initial equity ($e_t$) and end-of-period net asset value ($\pi_t$). The middle panels illustrate the probability distribution of end-of-period net asset values for a small bank (initial equity = 1), a mid-sized bank ($e_0 = 1.5$) and a large bank ($e_0 = 2$). Notice that the most left observations refer to $\text{Prob}(\pi_t \leq 0)$ rather than $\text{Prob}(\pi_t = 0)$. The bottom panels illustrate the cumulative distribution functions of the three banks’ equity positions at the beginning of next period.
Figure 3: Equilibrium outcomes under the baseline economy

Note: The figure plots the equilibrium outcomes under the baseline economy, including banks’ franchise values ($V$), the volume of loan portfolio ($L$), equity/asset ratios, asset return volatility ($\sigma$), monitoring efforts ($\mu$) and the probability of bank failure.
Figure 4: The banking sector in the United States
Figure 5: Equilibrium outcomes under a flat-rate capital requirement

Note: The figure plots the equilibrium outcomes under a flat-rate capital regime, which imposes a minimum capital/asset ratio of 8% on all banks. Definitions of variables follow Figure 3.
Figure 6: Lending cycles under two capital regimes

Note: The figure plots the level of lending revenues for each bank under two distinctive capital regimes, i.e. a uniform capital/asset ratio of 8% (dashed lines) or a risk-based capital standard (dotted lines). The left panels refer to the results if banks choose portfolios as indicated by the minimum capital requirements, and the right panels refer to equilibrium outcomes under both capital regimes (equilibrium outputs under the baseline economy are also shown in dark lines for comparison purpose). The bottom panels plot the ratio of outputs in the good state vs. in the bad state, a measure for the magnitude of lending cycles.
Figure 7: Equilibrium outcomes under a risk-based capital requirement
Figure 8: Comparison of capital holdings under two capital regimes

*Note:* In the top two panels, the dotted and dash-dotted lines plot the minimum capital requirements under two distinctive capital regimes, i.e. a uniform capital/asset ratio of 8% or a risk-based capital standard. The solid and dashed lines plot banks’ *actual* capital holdings under the two capital regimes. The bottom panel shows the average changes in regulatory capital (the solid line) and actual capital (the dashed line) if the flat-rate capital rule is replaced by the risk-sensitive capital requirement.
Figure 9: Comparison between the two capital regimes: a micro-perspective

Note: This figure plots expected franchise value and expected probability of failure for a very small bank (initial equity of 0.5, the top panel), a small bank (initial equity of 1, the middle panel) and a large bank (initial equity of 2, the bottom panel), with the good and bad states equally likely to occur in the long run. The circles refer to equilibrium outcomes under flat-rate capital requirements, with minimum capital ratios varying from 4% to 14%. The “+” symbols refer to equilibrium outcomes under the credit VaR-based capital regulation, with threshold default rates varying from 1% to 0.01%.
Figure 10: Comparison between the two capital regimes: a macro-perspective

Note: This figure plots expected franchise value and expected probability of failure for a group of banks that consist of 5 large banks (initial equity of 2) and 50 small banks (initial equity of 1), with the good and bad states equally likely to occur in the long run. The horizontal axis represents the probability that more than 5% of total bank assets default in one period and the vertical axis represents total franchise value. The circles refer to equilibrium outcomes under flat-rate capital requirements, with minimum capital ratios varying from 4% to 14%. The “+” symbols refer to equilibrium outcomes under the credit VaR-based capital regulation, with threshold default rates varying from 1% to 0.001%.