A Parametric Model to Estimate Risk in a Fixed Income Portfolio*

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Abstract: In this paper we propose a methodology that let us to calculate the variance and covariance matrix of a very large set of interest rate changes at a very low computational cost. The proposal uses the parametization of interest rates that underlies the model of Nelson and Siegel (1987) to estimate the yield curve. Starting with that model, we are able to obtain the variance-covariance matrix of a vector of *k* interest rates by estimating the variance of the principal components of the four parameters of the model. We used the methodology we propose to calculate risk in a fixed income portfolio, in particular to calculate Value at Risk (VaR). The results of the paper indicate that the application of our method to calculate VaR provides a precise measure of risk when compared to other parametric methods.

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1. Introduction

One of the most important tasks facing financial institutions is the evaluation of the degree to which they are exposed to market risk. This risk appears as a consequence of the changes in the market prices of the assets that compose their portfolios. One way to measure this risk is to evaluate the possible losses that can occur from changes in market prices. This is precisely what the VaR (value at risk) methodology does. This methodology has been very widely used recently, and it has become a basic tool for market risk management of many investment banks, trading banks, financial institutions and some non-financial corporations. Also, the Basel Committee on Banking Supervision (1996) at the Bank for International Settlements uses VaR to require financial institutions such as banks and investment firms to meet capital requirements to cover the market risk that they incur as a result of their normal operations.

The VaR of a portfolio is a statistical measure that tells us what is the maximum amount that an investor may lose over a given time horizon and with a given probability. Alternatively, the VaR of a portfolio can be defined as the amount of funds that a financial institution should have in order to cover the portfolio losses in almost all circumstances, except for those that occur with a very low probability.

Although VaR is a simple concept, its calculation is not trivial. Formally, VaR (α%) is the percentil α of the probability distribution of the changes in value of a portfolio, that is, it is the value for which α% of the values lie to the left on the distribution. Consequentially, in order to calculate VaR we must firstly estimate the probability distribution of the changes in value of the portfolio.

Several methods have been developed to do this: Monte Carlo Simulation, Historical Simulation, Parametric Models, and Stress Testing. See Jorion(2000) to get a general vision of this methodologies. Among all of these, the most widely used methods are those based on the parametric approach, or on variance and co-variance. We can see some applications of this method in Morgan(1995), García-Donato at all(2001) Gento(2001), Gento(2000), Benito and Novales(2005), Alex NcMain(2001).

The parametric approach is based on the assumption that the changes in value of a portfolio will follow a known distribution, which is generally assumed to be Normal. Under such an assumption, the only relevant parameter for the calculation of VaR is the variance conditional on the changes in value of the portfolio, assuming that on average these are zero.

The estimation of this variance is not trivial, since it requires estimating the variance co-variance matrix of the assets that make up the portfolio.

The estimation of this matrix poses two types of problems: (1) a dimensionality problem and (2) a viability problem. The first appears due to the large dimension of the matrix, which makes it difficult to estimate. For example, in order to estimate the variance of the return of a portfolio that is made up of five assets, it is necessary to estimate five variances and fifteen covariances, that is a total of twenty variables. This problem becomes especially important in fixed income portfolios in which the value depends on a large number of different interest rates, for different time horizons. The second problem has to do with the difficulty of estimating the conditional covariances if one uses sophisticated models, such as multivariate GARCH models. The estimation of such models is both very costly in terms of computation, and is also generally not even possible when the dimension of the matrix is greater than three. It is for this reason that these models have not been at all popular for financial management.

In the recent literature, these problems are tackled using the assumption that there exist common factors in the volatility of the interest rates, and that these same factors explain the changes in the temporal structure of the interest rates (TSIR). Under these two assumptions, it becomes theoretically possible to obtain the variance-covariance matrix of a wide range of interest rates using a factor model of TSIR. For example, Alexander (2001) and Gento (2000) show that if we begin with a principal components model (Alexander 2001) or a regression model (Gento, 2000), then we can get the variance-covariance matrix from a vector of interest rates at a low calculation cost.

The present paper proposes an alternative method of estimating the variance-covariance matrix of interest rates at a low computational cost. We take as our starting point the model of Nelson and Siegel (1987), which was developed initially to estimate the TSIR. This model provides an expression of the interest rates as a function of four parameters. Starting with this model, we can obtain the variance-covariance matrix of the interest rates by calculating the variances of only four variables – the principal components of the changes in the four parameters.

This paper continues as follows. In section 2 we present the method proposed to estimate the variance-covariance matrix for a large vector of interest rates at a low computational cost. The next sections evaluate the proposed method for a sample of data from the Spanish market. In section 3 we briefly describe the data that we use, and we apply the proposed method to obtain the variance-covariance matrix of a vector of interest rates. In section 4 we evaluate the proposed methodology to calculate the VaR in fixed income portfolios, and we compare the results with those that are obtained from standard methods of calculation. Finally, section 5 presents the main conclusions of the paper.

2. A parametric model for estimating risk.

In this section we present a methodology to calculate the variance-covariance matrix for a large vector of interest rates at a low computational cost. To do this we take as our starting point the model proposed by Nelson and Siegel (1987), designed to estimate the yield curve (TSIR).

The Nelson and Siegel formulation specifies a parsimonious representation of the forward rate function given by:

$$\varphi_m^t = \beta_0 + \beta_1 e^{\left(-\frac{m}{\tau}\right)} + \beta_2 \frac{m}{\tau} e^{\left(-\frac{m}{\tau}\right)} \tag{1}$$

This expression allows one to accommodate the different forms that may characterise (level, positive or negative slope, and greater or lower curvature) as a function of four parameters (β_0, β_1, β_2 and τ).

Bearing in mind the fact that the spot interest rate for a term of *m* can be expressed as the sum of the instantaneous *forward* interest rates from 0 up to *m*, that is, by integrating the expression that defines the instantaneous *forward* rate:

$$r_t(m) = \int_0^m \varphi_u^t d_u \tag{2}$$

we obtain the following expression for the spot interest rate for a term of *m*:

$$r_t(m) = \beta_0 - \beta_1 \frac{\tau}{m} e^{\left(-\frac{m}{\tau}\right)} + \beta_1 \frac{\tau}{m} + \beta_2 \frac{\tau}{m} - \beta_2 e^{\left(-\frac{m}{\tau}\right)} - \beta_2 \frac{\tau}{m} e^{\left(-\frac{m}{\tau}\right)} \tag{3}$$

Equation (3) shows that spot interest rates are a function of only four parameters. Consequentially the changes in these parameters are the variables that determine the changes in the interest rates. Using a linear approximation we can estimate the change in the zero coupon rate of term *m* from the following expression:

$$dr_t(m) \approx \begin{bmatrix} \frac{\partial r_t(m)}{\partial \beta_0} & \frac{\partial r_t(m)}{\partial \beta_1} & \frac{\partial r_t(m)}{\partial \beta_2} & \frac{\partial r_t(m)}{\partial \tau} \end{bmatrix} \begin{bmatrix} d\beta_{0,t} \\ d\beta_{1,t} \\ d\beta_{2,t} \\ d\tau_t \end{bmatrix} \quad (4)$$

In a multivariate context, the changes in the vector of interest rates that make up the TSIR can be expressed by generalizing equation (4) in the following way:

$$dr_t = G_t \, d\beta_t \quad (5)$$

where $dr_t = \left[dr_t(1)\, dr_t(2) \quad . \quad dr_t(k) \right]$, $d\beta_t^{'} = \left[d\beta_{0,t} \quad d\beta_{1,t} \quad d\beta_{2,t} \quad d\tau_t \right]$ and

$$G_t = \begin{bmatrix} \frac{\partial r_t(1)}{\partial \beta_0} & \frac{\partial r_t(1)}{\partial \beta_1} & \frac{\partial r_t(1)}{\partial \beta_2} & \frac{\partial r_t(1)}{\partial \tau} \\ \frac{\partial r_t(2)}{\partial \beta_0} & \frac{\partial r_t(2)}{\partial \beta_1} & \frac{\partial r_t(2)}{\partial \beta_2} & \frac{\partial r_t(2)}{\partial \tau} \\ . & . & . & . \\ \frac{\partial r_t(k)}{\partial \beta_0} & \frac{\partial r_t(k)}{\partial \beta_1} & \frac{\partial r_t(k)}{\partial \beta_2} & \frac{\partial r_t(k)}{\partial \tau} \end{bmatrix}.$$

This approximation (equation (5)) has been used with some success in interest rate risk management for fixed income assets (Gómez, 1999) and for portfolio immunization (Gómez, 1998).

In the context of this model, and using expression (5), we can calculate the variance-covariance matrix of a vector of changes in the k interest rates using the following expression:

$$\text{var}(dr_t) = G_t \Psi_t G_t^{'} \quad (6)$$

where:

$$\Psi_t = \begin{bmatrix} \text{var}(\beta_{0,t}) & \text{cov}(\beta_{0,t}\beta_{1,t}) & \text{cov}(\beta_{0,t}\beta_{2,t}) & \text{cov}(\beta_{0,t}\tau_t) \\ & \text{var}(\beta_{1,t}) & \text{cov}(\beta_{1,t}\beta_{2,t}) & \text{cov}(\beta_{1,t}\tau_t) \\ & & \text{var}(\beta_{2,t}) & \text{cov}(\beta_{2,t}\tau_t) \\ & & & \text{var}(\tau_t) \end{bmatrix}$$

At this point we note that we have arrived at an important simplification in the dimension of the variance-covariance matrix that we need to estimate. Note that for a vector of k interest rates, instead of having to estimate $k(k+1)/2$ variances and covariances, we only need to estimate 10 second order moments. However, the problem associated with the difficulty of the estimation of the covariances still remains.

But we can still simplify the calculation of the variance-covariance matrix even further, by applying principal components to the vector of the changes in the parameters ($d\beta_t$). In this way, the vector of changes in the parameters of the model of Nelson and Siegel (1987) can be expressed as:

$$d\beta_t = AF_t \tag{7}$$

$$F_t = \begin{bmatrix} f_{1,t} & f_{2,t} & f_{3,t} & f_{4,t} \end{bmatrix} \text{ and } A = \begin{bmatrix} a_{\beta_0}^1 & a_{\beta_0}^2 & a_{\beta_0}^3 & a_{\beta_0}^4 \\ a_{\beta_1}^1 & a_{\beta_1}^2 & a_{\beta_1}^3 & a_{\beta_1}^4 \\ a_{\beta_2}^1 & a_{\beta_2}^2 & a_{\beta_2}^3 & a_{\beta_2}^4 \\ a_{\tau}^1 & a_{\tau}^2 & a_{\tau}^3 & a_{\tau}^4 \end{bmatrix}$$

where F_t is the vector of principal components associated with the vector $d\beta_t$ and A is the matrix of constants that form the eigenvectors associated with each one of the four eigenvalues of the variance-covariance matrix of the changes in the parameters of the Nelson and Siegel model ($d\beta_t$).

Substituting equation (7) into equation (5) and given that each principal component is orthogonal to the rest, we can express the variance-covariance matrix of the interest rates as follows:

$$\text{var}(dr_t) = G_t^{*}\Omega_t G_t^{*'} \tag{8}$$

where: $$\Omega_t = \begin{bmatrix} \text{var}(f_{1,t}) & 0 & 0 & 0 \\ 0 & \text{var}(f_{2,t}) & 0 & 0 \\ 0 & 0 & \text{var}(f_{3,t}) & 0 \\ 0 & 0 & 0 & \text{var}(f_{4,t}) \end{bmatrix}$$ and $G_t^{*} \approx G_t \times A$.

Therefore, equation (8) gives us an alternative method to estimate the variance-covariance matrix of the changes in a vector of k interest rates using the estimation of the four principal components of the changes in the parameters of the Nelson and Siegel (1987) model. In this way, the dimensionality problem associated with the calculation of the covariances has been solved.

In the following sections we evaluate this method, both to calculate the variance matrix of a vector of interest rates, and to calculate the VaR of fixed income portfolios.

3. Estimating the variance-covariance matrix

3.1. The data

To examine the method proposed in this paper, we estimate a daily term structure of interest rates using actual mean daily Treasury transactions prices. The original data set consists of daily observations derived from actual transactions in all bonds traded on the Spanish government debt market. The database of bonds traded on the secondary market of Treasury debt covers the period from September, 1 1995 to October, 29 1997. We use this daily database to estimate the daily term structure of interest rates. We fit Nelson and Siegel's (1987) exponential model for the estimation of the yield curve and minimise price errors weighted by duration. We work with daily data for interest rates at 1, 2,…, 15 year maturities.

3.2. The results

In this section we examine this new approach to variance and covariance matrix estimation. The first section begins by comparing the changes in the estimated and observed interest rates. The changes in interest rates are modelled by equation (5), and then we compare these changes with the observed ones[1].

Then we estimate the variance-covariance matrix of a vector of 10 types of interest rate, using the methodology proposed in the previous section, and we compare these estimations (*Indirect Estimation*) with those obtained using some habitual univariate procedures (*Direct Estimation*).

Both in direct and indirect estimation we need a method for estimating variances and covariances. For the case of indirect estimation the estimation method gives us the variances of the four principal components of the changes in the parameters of the Nelson and Siegel model, which allow us to obtain, from equation (8), the variance-covariance matrix of the interest rates.

In order to estimate the variance-covariance matrix of the interest rates changes and the variance of the principal components, we use two alternative measures of volatility: exponentially weighted moving average (EWMA) and Generalized Autoregressive Conditional Heteroskedasticity models (GARCH).

(1) Under the first alternative, the variance-covariance matrix is estimated using the RiskMetrics methodology, developed by J.P. Morgan. RiskMetrics uses the so called

[1] The software we used in this application is MATLAB.

exponentially weighted moving average (EWMA) method. Accordingly, the estimator for the variance is:

$$\mathrm{var}(dx_t) = (1-\lambda)\sum_{j=0}^{N-1} \lambda^j (dx_{t-j} - \bar{d}x)^2 \tag{9}$$

the estimator the covariance is:

$$\mathrm{cov}(dx_t dy_t) = (1-\lambda)\sum_{j=0}^{N-1} \lambda^j (dx_{t-j} - \bar{d}x)(dy_{t-j} - \bar{d}y) \tag{10}$$

J.P. Morgan uses the exponentially weighted moving average method to estimate the VaR of its portfolios. On a widely diversified international portfolio, RiskMetrics found that the value $\lambda = 0.94$ with $N = 20$ produces the best backtesting results. In this paper, we use both of these values.

Therefore, we obtain the direct estimations of the variance-covariance matrix (D_EWMA) of the interest rates from equations (9) and (10) where x_t and y_t are the interest rates at different maturities . For the case of indirect estimation of the variance-covariance matrix (I_EWMA), we use equation (9) to obtain the variances of the principal components (where x_t are now these principal components) and, from there, equation (8) gives us the relevant matrix.

(2) The EWMA methodology, which is currently used for the Riskmetrics^{TM} data, is quite acceptable for calculating VaR measures, but some authors suggest that one alternative is to use variance-covariance matrices obtained using Multivariate Generalized Autoregressive Conditional Meteroskedasticity Models (GARCH). Nevertheless, the large variance-covariance matrices used in VaR calculations could never be estimated directly using a full multivariate GARCH model, because the computational complexity would be insurmountable. For this reason we only compute the variances of interest rates changes using univariate GARCH models and do not compute the covariance.

Given that indirect estimation (I_GARCH) does not require the estimation of covariances, we estimate the condicional variance of the principal components of the changes in the parameter of the Nelson and Siegel model using univariate GARCH models.

In the sub-section two of this section, we compare the alternative estimations of the variance-covariance matrix described above. This comparison is summarised in Table 1.

Table 1. Type of variance-covariance matrix estimation

		Type of variance models	
		EWMA	**GARCH**
Type of estimation	**Direct Estimation**	D-EWMA	~~D-GARCH~~*
	Indirect Estimation	I-EWMA	I-GARCH

* We have not estimated multivariate GARCH model because of the computational complexity are insurmountable, so that only present the result of the variances which have been estimated unsing univariate GARCH models.

What is relevant is that the estimation of the variance-covariance matrix using the methodology proposed here (indirect estimation) involves a minimum calculation cost, since it is only necessary to estimate the variance of four variables (the principal components of the daily changes in the parameters of the Nelson and Siegel model).

3.2.1. Comparing the changes of interest rates

Firstly, we have evaluated the ability of the model that we propose here to estimate the daily changes in a vector of interest rates. To do this, we compare the observed interest rates with their estimations from equation (5). In Illustration 1 in the Appendix, we show the scatter diagrams that relate the observed changes with the estimated changes in interest rates at 1, 3, 5 and 10 years. As can be seen, independently of the period considered, the relationship is very close.

[Insert Illustration 1]

In Table 2 we report some descriptive statistics of the errors of estimation of the interest rate. The average error is very small, about five basic points for all maturities. This error represents, in relative terms, 0.5% of the interest rates. Furthermore, we observe too that both the average error and the standard deviation are very similar in all period lengths so that the accurate of the model seems good for all maturities.

Table 2. Estimation errors in interest rates. Descriptive statistics.

	1 year	2 years	3 years	4 years	5 years	6 years	7 years	8 years	9 years	10 years
Mean [(a)]	3.1	4.3	4.8	5.0	5.1	5.0	5.0	5.0	5.0	5.0
Standard deviation	4.3	5.6	6.2	6.4	6.5	6.5	6.4	6.4	6.4	6.4
Maximum error	30.6	25.2	26.2	26.6	25.7	25.2	25.2	25.1	25.0	25.0
Minimum error	-17.5	-21.9	-24.0	-24.5	-24.4	-24.3	-24.3	-24.4	-24.5	-24.7

Note: The sample period is from 1/9/1995 to 29/10/1997. The errors (and all statistics) are expressed in basic points. [(a)] The average error is calculated in absolute value.

Therefore, these results imply that the degree of error committed when estimating the changes in zero coupon rates using equation (5) are practically non-existent. In what follows, we evaluate the differences in the estimation of the variance-covariance matrix using the different alternatives.

3.2.2. Comparing the estimations of variance-covariance matrix

In Illustration 2 we show the conditional variances of the interest rates at 1, 3, 5 and 10 years, as estimated using the exponentially weighted moving average method, both directly and indirectly: D_EWMA versus I_EWMA. In Illustration 3 we show the direct estimation of the conditional variances of these same interest rates using the GARCH (D_GARCH) models, and the indirect estimation of the same data (I_GARCH). As can be seen in both illustrations, in most of the time horizons considered, the variances estimated using the method proposed in this paper are very similar to the direct estimates.

[Insert Illustration 2]

[Insert Illustration 3]

The descriptive statistics of the differences between the standard deviations that are estimated using both procedures are reported in Table 3. We compare the direct and indirect estimation methods using an EWMA model in panel (a), and using a GARCH model in panel (b). Panel (a) shows that the differences in absolute value for EWMA specification oscillate between 0.62 and 1 base point. This average difference represents between 10% and 20% of the size of the estimated series.

Panel (b) of Table 3 also shows that the average difference in absolute value for GARCH specification is quite small, even though greater than those of panel (a). However, as a percentage of the estimated conditional variance series, these differences are smaller than those of panel (a). In both comparisons, we can note that the range of differences between each pair of estimations is far greater for a one year rate than for the other horizons. We also note that the range of the estimation error is also greater for the case of one year than for the other interest rates (Table 2).

Table 3. Differences in the estimation of the standard deviation of interest rates. Descriptive statistics.

	1 year	2 years	3 years	4 years	5 years	6 years	7 years	8 years	9 years	10 years
	Panel (a): Comparing D_EWMA vs. I_EWMA.									
Mean [(a)]	0.74	0.62	0.77	0.9	0.96	0.99	1	1.01	1	0.99
Standard deviation	1.28	0.83	1.02	1.17	1.25	1.29	1.31	1.32	1.32	1.31
Maximum error	1.12	1.71	1.92	2.03	2.1	2.1	2.05	1.97	1.88	1.78
Minimum error	-14.48	-5.31	-3.12	-3.88	-4.56	-5.25	-5.61	-5.77	-5.79	-5.74
	Panel (b): Comparing D_EGARCH vs. I_EGARCH.									
Mean [(a)]	0.89	1.21	0.96	0.86	0.83	0.81	0.77	0.87	0.84	0.8
Standard deviation	1.35	1.1	1.29	1.28	1.24	1.28	1.25	1.26	1.25	1.23
Maximum error	17.6	5.31	7.72	7.61	7.34	8.21	7.91	7.2	7.44	7.58
Minimum error	-0.8	-3.37	-2.15	-1.61	-1.46	-1.27	-1.29	-2.58	-2.47	-2.49

Note: Sample period from 29/9/1995 to 29/10/1997 (515 observations). I_EWMA indirect estimation (equation (8)) and D_EWMA direct estimation. Riskmetrics methodology (EWMA). I_GARCH: indirect estimation (equation (8)) and D_GARCH direct estimation. Conditional autoregressive volatility models (GARCH). [(a)] The average of the differences has been calculated in absolute value. Differences measured in base points.

We now compare the covariances estimated directly with those obtained from the procedure suggested in this paper. As we have mentioned above, given the extreme complexity of the GARCH multivariate model estimations, the direct estimation of the covariances was only done using EWMA models.

[Insert Illustration 4]

Illustration 4 shows the estimated covariances between the different pairs of interest rates, using both procedures: D_EWMA versus I_EWMA. As can be seen in the graphs, the estimated covariances have very similar behaviour, although we can note that there are greater differences than for the variances. In Table 4 we report some of the descriptive statistics of the estimated covariances. The average absolute value difference is very small, between 0.0007 and 0.0019. However, this does represent about 40% of the average estimated covariance.

Table 4. Differences in the estimation of covariances between interest rates. Descriptive statistics.

Comparing D_EWMA vs. I_EWMA

	1 year			3 years		5 years
	3 years	**5 years**	**10 years**	**5 years**	**10 years**	**10 years**
Mean [(a)]	0.0008	0.0007	0.0007	0.0019	0.0018	0.0017
Standard deviation	0.0013	0.0013	0.0015	0.0020	0.0021	0.0022
Maximum error	0.0042	0.0078	0.0131	0.0032	0.0037	0.0046
Minimum error	-0.0140	-0.0120	-0.0138	-0.0162	-0.0170	-0.0172

Note: Sample period from 29/9/1995 to 29/10/1997 (515 observations). I_EWMA indirect estimation (equation (8)) and D_EWMA direct estimation. Riskmetrics methodology (EWMA). [(a)] The average difference is calculated in absolute value.

To sum up this section, we have shown that the procedure proposed in this paper to estimate the variance-covariance matrix of a large vector of interest rates generates results that are quite satisfactory, above all as far as variances are concerned. For the case of co-variances, we have detected some differences that could be important. In the following section we evaluate whether these differences are important for risk management. To do this, we apply the methodology to the calculation of Value at Risk (VaR) in several fixed income portfolios.

4. Estimating the Value at Risk

In this section we evaluate the utility of the proposed method for risk management of fixed income portfolios, by constructing a parametric measure of VaR as an indicator of the risk of a given portfolio.

4.1. Value at Risk

The VaR of a portfolio is a measure of the maximum loss that the portfolio may suffer over a given time horizon and with a given probability. Formally, the VaR measure is defined as the lower limit of the confidence interval of one tail:

$$\Pr\left[\Delta V_t(\tau) < VaR_t\right] = \alpha \qquad (10)$$

where α is the level of confidence and $\Delta V_t(\tau)$ is the change in the value of the portfolio over the time horizon τ.

The methods that are based on the parametric, or variance-covariance, approach start with the assumption that the changes in the value of a portfolio follow a Normal distribution. Assuming that the average change is zero, the VaR for one day of portfolio *j* is obtained as:

$$VaR_{j,t}(\alpha\%) = \sigma_{t,dV_j} \cdot k_{\alpha\%} \quad (11)$$

where $k_{\alpha\%}$ is the α percentile of the Standard Normal distribution, and the parameter to estimate is the standard deviation conditional upon the value of portfolio *j* (σ_{t,dV_j}).

In a portfolio that is made up of fixed income assets, the duration can be used to obtain the variance of the value of portfolio *j* from the variance of the interest rates in the following way (Jorion, 2000):

$$\sigma^2_{t,dV_j} = D_{j,t}\,\Sigma_t\,D'_{j,t} \quad (12)$$

where Σ_t is the variance-covariance matrix of the interest rates and $D_{j,t}$ is the vector of the duration of portfolio *j*. This vector represents the sensitivity of the value of the portfolio to changes in the interest rates that determine its value.

In this section, value at risk measures are calculated and compared. In the parametric approach, we use the estimations of the variance-covariance matrix as obtained in the previous section (see Table 1). Table 5 illustrates the four measures of VaR that we obtain from the four variance-covariance models:

Table 5. Type of VaR measures

	Type of variance- covariance matrix estimation	Type of VaR measure
Direct Estimation	D_EWMA	VaR_D_EWMA
	D_GARCH	VaR_D_GARCH*
Indirect Estimation	I_EWMA	VaR_I_EWMA
	I_GARCH	VaR_I_GARCH

* We did not compute VaR_D_GARCH because of the impossibility to estimate a multivariate GARCH model with 10 variables.

In the case of the first VaR measure, VaR_D_EWMA, the VaR is obtained by directly estimating Σ_t with an EWMA model. This is a popular approach to measuring market risk, and it is used by JP Morgan (RiskMetric^{TM}). The second VaR measure, VaR_D_GARCH, is also obtained by directly estimating the variance-covariance matrix, but in this case the second order moments are estimated using GARCH models. This VaR measure has not been calculated, given that the large variance-covariance matrices used in VaR calculations could never be estimated directly using a full multivariate GARCH model, because the computational complexity would be insurmountable.

The final two VaR measures are calculated by estimating the variance-covariance matrix of the interest rates using the procedure described in Section 2. We can estimate the variance-covariance matrix of interest rates indirectly, by substituting equation (8) into equation (12) to obtain a new expression for the variance of the changes in the value of the portfolio:

$$\sigma^2_{t,dV_j} = D_{j,t}\, G^*\, \Omega_t\, G^{*'}\, D^{'}_{j,t} = D^m_{j,t}\, \Omega_t\, D^{m'}_t \tag{13}$$

In indirect estimation, Ω_t is a diagonal matrix that contains on its principal diagonal the conditional variance of the principal components of the changes in the four parameters of the Nelson and Siegel model, and $D^m_{j,t}$ is the modified vector of durations of portfolio *j* (of dimension 1x4) which represents the sensitivity of the value of the portfolio to changes in the principal components of the four parameters of the Nelson and Siegel model. In the VaR_I_EWMA, we use an EWMA model to estimate the variance of the principal components; and we use a GARCH model to estimate these variances in the case of the calculation of the VaR_I_GARCH measure.

4.2. The portfolios

In order to evaluate the procedure proposed in this paper for calculating VaR we have considered 4 different portfolios made up of theoretical bonds with maturities at 3, 5, 10 and 15 years, constructed from real data from the Spanish debt market. In each portfolio, the bond coupon is 3.0%. The period of analysis is from 29/9/1995 to 29/10/1997, which allows us to perform 516 estimations of daily VaR for each portfolio.

In order to estimate the daily VaR we have assumed that the characteristics of each portfolio do not change over the dates of the period of analysis: the initial value of the portfolio, the maturity date and the coupon rate. In this way, the results are comparable over the entire period of analysis since we avoid both the pull to par effect (the value of the bonds tends to par as the maturity date of the bond approaches) and the roll down effect (the volatility of the bond decreases over time).

4.3. Comparing VaR measures

In this section value at risk measures are compared. For all portfolio considered we calculate daily VaR at a 5%, 4%, 3%, 2% and 1% confidence level. Firstly, before formally evaluating the precision of the VaR measures under comparison, we examine actual daily portfolio value changes as implied by daily fluctuations in the zero cupon interest rate and compare them with the 5% VaR. In Illustration 5 we show the actual change in a 10 year portfolio together with the VaR at 5% for the three measures of VaR that we consider: VaR_D_EWMA (Figure 1), VaR_I_EWMA (Figure 2) and VaR_I_GARCH (Figure 3). In Figures 1 and 2 we observe that the value of the portfolio falls below the VaR on more occasions than in Figure 3. In all case, the number of times that the value of the portfolio falls below the VaR is closer to its theoretical level. This result is also evident in the other portfolios that we consider, but that we have not reported due to space considerations. This preliminary analysis suggests that the estimations of VaR that are obtained from both models, both directly and indirectly are very precise. However, a more rigorous evaluation of the precision of the estimations is required.

We then compare VaR measures the actual change in portfolio value on day t+1, denoted as ΔV_{t+1}. If $\Delta V_{t+1} <$ VaR, then we have an exception. For testing purposes, we define the exception indicator variable as

$$I_{t+1} = \begin{cases} 1 & if\ \Delta V_{t+1} < VaR \\ 0 & if\ \Delta V_{t+1} \geq VaR \end{cases} \tag{14}$$

a) <u>*Testing the Level*</u>

The most basic test of a value at risk procedure is to see if the stated probability level is actually achieved. The mean of the exception indicator series is the level of the procedure that is achieved. If we assume the probability of an exception is constant, then the number of exceptions follows the binomial distribution. Thus it is possible to form confidence intervals for the level of each VaR measure (see Kupiec (1995)).

Table 6. Testing the Level

VaR measures	Number of exceptions 3- years	5-years	10-years	15-years	Confidence intervals at the 95% level
VaR_D_EWMA (1%)	5	5	8	10	(1 - 10)
VaR_D_EWMA (2%)	6	10	12	12	(5 - 17)
VaR_D_EWMA (3%)	10	11	14	14	(8 - 23)
VaR_D_EWMA (4%)	14	16	18	18	(12 - 30)
VaR_D_EWMA (5%)	21	23	22	24	(17 - 36)
VaR_I_EWMA (1%)	6	9	9	11	(1 - 10)
VaR_I_EWMA (2%)	7	14	14	14	(5 - 17)
VaR_I_EWMA (3%)	17	16	18	19	(8 - 23)
VaR_I_EWMA (4%)	19	24	23	24	(12 - 30)
VaR_I_EWMA (5%)	21	29	29	28	(17 - 36)
VaR_I_GARCH (1%)	3	5	5	4	(1 - 10)
VaR_I_GARCH (2%)	5	9	7	5	(5 - 17)
VaR_I_GARCH (3%)	7*	9	13	11	(8 - 23)
VaR_I_GARCH (4%)	11*	13	14	14	(12 - 30)
VaR_I_GARCH (5%)	13*	17	17	17	(17 - 36)

Note: Sample period 29/9/1995 to 29/10/1997. Confidence intervals derived from the number of exceptions follows the binomial distribution (516, x%) for x=1, 2, 3, 4 and 5. An * indicates the cases in which the number of exceptions is out of the confidence interval, so that, we obtain evidence to reject the null hypothesis at the 5% level type I error rate.

Table 6 shows the level that is achieved and a 95% confidence interval for each of the 1-day VaR estimates. An * indicates the cases in which the number of exceptions is out of the confidence interval, so that, we obtain evidence to reject the null hypothesis at the 5% confidence level. For the three measures and almost all portfolios considered, the number of exceptions is inside the interval confidence, so that the VaR estimation (direct and indirect) seems to be good.

We find just only three cases in which the number of exceptions is out of the confidence interval. This happen for VaR_I_GARCH measure for 3%, 4% and 5% confidence level of the portfolio at 3 years. In those cases the number of exceptions are much lower than the theoretical level, so that it seems that this measure is overestimating the risk of short-term portfolio.

b) *Testing Consistency of Level*

We want the level of the VaR that is found to be the stated level on average, but we also want to find the stated level at all points in time. One approach to testing the consistency of the level is to use the Ljung-Box portmanteau test (Ljung and Box, 1978) on the exception indicator variable of zeros and ones. When using Ljung-Box tests, there is a choice of the number of lags in which to look for autocorrelation. If the test uses only a few lags but autocorrelation occurs over a long time frame, the test will miss some of the autocorrelation. Conversely should a large number of lags be used in the test when the autocorrelation is only in a few lags, then the test won't be as sensitive as if the number of lags in the test matched the autocorrelation.

Different lags have been used for each estimate in order to try to get a good idea of the autocorrelation. Table 7 shows the Ljung-Box statistics at lags of 4 and 8.

Table 7. Testing Consistency of Level

	Lags	3- years	5-years	10-years	15-years
VaR_D_EWMA (1%)	**4**	0,20 (0,995)	0,20 (0,995)	0,52 (0,971)	0,82 (0,936)
	8	0,41 (1,000)	0,41 (1,000)	1,06 (0,998)	5,07 (0,750)
VaR_D_EWMA (2%)	**4**	0,29 (0,990)	0,82 (0,936)	1,19 (0,879)	1,19 (0,879)
	8	0,59 (1,000)	4,98 (0,760)	13,60* (0,093)	4,10 (0,848)
VaR_D_EWMA (3%)	**4**	4,13 (0,389)	1,00 (0,910)	1,64 (0,802)	2,30 (0,681)
	8	4,85 (0,773)	4,46 (0,814)	11,22 (0,189)	5,41 (0,713)
VaR_D_EWMA (4%)	**4**	2,96 (0,565)	2,16 (0,707)	2,30 (0,681)	5,29 (0,258)
	8	5,26 (0,729)	4,41 (0,819)	7,30 (0,504)	7,17 (0,518)
VaR_D_EWMA (5%)	**4**	2,66 (0,617)	2,16 (0,707)	4,44 (0,349)	6,04 (0,196)
	8	10,66 (0,222)	6,59 (0,581)	7,90 (0,444)	7,33 (0,502)
VaR_I_EWMA (1%)	**4**	0,29 (0,990)	0,66 (0,956)	0,66 (0,956)	3,37 (0,498)
	8	0,59 (1,000)	5,91 (0,657)	5,91 (0,657)	6,77 (0,562)
VaR_I_EWMA (2%)	**4**	0,40 (0,983)	2,30 (0,681)	2,30 (0,681)	8,57* (0,073)
	8	0,80 (0,999)	5,27 (0,728)	4,63 (0,796)	10,90 (0,207)
VaR_I_EWMA (3%)	**4**	2,20 (0,700)	2,16 (0,707)	4,84 (0,305)	4,28 (0,369)
	8	7,79 (0,454)	4,34 (0,826)	6,71 (0,568)	5,47 (0,706)
VaR_I_EWMA (4%)	**4**	2,44 (0,656)	1,28 (0,864)	3,30 (0,509)	4,03 (0,403)
	8	6,75 (0,564)	2,62 (0,956)	4,48 (0,812)	6,82 (0,557)
VaR_I_EWMA (5%)	**4**	1,93 (0,748)	0,75 (0,945)	3,53 (0,474)	3,78 (0,436)
	8	5,55 (0,698)	2,49 (0,962)	7,87 (0,447)	14,77* (0,064)
VaR_I_GARCH (1%)	**4**	0,07 (0,999)	0,20 (0,995)	0,20 (0,995)	0,13 (0,998)
	8	0,15 (1,000)	0,41 (1,000)	0,41 (1,000)	0,26 (1,000)
VaR_I_GARCH (2%)	**4**	0,20 (0,995)	0,66 (0,956)	0,40 (0,983)	0,20 (0,995)
	8	0,41 (1,000)	5,91 (0,657)	9,68 (0,288)	19,69* (0,012)
VaR_I_GARCH (3%)	**4**	0,40 (0,983)	0,66 (0,956)	2,51 (0,643)	3,37 (0,497)
	8	0,80 (0,999)	5,91 (0,657)	6,15 (0,630)	9,15 (0,330)
VaR_I_GARCH (4%)	**4**	1,00 (0,910)	1,40 (0,843)	2,29 (0,682)	2,29 (0,682)
	8	4,40 (0,820)	5,04 (0,753)	5,28 (0,727)	5,93 (0,655)
VaR_I_GARCH (5%)	**4**	2,51 (0,643)	1,95 (0,746)	2,19 (0,701)	1,95 (0,745)
	8	5,05 (0,752)	3,66 (0,886)	3,91 (0,865)	3,67 (0,886)

Note: Sample period 29/9/1995 to 29/10/1997. The Ljung-Box Q-statistics on the exception indicator variable and their p-values. The Q-statistic at lag 4 (8) for the null hypothesis that there is no autocorrelation up to order 5 (10). An * indicates that there is evidence to reject the null hypothesis at the 5% level type I error rate.

We only detect the existence of autocorrelation in the portfolios at 10 years with the VaR_D_EWMA (2%) estimate, in the portfolio at 15 years with the measures VaR_I_EWMA (2%) and (5%) and the portfolio at 15 years with the measures VaR_I_GARCH (2%). In general, the results of the Ljung-Box comparison indicate that autocorrelation is not present. When we consider other lags, that are not reported here in the interests of space, the result are pretty the same, so that the VaR estimate also seems to be good using this test.

c) *Unconditional Coverage Tests*

Assuming that a set of VaR estimates and their underlying model are accurate, the exceptions can be modeled as independent draws from a binomial distribution with a probability of occurrence equal to α percent. Accurate VaR measures should exhibit the property that their unconditional coverage $\hat{\alpha} = x/T$ equals α percent, where *x* is the number of exceptions and *T* the number of observations. The likelihood ratio statistic for testing whether $\hat{\alpha} = \alpha$ is

$$LR = 2\left[\log\left(\hat{\alpha}^{x}\left(1-\hat{\alpha}\right)^{T-x}\right) - \log\left(\alpha^{x}\left(1-\alpha\right)^{T-x}\right)\right]$$

which has an asymptotic χ^2 (1) distribution.

Table 8. Unconditional Coverage Tests and The Back-testing Criterion

	% of exceptions [(a) (b)]			
	3- years	**5-years**	**10-years**	**15-years**
VaR D EWMA (1%)	1,0%	1,0%	1,6%*	1,9%*+
	(0,002)	(0,002)	(0,587)	(1,563)
	[-0,071]	[-0,071]	[1,257]	[2,141]
VaR D EWMA (2%)	1,2%*	3,1%	3,9%+	4,3%+
	(0,942)	(-1,156)	(-1,565)	(-2,340)
	[-1,358]	[1,786]	[3,044]	[3,673]
VaR D EWMA (3%)	1,9%*	2,1%*	2,7%	2,7%
	(0,990)	(0,644)	(0,065)	(0,065)
	[-1,414]	[-1,156]	[-0,382]	[-0,382]
VaR D EWMA (4%)	2,7%*	3,1%*	3,5%	3,5%
	(1,086)	(0,510)	(0,159)	(0,159)
	[-1,492]	[-1,042]	[-0,593]	[-0,593]
VaR D EWMA (5%)	4,1%*	4,5%	4,3%	4,7%
	(0,435)	(0,144)	(0,269)	(0,059)
	[-0,970]	[-0,566]	[-0,768]	[-0,364]
VaR I EWMA (1%)	1,2%	1,7%*	1,7%*	2,1%*+
	(0,057)	(1,026)	(1,026)	(2,189)
	[0,372]	[1,699]	[1,699]	[2,584]
VaR I EWMA (2%)	1,4%*	2,7%*	2,7%*	2,7%*
	(0,533)	(0,524)	(0,524)	(0,524)
	[-1,044]	[1,157]	[1,157]	[1,157]
VaR I EWMA (3%)	3,3%	3,1%	3,5%	3,7%*
	(0,065)	(0,008)	(0,175)	(0,335)
	[0,392]	[0,134]	[0,650]	[0,908]
VaR I EWMA (4%)	3,7%	4,7%	4,5%	4,7%
	(0,061)	(0,236)	(0,118)	(0,236)
	[-0,368]	[0,755]	[0,530]	[0,755]
VaR I EWMA (5%)	4,1%*	5,6%	5,6%	5,4%
	(0,435)	(0,175)	(0,175)	(0,084)
	[-0,970]	[0,646]	[0,646]	[0,444]
VaR I GARCH (1%)	0,6%*	1,0%	1,0%	0,8%
	(0,467)	(0,002)	(0,002)	(0,124)
	[-0,956]	[-0,071]	[-0,071]	[-0,513]
VaR I GARCH (2%)	1,0%*	1,7%	1,4%*	1,0%*
	(1,498)	(0,078)	(0,533)	(1,498)
	[-1,673]	[-0,415]	[-1,044]	[-1,673]
VaR I GARCH (3%)	1,4%*+	1,7%*	2,5%	2,1%*
	(2,602)	(1,425)	(0,188)	(0,644)
	[-2,188]	[-1,672]	[-0,640]	[-1,156]
VaR I GARCH (4%)	2,1%*+	2,5%*	2,7%*	2,7%*
	(2,441)	(1,467)	(1,086)	(1,086)
	[-2,166]	[-1,716]	[-1,492]	[-1,492]
VaR I GARCH (5%)	2,5%*+	3,3%*	3,3%*	3,3%*
	(3,522)	(1,552)	(1,552)	(1,552)
	[-2,585]	[-1,778]	[-1,778]	[-1,778]

Note: Sample period 29/9/1995 to 29/10/1997. (a) Between parentheses Unconditional Coverage Tests: The LR statistic for testing whether the percentage of exceptions ($\hat{\alpha} = x/T$) is α percent. An * indicates that there is evidence to reject the null hypothesis at the 5% level type I error rate. (b) Between square brackets Back-testing Criterion: The Z statistic for determining the significance of departure for $\hat{\alpha} = x/T$ from α%. An + indicates that there is evidence to reject the null hypothesis at the 5% level type I error rate.

Table 8 reports the percentage of exceptions observed for the 1%, 2%, 3%, 4% and 5% quantiles over the entire sample period. In parentheses, Table 8 reports the LR statistic for testing whether the percentage of exceptions is the quantile. For the case of measures obtained from VaR_ I_ EWMA, independently of the quantile considered, we reject the null hypothesis that the percentage of exceptions coincides with the corresponding quantile in 40% of the cases. The result with VaR_I_EWMA measure are pretty the same (45%). It is worth to note that the measure VaR_D_EWMA produce a slight underestimation the risk for 1% confidence level and overestimate the risk for 2%, 3%, 4% and 5% confidence level. However, the measure VaR_I_EWMA underestimate the risk for 1%, 2% and 3% and overestimate for 5% confidence level.

The result are worst with the measure VaR_I_GARCH. For this measure reject the null hypothesis in 75% of the times. With this measure overestimating the risk in all cases. It seems that whem the aim is to calculate value at risk, the GARCH models doesn't produce a good estimation of the volatility.

d) *The Back-testing Criterion*

The back-testing criterion is used to evaluate the performance of these VaR measures. The most popular back-testing measure for accuracy of the quantile estimator is the percentage of returns falling below the quantile estimate, denoted as $\hat{\alpha}$. For an accurate estimator of an α quantile, $\hat{\alpha}$ will be very close to α%. In order to determine the significance of departure of $\hat{\alpha}$ from α%, the following test statistic is used[2]:

$$\boldsymbol{Z} = (T\hat{\alpha} - T\alpha\%)\Big/\sqrt{T\alpha\%(1-\alpha\%)} \xrightarrow{d} N(0,1)$$

where T is the simple size.

Table 8 presents the Z statistic for VaR measures in square brackets. For the case of measures obtained from EWMA, (independently of the quantile considered), we reject the null hypothesis that the percentage of exceptions coincides with the corresponding quantile in three cases with the VaR_D_EWMA measure and one time with the VaR_I_EWMA measure. On the other hand, with this test just only in three cases of the VaR_I_GARCH measure reject the hypothesis.

In summary, we can say that the VaR measures we obtain using the simplification proposed in this paper are so good as that we obtain from Riskmetrics method

(VaR_D_EWMA). The advantage of the method we propose is that the computational cost to calculate value at risk is much lower. Additionally, we find slight evidence that to estimate value at risk the EWMA model seem more accurate that the autoregressive conditional volatility models (GARCH models).

5. Conclusion

In this paper we propose a method for calculating the variance-covariance matrix of a large set of interest rates with a low computational cost. The methodology suggested exploits the parametrization of the underlying interest rates that was proposed by Nelson and Siegel (1987) for estimating the yield curve. The method proposed in this paper turns out to be useful for estimating VaR, since it simplifies considerably the calculation of this measure.

Following the papers of Alexander (2001) and Gento (2000), the starting point for our method is an explanatory model of the interest rates. However, contrary to those authors, our model is based on that of Nelson and Siegel (1987) whose objective was to estimate the TSIR. Using a linear approximation, this model provides a relationship in which the changes in interest rates are a function of the changes in four parameters. Although this approximation reduces the dimension of the variance-covariance matrix, it still requires covariances to be estimated. In order to solve this problem, we propose the application of principal components of the changes in the four parameters of the Nelson and Siegel model (1987). Given the orthogonality of the principal components among themselves, the resulting variance-covariance matrix has a smaller dimension since it is diagonal, that is, all the covariances are zero.

The procedure that we propose in this paper has been contrasted using data from the Spanish debt market. The results of this application of our methodology are very satisfactory. On the one hand, the variances that we estimate with our procedure and those that are given by a direct estimation are quite similar, independently of the method used to estimate them (Exponentially Weight Moving Average Model (EWMA) vs. autoregressive conditional volatility models).

Concerning the calculation of VaR, the estimations that we obtain using EWMA models, both under direct and indirect estimation (following the procedure proposed here) are quite precise. The estimations of VaR get worse when we directly estimate the variance-

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[2] This criterion has been used by Alexander and Leigh (1997), etc.

covariance matrix of the interest rates using GARCH models. These results not only validate the methodology proposed in this paper, but they also point out that the use of EWMA models for calculating VaR yields superior results to those obtained using GARCH models.

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Illustration 1. Comparing the changes of interest rates observed with the estimated changes (equation (5)).

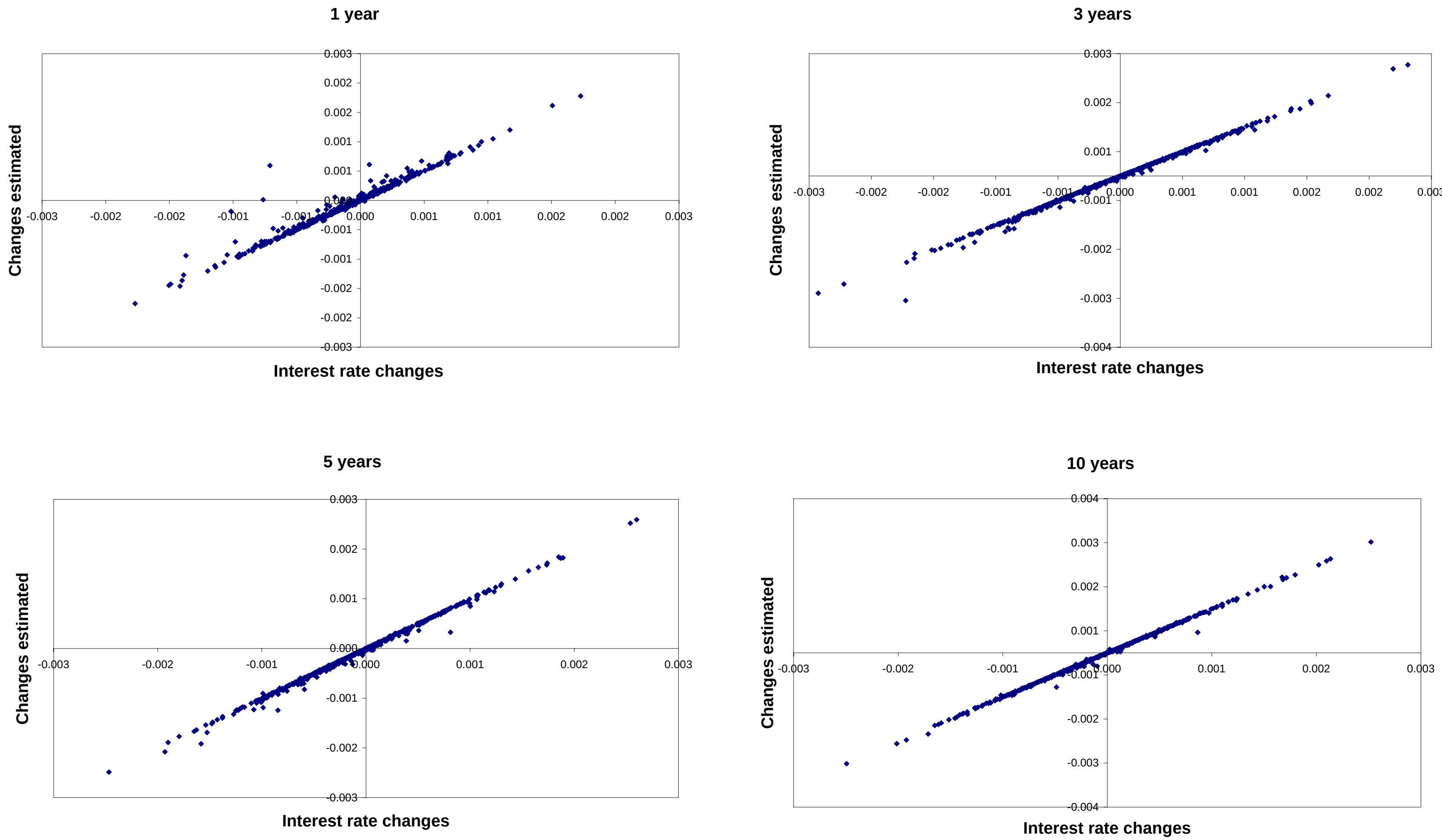

Illustration 2. Comparing the variance of changes of interest rate: Direct and indirect estimation using exponentially weighted moving average model.

Figure 1(a). Conditional Standar Deviation, 1 year

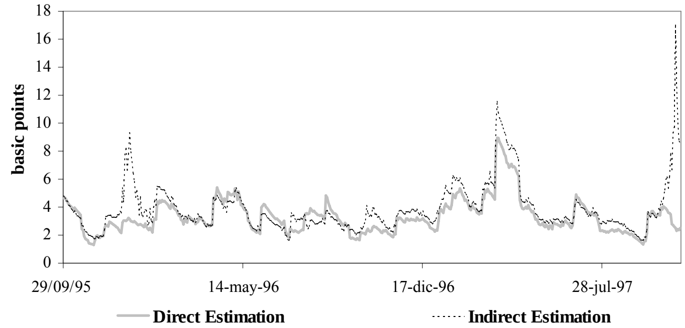

Figure 1(b). 1 year

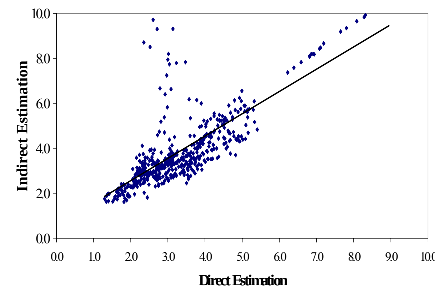

Figure 2(a). Conditional Standar Deviation, 3 years

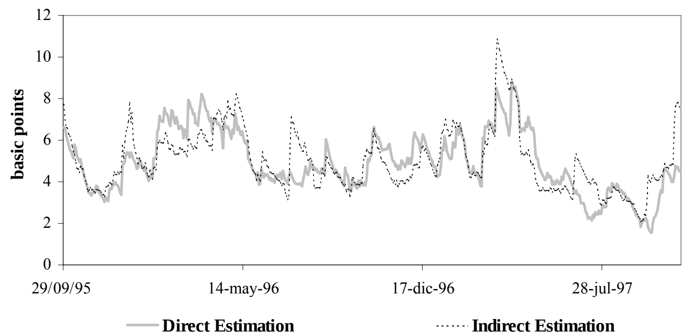

Figure 2(b). 3 years

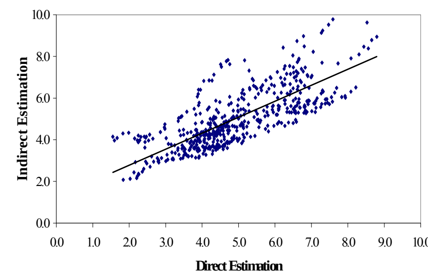

Figure 3(a). Conditional Standar Deviation, 5 years

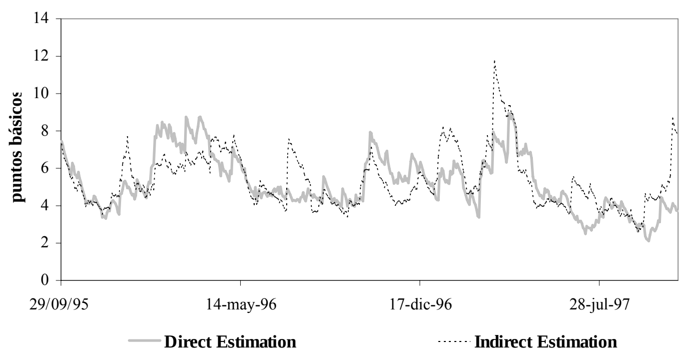

Figure 4(b). 10 years

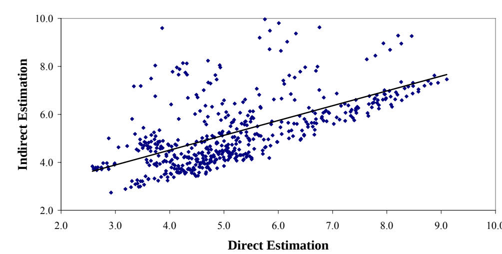

Figure 4(a). Conditional Standar Deviation, 10 years

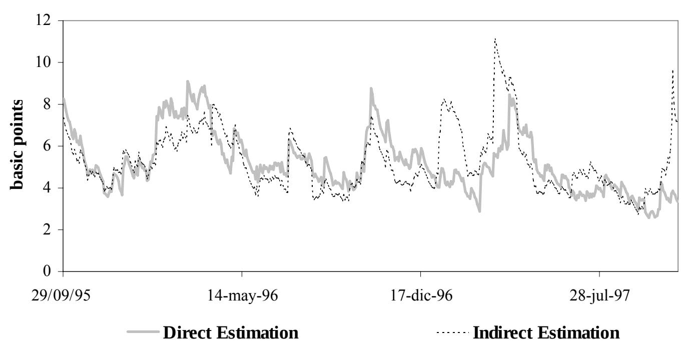

Figure 4(b). 10 years

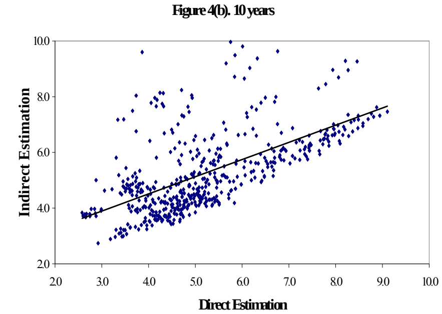

Illustration 3. Comparing the variance of changes of interest rate: Direct and indirect estimation using GARCH model.

Figure 1(a). Conditional Standar Deviation, 1 year

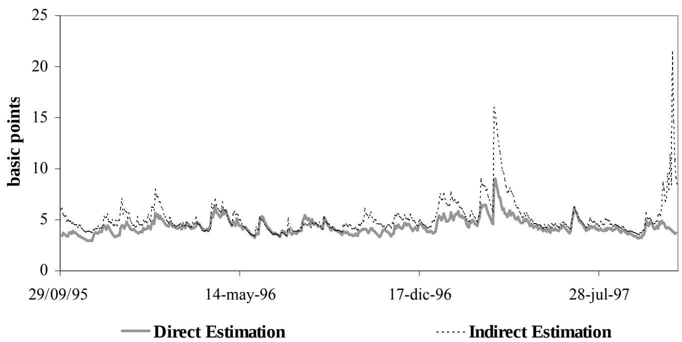

Figure 1(b). 1 year

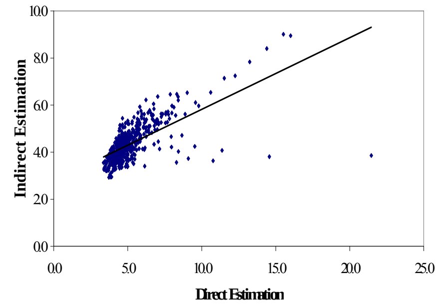

Figure 2(a). Conditional Standar Deviation, 3 years

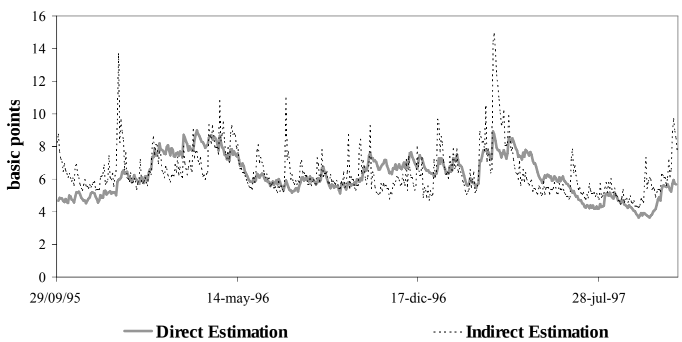

Figure 2(b). 1 year

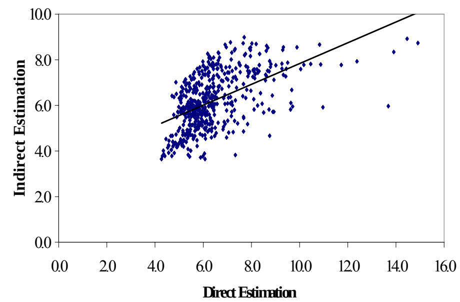

Figure 3(a). Conditional Standar Deviation, 5 years

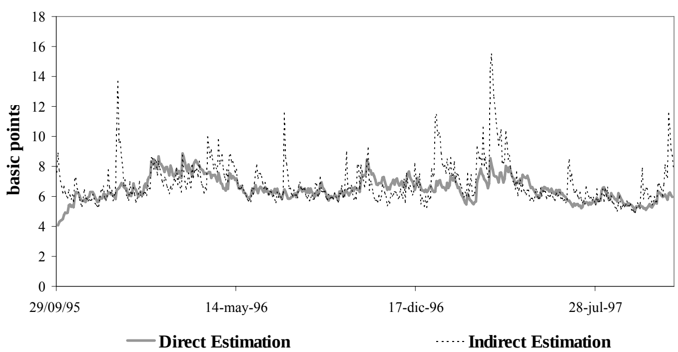

Figure 3(b). 5 years

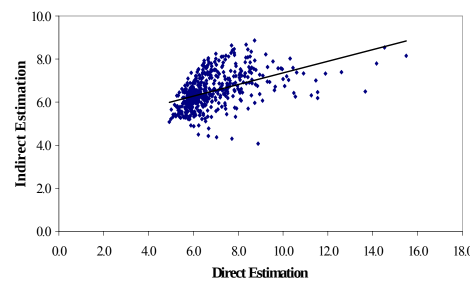

Figure 4(a). Conditional Standar Deviation, 10 years

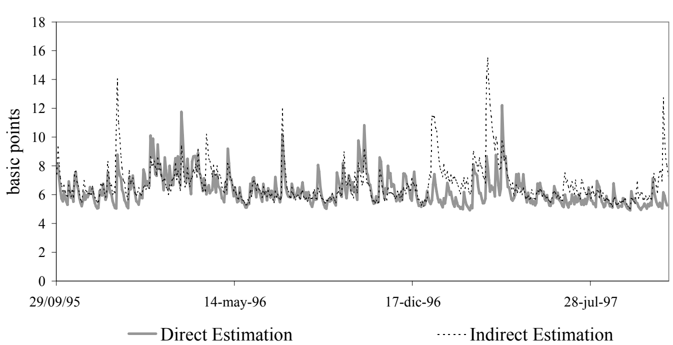

Figure 4(b). 10 years

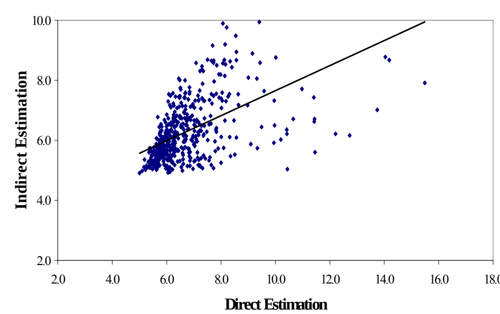

Illustration 4. Comparing the covariance between interest rate: Direct and indirect estimation using exponentially weighted moving average model.

Figura 1(a). Conditional Covariance, 1 year - 3 years

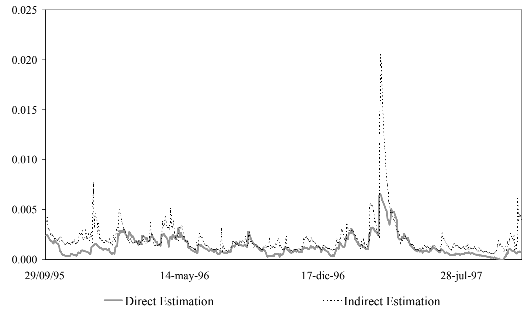

Figure 1(b). 1 year - 3 years

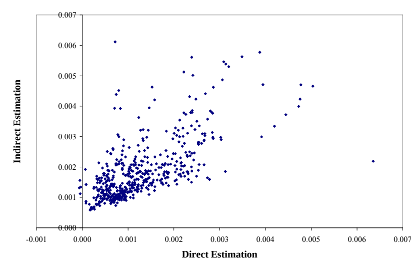

Figura 2(a). Conditional Covariance, 3 years - 5 years

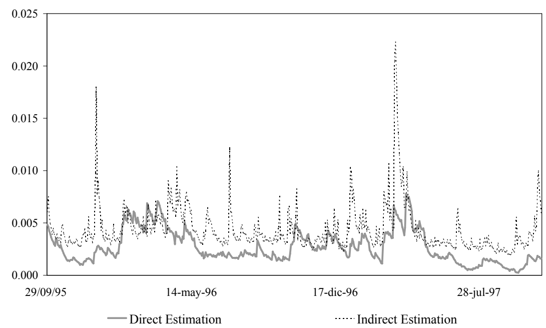

Figure 1(b). 3 year - 5 years

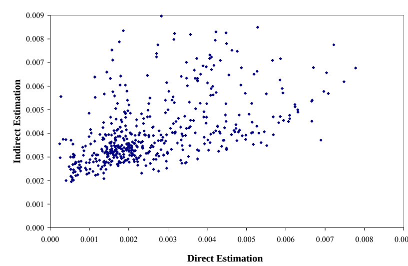

Figura 3(a). Conditional Covariance, 3 years - 10 years

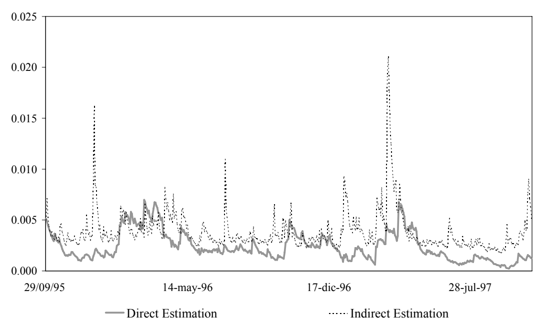

Figure 1(b). 3 year - 10 years

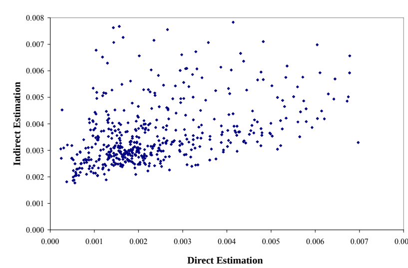

Figura 4(a). Conditional Covariance, 5 years - 10 years

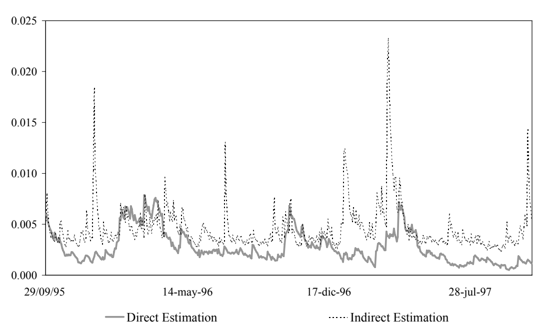

Figure 1(b). 3 year - 10 years

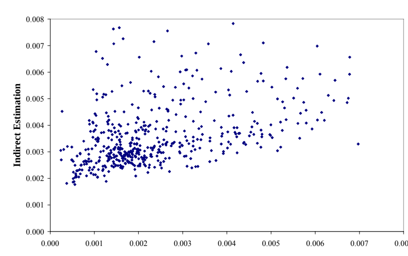

Illustration 5. The 5% one day VaR for a 10 year portfolio. Direct estimation using an exponentially weighted moving average model, VaR_D_EWMA(5%), indirect estimation using an exponentially weighted moving average model, VaR_I_EWMA(5%), and indirect estimation using a GARCH model, VaR_I_GARCH(5%).