Hedge fund portfolio construction: A comparison of static and dynamic approaches

Daniel Giamouridis, Ioannis D. Vrontos

Abstract

This article studies the impact of modelling time varying covariances/correlations of hedge fund returns in terms of hedge fund portfolio construction and risk measurement. We use a variety of static and dynamic covariance/correlation prediction models and compare the optimized portfolios’ out-of-sample performance. We find that dynamic covariance/correlation models construct portfolios with lower risk and higher out-of-sample risk-adjusted realized return. The tail-risk of the constructed portfolios is also lower. Using a mean-conditional-value-at-risk framework we show that dynamic covariance/correlation models are also successful in constructing portfolios with minimum tail-risk.

JEL classification: G11; G12

Keywords: Hedge fund portfolios; dynamic covariances/correlations; multivariate GARCH; regime switching; CVaR

*Corresponding author. Department of Accounting and Finance, Athens University of Economics and Business, Patission 76, GR-10434 Athens, Greece. Tel.: +30-210-8203925; fax: +30-210-8203936. Email: dgiamour@aueb.gr
1 Introduction

The hedge fund industry has been growing rapidly over the last years. Individual hedge funds and funds of funds have been traditionally available to high net-worth individuals or institutional investors seeking exposure in the so-called alternative investments arena. With the recent launch of investable hedge fund indices, small- to medium-sized investors also gained access to this asset class, either directly or via index-linked products\. These developments have attracted a substantial amount of business and give an additional boost to interest in studying hedge fund investments.

To date research in hedge fund investing has mainly focused on determining the right proportion to allocate in hedge funds (see, e.g. Terhaar et al., 2003, Cvitanić et al., 2003, Popova et al., 2003, Amin and Kat, 2003), on identifying hedge fund risks (see, e.g. Fung and Hsieh, 1997, Ackermann et al., 1999, Brown et al., 1999, Edwards and Caglayan, 2001, Liew, 2003, Agarwal and Naik, 2004), and on constructing optimal hedge fund portfolios (see, e.g. McFall Lamm, 2003, Kat, 2004, Agarwal and Naik, 2004, Alexander and Dimitriu, 2004, Morton et al., 2005). These studies, despite, (a) the nature of hedge fund investments i.e. dynamic trading strategies, derivatives, and leverage used by fund managers, (b) the well known fact that the variance and covariance of most financial time series - the funds’ underlying assets - are time-varying, and (c) empirical evidence for volatility clustering and high kurtosis in the time series of fund returns (see, e.g. McFall Lamm, 2003, Morton et al., 2005), do not account for possible time-varying variances and covariances/correlations of hedge fund returns. They assume constant - static - covariance/correlation structure through time. As a result hedge fund return variances and covariances/correlations may not be measured accurately, with potential important impacts in terms of asset allocation, pricing, and portfolio construction, but also in terms of risk measurement, i.e. computation of the Value at Risk (VaR), the Conditional Value at Risk (CVaR).

In this paper, we address the issue of time-varying variances and covariances/correlations of hedge fund returns and concentrate on the potential impacts in terms of hedge fund portfolio construction and risk measurement. We start with the case where the hypothetical investor is concerned with the volatility

---

of the portfolio. We compare the performance of different methods of forecasting variances and covariances/correlations, with an eye to judge which model improves our ability to optimize hedge fund portfolio risk. More specifically we compare the performance of five different forecasting models: (a) a sample covariance model, (b) an implicit factor model (Fung and Hsieh, 1997, Amenc and Martellini, 2002, Alexander and Dimitriu, 2004), (c) an implicit factor GARCH model (Alexander, 2001), (d) an implicit full-factor GARCH model (Vrontos et al., 2003), and (e) a regime switching dynamic correlations model (Pelletier, 2005). The different models are evaluated, out-of-sample, in a case study which examines the portfolio risk, but also realized return, risk-adjusted realized return, and tail-risk. We then hypothesize that the investor is concerned with the portfolio tail-risk. We set up an additional case study where the different risk models are evaluated, out-of-sample, for their ability to construct hedge fund portfolios with optimal tail-risk.

Our empirical analysis provides three main findings. First, we find that dynamic covariance/correlation prediction models improve our ability to optimize hedge fund portfolios. They are able to construct portfolios with lower average risk and higher average risk-adjusted realized return. These results are significant on a statistical basis. Second, we find that the dynamic models provide a more accurate tool for tail-risk measurement. They are able to construct hedge fund portfolios which exhibit significantly lower tail-risk. Third, we find that the allocation determined by the dynamic covariance/correlation prediction models is not very similar with that computed from the other models. In fact, this difference is substantial for certain funds, suggesting that a more accurate risk model improves our ability to select the right fund for the portfolio. Our findings have important implications for hedge fund style allocation decisions and risk measurement. They also provide useful insights for managers wishing to adopt a dynamic approach for fund selection and allocation purposes.

The contributions of this article are several. We model time-varying covariances/correlations of hedge fund returns for the first time to our knowledge. In the dynamic environment that hedge fund investments are determined, variances and covariances/correlations are expected to change over time, thus, making it sensible to seek an appropriate covariance specification in this class of models. In addition, we perform a comparative study to evaluate the ability of different covariance prediction models to construct optimal hedge
fund portfolios. This analysis is executed out-of-sample and the results are validated on a statistical basis. Also, we provide a thorough analysis of coherent tail-risk measures. Hedge fund tail-risk measurement is of particular importance given the impact of extreme events in hedge fund investments. Agarwal and Naik (2004) and Krokhmal et al. (2002) are the only studies we know of that touch upon the issue of hedge fund portfolio tail-risk measurement. We extend their analysis in that we measure hedge fund portfolio tail-risk for a number of different covariance/correlation models. The entire study is carried out on portfolios constructed with different optimization benchmarks to study performance under different investment objectives. The sensitivity of our conclusions to the choice of parameters such as the rebalancing frequency or the size of the estimation period is also examined. This analysis is also new, to our knowledge, to the hedge fund portfolio construction literature.

The remainder of this article proceeds as follows. Section 2 discusses the approaches used to construct optimal hedge fund portfolios. Section 3 outlines the covariance prediction models used in our empirical analysis. Section 4 discusses the data. Section 5 presents the metrics used in our empirical analysis and reports the results of our investment exercise. Section 6 concludes.

2 Optimal hedge fund portfolios

The standard Markowitz (1952) mean-variance analysis in the construction of portfolios involving hedge funds has been subject to criticisms in the literature. While Amenc and Martellini (2002), Terhaar et al. (2003), Alexander and Dimitriu (2004) concentrate on hedge fund allocation within mean-variance opportunity sets, alternative approaches have been proposed. Barès et al. (2002) and Cvitanić et al. (2003), for example, determine the optimal asset allocation in an expected utility framework. Amin and Kat (2003) discuss the issues arising in mean-variance allocation when the distribution of asset returns is not symmetric. Popova et al. (2003), Hagelin and Pramborg (2003), and Davies et al. (2005) deal with these issues by employing higher moment analysis. McFall Lamm (2003) also addresses asymmetry and fat-tailness in the returns distributions with Duarte’s (1999) generalized approach. Krokhmal et al. (2002), on the other hand, construct optimal portfolios using alternative - to the standard variance - risk objectives.
which control different types of risks, i.e. CVaR and CDaR (Conditional Drawdown at Risk) among others. In the same vain, Agarwal and Naik (2004) seek optimal mean-CVaR portfolios. Finally, Rockafellar et al. (2005) propose generalized measures as substitutes for standard deviation.

The criticisms against the mean-variance framework stress that it is appropriate only for normally distributed returns or for investors having quadratic preferences, thus, making it not perfectly applicable to hedge funds. Chambers and Quiggin (2005), however, prove that ‘...much of the standard mean-standard deviation analysis can be extended to general invariant preferences, without requiring...[the preferences to be]...neutral with respect to...higher moments’. In addition to that, the mean-variance analysis is a widely used portfolio construction approach in practice. We, thus, focus on the mean-variance approach and investigate if an appropriate covariance model improves our ability to construct optimal portfolios. In the mean-variance framework portfolios are constructed through the following optimization:

$$\min_{X} \text{Var}(R_p)$$

s.t. $x_i \geq 0$, $i = 1, \ldots, n$, $\sum_{i=1}^{n} x_i = 1$, and $E(R_p) \geq \text{Target return}$

where $R_p$ and $\text{Var}(R_p) = XX'X$ are the $n-$assets hedge fund portfolio return and variance respectively, $X = (x_1, x_2, \ldots, x_n)'$ is the vector containing the funds’ weights in the portfolio, $V$ is the $n \times n$ covariance matrix, and $E(R_p)$ is the expected return of the fund portfolio. We impose no-short-sales constraints since hedge funds cannot be shorted in practice.

Optimal portfolios can alternatively be constructed using other risk objectives. Given the increasing emphasis on risk management and its potential payoffs, there is a proliferation of measures capturing different types of risks. One such measure is the CVaR - the expectation of the losses greater than or equal to the VaR - which measures the risk in the tail of the loss distribution. Krokhmal et al. (2002) employ a number of risk management methodologies to construct optimal hedge fund portfolios. They conclude that CVaR demonstrates the most solid out-of-sample performance in their data set. We also investigate how alternative econometric specifications perform in constructing hedge fund portfolios with minimum tail risk. We employ Rockafellar and Uryasev’s (2000) convex programming formulation. The problem is expressed
mathematically as follows:

\[
\min_{\mathbf{x}} \text{CVaR}(F_{R_p}, \alpha) \quad (2)
\]

s.t. \( x_i \geq 0, \ i = 1, ..., n, \ \sum_{i=1}^{n} x_i = 1, \) and \( E(R_p) \geq \text{Target return} \)

where:

\[
\text{CVaR}(F_{R_p}, \alpha) = -E(R_p | R_p \leq -\text{VaR}) = -\frac{\int_{-\infty}^{-\text{VaR}} z f_{R_p}(z) dz}{F_{R_p}(-\text{VaR})} \quad (3)
\]

\[
\text{VaR}(F_{R_p}, \alpha) = -F_{R_p}^{-1}(1 - \alpha), \ f_{R_p} \text{ and } F_{R_p} \text{ denote the probability density and the cumulative density of } R_p \text{ respectively, and } \alpha \text{ is a probability level}. \]

Rockafellar and Uryasev (2002) provide a thorough analysis of the properties of CVaR in risk measurement and portfolio optimization exercises.

3 Predicting hedge fund return covariances

Given our focus on hedge fund portfolio risk or tail-risk optimization, we concentrate on predicting fund return covariances. The fact that hedge fund investments are determined in a dynamic manner makes fund returns and covariances modelling not a straightforward task. In addition to that, a short history of data is normally available for hedge funds, thus, making the estimation of data-demanding models not always easy.

The ‘cleaning’ of the covariance matrix by imposing some structure is the common thread in the literature. Amenc and Martellini (2002) discuss the trade-off between model risk and estimation risk in alternative approaches. Explicit factor models (e.g. Agarwal and Naik, 2004) are claimed to involve high specification error and low sampling error while models imposing optimal structure (e.g. Ledoit and Wolf, 2003) engage medium, specification and sampling, errors. An alternative option is to use implicit factors. Amenc and Martellini (2002) discuss the advantages of using implicit factor models. These involve low specification error, as they exploit the information contained in the empirical correlation, and low sampling error due to the - little - structure that is imposed.

Our empirical investigation uses a variety of covariance models including static and dynamic specifications. The static specifications include the sample covariance model and an implicit factor model which has been used successfully in hedge fund pricing and portfolio construction studies (Fung and Hsieh, 1997,
Amenc and Martellini, 2002, Alexander and Dimitriu, 2004). A natural step towards dynamic modeling is to consider an implicit factor GARCH model, which essentially models each factor as a GARCH processes (Alexander, 2001). To model time-varying covariances/correlations we consider two additional dynamic specifications. The full-factor multivariate GARCH model proposed by Vrontos et al. (2003) which falls in the class of implicit factor models; and the regime switching dynamic correlations model proposed by Pelletier (2005) which is a successful alternative - to GARCH models - for modeling conditional covariances/correlations. The following paragraphs present the details of each model.

Throughout the paper we consider having observed returns

\[ \mathbf{R}_t, \ t = 1, \ldots, T \]

where each

\[ \mathbf{R}_t = (R_1, R_2, \ldots, R_n) \]

is a \( n \times 1 \) vector of hedge fund returns in period \( t \).

### 3.1 Sample covariance model

The sample covariance matrix during the estimation period corresponds to the basis model (SAM hereafter) for our analysis. The sample covariance matrix is calculated through:

\[
V^{\text{SAM}} = \frac{1}{T-1} \sum_{t=1}^{T} (\mathbf{R}_t - \bar{\mathbf{R}})(\mathbf{R}_t - \bar{\mathbf{R}})^\prime
\]  

(4)

where \( T \) is the sample size and \( \bar{\mathbf{R}} \) is a \( n \times 1 \) vector containing the means of the return vectors. This model involves low specification error and high sampling error (Amenc and Martellini, 2002).

### 3.2 Implicit factor model

An implicit factor model involves extracting explanatory factors through principal components analysis of the funds’ returns data. With \( n \) fund return series under study there are \( n \) principal components, and these principal components are just linear combinations of the fund returns. The principal components are
constructed to be orthogonal to each other and to be normalized to have unit length. The implicit-factor model (IFAC hereafter) is formulated as follows (see, Zivot and Wang, 2002):

\[ R_{i,t} = \alpha_i + \sum_{k=1}^{K} \beta_{ik} f_{k,t} + \varepsilon_{i,t} \]  \hspace{1cm} (5)

where \( \alpha_i \) is a constant term, \( \beta_{ik} \) is the loading on factor \( k \) for the return of asset \( i \), \( f_{k,t} = R_0^* x_k^*, \) \( k = 1, ..., K \), \( x_k^* \) is the \( k^{th} \) \((n \times 1)\) eigenvector of the return covariance matrix. We select the number of principal components, \( K \), so that at least 85\% of the covariation in the portfolio is explained. Amenc and Martellini (2002) discuss alternative approaches for selecting \( K \). The covariance matrix of returns on a set of \( n \) hedge funds, \( V^{IFAC} \), is given by:

\[ V^{IFAC} = \mathbf{B} \Omega^{IFAC} \mathbf{B}' + \mathbf{D} \]  \hspace{1cm} (6)

where \( \mathbf{B} \) is the matrix of factor loadings, \( \Omega^{IFAC} \) is the diagonal covariance matrix of the implicit factors and \( \mathbf{D} \) is a diagonal matrix with elements in the main diagonal \( \sigma_i^2 = var(\varepsilon_{i,t}) \).

### 3.3 Implicit factor GARCH model

A modification of the standard implicit factor model has been proposed by Alexander (2001) to benefit from the success of GARCH models in financial markets volatility forecasting. In Alexander's (2001) spirit, we use principal component analysis to extract key implicit risk factors and estimate their volatilities on the basis of a univariate GARCH model. Given a GARCH (1,1) representation the variances of the implicit factors at time \( t \) are defined by:

\[ \sigma_{k,t}^2 = \omega_k + \gamma_k f_{k,t-1}^2 + \delta_k \sigma_{k,t-1}^2, \quad k = 1, ..., K, \quad t = 1, ..., T \]  \hspace{1cm} (7)

where \( \omega > 0 \), and \( \gamma, \delta \geq 0 \). The implicit factor GARCH models (IFAC-G hereafter) follows the formulation of Equation (5). The covariance matrix of the returns on a set of \( n \) hedge funds, \( V_t^{IFAC-G} \), is given by:

\[ V_t^{IFAC-G} = \mathbf{B} \Omega_t^{IFAC-G} \mathbf{B}' + \mathbf{D} \]  \hspace{1cm} (8)

where \( \Omega_t^{IFAC-G} \) is a diagonal covariance matrix containing the GARCH variances given by Equation (7).
3.4 Full-factor multivariate GARCH model

The full-factor multivariate GARCH model (FFMG hereafter) can be considered as an implicit factor model thus preserving implicit factor models’ properties. Moreover, it is capable of modeling the dynamics of time-varying variances and covariances/correlations. It is also flexible enough to capture many kinds of heteroscedastic behavior in hedge fund return series, i.e. volatility clustering, fat tails (high kurtosis). The FFMG model is defined by:

\[ R_t = \mu + \varepsilon_t \]
\[ \varepsilon_t = WF_t \]
\[ F_t|\Phi_{t-1} \sim N_n(0, \Sigma_t) \]

where \( \mu \) is a \( n \times 1 \) vector of constants, \( \varepsilon_t \) is a \( n \times 1 \) innovation vector, \( W \) is a lower triangular \( n \times n \) parameter matrix, \( F_t \) is a \( n \times 1 \) vector of factors with elements \( f_{i,t}, i = 1, ..., n \), and \( \Phi_{t-1} \) is the information available up to time \( t - 1 \). \( \Sigma_t \) is \( n \times n \) diagonal variance covariance matrix given by \( \Sigma_t = diag(\sigma^2_{1,t}, ..., \sigma^2_{n,t}) \) with

\[ \sigma^2_{i,t} = \alpha_i + b \sigma^2_{i,t-1} + g \sigma^2_{i,t-1}, \ i = 1, ..., n, \ t = 1, ..., T \]

where \( \sigma^2_{i,t}, i = 1, ..., n \) is the variance of the \( i^{th} \) factor at time \( t \), \( \alpha_i > 0, b \geq 0, g \geq 0, i = 1, ..., n \). Within this setting the factors \( f_{i,t}, i = 1, ..., n \) are GARCH(1,1) processes, and the vector \( \varepsilon_t \) is a linear combination of the factors \( f_{i,t}, i = 1, ..., n \). The conditional covariance matrix \( \Sigma_t \) of the returns on a set of \( n \) hedge funds is given by:

\[ \Sigma_t^{FFMG} = W \Sigma_t W' \] (10)

while the conditional correlations are given by:

\[ \rho_{j,k,t} = \frac{\min\{j,k\} \sum_{i=1}^{\min\{j,k\}} w_{ji} w_{ki} \sigma^2_{i,t}}{\left( \sum_{i=1}^{j} w^2_{ji} \sigma^2_{i,t} \right)^{1/2} \left( \sum_{i=1}^{k} w^2_{ki} \sigma^2_{i,t} \right)^{1/2}} \]

(11)

The conditional correlation is a non-linear function of all \( \sigma^2_{i,t}, i = 1, ..., \max\{j,k\} \) which also preserves the positive-definiteness of \( \Sigma_t \). Estimation of the parameter vector \( \theta = (\mu, \alpha_i, b, g, w_{ij}), i, j = 1, ..., n \) is achieved by using classical techniques. Maximum likelihood estimation can be implemented using the Fisher scoring
algorithm, since the gradient and the expected information matrices are available in closed form. A detailed discussion of the theoretical as well as the empirical properties of the FFMG model can be found in Vrontos et al. (2003).

3.5 Regime switching dynamic correlations model

Another class of models that is capable of capturing nonlinearities in financial time series is the class of regime switching models. Regime switching models have been used in the literature of hedge funds in a number of contexts including measuring the systemic risk (Chan et al., 2005), studying serial correlations (Getmansky et al., 2004), detecting switching strategies (Alexander and Dimitiu, 2005).

We employ a Regime Switching Dynamic Correlations (RSDC hereafter) model in the spirit of Pelletier’s (2005). This specification can accommodate time-varying variances and covariances/correlations. The RSDC model is defined by:

\[ \mathbf{R}_t = \mu + \varepsilon_t \]

\[ \varepsilon_t | \Phi_{t-1} \sim N_n (0, \mathbf{V}_t^{RSDC}) \]

where \( \mu \) is a \( n \times 1 \) vector of constants, \( \varepsilon_t \) is a \( n \times 1 \) innovation vector, \( \Phi_{t-1} \) is the information available up to time \( t-1 \). \( \mathbf{V}_t^{RSDC} \) is the \( n \times n \) covariance matrix which can be decomposed into

\[ \mathbf{V}_t^{RSDC} = \Sigma_t P_t \Sigma_t \]

where \( \Sigma_t \) is a diagonal matrix composed of the standard deviations \( \sigma_{i,t}, i = 1, \ldots, n \) and \( P_t \) is the correlation matrix. Both matrices are time varying. In particular, the conditional variances are modelled using a GARCH(1,1) specification of the form

\[ \sigma_{i,t}^2 = \alpha_i + b_i \varepsilon_{i,t-1}^2 + g_i \sigma_{i,t-1}^2, \ i = 1, \ldots, n, \ t = 1, \ldots, T \]

while the correlation matrix \( P_t \) is modelled in a dynamic framework by using

\[ P_t = \sum_{k=1}^{K} I(S_t = k) P_k \]
where \( I \) is the indicator function, \( S_t \) is an unobserved Markov chain process independent of \( \varepsilon_t \) which can take \( K \) possible values \((S_t = 1, 2, \ldots, K)\) and \( P_k \) are correlation matrices with \( P_k \neq P_{k'} \) for \( k \neq k' \). Regime switches in the state variable, \( S_t \), are assumed to be governed by the transition probability matrix \( \Pi \). The transition probabilities between states are assumed to follow a first order Markov chain and remain constant through time:

\[
\Pr(S_t = j | S_{t-1} = i, S_{t-2} = l, \ldots) = \Pr(S_t = j | S_{t-1} = i) = \pi_{ij}, \quad i, j = 1, \ldots, K
\]

We assume \( K = 2 \). The estimation of the RSDC model is achieved by using a two-step procedure (Engle, 2002). In the first step we estimate the univariate GARCH model parameters and in the second step we estimate the parameters in the correlation matrix and the transition probabilities \( \pi_{ij} \) conditional on the first step estimates. Details of the estimation procedure can be found in Pelletier (2005).

4 The data

We carry out our empirical investigation by using hedge fund index data from Hedge Fund Research (HFR hereafter). This choice makes our empirical analysis more relevant to style allocation decisions as in Amenc and Martellini (2002), McFall Lamm (2003), Agarwal and Naik (2004) and Morton et al. (2005). We consider eight HFR single strategy indices: Equity Hedge, Macro, Relative Value Arbitrage, Event-Driven, Convertible Arbitrage, Distressed Securities, Equity Market Neutral, and Merger Arbitrage\(^2\). The sample consists of monthly returns from January 1990 through to August 2005. It includes a number of crises that occurred in the ’90s, i.e. the Mexican, Asian, Russian, the LTCM crises as well as the IT bubble and the corporate scandals periods of the early ’00s. Crises cause large volatility variation and high kurtosis in the returns data.

Table 1 reports summary statistics for the HFR single strategy hedge fund index returns over the period of our analysis. Panel A in particular presents the mean, standard deviation, median, interquartile range,

\(^2\)These are the 8 single strategy indices for which HFR constructs investable counterparts called ‘HFRX investable strategy indices’. Details can be found in www.hfr.com.
skewness, kurtosis, minimum, and the maximum for fund index returns. We observe that the returns of the eight hedge fund strategies are very heterogeneous: there are strategies with relatively high volatilities and high average returns i.e. the Equity Hedge, Macro, Event Driven, and the Distressed Securities strategies. Amenc and Martellini (2002) highlight that these strategies act as return enhancers and could substitute some fraction of the portfolio’s equity holding. The Relative Value Arbitrage, Convertible Arbitrage, Equity Market Neutral, and the Merger Arbitrage strategies exhibit low volatilities and low average returns. They could be used in a portfolio to substitute some percentage of the fixed income or cash holdings. Differences in the higher order moments are also present. The kurtosis of the eight indexes’ returns ranges from 3.34 to 14.29, indicating fat-tailness in most of the return distributions. Panel B reports correlation coefficients computed for the HFR single strategy hedge fund index returns. We find that fund returns exhibit low to medium pairwise correlation in general. This is a desirable property in the context of efficient portfolio construction. Fund returns correlations range from a minimum of 0.17 between Equity Market Neutral and Distressed Securities, to a maximum of 0.79 for Event Driven and Distressed Securities. The average pairwise correlation is 0.47. Low correlation indicates a potential for risk diversification in hedge fund investment portfolios.

High kurtosis in hedge fund returns motivates, in principle, the use of dynamic covariance specifications. Further analysis of the data reveals time-variation of covariances and correlations, thus, providing additional support for the use of dynamic covariance/correlation models. To examine the variation of pairwise correlations through time, we consider the data covering the period January 1990 through to December 1999 and compute pairwise correlations. By sequentially adding index returns of subsequent months in the initial dataset we compute a series of 68 correlation coefficients for each possible pair. Figure 1 plots correlation coefficients for selected index pairs. We observe that pairwise correlations vary through time suggesting that modeling time-varying correlations may improve our ability to construct optimal portfolios. Within the set of models we compare, time-varying correlations can be modeled only with the FFMG specification (see Equation 11) or with the RSDC model (see Equation 14).

INSERT FIGURE 1 ABOUT HERE
Our preliminary analysis of the data concludes that hedge fund index returns generally exhibit high kurtosis and time-varying variances and covariances/correlations. These findings motivate the use of dynamic covariance/correlation specifications such as the FFMG and the RSDC.

5 Hedge fund portfolio performance

The objective of this study is to examine the benefits of introducing dynamic structures for the variances and covariances/correlations of hedge fund returns in hedge fund portfolio construction and risk measurement. This is achieved through an investment exercise which compares the empirical out-of-sample performance of the different methods of forecasting variances and covariances/correlations presented in Section 3. The setup of our experiments is as follows. We use a history of data covering the period January 1990 to December 2001 to estimate the parameters of the SAM, IFAC, IFAC-G, FFMG, and RSDC covariance matrixes. This period contains 144 return observations for each asset. We then construct optimal hedge fund portfolios. Two portfolios are constructed: (a) a conservative (no average annual return target - minimum variance portfolio3), and (b) an aggressive (average annual return target of 15.5%). Given the optimized weights we calculate buy-and-hold returns on the portfolio for a holding period of 1 month, at the end of which the estimation and optimization procedures are repeated until the dataset is exhausted. The estimation period grows by one data point every time we perform the optimization as in Krokhmal et al. (2002) in order to utilize all available information. This exercise produces 44 out-of-sample observations that cover a period of just over three and a half years, January 2002 to August 2005.

We assess the empirical performance of the covariance prediction models on several grounds.

First, we examine the realized returns of the constructed portfolios. Given the fund weights $X_t = (x_1, x_2, ..., x_n)_t$ at time $t$ and the realized returns of the individual $n$ assets at time $t+1$ in our sample, $R_{t+1} = (R_1, R_2, ..., R_n)^{\prime}_{t+1}$, the realized return $R_p$ of the portfolio at time $t+1$ is computed as

3This portfolio requires no estimate of the expected return, thus, allowing performance evaluation of competing covariance matrix estimators alone. For a discussion of this issue see Chan et al. (1999), for traditional assets, and Amenc and Martellini (2002), Alexander and Dimitriu (2004) for alternative investments.
We also calculate and discuss the cumulative returns for the entire period.

Second, we compare the return per unit of risk. Portfolio optimization will generally arrive at a different minimum variance for each covariance prediction model. As a result the realized return will not be comparable across models since it will represent portfolios bearing different risk. We define a measure similar in spirit to the Sharpe Ratio by standardizing the realized returns with the risk of the portfolio when it is constructed. We call this measure a Conditional Sharpe Ratio (CSR hereafter) and calculate it through:

\[
\text{CSR}_{p,t+1} = \frac{R_{p,t+1}}{\sqrt{\text{Var}(R_p)_t}}
\]

where \( \text{Var}(R_p)_t \) is determined through Equation 1.

Next, we set out to incorporate transaction costs. Transaction costs associated with hedge funds, however, are not generally easy to compute given the variation in early redemption, management or other types of fees (Alexander and Dimitriu, 2004). Nevertheless, if the gain in the performance does not cover the extra transaction costs, less accurate, but less variable weighting strategies would be preferred. To study this issue we define portfolio turnover as (Greyserman et al., 2005):

\[
\text{PT}_{t+1} = \sum_{i=1}^{n} |w_{i,t+1} - w_{i,t}|
\]

that is, the portfolio turnover in a given month is the sum of the absolute changes in the portfolio weights from the previous month to that month. This metric intuitively represents the fraction (in percentage terms) of the portfolio value that has to be liquidated/reallocated at the point of rebalancing.

Finally, we investigate the capacity of the different covariance prediction models to assess tail-risk. Agarwal and Naik (2004) focus on CVaR as a superior risk management tool to control the tail risk. The intuition of CVaR is as follows. Suppose a hedge fund portfolio is managing $1 billion. A CVaR of 1% at the 95% confidence level means that there is 5% probability that the average portfolio loss greater than or equal to the VaR can exceed $10 million. A relatively higher CVaR, 1.1% for example, calculates the same loss as
$11 million which suggests an economically significant difference. To compute CVaR in Equation 3, one can either impose a distributional assumption on the fund returns or use the empirical distribution of fund returns. We include both approaches in our analysis. We assume that portfolio returns follow a multivariate normal distribution with means and covariances determined by the respective covariance model. We also calculate CVaR by using the empirical distribution. The CVaR is calculated at the 90%, 95%, and 99% confidence levels.

Sections 5.1 and 5.2 present the results of two distinct case studies: the mean-variance case study and the mean-CVaR case study.

In the mean-variance case study optimal portfolios are constructed with the (a) sample covariance model, SAM, (b) implicit factor model, IFAC, (c) implicit factor GARCH model, IFAC-G, (d) full-factor multivariate GARCH model, FFMG, and (e) the regime switching dynamic correlations model, RSDC. The sample covariance model produces mean-variance portfolios whose weights are independent of the distributional assumption on the fund returns. As a result all performance metrics are also independent of the distributional assumption. Given the two approaches in CVaR calculation, however, the CVaR is calculated under the empirical distribution (this model is termed SAM-E), but also under the multivariate normal distributional assumption (this model is termed SAM-N).

In the mean-CVaR case study portfolios are constructed only with the sample covariance model and the empirical distribution of hedge fund returns. Rockafellar and Uryasev (2000) show that for normal loss distributions the mean-CVaR methodology is equivalent to the standard mean-variance approach. As a result the portfolios constructed in the mean-variance case study with returns assumed multivariate normal are also mean-CVaR optimal portfolios and can be used for comparison in this case study.

5.1 Out-of-sample performance of mean-variance optimal portfolios

Table 2 reports results of the out-of-sample performance of mean-variance efficient portfolios. Panel A, provides average and median values of the metrics presented above, calculated in the conservative portfolio construction exercise. Panel B, presents the average and median metrics’ values from the aggressive portfolio
construction exercise. Differences in the mean and median values of the metrics are examined through standard \(t\)- and Wilcoxon signed-rank tests. Table 3 presents results of these tests for mean and median ‘CSR’. Due to space limitations the remaining results of the \(t\)- and Wilcoxon signed-rank tests are not presented in detail.

First, we examine portfolio performance in terms of the cumulative returns. The top half of Figure 2 plots cumulative returns of the conservative and the aggressive portfolios in the out-of-sample period. The respective portfolio standard deviation is depicted on the bottom half of Figure 2. We find that the RSDC covariance prediction model determines the conservative structure with the highest cumulative return, 22.02%. The second best model for constructing conservative portfolios is the FFMG with out-of-sample cumulative return of 17.25%. The IFAC-G and IFAC covariance models achieve 15.37% and 15.29% respectively. The SAM cumulative returns are 14.95%. For aggressive portfolio construction, the FFMG model ranks first with a cumulative return of 37.52%, followed by IFAC with 36.17%, IFAC-G with 36.04, SAM with 35.98%, and RSDC with 34.95%. We should note here that this comparison penalizes models with low realized returns ignoring the risk of the constructed portfolios.

**INSERT FIGURE 2 ABOUT HERE**

**INSERT TABLE 2 ABOUT HERE**

Next, we focus on ‘Return’, ‘Risk’, and ‘CSR’. Table 2 reports mean and median values of these metrics. For the conservative portfolio construction exercise we find that the RSDC model computes structures that realize the highest average and median out-of-sample ‘Return’. The second best model is the FFMG. In terms of average and median ‘Risk’, FFMG ranks first and RSDC second. The ‘CSR’, ranks RSDC first and FFMG second. IFAC-G ranks third by means of ‘Return’, ‘Risk’, and ‘CSR’. Examination of the significance of the difference in the mean and median metrics’ values yields the following. The mean and median ‘Return’ of RSDC are statistically different from all other models’. The mean and median ‘Return’ of FFMG are also statistically different from all other models’. The mean and median ‘Return’ of IFAC and IFAC-G are not significantly different but are both different from SAM. The mean and median ‘Risk’ of RSDC are not different from the mean and median ‘Risk’ of FFMG but are different all other models’. The
mean and median ‘CSR’ are statistically different in all models. For the aggressive portfolio construction exercise we find that the mean and median ‘Return’ is the same for all models. The mean ‘Risk’ of RSDC is statistically different from the mean ‘Risk’ of all other models. The same holds for the mean ‘Risk’ of FFMG. The mean ‘Risk’ of IFAC-G, IFAC, and SAM are not statistically different. The median ‘Risk’ is different in all models. The mean ‘CSR’ is statistically different in all models. The median ‘CSR’ is the same for IFAC-G, IFAC, and SAM, but is different for RSDC and FFMG.

**INSERT TABLE 3 ABOUT HERE**

We also compare the capacity of the different covariance prediction models to assess tail risk and construct optimal portfolios with minimal tail risk. In line with Agarwal and Naik (2004), we compute the 90%, 95%, and 99% CVaR of mean-variance optimal portfolios and report mean and median values in Table 2. Mean and median CVaR are consistently lower when the RSDC specification is used. FFMG ranks second and IFAC-G third. For conservative portfolios we find that the mean and median 90%, 95%, and 99% CVaR are significantly different in all models. For aggressive portfolios we find that the mean 90%, 95%, and 99% CVaR of RSDC are different from FFMG’s and are both different from the means of all other models. Median 90%, 95%, and 99% CVaR, however, are significantly different in all models.

To this point, the general conclusion is that the RSDC model improves our ability to construct optimal hedge fund portfolios and to measure tail-risk. FFMG does also very well in that respect. The last column of Table 2 reports average and median PT values to study the cost of rebalancing strategies implied by the different covariance prediction models. It appears that the RSDC specification requires a higher proportion of the portfolio to be restructured at each rebalancing point which imposes a higher transaction cost. Median values of 55.61% and 61.73% are calculated for conservative and aggressive portfolios respectively. FFMG is the second most expensive with respective values of 12.84% and 14.97%. IFAC-G ranks third with 4.36% and 7.67%. Static specifications imply very similar rebalancing strategies costwise. We find that the median PT of static rebalancing schemes for conservative portfolios is only a small fraction of the respective PT of dynamic schemes and increases substantially - but remains lower than the dynamic models’ - for aggressive portfolios. We can not conclude with certainty whether transaction costs are compensated for in more
variable weighting strategies since the actual transaction cost is not easy to estimate in the case of hedge funds and funds of funds. Moreover transaction costs may vary with the ‘buyer’, i.e. size of a fund of funds, and the ‘seller’, i.e. liquid vs. less liquid strategies, which makes it even more difficult to create a uniform decision rule.

Finally, we discuss the ‘average’ structure of hedge fund portfolios constructed with the different covariance prediction models. Table 4 provides average weights of conservative portfolios in Panel A and aggressive portfolios in Panel B. The weights of the assets in the conservative portfolio are similar for the different covariance models. We note that four strategies, Equity Hedge, Macro, Event Driven, and Distressed Securities, are not included in the ‘average’ structure of any covariance model. This does not surprise us since these strategies exhibit the highest volatility (see Table 1). Conversely, the largest fraction of the portfolio is allocated in Equity Market Neutral which has the lowest volatility over the study period. Amenc and Martellini (2002) report similar results. We also note that RSDC and FFMG favor funds that exhibit high kurtosis, i.e. Relative Value Arbitrage and Merger Arbitrage relative to other models while the opposite holds for Equity Market Neutral which has almost no kurtosis. Aggressive portfolio construction yields some very interesting results. SAM, IFAC, and IFAC-G, try to achieve the benchmark return by using assets with high volatilities, i.e. Equity Hedge, Macro, Distressed Securities, and as a result the constructed portfolio exhibits high risk. On the other hand, RSDC favors less volatile assets. Almost zero investment in Macro is calculated and Convertible Arbitrage is preferred to Relative Value Arbitrage and Equity Market Neutral. FFMG is somewhere in the middle. Almost zero capital is allocated to Macro which reduces the risk of the portfolio relative to SAM, IFAC, and IFAC-G. Event Driven is selected as with RSDC. Equity hedge is also selected as with SAM, IFAC, and IFAC-G. These results are in line with Morton et al.’s (2005) approach which uses a data set of CSFB/Tremont hedge fund indexes.

\textbf{INSERT TABLE 4 ABOUT HERE}\n
In summary, we have found that modeling time varying variances and covariances/correlations of hedge fund returns improves our ability to optimize hedge fund portfolio risk. This is reflected in the reduced risk of the portfolios constructed with the dynamic covariance models relative to the risk of the portfolios
constructed with the other models. It is also reflected in the portfolio ‘CSR’, which ranks RSDC first, FFMG second, and IFAC-G, IFAC, and SAM, third, fourth, and fifth respectively. The difference in the mean and median ‘CSR’ is almost always significant at the 5% level. In addition, we have shown that the RSDC covariance model improves our ability in risk measurement and confirmed that this result is statistically significant. The overall ranking of covariance models in terms of risk measurement resembles the ‘CSR’ ranking. We have found, however, that RSDC imposes a substantially more variable weighting strategy than other models. FFMG also requires variable rebalancing but its cost is closer to that imposed by the other covariance models. IFAC-G’s cost of rebalancing is even closer to the static models’.

5.2 Out-of-sample performance of mean-CVaR optimal portfolios

This case study involves constructing portfolios with the sample covariance model and the empirical distribution of fund returns. The results of this case study are summarized in Table 5. These results correspond to portfolios constructed through Equation 2 for a target return of 15.5% as in Section 5.1, an aggressive portfolio, and for probability levels of 90%, 95%, and 99%. A portfolio with minimum 90% CVaR subject to the target mean is denoted with ‘mean-CVaR90’. Portfolios constructed at different probability levels are denoted accordingly. We note that a conservative, minimum-variance portfolio cannot be constructed within the mean-CVaR framework.

First we examine cumulative returns. The out-of-sample performance of aggressive mean-CVaR optimal portfolios is depicted on the top half of Figure 2. The mean-CVaR90 aggressive portfolio yields cumulative returns of 34.12%. These returns are lower than the cumulative returns of mean-variance optimal portfolios constructed with the same covariance model, SAM, indicating that minimization of tail-risk - under the empirical distribution - comes at the expense of some fraction of the cumulative return.

Our discussion of performance metrics focuses on ‘CSR’. We refer to the results presented in Table 5 and those presented in Panel B of Table 2. We find that the average and median ‘CSR’ of portfolios is statistically different from the average and median ‘CSR’ of the mean-variance RSDC, FFMG, and IFAC-G
portfolios. Also, the average and median ‘CSR’ of mean-CVaR95 portfolios is statistically different from the average and median ‘CSR’ of the mean-variance RSDC, FFMG, and IFAC-G, IFAC, and SAM portfolios but not different from the mean and median ‘CSR’ of mean-CVaR90 portfolios. Finally, the average and median ‘CSR’ of mean-CVaR99 portfolios is statistically different from the average and median ‘CSR’ of the mean-variance RSDC, FFMG, and IFAC-G, IFAC, and SAM portfolios and mean-CVaR90, mean-CVaR95 portfolios.

By referring to the same tables we also compare the CVaR of mean-variance optimal portfolios with the CVaR of mean-CVaR90, 95, 99 portfolios. For the empirical distribution, the general rule is that the CVaR of mean-CVaR portfolios is lower only when it is the optimization’s objective, i.e. the CVaR90 of mean-CVaR90 portfolios is lower than the CVaR90 of portfolios constructed with SAM-E. This result is statistically significant. When compared with the CVaR of mean-variance IFAC-G, IFAC, and SAM-N portfolios the CVaR of mean-CVaR portfolios is most of times higher and significant. The RSDC and FFMG, on the other hand, compute consistently lower average and median CVaR values. The difference in the mean and median CVaR is statistically significant.

Following the discussion in Section 5.1 we examine the general cost of maintaining a portfolio with minimal tail risk. The last column of Table 5 provides average and median \( PT \) values. We find that the average cost of constructing mean-CVaR efficient portfolios is almost the same as the cost of constructing mean-variance efficient portfolios with any of the static covariance prediction models. The cost is very low for mean-CVaR99 portfolios. This is due to a very conservative, stable weighting strategy imposed.

Finally, we discuss the ‘average’ structure of mean-CVaR efficient hedge fund portfolios. Table 4 provides average weights of aggressive portfolios. We note that the structure of aggressive portfolios is similar in the mean-variance (SAM-E and SAM-N, IFAC, and IFAC-G) and the mean-CVaR cases when the objective is to minimize 90% or 95% CVaR. At the 99% most of the capital is invested in Equity Market Neutral, Macro, and Equity Hedge. These strategies contribute positive skewness in the portfolio and at the same time reduce its kurtosis. We generally observe that, as our objective becomes more conservative, i.e. minimizing 99% relative to minimizing 90% CVaR, less diversified structures are computed. One explanation to this
feature can be the following. The mean-CVaR algorithm seeks minimal tail risk portfolios or equivalently, portfolios exhibiting minimal kurtosis. As a result, the more conservative the objective becomes, the more likely it is that assets exhibiting high kurtosis are excluded and assets exhibiting low kurtosis are included in the portfolio.

In summary, we have shown that the cumulative returns of mean-CVaR optimal portfolios are generally lower than the cumulative returns of mean-variance optimal portfolios. We have also shown that mean and median ‘CSR’ of mean-CVaR optimal portfolios are lower than mean and median ‘CSR’ of mean-variance optimal portfolios and that this difference is most of the times statistically significant at the 5% level. Also, the CVaR of mean-CVaR optimal portfolios is only lower than the CVaR of mean-variance portfolios constructed with the SAM-E model. We have found that the cost of maintaining a mean-CVaR optimal portfolio is similar on average with the cost of maintaining a mean-variance (constructed with the SAM and IFAC covariance models) efficient portfolio with the same benchmark return. Finally, we have found similar allocations between mean-CVaR and mean-variance (SAM, IFAC, and IFAC-G) efficient portfolios. The portfolio structure changes significantly for more conservative structures i.e. mean-CVaR99 portfolios, where the largest fraction of the portfolio is invested in assets exhibiting low kurtosis.

5.3 Additional results

In additional, unreported work (available from the authors upon request) we have extended our analysis to incorporate: (a) one additional target return, 14.5%, (b) one alternative rebalancing frequency, 3 months, (c) one shorter estimation period, 120 months, and one longer estimation period, 168 months, 2 years apart each from the 144 months period used in the experiments reported in the previous sections, (d) returns in the excess of the risk free rate\(^4\) as opposed to absolute returns used in the reported results. In addition we have performed our experiments under various combinations of the latter.

The study of mean-variance and mean-CVaR optimal portfolios targeting annual expected return of 14.5% neither offers any additional evidence nor it contradicts any of the conclusions that drawn in this analysis. In fact, the results are veryy similar with those presented for the benchmark return 15.5%.

\(^4\)We use the 3-month US Treasury-bill as a proxy for the risk free rate.
The findings for the three months rebalancing are summarized as follows. For conservative portfolios the cumulative return of the RSDC model is 20.46%, the FFMG model is 15.81%, the IFAC-G 13.70%, the IFAC 13.64%, and the SAM-E and SAM-N 13.30%. These are lower than the cumulative returns achieved with the respective models and 1-month rebalancing. The mean and median CSR of the RSDC are 1.44 and 1.06 respectively. The FFMG mean and median CSR are 1.07 and 1.10 respectively. These values are not statistically different but they are different - and higher - from the respective figures of IFAC-G, IFAC, and SAM portfolios. These are also the conclusions drawn in Section 5.1.

The use of longer/shorter estimation periods and/or excess returns also does not alter our conclusions. For example, in the case that the first 120 months of excess returns are used to estimate the different covariance models, and aggressive portfolios are constructed with monthly rebalancing. The cumulative excess returns rank RSDC first with 38.18%, FFMG second with 36.22%, IFAC, SAM, and IFAC-G, with 35.14%, 34.94%, and 34.93% respectively. The ‘CSR’ ranks RSDC first with statistically different mean and median with all other models.

In summary, we examined if certain preferences in our investment exercise, i.e. target returns, frequency of rebalancing, size of the estimation period, excess returns, have an impact in the conclusions of Sections 5.1 and 5.2. We have found that the notion of our results remains unchanged.

6 Conclusion

Despite the facts that hedge funds are dynamic investments, the variance and covariance of most financial time series - the funds’ underlying assets - are time-varying, the time series of fund returns exhibit volatility clustering and high kurtosis, to date studies do not account for possible time-variance of the variances and covariances/correlations of hedge fund returns.

This article addresses the issue of time-varying variances and covariances/correlations of hedge fund returns and concentrate on the potential impacts in terms of hedge fund portfolio construction and risk measurement. We compare the performance of different methods of forecasting variances and covariances/correlations and judge which model improves our ability to construct optimal hedge fund portfolios.
and measure tail-risk.

We find that a regime switching dynamic correlations model, RSDC, reduces portfolio risk and improves the out-of-sample risk-adjusted realized returns. We also find that the CVaR of the portfolio constructed with the RSDC model is the lowest among alternative covariance models. This suggests that the RSDC covariance model represents a more accurate tool for tail-risk measurement. These results are statistically significant. The full-factor multivariate GARCH model, FFMG ranks second with significant differences. The implicit factor GARCH, implicit factor, and sample covariance models rank third, fourth, and fifth with average and median metrics’ values that in most cases are not statistically different. When we study the cost of rebalancing we find that the RSDC imposes substantially higher transaction costs than the FFMG which is the second most variable weighting strategy. Changing various preferences in our investment exercise, i.e. portfolio rebalancing period, estimation period, did not alter the overall verdict that the RSDC and the FFMG models provide a superior tool for portfolio choice and risk measurement among the considered methodologies.

Acknowledgements

The authors thank the two referees for their detailed and constructive advice. We also thank Elias Tzavalis for his comments and advice, Anca Dimitriu, George Leledakis, Loukia Meligkotsidou, George Skiadopoulos for their suggestions, and Dimitris Alexopoulos, George Chalamandaris, Dimitris Flamouris, Manolis Liodakis, Vassilios Siokis, Michael Stelianos, for helpful discussions on practical issues. A previous version of this paper was presented in 2005 at the Quant Congress USA, International Summer School in Risk Measurement and Control, 12th Annual Conference of the Multinational Finance Society, and the 2nd Advances in Financial Forecasting International Symposium. We thank participants at these meetings for their comments.
References


[31] Popova, I., D.P. Morton, and E. Popova, 2003, Optimal hedge fund allocation with asymmetric preferences and distributions, Working paper, University of Texas at Austin


A Tables and Figures
Table 1: Summary statistics

Panel A: HFR single strategy hedge fund index descriptive statistics

<table>
<thead>
<tr>
<th>Hedge fund strategy</th>
<th>Mean</th>
<th>SD</th>
<th>Median</th>
<th>P75%-P25%</th>
<th>Skew</th>
<th>Kurtosis</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity hedge</td>
<td>1.36</td>
<td>2.55</td>
<td>1.40</td>
<td>3.12</td>
<td>-0.09</td>
<td>4.34</td>
<td>-7.65</td>
<td>0.88</td>
</tr>
<tr>
<td>Macro</td>
<td>1.25</td>
<td>2.40</td>
<td>0.84</td>
<td>2.80</td>
<td>0.24</td>
<td>3.52</td>
<td>-6.40</td>
<td>7.88</td>
</tr>
<tr>
<td>Relative value arbitrage</td>
<td>0.95</td>
<td>1.04</td>
<td>0.92</td>
<td>1.18</td>
<td>0.06</td>
<td>13.15</td>
<td>-5.80</td>
<td>5.72</td>
</tr>
<tr>
<td>Event-driven</td>
<td>1.17</td>
<td>1.89</td>
<td>1.36</td>
<td>1.91</td>
<td>-0.16</td>
<td>7.67</td>
<td>-8.90</td>
<td>5.13</td>
</tr>
<tr>
<td>Convertible arbitrage</td>
<td>0.81</td>
<td>1.03</td>
<td>1.01</td>
<td>1.11</td>
<td>-0.14</td>
<td>4.87</td>
<td>-3.19</td>
<td>3.33</td>
</tr>
<tr>
<td>Distressed securities</td>
<td>1.21</td>
<td>1.75</td>
<td>1.14</td>
<td>1.90</td>
<td>0.08</td>
<td>8.48</td>
<td>-8.50</td>
<td>7.06</td>
</tr>
<tr>
<td>Equity market neutral</td>
<td>0.74</td>
<td>0.91</td>
<td>0.66</td>
<td>1.09</td>
<td>0.08</td>
<td>3.34</td>
<td>-1.67</td>
<td>3.59</td>
</tr>
<tr>
<td>Merger arbitrage</td>
<td>0.82</td>
<td>1.23</td>
<td>1.04</td>
<td>1.15</td>
<td>-0.12</td>
<td>14.29</td>
<td>-6.46</td>
<td>2.90</td>
</tr>
</tbody>
</table>

Minimum | 0.74 | 0.91 | 0.66 | 1.09 | -0.16 | 3.34 | -8.90 | 2.90 |
Maximum  | 1.36 | 2.55 | 1.40 | 3.12 | 0.24  | 14.29| -1.67 | 10.88 |

Panel B: HFR single strategy hedge fund index correlations

<table>
<thead>
<tr>
<th>Hedge fund strategy</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity hedge</td>
<td>(1)</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Macro</td>
<td>(2)</td>
<td>0.59</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relative value arbitrage</td>
<td>(3)</td>
<td>0.53</td>
<td>0.39</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Event-driven</td>
<td>(4)</td>
<td>0.77</td>
<td>0.55</td>
<td>0.62</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Convertible arbitrage</td>
<td>(5)</td>
<td>0.45</td>
<td>0.40</td>
<td>0.59</td>
<td>0.56</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distressed securities</td>
<td>(6)</td>
<td>0.58</td>
<td>0.46</td>
<td>0.67</td>
<td>0.79</td>
<td>0.54</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>Equity market neutral</td>
<td>(7)</td>
<td>0.36</td>
<td>0.25</td>
<td>0.25</td>
<td>0.21</td>
<td>0.19</td>
<td>0.17</td>
<td>1.00</td>
</tr>
<tr>
<td>Merger arbitrage</td>
<td>(8)</td>
<td>0.49</td>
<td>0.31</td>
<td>0.45</td>
<td>0.73</td>
<td>0.45</td>
<td>0.51</td>
<td>0.23</td>
</tr>
</tbody>
</table>

This table presents summary statistics of monthly returns for eight HFR indexes from January 1990 through to August 2005. The summary statistics include mean, standard deviation (SD), median, interquartile range (P75%-P25%), skewness (Skew), kurtosis, minimum (Min) and maximum (Max), minimum and maximum values of each of the statistic among the indexes in percentage terms (Panel A), and correlations (Panel B). It is noted that the skewness is measured with the quartile or Bowley skewness coefficient to avoid misinterpretations due to outliers in the data set.
Table 2: Out-of-sample performance of Mean-Variance efficient portfolios with 1-month rebalancing

Panel A: Conservative (minimum variance)

<table>
<thead>
<tr>
<th>Model</th>
<th>Return</th>
<th>Risk</th>
<th>CSR</th>
<th>CVaR90</th>
<th>CVaR95</th>
<th>CVaR99</th>
<th>PT</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAM-E</td>
<td>0.34 [0.39]</td>
<td>0.71 [0.71]</td>
<td>0.48 [0.55]</td>
<td>0.46 [0.46]</td>
<td>0.79 [0.79]</td>
<td>2.06 [1.89]</td>
<td>0.99 [0.64]</td>
</tr>
<tr>
<td>SAM-N</td>
<td>0.34 [0.39]</td>
<td>0.71 [0.71]</td>
<td>0.48 [0.55]</td>
<td>0.37 [0.36]</td>
<td>0.59 [0.58]</td>
<td>1.01 [1.00]</td>
<td>0.99 [0.64]</td>
</tr>
<tr>
<td>IFAC</td>
<td>0.35 [0.41]</td>
<td>0.69 [0.69]</td>
<td>0.50 [0.58]</td>
<td>0.34 [0.34]</td>
<td>0.55 [0.55]</td>
<td>0.97 [0.97]</td>
<td>0.81 [0.55]</td>
</tr>
<tr>
<td>IFAC-G</td>
<td>0.35 [0.43]</td>
<td>0.68 [0.68]</td>
<td>0.52 [0.59]</td>
<td>0.32 [0.31]</td>
<td>0.53 [0.52]</td>
<td>0.94 [0.94]</td>
<td>6.38 [4.36]</td>
</tr>
<tr>
<td>FFGM</td>
<td>0.39 [0.43]</td>
<td>0.60 [0.60]</td>
<td>0.64 [0.70]</td>
<td>0.14 [0.15]</td>
<td>0.32 [0.32]</td>
<td>0.69 [0.69]</td>
<td>16.97 [12.84]</td>
</tr>
<tr>
<td>RSDC</td>
<td>0.50 [0.50]</td>
<td>0.63 [0.62]</td>
<td>0.85 [0.74]</td>
<td>0.08 [0.06]</td>
<td>0.27 [0.26]</td>
<td>0.65 [0.65]</td>
<td>71.50 [55.61]</td>
</tr>
</tbody>
</table>

Panel B: Aggressive (target expected return 15.5%)

<table>
<thead>
<tr>
<th>Model</th>
<th>Return</th>
<th>Risk</th>
<th>CSR</th>
<th>CVaR90</th>
<th>CVaR95</th>
<th>CVaR99</th>
<th>PT</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAM-E</td>
<td>0.82 [0.89]</td>
<td>1.79 [1.81]</td>
<td>0.44 [0.48]</td>
<td>1.80 [1.85]</td>
<td>2.49 [2.55]</td>
<td>4.85 [4.67]</td>
<td>7.29 [6.93]</td>
</tr>
<tr>
<td>SAM-N</td>
<td>0.82 [0.89]</td>
<td>1.79 [1.81]</td>
<td>0.44 [0.48]</td>
<td>1.85 [1.89]</td>
<td>2.40 [2.45]</td>
<td>3.47 [3.54]</td>
<td>7.29 [6.93]</td>
</tr>
<tr>
<td>IFAC</td>
<td>0.82 [0.90]</td>
<td>1.78 [1.81]</td>
<td>0.45 [0.48]</td>
<td>1.84 [1.88]</td>
<td>2.39 [2.44]</td>
<td>3.47 [3.53]</td>
<td>7.34 [6.84]</td>
</tr>
<tr>
<td>IFAC-G</td>
<td>0.82 [0.92]</td>
<td>1.73 [1.74]</td>
<td>0.47 [0.51]</td>
<td>1.75 [1.76]</td>
<td>2.28 [2.29]</td>
<td>3.33 [3.34]</td>
<td>9.70 [7.67]</td>
</tr>
<tr>
<td>FFGM</td>
<td>0.85 [0.83]</td>
<td>1.49 [1.47]</td>
<td>0.58 [0.65]</td>
<td>1.33 [1.28]</td>
<td>1.79 [1.73]</td>
<td>2.69 [2.62]</td>
<td>17.33 [14.97]</td>
</tr>
<tr>
<td>RSDC</td>
<td>0.79 [0.81]</td>
<td>1.02 [0.99]</td>
<td>0.88 [0.84]</td>
<td>0.50 [0.44]</td>
<td>0.81 [0.75]</td>
<td>1.43 [1.34]</td>
<td>73.68 [61.73]</td>
</tr>
</tbody>
</table>

This table presents out-of-sample results of the mean-variance portfolio construction case study. These include mean and median values of realized return (Return), portfolio standard deviation (Risk) in percentage terms, Conditional Sharp Ratio (CSR), Conditional Value at Risk (CVaR) at the 90% (CVaR90), 95% (CVaR95) and 99% (CVaR99), and portfolio turnover (PT) in percentage terms in the period January 2002 to August 2005. Mean and median values are reported for a minimum variance portfolio, the ‘Conservative’ portfolio (Panel A) and a portfolio with a target expected annual return of 15.5%, the ‘Aggressive’ portfolio (Panel B). The Return, Risk, CSR, and PT of SAM-E and SAM-N are identical since both models determine allocations on the basis of the mean and covariance matrix of the sample data. Median values are reported in brackets.
Table 3: $t$-test and Wilcoxon signed rank test for the difference in mean and median CSR

Panel A: Conservative (minimum variance)

<table>
<thead>
<tr>
<th>Model</th>
<th>SAM</th>
<th>IFAC</th>
<th>IFAC-G</th>
<th>FFMG</th>
<th>RSDC</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAM-E/N</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IFAC</td>
<td>1 (1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IFAC-G</td>
<td>1 (1)</td>
<td>1 (1)</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FFMG</td>
<td>1 (1)</td>
<td>1 (1)</td>
<td>1 (1)</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>RSDC</td>
<td>1 (1)</td>
<td>1 (1)</td>
<td>1 (1)</td>
<td>1 (1)</td>
<td>-</td>
</tr>
</tbody>
</table>

Panel B: Aggressive (target expected return 15.5%)

<table>
<thead>
<tr>
<th>Model</th>
<th>SAM</th>
<th>IFAC</th>
<th>IFAC-G</th>
<th>FFMG</th>
<th>RSDC</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAM-E/N</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IFAC</td>
<td>1 (1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IFAC-G</td>
<td>1 (0)</td>
<td>1 (0)</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FFMG</td>
<td>1 (1)</td>
<td>1 (1)</td>
<td>1 (1)</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>RSDC</td>
<td>1 (1)</td>
<td>1 (1)</td>
<td>1 (1)</td>
<td>1 (1)</td>
<td>-</td>
</tr>
</tbody>
</table>

This table presents results of the $t$- and Wilcoxon signed-rank tests at the 5%. Wilcoxon signed-rank test results are reported in parenthesis. A value of 0 indicates that the null hypothesis ‘the difference between the mean (median) CSR, of the respective models, is zero’ cannot be rejected at the 5%. A value of 1 indicates that the null hypothesis can be rejected.
Table 4: Portfolio composition with 1-month rebalancing

Panel A: Conservative (minimum variance)

<table>
<thead>
<tr>
<th>Hedge fund strategy</th>
<th>SAM</th>
<th>IFAC</th>
<th>IFAC-G</th>
<th>FFMG</th>
<th>RSDC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity hedge</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Macro</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.57</td>
</tr>
<tr>
<td>Relative value arbitrage</td>
<td>13.71</td>
<td>17.63</td>
<td>18.35</td>
<td>23.48</td>
<td>20.74</td>
</tr>
<tr>
<td>Event-driven</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Convertible arbitrage</td>
<td>31.13</td>
<td>30.46</td>
<td>30.39</td>
<td>26.85</td>
<td>29.82</td>
</tr>
<tr>
<td>Distressed securities</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.53</td>
</tr>
<tr>
<td>Equity market neutral</td>
<td>46.56</td>
<td>46.55</td>
<td>44.85</td>
<td>39.14</td>
<td>39.99</td>
</tr>
<tr>
<td>Merger arbitrage</td>
<td>8.60</td>
<td>5.36</td>
<td>6.40</td>
<td>10.53</td>
<td>8.34</td>
</tr>
</tbody>
</table>

Panel B: Aggressive (target expected return 15.5%)

<table>
<thead>
<tr>
<th>Hedge fund strategy</th>
<th>SAM</th>
<th>IFAC</th>
<th>IFAC-G</th>
<th>FFMG</th>
<th>RSDC</th>
<th>SAM-E 90%</th>
<th>95%</th>
<th>99%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity hedge</td>
<td>36.10</td>
<td>36.45</td>
<td>36.81</td>
<td>39.49</td>
<td>1.92</td>
<td>31.59</td>
<td>42.72</td>
<td>40.14</td>
</tr>
<tr>
<td>Macro</td>
<td>22.66</td>
<td>21.88</td>
<td>21.01</td>
<td>0.16</td>
<td>1.37</td>
<td>31.77</td>
<td>21.82</td>
<td>43.91</td>
</tr>
<tr>
<td>Relative value arbitrage</td>
<td>18.43</td>
<td>18.35</td>
<td>18.49</td>
<td>16.55</td>
<td>5.81</td>
<td>19.09</td>
<td>16.57</td>
<td>0.00</td>
</tr>
<tr>
<td>Event-driven</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>6.30</td>
<td>7.93</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Convertible arbitrage</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>5.03</td>
<td>40.20</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Distressed securities</td>
<td>22.81</td>
<td>23.31</td>
<td>23.67</td>
<td>32.21</td>
<td>30.65</td>
<td>17.49</td>
<td>14.22</td>
<td>0.00</td>
</tr>
<tr>
<td>Equity market neutral</td>
<td>0.00</td>
<td>0.00</td>
<td>0.01</td>
<td>0.00</td>
<td>0.78</td>
<td>0.07</td>
<td>4.66</td>
<td>15.95</td>
</tr>
<tr>
<td>Merger arbitrage</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.25</td>
<td>11.35</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

This table shows average weights of the eight hedge fund indexes in Mean-Variance and Mean-CVaR optimal portfolios in the period January 2002 to August 2005. Mean-CVaR optimal portfolios are constructed under two minimization objectives which correspond to the 90%, 95%, and 99% CVaR minimization. Panel A presents ‘Conservative’ and Panel B ‘Aggressive’ portfolio average optimal weights.
Table 5: Out-of-sample performance of Mean-CVaR efficient portfolios with 1-month rebalancing

Aggressive (target expected return 15.5%)

<table>
<thead>
<tr>
<th>Model</th>
<th>Return</th>
<th>Risk</th>
<th>CSR</th>
<th>CVaR90</th>
<th>CVaR95</th>
<th>CVaR99</th>
<th>PT</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAM-E</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CVaR90</td>
<td>0.78 [0.81]</td>
<td>1.80 [1.83]</td>
<td>0.42 [0.44]</td>
<td>1.78 [1.82]</td>
<td>2.50 [2.55]</td>
<td>4.71 [4.43]</td>
<td>9.07 [5.46]</td>
</tr>
<tr>
<td>CVaR95</td>
<td>0.72 [0.84]</td>
<td>1.82 [1.83]</td>
<td>0.39 [0.43]</td>
<td>1.83 [1.87]</td>
<td>2.45 [2.48]</td>
<td>4.64 [4.44]</td>
<td>7.51 [3.50]</td>
</tr>
<tr>
<td>CVaR99</td>
<td>0.62 [0.69]</td>
<td>1.97 [1.98]</td>
<td>0.31 [0.35]</td>
<td>1.95 [1.94]</td>
<td>2.60 [2.57]</td>
<td>4.11 [4.03]</td>
<td>2.39 [1.77]</td>
</tr>
</tbody>
</table>

This table presents results of the Mean-CVaR portfolio construction case study. Mean-CVaR optimal portfolios are constructed under three minimization objectives which correspond to the 90%, 95%, and 99% CVaR minimization. The results include mean and median values in percentage terms of realized return (Return), portfolio standard deviation (Risk), Conditional Sharp Ratio (CSR), Conditional Value at Risk (CVaR) at the 90% (CVaR90), 95% (CVaR95) and 99% (CVaR99) levels, and portfolio turnover (PT) in the period January 2002 to August 2005. Mean and median values are reported for a portfolio with a target expected annual return of 15.5%, the ‘Aggressive’ portfolio. Median values are reported in brackets.
Figure 1: Correlation coefficients for selected pairs including the Equity hedge (EH), Relative value arbitrage (RVA), Event driven (ED), Convertible arbitrage (CA), Distressed securities (DS), Equity market neutral (EMN), and the Merger arbitrage (MA) hedge fund indexes. The first point of each series corresponds to the respective correlation during January 1990 to December 1999. Subsequent points are computed by sequentially adding index returns in the initial sample.
Figure 2: Cumulative returns of ‘Conservative’ and ‘Aggressive’ portfolios in the out-of-sample period January 2002 to August 2005 are presented on the top half. The bottom half presents the respective portfolio standard deviation.