Projections of Pension Fund Solvency under Alternative Accounting Regimes

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Abstract

This paper examines the impact of three alternative accounting regimes on perceived pension fund solvency. *Deterministic accounting* assumes actuarially smoothed valuation of assets and liabilities. *National accounting* is based on market valuation of assets and on actuarial valuation of liabilities. *International accounting* books assets and liabilities to market values. Using closed-form methods based on the funding ratio return, we exemplify the dramatic effect that the choice of accounting approach has on long-horizon solvency projections.

Key words: pension fund, solvency, long-horizon return, asset liability management, accounting standards

JEL classification: G23, G28, G11, O21

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1 Introduction

A pension liability of defined-benefit type consists of promised future benefit payments to scheme participants according to a pre-specified formula. The formula captures the way in which payments are calculated, but the actual level of future cash-flows is subject to uncertainty of different kinds: scheme participants' career paths, mortality, development of prices, etc. The present value of future liabilities is obtained by discounting expected future benefit payments. A discount rate can be linked to government bonds, corporate bonds, the expected rate of return on the asset portfolio or a fixed actuarial rate depending on modeller, country or accounting system[1]. The role of the discount rate in the valuation exercise is difficult to overstate. A high rate will result in a lower liability value and a better funded status than a low rate. Additionally, a deterministic rate will result in a quite smooth development of the liability value whereas a market-based rate is stochastic.

To beat a liability return[2], a pension fund manager must understand its statistical and economic properties. This is a precondition for the derivation of an efficient frontier in an asset and liability management (ALM) context. Elton and Gruber (1988) and Sharpe and Tint (1990), among others, apply classical mean-variance techniques to solve the ALM problem. Further work by e.g. Cariño *et al.* (1994), Consigli and Dempster (1998), and Kouwenberg (2000) employ more complicated simulation-based methods for handling alternative objective functions, constraints and distributions. A general result in ALM studies is that the liability return should effectively be a passive representation of the pension fund's investment process and will as such define the minimum risk strategy within an ALM framework.

Sharpe (2002) states, that assets and liabilities should be measured in terms of market values but liabilities are often determined actuarially. The practice of actuarial valuation smoothing results in a diminished impact of market-related liability risks on optimization solutions of ALM models. Ponds and Quix (2003, p. 223) confirm that this framework ignores risks and "leads to a self-constructed picture of the financial solidity of a pension fund without any link to financial markets". Asset allocation studies within this smoothing framework are likely to give results quite similar to those of an asset-only approach in which liabilities are ignored and the risk minimizing asset class is cash.

On the other hand, with a market rate used for discounting liabilities, the liability value will fluctuate with market forces. Arnott and Bernstein (1988), using a pension surplus framework, make a case for equities and long-dated bonds as the risk minimizing asset classes and point out the high risk inherent in cash under the Financial Accounting Standards Board ruling no. 87. Similar reasoning can be found in Peskin (1997), who uses simulation techniques and an objective function that minimizes expected future pension contributions.

Whatever the accounting method for assets and liabilities, pension funds have been set up for one reason only: to make benefit payments to people who have ended their active career. A logical minimum requirement in solvency considerations is that the present value of assets is at least as high as the present value of liabilities. Imposing this constraint on the financial solidity of a pension fund sounds straightforward when stated in so many words, but a plethora of valuation and accounting methods render unclear the optimal hedge against liability developments as well as the minimal present value of assets that is sufficient to fund future benefits.

This paper focuses on the question of pension fund solvency under different accounting regimes. We present three regimes and draw parallels to the Finnish pension system to support their practical relevance. Using the funding ratio return, first introduced by Leibowitz *et al.* (1994), we analyze the impact of alternative accounting approaches on the projected future solvency position of a hypothetical pension fund over investment periods of 1-30 years.

The paper is structured as follows. Section 2 describes the accounting regimes. Section 3 presents a method for projecting the solvency position of the pension fund. Section 4 presents results and Section 5 provides concluding remarks.

2 Description of three alternative accounting regimes

We refer to *deterministic accounting* when asset and liability returns are subject to accounting smoothing, i.e. risk is ignored or disguised to produce smooth development patterns for portfolio and liability values. This type of modelling is used e.g. by the Finnish Centre for Pensions (FCP) to produce long-term funded status projections for the Finnish pension system[3]. See Biström *et al.* (2004) for a more detailed description.

We refer to *national accounting* if assets are booked at market values but liabilities are valued using actuarial smoothing techniques such as a fixed discount rate. In this case the pension fund manages market-valued assets to compete against a very smooth development of the liability value. This accounting regime closely resembles reality for many Finnish pension funds in that they manage assets under a national accounting regime in which an administratively set rate, the technical interest rate, determines the liability return (see www.etk.fi).

As of year 2005, EU companies that have securities traded on any EU regulated market should publish consolidated financial statements that comply with the International Financial Reporting Standards (IFRS). Cairns (2004, p. 107) succinctly words one of the aims of the IFRS: ”IFRS financial statements should report volatility in entity performance when it happens and not when the company management chooses to report it". One aspect of this is that actuarial assumptions should reflect market expectations on the balance sheet date. The expected returns on financial assets, the future level of interest rates, and the future rate of inflation are all uncertain. Therefore, a third accounting regime considers the case in which assets and liabilities are booked to market and their returns are modelled stochastically. We refer to this regime as *international accounting*.

3 Modelling solvency

Section 3.1 discusses measures for projecting financial solidity and motivates our preference for funding ratio return. Section 3.2 presents closed-form expressions for the expected value and variance of the n-year funding ratio return. Section 3.3 explains how the long-term solvency position of a pension fund is projected under alternative accounting regimes.

3.1 Measures of pension fund performance

There are two commonly used measures of financial solidity for pension funds: funding ratio and surplus. The funding ratio is defined as the ratio of the value of assets to the value of liabilities. The surplus is defined as the value of assets minus the value of liabilities. We follow Leibowitz *et al.* (1994) to define funding ratio return (FRR) as the percentage change in the funding ratio over a one-year period. Similarly, surplus return can be defined as the percentage change in the surplus over a one-year period.

Waring (2004) points out that, the above definition of surplus return is not a robust single-number measure of pension fund performance. First, if the initial surplus is zero there will be a zero divide problem. Second, if the initial surplus is small, a small change in its value will result in a high surplus return which has no meaningful interpretation without some knowledge of the initial funding ratio. Third, in the event of the surplus turning into a deficit the result will be nonsensical. Sharpe and Tint (1990) propose a surplus return measure that is scaled to the initial value of assets. Leibowitz *et al.* (1992) choose to scale it to the initial value of the liability. In fact, the change in surplus could be scaled back to any convenient base as long as the base is large and stable enough to avoid the above mentioned problems.

The rescaling does not eliminate the dependency between surplus return and initial funding ratio however. Consider the surplus return measure of Sharpe and Tint, defined as the change in surplus divided by initial assets. This measure is dependent on both the initial surplus and initial assets.

The FRR is more appealing as a general measure of pension fund performance: it doesn't suffer from the zero-divide problem or the sign change problem, and it is not dependent on the initial funding ratio. Table 1 illustrates the difference between FRR and surplus return. It compares them in assumption of a 10 percent portfolio return and a 20 percent liability return. Unlike the surplus return, the FRR does not change with the initial funding ratio.

Table 1

FRR and surplus return.

This table compares the FRR to the surplus return measure of Sharpe and Tint (1990). Surplus return is defined as the change in surplus over a one-year period divided by initial assets. FRR is defined as the change in the funding ratio over a one-year period divided by the initial funding ratio.

Initial assets (MEUR)	1400.0	1200.0	1000.0	800.0
Initial liability (MEUR)	1000.0	1000.0	1000.0	1000.0
Initial surplus (MEUR)	400.0	200.0	0.0	-200.0
Initial funding ratio	1.4	1.2	1.0	0.8
Return on assets	10.0 %	10.0 %	10.0 %	10.0 %
Return on liability	20.0 %	20.0 %	20.0 %	20.0 %
Final assets (MEUR)	1540.0	1320.0	1100.0	880.0
Final liability (MEUR)	1200.0	1200.0	1200.0	1200.0
Final surplus (MEUR)	340.0	120.0	-100.0	-320.0
Final funding ratio	1.3	1.1	0.9	0.7
Surplus return	-4.3 %	-6.7 %	-10.0 %	-15.0 %
FRR	-8.3 %	-8.3 %	-8.3 %	-8.3 %

3.2 The n-year FRR distribution

Having made a preference for the FRR, we proceed by assuming a model of asset and liability returns, in which successive annual returns are independent and have lognormal probability distributions[4]. The FRR distribution is a ratio of two lognormal distributions and is lognormal itself[5]. It is skewed to the right and has heavy tail. Let us denote the expected value and variance of the yearly FRR of portfolio P as $E(FRR^P_{t-1,t})$ and $\mathrm{var}(FRR^P_{t-1,t})$ respectively. It can be shown that

$$E(FRR^P_{t-1,t}) = \exp(\log E_P - \log E_L + \sigma_L^2 - \sigma_{PL}) - 1 \qquad (1)$$

$$\mathrm{var}(FRR^P_{t-1,t}) = \exp\left[2(\log E_P - \log E_L + \sigma_L^2 - \sigma_{PL})\right] \times \left[\exp(\sigma_P^2 + \sigma_L^2 - 2\sigma_{PL}) - 1\right] \qquad (2)$$

where $E_P = E(1+R^P_{t-1,t})$, is the expected yearly gross return for the portfolio, $E_L = E(1+R^L_{t-1,t})$, is the expected yearly gross return for the liability, σ^2_P is the variance of the continuously compounded yearly rate of portfolio return, σ^2_L is the variance of the continuously compounded yearly rate of liability return, and σ_{PL} is the covariance between the continuously compounded yearly rates of return of the portfolio and the liability.

Equations (1) and (2) point out that although the pursuit of positive covariance between asset and liability returns is generally regarded as prudent investing, not all sponsors like positive liability covariance. A high covariance decreases the variance of the FRR, but it also reduces the expected FRR. Furthermore, a plan sponsor does not necessarily dislike variance of liability returns. A higher liability uncertainty does add to the variance of the FRR, but it also increases the expected FRR. Whether on balance the sponsor dislikes a high covariance between portfolio and liability returns or volatility of liability returns depends on his relative risk aversion.

Because pension obligations are long-term commitments and the planning horizon of a pension fund often spans several decades, our next task is to extend the definition of FRR to a longer horizon. For portfolio P, let us denote the expected value of the n-year horizon FRR as $E(FRR^P_{0,n})$, and the standard deviation of the n-year horizon FRR as $\sigma(FRR^P_{0,n})$. Building on de La Grandeville's (1998) argumentation, we can calculate exactly the first two moments of the n-year horizon FRR as

$$E(FRR^P_{0,n}) = \left[1+E(FRR^P_{t-1,t})\right]\left(1+\frac{\operatorname{var}(FRR^P_{t-1,t})}{\left[1+E(FRR^P_{t-1,t})\right]^2}\right)^{(1/2)((1/n)-1)} -1 \qquad (3)$$

$$\sigma(FRR^P_{0,n}) = \left[1+E(FRR^P_{0,n})\right]\left[\left(1+\frac{\operatorname{var}(FRR^P_{t-1,t})}{\left[1+E(FRR^P_{t-1,t})\right]^2}\right)^{1/n} -1\right]^{1/2}. \qquad (4)$$

These formulae reveal important properties of the n-year horizon FRR. First, there are two special cases for which the expected value of the FRR does not depend upon the measurement horizon, i.e. $E(FRR^P_{0,n}) = E(FRR^P_{t-1,t})$ for all $n \in [0,\infty]$, implying zero standard deviation of the FRR. This happens when the standard deviation of asset and liability returns is zero as well as when asset and liability returns are perfectly correlated with equal standard deviations. Second, both the standard deviation and the expected value of the n-year horizon FRR are decreasing functions of the horizon.

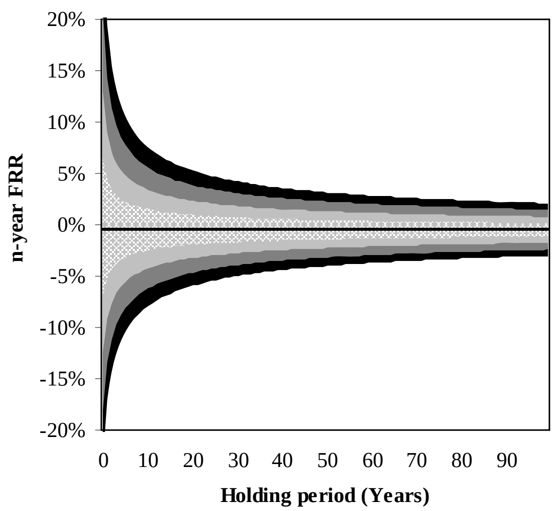
In the limit, the standard deviation of the n-year horizon FRR will tend to zero and the expected value of the n-year horizon FRR will converge to

$$E(FRR_{o,\infty}) = \frac{\left[1 + E(FRR_{t-1,t})\right]^2}{\sqrt{\left[1 + E(FRR_{t-1,t})\right]^2 + \mathrm{var}(FRR_{t-1,t})}} - 1. \tag{5}$$

Let us assume an expected value of the yearly FRR of 0.0 percent and a standard deviation of 10.0 percent. Panel A in Figure 1 shows how the standard deviation of the n-year horizon FRR decreases with time. Following Dimson *et al.* (2004), we estimate and plot the 0.5^{th}, 2.5^{th}, 10^{th}, 25^{th}, 50^{th}, 75^{th}, 90^{th}, 97.5^{th}, and 99.5^{th} percentiles of the FRR distribution for investment horizons of 1-100 years. Panel B in Figure 1 illustrates that as the standard deviation of the n-year horizon FRR approaches zero in the limiting case, the expected n-year FRR approaches its geometric mean of -0.50 percent given by equation (5). It is a straight horizontal line in Panel B. The expected value of the FRR is 0.0 percent when n equals one (n = 1) and -0.49 for a horizon of one hundred years (n = 100).

A. Projected distribution of the n-year FRR

$E(FRR_{t-1,t}) = 0.0\%$; $\sigma(FRR_{t-1,t}) = 10.0\%$



B. Expected value of the n-year FRR

$E(FRR_{t-1,t}) = 0.0\%$; $\sigma(FRR_{t-1,t}) = 10.0\%$

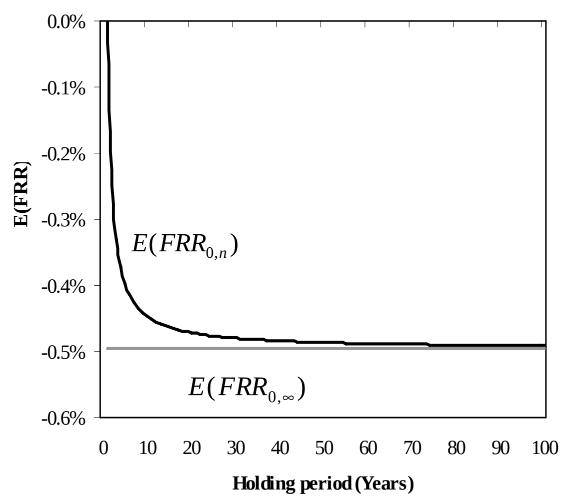


Fig. 1. The n-year horizon FRR.

This figure illustrates the dependence of the first two moments of the n-year FRR on the time horizon. Panel A shows the projected distribution of the n-year FRR. The black area of the distribution represents the 99 percent, the dark grey area is the 95 percent, the light grey area depicts the 80 percent, and the white area shows the 50 percent confidence intervals. The dark bolded line in the middle represents the median. Panel B shows the expected value of the n-year FRR over holding periods of up to 100 years.

3.3 Projecting the n-year funding ratio under different accounting regimes

It is straightforward to calculate the distribution of the funding ratio at the end of year n through the FRR. The n-year FRR is the annual rate of growth that equates initial funding ratio to final funding ratio. End-of-period funding ratio distribution percentiles can be obtained in a similar manner to the percentiles of the FRR distribution by utilizing the properties of lognormal variables.

Table 2 gives a summary of inputs needed for each of the three accounting regimes. Under deterministic accounting, expected portfolio and liability returns are the input required by the model, as standard deviations and correlation coefficients are all equal to zero by construction. Under national accounting, our model requires estimates of expected portfolio and liability returns, as well as an estimate of the standard deviation of the portfolio return. Note that the liability return has zero variability in this case. Under international accounting we need estimates of expected portfolio and liability returns, their standard deviations and the correlation coefficient between the returns.

This setup provides a simple framework for measuring the impact of alternative accounting regimes on projections of the future funding ratio. It captures essential differences in dynamics, which can not be overlooked when more detailed and sophisticated models are considered and optimization procedures applied. The findings of the next chapter suggest that agreement on the choice of accounting regime is a prerequisite for agreement on optimal ALM strategy.

Table 2

Accounting regimes.

Required input parameter estimates (marked with an X) for the calculation of FRR.

	ACCOUNTING REGIME		
	Deterministic	**National**	**International**
Return generating process			
Assets	Deterministic	Stochastic	Stochastic
Liability	Deterministic	Deterministic	Stochastic
Required estimates			
Expected return (Assets)	X	X	X
Volatility (Assets)	-	X	X
Expected return (Liability)	X	X	X
Volatility (Liability)	-	-	X
A/L correlation	-	-	X

Note: A/L correlation = the correlation coefficient between the yearly returns of the asset portfolio and the liability.

4 Projections of pension fund solvency

This section implements the methods of section 3 to project the future funding ratio of a hypothetical defined-benefit pension fund over investment horizons of 1-30 years. We project the future funding ratio from three points of view. First, we project the funding ratio under *deterministic accounting*, i.e. when portfolio and liability returns are modelled deterministically. Second, we project the funding ratio under *national accounting*, i.e. when the portfolio return is assumed to be stochastic but the liability return is assumed to be deterministic. Third, we project the funding ratio under *international accounting*, i.e. when portfolio and liability returns are stochastic. Figure 2, Figure 3 and Figure 4 illustrate how alternative accounting approaches can lead to significant differences in the perceived long-term financial solidity of the pension fund.

Let us assume an initial funding ratio of 1.2, an expected annual liability return of 3.5 percent, and 20 percent standard deviation of annual liability returns. We further assume a 3.5 percent expected annual portfolio return and 10 percent standard deviation of annual portfolio returns. Measured in terms of market values, the correlation coefficient between annual portfolio and liability returns is assumed to be 0.5. Figure 4, which represents international accounting, uses the whole information set for expected returns, standard deviations and correlation whereas Figure 2 (deterministic accounting) and Figure 3 (national accounting) neglect information as outlined in Table 2.

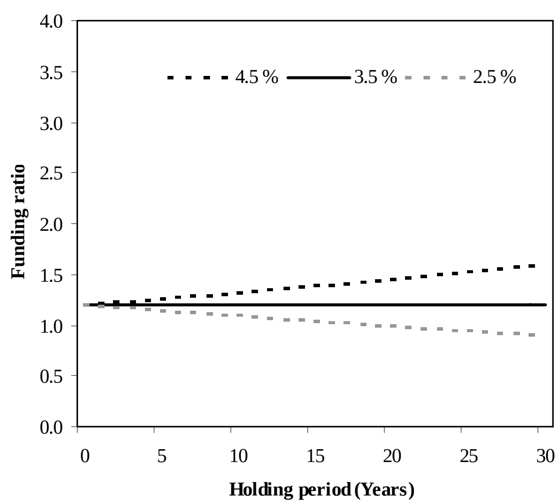


Fig. 2. Projected future funding ratio under deterministic accounting.

This figure projects the future funding ratio over holding periods of 1-30 years under three deterministic scenarios. The baseline scenario is represented by the black 3.5%-line. The optimistic scenario is represented by the dotted black 4.5%-line and the pessimistic scenario is represented by the dotted grey 2.5%-line.

Figure 2 shows the projected distribution of the future funding ratio under deterministic accounting. A 3.5 percent expected annual portfolio return results in an expected one-year FRR of 0.0

percent. The standard deviation of the FRR is 0.0 percent as well. Because there is no uncertainty involved, the funding ratio develops along a straight line from a level of 1.2 today to a level of 1.2 at the end of year 30 (the bolded 3.5%-line). Based on the deterministic projection, a yearly portfolio return of 3.5 percent will guarantee a funding ratio of 1.2 over the long-term.

We further stress test the projection by assuming that the performance of the asset portfolio might disappoint on the downside or surprise on the upside. The pessimistic scenario assumes a 2.5 percent annual portfolio return (the 2.5%-line) and the optimistic scenario assumes a 4.5 percent annual portfolio return (the 4.5%-line). The liability return is 3.5 percent for all scenarios. The pessimistic scenario gives a funding ratio of 0.9 at the end of year 30 and the optimistic scenario results in a funding ratio of 1.6 at the end of year 30. The naive way of interpreting Figure 2 is that the funding ratio is expected to stay flat over a 30-year period, and furthermore, is highly likely to be in a range of 0.9 to 1.6 at the end of year 30. Figure 2 is not just a case of theoretical artifice. Deterministic models are used in practice (see section 2). Based on the results of Figure 2, pension policies might be planned and pension reform implemented.

$$E(FRR_{t-1,t}) = 0.0\%\text{;}\ \sigma(FRR_{t-1,t}) = 9.6\%$$

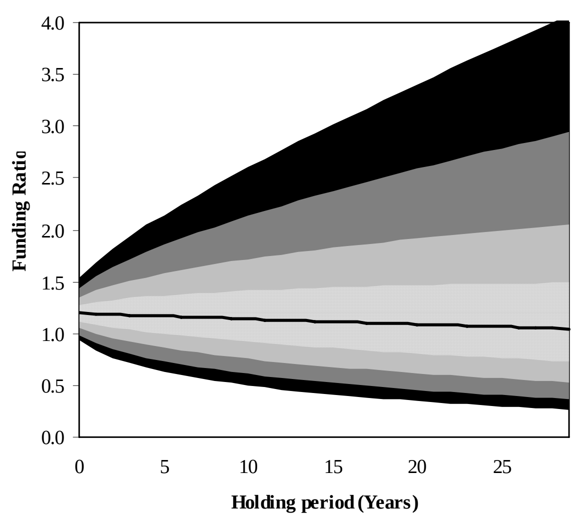


Fig. 3. Projected distribution of future funding ratio under national accounting.

This figure shows the projected distribution of the future funding ratio. The black area of the distribution represents the 99 percent, the dark grey area is the 95 percent, the light grey area depicts the 80 percent, and the white area shows the 50 percent confidence intervals. The dark bolded line in the middle represents the median.

In Figure 3, which represents national accounting, we have added 10 percent volatility to portfolio returns and re-run the deterministic baseline projection of Figure 2. The FRR of the portfolio now has a one-year expectation of 0.0 percent and an expected standard deviation of 9.6 percent. As a first thought, an expected yearly FRR of 0.0 percent might seem satisfactory and might lead us to conclude that the pension fund is well on track to reach the 1.2 funding ratio target shown in the deterministic projection. In fact, it is not. The 30-year expected annual FRR is -0.44 percent, which translates into an expected funding ratio of 1.05 at the end of year 30. This is quite a large difference to have in the central tendency of such an important projection. Furthermore, the percentiles of the distribution indicate that there is roughly a 60 percent chance that the funding ratio of the pension fund will decrease over the holding period and that the bottom decile of the distribution runs from 0.27 to 0.53.

The projection in Figure 3 makes clear that an expected yearly 3.5 percent portfolio return neither gives guarantees nor is enough to preserve the funding ratio under national accounting. The explanation to this result, which we will come back to later, lies in the relative levels of return volatility between the asset portfolio and the liability. Equations (1) through (5) confirm that with equal expected portfolio and liability returns, but with positive portfolio return volatility and zero liability return volatility, the expectation is that the portfolio will loose to the liability in the long-term.

$E(FRR_{t-1,t}) = 2.8\%$; $\sigma(FRR_{t-1,t}) = 17.2\%$

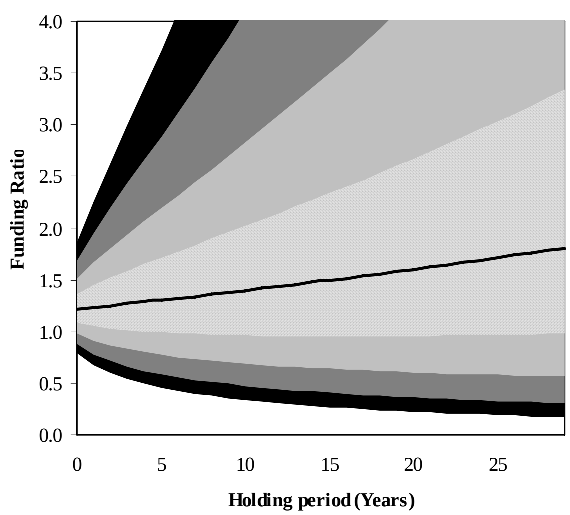


Fig. 4. Projected distribution of future funding ratio under international accounting.

This figure shows the projected distribution of the future funding ratio. The black area of the distribution represents the 99 percent, the dark grey area is the 95 percent, the light grey area depicts the 80 percent, and the white area shows the 50 percent confidence intervals. The dark bolded line in the middle represents the median.

However, it is not always obvious that the expected volatility of the liability return is zero. In Figure 3, national accounting imposed that assumption on us by brute force. A company that operates under international accounting would argue that of course the pension liability, like any other stream of cash flows, can be thought of in economic terms and is subject to the same laws of risk and return as other financial assets. Therefore, the projection must incorporate the 20 percent volatility estimate for the liability return and the 0.5 estimate of the correlation coefficient between asset and liability returns.

Figure 4 shows the projected distribution of the future funding ratio under international accounting. Indeed, the projection looks different from the ones in Figure 2 and Figure 3. The expected 30-year funding ratio now is 1.83 and the probability of a funding ratio decline is 32.6 percent on the 30-year horizon, with the bottom decile running from 0.17 to 0.56.

To grasp why the differences in Figure 2, Figure 3, and Figure 4 are so large one must understand the effect of volatility on the long-horizon rate of return. First of all, volatility creates uncertainty about what the future cumulative return will be at a certain point in time. Obviously, a deterministic model is unable to capture that uncertainty and might therefore lead to recommendations that prove hazardous to the performance of any long-term investor. Second, volatility has a way of reducing returns over the long run. If the fund has a year with a minus 50 percent FRR, it needs one year with a FRR of 100 percent, two consecutive years of 41.4 percent FRRs, or 70 consecutive years of 1.0 percent FRRs to break even. Under national accounting the liability grows deterministically whereas asset returns are volatile. In such a setting the pension manager will struggle to beat the liability return because it never experiences bad years whereas the portfolio is bound to do so sooner or later. This phenomenon should be reflected in the expected value of the FRR and as Figure 3 shows, indeed it is. On the other hand, if expected liability uncertainty is higher than expected portfolio uncertainty things will work in the opposite direction as shown in Figure 4.

This is an important result with an important policy implication: in order to preserve the funding ratio of a pension fund or system, a national accounting regime requires a higher expected portfolio return than either a deterministic or international regime. Higher return means higher risk and if portfolio risks materialize, recovery time will be long. In contrast, market valuation of liabilities make liability covariance an important consideration in portfolio decisions and can by itself reduce FRR volatility. Furthermore, variance of liability returns allow pension managers to target higher future funding ratios without necessarily increasing the risk of shortfalls.

5 Concluding remarks

This paper evaluates the financial solidity of a pension fund under different accounting regimes. We examine three regimes: a deterministic regime under which asset and liability returns are deterministic, a national regime under which the asset returns are stochastic but liability returns are deterministic, and an international regime under which both asset and liability returns are stochastic. To illustrate the impact of alternative accounting approaches on the perceived solvency of the pension fund we use a closed-form method to project its long-term funding ratio, which is based on the concept of funding ratio return.

Our projections suggest that alternative accounting approaches indeed may result in confusion between the parties involved with the fund. Deterministic accounting can be very misleading if the true accounting regime is either national or international. It will surely lead to a false sense of security but might even result in an incorrect estimate of the expected value of the future funding ratio. Hence, deterministic modelling is bad policy support. A national regime, on the other hand, is likely to be overly pessimistic about the pension fund's long-term solvency position and can therefore result in an unnecessarily aggressive investment policy. However, aggressive investment strategies are difficult to implement due to deterministic liability growth. One must first accumulate assets to attain an initial funding ratio high enough to cope with a combination of uncertain asset returns and deterministic liability growth. Asset accumulation can be costly and must be repeated each time the funding ratio deteriorates to dangerously low levels.

The alignment of short- and long-term goals is achieved more naturally under international accounting, as market valued liabilities provide opportunities for hedging short-term liability-related risks on financial markets and as liability uncertainty is not solely a negative factor within this framework. Replacing a national regime with an international one might reduce the probability of negative surprises and subsequent unpleasant political decisions without the need for a higher initial funding ratio.

Notes

[1] See the European Actuarial Consultative Group (2004) for a comparison of how liabilities in respect of retirement benefit arrangements are typically valued in different countries.

[2] We use "liability return" as a generic term in referring to the annual rate of change in the value of whatever liability definition the pension fund has chosen, or is required, to consider as a benchmark for its portfolio management activities.

[3] The FCP is one of the most influential providers of pension research in Finland. It publishes projections on e.g. future pension premiums, the size of pension funds and payments up to year 2075.

[4] This simplifying assumption allows us to use closed-form properties of lognormal distributions in modelling the yearly FRR. With other distributions, we would have to resort to more complicated simulation techniques, a subject that is beyond the scope of this treatment. For the n-year rate of return (see section 3.2), de La Grandeville (1998) has shown that the normality assumption of the continuously compounded yearly rate of return can be dropped because the independence assumption (and finite variance) is enough to guarantee that the n-year rate of return will converge to the lognormal probability distribution.

[5] The FRR can be expressed as $(1+R^P)/(1+R^L)-1$, where R^P and R^L denote the annual rates of portfolio and liability return.

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