Information Precision, Transaction Costs, and Trading Volume: A Closer Look

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Abstract

We decompose the relation between trading volume and the precision of the public announcement into three parts: the first part denotes the influence of the differential interpretation for the public announcement; the second denotes the contribution of the different precision of the previous private information; and the third denotes the impact of proportional transaction costs. The first part is always positive, the third is always negative and the second may be positive or negative.

In the economy with common interpretation and without transaction costs, while our model may generate the traditional positive relation between trading volume and the precision, this relation may also be monotonically decreasing in our model. Moreover, whether the public information is identically interpreted or not, the effect of the transaction costs may dominate the others so that trading volume is firstly increasing and then decreasing in the precision. Our model helps reconcile several puzzling empirical results involving trading volume reactions to public announcement (Morse (1981), Bamber and Cheon (1995) vs Ziebart (1990), Barron (1995)).

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The conventional wisdom holds that meaningful news generates trading volume, and that trading volume is an useful measure of a public announcement’s information content. In the one hand, most theoretical models (Grundy and McNichols (1989), Holthausen and Verrecchia (1990), and Kim and Verrecchia (1991), for example) suggest that trading volume increases with the precision of public information. In the other hand, earnings announcements accompanied by high trading volume are interpreted to convey more information to traders than announcements with low trading volume in many empirical papers (Beaver (1968), Morse (1981), Bamber (1986), and Bamber and Cheon (1995), for example.).

However, Ziebart (1990) and (?) report that trading volume negatively related to the convergence in analysts’ forecasts around earnings releases. Barron (1995) also finds that trading volume is negatively related to convergence in analysts’ forecast in general. These finding are inconsistent with the above conventional notion that trading volume increases with information precision and thus the convergence of beliefs.

To help reconcile these empirical results, Barron and Karpoff (2004) construct a simple model with transaction costs and show that the relation between information precision and trading volume is ambiguous and can be negative. However, they do not study the equilibrium price at all. To remove the equilibrium price from the demand function of each trader, they suppose that there are only two traders in the economy. Their model is very particular because when we make any modification about the economy (noisy supply in the second period, a large number of traders, differential interpretation of traders, etc.), their model does no longer work.1

We develop a two periods model in which traders have different prior expectation in the second period. Instead of the economy with only two traders, we
study the equilibrium with a large number of traders. While our model may generate the traditional positive relation between trading volume and the precision in the economy with common interpretation and without transaction costs, this relation may also be monotonically decreasing. We also show that for any given public announcement and a (non infinite small) proportional transaction cost, when the precision is high enough, trading volume is decreasing in the precision. After introducing per capita supply in the second period, we report that our result above on the relation between trading volume and the precision is robust.

We further incorporate the differential interpretation to the public announcement. We find that the relation can be decomposed into three parts: the first part denotes the influence of the differential interpretation for the public announcement; the second denotes the contribution of different precision of the previous private information; and the third denotes the influence of proportional transaction costs. The first part is always positive, the third is always negative, and the second may be positive or negative. Whether total trading volume is increasing in the precision or not depends on which effect dominates.

The motivation for incorporating also the differential interpretation is that it is more descriptive of real market settings. As Kim and Verrecchia (1997) shown, when there exists exclusively preannouncement private information, trading volume has a linear relation to absolute value of price change, with a zero intercept. This relation is contrary to the empirical evidence (e.g. Kandel and Pearson (1995), Bamber et al. (1999)) that trading volume exists without price change. When there exists exclusively event-period private information (differential interpretation), trading volume is independent of absolute value of price change. This result is inconsistent with the positive relation between trading volume and absolute value of price change documented throughout the empirical literature
(e.g. Karpoff (1987), Gallant et al. (1992)). Most typically, models of trade eschew this issues by assuming that traders employ information either in anticipation of or in conjunction with public announcement. Several prior attempt can be found in Dantoh and Ronen (1993) and Kim and Verrecchia (1997).

The rest of the article is structured as follows. In section 1 we describe the basic economy. In section 2 we study the equilibriums in the simplest economy (with different prior expectation, common interpretation, and without noisy supply in the second period) and the relation between trading volume and the precision. In section 3, we add the differential interpretation and discuss the decomposition of the relation with numerical results. We conclude the paper in section 4 with several empirical implications.

1 Economy

The securities market model we suggest is one of pure exchange with two periods and two assets in the economy. One unit of riskless asset pays off one unit of consumption good in the end of the second period (at date 3). The return of the risky asset is a random variable, denoted by $\psi$, and is realized in the end of the second period. All traders have the identical prior expectation about the payoff of the risky asset with mean $\psi_0$ and precision $\rho_0$ at the very beginning (at date 0).

There is a countably infinite number of traders in the economy and they can be divided into two groups with $N_I = uN$ traders in group 1 and $N_U = (1-u)N$ traders in group 2. All traders have constant absolute risk aversion. Each trader maximizes his expected utility of consumption in the end of the second period

$$E \left[ U(w_{3,i}^j) \bigg| f_{t,i}^j \right] = E \left[ -\exp(-Rw_{3,i}^j) \bigg| f_{t,i}^j \right]$$
where $w^j_{3,i}$ is trader $j$ in group $i$’s wealth at date 3, the common absolute risk aversion $R$ is simplified to 1, and $F^j_{t,i}$ is the information set available to trader $j$ in group $i$ at date $t$.

Three events occur in the first period (at date 1). First, trader $j$ is endowed with $z_0$ units of riskless asset and zero risky asset. Second, trader $j$ in group $i$ observes his private information $y^j_{1,i} = \psi + e^j_i$, where $e^j_i$ is normally distributed with mean 0 and precision $\rho_i$. We suppose $\rho_1 > \rho_2$. Then the private information that traders in group 1 receive is more reliable than those traders in group 2 receive. We refer to traders in group 1 as informed traders and traders in group 2 as uninformed traders. Third, the market opens and all traders buy and sell assets at competitive market prices. By the Strong Law of Large Numbers, the mean signal in each group, $\bar{y}_{1,1}$ and $\bar{y}_{1,2}$ converges almost surely to $\psi$ as $N \to \infty$. The supply of the risky asset, denoted by $x_t$ ($t = 1, 2$), is a random variable normally distributed with mean 0 and precision $t$.

In the second period (at date 2), there is a public announcement of a signal $y^j_2 = \psi + w + \phi \epsilon^j$, where $w$ is normally distributed with mean 0 and precision $\rho_w$ and $\epsilon^j$ is an idiosyncratic noise term that has also a normal distribution with mean 0 and precision $n$. It is assumed that all random variables are mutually independent. This form for the information release is chosen because we incorporate the effects of heterogeneous interpretation of a public information release$^{3,4}$. $\phi$ is zero when all traders interpret the public announcement in the same way and is 1 when each trader have his own differential interpretation. In the latter case, each trader observes the same public signal (e.g., an earnings announcement) but each trader’s interpretation of what the signal implies about the value of the risky asset varies because of the idiosyncratic noise term. What is relevant is not the public announcement per se, but the implication of the release for the payoff of the risky asset$^5$. 

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In our model, traders differ in terms of their private information in period 1 ($y_{1,i}$), its precision ($\rho_i$), and perhaps their differential interpretation in the second period ($\varepsilon_{2,j}$). Hence we model the simple observation that some traders are better informed than others and traders hold different expectations. This difference in information quality plays a central role on the relation between trading volume and the precision of public announcement in sections 2.

Following Blume et al. (1994) and Barron and Karpoff (2004), we assume that traders have myopic, or naive, demands so that each trader chooses his demand to maximize his expected utility at date 3 without taking possible future transactions into account. We also suppose that trading volume is not publicly available and each trader makes his decision conditional on all information up to but not including the equilibrium price in the actual period. Then the information sets of each trader are $F_{1,i} = \{\psi_0, y_{1,i}\}$ at date 1, $F_{2,i} = \{\psi_0, y_{1,i}, p_1, y_{2,i}\}$ at date 2.

To examine the effect of transaction costs on the equilibrium price and trading volume, we introduce a per-share fee, $c$, for each share bought or sold of the risky asset in the second period. Thus total transaction costs increase with the number of shares traded. This is consistent with most theoretical models, e.g., Vayanos (1998), Barron and Karpoff (2004), and empirical evidence, e.g., Brennan and Chordia (1993).

2 Public Information with Common Interpretation

We begin our analysis from the simplest setting: there is not noisy supply (change) in the second period and all traders interpret the public information in the same way.
2.1 Market Equilibrium

The analysis required to obtain the equilibrium in the first period is standard. Conditional on the common prior expectation and his private information, each trader maximizes his final wealth in the end of the second period. The conditional expectation and variance are 

\[ \psi_{1,i}^j = E\left(\psi \mid \psi_{0,i}^y, y_{1,i}^j\right) = \frac{\rho_0 \psi_{0} + \rho_i y_{1,i}^j}{\rho_0 + \rho_i} \]

and 

\[ K_{1,i} = Var^{-1}\left(\psi \mid \psi_{0,i}^y, y_{1,i}^j\right) = \rho_0 + \rho_i. \]

The demand of each trader in the first period is 

\[ d_{1,i}^j = \rho_0(\psi_{0} - p_1) + \rho_i(y_{1,i}^j - p_1). \]

By the market clearing condition, the equilibrium price in the first period can be easily calculated and is

\[ p_1 = \frac{\rho_0 \psi_{0} + (u \rho_1 + (1 - u) \rho_2) \psi - x_1}{\rho_0 + u \rho_1 + (1 - u) \rho_2} = \frac{\rho_0 \psi_{0} + (u \rho_1 + (1 - u) \rho_2) \psi - x_1}{K_1} \]

where \( K_1 \) is the weighted precision of all traders at date 1.

Because of the uncertainty of the noisy supply, the equilibrium price in the first period is not revealing. However, it helps each trader to revise his expectation on the payoff of the risky asset. In the second period, the information sets become \( \{\psi_{0,i}^y, y_{1,i}^j, p_1, y_2\} \). Following Kim and Verrecchia (1991) and Kim and Verrecchia (1997), we write the signal \( p_1 \) in another way

\[ q = \frac{K_1 p_1 - \rho_0 \psi_{0}}{u \rho_1 + (1 - u) \rho_2} = \psi - G x_1 \]

where \( G^{-1} = u \rho_1 + (1 - u) \rho_2 \) and \( q \) has the precision \( t / G^2 \). The information set \( \{\psi_{0,i}^y, y_{1,i}^j, p_1, y_2\} \) is equivalent to \( \{\psi_{0,i}^y, q, y_2\} \) because one can be generated from the other.

As in the first period, each trader maximizes his expected wealth at the end
of the second period:

\[ w^j_3 = d^j_{2,i} (\psi - p_2) + d^j_{1,i} (p_2 - p_1) + z_0 - (d^{j}_{2,i} - d^{j}_{1,i}) c \]

\[ = d^j_{2,i} (\psi - p_2) + d^j_{1,i} (p_2 - p_1) + z_0 - \left( d^j_{2,i} - d^j_{1,i} \right) c \text{ for buyer at date 2} \]

\[ d^j_{2,i} (\psi - p_2) + d^j_{1,i} (p_2 - p_1) + z_0 + \left( d^j_{2,i} - d^j_{1,i} \right) c \text{ for seller at date 2} \]

The conditional expectation and variance at date 2 are

\[ \psi^j_{2,i} = E \left( \psi \mid y^j_{0,i}, q, y^j_{2} \right) \]

\[ = \rho_0 \psi_0 + \rho_i y^j_{1,i} + \frac{tq}{G^2} + y_2 \rho_w \]

\[ K^j_{2,i} = Var^{-1} \left( \psi \mid y^j_{0,i}, q, y^j_{2} \right) \]

\[ = \rho_0 + \rho_i + \frac{t}{G^2} + \rho_w \]

With negative exponential utility, we have the demand of each trader in the second period

\[ d^j_{i,2} = \begin{cases} 
\rho_0 \psi_0 + \rho_i y^j_{1,i} + \rho_w y_2 + \frac{tq}{G^2} - (p_2 + c) K^j_{2,i} & \text{for buyer} \\
\rho_0 \psi_0 + \rho_i y^j_{1,i} + \rho_w y_2 + \frac{tq}{G^2} - (p_2 - c) K^j_{2,i} & \text{for seller} 
\end{cases} \]

Because of transaction costs in the second period, traders trade only when the gains from transaction is high enough to compensate transaction costs. It is easy to show that this condition is verified if the demand of trader \( j \) in group \( i \) \( \left( d^j_{2,i} \right) \) calculated above is higher (lower) than that \( \left( d^j_{1,i} \right) \) in the first period for buyers (sellers)\(^6\). Then the conditions that traders trade in the second period are

\[ \rho_0 \psi_0 + \rho_i y^j_{1,i} + \rho_w y_2 + \frac{tq}{G^2} - (p_2 + c) K^j_{2,i} \]

\[ > \rho_0 (\psi_0 - p_1) + \rho_i (y^j_{1,i} - p_1) \quad \text{for buyer} \]

\[ \rho_0 \psi_0 + \rho_i y^j_{1,i} + \rho_w y_2 + \frac{tq}{G^2} - (p_2 - c) K^j_{2,i} \]

\[ < \rho_0 (\psi_0 - p_1) + \rho_i (y^j_{1,i} - p_1) \quad \text{for seller} \]
After several arrangement, we have the critical points above or below which traders will trade:

\[ y_{up}^i = \frac{1}{\rho_w} \left[ (p_2 + c) K_{2,i} - p_1 K_{1,i} - \frac{tq}{G^2} \right] \]

\[ = \frac{1}{\rho_w} \left[ (p_2 + c) \left( \rho_0 + \frac{t}{G^2} + \rho_w \right) - \frac{tq}{G^2} - p_1 \rho_0 + (p_2 + c - p_1) \rho_i \right] \]

\[ y_{down}^i = \frac{1}{\rho_w} \left[ (p_2 - c) K_{2,i} - p_1 K_{1,i} - \frac{tq}{G^2} \right] \]

\[ = \frac{1}{\rho_w} \left[ (p_2 - c) \left( \rho_0 + \frac{t}{G^2} + \rho_w \right) - \frac{tq}{G^2} - p_1 \rho_0 + (p_2 - c - p_1) \rho_i \right] \]

Note that these critical points do not depend on the private information in the first period. In other words, they are identical to all traders in the same group and each trader in the same group buy or sell exactly the same amount of the risky asset. By the market clearing condition, one group should buy and the other should sell.

Recall that \( \rho_1 > \rho_2 \). Then when \( p_2 > p_1 - c \), we have \( y_{up}^1 > y_{up}^2 \); and when \( p_2 < p_1 + c \), we have \( y_{down}^1 < y_{down}^2 \). Because \( y_{up}^i > y_{down}^i \), it is impossible that when \( p_2 > p_1 - c \), both \( y_2 > y_{up}^1 \) and \( y_2 < y_{down}^1 \) hold; when \( p_2 < p_1 + c \), both \( y_2 < y_{down}^1 \) and \( y_2 > y_{up}^1 \) hold. Moreover, when \( p_1 - c < p_2 < p_1 + c \), \( y_{up}^2 < y_2 < y_{down}^1 \) and \( y_{up}^1 < y_2 < y_{down}^2 \) can not hold at the same time. It means that to trading volume in the second period not to be zero, the following condition should be verified

\[ y_{up}^2 < y_2 < y_{down}^1 \text{ when } p_2 > p_1 + c \]

\[ y_{up}^1 < y_2 < y_{down}^2 \text{ when } p_2 < p_1 - c \]

Now we construct the equilibrium in the second period according to the relation between \( p_2 \) and \( p_1 \). In the case of \( p_2 > p_1 + c \), traders in group 1 sell and traders in group 2 buy. From the demand functions in the first and second period, the
sell (buy) order of traders in group 1 (2) are

\[
\begin{align*}
    d_{\text{sell}}^1 &= \rho_w y_2 - (p_2 - c) K_{2,1} + K_{1,1} p_1 + \frac{tq}{G^2} \\
    d_{\text{buy}}^2 &= \rho_w y_2 - (p_2 + c) K_{2,2} + K_{1,2} p_1 + \frac{tq}{G^2}
\end{align*}
\]

By market clearing condition, the sum of the whole sell orders and the whole buy orders should be equal to zero \((u d_{\text{sell}}^1 + (1 - u) d_{\text{buy}}^2 = 0)\). After substituting from the equations about \(d_{\text{sell}}^1\) and \(d_{\text{buy}}^2\) and several arrangements, we have the equilibrium price in the second period

\[
p_2 = \frac{1}{K_2} [\rho_w y_2 + p_1 K_1 + \frac{tq}{G^2} + c \left( (2u - 1) \left( \rho_0 + \frac{t}{G^2} \right) + u \rho_1 - (1 - u) \rho_2 \right)]
\]

where \(K_2\) denotes the weighted precision in the second period. Now we get the upper critical point of \(y_2\) for trading volume to be positive

\[
y_2^{up} < y_2 < y_1^{down} \text{ when } p_2 > p_1 + c
\]

\[
\iff \quad y_2 > y_2^{up} = p_1 \left( 1 + \frac{t}{G^2 \rho_w} \right) + \frac{2cK_{2,1}K_{2,2}}{(\rho_1 - \rho_2) \rho_w} - \frac{tq}{G^2 \rho_w}
\]

Recall that we study the case of \(p_2 > p_1 + c\). We need check whether this condition is verified

\[
p_2 > p_1 + c
\]

\[
\iff \quad y_2 > p_1 \left( 1 + \frac{t}{\rho_w G^2} \right) - \frac{tq}{\rho_w G^2} + \frac{2c(1 - u) K_{2,2}}{\rho_w}
\]

Since it is clear that \(\frac{2cK_{2,1}K_{2,2}}{\rho_1 - \rho_2 \rho_w} > \frac{2c(1 - u) K_{2,2}}{\rho_w}\), the condition \((p_2 > p_1 + c)\) is automatically verified.

In the case of \(p_2 < p_1 - c\), informed traders in group 1 buy and uninformed
traders in group 2 sell. In the same way, we can calculate the equilibrium price in the second period

\[ p_2 = \frac{1}{K_2} [\rho_w y_2 + p_1 K_1 + tq \frac{G}{G^2} - c \left( (2u - 1) \left( \rho_0 + \rho_w + \frac{t}{G^2} \right) + u \rho_1 - (1 - u) \rho_2 \right)] \]

and the lower critical point of \( y_2 \) for trading volume to be positive

\[ y_1^{up} < y_2 < y_2^{down} \text{ when } p_2 < p_1 - c \]
\[ \Leftrightarrow \quad y_2 < y_2^{down} = \rho_1 \left( 1 + \frac{t}{G^2 \rho_w} \right) \frac{2c K_2.1 K_2.2}{(\rho_1 - \rho_2) \rho_w} - \frac{tq}{G^2 \rho_w} \]

As in the case of \( p_2 > p_1 + c \), we check whether the condition \( (p_2 < p_1 - c) \) is verified

\[ p_2 < p_1 - c \]
\[ \Leftrightarrow \quad y_2 < p_1 \left( 1 + \frac{t}{\rho_w G^2} \right) \frac{\rho_w}{p_2} - \frac{2c K_2.1}{\rho_w} \]

Since it is clear that \( \frac{2c K_2.1 K_2.2}{(\rho_1 - \rho_2) \rho_w} > \frac{2c K_2.1}{\rho_w} \), the above condition is also automatically verified.

While trading volume can be calculated either by the orders of informed traders or by those of uninformed traders, it is relatively easier to be calculated by the buy orders. In the case of \( y_2 > y_1^{up} \), \( V = (1 - u) \left| d_{buy}^2 \right| = (1 - u) d_{buy}^2 \). In the case of \( y_2 < y_2^{down} \), \( V = u \left| d_{buy}^1 \right| = u d_{buy}^1 \). Substituting from the equation about \( d_{buy}^i \) and \( p_2 \) yields

\[ V = (1 - u) \left[ \rho_w y_2 + \frac{tq}{G^2} + p_1 (\rho_0 + \rho_2) - (p_2 + c) \left( \rho_0 + \frac{t}{G^2} + \rho_2 + \rho_w \right) \right] \]
\[ = \frac{(1 - u) \lambda}{K_2} \left[ \rho_w y_2 + \frac{tq}{G^2} - p_1 \left( \rho_0 + \frac{t}{G^2} \right) \right] (\rho_1 - \rho_2) - 2c K_2.1 K_2.2 \]
where $\lambda$ is 1 if $y_2 > y^{up}_*$, and is -1 if $y_2 < y^{down}_*$. When $y^{up}_* > y_2 > y^{down}_*$, the equilibrium price cannot be calculated and trading volume is zero. For this reason, the interval $[y^{up}_*, y^{down}_*]$ is called "No volume area". The length of this interval is $\frac{4cK_2}{(\rho_1 - \rho_2)\rho_w}$ and is increasing in proportional transaction costs. These results are now restated as a Proposition.

**Proposition 1** In the second period of the economy in this section,

(1) the equilibrium price is

\[
p_2 = \frac{1}{K_2} \left[ \rho_w y_2 + p_1 \right] \\
+ \frac{tq}{G^2} + \lambda c \left( (2u - 1) \left( \rho_0 + \rho_w + \frac{t}{G^2} \right) + u \rho_1 - (1 - u) \rho_2 \right)
\]

(2) and given $y_2$, trading volume is

\[
V = (1 - u) \left[ \rho_w y_2 + \frac{tq}{G^2} + p_1 (\rho_0 + \rho_2) - (p_2 + c) \left( \rho_0 + \frac{t}{G^2} + \rho_2 + \rho_w \right) \right]
\]

\[
= \frac{(1 - u)u}{K_2} \left[ \lambda \left( \rho_w y_2 + \frac{tq}{G^2} - p_1 \left( \rho_w + \frac{t}{G^2} \right) \right) (\rho_1 - \rho_2) - 2cK_2,1K_2,2 \right]
\]

where $\lambda$ is 1 if $y_2 > y^{up}_*$, and is -1 if $y_2 < y^{down}_*$.

**Proof.** See the discussion above and Appendix A. ■

### 2.2 Trading Volume, Information Precision and Transaction Costs

The partial derivative of trading volume ($V$) with respect to the precision of public information ($\rho_w$) is

\[
\frac{\partial V}{\partial \rho_w} = \frac{u(1 - u)}{(K_2)^2} \left\{ \lambda (\rho_1 - \rho_2) \left[ (y_2 - p_1) (K_2 - \rho_w) + \frac{t}{G^2} (p_1 - q) \right] \\
- 2c \left[ u (K_2,1)^2 + (1 - u) (K_2,2)^2 \right] \right\}
\]
$\frac{u(1-u)}{(K_2)^2} \left\{ \lambda (\rho_1 - \rho_2) \left( \frac{t}{G^2 \rho_w} (p_1 - q) K_2 \right) + \frac{2cK_{2,1}K_{2,2}}{\rho_w} \left( K_1 + \frac{t}{G^2} \right) - 2c \left[ u (K_{2,1})^2 + (1-u) (K_{2,2})^2 \right] \right\}$

In a world without transaction costs, the term in the square brackets does not depend on the precision of common information, trading volume is always a monotonic function of the precision. If we suppose that the equilibrium price is not available or traders cannot infer information contained in the equilibrium price in the first period, then the term $\frac{t}{G^2} (p_1 - q)$ disappears. We have shown that when $y_2 > p_1$, $\lambda = 1$; and when $y_2 < p_1$, $\lambda = -1$. Trading volume is always increasing in the precision of common information, which is consistent with Kim and Verrecchia (1991)’s model.

If traders can infer information contained in the equilibrium price in the first period, $(y_2 - p_1)$ and $(p_1 - q)$ may have different signs and trading volume may be monotonically decreasing in the precision of common information. Furthermore, it is easy to show $p_1 - q = \frac{\rho_0(x_0 - p_1)}{\rho_1 + (1-u)\rho_2}$. If we interpret the initial identical expectation as the initial price, the fact that $(y_2 - p_1)$ and $(p_1 - q)$ have the same (inverse) signs means the positive (negative) price autocorrelation. Then our model suggests that in the case of price momentum, trading volume is decreasing in the precision and in the inverse case, trading volume may be increasing or decreasing in the precision.

When there are transaction costs, the second term is always negative and increases in magnitude with the size of transaction costs. At the first glance, it seems that the first term is independent of the precision of common information ($\rho_w$) and the second term is increasing in the precision. For the value of the common information given, even the first term is positive, if transaction costs are high enough, trading volume may be decreasing in the precision of common information.
From the definition of the upper \( y_{\text{up}}^* \) and lower \( y_{\text{low}}^* \) critical points, we know that for the common information given, the higher the precision, the higher (lower) the upper (lower) critical point, the larger the length of "No volume area" \( 3^cK^2,1 - K^2,2 \). However, the term \( \frac{2cK^2,1 - K^2,2}{\rho_w} (K^1_1 + \frac{1}{\rho_w}) \) is only 1 order of \( \rho_w \), and the term \( 2c \left[ u(K^2,1)^2 + (1 - u)(K^2,2)^2 \right] \) is 2 order of \( \rho_w \). It means that for any given (non infinite small) proportional transaction cost \( c \), when the precision of the common information increases, trading volume may be decreasing in the precision. For any common information given, when its precision is high enough that \( y_2 \) is no longer superior than \( y_{\text{up}}^* \), there is not any transaction at all.

**Proposition 2** In the large economy in this section, trading volume may be monotonically decreasing, monotonically increasing or firstly increasing and then decreasing in the precision of public announcement. For any given public announcement and a (not infinite small) proportional transaction cost, when the precision is high enough, trading volume is decreasing in the precision. In the extreme case with the infinite precision, trading volume is zero.

**Proof.** See the discussion above and Appendix A.

### 2.3 A discussion of Barron & Karoff (2004)’s model

The Barron and Karoff (2004)’s model is similar to the one outlined in this section\(^8\). Trading volume in their model (equ.11, page 1212) is

\[
V_{BK} = \max (W [Y + Z] - X, 0)
\]
where

\[ W = \frac{(K_{b,1} - K_{s,1})\rho_w}{K_{s,2} + K_{b,2}} \]

\[ Y = y_2 - \frac{K_{b,1}\psi_{b,1} + K_{s,1}\psi_{s,1}}{K_{b,1} + K_{s,1}} \]

\[ Z = \frac{x_1}{K_{b,1} + K_{s,1}} \]

\[ X = \frac{2cK_{2,1}K_{2,2}}{K_{s,2} + K_{b,2}} \]

It is interesting to compare their trading volume with ours'. Because they do not study the equilibrium price, the information sets in the second period do not include the equilibrium price in the first period. Trading volume in our model becomes

\[ V = \max \left( \frac{(1-u)u}{K_2} [\lambda \rho_w (y_2 - p_1) (\rho_1 - \rho_2) - 2cK_{2,1}K_{2,2}], 0 \right) \]

\[ = \max \left( \frac{(1-u)u}{K_2} \lambda \rho_w \left( y_2 - \frac{\rho_0 \psi_0 + (u \rho_1 + (1-u) \rho_2) \psi}{K_1} + \frac{x_1}{K_1} \right) \right) (\rho_1 - \rho_2) \]

\[ -2cK_{2,1}K_{2,2} \frac{(1-u)u}{K_2}, 0 \right) \]

The main difference between these two formula is the signs of the terms \( WY \) and \( \left( y_2 - \frac{\rho_0 \psi_0 + (u \rho_1 + (1-u) \rho_2) \psi}{K_1} \right) \). Barron and Karpoff (2004) argue that "when information surprise is positive (\( Y > 0 \)), investor b is the buyer at date 2 because his or her precision is higher (\( K_{b,1} > K_{s,1} \)), implying that \( W \) also is positive. Conversely, when the information surprise is negative (\( Y < 0 \)), investor b is the buyer because his or her precision is lower (\( K_{b,1} < K_{s,1} \)). Thus \( W \) and \( Y \) always have the same sign. Our assumption that trading volume is non-negative therefore rules out only such idiosyncratic situations as when \( W \) and \( Y \) are negative and \( Z > |Y| \)." (page 1213).

Our analysis above shows that in the second period, when the information
surprise is positive \(y_2 > y_{up}^*)\), traders with higher precision \((K_{1,1} > K_{1,2})\) are sellers, and that when the information surprise is negative \(y_2 < y_{down}^*\), traders with higher precision \((K_{1,1} > K_{1,2})\) are buyers. We do think our results are consistent with the financial intuition. In the first period, when information is positive, traders with higher precision buy and those with lower precision sell in general. If there are only two traders, both of them know who owns higher or lower precision. In the second period, when information surprise is positive (negative), trader with lower precision will never wants to sell (buy) again, even though trader with higher precision wants to buy (sell). The only one possibility of transaction is that when information surprise is positive (negative), trader with higher precision wants to sell and trader with lower precision wants to buy.

We can also explain it in another way. Recall that the conditional variances in the first and second period are \(K_{1,i}^1 = (\rho_0 + \rho_i)^{-1}\) and \(K_{2,i}^2 = (\rho_0 + \rho_1 + \rho_w)^{-1}\), respectively. It means that the relative precision advantage of informed traders decreases from the first to the second period. Then it should be informed traders (not uninformed traders) that sell (buy) in case of positive (negative) information surprise. Our analysis also shows that \(W[Y + Z]\) (namely, \(\lambda(y_2 - p_1)(\rho_1 - \rho_2)\)) is always non-negative and that the situation when \(W\) and \(Y\) are negative and \(Z > |Y|\) do not exist.

The partial derivative of trading volume with respect to the precision in Barron and Karpooff (2004)’s model is

\[
\frac{\partial V_{BK}}{\partial \rho_w} = \frac{(K_{b,1} + K_{s,1})(K_{b,1} - K_{s,1})}{(K_2)^2} [Y + Z] - 2 \frac{(K_{2,1})^2 + (K_{2,2})^2}{(K_2)^2}
\]

They argue that the first term on the right hand side is positive and the second term on the right hand side is negative. The latter increases in magnitude with the size of transaction costs. Thus for some transaction cost and precision levels, the second term dominates and information precision is negatively related
to trading volume.

Although we shows the similar results, their analysis is not quite rigorous. To make the formula above on the partial derivative to hold, the condition $y_2 > y_{*up}$ or $y_2 < y_{*down}$ should be verified. Because $y_{*up}$ and $y_{*down}$ depends on also the precision of common information, we can not tell whether the second affect would dominate or not from the formula in Barron and Karpoff (2004)'s paper (equ.12, page 1213). This drawback comes from the fact that they do not study the equilibrium price and thus they can not explicitly get the relation between the equilibrium price and the common information.

3 Public Information with Differential Interpretation

While holding all the other assumptions unchanged, we assume that traders interpret the same public announcement in the differential ways and there is not noisy supply in the second period. The signal in the second period becomes $y_2^j = \psi + w + \epsilon^j$. To simplify the notation, we write each trader’s information and interpretation structures of the risky asset in the second period in the following way as $y_2^j \sim N(\psi, 1/\rho)$ where $\rho = \rho_w n / (n + \rho_w)$. Conditional on $w$, $y_2^j | w \sim N(\theta, 1/\rho)$ where $\theta = \psi + w$ denotes public announcement to all traders. So by the Strong Law of Large Numbers, the mean interpretation in each group converges almost surely to $\theta$ as $N \to \infty$.

3.1 Market Equilibrium

This change of the information environment does not influence the equilibrium in the first period and all the analysis about the first period applies. According to his own differential interpretation, each trader maximizes his expected utility
in the end of the second period as in the previous sections. The transaction of each trader in the second period is the difference of this demand at date 2 and that at date 1

\[ \Delta_{2,j}^i(y_j^2, y_{1,i}^j) = \begin{cases} 
\rho y_j^2 - (K_{1,i} + \rho + \frac{t_q}{C^c}) (p_2 + c) + K_{1,i} p_1 + \frac{t_q}{C^c} & \text{if } y_j^2 > y_i^{up} \\
0 & \text{if others} \\
\rho y_j^2 - (K_{1,i} + \rho + \frac{t_q}{C^c}) (p_2 - c) + K_{1,i} p_1 + \frac{t_q}{C^c} & \text{if } y_j^2 < y_i^{down}
\end{cases} \]

where

\[ y_i^{up} = \frac{(K_{1,i} + \rho + \frac{t_q}{C^c}) (p_2 + c) - p_1 K_{1,i} - \frac{t_q}{C^c}}{\rho} \]

\[ y_i^{down} = \frac{(K_{1,i} + \rho + \frac{t_q}{C^c}) (p_2 - c) - p_1 K_{1,i} - \frac{t_q}{C^c}}{\rho} \]

Because of differential interpretation, traders in the same group do no longer have the same demand, which induces the different equilibrium price and trading volume.

**Proposition 3** In the economy of this section,

1. the equilibrium price in the second period is

\[ p_2 = B \left( \rho \theta + \frac{t_q}{C^c} \right) - C c + D + E p_1 \]
where

\[ A = uK_{2,1} (1 - \Phi (m_1) + \Phi (n_1)) \]
\[ + (1 - u) K_{2,2} (1 - \Phi (m_2) + \Phi (n_2)) \]
\[ B = u (1 - \Phi (m_1) + \Phi (n_1)) + (1 - u) (1 - \Phi (m_2) + \Phi (n_2)) \]
\[ C = uK_{2,1} (1 - \Phi (m_1) - \Phi (n_1)) \]
\[ + (1 - u) K_{2,2} (1 - \Phi (m_2) - \Phi (n_2)) \]
\[ D = u \frac{\rho}{\sqrt{n}} (\phi (m_1) - \phi (n_1)) + (1 - u) \frac{\rho}{\sqrt{n}} (\phi (m_2) - \phi (n_2)) \]
\[ E = uK_{1,1} (1 - \Phi (m_1) + \Phi (n_1)) \]
\[ + (1 - u) K_{1,2} (1 - \Phi (m_2) + \Phi (n_2)) \]
\[ m_i = \sqrt{\frac{n}{\rho}} \left( K_{1,i} (p_2 + c - p_1) + \rho (p_2 - \theta + c) + \frac{t}{G^2} (p_2 + c - q) \right) \]
\[ n_i = \sqrt{\frac{n}{\rho}} \left( K_{1,i} (p_2 - c - p_1) + \rho (p_2 - \theta - c) + \frac{t}{G^2} (p_2 - c - q) \right) \]

(2) and given \( \theta \), trading volume in the second period is

\[ V = \frac{u}{2} V^1 + \frac{(1 - u)}{2} V^2 \]

where

\[ V^i = \frac{\rho}{\sqrt{n}} (\phi (-m_i) - m_i \Phi (-m_i) + \phi (n_i) + n_i \Phi (n_i)) \]

**Proof.** See Appendix B. ■
3.2 Trading Volume, Information Precision and Transaction Costs

In the general case, the partial derivative of trading volume with respect to the precision can be written as (See Appendix B)

$$\frac{\partial V}{\partial \rho_w} = DV_1 \frac{\partial \left( \frac{\rho_w}{\sqrt{n}} \right)}{\partial \rho_w} + DV_2 \cdot \frac{u (1 - u)}{A} \frac{\partial \rho}{\partial \rho_w} + DV_3 \cdot \frac{c}{A} \frac{\partial \rho}{\partial \rho_w}$$

where

$$DV_1 = u (1 - u) K_{2,1} [\phi(m_2) \Phi(n_1) + \phi(n_2) (1 - \Phi(m_1))] + u (1 - u) K_{2,2} [\phi(m_1) \Phi(n_2) + \phi(n_1) (1 - \Phi(m_2))] + K_{2,1} u^2 [\phi(m_1) \Phi(n_1) + \phi(n_1) (1 - \Phi(m_1))] + K_{2,2} (1 - u)^2 [\phi(m_2) \Phi(n_2) + \phi(n_2) (1 - \Phi(m_2))]$$

$$DV_2 = (p_2 - \theta) (\rho_1 - \rho_2) [\Phi(n_2) (1 - \Phi(m_1)) - \Phi(n_1) (1 - \Phi(m_2))]$$

$$DV_3 = -u^2 K_{2,1} (1 - \Phi(m_1)) \Phi(n_1) - (1 - u)^2 K_{2,2} (1 - \Phi(m_2)) \Phi(n_2) - u (1 - u) (K_{2,1} + K_{2,2}) [(1 - \Phi(m_2)) \Phi(n_1) + (1 - \Phi(m_1)) \Phi(n_2)]$$

It is difficult to analyze the sign of this formula directly. Two special cases help us understand the relation between trading volume and the precision.

3.2.1 Zero Transaction Cost and Identical Private Information Precision

In this special case, although traders still observe different information in the first period, the precision of this private information is identical to all traders.
(\rho_1 = \rho_2). The partial derivative simplifies to

$$\frac{\partial V}{\partial \rho_w} = \frac{1}{A\sqrt{n}} \frac{\partial \rho}{\partial \rho_w} \phi(m_1)$$

This terms captures the influence of the differential interpretation. Since we have same framework as Holthausen and Verrecchia (1990)'s model, it is not surprising that we have traditional result that trading volume is monotonically increasing in the precision\(^{12}\).

### 3.2.2 Zero Transaction Cost and Different Private Information Precision

When the previous private information precision are different (\(\rho_1 > \rho_2\)), the second term is added to the partial derivative which becomes

$$\frac{\partial V}{\partial \rho_w} = (u\phi(m_1) + (1-u)\phi(m_2)) \frac{\partial \left(\frac{\rho}{\sqrt{n}}\right)}{\partial \rho_w} + (p_2 - \theta) (\rho_1 - \rho_2) (\Phi(n_2) - \Phi(n_1)) \frac{u(1-u)}{A} \frac{\partial \rho}{\partial \rho_w}$$

Thus the second term presents the influence of the previous different precision of the private information on the relation. This term has the same sign as the following term \((n_2 - n_1)(p_2 - \theta)\). From the equilibrium price with zero transaction cost \((p_2 = \frac{1}{K_2} \left(\rho \theta + \frac{t}{G^2} + K_1 p_1\right))\), we have

$$= \frac{(n_2 - n_1)(p_2 - \theta)}{(K_2)^2 \rho} \left(\rho (\theta - p_1) + \frac{t}{G^2} (q - p_1)\right) \left(K_1 (p_1 - \theta) + \frac{t}{G^2} (q - \theta)\right)$$

Recall that in the economy where traders interpret the common information in the same way, trading volume is always a monotonic function of the precision. If we suppose that the equilibrium price is not available or traders cannot infer
information contained in the equilibrium price in the first period, then the term \( \frac{1}{\sigma} \) disappears and trading volume induced by the different precision of their private information is always increasing in the precision.

However, when traders can infer the information contained in the previous period, even though transaction is costless, trading volume is no longer a monotonic function of the precision. Note that while the second term does not depend on the precision, the first term may change the sign whenever \( \theta - p_1 \) and \( q - p_1 \) have different signs. Recall that \( q - p_1 > 0 \) means that the equilibrium price in the first period is higher than the prior identical expectation before the first period. Under this condition, \( \theta - p_1 > 0 \) means that \( p_2 > p_1 \). On the contrary, \( q - p_1 < 0 \) means that the equilibrium price in the first period is lower than the prior identical expectation before the first period. Under this condition, \( \theta - p_1 < 0 \) means that \( p_2 < p_1 \). In other words, when there exists the positive (negative) momentum of the prices, trading volume induced by the different precision is a monotonic increasing (decreasing) function of the precision of public announcement.

3.2.3 Proportional Transaction Costs and Different Private Information Precision

When we further suppose that there are proportional transaction costs, we have the formula expressed in the beginning of this section. \( DV_1 \) denotes the influence of the differential interpretation, \( DV_2 \) denotes the influence of the different precision of the previous private information, and \( DV_3 \) denotes the influence of transaction costs. \( DV_3 \) is always negative, which means that transaction costs decrease trading volume. Although \( DV_1 \) and \( DV_2 \) becomes more complicated because of transaction costs, the analysis above always applies. Whether total trading volume is increasing in the precision or not depends on which effect dominates. These results are restated in the following Proposition.
Proposition 4 In the large economy of this section, the relation between the precision of public announcement and trading volume can be decomposed into three parts:

(1) the first part denotes the contribution of the differential interpretation and it is always positive;

(2) the second part denotes the contribution of the different precision of the pervious private information and it may be negative or positive;

(3) and the third part denotes the contribution of proportional transaction costs and it is always negative.

Whether total trading volume is increasing in the precision or not depends on which effect dominates.

Proof. See the discussion above and Appendix B.

3.3 Numerical Results

This relationship between trading volume, the precision and transaction costs can perhaps best be investigated by examining the actual equilibrium outcomes for a specific economy. The values of the parameters are set to be: $\psi_0 = 1$, $\rho_0 = 1$, $\rho_1 = 1$, $\rho_2 = 0.001$, and $t = 1$. The values of $x_1$, $\psi_1$, and $\theta$ are randomly chosen to be 0.434, 0.615, and 1.272, respectively. The value of $\rho_w$ varies from 1 to 150. We think that the higher the precision, the lower the degree of differential interpretation. Then we let $n$ be in proportion with $\rho_w$. In this example, we set $n = 0.8\rho_w$.

Figure 1 shows trading volume as a function of the precision with different value of transaction costs. The solid line represents trading volume under no transaction costs and is always increasing in the precision. The dot, dash and
dot-dash lines plot trading volume with proportional transaction costs 0.05, 0.1, and 0.15, respectively. The turnaround point of the precision from which trading volume begins to decrease is clearly lower when proportional transaction costs are higher than that when proportional transaction costs are lower.

4 Conclusions

In this paper, we investigate the relation between trading volume and public announcement under proportional transaction costs. We develop a two period model in which traders have different prior expectation in the second period. In the economy with common interpretation and without transaction costs, while our model may generate the traditional positive relation between trading volume and the precision, this relation may also be monotonically decreasing in our model. We also show that for any given public announcement and a (non infinite small) proportional transaction cost, when the precision is high enough, trading volume is decreasing in the precision. The introduction of noisy capita supply in the second period complicates the analysis and helps to calculate the equilibrium price for any value of public announcement but does not change the relation between trading volume and the precision.

Then we further incorporate the differential interpretation to the public announcement. We find that the relation can be decomposed into three parts: the first part denotes the influence of the differential interpretation for the public announcement; the second denotes the contribution of different precision of the previous private information; the third denotes the impact of proportional transaction costs. The first part is always positive, and the third is always negative and the second may be positive or negative. Whether total trading volume is increasing in the precision or not depends on which effect dominates. As in the case where the public information is identically interpreted, the effect of trans-
action costs may dominate others and trading volume is firstly increasing and then decreasing in the precision. Our model helps reconcile several puzzling empirical results involving trading volume reactions to public announcement (Morse (1981), Bamber and Cheon (1995) vs Ziebart (1990), Barron (1995)).

Our analysis yields naturally several implications for empirical results. The fact that the degree of differential interpretation depends on the nature of public information implies that the reaction of trading volume to public announcements will differ according to public announcements. The second empirical implication rests on the different transaction costs across both assets and time.

Notes

1Several errors of this paper are discussed in subsection 2.3.

2Examples of papers whose analyses are based exclusively on information anticipataion of an announcement include: Kim and Verrecchia (1991), Demski and Feltham (1994), McNichols and Trueman (1994), and Barron and Karpoff (2004). Examples of papers whose analyses are based exclusively on information in conjunction with the announcement include: Holthausen and Verrecchia (1990), Indjejikian (1991), Harris and Raviv (1993), and Kim and Verrecchia (1994).

3It is also appriocrate to interpret the information signal as private inforation (for exemple, Grundy and McNichols (1989), Blume et al. (1994), etc). However, because we are primarily interestd in heterogeneous interpretations of a public information signal, we do not explicitly recognize the private information interpretation.
An alternative way to model information processing would be to assume that there are two signals in the second period: 
\[ y_2 = \psi + w, \] and \[ O^j = w + \varepsilon^j, \]
where \( w \) and \( \varepsilon^j \) are also independent. These two signals generate a signal \( y_2 - O^j = \psi - \varepsilon^j \) and this provides information about the payoff of the risky asset. This method is used by Kim and Verrecchia (1994) and Kim and Verrecchia (1997).


This is true only when transaction costs are proportional. When transaction costs are fixed, we need compare the expected wealth with transaction costs and that without transaction costs.

In Barron and Karpoff (2004)’s model, traders can not infer the information contained in the equilibrium price in the previous period because they do not study the equilibrium price at all.

The first small distinction between their model and the one here is that they assume that there are only two investors. The second is that traders in their model do not have any prior expectation but observe an additional common signal in the first period. The role of this common signal is the same as the prior idential expectation in our model. We do not expect that any of them changes the result on the relation.

Mathematically, there is also an error. If we suppose that the intuition in Barron and Karpoff (2004)’s model is true, we cannot get the equ.11 in page 1212. However, if we suppose our intuition is correct, we get the equ.11.

We think that the coefficient of the transaction cost in the equ.12 in page
1213 should be \(2\frac{(K_{2,1})^2 + (K_{2,2})^2}{(K_2)^2}\) (not \(2\frac{(K_{2,1})^2(K_{2,2})^2}{(K_2)^2}\)).

The introduction of noisy capita supply in the second period complicates the analysis and helps to calculate the equilibrium price for any value of public announcement but does not change the relation between trading volume and the precision. The proof is available upon request.

In the economy of section 2, if we suppose that traders have the same precision of their private information, trading volume will be zero in the second period.
5 Appendix

5.1 Figure

Figure 1: Relation between trading volume and the precision of common information. The solid line plots trading volume with $c = 0$, the dot line plots trading volume with $c = 0.05$, the dash line plots trading volume with $c = 0.1$, and the dot-dash line plots trading volume with $c = 0.15$. 
5.2 Appendix A: Market Equilibrium with Identical Interpretation

Proof. of Proposition 1: We present only the proof for the case of $p_2 > p_1 + c$ since it can be proved in exactly the same way in the inverse case. The equilibrium price is easily calculated and its proof is omitted. From this equilibrium price, we get the condition on $y_2$ without noisy supply

\[
y_2 > y_2^{up} = \frac{(p_2 + c) K_{2,2} - p_1 K_{1,2} - \frac{tq}{G^2 \rho_w}}{\rho_w}
= \frac{1}{K_2} \left[ \rho_w y_2 + \frac{tq}{G^2 \rho_w} + p_1 K_1 + cu2K_{2,1} \right] K_{2,2} - p_1 K_{1,2} - \frac{tq}{G^2 \rho_w}
\]
\[
\Leftrightarrow (K_2 - K_{2,2}) y_2 \rho_w
> p_1 (K_1 K_{2,2} - K_{1,2} K_2) + 2cu K_{2,1} K_{2,2} + \frac{tq}{G^2} (K_{2,2} - K_2)
\]
\[
\Leftrightarrow y_2 > p_1 (1 + \frac{t}{G^2 \rho_w}) + \frac{2c K_{2,1} K_{2,2}}{(\rho_1 - \rho_2) \rho_w} - \frac{tq}{G^2 \rho_w}
\]

and

\[
y_2 < y_1^{down} = \frac{(p_2 - c) K_{2,1} - p_1 K_{1,1} - \frac{tq}{G^2 \rho_w}}{\rho_w}
= \frac{1}{K_2} \left[ \rho_w y_1 + \frac{tq}{G^2 \rho_w} + p_1 K_1 + 2c (u - 1) K_{2,2} \right] K_{2,1} - p_1 K_{1,1} - \frac{tq}{G^2 \rho_w}
\]
\[
\Leftrightarrow (K_2 - K_{2,1}) y_2 \rho_w
< p_1 (K_1 K_{2,1} - K_{1,1} K_2) + 2c (u - 1) K_{2,2} K_{2,1} + \frac{tq}{G^2} (K_{2,1} - K_2)
\]
\[
\Leftrightarrow y_2 > p_1 (1 + \frac{t}{G^2 \rho_w}) + \frac{2c K_{2,1} K_{2,2}}{(\rho_1 - \rho_2) \rho_w} - \frac{tq}{G^2 \rho_w}
\]
Then, we check whether the condition \((p_2 > p_1 + c)\) is verified

\[
p_2 > p_1 + c
\]

\[
\Leftrightarrow 0 < \frac{1}{K_2} \left[ \rho_w y_2 + p_1 (K_1 - K_2) + \frac{tq}{G^2} \right.
\]

\[
+ c \left( (2u - 1) \left( \rho_0 + \rho_w + \frac{t}{G^2} \right) + u \rho_1 - (1 - u) \rho_2 \right) - K_2 \left. \right]
\]

\[
\Leftrightarrow y_2 > p_1 \left( 1 + \frac{t}{\rho_w G^2} \right) - \frac{tq}{\rho_w G^2} + \frac{2c(1 - u)K_2}{\rho_w}
\]

Since it is clear that \(\frac{2cK_2,2}{(\rho_1 - \rho_2)\rho_w} > \frac{2c(1-u)K_2,2}{\rho_w}\), the condition \((p_2 > p_1 + c)\) is automatically verified.

Trading volume is the total buy order of uninformed traders

\[
V = (1 - u) \left[ \rho_w y_2 + \frac{tq}{G^2} + p_1 (\rho_0 + \rho_2) - (p_2 + c) \left( \rho_0 + \frac{t}{G^2} + \rho_2 + \rho_w \right) \right]
\]

\[
= (1 - u) \left[ \rho_w y_2 + \frac{tq}{G^2} + p_1 K_{1,2}
\]

\[
- \frac{1}{K_2} \left( \rho_w y_2 + \frac{tq}{G^2} + p_1 K_1 + 2cuK_{2,1} \right) K_2,2 \right]
\]

\[
= \frac{(1 - u)}{K_2} \left[ \left( \rho_w y_2 + \frac{tq}{G^2} \right) (K_2 - K_{2,2})
\]

\[
- p_1 \left( \rho_w + \frac{t}{G^2} \right) u (\rho_1 - \rho_2) - 2cuK_{2,1}K_{2,2} \right]
\]

\[
= \frac{(1 - u)u}{K_2} \left[ \left( \rho_w y_2 + \frac{tq}{G^2} - p_1 \left( \rho_w + \frac{t}{G^2} \right) \right) (\rho_1 - \rho_2) - 2cK_{2,1}K_{2,2} \right]
\]

\[
\]

**Proof.** of Proposition 2: In the case of \(y_2 > y_u^P\), the partial derivative of
trading volume \((V)\) with respect to the precision \((\rho_w)\) is

\[
\frac{\partial V}{\partial \rho_w} = \frac{u(1-u)}{(K_2)^2} \left\{ \left( \rho_1 - \rho_2 \right) \left[ (y_2 - y_1) K_2 - \left( \rho_w y_2 + \frac{t q}{G^2} - p_1 (\rho_w + \frac{t}{G^2}) \right) \right] \\
-2c \left\{ (K_{2,1} + K_{2,2}) K_2 - K_{2,1} K_{2,2} \right\} \right\}

\[
= \frac{u(1-u)}{(K_2)^2} \left\{ \left( \rho_1 - \rho_2 \right) \left[ (y_2 - y_1) (K_2 - \rho_w) + \frac{t q}{G^2} (p_1 - q) \right] \\
-2c \left( u(K_{2,1})^2 + (1-u) (K_{2,2})^2 \right) \right\}

\[
> \frac{u(1-u)}{(K_2)^2} \left\{ \left( \rho_1 - \rho_2 \right) \left( \frac{p_1 t}{G^2 \rho_w} + \frac{2c K_{2,1} K_{2,2}}{\rho_w} - \frac{t q}{G^2 \rho_w} \right) (K_2 - \rho_w) \\
+ \frac{t}{G^2} (p_1 - q) (\rho_1 - \rho_2) - 2c \left[ u(K_{2,1})^2 + (1-u) (K_{2,2})^2 \right] \right\}

\[
= \frac{u(1-u)}{(K_2)^2} \left\{ \left( \rho_1 - \rho_2 \right) \left( \frac{t}{G^2 \rho_w} (p_1 - q) K_2 \right) \\
+ \frac{2c K_{2,1} K_{2,2}}{\rho_w} \left( K_1 + \frac{t}{G^2} \right) - 2c \left[ u(K_{2,1})^2 + (1-u) (K_{2,2})^2 \right] \right\}
\]

In the last inequity, the fact that \(y_2\) should be superior to \(y_{up}^{\text{up}}\) is used. We can prove the case of \(y_2 < y_{down}^{\text{down}}\) in the same way. ■

### 5.3 Appendix B: Public Information and Differential Interpretation

**Lemma 5** \(u = \alpha + \beta y\) and \(y \sim N(\theta, 1/\rho)\), then

\[
\int_{-\infty}^{0} |u| f(u) du = \frac{\beta}{\sqrt{\rho}} \Phi \left( \frac{\sqrt{\rho}}{\beta} (\alpha + \beta \theta) \right) - (\alpha + \beta \theta) \Phi \left( -\frac{\sqrt{\rho}}{\beta} (\alpha + \beta \theta) \right)
\]

\[
\int_{0}^{+\infty} u f(u) du = \frac{\beta}{\sqrt{\rho}} \Phi \left( \frac{\sqrt{\rho}}{\beta} (\alpha + \beta \theta) \right) + (\alpha + \beta \theta) \Phi \left( \frac{\sqrt{\rho}}{\beta} (\alpha + \beta \theta) \right)
\]

**Proof.** of lemma 5: \(u\) is a function of \(y\), we can calculate the integral of \(u\)
without knowing its density function

\[
\int_{-\infty}^{0} |u| f(u) du = \int_{-\infty}^{\frac{\pi}{2}} |u(y)| f(y) dy \\
= - \int_{-\infty}^{\frac{\pi}{2}} u(y) f(y) dy \\
= - \int_{-\infty}^{\frac{\pi}{2}} (\alpha + \beta y) \frac{1}{\sqrt{2\pi}} e^{-\frac{(y-\theta)^2}{2\rho}} dy \\
= - \int_{-\infty}^{\frac{\pi}{2}} (\alpha + \beta \theta + \beta (y-\theta)) \frac{\sqrt{\rho}}{\sqrt{2\pi}} \left(-\frac{1}{\beta}(\sqrt{\pi(y-\theta)})^2\right) dy \\
= - (\alpha + \beta \theta) \Phi \left(-\frac{\sqrt{\rho}(\alpha + \beta \theta)}{\beta}\right) \\
- \int_{-\infty}^{\frac{\pi}{2}} \beta (y-\theta) e^{-\frac{(\sqrt{\pi(y-\theta)})^2}{2\rho}} d\sqrt{\rho} (y-\theta) \\
= - (\alpha + \beta \theta) \Phi \left(-\frac{\sqrt{\rho}(\alpha + \beta \theta)}{\beta}\right) + \frac{\beta}{\sqrt{\rho}} \phi \left(-\frac{\sqrt{\rho}(\alpha + \beta \theta)}{\beta}\right)
\]

The second formula can be proven in the same way. ■

**Proof.** of Proposition 3: The conditional expectation and conditional variance at date 2 are

\[
E \left( \psi \mid \mathcal{F}_{2,i} \right) = \frac{K_{1,i} \psi_{1,i} + \rho y_2^i + \frac{t_q}{\sqrt{\pi}}}{K_{1,i} + \rho + \frac{t_q}{\sqrt{\pi}}} \\
V \left( \psi \mid \mathcal{F}_{2,i} \right) = \frac{1}{K_{1,i} + \rho + \frac{t_q}{\sqrt{\pi}}}
\]

Substituting from the demand functions the conditional expectation and conditional variance gives

\[
d_{2,i}^j = \begin{cases} 
K_{1,i} (\psi_{1,i} - p_2 - c) + \rho (y_2^i - p_2 - c) & \text{for buyer at date 2} \\
+ \frac{t_q}{\sqrt{\pi}} (q - p_2 - c) \\
K_{1,i} (\psi_{1,i} - p_2 + c) + \rho (y_2^i - p_2 + c) & \text{for seller at date 2} \\
+ \frac{t_q}{\sqrt{\pi}} (q - p_2 + c)
\end{cases}
\]

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Thus the demand for the risky asset $d_{2,i}(y^{j}_{2}, y^{j}_{1}, i)$ is the maximum (minimum) of $d_{2,i}^j$ and $d_{1,i}^j$ for buyer (seller) in the period 2. The order of each trader is

$$d_{2,i}^j = \begin{cases} \rho \left( y^j_2 - p_2 - c \right) - K_{1,i} (p_2 - p_1 + c) & \text{for buyer at date 2} \\ + \frac{t}{G_2} (q - p_2 - c) & \\ \rho \left( y^j_2 - p_2 + c \right) - K_{1,i} (p_2 - p_1 - c) & \text{for seller at date 2} \\ + \frac{t}{G_2} (q - p_2 + c) \end{cases}$$

To calculate the equilibrium price, we introduce a function

$$f(y^j_2, y^j_1, i) = \begin{cases} +1 & \text{if } y^j_2 > y^{ap}_i \\ \frac{\rho y^j_2 + p_1 K_{1,i} + \frac{t}{G_2} - (K_{1,i} + \rho + \frac{t}{G_2})p_2}{c(K_{1,i} + \rho + \frac{t}{G_2})} & \text{if others} \\ -1 & \text{if } y^j_2 < y^{down}_i \end{cases}$$

where

$$y^{ap}_i = \frac{(K_{1,i} + \rho + \frac{t}{G_2}) (p_2 + c) - p_1 K_{1,i} - \frac{t}{G_2}}{\rho}$$

$$y^{down}_i = \frac{(K_{1,i} + \rho + \frac{t}{G_2}) (p_2 - c) - p_1 K_{1,i} - \frac{t}{G_2}}{\rho}$$

Then the demand of each trader can be expressed in the following way

$$d_{2,i}(y^j_2, y^j_1, i) = K_{1,i}p_1 + \rho y^j_2 + \frac{tq}{G_2} - \left( K_{1,i} + \rho + \frac{t}{G_2} \right) \left( p_2 + f(y^j_2, y^j_1, i) c \right)$$
The total demand of the market is
\[
\sum_{j=1}^{N} d_2(y^j_2, y^j_1) = \sum_{j=1}^{uN} d_{2,1}(y^j_2, y^j_1, 1) + \sum_{j=uN+1}^{N} d_{2,2}(y^j_2, y^j_1, 2)
\]
\[
= \sum_{j=1}^{uN} \left( K_{1,1}p_1 + \rho y^j_2 + \frac{tq}{G^2} - K_{2,1} \left( p_2 + f(y^j_2, y^j_1) \right) \right) 
+ \sum_{j=uN+1}^{N} \left( K_{1,2}p_1 + \rho y^j_2 + \frac{tq}{G^2} - K_{2,2} \left( p_2 + f(y^j_2, y^j_1) \right) \right)
\]

By the Strong Law of Large Numbers as \( uN \) and \( (1 - u)N \to \infty \), the total demand of the market can be rewritten as
\[
\sum_{j=1}^{N} d_2(y^j_2, y^j_1) = uN K_{1,1}p_1 + (1 - u) N K_{1,2}p_1 + \frac{tq}{G^2} N 
- N \left( uK_{1,1} + (1 - u) K_{1,2} + \rho + \frac{tq}{G^2} \right) p_2 
+ N \rho \theta_2 - NuK_{2,1}E \left[ f(y^j_2, y^j_1) \right] c 
- N (1 - u) K_{2,2}E \left[ f(y^j_2, y^j_1) \right] c
\]

By the market clearing condition in the second period, the equilibrium price converges almost surely to
\[
p_2 = \frac{1}{K_2} \left[ uK_{1,1}p_1 + (1 - u) K_{1,2}p_1 + \rho \theta + \frac{tq}{G^2} 
- \left( uK_{2,1}E \left[ f(y^j_2, y^j_1) \right] + (1 - u) K_{2,2}E \left[ f(y^j_2, y^j_1) \right] \right) c \right]
\]
where

\[
E \left[ f(y^2_j, y^1_i) \right] =
\int_{y^1_i}^{y^2_i} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} \left( x - \theta \right)^2} \, dx
\]

Substituting these two expectations gives the coefficients \( A, B, C, D \) and \( E \) and the equilibrium price follows after several arrangements.
The transaction of each trader in the second period is

\[
\Delta_{2,i}^{j}(y_{2}^{j},y_{1}^{i}) = \begin{cases} 
K_{1,i} (\psi_{1,i} - p_{2} - c) + \rho (y_{2}^{j} - p_{2} - c) & \text{for buyer} \\
0 & \text{for others} \\
K_{1,i} (\psi_{1,i} - p_{2} + c) + \rho (y_{2}^{j} - p_{2} + c) & \text{for seller} \\
-\frac{\rho}{\sigma_{x}} (q - p_{2} - c) - d_{1}(y_{1}^{i}) & \text{for others} \\
-\frac{\rho}{\sigma_{x}} (q - p_{2} + c) - d_{1}(y_{1}^{i}) & \text{for buyer} \\
\rho y_{2}^{j} - (K_{1,\bar{i}} + \rho + \frac{1}{\sigma_{x}}) (p_{2} + c) + K_{1,\bar{i}} p_{1} + \frac{t_{u}}{\sigma_{x}} i f \ Y_{i}^{j} > y_{i}^{up} \\
0 & \text{if others} \\
\rho y_{2}^{j} - (K_{1,\bar{i}} + \rho + \frac{1}{\sigma_{x}}) (p_{2} - c) + K_{1,\bar{i}} p_{1} + \frac{t_{u}}{\sigma_{x}} i f \ Y_{i}^{j} < y_{i}^{down} 
\end{cases}
\]

Then the per capita trading volume is

\[
V = \frac{1}{2N} \left( \sum_{j=1}^{uN} \Delta_{2,1}^{j}(y_{2}^{j},y_{1,1}^{i}) + \sum_{j=1+uN}^{N} \Delta_{2,2}^{j}(y_{2}^{j},y_{1,2}^{i}) \right) \\
= \frac{1}{2} \left( u E \left( \Delta_{2,1}^{j}(y_{2}^{j},y_{1,1}^{i}) \right) + \left( 1 - u \right) E \left( \Delta_{2,2}^{j}(y_{2}^{j},y_{1,2}^{i}) \right) \right) \\
= \frac{1}{2} \left( u V^{1} + \left( 1 - u \right) V^{2} \right)
\]

where

\[
V^{i} = \int_{-\infty}^{y_{i}^{down}} \left| \Delta_{2,1}^{j}(x_{1}^{i}) \right| f_{i}(x_{1}^{i}) dx_{1}^{i} + \int_{y_{i}^{up}}^{\infty} \left| \Delta_{2,1}^{j}(x_{1}^{i}) \right| f_{i}(x_{1}^{i}) dx_{1}^{i}
\]
Using the lemma 5 with
\[ y = \alpha + \beta y_2 \]
\[ \alpha = (K_{1,i} + \rho) \left( p_2 \pm c + \frac{t}{G^2} \right) - K_{1,i} p_1 - \frac{t q}{G^2} \]
\[ \beta = \rho \]
\[ y_2 \sim N(\theta, (n)^{-1}) \]

we have trading volume expressed in the proposition 3. ■

**Proof.** of Proposition 4: Differentiating trading volume expressed in the Proposition 3 with respect to \( \rho_w \), yields
\[
\frac{\partial V}{\partial \rho_w} = \frac{u}{2} \frac{\partial V_1}{\partial \rho_w} + \frac{1-u}{2} \frac{\partial V_2}{\partial \rho_w}
\]

where
\[
\frac{\partial V_i}{\partial \rho_w} = -\frac{\partial (\rho \sqrt{n} \rho) \Phi (-m_i)}{\partial \rho_w} \Phi (n_i) + \frac{\partial (\rho \sqrt{n} \rho)}{\partial \rho_w} \Phi (n_i) \]
\[
+ \frac{\partial (n \rho \sqrt{n} \rho)}{\partial \rho_w} (\phi (m_i) + \phi (n_i)) \]
\[
= -\left( K_{2,i} \frac{\partial p_2}{\partial \rho_w} + \frac{\partial p}{\partial \rho_w} (p_2 - \theta) \right) (1 - \Phi (m_i) - \Phi (n_i)) \]
\[
+ \frac{\partial (\rho \sqrt{n} \rho)}{\partial \rho_w} (\phi (m_i) + \phi (n_i)) - c (1 - \Phi (m_i) + \Phi (n_i)) \frac{\partial p}{\partial \rho_w} \]

Before calculating \( H_i = K_{2,i} \frac{\partial p_2}{\partial \rho_w} + \frac{\partial p}{\partial \rho_w} (p_2 - \theta) \), we calculate \( \frac{\partial F}{\partial \rho_w} = \frac{\partial A}{\partial \rho_w} p_2 - \)
\[
\frac{\partial B}{\partial \rho_w} \rho_w + \frac{\partial C}{\partial \rho_w} - \frac{\partial D}{\partial \rho_w} \rho_w + \frac{\partial E}{\partial \rho_w} p_1 \text{ where }
\]

\[
+ \frac{\partial A}{\partial \rho_w} p_2 = -u \frac{K_{1,1}}{\rho_w} \left( \phi (m_1) \frac{\partial m_1}{\partial \rho_w} - \phi (n_1) \frac{\partial n_1}{\partial \rho_w} \right) p_2
\]

\[
- (1 - u) \frac{K_{1,2}}{\rho_w} \left( \phi (m_2) \frac{\partial m_2}{\partial \rho_w} - \phi (n_2) \frac{\partial n_2}{\partial \rho_w} \right) p_2
\]

\[
+ \frac{\partial K_{2,1}}{\partial \rho_w} u (1 - \Phi (m_1) + \Phi (n_1)) p_2
\]

\[
+ \frac{\partial K_{2,2}}{\partial \rho_w} (1 - u) (1 - \Phi (m_2) + \Phi (n_2)) p_2
\]

\[
- \frac{\partial B}{\partial \rho_w} \left( \rho \theta + \frac{t q}{G^2} \right) = u \left( \phi (m_1) \frac{\partial m_1}{\partial \rho_w} - \phi (n_1) \frac{\partial n_1}{\partial \rho_w} \right) \left( \rho \theta + \frac{t q}{G^2} \right)
\]

\[
+ (1 - u) \left( \phi (m_2) \frac{\partial m_2}{\partial \rho_w} - \phi (n_2) \frac{\partial n_2}{\partial \rho_w} \right) \left( \rho \theta + \frac{t q}{G^2} \right)
\]

\[
- \frac{\partial D}{\partial \rho_w} u (1 - \Phi (m_1) + \Phi (n_1)) \theta
\]

\[
- \frac{\partial D}{\partial \rho_w} (1 - u) (1 - \Phi (m_2) + \Phi (n_2)) \theta
\]

\[
+ \frac{\partial C}{\partial \rho_w} c = -u \frac{K_{1,1}}{\rho_w} \left( \phi (m_1) \frac{\partial m_1}{\partial \rho_w} + \phi (n_1) \frac{\partial n_1}{\partial \rho_w} \right) c
\]

\[
- (1 - u) \frac{K_{1,2}}{\rho_w} \left( \phi (m_2) \frac{\partial m_2}{\partial \rho_w} + \phi (n_2) \frac{\partial n_2}{\partial \rho_w} \right) c
\]

\[
+ \frac{\partial K_{2,1}}{\partial \rho_w} u (1 - \Phi (m_1) - \Phi (n_1)) c
\]

\[
+ \frac{\partial K_{2,2}}{\partial \rho_w} (1 - u) (1 - \Phi (m_2) + \Phi (n_2)) c
\]

\[
- \frac{\partial E}{\partial \rho_w} p_1 = u \frac{K_{1,3}}{\rho_w} \left( \phi (m_1) \frac{\partial m_1}{\partial \rho_w} - \phi (n_1) \frac{\partial n_1}{\partial \rho_w} \right) p_1
\]

\[
+ (1 - u) \frac{K_{1,2}}{\rho_w} \left( \phi (m_2) \frac{\partial m_2}{\partial \rho_w} - \phi (n_2) \frac{\partial n_2}{\partial \rho_w} \right) p_1
\]

\[
- \frac{\partial D}{\partial \rho_w} \rho \sqrt{n} \left( \phi (m_1) \frac{\partial m_1}{\partial \rho_w} - \phi (n_1) \frac{\partial n_1}{\partial \rho_w} \right)
\]

\[
- (1 - u) \frac{\rho}{\sqrt{n}} \left( \phi (m_2) \frac{\partial m_2}{\partial \rho_w} - \phi (n_2) \frac{\partial n_2}{\partial \rho_w} \right)
\]

\[
- \frac{\partial E}{\partial \rho_w} \rho \sqrt{n} \left( \phi (m_1) - \phi (n_1) \right)
\]

\[
- (1 - u) \frac{\partial}{\partial \rho_w} \left( \rho \sqrt{n} \right) \left( \phi (m_2) - \phi (n_2) \right)
\]

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Thus we have

\[
\frac{\partial F}{\partial \rho_w} = \frac{\partial \rho}{\partial \rho_w} [u(1 - \Phi(m_1) + \Phi(n_1)) + (1 - u)(1 - \Phi(m_2) + \Phi(n_2))] p_2 - \frac{\partial \rho}{\partial \rho_w} [u(1 - \Phi(m_1) + \Phi(n_1)) + (1 - u)(1 - \Phi(m_2) + \Phi(n_2))] \theta \\
+ \frac{\partial \rho}{\partial \rho_w} [u(1 - \Phi(m_1) - \Phi(n_1)) + (1 - u)(1 - \Phi(m_2) - \Phi(n_2))] c \\
- \frac{\partial \left( \frac{\rho}{\sqrt{n}} \right)}{\partial \rho_w} (u(\phi(m_1) - \phi(n_1)) + (1 - u)(\phi(m_2) - \phi(n_2)))
\]

For \( i = 1 \), we have

\[
H_1 = -\frac{K_{2,1}}{A} \frac{\partial F}{\partial \rho_w} + \frac{\partial \rho}{\partial \rho_w} (p_2 - \theta) \\
= -\frac{1}{A} (1 - u)(p_2 - \theta)(1 - \Phi(m_2) + \Phi(n_2)) (\rho_1 - \rho_2) \frac{\partial \rho}{\partial \rho_w} \\
- \frac{K_{2,1}}{A} \frac{\partial \rho}{\partial \rho_w} [u(1 - \Phi(m_1) - \Phi(n_1)) + (1 - u)(1 - \Phi(m_2) - \Phi(n_2))] c \\
+ \frac{K_{2,1}}{A} \frac{\partial \left( \frac{\rho}{\sqrt{n}} \right)}{\partial \rho_w} [u(\phi(m_1) - \phi(n_1)) + (1 - u)(\phi(m_2) - \phi(n_2))]
\]

and

\[
\frac{\partial V^1}{\partial \rho_w} = -H_1 (1 - \Phi(m_1) + \Phi(n_1)) + \frac{\partial \left( \frac{\rho}{\sqrt{n}} \right)}{\partial \rho_w} (\phi(m_1) + \phi(n_1)) \\
- c (1 - \Phi(m_1) + \Phi(n_1)) \frac{\partial \rho}{\partial \rho_w}
\]
For $i = 2$, we have

\begin{align*}
H_2 &= - \frac{K_{2,2}}{A} \frac{\partial F}{\partial \rho_w} + \frac{\partial \rho}{\partial \rho_w} (p_2 - \theta) \\
&= - \frac{1}{A} [u (p_2 - \theta) (1 - \Phi (m_1) + \Phi (n_1)) (\rho_2 - \rho_1) \frac{\partial \rho}{\partial \rho_w} \\
&\quad - \frac{K_{2,2}}{A} \frac{\partial \rho}{\partial \rho_w} [u (1 - \Phi (m_1) - \Phi (n_1)) + (1 - u) (1 - \Phi (m_2) - \Phi (n_2))] c \\
&\quad + \frac{K_{2,2}}{A} \partial \left( \frac{\phi}{\sqrt{n}} \right) [u (\phi (m_1) - \phi (n_1)) + (1 - u) (\phi (m_2) - \phi (n_2))] \\
\end{align*}

and

\begin{align*}
\frac{\partial V^2}{\partial \rho_w} &= - H_2 (1 - \Phi (m_2) - \Phi (n_2)) + \frac{\partial \left( \frac{\phi}{\sqrt{n}} \right)}{\partial \rho_w} (\phi (m_2) + \phi (n_2)) \\
&\quad - c (1 - \Phi (m_2) + \Phi (n_2)) \frac{\partial \rho}{\partial \rho_w} \\
\end{align*}

The partial derivative of the total trading volume with respect to the precision can thus be written as

\begin{align*}
\frac{\partial V}{\partial \rho_w} &= u \frac{\partial V^1}{\partial \rho_w} + \frac{(1 - u)}{2} \frac{\partial V^2}{\partial \rho_w} \\
&= - \frac{u}{2} H_1 (1 - \Phi (m_1) - \Phi (n_1)) - \frac{1 - u}{2} H_2 (1 - \Phi (m_2) - \Phi (n_2)) \\
&\quad + \frac{u}{2} \frac{\partial \left( \frac{\phi}{\sqrt{n}} \right)}{\partial \rho_w} (\phi (m_1) + \phi (n_1)) + \frac{1 - u}{2} \frac{\partial \left( \frac{\phi}{\sqrt{n}} \right)}{\partial \rho_w} (\phi (m_2) + \phi (n_2)) \\
&\quad - \frac{u}{2} c (1 - \Phi (m_1) + \Phi (n_1)) \frac{\partial \rho}{\partial \rho_w} - \frac{1 - u}{2} c (1 - \Phi (m_2) + \Phi (n_2)) \frac{\partial \rho}{\partial \rho_w} \\
&= DV_1 \frac{\partial \left( \frac{\phi}{\sqrt{n}} \right)}{\partial \rho_w} + DV_2 \cdot \frac{u (1 - u)}{2A} \frac{\partial \rho}{\partial \rho_w} + DV_3 \cdot \frac{c}{2A} \frac{\partial \rho}{\partial \rho_w} \\
\end{align*}
where

\[ DV_1 = - [K_{2,1} (\phi (m_2) - \phi (n_2)) (1 - \Phi (m_1) - \Phi (n_1))] u (1 - u) \]
\[ - [K_{2,2} (\phi (m_1) - \phi (n_1)) (1 - \Phi (m_2) - \Phi (n_2))] u (1 - u) \]
\[ - [K_{2,1} (\phi (m_1) - \phi (n_1)) (1 - \Phi (m_1) - \Phi (n_1))] u^2 \]
\[ - [K_{2,2} (\phi (m_2) - \phi (n_2)) (1 - \Phi (m_2) - \Phi (n_2))] (1 - u)^2 \]
\[ + [u (\phi (m_1) + \phi (n_1)) + (1 - u) (\phi (m_2) + \phi (n_2))] A \]
\[ = 2u (1 - u) K_{2,1} [\phi (m_2) \Phi (n_1) + \phi (n_2) (1 - \Phi (m_1))] \]
\[ + 2u (1 - u) K_{2,2} [\phi (m_1) \Phi (n_2) + \phi (n_1) (1 - \Phi (m_2))] \]
\[ + 2K_{2,1} u^2 [\phi (m_1) \Phi (n_1) + \phi (n_1) (1 - \Phi (m_1))] \]
\[ + 2K_{2,2} (1 - u)^2 [\phi (m_2) \Phi (n_2) + \phi (n_2) (1 - \Phi (m_2))] \]

\[ DV_2 = (p_2 - \theta) (p_1 - p_2) [(1 - \Phi (m_1) - \Phi (n_1)) (1 - \Phi (m_2) + \Phi (n_2)) \]
\[ - (1 - \Phi (m_1) + \Phi (n_1)) (1 - \Phi (m_2) - \Phi (n_2))] \]
\[ = 2 (p_2 - \theta) (p_1 - p_2) [\Phi (n_2) (1 - \Phi (m_1)) - \Phi (n_1) (1 - \Phi (m_2))] \]

\[ DV_3 = K_{2,1} u^2 (1 - \Phi (m_1) - \Phi (n_1))^2 \]
\[ + K_{2,2} (1 - u)^2 (1 - \Phi (m_2) - \Phi (n_2))^2 \]
\[ + (K_{2,1} + K_{2,2}) u (1 - u) (1 - \Phi (m_2) - \Phi (n_2)) (1 - \Phi (m_1) - \Phi (n_1)) \]
\[ - [u (1 - \Phi (m_1) + \Phi (n_1)) + (1 - u) (1 - \Phi (m_2) + \Phi (n_2))] A \]
\[ = -2u^2 K_{2,1} (1 - \Phi (m_1)) \Phi (m_1) \]
\[ - 2 (1 - u)^2 K_{2,2} (1 - \Phi (m_2)) \Phi (n_2) \]
\[ - 2u (1 - u) (K_{2,1} + K_{2,2}) [(1 - \Phi (m_2)) \Phi (n_1) + (1 - \Phi (m_1)) \Phi (n_2)] \]
References


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