Migration and concentration risks in bank lending: new evidence from credit portfolio data

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Abstract

In this paper, we analyze migration and concentration risks by means of individual borrower data from a commercial lending portfolio of a German universal bank during the period 2001-2004. With respect to migration risk, we find that default is non-absorbing, rating stability does not decline monotonously by rating grades, and alternative ways of calculating migration matrices lead to very different outcomes. Analyzing concentration risk, we find that the evolution of concentration on single names differs across measures whereas concentration on industries has consistently increased. Moreover, the credit portfolio gets more similar to benchmark portfolios during the sampling period. Finally, stress tests reveal that the bank's credit value at risk is very sensitive to the rating distribution, leading to +/-50% changes of its current economic capital.

EFM classification codes: 510, 450, 370; JEL classification: G11, G21

Keywords: credit risk, credit ratings, migration matrix; concentration, stress tests

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1. Introduction

During the last decade, banks has increasingly focused on the analysis and management of their credit portfolios. This development has been fostered by the emergence of credit portfolio models and its commercial applications in banking practice as well as by the discussion and the final decision on a new regulatory capital framework for banks (Basel II, see Basel Committee on Banking Supervision (2005)). Banks need to assess the credit risk of their prospects and actual borrowers by means of internal rating systems not only to make correct credit approvals but also for pricing, monitoring, loan loss provisioning, and credit risk transfer purposes. It has become evident that credit risk has not only to be assessed and managed at the single borrower-level but at the portfolio level as well.

In this paper, we analyze in detail two particular aspects of credit risk: (i) migration risk (or transition risk), i.e. the risk that a borrower that is included in a credit portfolio will change its creditworthiness within a given time horizon, and (ii) concentration risk, i.e. the risk of the overall portfolio is caused by a relatively small number of portfolio constituents (lump risk). While the literature on migration risks is predominantly based on external ratings (agency ratings), there is still relatively little evidence on the empirical behavior of bank internal ratings so far. Unfortunately, results for agency ratings are not directly transferable to internal ratings because both differ in several ways (see, for example, Löffler (2004)). Some exceptions are, for example, the studies of bank internal rating systems of Treacy and Carey (2000), Crouhy et al. (2001), Araten et al. (2004), and Gloy et al. (2004) for the US, Machauer and Weber (1998), Weber et al. (1999) and Grunert et al. (2005) for Germany, and Jacobson et al (2003) for Sweden. More recently, Trück and Rachev (2005) propose a new concept of measuring differences in migration matrices and Mählmann (2005) underlines that the time period between rating changes has to be taken into account when calculating migration matrices. Jafry and Schuermann (2004) provide an overview of estimations and comparison issues related with migration matrices.

To our knowledge, the empirical literature on the analysis of concentration risks in actual bank portfolios is very scarce so far. Theoretically, the importance of this issue has been emphasized by Winton (1999). First empirical evidence on industry and geographical diversification of loan portfolios has been provided by Acharya et al. (2004) using data from Italian banks. For Germany, Pfingsten and Rudolph (2004) and Kamp et al. (2005) measure the distance between banking sectors/individual banks and the overall German credit market to study the evolution of credit portfolio diversification. Recently, Heitfield et al. (2005) examine the impact of systematic and idiosyncratic risk on credit portfolio losses for US banks in a simulation study while Bonti et al (2005) propose new methods for measuring and stressing risk concentrations in credit portfolios. Furthermore, note that measuring concentration is not only relevant in primary lending but also of major importance for the securitization of loans. Typically, investors that face a noticeable degree of concentration in a loan pool will require additional credit enhancements that make the securitization transaction more costly for the originator.

We think that, in a first step, research should explain how to identify and measure concentration in general and then, in a second step, deal with measuring and managing diversification, including the big issue of assessing cross-sectional and serial correlations in various characteristics of portfolio constituents. Note that considering concentration risk is explicitly required under the future Basel II capital adequacy framework in pillar 2 and the supervisors are allowed to respond with an increase of capital charges to abnormally high credit risk concentrations or adverse results of stress tests. More important, given that many large banks already rely on internal credit portfolio models to determine their economic capital and that the recognition of these models to determine regulatory capital requirements may be included in future regulatory capital adequacy frameworks, it is obvious that there is a need for empirical research on migration and, even more important, on concentration risk in bank lending. This paper represents an attempt in that direction.

The remainder of the paper is organized as follows. Section 2 briefly describes the dataset. In Section 3, we provide a summary of related empirical studies on migration risk, present our research questions before reporting the main results. In section 4, we first explain basic concepts of measuring relative and absolute concentration before applying these concepts to actual data. Section 5 concludes.

2. Description of the dataset

The data covers the entire commercial lending portfolio of a German universal bank, which requests to stay anonymous, over the period 2001-2004. Borrowers are predominantly small and medium-sized firms. For each borrower we observe the gross loan volume (LOAN), the amount of collateral (COLLAT), the amount of specific loan loss provisions (PROV), the net credit exposure (NET), the amount of economic capital calculated from a borrower's contribution to the overall portfolio credit value at risk (CVAR), and the bank's internal rating grade (RATING). The variable NET is calculated as LOAN minus COLLAT minus PROV. Note that for the CVAR calculations, we gained access to the internal credit portfolio model used by the bank. All variables are as of December 31 of each year in the sampling period. In total, we consider 24,409 rating observations in our analysis. The portfolio analyzed here represents 41-53% of total assets of the bank. The bank is among the largest 5% by total assets in the category of comparable banks, as defined by the Deutsche Bundesbank.

The bank's internal rating system consists of six grades with grade 1 being the highest creditworthiness. Note that this system is currently modified towards a higher number of grades to meet the minimum Basel II requirements (i.e. eight rating grades) for the IRB approach in the future. Rating grade 5 includes borrowers for which the bank has established a specific loan loss provision while borrowers in grade 6 have declared bankruptcy. Consistent with the Basel II default definition, the bank considers borrowers which have been assigned a grade of 5 or 6 as being in default. Figure 1, panel A, displays the number of borrowers, the gross loan volume, and the net credit exposure by years. Figure 1, panel B, shows the distribution of internal rating grades over the sampling period.

Insert Figure 1 here

3. Empirical analysis of migration risk

3.1. Migration matrices and research questions

Sound practice in modern banking as well as banking supervisors require that banks do not only assign credit ratings at the time of credit approval but that they track (and potentially adjust) each borrower's creditworthiness at least annually to ensure a timely assessment of credit risks in their credit portfolios. The results of such a re-rating can be documented with a migration (or transition) matrix. The latter may be interpreted as an estimate of the probability for a particular rating change from grade x in t to grade y in t+1. The diagonal of a migration matrix displays the relative frequency of borrowers that will not experience a rating change. Borrowers in cells above the main diagonal have been downgraded, while those in cells below the main diagonal have been upgraded. The right-most columns indicate the relative frequency of borrowers that enter the default state directly. In addition, a column for withdrawn ratings indicates the percentages of firms that are not assigned a rating anymore (mainly firms that are no longer borrowers of a bank).[1]

In practice, banks rely on one-year migration matrices (see Bangia et al. (2002), Hanson and Schuermann (2004)) that refer to the year-end rating distribution. In addition, banks calculate borrower-weighted averages of one-year matrices to obtain more robust migration measures (see Berthault et al. (2000), Mah and Verde (2004)). As the maturity of loans usually exceeds the horizon of one year, bankers are interested in the mid- and long-term migration behavior in their credit portfolios as well. To obtain multi-year migration matrices, there a two basic approaches: (i) historical matrices, (ii) multiplying one-year matrices. In the case of historical matrices, the migration behavior of a cohort that is defined at the starting date is, similar to a life-cycle analysis, empirically tracked. The advantage of this approach is that, with the exception of independence between annual rating changes, no further assumption is needed. The exponentiating approach requires additional assumptions. The migration behavior has to follow a Markov process with fixed, time-invariant transition probabilities, generating stationary one-year matrices. Accordingly, the n-year migration matrix is simply obtained by exponentiating the one-year matrix n times. The column default displays the cumulative default rate over the considered time horizon (see Brady (2004)). A major disadvantage of this approach is its high sensitivity to errors in the one-year matrix which may be due to sampling or estimation techniques. Exponentiating aggravates the initial errors severely and can generate considerable problems. Finally, note that assuming the Markov property for rating changes and stationarity for transition probabilities is not consistent with empirical evidence on industry and macro-economic cycle-

[1] See Bangia et al. (2002), Cantor and Hu (2003), Araten et al. (2004), Mah and Verde (2004) for alternative ways how to deal with withdrawn ratings.

effects (see, for example, Altman and Kao (1992), Weber et al. (1999), Nickell et al. (2000), Kavvathas (2001), Lando and Skodeberg (2002), Fabozzi et al. (2004)).

Subsequently, we intend to provide evidence for bank internal ratings on the following questions that relate to migration risk in credit portfolios: (i) Can we reproduce typical empirical properties of migration matrices?, (ii) What is the impact of alternative approaches of calculating migration matrices?, (iii) What are the characteristics of rating changes by direction, magnitude and volatility? and (iv) How does the credit portfolio respond to stress scenarios?

3.2. Results

(i) Empirical properties of migration matrices

In a first step, we examine whether the calculated migration matrices exhibit the typical empirical properties. If this is not the case they might be uninformative, i.e. the bank cannot track the migration risk of its credit portfolio correctly which, in turn, might provoke wrong portfolio management decisions. According to the empirical literature on rating systems, the following properties are regarded as typical for migration matrices: 1. the probability of default increases by rating grades, 2. most of the probability mass is located on the main diagonal of the migration matrix, i.e. the probability that the rating in t remains the same in t+1 is higher than the probability of a rating change, 3. borrowers in high grades exhibit a higher probability of maintaining their rating than borrowers in lower grades, i.e. rating stability decreases by rating grades, 4. the more one departs from the main diagonal the lower the probabilities, i.e. the stronger the rating change the less likely it is, 5. the likelihood of a rating improvement is higher for low grade borrowers than for high grade borrowers (ceiling effect).

Table 1 displays the one-year migration matrix for 2004 (panel A), the three-year migration matrix for 2001-2004 (panel B) and the weighted average one-year migration matrix (panel C).[2] The last column of the one- and three-year matrices indicates the number of borrowers included in each rating class at the beginning of a period. The column “default” includes all firms that have been assigned a rating 5 or 6, i.e. are considered by the bank as being in default. It can be seen that grades 1-4 of the rating system display all typical properties mentioned above except the decreasing rating stability by grades. The latter is only observed for the one-year migration matrix for 2002. All other one-year, two-year, and three-year migration matrices exhibit a higher rating stability for grades 2 and 3 in comparison to grade 1. Interestingly, this finding is in line with evidence from Jacobsen et al. (2003) for Sweden and Araten et al. (2004) for the United States.

Insert Table 1 here

With regard to monotony, we detect inconsistencies in rating grade 5. For example, the probability of moving from grade 5 to grade 1 was 0.80% higher than for a move to grade 2 in 2003. However, it may be possible that this result is due to the small number of observations in the considered grades. The migration behavior in grades 5 and 6 can be characterized as follows: On average, 15% of the borrowers with a rating 5 leave the default state and raise to better rating grades within one year. Note that this percentage even amounts to 29.1% for the three-year-horizon. This finding is clear counter-evidence for the widely used assumption in migration

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[2] Note that the calculation of migration matrices is based on observations from all borrowers in the credit portfolio. In particular, we also include firms with a net credit exposure of zero. On the one hand, one might think that these observations should be omitted because the bank will not incur a loss if these firms default. On the other hand, note the bank still may face considerable costs if these borrowers default (depreciation of collateral, cost for the realization of collateral etc.).

matrix modeling that default is as an absorbing state (see, for example, Bangia et al. (2002) or Jarrow et al. (1997)). The reason for the fact that default represents a non-absorbing state in our case is the definition of default itself. It can be seen that most of the probability mass for upgrades from the default state come from rating grade 5 (i.e. a suspension of a specific loan loss provision) while the probability for upgrades from grade 6 to non-default states is only 0.40%. In comparison to the weighted average migration matrix for 2001-2004 the one-year matrix for 2004 exhibits lower default and migration probabilities, indicating a decreasing number of defaults and higher degree of rating stability. Furthermore, note that no borrower directly migrates from grade 1 to 5 or 6 during the sampling period. Accordingly, the actual default rate of grade 1 is 0.00%. However, this finding is somehow limited due to the fact that borrowers from grade 1 disproportionately often withdrew their ratings in comparison to borrowers from grades 2-4 from the same bank. It may be possible that very healthy firms more frequently switch between banks to obtain optimal financing conditions.

Moreover, comparing the calculated migration matrices for the internal rating system with those from rating agencies leads to the following results. First, the fraction of withdrawn ratings is higher which is consistent with Araten et al. (2004). Second, the default rates are higher for internal ratings than for external ratings which is the opposite of Machauer and Weber (1998). Moreover, the stability of internal ratings is not lower than the stability of external ratings as suggested by Machauer and Weber (1998). Finally, note that a bank's internal definition of default (which is usually met more frequently and some time before an actual bankruptcy) may lead to higher default rates and to higher transition probabilities from default to non-default states than migration matrices based on external ratings.

(ii) Comparing alternative approaches of calculating migration matrices

Subsequently, we compare different approaches to calculate migration matrices. Table 2, panel A reports the three-year migration matrix that is obtained by multiplying the one-year matrix of 2002.

Insert Table 2 here

This matrix exhibits a higher rating stability for grades 1, 2, and 4 than the empirically observed three-year migration matrix. The obtained default rates are relatively similar for grades 1, 2, and 6 with the observed empirical ones, they are higher in grade 3 and below the empirical ones in grades 4. From a credit risk management perspective, these deviations can be judged as problematic because they may induce too high or too low calculations for credit spreads in loan rates which can be a disadvantage in comparison to competing banks. Moreover, considering the method of exponentiating a one-year matrix to obtain a multi-year matrix multiplies errors or inconsistencies that exist in the one-year matrix. In our case, the one-year matrix of 2002 has an inconsistency with respect to monotony in grade 5. This problem is aggravated in the three-year matrix where the probability of moving from grade 5 to grade 3 amounts to 29.95% which is three times higher than moving to the adjacent grade 4. Note that this inconsistency disappears in the empirically observed three-year matrix.

Table 2, panel B shows the average one-year migration matrix for the years 2001-2004 which has been calculated as the arithmetic mean of the one-year migration matrices. Comparing this matrix with the one displayed in Table 1 does not reveal important differences. Default rates for grades 1-3 are identical and for grade 4 there is only a small difference of 0.12%. Note that the same is true for the migration probabilities of grades 1-4. Hence, taking the arithmetic mean generates a result that is very similar to the weighting approach. This is mainly due to the relatively constant number of borrowers in each grade and year which impedes a major deviation of the arithmetic mean matrix from the weighted average matrix.

Finally, we consider two additional approaches to obtain migration matrices. More specifically, migration probabilities can be calculated on the basis of volumes (LOAN or NET) rather than names. Table 2, panel C reports the weighted average one-year migration matrix based on LOAN (see Appendix I for additional tables). A comparison with the previously calculated matrices that are based on the number of borrowers reveals that default rates are considerably higher in matrices based on NET and LOAN. For example, the default rate of grade 3 in the matrix based on LOAN is more than three times higher than in the one based on the number of borrowers. In addition, on the one hand these alternative matrices exhibit a lower rating stability. On the other hand, they display a monotonous rating stability by grades and lower percentages of withdrawn ratings than the borrower-based matrix. In summary, the volume-based matrices display more typical properties of migration matrices than the borrower-based matrix which is in contrast to banking practice where we can observe a wide-spread use of the latter type of matrices.

(iii) Direction, magnitude and volatility of rating changes

Table 3 summarizes the frequency and magnitude of rating changes in the analyzed credit portfolio. It can be seen that, on average, 12.79% of all borrowers experienced at least one rating change per year. Additionally, in every year more borrowers were downgraded than upgraded. Accordingly, the ratio of downgrades to upgrades is above one in all years. For example, in 2003 the ratio amounts to 1.99 (weighted by the magnitude of rating changes: 2.59). In 2004, the ratio declines to 1.59. Interestingly, this finding is consistent with the ratio of down- and upgrades of agency rating changes in 2002 and 2003.[3] The highest ratio for agency ratings is observed in 2002, i.e. one year before the maximum of the ratio for internal rating changes. This may be evidence for the hypothesis that external rating changes, which mainly refer to relatively large companies, lead internal rating changes in the course of time. The economic rationale could be that large firms may experience changes in the macroeconomic conditions earlier than small firms.

Insert Table 3 here

Moreover, we examine the rating volatility. We define the latter as fraction of borrowers that experience a one-notch rating change.[4] In the period 2002-2004 this is true for every fifth to seventh borrower, while there is no clear trend of an increase or decrease of the rating volatility.

The rating drift which is an indicator for the direction of all rating changes exhibits a negative sign in each of the years. This is evidence for a gradual deterioration of the credit portfolio over the sampling period. Table 4 summarizes the volatility and magnitude of rating changes.

Insert Table 4 here

Note that in more than 75% of all rating changes we observe a one-notch change, i.e. a migration to an adjacent rating grade. Changes by more than two notches occur in 6.82% of all cases. Considering all borrowers (with and without rating changes) in 2004, the average

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[3] See Mah and Verde (2004), Cantor et al. (2005).

[4] See Cantor and Hu (2003).

magnitude of rating changes was 0.14 notches, and the mean rating drift per borrower is -0.04. Consistent with prior findings, the lowest value for the rating drift per borrower (-0.09) is observed in 2003. The mean magnitude of rating changes slightly decreases from 1.31 notches in 2002 to 1.24 notches in 2004. The mean magnitude of rating downgrades is higher than the one of rating upgrades in each year. However, the difference between both measures declines because the mean magnitude of downgrades has dropped from 1.42 to 1.31. Table 5 revisits the rating stability issue from another perspective. We report the percentage of borrowers in grades 1-4 respectively that do not leave their initial rating grade within one year.

Insert Table 5 here

The percentages range between 85.46% in 2003 and 89.34% in 2004. Interestingly, the stability in rating grades 1-3 is relatively high while it is much lower in grade 4. Accordingly, grade 4 can be considered as a rather volatile category that includes borrowers whose creditworthiness is not very stable and near to default ("borderline risks"). In summary, there is evidence for a decline of the average credit portfolio quality which is indicated by a negative rating drift and a downgrade-upgrade ratio that lays considerably above one. However, while 2003 marks the worst year, we detect a consistent slowdown of the deterioration during 2004.

(iv) Stress testing

In the remainder of this section, we perform three types of stress tests to study the impact of sudden changes in different risk parameters on the portfolio credit value at risk (CVAR). We define the CVAR as difference between a loss that will not be exceeded with a probability of 99% and the expected loss of the portfolio within a time horizon of one year. The CVAR corresponds to the economic capital the banks holds as a buffer against unexpected losses. Input parameters for the calculation of CVAR are PD, NET, and the asset correlation between borrower industry classifications.[5] We take the actual credit portfolio as of December 31, 2004 as a base case.

First, we assume a shock in default rates. More specifically, we replace the calibration PDs by different empirical or assumed PDs. Table 6, panel A displays the results for the corresponding CVAR-levels.

Insert Table 6 here

The CVAR mostly increases if we include the empirical PDs for 2003. The main reason for this finding is that the empirical PDs of that year are higher (or equal) than those in other years or the average values.

Second, we consider an increase of default rates and a decrease of the value of collateral. While default rates are raised by 20%, 40%, ... , 200% of their original value, the value of collateral is simultaneously varied from 100% discretely down to 80%, 60%, ..., 0% of the actual value. This is equivalent to increasing the net credit exposure (NET) gradually to the value of the gross loan volume per borrower (after potential specific loan loss provision). Note that we assume a PD of 100% for grades 5 and 6 in all scenarios. Figure 2 presents the change of the CVAR as a function of the value of collateral while keeping the default rates constant.

Insert Figure 2 here

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[5] The bank's correlation matrix is based on default statistics from the Federal Statistic Bureau of Germany for the period 1989-1998.

Surprisingly, the CVAR does not increase monotonously if default rates increase and the value of collateral decreases. Instead, we observe a kink. The CVAR increases if the default rates are raised by more than 40% before it declines if the value of collateral is depreciated by more than 30%. For higher depreciations of the value of collateral the CVAR-function starts growing again. Similar findings are observed for variations of the value of collateral, i.e. for depreciations above 20% the CVAR declines. What are the reasons for this finding? The explanation for the partial decrease in the CVAR, although the risk parameters increase, is the disproportionate rise of the expected loss. The latter is calculated as the product of PD and NET. If both risk parameters increase considerably, the expected loss rises strongly as well, making unexpected losses less likely. Accordingly, the CVAR partially declines if default rates increase and the value of collateral decreases.

Third, we analyze the impact of an adverse change of the rating distribution of the credit portfolio within a time horizon of two years. Supposed that 15% of all borrowers are subject to a rating change in each year of the sampling period. The rating volatility is 17.9% in the first and 19.2 % in the second year. The ratio of downgrades and upgrades rises from 2 in the first year to 3.3 in the second year, the rating drift from –5% down to –10%. The value of collateral is assumed to remain constant or to decrease by 10% and 20%. Table 6, panel B summarizes the results for this stress scenario. The CVAR increases by 16.9% in the first year due to the assumed rating drift of – 5% and by additional 8.4% due to the assumed rating drift –10% in the second year. It rises even more if the value of collateral declines. The extreme case, i.e. a depreciation of collateral by 20%, causes a rise of the CVAR in the second year by 53.3%.

Comparing the CVAR values for the base case and the stress scenario, we find that the assumed rating drift of –5% corresponds to an increase of default rates by 60% in a one-year scenario. In the two-year stress scenario, the drift combination of –5% and –10% produces the same effect as a rise of the default rates by 80%. In other words: the assumed decline of the credit portfolio quality corresponds to a sharp increase of the portfolio loss rate from 1.44% to 2.6%. A comparison of the latter value with the actual portfolio loss rate of 2003 (2.96%) shows that the stress scenario is an absolutely realistic one. Hence, the bank may face an increase in economic capital of more than 50% in times of significant macroeconomic downturns.

4. Empirical analysis of concentration risk

4.1. Alternative measures of concentration risk and research questions

The statistical analysis of concentration distinguishes between two basic concepts. First, relative concentration describes what share of, for example, the item total gross loan volume is attributed to what share of borrowers. If a large share of the item total is attributed to a small share of borrowers, we observe high degree of concentration or disparity. Second, absolute concentration measures what share of the total gross loan volume is observed at how many borrowers. In the remainder, we present various approaches how to measure both types of concentration in credit portfolios (for a detailed overview see Piesch (1975)).

One of the most well-known methods to describe the degree of relative concentration graphically is the Lorenz curve. In a corresponding diagram, the x-axis displays the cumulative relative frequencies of all borrowers (starting at the left with borrowers with the smallest gross loan volume), while the y-axis displays the according cumulative share of the total gross loan volume. The degree of relative concentration can be measured by the ratio of the area between the diagonal and the Lorenz curve (F) and the sum of the latter and the area under the Lorenz curve (F+L). This metric is known as the Gini coefficient (GC):

$$GC = \frac{F}{F + L} \tag{1}$$

With a data list sorted by the gross loan volume, the Gini coefficient can be also calculated as follows:

$$GC = \frac{2\sum_{i=1}^{n} i x_i}{n\sum_{i=1}^{n} x_i} - \frac{n+1}{n} \qquad (2)$$

with i: rank of the borrower i the volume-sorted data list, x_i: gross loan volume of borrower i.

The minimum of the Gini coefficient is zero (indicating no concentration or an equal distribution) and the maximum is (n-1)/n. These properties show that the value of the Gini coefficient depends on the number of observations. This problem can easily be solved by transforming the Gini coefficient into the standardized Lorenz-Münzner coefficient in the case of small samples. However, this procedure is not necessary in this paper because the analyzed credit portfolio, including more than 5,000 borrowers per year, can be considered as a relatively large sample.

A simple way to measure absolute concentration is the concentration rate C_m, which indicates what share of the total gross loan volume is attributed to the m largest borrowers. It is calculated as:

$$C_m = \sum_{i=n-m+1}^{n} p_i \qquad (3)$$

with p_i: i-th borrower's share of the total gross loan volume.

A graphical representation of absolute concentration is the concentration curve. The difference between the latter and the Lorenz curve is that instead of relative frequencies the absolute frequency (number of borrowers) is displayed on the x-axis. The graph is monotonously increasing and concave. The closer the curve to the diagonal, the more equal the distribution of the gross loan volume. Alternatively stated, the higher the absolute concentration, the smaller the area above the concentration curve.

Another measure based on this concept is the Rosenbluth index (R). This index increases as concentration increases and is defined for values between 1/n and 1. For an inversely sorted data list, starting with the largest borrowers, it is calculated as follows:

$$R = \frac{1}{2I} = \frac{1}{2\left(\sum_{i=1}^{n} i p_i\right) - 1} \tag{4}$$

Finally, a widely used measure for absolute concentration is the Herfindahl index (H), which is calculated as the sum of squared share of total gross loan volume for borrower i (p_i). The inverse Herfindahl index can be interpreted as the effective number of borrowers in a credit portfolio. It is calculated as follows:

$$H = \sum_{i=1}^{n} p_i^{\,2} \tag{5}$$

The Herfindahl index can take values between 1/n and 1 and decreases in the number of borrowers and in the homogeneity of the individual gross loan portfolios.

On the one hand, some of these previously presented measures depend on the number of borrowers and the portfolio size which both can vary over time. On the other hand, they are compact because the degree of concentration is condensed to one single number. However, the interpretation is difficult concerning information on different credit portfolios. For example, the Gini coefficient may take the same value for two credit portfolios that exhibit very different compositions. Hence, graphical approaches convey more information than index measures. The advantage of the concentration rate is that it is very simple and easy to interpret but it suffers from the more or less arbitrary need to choose a value for m. The Herfindahl index reveals information about all borrowers but it shows the disadvantage of being relatively invariant to an addition or omission of many borrowers with small loan amounts. Accordingly, the weighting of large and small exposures differs considerably. Furthermore, if the number of observations in the portfolio changes, the lower bound of the Herfindahl and Rosenbluth index change as well which may restrict comparability. In summary, for concentration analysis we advocate to rely on both graphical and index measures as well.

In the next section, we demonstrate how concentration measures can be used to answer the following empirical questions: (i) What is the degree of concentration on single names and industries in the credit portfolio?, (ii) How similar is the credit portfolio to benchmark portfolios in terms of industry composition?, (iii) How strong is the concentration on names within industries?, (iv) What is the rank correlation between different risk parameters like LOAN, NET and CVAR?, (v) How does the credit portfolio respond to sudden changes in the degree of concentration?

4.2. Results

(i) Concentration on single names

The Lorenz curves for the gross loan volume (LOAN) and the credit value at risk (CVAR) for 2004 are displayed in Figures 3, panel A and B. Table 7 reports concentration rates, the Gini coefficient, the Rosenbluth index, and the Herfindahl index for 2001 and 2004.

Insert Figure 3 here

The strong curvature of the Lorenz curves indicates a relative high degree of concentration. For example, 85% of the total gross loan volume in 2004 is attributed to the 20% largest borrowers. Even more pronounced, 20% of the borrowers account for 95% of the CVAR.

This first impression of a relatively high degree of concentration is confirmed by the Gini coefficient displayed in Table 7. Taking values of above 0.8, it lays at the upper end of the defined range. Moreover, it can be noticed that the degree of concentration based on LOAN and NET (CVAR) has decreased (increased) over the period 2001-2004.

Insert Table 7 here

The concentration rates shown in Table 7 indicate that the absolute concentration in LOAN and NET slightly decreases in the course of time. Only the measure C_{1000} increases for LOAN but decreases for NET. The decline is very pronounced for the concentration rates for the 3, 5, 10 and 25 largest borrowers. While the three largest borrowers in 2001 account for 6.15% (12.8%) of the total gross loan volume (total net credit exposure), these rates drop to 3.22% (5.21%) in 2004. This finding may be interpreted in the sense that the bank has reduced its lending business with the largest borrowers. However, analyzing concentration based on CVAR changes the picture. While 33.33% of the total CVAR is attributed to the 25 largest borrowers in 2001, this share amounts to 47.18% in 2004. Interestingly, the concentration rates of NET and CVAR for the 3 to 25 largest borrowers are very similar in 2001 but then they diverge until 2004. Although the bank has clearly reduced the concentration in LOAN and NET, it has to face an increase in the absolute concentration in CVAR. We admit that it cannot be ruled out that this finding is due to an inadequate calculation of the CVAR because the latter is based on the relatively old German industry classification WZ 93. Combined with non-recent time series of default statistics, this may adversely affect the industry correlation matrix which is an important input for the CVAR calculation.

In addition, Table 7 displays the Rosenbluth and Herfindahl index for 2001 and 2004. Since the potential minimum of both indices is very small and not constant over time, we report the ratio of the empirically observed index and its theoretical minimum. These ratios confirm the previous results that the absolute concentration is decreasing in LOAN and NET but increasing in CVAR. The Rosenbluth index for NET and the Herfindahl index for LOAN and NET slightly decrease over the sampling period, while the Rosenbluth index for LOAN rises marginally. In addition, the ratio of the indices to their minimum decrease for LOAN and NET. The opposite is observed for CVAR, i.e. the indices and the ratios have risen over time, indicating an increase in absolute concentration.

Comparing the index ratios for NET and CVAR in 2001 reveals that they are very close to each other. The ratios for the Herfindahl index in 2001 amount to 44.83 for CVAR and 46.15 for NET while they take values of 65.91 for CVAR and 16.75 for NET in 2004. This finding shows that the reduced concentration in LOAN and NET was accompanied with an increased concentration of unexpected losses.

For a more accurate comparison of concentration measures at different points in time, we divide the sorted data list of borrowers in groups which include 10% of all borrowers each. We then display the share of LOAN and CVAR of each decile in Figure 4 on a logarithmic scale.

Insert Figure 4 here

The decrease of relative concentration in LOAN is mainly due to a rising share of the 40% smallest borrowers. In contrast, for the CVAR, the 90% smallest borrowers have lost some of their initial share to the 10% largest borrowers.

Figure 5 displays the concentration curves to illustrate the overall degree of concentration for LOAN (panel A) and CVAR (panel B). As the curve is far away from the diagonal in both figures we can conclude that the absolute concentration is relatively high.

Insert Figure 5 here

In summary, we find empirical evidence for a rather high degree of relative and absolute concentration in the credit portfolio. During the sampling period, the concentration has decreased in LOAN and NET but increased in CVAR.

(ii) Concentration on industries

Before turning to the analysis of industry concentration, we compare the industry composition of the bank's credit portfolio with two benchmark cases. The first benchmark is the entire credit market portfolio in Germany, the second is the average credit portfolio of a matching group. This may help interpreting the results from the concentration analysis. Following Pfingsten and Rudolph (2004) and Kamp et al. (2005), we base this preliminary analysis on the gross loan volume and transform the 30 industry categories into the following 8 industry groups: 1. Agriculture, forestry and fishing, 2. Energy, water and mining, 3. Manufacturing, 4. Construction works, 5. Commerce, 6. Transportation and media, 7. Financial services and insurance, 8. Other services. For these industry groups we calculate three distance measures (D_1: standardized sum of absolute difference, D_2: mean relative difference, D_3: mean squared relative difference). While D_2 and D_3 represent relative measures, D_1 is an absolute measure that is standardized to the interval [0;1]. Hence, D_1 will be more affected by the large industries than the other measures.

Table 8 summarizes the calculated distance measures between the bank's credit portfolio and 1. the entire German credit market and 2. the average credit portfolio of comparable banks (banks that are classified in the same category by the Deutsche Bundesbank). Interestingly, the difference to the market portfolio measured by D_1 decreases continuously during the sampling period while the trend for D_2 and D_3 is not clear.

Insert Table 8 here

The measures increase from 2001 to 2004 but they vary a lot over time. For example, D_2 rises by 38% and D_3 by 61% in 2003. Note that the difference between the market portfolio and the average portfolios of the matching group increases monotonously during the sampling period. Not surprisingly, the bank's portfolio is subject to stronger fluctuations than the mean portfolio of the matching group. For example, the industry groups construction and other services exhibit a higher share and change in a different manner than in the mean portfolio of the matching group.

Interestingly, the measures D_2 and D_3 for the considered bank are very close to those for the matching group in 2003 and 2004 while they are below this value in 2001 and 2002. Equally, D_1 has steadily become closer to the one of the matching group over the sampling period. These findings lead to the conclusion that the bank's credit portfolio has become increasingly similar to the portfolio of the matching group, indicated by reduced distance of 34% to 44% from 2001 to 2004. The distance measure D_1 offers an interesting economic interpretation. It represents the share of the credit portfolio that has to be changed to obtain the market (matching group) portfolio structure . While this value is 20.6% (14.2%) in 2001, it is reduced to 18.1% (9.0%) in 2004. In summary, the distance of the bank's credit portfolio to both benchmarks has declined during the period 2001-2004. It remains an open question whether this observation is due to an explicit credit portfolio strategy of the bank management or whether other factors drive this finding.

In a next step, we analyze the industry concentration within the credit portfolio. At this stage, we take into account the diversity of the credit portfolio as much as possible by considering all 30 industries separately. Figure 6 displays the Lorenz curves based on LOAN (panel A) and CVAR (panel B) for the year 2004. Table 9 reports additional concentration measures for 2001 and 2004.

Insert Figure 6 here

The relative concentration on industries is smaller than on borrowers (compare with Figure 3). The Lorenz curves based on industries are closer to the diagonal than those based on borrowers and the Gini coefficients are, assuming values between 0.61 and 0.72, smaller than before. However, we find that the Gini coefficients for LOAN, NET and CVAR rise consistently.

Insert Table 9 here

In opposite to the concentration on single names the absolute concentration on industries has consistently risen during the sampling period. The effect is particularly strong if we calculate the measures based on CVAR. For example, the three (five) largest industries comprise 43.7% (58.03%) of the total CVAR in 2001, whereas the corresponding value is 61.11% (70.82%) in 2004. In addition, the analysis of concentration curves as well as the Rosenbluth and Herfindahl indices clearly indicate a lower absolute concentration on industries than on single names.

Furthermore, a look at the index ratios confirms the previous finding that the absolute concentration has increased more strongly for CVAR than for LOAN and NET. For example, the ratio for the Rosenbluth index (Herfindahl index) has risen from 2001 to 2004 by 70.7% (41.8%) while the same measures increase for LOAN by 7% (9,6%) and for NET by 2.3% (1.4%). Figure 7 illustrates for LOAN (panel A) and CVAR (panel B) why the absolute concentration has increased. We display the share of each industry of total gross loan value and total credit value at risk in 2001 and 2004, starting with the largest industry.

Insert Figure 7 here

The increase in absolute concentration for LOAN is mainly due to higher shares of the six largest industries in 2004 compared to 2001. For CVAR, the increase in concentration is caused by a larger share of the two largest industries while all other industries exhibit a smaller share. In summary, the relative and absolute concentration on industries has increased for LOAN, NET, and CVAR during the period 2001-2004. The increase is highest in CVAR compared to the other variables. A potential explanation may be that the bank's credit portfolio has become more similar to the one of comparable banks, accepting a higher degree of concentration for the sake of comparability or benchmarking.

(iii) Concentration within industry

Previously, we have found that the concentration of the bank's credit portfolio has increased. Consequently, the bank's dependence on the evolution within the largest industries has risen as well. For this reason, we examine the degree of concentration within the largest industries subsequently. The two largest industries in each year of the sampling period and in terms of LOAN, NET, and CVAR are "construction" and "wholesale/retail". Consistent with the result for the overall credit portfolio, the concentration measures based on LOAN and NET for "wholesale/retail" has decreased while the concentration for CVAR has increased. In the case of "construction", we observe an increase in all three concentration measures which is particularly strong for CVAR. The Rosenbluth index roughly doubled and the Herfindahl index even tripled.

Examining different concentration rates, we find that the increase is especially pronounced for the three to five largest borrowers within this industry. Note that in 2004 more than 50% of the CVAR in the sub-portfolio "construction" is attributed to the five largest borrowers within this industry. Obviously, the high degree of absolute concentration in the overall credit portfolio is due to high concentration within industries.

(iv) Correlation between measures of concentration

To quantify the degree of association between the different portfolio parameters, we sort the data list and calculate pair-wise rank correlations. Table 10 reports Spearman's rank correlation coefficients calculated at the borrower and industry level.

Insert Table 10 here

It turns out that the correlation is significantly positive in all cases. At the borrower level, the rank correlation between LOAN and NET is 0.3866 in 2004, exceeding the one between LOAN and CVAR which amounts to 0.2710. Note that the correlation between NET and CVAR is considerably higher, assuming a mean of 0.7579. Moreover, it can be seen that the rank correlation at the borrower level has decreased for all variable combinations during the sampling period. At the industry level, all rank correlation coefficients are slightly below 1, indicating a much stronger degree of ordinal association than at the borrower level. The rank correlation between LOAN and CVAR is 0.9355 in 2004 which falls short of the one between LOAN and NET (0.9631). In contrast to the borrower level analysis, the rank correlation between NET and CVAR (0.9008) is lower than the other previously mentioned values. In summary, we find that rank correlations of risk parameters for industries are considerably higher than those for single names.

(v) Stress testing

Similar to section 3.2, we carry out different stress tests to study the sensitivity of the credit portfolio to changes in its concentration. Remind that stress tests for portfolio concentration are explicitly required under the future Basel II capital requirements for banks (see Basel Committee on Banking Supervision (2004), paragraph 775).

First, we change the distribution of NET for the rating grades 1-4 to an equal distribution while the shares of grades 5 and 6 are left unchanged. Using the calibration PDs, the CVAR drops by 55% to 5,428,125 EUR while the decline is even more pronounced (almost 100%) if we consider the empirical PDs for 2004. Hence, that bank can save more than 50% of its economic capital if the portfolio is changed in the described manner. Note that this saving can also be interpreted as the opportunity cost of maintaining a relatively non-granular credit portfolio.

Second, we investigate the impact on relative and absolute concentration measured by NET if the value of collateral declines by 10% to 100%. Table 11 reports the impact on various concentration measures of this stress test. The row CVAR indicates the absolute CVAR in EUR, based on empirical PDs from 2004 while that last row displays the percentage change in comparison to the base case.

Insert Table 11 here

Overall findings are mixed. The Gini coefficient indicates a partial decrease in the degree of relative concentration if the value of collateral declines, reaching its minimum at a depreciation rate of 40%. Then, it slightly rises again but ranges still below its initial value. The Rosenbluth index continuously decreases down to 60% of its initial value if the collateral is depreciated. Accordingly, the degree of absolute concentration in NET is reduced because NET has been increased if the value of collateral has been decreased. Conversely, the Herfindahl index increases by 300%, signaling a dramatic increase in absolute concentration if the value of collateral declines. Hence, the degree of relative and absolute concentration in the credit portfolio is considerably affected by variations of the value of collateral which, in turn, changes NET.

Third, we consider the 130 largest borrowers by NET in the credit portfolio and vary their default rates and the value of collateral simultaneously. The results are shown in Figure 8.

Insert Figure 8 here

In contrast to findings from section 3.2, the calculated CVAR is continuously increasing if the defaults rates of the 130 largest borrowers have been risen. At the limit, i.e. a complete loss of the collateral and tripling the default rates, the CVAR rises by 130.7%. Interestingly, a moderate reduction of the value of collateral by 20% has a stronger impact on the CVAR than a moderate increase of the default rates. The CVAR rises by 14.5% in the first case which can also be obtained by doubling the default rates. On the one hand, this may indicate that the largest borrowers post relatively more collateral than smaller ones which results in an disproportionately high increase in NET if the value of the collateral is reduced. On the other hand, it may be more likely to observe large borrowers in relative good rating grades, resulting in a small impact of an increase of default rates on CVAR. In summary, we like to emphasize that variation of portfolio risk parameters may have a very different impact on the CVAR of a credit portfolio.

5. Conclusions

In this study, we provide new evidence on migration and concentration risks in bank lending based on actual data. More specifically, we analyze the entire commercial lending portfolio of a German universal bank during the period 2001-2004, including more than 20,000 borrower-year observations.

With respect to migration risk, we obtain the following results. First, examining various migration matrices, we find that default is not an absorbing state which is frequently assumed in credit portfolio models. In each of the years during the sampling period, a considerable fraction of borrowers recover from default to the non-default state which is, among other factors, a consequence of a rigorous application of the default definition. Second, we do not consistently observe a decreasing rating stability by rating grades. Third, alternative ways of calculating one-year and multi-year migration matrices reveal considerable differences and inconsistencies between the exponentiating approach and a consideration of actual empirical default rates. Accordingly, we recommend to rely on empirical default rates to obtain multi-year migration matrices rather than exponentiating a one-year matrix. Fourth, migration matrices differ if they are calculated for the number of borrowers or based on the gross loan volume or net credit exposure. Finally, the analysis of rating changes indicates that changes of agency ratings tend to lead changes of bank internal ratings.

With respect to concentration risk, we obtain the following results. First, the portfolio exhibits a relatively high degree of relative and absolute concentration on single names which is decreasing during the sampling period if the calculations are based on the gross loan volume and the net credit exposure but it is increasing for the credit value at risk contributions. Second, we detect a consistent increase in concentration on industries for all risk parameters on the one hand. On the other hand, comparing the bank's credit portfolio with the German credit market and with the portfolio of comparable banks, we find that the bank's portfolio is gradually becoming more similar to both benchmark portfolios in terms of its industry composition. Third, the analysis of concentration on single names within the largest industries uncovers a high degree of concentration on single borrowers in the sub-portfolios, indicating that large borrowers come from industries that exhibit a large share of the overall credit portfolio. Finally, stress tests reveal that a shift to an equal rating grade distribute could save the bank more than 50% of its economic capital while an extrapolation of the current rating drifts may cause an increase of 50% of its economic capital. In summary, we conclude that a concentration analysis of credit portfolio should rely on various concentration concepts (relative vs. absolute measures, graphs vs. indices). We believe that banks should carry out concentration analyses in their own interest on a regular basis for credit portfolio monitoring and management purposes and be able to report concentration risks to banking supervisory authorities on demand. Finally, note that the methodology of measuring credit risk concentration presented in this paper can easily be applied to an analysis of single borrower or industry profitability (or risk-adjusted performance measures) to obtain a picture of concentration that is richer in terms of credit risk and return.

Appendix I: Bank internal one-year and multi-year migration matrices

Panel A: One-year migration matrix (2002)

From/To	1	2	3	4	5	6	Default	W.R.	Obs.
1	90.93%	8.24%	0.82%	0.00%	0.00%	0.00%	0.00%	14.35%	425
2	0.69%	90.73%	7.04%	1.03%	0.11%	0.40%	0.52%	9.62%	1933
3	0.39%	6.32%	87.36%	3.49%	0.85%	1.59%	2.44%	14.43%	3014
4	0.00%	1.72%	14.59%	78.11%	2.79%	2.79%	5.58%	17.23%	563
5	0.00%	0.56%	15.64%	5.03%	66.48%	12.29%	78.77%	20.44%	225
6	0.00%	0.00%	0.00%	0.00%	0.70%	99.30%	100.00%	42.42%	195

Panel B: One-year migration matrix (2003)

From/To	1	2	3	4	5	6	Default	W.R.	Obs.
1	83.60%	13.88%	1.89%	0.63%	0.00%	0.00%	0.00%	19.95%	396
2	1.61%	86.61%	8.67%	1.94%	0.78%	0.39%	1.17%	9.27%	1984
3	0.32%	5.06%	88.30%	3.62%	1.53%	1.17%	2.69%	12.49%	2843
4	0.00%	2.37%	18.01%	65.17%	6.87%	7.58%	14.45%	17.42%	511
5	0.80%	0.00%	3.20%	4.80%	63.20%	28.00%	91.20%	22.84%	162
6	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%	100.00%	42.42%	231

Panel C: Two-year migration matrix (2001-2002)

From/To	1	2	3	4	5	6	Default	W.R.	Obs.
1	75.80%	20.38%	3.18%	0.64%	0.00%	0.00%	0.00%	26.12%	425
2	2.01%	79.62%	13.73%	2.51%	1.32%	0.82%	2.13%	17.49%	1933
3	0.57%	9.48%	80.13%	5.20%	1.83%	2.79%	4.63%	24.02%	3014
4	0.00%	4.94%	29.14%	52.84%	5.68%	7.41%	13.09%	28.06%	563
5	0.00%	0.74%	11.76%	11.03%	47.79%	28.68%	76.47%	39.56%	225
6	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%	100.00	56.92%	195

Appendix I: Bank internal one-year and multi-year migration matrices (continued)

Panel D: Two-year migration matrix (2003-2004)

From/To	1	2	3	4	5	6	Default	W.R.	Obs.
1	75.35%	20.14%	3.82%	0.69%	0.00%	0.00%	0.00%	27.27%	396
2	3.07%	79.08%	14.04%	2.41%	0.90%	0.48%	1.39%	16.38%	1984
3	0.66%	8.23%	81.78%	5.79%	1.02%	2.52%	3.54%	20.47%	2843
4	0.00%	4.75%	24.58%	53.07%	7.82%	9.78%	17.60%	29.94%	511
5	0.97%	0.00%	4.85%	13.59%	44.66%	35.92%	80.58%	36.42%	162
6	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%	100.00%	58.87%	231

Panel E: Weighted mean two-year migration matrix (2001-2004)

From/To	1	2	3	4	5	6	Default	W.R.
1	75.58%	20.27%	3.49%	0.66%	0.00%	0.00%	0.00%	26.67%
2	2.55%	79.35%	13.89%	2.46%	1.11%	0.65%	1.75%	19.63%
3	0.62%	8.86%	80.95%	5.49%	1.43%	2.66%	4.09%	22.30%
4	0.00%	4.85%	27.00%	52.95%	6.68%	8.52%	15.20%	28.96%
5	0.42%	0.42%	8.79%	12.13%	46.44%	31.80%	78.24%	38.24%
6	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%	100.00%	57.98%

Panel F: Weighted average of exponentiated two-year migration matrices (2001-2004)

From/To	1	2	3	4	5	6	Default
1	77.72%	18.32%	3.16%	0.69%	0.06%	0.06%	0.12%
2	2.46%	79.98%	13.49%	2.63%	0.66%	0.79%	1.45%
3	0.77%	9.30%	79.88%	5.73%	1.49%	2.83%	4.32%
4	0.10%	4.30%	25.36%	54.18%	7.67%	8.39%	16.06%
5	0.38%	0.95%	13.78%	10.19%	40.33%	34.37%	74.70%
6	0.00%	0.01%	0.11%	0.81%	0.76%	98.31%	99.07%

Panel G: Weighted average one-year migration matrix based on NET (2001-2004)

From/To	1	2	3	4	5	6	Default	W.R.
1	92.53%	6.23%	1.18%	0.06%	0.00%	0.00%	0.00%	6.49%
2	0.79%	82.55%	13.86%	1.81%	0.76%	0.24%	0.99%	8.29%
3	0.42%	10.06%	73.59%	9.11%	2.85%	3.96%	6.82%	6.29%
4	0.00%	1.03%	15.94%	66.02%	13.72%	3.28%	17.01%	7.58%
5	0.05%	0.01%	9.07%	14.30%	63.39%	13.18%	76.58%	4.18%
6	0.00%	0.00%	0.00%	0.03%	0.11%	99.86%	99.97%	19.06%

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Table 1: Empirical migration matrices (part I)

Panel A: One-year migration matrix (2004)

From/To	1	2	3	4	5	6	Default	W.R.	Borrower
1	89.33%	9.00%	1.33%	0.33%	0.00%	0.00%	0.00%	16.90%	361
2	1.79%	90.07%	6.56%	1.35%	0.11%	0.11%	0.22%	8.14%	1941
3	0.47%	4.08%	90.81%	3.26%	0.16%	1.22%	1.37%	10.63%	2850
4	0.00%	2.49%	13.02%	75.35%	7.48%	1.66%	9.14%	17.01%	435
5	0.00%	0.00%	3.57%	12.14%	58.57%	25.71%	84.29%	16.67%	168
6	0.00%	0.00%	0.00%	1.20%	0.60%	98.20%	98.80%	30.71%	241

Panel B: Three-year migration matrix (2001-2004)

From/To	1	2	3	4	5	6	Default	W.R.	Borrower
1	68.71%	25.85%	5.10%	0.34%	0.00%	0.00%	0.00%	30.82%	425
2	3.33%	73.25%	17.72%	3.73%	1.15%	0.81%	1.97%	23.80%	1933
3	0.68%	12.37%	75.27%	6.84%	1.60%	3.25%	4.85%	31.59%	3014
4	0.29%	6.38%	36.81%	41.16%	5.80%	9.57%	15.36%	38.72%	563
5	0.00%	1.82%	11.82%	15.45%	37.27%	33.64%	70.91%	51.11%	225
6	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%	100.00%	67.18%	195

Panel C: Weighted average one-year migration matrix (2001-2004)

From/To	1	2	3	4	5	6	Default	W.R.
1	88.07%	10.30%	1.33%	0.31%	0.00%	0.00%	0.00%	11.84%
2	1.37%	89.12%	7.43%	1.44%	0.34%	0.30%	0.64%	5.96%
3	0.39%	5.16%	88.82%	3.45%	0.84%	1.33%	2.17%	8.03%
4	0.00%	2.16%	15.29%	72.94%	5.52%	4.08%	9.61%	11.66%
5	0.23%	0.23%	8.33%	7.21%	63.06%	20.95%	84.01%	15.68%
6	0.00%	0.00%	0.00%	0.45%	0.45%	99.10%	99.55%	46.93%

Table 2: Empirical migration matrices (part II)

Panel A: Three-year migration matrix (2001-2004)

From/To	1	2	3	4	5	6	Default
1	75.36%	20.58%	3.55%	0.32%	0.05%	0.14%	0.19%
2	1.78%	76.10%	17.28%	2.87%	0.45%	1.53%	1.98%
3	1.05%	15.36%	69.51%	7.48%	1.81%	4.79%	6.60%
4	0.18%	6.13%	31.51%	49.29%	4.75%	8.13%	12.89%
5	0.17%	3.73%	29.95%	9.24%	30.20%	26.72%	56.92%
6	0.00%	0.02%	0.28%	0.09%	1.47%	98.13%	99.61%

Panel B: One-year migration matrix (arithmetic mean, 2001-2004)

From/To	1	2	3	4	5	6	Default	W.R.
1	87.95%	10.37%	1.35%	0.32%	0.00%	0.00%	0.00%	17.07%
2	1.36%	89.14%	7.42%	1.44%	0.33%	0.30%	0.64%	9.01%
3	0.39%	5.16%	88.83%	3.46%	0.85%	1.32%	2.17%	12.52%
4	0.00%	2.19%	15.21%	72.87%	5.71%	4.01%	9.73%	17.22%
5	0.27%	0.19%	7.47%	7.32%	62.75%	22.00%	84.75%	19.98%
6	0.00%	0.00%	0.00%	0.40%	0.43%	99.17%	99.60%	33.44%

Panel C: One-year weighted average migration matrix calculated on loan basis (2001-2004)

From/To	1	2	3	4	5	6	Default	W.R.
1	85.23%	10.08%	4.59%	0.10%	0.00%	0.00%	0.00%	6.13%
2	1.30%	83.54%	12.56%	1.57%	0.90%	0.14%	1.04%	5.87%
3	0.37%	9.12%	73.19%	9.46%	3.76%	4.10%	7.86%	5.38%
4	0.00%	1.79%	14.77%	68.10%	11.73%	3.61%	15.34%	6.24%
5	0.01%	0.00%	8.61%	4.11%	66.68%	20.59%	87.27%	2.99%
6	0.00%	0.00%	0.00%	0.11%	1.92%	97.97%	99.89%	10.09%

Table 3: Frequency and direction of rating changes (2002-2004)

Year	Concerned borrowers	Downgraded borrowers		Upgraded borrowers		Down-/Upgrade-Ratio		Rating volatility	Rating drift
		Share	Weighted share	Share	Weighted share	Total	Weighted		
2002	12.49%	7.01%	9.99%	5.48%	6.35%	1.28	1.57	16.34%	-3.63%
2003	14.70%	9.78%	13.98%	4.92%	5.39%	1.99	2.59	19.38%	-8.59%
2004	11.19%	6.87%	8.98%	4.32%	4.85%	1.59	1.85	13.84%	-4.13%
Average	12.79%	7.88%	10.97%	4.91%	5.54%	1.60	1.98	16.51%	-5.43%

Table 4: Magnitude of rating changes (2002-2004)

Mean magnitude of rating changes				Rating changes in percent of all rating grade changes			
	2002	2003	2004	Rating change of …	2002	2003	2004
Magnitude of all rating changes	1.31	1.32	1.24				
Magnitude of downgrades	1.42	1.43	1.31	1 grade	77.63%	76.06%	82.80%
Magnitude of upgrades	1.16	1.10	1.12	2 grades	14.91%	17.12%	11.13%
Rating change per borrower	0.16	0.19	0.14	3 grades	6.43%	5.79%	5.73%
Rating drift per borrower	-0.04	-0.09	-0.04	4 grades	1.02%	1.03%	0.34%

Table 5: Rating stability in grades 1 to 4 (2002-2004)

Rating grade \ year	2002	2003	2004
1	90.93%	83.60%	89.33%
2	90.73%	86.61%	90.07%
3	87.36%	88.30%	90.81%
4	78.11%	65.17%	75.35%
1-4	87.91%	85.46%	89.34%

Table 6: Stress test results

Panel A: Change of CVAR as a function of default rates

Default rates based on …	ΔCVAR
2002	11.89%
2003	42.34%
2004	base case
Mean of 2001-2004	19.44%
Assumptions at rating system introduction	-14.45%
Master scale	16.78%

Panel B: CVAR results for a two-way scenario stress test

CVAR (EUR, percentage change)		Scenario	
		First year	Second year
Loss of collateral by …	0%	16.94%	26.80%
	10%	32.72%	43.47%
	20%	51.88%	53.46%

Table 7: Concentration on single borrowers (2001, 2004)

	LOAN		NET		CVAR	
	2001	2004	2001	2004	2001	2004
GC	0.8294	0.8182	0.8527	0.8283	0.8877	0.9208
C_3	6.15%	3.22%	12.80%	5.21%	12.15%	13.51%
C_5	7.89%	4.73%	16.27%	7.58%	16.52%	20.72%
C_{10}	11.63%	7.88%	21.65%	12.11%	22.66%	31.68%
C_{25}	19.42%	15.73%	31.62%	22.23%	33.33%	47.18%
C_{50}	28.70%	25.62%	41.60%	33.26%	43.93%	57.97%
C_{100}	39.99%	37.87%	52.56%	46.38%	56.66%	69.32%
C_{250}	56.20%	54.98%	66.80%	64.28%	73.00%	82.64%
C_{500}	68.87%	68.28%	77.78%	76.64%	84.09%	90.26%
C_{1000}	81.63%	82.04%	88.29%	87.65%	92.34%	95.26%
R	0.000922	0.000927	0.001337	0.001221	0.001767	0.002674
(R/min)	5.86	5.5	6.79	5.82	8.91	12.63
H	0.003139	0.002138	0.009082	0.003515	0.008892	0.013951
(H/min)	19.95	12.68	46.15	16.75	44.83	65.91

Table 8: Distance measures between credit portfolios (2001-2004)

	Bank vs. German credit market			Bank vs. comparable banks			Comparable banks vs. German credit market		
	D_1	D_2	D_3	D_1	D_2	D_3	D_1	D_2	D_3
2001	0.2064	0.2638	0.1116	0.1422	0.2604	0.1457	0.1686	0.3160	0.1473
2002	0.1944	0.2349	0.0966	0.1298	0.2461	0.1271	0.1718	0.3204	0.1474
2003	0.1867	0.3250	0.1560	0.1013	0.1579	0.0743	0.1659	0.3235	0.1502
2004	0.1809	0.3228	0.1461	0.0895	0.1709	0.0818	0.1669	0.3271	0.1521
Mean	0.1921	0.2866	0.1275	0.1157	0.2088	0.1072	0.1683	0.3217	0.1492

Table 9: Concentration on industries (2001, 2004)

	LOAN		NET		CVAR	
	2001	2004	2001	2004	2001	2004
GC	0.6290	0.6536	0.6196	0.6536	0.6091	0.7242
C_3	45.37%	47.75%	41.35%	43.48%	43.70%	61.11%
C_5	61.50%	65.44%	57.34%	61.12%	58.03%	70.82%
C_{10}	82.37%	84.10%	82.05%	81.75%	80.99%	88.58%
R	0.089839	0.096229	0.087621	0.089519	0.085263	0.120856
(R/min)	2.70	2.89	2.63	2.69	2.63	4.49
H	0.094025	0.102877	0.090658	0.092053	0.087804	0.149829
(H/min)	2.82	3.09	2.72	2.76	2.56	3.63

Table 10: Spearman's rank correlation of LOAN, NET and CVAR (2001-2004)

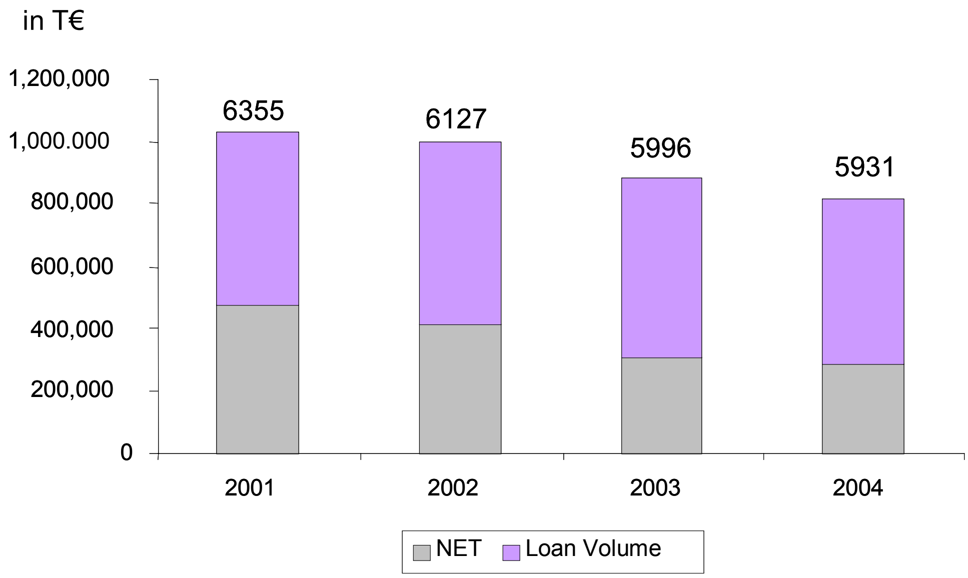
	Borrowers			Industries		
	ρ_B (LOAN, CVAR)	ρ_B(LOAN, NET)	ρ_B(NET, CVAR)	ρ_I(LOAN, CVAR)	ρ_I(LOAN, NET)	ρ_I(NET, CVAR)
2001	0.3119	0.4484	0.7712	0.8945	0.9760	0.8696
2002	0.2690	0.3955	0.7650	0.9017	0.9724	0.8941
2003	0.2767	0.3988	0.7442	0.9075	0.9502	0.8696
2004	0.2710	0.3866	0.7510	0.9355	0.9631	0.9008
Mean	0.2822	0.4073	0.7579	0.9654	0.9654	0.8822

Table 11: Concentration measures based on NET by loss of collateral

Loss	0%	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
GC	0.8283	0.8245	0.8137	0.8096	0.8084	0.8086	0.8096	0.8109	0.8123	0.8139	0.8154
R	0.001221	0.000961	0.000788	0.000682	0.000608	0.000552	0.000508	0.000471	0.000440	0.000413	0.000390
(R/min)	5.82	5.70	4.67	4.05	3.61	3.28	3.01	2.79	2.61	2.45	2.32
H	0.003515	0.002927	0.003422	0.004034	0.004762	0.005606	0.006566	0.007642	0.008834	0.010142	0.011566
(H/min)	16.75	17.36	20.30	23.93	28.24	33.25	38.94	45.32	52.39	60.15	68.60
CVAR (EUR)	10.500.445	12.056.204	14.235.329	16.617.318	18.973.947	21.512.008	24.123.997	26.549.931	29.096.277	31.492.111	34.049.560
CVAR (% change)	0.0%	14.8%	35.6%	58.3%	80.7%	104.9%	129.7%	152.8%	177.1%	199.9%	224.3%

Figure 1: Borrowers, loan volume, net credit exposure and ratings by years

Panel A: Number of borrowers, LOAN and NET by years



Panel B: Rating distribution by grade and year

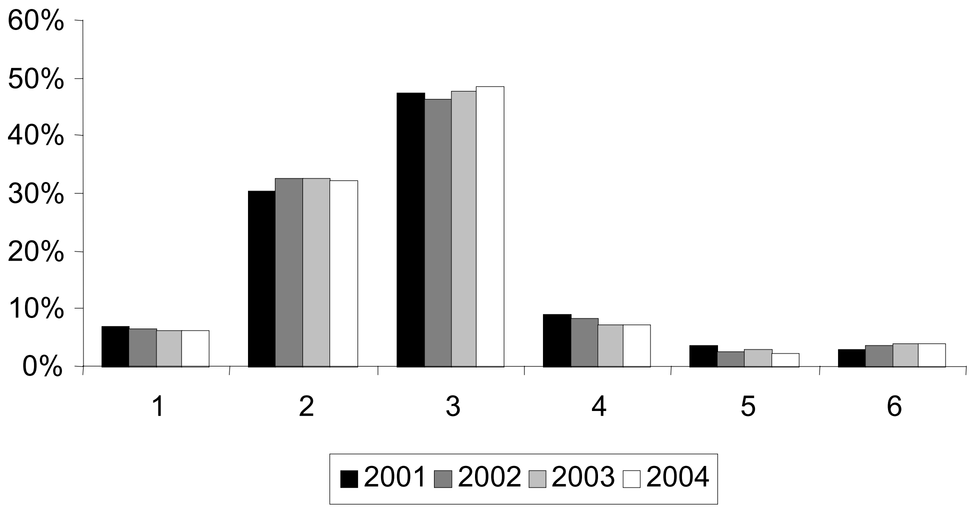


Figure 2: CVAR-change for constant increases of default rate and depreciation of collateral

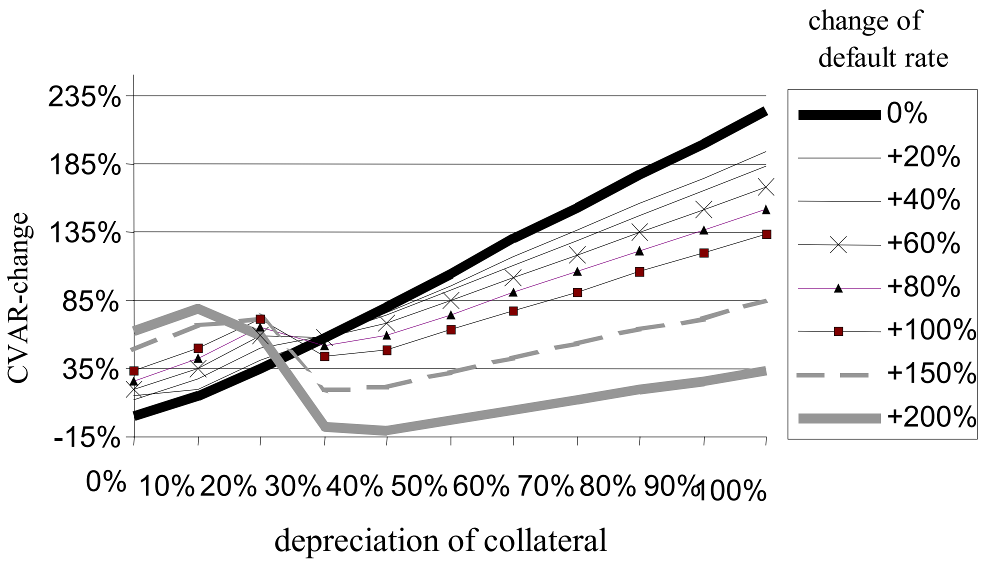
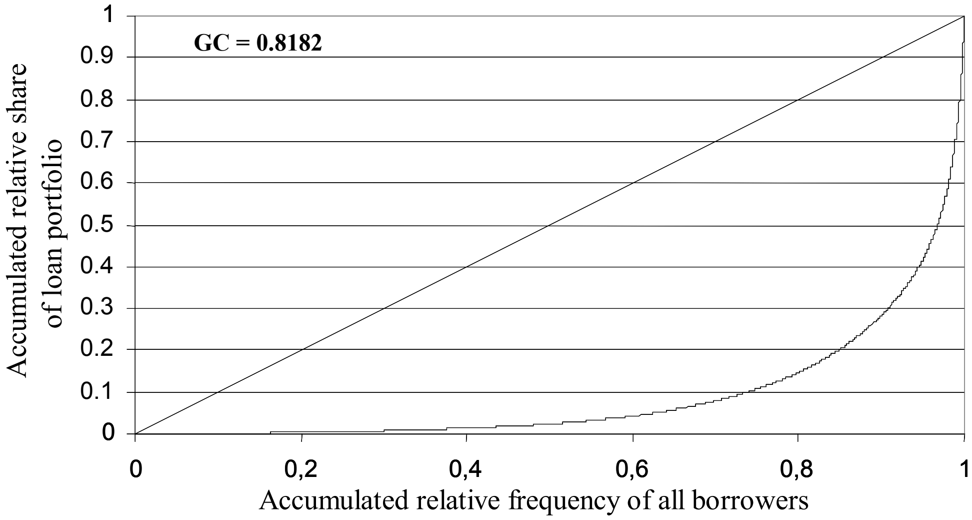


Figure 3: Lorenz curves

Panel A: Lorenz curve for all borrowers based on LOAN (2004)



Panel B: Lorenz curve of all borrowers based on CVAR (2004)

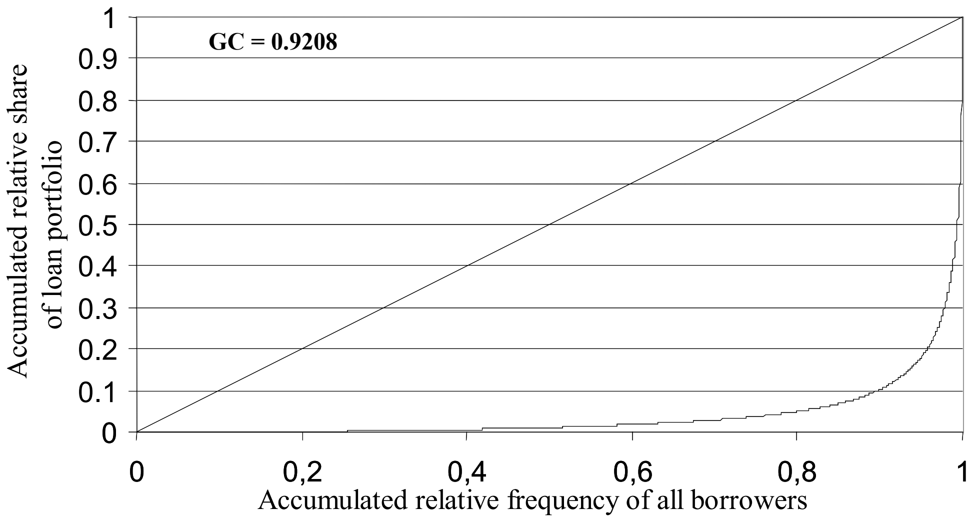
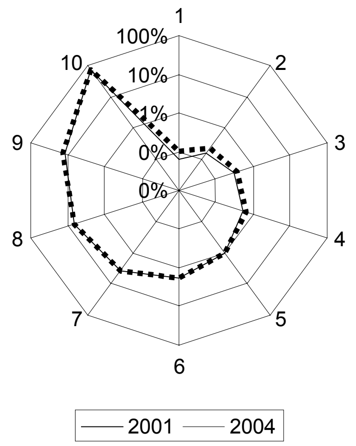


Figure 4: Evolution of relative concentration from 2001 to 2004 by borrower deciles

Panel A: Evolution of relative concentration in LOAN from 2001 to 2004



Panel B: Evolution of relative concentration in CVAR from 2001 to 2004

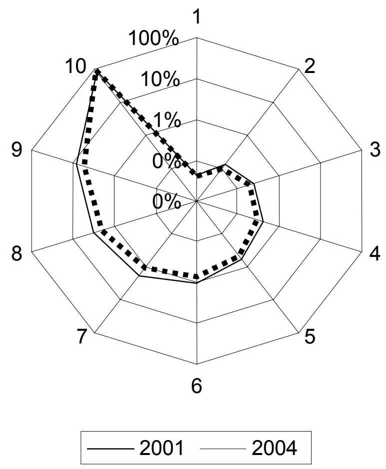
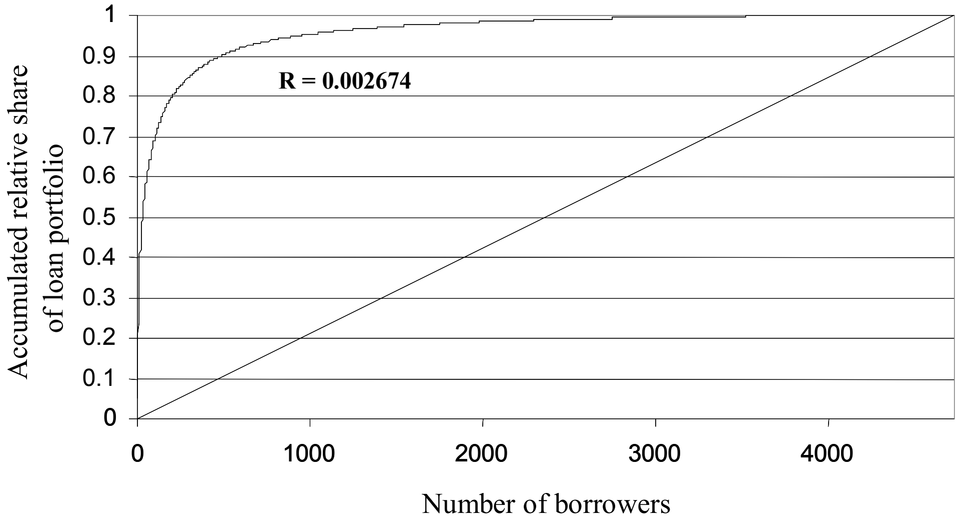


Figure 5: Concentration curves in 2004

Panel A: Concentration curve for all borrowers based on LOAN (2004)



Panel B: Concentration curve for all borrowers based on CVAR (2004)

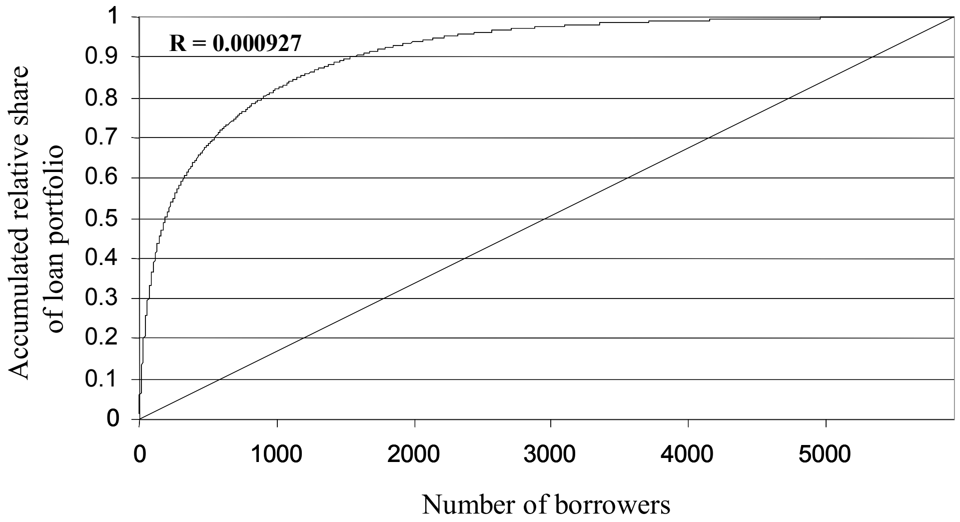
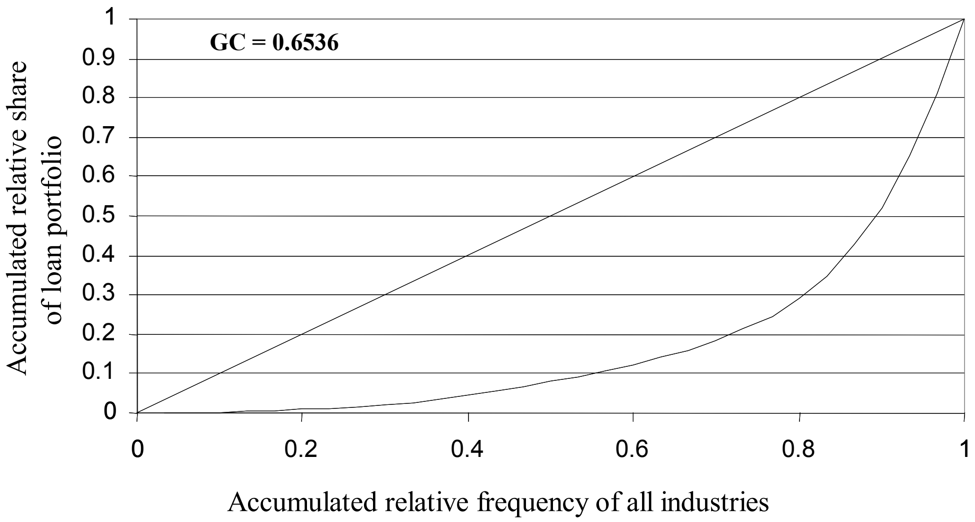


Figure 6: Lorenz Curves

Panel A: Lorenz curve for all industries based on LOAN (2004)



Panel B: Lorenz curve for all industries based on CVAR (2004)

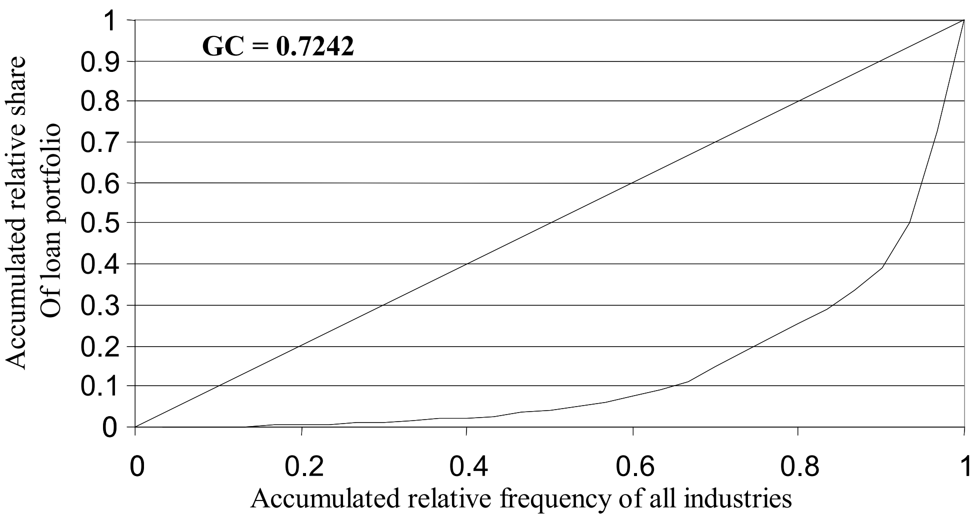
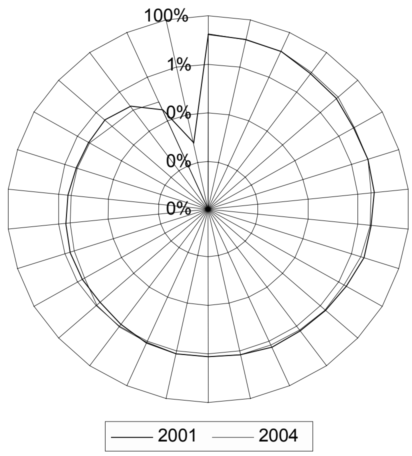


Figure 7: Evolution of absolute concentration by industry groups

Panel A: Evolution of absolute concentration in LOAN from 2001 to 2004



Panel B: Evolution of absolute concentration in CVAR from 2001 to 2004

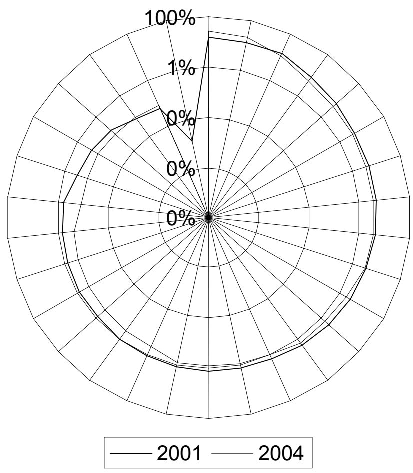


Figure 8: CVAR-change for constant changes of default rates for large borrowers