Private Equity Returns: Is there really a Benefit of low Co-movement with Public Equity Markets?*

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Abstract

In this paper we show how cross-sectional correlations between Private Equity (PE) and Public Market Equity (PM) returns can be approximated, resolving the lack of time series PE data. Based on a sample comprising 2,380 realized PE projects, we observe low cross-sectional correlations between PE and stock index returns (e.g. 0.18 for Nasdaq Composite Index). Despite overall low correlations, we find that investor can not benefit from a diversification effect when they really need it - during times of low PM returns.

We point out differences between early and later stage PE investments regarding hedging capabilities triggered by distinct valuation approaches and exit channels. In PE, financial statements become more relevant for valuation purposes as firms mature. As early stage investors do not have meaningful fundamental information they rely more on comparable market valuation when pricing acquisitions. Furthermore, IPOs are their most important exit channel. In contrast, later stage investors price projects based on fundamental information and have better functioning alternative exit channels, such as trade sales or secondary transactions. They thus are less dependent on PM developments. We find significant statistical evidence for the negative correlation between early stage investments and excess returns over PM in bearish market conditions. Later stage investments exhibit a significant positive relation with excess returns over low PM returns.

Investors seeking diversification benefits in PE should not solely rely on correlation coefficients, but need to carefully consider a fund’s investment strategy. The conflict of objectives for Venture funds between active market timing and smoothing of PM returns is resolved in favor of active market timing. Buyout funds’ returns are less dependent on PM developments and are thus the better complement for PM portfolios if investors seek diversification.

* All Private Equity data for this project were obtained from the CEPRES Center of Private Equity Research (www.cepres.de). Market quotes and exchange rates were obtained form Bloomberg and Datastream. I gratefully acknowledge the participants of the 18th Australasian Conference of Banking and Finance in Sydney and of the finance seminar at the University of Lugano for their helpful comments.

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1 Introduction

The interest of investors in Private Equity (PE) substantially increased over the past decade.\(^1\) Total PE Funds raised p.a. in Europe increased by 310% from \(m\)€ 6,693 in 1994 to \(m\)€ 27,451 in 2004.\(^2\) In general investors are attracted by high returns and diversification potential for their portfolios. While it has been widely discussed that investment decisions need to be taken in a portfolio context, this is not straightforward in case of PE. In comparison to public market equity (PM) there are two major characteristics of PE: No liquid secondary markets and very restrictive disclosure politics of market participants. Time series of market values are thus not observable for investments. But portfolio construction in a mean-variance framework requires expected returns, variance and correlation factors between asset classes as input factors.\(^3\) In this paper we strive to analyze the co-movement between PE and PM returns and approximate cross-sectional correlation factors. Furthermore we want to evaluate whether there is indeed a benefit from a loose relationship between PE and PM returns. Do PE projects still generate solid profits in times of low public stock index returns? Considering that timing of IPOs to hit valuation peaks of public stock markets is an important success factor for Venture Capital (VC) funds, it is questionable whether investor can benefit from a diversification effect when they really need it - during times of low PM returns.\(^4\) Furthermore, the VC market is highly cyclical.\(^5\) Our empirical analysis is based on a unique data sample of 2,380 realized PE projects derived from the records of the Center of Private Equity Research (CEPRES).\(^6\)

Comparable pairs of PE and PM returns form the foundation of our analysis. We apply a simulation approach introduced in our previous work to generate cash flow

\(^1\) Private Equity investments are equity capital investments in privately held, non-quoted companies. This definition includes both Venture Capital (VC) and Buyout (BO) investments.
\(^2\) After the peak in 2000 with \(m\)€ 48,023 fundraising dropped back to \(m\)€ 27,533 in 2002 and remained stable since then. The split between VC and BO in 2004 was 32% and 65% respectively. Source: EVCA Yearbook (2005)
\(^3\) See Markowitz (1952)
\(^4\) Lerner (1994) was one of the first to underpin the importance of divestment timing in the Venture Capital market. Refer to Nowak et al. (2004) for a more recent analysis.
\(^5\) See Gompers/Lerner (1999)
\(^6\) We thank CEPRES for delivering the data. www.cepres.de
patterns of simultaneous investments in PE and quoted indices. Resulting index investments mimic our PE projects with respect to timing and amount of investments.\(^7\) All cash flow based performance measures can then be calculated for both investments, while approximating cross-sectional correlation factors between PE and PM returns becomes feasible. In contrast, a time series correlation analysis would need to rely on subjective Net Asset Values (NAV) reported by PE funds rather than on market quotes. To avoid potential biases we focus on our cross sectional correlation analysis.\(^8\)

Overall we observe low cross-sectional correlation factors between PE and PM investments. Relative to broad equity indices, the PE sample returns exhibit the highest correlation with the Nasdaq Composite Index (0.18) and the lowest with the S&P 500 (0.05). Looking at co-movement with industry and local equity indices, we observe a mixed picture. While specific indices add explanatory power for some clusters (consumer discretionary, industrial production, healthcare, UK investments) they fail to do so for others (IT, telecommunications, German investments). Based on the observed low levels of co-movement and thus explanatory power between PE and PM returns, the selection of a benchmark index seems to be arbitrary while PE offers substantial diversification potential for pure PM portfolios.

To evaluate PE’s hedging potential, we examine excess returns of PE over PM investments for different PM performance levels.\(^9\) These excess returns enable a split of the dataset into: PE projects outperforming PM investments and PE projects underperforming relative to PM investments. To analyze the PE performance for different levels of PM returns, we cluster the PE projects in descending order of simultaneous PM performance. Looking at the worst performing clusters (lowest 20% of PM returns) we observe negative excess returns for the corresponding PE investments and thus overall limited hedging capabilities.

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\(^7\) For an illustration of the simulation approach, please refer to appendix A. Also see Ick (2005)

\(^8\) Meaningful, frequently updated NAVs are not available for our data set. Furthermore track record is an important success factor for PE fundraising. Thus, funds face the incentive to overstate NAVs of portfolio companies and mark-up returns of their unrealized funds. For theoretical discussion and empirical evidence see Gompers/Lerner (1999) and Cumming/Walz (2004) respectively.

\(^9\) For our analysis “hedging” relates to PE generating excess returns over PM in times of low PM returns, rather than to “lock in” a certain return level.
Both, entry valuation levels and likely exit channels indicate a stronger impact of low PM returns for early stage rather than later stage PE investments. While valuations of later stage investments tend to rely on fundamental analysis, early stage companies mainly consist of growth opportunities and suffer from less meaningful financial statements.\textsuperscript{10} Besides following PM more strictly with respect to the appraisal of investment opportunities, availability of VC funds’ preferred and most profitable exit channel (IPOs) again strongly depends on PM conditions.\textsuperscript{11} Ultimately, returns of early stage PE investments are less likely to constitute a hedge than later stage investments. To test this hypothesis we use discriminant and regression models.

The discriminant model reveals a significant positive relation between being a later stage PE investment and constituting a hedge against low PM returns. Early stage investments are on the other hand significantly negatively related with the likelihood of constituting a hedge. Investment size and duration also exhibit a positive relation with hedging capabilities. To analyze both probability and magnitude of PE’s excess returns in bearish PM conditions, we use a regression model. Again, we observe a positive (negative) relation between being a later (early) stage investment and excess returns in bearish market conditions. Overall we find broad statistically significant evidence for our hypotheses, and conclude that later stage PE investments (BO) constitute a better hedge against downturns of public equity markets than early stage PE investments (VC).\textsuperscript{12}

\section{Related Literature}

Recently, the empirical work on performance and risk-return relationship of PE investments has been a very active research area. Besides increasing interest in the asset class, availability of more detailed and reliable datasets facilitates this development.

\textit{Cochrane} (2003) focuses on the individual portfolio company level and infers the aggregate performance of PE investments. He stresses the importance of adjusting for

\footnotesize
\textsuperscript{10} See Hand (2005)
\textsuperscript{11} See Black / Gilson (1998)
\textsuperscript{12} Our classification in Buy-out (BO) and Venture Capital (VC) refers to the typical investment strategy pursued by these funds. Anyhow there are BO and VC funds that invest in PE projects of all stages.
survivorship bias, which potentially arises due to the high failure rate of PE investments. In his paper he uses a maximum likelihood estimate that corrects for selection bias. He finds mean log returns of individual portfolio investments are around 15% percent, though arithmetic mean returns are much higher and generate an arithmetic alpha of 32%.  

Kaplan and Schoar (2003) analyze the performance of 746 PE funds obtained from VentureEconomics. They find a large heterogeneity in fund returns. Using average fees and carried interest figures they conclude that on average PE outperformed the S&P 500 gross of fees. Focusing on dynamics of fund returns, they find a strong persistence of fund returns and improving returns with increasing experience of PE funds.

Ljungqvist and Richardson (2003a) study the returns to investments in 73 PE funds. They calculate IRR, TVPI and Excess IRR for investments on fund level. They develop a risk-adjusted profitability index discounting cash inflows at the cost of capital, which they estimate using Fama and French’s industry cost of equity figures. Doing so, they observe excess alpha returns on the PE fund level. Their focus is on a general analysis of private equity fund’s cash flow patterns, draw down rates and performance determinants.

Jones and Rhodes-Kropf (2003) use data from 1,245 funds to investigate whether and how idiosyncratic risk is priced in VC markets. They find that unavoidable principal-agent problems result in fund returns that are increasing in the amount of idiosyncratic risk. Thus, in a competitive model for VC funds, total risk rather than only systematic risk is priced. They show that VC investments earn alpha returns over CAPM returns on the project level, while investors in VC funds earn no abnormal returns. Alpha returns compensate VC funds for the level of idiosyncratic risk they can not diversify away.

The analysis of PE in the portfolio context is challenging due to the lack of time-series data. Following papers have recently addressed issues of co-movement and diversification benefits.

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14 See Kaplan/Schoar (2003)
15 See Fama/French (1997)
16 See Ljungqvist/Richardson (2003a)
Kaserer and Diller (2004) analyze the risk-/return relationship of 794 European funds to assess the role of PE in the asset allocation. Their results reveal a slight underperformance of the average realized fund relative to the MSCI Europe. BO funds exhibit consistently higher performance figures than VC funds. They approximate correlations between PE and the public benchmarks to be 0.8 (MSCI Europe) and 0.1 (Bond index) derived from calculated Public Market Equivalents (PME) and Bond Market Equivalent (BME) figures. Both measures assume a reinvestment of intermediate cash flows in the PM and thus are tend to overstate correlation factors. They show that adding PE to a portfolio comprising MSCI Europe and J.P. Morgan Government Bond Index shares triggers diversification effects.\footnote{See Kaserer/Diller (2004)}

Gottschalg et al. (2004) analyze returns of more than 500 PE funds, derived from the records of VentureEconomics. Based on net cash flows they find that realized funds underperform public stock-markets. As they find PE performance to be pro-cyclical relative to public markets, desirable hedging properties do not justify a return discount of PE compared to PM. According to their analysis PE funds are exposed to non-negligible risk and should command a return premium over public markets rather than the observed discount. Gottschalg et al. call this an illiquidity puzzle of PE.\footnote{See Gottschalg et al. (2004)}

Schmidt (2004) analyzes 642 U.S. PE investments and investigates how they can be used for diversification purposes. To assess the relationship between PE and PM investments he generates cash flow streams of public market investments mimicking cash flow patterns of PE investments. He performs a bootstrap simulation to observe risk-return characteristics of portfolios comprising PE investments. He finds PE to bare higher levels of non-systematic risk compared to PM investments. As correlations factors with public market investments are overall low, he concludes that PE as an asset class offers diversification potential.\footnote{See Schmidt (2004)}

The third field of PE research related to our work addresses the impact of market structure and timing on returns.

Lerner (1994) analyzes 350 privately held venture-backed biotechnology firms. He shows that these companies tend to go public when equity valuations are high and are
privately financed otherwise. Experienced VCs appear to be proficient at taking companies public near market peaks.\textsuperscript{21}

Ljungqvist and Richardson (2003b) analyze the investment behavior of PE fund managers. They show that fund manager time their investment and exit decisions in response to PE market conditions. Especially availability of investment opportunities and competition for deal flow affect the time PE funds take to draw-down, invest and return omitted capital.\textsuperscript{22}

Nowak et al. (2004) investigate the market timing abilities of PE fund managers using a dataset similar to ours. They find that investments timing does impact VC returns, while it does not for later stage returns. Individual experience of fund managers is the more important and significant factor determining returns.\textsuperscript{23}

Inderst and Müller (2005) show that success probability, financial contracting, valuation, and value created in a start-up firm depend strongly on characteristics of the capital market in which the start-up raises finance. They find that VC’s screen potential projects more intensely if level of capital supply is low, and less if it is high.\textsuperscript{24}

3 Data

3.1 Data Description

The dataset we use provides information on 86 PE companies, 243 PE funds and their 5,991 investments in 4,819 different companies.\textsuperscript{25} The investments span over a time of 28 years (1975 – 2003) and cover 51 Countries. Information is completely anonymous, but provides characteristics of both, the investing fund and the portfolio company. Names of neither funds nor firms are disclosed. The key advantage of the dataset is cash flow and write-off information provided for each individual investment. These cash streams between the portfolio company and the PE fund are reported gross of fees and thus are not biased by any externalities, especially management fees and carried interest. CEPRES

\textsuperscript{21} See Lener (1994)
\textsuperscript{22} See Ljungqvist/Richardson (2003b)
\textsuperscript{23} See Nowak et al. (2004)
\textsuperscript{24} See Inderst/Müller (2005)
\textsuperscript{25} The dataset is derived from the records of CEPRES (www.cepres.de).
contains all investments pursued by included private equity funds, thus the sample does not suffer from severe selection biases.\textsuperscript{26} In contrast to other databases, such as VentureEconomics or VentureOne, the data is not based on unsolicited reported data but contains all pursued investments.\textsuperscript{27} As we are interested in the general relation between PE as an asset class and PM returns, we analyze portfolios comprising all PE projects rather than focusing on fund-level data.\textsuperscript{28}

As we strive to explore co-movement between private equity and public market returns, it is crucial to only include unbiased returns. To avoid estimation biases due to subjective valuation treatment we concentrate on completely liquidated investments with comprehensive cash flow history. This is in line with Cochrane’s approach.\textsuperscript{29} For partially and unrealized investments the dataset includes the NAV at valuation date. The NAV is used to calculate the Modified Internal Rate of Return (MIRR), a key performance figure for not fully liquidated funds. And as performance of historic and current funds is an important marketing instrument of PE companies during fund raising periods, there is an incentive to overstate NAVs.\textsuperscript{30} In the analysis we compare correlation between PE and PM investments for a variety of public equity indices. To do so, we drop PE projects which can not be compared to all relevant benchmarks.\textsuperscript{31} In a last step we eliminate all investments with missing or obviously incomplete data. Looking at the cash flow entries we require investments to cover at least half a year of information. Our assumption that investments with shorter investment horizons are incomplete entries is

\textsuperscript{26} As CEPRES data comprise mainly private equity-managers reporting performance over recent years it might face a certain survivorship bias, because it contains no information on PE companies which ran out of business prior to the mid 90ties. For a detailed discussion of potential biases in the CEPRES dataset please refer to Schmidt (2004).
\textsuperscript{27} Kaplan et al. (2002) find a tendency of overstated returns in VentureEconomics and VentureOne if the data is compared to real life data.
\textsuperscript{28} With respect to timing of distributions of funds it is reasonable to assume a close matching between the realization of a profit by the fund (via exit) and its distribution to investors. See for example Kaserer/Diller (2004), p. 30 and Ljungqvist/Richardson (2003a), p. 28 who find an almost uniformly distribution of profits over a PE funds lifetime.
\textsuperscript{29} See Cochrane (2003), pp 3.
\textsuperscript{30} See Gompers / Lerner (1999) pp. 17 – 29 for a detailed discussion of the fund raising process and this incentive structure.
\textsuperscript{31} Market quotes of some public indices are not available for the time span of early PE investments. These investments are thus dropped.
supported by missing information regarding most other characteristics of these investments.\textsuperscript{32}

### 3.2 Summary Statistics

The resulting dataset comprises of 2,380 fully realized investments. Vintage years of the investments span over 15 years (1987 to 2002). Exhibit 1 summarizes the distribution over time.

The dataset can be considered as a good sample of the overall realized PE investments. As a matter of fact, only few investments pursued after the burst of the “internet bubble” have materialized sufficiently to be included in our sample. But as we focus on the co-movement of PE and PM returns rather than on absolute returns, this does not weaken our analysis.

\begin{figure}[h]
  \centering
  \includegraphics[width=\textwidth]{chart.png}
  \caption{Number of PE investments per vintage year}
\end{figure}

Geographically the sample covers PE investments in four continents: North America, South America, Europe and Asia. Most transactions are from the U.S. (1,212), the United Kingdom (342), France (264) and Germany (104). The remaining investments were

\footnote{Results of following analysis for the larger sample including these investments do not differ significantly and are available upon request.}
pursued in 28 other countries. Overall, the geographic distribution of the investments roughly reflects the size of the corresponding PE market at that point in time. Developing PE markets, such as China, are not represented in the sample.

The dataset includes a variety of company, fund and investment specific characteristics. Most important for our following analysis are investment stage, industry classification, investment horizon and real investment value. PE investments are often clustered according to the stage of the private company. Stage information is available for 1,205 investments. The missing stage observations are not related to any other information or characteristic of the portfolio company. We cluster the sample investments into four distinct stages:

- Early stage: seed and early (initial) financing of private start-up companies mainly by Venture Capital funds
- Expansion: organic growth and acquisition financing of private companies by Venture Capital and Buyout funds
- Later stage: financing of established companies, especially Leveraged Buyouts, Management Buyouts/-ins and public to private transactions mainly by Buyout funds
- Turnaround: recapitalization and other turnaround investments mainly by Buyout funds

To enable a comparison with specific public equity indices, we distinguish seven industry clusters. 606 investments can not be assigned to a cluster and are categorized as “other industries”. Table 1 summarizes the distribution across the industries. The frequency distributions of investment stages per industry cluster reveal a larger share of

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33 Argentina, Austria, Belgium, Benelux, Brazil, Canada, China, Denmark, Finland, Iceland, Indonesia, Ireland, Israel, Italy, Japan, Korea, Luxemburg, Netherlands, Norway, Philippines, Portugal, Puerto Rico, Russia, Singapore, Spain, Sweden, Switzerland.
34 See for example the annual statistics of DowJones’ VentureSource (2005).
35 See e.g. Lerner / Schoar (2004) who focus on PE transactions in developing markets
36 Starting from 25 different industry classifications provided from CEPRES, we aggregate sub-cluster to form 7 larger and homogenous clusters. Remaining investments are gathered in the “other industry” clusters. Following are the clusters with comprising sub-clusters in brackets: Consumer Discretionary (Hotel, Leisure, Retail, Textile), Healthcare, Industrial Production (Construction, Traditional Business), Information Technology (High-tech, Semiconductor, Software), Internet & Media, Services (Environment, Logistics, Waste, Recycling), Telecommunication.
early stage investments for high-tech driven industries. This relates to IT, Internet & Media, Healthcare and Telecommunications. On the other hand non-high-tech industries are investment targets of later stage and turnaround investors. In the sample this is the case for the Consumer Discretionary, Industrial Production and Services industry clusters.

The time horizons of the sample investments range from 6 to 180 months, with a mean holding period of 48.3 months and a standard deviation of 30 months. Table 2 illustrates the distribution of investment horizons per industry cluster. As expected, there are only minor differences regarding the share of short, medium and long term investments between industry clusters.\(^37\)

The real investment amounts are calculated in terms of 2004 U.S. Dollars.\(^38\) Investment amounts range from $1,600 to $340. With an arithmetic mean of m$ 11.7 and a standard deviation of m$ 23.3 the wide dispersion of investment amounts becomes apparent. Table 3 summarizes the investment amount for the different investment stages. Early stage investments have a larger share (28%) of small investments, while the other stages are pretty similar regarding their amount distribution.\(^39\) This is in line with the general investment strategy of funds focusing on early stage (VC) and later stage (BO) PE investments.

Performance of our PE sample is summarized in table 4. Internal Rates of Return (IRR) are provided for the full sample and alternative perspectives (industry, stage, investment amount).\(^40\) Overall our PE sample generated a mean annual IRR of 50.5% with a median of 18.1% and a standard deviation of 354.2%. These figures are in line with the findings of Cochrane after he corrected for survivorship biases.\(^41\) In general high-tech industries (IT, Telecommunications, Internet & Media) exhibit higher mean

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37 short term: up to 24 months; medium term: between 25 and 72 months; long term: at least 73 months
38 The inflation adjustment is based on Consumer Price Index (CPI) data for all urban households and all items. Data is derived from the records of U.S. Department of Labor (www.bls.gov).
39 small amount: up to $1 Mio; medium amount: between $1 Mio and $10 Mio; large amounts: exceed $10 Mio
40 IRR can be regarded as standard performance measure in the PE industry. Refer to Ick (2005) for a critical discussion of IRR and alternative measures better fitting PE’s characteristics. He also provides in-depth performance statistics of the CEPRES data sample.
IRRs with larger dispersion levels than non-high-tech industries (Consumer Discretionary, Industrial Production, Services). Regarding stage perspective, early stage investments generated the lowest mean IRR with an even negative median. Return distributions of later stage and turnaround investments have higher means and medians, with larger standard deviations. The investment amount perspective reveals descending mean returns with increasing size, in line with the relation Fama and French carve out for public stocks.\(^{42}\) We will analyze whether this relations is significant for the explanation of cross-sectional differences with respect to hedging capabilities for our PE sample.

4 Cross-sectional Correlation

Correlation factors are of interest to describe the relation between PE and PM returns and to appraise diversification potential of PE as an asset class relative to PM. Besides illustrating an approach to generate inputs for portfolio composition, we are interested in the explanatory power of different PM benchmarks for PE returns. The question is whether specific stock indices are closer related to PE returns than broad global indices and should thus be preferred for relative performance appraisal.\(^{43}\)

4.1 Dealing with the lack of time-series data

Unlike for investments in listed securities, market values for PE projects can only be observed when transactions occur. In-between these investments and divestments no market values can be obtained. Due to this episodic nature of data, time-series market data is thus not available for PE investments. Woodward and Hall interpolate valuation levels between trades to derive an index of U.S. venture capital investment returns. As it remains an index of estimated values opposed to an index of trade based values for public stock indices (e.g. S&P 500), their focus is on performance appraisal of PE funds.\(^{44}\) Despite the availability of index data, its application for standard time-series correlation analysis between PE and PM investments remains questionable. To anyhow estimate the

\(^{42}\) This relates to the SMB factor in their Three-factor model. See Fama/French (1993)

\(^{43}\) Especially as the benchmark selection can substantially impact the relative performance appraisal for PE investments. See Ick (2005) pp. 20–22

\(^{44}\) See Woodward/Hall (2003)
co-movement of PE project’s returns with that of public equity indices, we calculate cross-sectional correlation coefficients. In general we follow the spirit of cross-sectional analysis of returns.\textsuperscript{45} We analyze return pairs for PE and PM investments with corresponding investment timing and amount. But as investment and divestment dates differ widely between our PE projects, we measure the “general” relation between returns, rather than periodic correlation factors for a specific time period. Doing so, we compose portfolios of PE investments and portfolios of corresponding PM investments and measure correlation between their returns. These portfolios comprise PE investments with heterogeneous timing, while timing and investment amount are matched with PM investments. The resulting cross-sectional correlation thus measures the co-movement between simultaneous investment in PE and PM but over a broad time horizon.\textsuperscript{46} It is important to recognize this relation when using cross-sectional correlation factors for portfolio composition.\textsuperscript{47} The cross-sectional approach picks-up the specific problem of missing time-series data and \textit{approximates} correlation factors. Sample size is an issue as samples need to be large and representative with respect to total population in order to generate valid correlation factors. To address this concern we perform a sensitivity analysis for our results.

We overcome the usual problems of non-performance comparability between PE and PM investments, with the simulation of comparable cash flow patterns for both asset classes. The approach has been introduced in our previous work and is similar to the one used by Schmidt (2004).\textsuperscript{48} We start from the PE’s cash flow patterns. Whenever an investment into the PE project occurs, we simulate a concurrent purchase of shares of the PM benchmark investment. Alternatively shares of the benchmark are sold when a distribution occurs for the PE project. The cash flow amounts are translated into number of shares being purchased and sold according to PM’s market quotes. Using the obtained

\textsuperscript{45} See e.g. Fama/French (1992)
\textsuperscript{46} Our sample covers investment dates between 1987 and 2002.
\textsuperscript{47} The mean-variance framework for portfolio composition requires consistency of all input factors with respect to timing. Thus all other input data need to be determined consistently. If one wants to approximate cross-sectional correlations for a specific period, the sample of PE projects with matching timing needs to be available. Portfolio composition and optimization is not in the scope of this paper. The following papers deal with portfolio composition including PE investments: Artus/Teiletche (2004), Kaserer/Diller (2004), Schmidt (2004)
cash flow patterns, we calculate the $\text{IRR}_i$ of the PE project $i$ and of its counterpart investment in the public benchmark $b \text{IRR}_b$. Appendix A illustrates the simulation model in detail.

4.2 Empirical Analysis

As outlined we calculate cross sectional correlations based on pairs of comparable Internal Rate of Return (IRR) figures. We therefore first simulate IRR figures for PM investments in five broad, three local and seventeen industry specific equity indices, mimicking timing and amount of our PE investments.\footnote{49} Based on resulting return pairs we calculate cross-sectional correlations for the entire sample and various sub-clusters. Conceptually, we construct portfolios comprising all investments of our PE sample and sub-samples. Key results of our analysis are summarized in table 5.\footnote{50} Besides estimates for correlation coefficients, we provide p-values. Low p-values can be triggered by both, a coefficient close to zero or small very heterogeneous sub-samples. We tackle this issue in our sensitivity analysis.

Overall we observe low levels of co-movement, indicating a loose relationship between PE and PM returns. Public market index returns do seem to have low explanatory power for our sample PE returns. On the other hand, results promise attractive diversification potential of PE as an asset class for pure PM portfolios. This is in line with the findings of Kaserer/Diller and Schmidt.\footnote{51} Investing in PE seems to be attractive due to high expected returns \textit{and} its smoothing effect on pure public equity portfolios.

Of the broad indices, the Nasdaq Composite Index (0.18) has the highest and the S&P 500 (0.05) the lowest correlation factor with our full PE sample, while all factors are significant at the 99% level. With all coefficients being relative low, selecting a reference


\footnote{50} Results for the other indices not covered in the table are available upon request.

benchmark seems to only marginally impact its explanatory power for PE returns. For the industry clusters, correlation factors with Nasdaq Composite Index range from 0.01 (services) to 0.44 (telecommunication) and not all are statistically significant. Correlation factors of the stage clusters range more narrowly around 0.17. Turnaround investments are the exception (0.07), but as this sample is small, cross-sectional correlation results are unstable (as indicated by the high p-value of 58%). Size (investment amount) seems to have little impact on cross-sectional correlation factors.

While specific and local indices add explanatory power for some clusters (consumer discretionary, industrial production, healthcare, UK investments) they fail to do so for others (IT, telecommunications, German investments). Although performance appraisal deviates relative to broad and specific benchmarks, it can not be concluded which benchmarks are per se the better fitting yardstick. Correlations are consistently on a low level and there seem to be little value added of selecting industry specific benchmarks to explain PE investment returns.

### 4.3 Sensitivity Check

We are concerned that sample size impacts validity of cross-sectional correlation analysis. If samples are small compared to the total population, results can be misleading as PE returns are very heterogeneous. We use a bootstrap approach to investigate the relevance for our sample. Basically, we compare correlation coefficient of the full sample with that of randomly drawn portfolios of different sizes. Our four bootstrap models randomly draw 100 portfolios each comprising 50, 100, 250 or 500 PE investments. Concurrently we form matching PM portfolios with our simulated benchmark investments. We then calculate cross-sectional correlations between the PE and PM portfolios. Appendix B illustrates the bootstrap models in detail.

For our sensitivity check we select the Nasdaq Composite Index as benchmark. The correlation coefficient of the full PE sample with this index is approximately 0.18, the reference value for the following analysis. Each of the four bootstrap models generates

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53 See for example Kaserer/Diller (2004), Ljungqvist/Richardson (2003a) for a discussion of PE return distributions.
100 correlation coefficients. We sort these coefficients in ascending order. Exhibit 2 illustrates the coefficient distribution of our four models.

**Exhibit 2:** Sensitivity of cross-sectional correlations with respect to sample size

Looking at the graph we see that dispersion substantially declines with sample size. Correlation factors for 50 randomly drawn investment pairs (orange graph) range from -0.28 to 0.62. In contrast, large estimation errors due to sample selection become unlikely if one draws 500 investment pairs (blue graph). Correlation factors only range from 0.06 to 0.38 and center much closer around the reference value of 0.18. Looking at the 25% and 75% cumulative density levels provided in table 6, one can see that for the 200 investments case (green graph) the estimates range from 0.13 to 0.24 and thus provide a better estimate of the reference value. The benefit associated with increasing sample size seems to decline with size as the extreme values of the 200 and 500 investment pair models are relatively close. We can conclude that in case of PE, sample size (relative to total population) indeed substantially impacts reliability of cross-sectional correlations estimates. This issue needs to be taken into account when using such estimates for further analysis. Small samples (e.g. n < 200) result in questionable correlation coefficients. Getting back to our estimates of table 5, this especially relates to the turnaround investment cluster (n = 63) for which we observed very large p-values.
5 Hedging Capabilities of PE Investments

5.1 Overview

The observed low levels of co-movement between PE and PM returns are attractive for PM investors, if PE investments generate profits when public equity investments fail to do so. In order to obtain further insights on the diversification potential of PE as an asset class, we sort our investments in descending order of their PM performance and form ten clusters. Our objective is to investigate whether PE investments in our sample on average still generate positive returns in times of low public stock market returns.

First we look at a portfolio comprising our full PE sample. Of the 10 clusters with descending PM returns, the 1st cluster contains the best and the 10th cluster contains the worst performing PM investments. Exhibit 3 shows that mean returns of our simulated investments in the Nasdaq Composite Index (blue bars) gradually decrease from 58% to -22%. Turning to IRR figures of corresponding PE returns (orange bars) we observe substantial variations of returns between the clusters, illustrating the low level of co-movement with PM returns.

Exhibit 3: Performance of PE investments for different PM market return conditions
But despite overall low correlations between return pairs, PE returns for the worst performing cluster (10) are very low. Thus, in times of very low returns of Nasdaq Composite Index investments, PE investments were not able to generate positive returns but even performed worse. This result is in line with the findings of Gottschalg et al., who observe low hedging capabilities of PE as an asset class relative to PM.\(^{54}\) Table 7 summarizes our analysis for the Nasdaq Composite Index and the S&P 500. Results are robust with respect to the chosen reference benchmark.\(^{55}\) Although our full sample fails to constitute a hedge against low PM returns, we are curious to check for differences between PE investment classes, especially VC and BO. Do different valuation approaches and exit channels trigger different results for early and later stage PE investments?

5.2 Determinants of Hedging Capabilities

PE returns depend, like every other investment, on realized purchase and sale prices. For our appraisal of PE’s hedging capabilities, it boils down to the question whether PM market developments differently influence either purchase or sale valuations of PE investment classes. We are especially interested in divergence between early stage (VC) and later stage (BO) investments. For VC funds, IPOs are the most important and most profitable exit channel.\(^{56}\) Black and Gilson even state that the health of the VC market depends on the existence of a lively public equity market that enables successful IPOs.\(^{57}\) BO funds on the other hand are less dependent on vibrant capital market conditions as trade sales and secondary buyouts are functioning alternative exit opportunities. Besides the mere opportunity of pursuing a successful IPO, VC funds also have an incentive to postpone it during times of low public market valuation. As pointed out by Barry et al. and Nowak et al., exit timing is a critical success factor for VC funds, even though they usually do not immediately sell all their shares.\(^{58}\) The question is whether investor can still benefit from low correlations between PE and PM markets, if as Lerner empirically

\(^{54}\) See Gottschalg et all. (2004), p. 17
\(^{55}\) Results for other benchmarks are available upon request.
\(^{56}\) See for example Gompers / Lerner (1999), pp. 203-205
\(^{57}\) See Black / Gilson (1998)
\(^{58}\) See Barry et al. (1990), Nowak et al. (2004)
shows, VCs only pursue IPOs when public equity values are high and use private financing when values are low. With respect to exit opportunities, we therefore expect higher returns for BO than for VC investment in bearish public market conditions.

Regarding ex ante valuation of potential projects, PE market participants suffer from the lack of ready to observe market values. Either fundamental analysis or relative valuation (e.g. multiples) can be used to price investment opportunities. Fundamental analysis in principal refers to systematic projection of future company cash flows or earnings based on historic developments and industry forecasts. In case of later stage PE transaction this seems to be an adequate way of assessing a company value as meaningful historic financial statements and supplementary data are available. In contrast, there is little historic information and mainly intangible growth opportunities on the asset side for early stage PE projects. Hand empirically investigates the value relevance of financial statements in PE markets. He points out that financial statements become more relevant for valuation purposes as firms mature. This is consistent with financial statements capturing more and more assets-in-place rather than pure future growth options. Furthermore, Hand shows a decreasing relevance of non-financial statement information with increasing company history. He argues that in a dynamic sense, financial statement and non-financial information are substitutes rather than complements. Summarizing, early stage investors are more likely to follow “comparable” market valuation levels rather than fundamental company data when pricing investment opportunities. Market valuations in this context refer to recent IPO companies, listed players within the same industry or users of similar technologies.

Both entry valuation levels and likely exit channels indicate a stronger dependence of public equity markets’ downturns for early stage rather than later stage PE investments. Based on our discussion we formulate following hypotheses:

**Hypothesis 1:** Stage information is an important factor for PE’s “hedging” capabilities

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59 See Lerner (1994), pp. 313-315
60 For an overview of fundamental analysis refer for example to Sharpe et al. (1999), pp. 751-768
62 see Hand (2005)
Hypothesis II: Early stage investments are negatively related with “hedging” capabilities

Hypothesis III: Later stage investments are positively related with “hedging” capabilities

Besides valuation levels, investment size is a potential factor of PE hedging capabilities. We observe a negative relation of size with returns. Small companies load up on the size factor and are expected to generate higher returns according to Fama and French’s three factor model. But it is questionable if this relation holds for PE investments during PM downturns. Small PE investments are likely to be early stage companies, as we have shown in table 3. Hypothesis II, if true, would predict weaker performance for small PE investments in times of low PM returns. As we expect the stage variable to be the governing factor of PE hedging capabilities, we believe that it dominates size effects and therefore state:

Hypothesis IV: Investment size is positively related with “hedging” capabilities

In the following sections we first check whether there are different hedging capabilities of early and later stage investments. We then use discriminant and regression models to test our hypotheses. The discriminant analysis identifies variables influencing the probability of a PE investment constituting a hedge. Supplementing, the regression analysis looks at PE variables influencing both, probability and magnitude, of PE excess returns over PM returns in times of low PM returns.

5.3 Differences between Early and Later Stage Investments

We now rerun our previous analysis separately for early stage and later stage investments. Can investors indeed benefit from distinct characteristics of VC and BO investments and if so, to what extent?

Table 8 covers results of our analysis. We again form ten clusters according to PM performance and compare mean returns (IRR) with that of corresponding PE investments. Our early stage sample comprises 426 investments and thus each cluster contains 42 or

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63 See Fama/French (1993)
43 investments. The later stage sample comprises 448 investment cluster size varies between 44 and 45. We again select the Nasdaq Composite Index as benchmark for both analyses. Its performance differs between the early and later stage investments due to different timing of PE investments. As every PM investment matches the timing of a PE investment, different Nasdaq returns result.

For early stage investments we observe underperformance relative to Nasdaq for clusters 7 to 10. Thus, these PE investments are not capable to generate positive returns in times of low PM returns even if PM returns are still positive. In contrast, later stage investments consistently achieve positive returns. These investments do not decline with Nasdaq returns. Exhibit 4 (early stage) and exhibit 5 (later stage) illustrate our results.

**Exhibit 4:** Performance of early stage investments for different PM market conditions (Nasdaq Comp.)

**Exhibit 5:** Performance of later stage investments for different PM market conditions (Nasdaq Comp.)
Our findings show substantial differences between PE investment classes with respect to hedging capabilities. Later stage PE investments seem to generate robust returns during PM downturns while early stage PE investments fail to do so. Result provide first empirically support for our hypotheses I, II and III.

5.4 Discriminant Analysis

In this section we explore factors describing characteristics of PE projects that generated excess returns over PM indices in times of bearish public markets. These PE projects can be regarded as successful “hedging” investments as they reduce losses of pure PM portfolios. In this context the discriminant analysis is a useful tool.

Based on the dataset it is straightforward to calculate excess returns of PE over PM investments. To identify “hedging” cases, we generate the variable “outperformance”. It is equal to one if the PE project generated a higher return than the Nasdaq Composite Index, and zero otherwise. In a second step we again cluster our 2,380 PE investment returns in descending order of corresponding Nasdaq Composite Index returns, with cluster one and ten comprising the best and worst investments respectively. As we are only interested in the worst performing clusters, we initially consider the 10th cluster only and use the “outperformance” variable as grouping variable: Group 0 underperforms and thus contains the “no-hedge” PE investments, while group 1 outperforms and contains the “hedge” PE investments against PM losses.

Based on our hypotheses we include the following independent variables in the discriminant model:

- Early stage: dummy variable; 1 for early stage investments; 0 otherwise
- Expansion: dummy variable; 1 for expansion investments; 0 otherwise
- Later: dummy variable; 1 for later stage investments; 0 otherwise
- Turnaround: dummy variable; 1 for turnaround investments; 0 otherwise
- Other stage: dummy variable; 1 for other or no stage information; 0 otherwise

64 We select the Nasdaq Composite Index as reference benchmark, because it exhibits the largest correlation of all PM equity indices with the PE returns.

65 To check the robustness of the results, we later expand the sample to include clusters 7 to 10.
• Real invest: total investment amount in terms of 2004 USD; measures size

In addition we want to control for the impact of investment duration, geographical diversification and investment focus of the fund. As we perform our analysis relative to Nasdaq Composite Index, PE projects outside the U.S. might benefit from a geographical diversification and thus could exhibit better hedging capabilities. PE funds focusing on a specific industry can be considered to be “insiders” and thus might focus more on fundamental information in their investments decisions resulting in better hedging capabilities. Our control variables therefore are:

• U.S.: dummy variable; 1 for investment in the U.S.; 0 otherwise
• Focus fund: dummy variable; 1 if fund focuses on an industry; 0 otherwise

The group statistics in Table 9 reveal that the mean values for the first three stage variables (early, expansion, later) differ substantially between the “non-hedge” group (0) and the “hedge” group (1). On the other hand the relatively high within group standard deviation decrease their explanatory power. Regarding the two remaining stage variables, the group statistics indicate lower relevance for our classification. Turnaround and other stage variables both vary marginally in mean values. Especially the other stage variable exhibits an extraordinary high within group standard deviation. For all other variables little explanatory power can be expected. While the size variable differs substantially in its group means (m$ 6.8 vs. m$ 1.4) its within group standard deviations are high.

The mean-differences tests between the two groups are summarized in table 10. The very high significance levels for the first three stage variable underpin their relevance for our grouping model. Furthermore, the test for the size variable (real investment amount) is significant at the 1% level. But with its high within group standard deviation its explanatory power remains doubtful. For all other variables significance levels are above 5%. The explanatory relevance of the geographical diversification (U.S.) (38%) variable can already be neglected.

When we check for within group correlation factors, the “other stage” variable fails the tolerance test and is excluded from our model. As this is only a summarizing variable for “other” stages this is uncritical for our analysis.

In table 11 we summarize goodness of model statistics. The eigenvalue of approximately 0.28 indicates that the between group deviation is only 28% of the within
group deviation. As there is a discrepancy between the groups, the model has explanatory power, but as expected not all the discrepancy can be explained. The very high significance level of the chi-square test underpins the overall adequacy of our model. Thus, the model finds statistical support for our hypothesis I, but its prediction precision is questionable.\footnote{Prediction” refers to the model that can be derived from the unstandardized structure matrix, which can be used to predict whether a PE investment is likely to constitute a hedge or not. Due to the very heterogeneous nature of PE returns, this is not surprising.}

The standardized discriminant coefficients are summarized in exhibit 6:

<table>
<thead>
<tr>
<th>Discriminant coefficients</th>
<th>Function = 1</th>
</tr>
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<tbody>
<tr>
<td>standardized</td>
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</tr>
<tr>
<td>earlystage</td>
<td>-0.29</td>
</tr>
<tr>
<td>expansion</td>
<td>0.57</td>
</tr>
<tr>
<td>later</td>
<td>0.68</td>
</tr>
<tr>
<td>turnaround</td>
<td>0.27</td>
</tr>
<tr>
<td>size</td>
<td>0.10</td>
</tr>
<tr>
<td>usa</td>
<td>-0.10</td>
</tr>
<tr>
<td>focus_fund</td>
<td>0.21</td>
</tr>
</tbody>
</table>

Exhibit 6: standardized discriminant coefficients for the hedge group (outperformance = 1)

The negative coefficient for the early stage variable (-0.291) indicates that early stage investments are not likely to belong to the “hedge” group. This is in line with our expectations and supports hypothesis II. On the other hand, the coefficients for all other stage variables are positive with later stage (0.675) exhibiting the highest likelihood of being a successful “hedge”. This provides statistical evidence for hypothesis III. The high coefficient for the expansion stage variable (0.569) shows, that these investments are closer to later rather than early stage investments with respect to “hedging” capabilities. As predicted by hypothesis IV we observe a positive size coefficient. Although not being significant, the negative coefficient of the US variable indicates better hedging capabilities of PE projects pursued outside the U.S. relative to the Nasdaq Composite Index.

The structure matrix (table 12) reveals that indeed later and early stage variables have the highest correlation with the dependent variable. This provides empirical support for our hypothesis I; the stage variable is indeed important to explain PE’s “hedging”
capabilities. Not only the expansion stage, but also the real investment amount variable has a correlation coefficient exceeding 0.4

To further assess the power of the model, we look at classification results (table 13). Overall approximately 77% of the cases are classified correctly by the discriminant model. Although this limits the predictability power of the model (23% are not correctly classified), it is no downturn for the cross sectional difference we strive to carve out.

As robustness check, we rerun the discriminant analysis including more clusters of weak PM performance: 7 to 10, 8 to 10 and 9 to 10. In all cases results are very similar and show that our findings are robust.\(^67\)

### 5.5 Regression Analysis

We now expand our focus and look at probability and magnitude of PE’s excess returns during times of bearish public market returns. The dependent variable of our OLS regression therefore is excess return of PE over PM. The independent variables remain the same as in our previous analysis. With the model we strive to empirically analyze the impact of stage and control variables rather than to fully explain excess returns.\(^68\)

Our model has the following form:

\[
\text{In Excess} = \beta_0 + \beta_{\text{early}} + \beta_{\text{epan}} + \beta_{\text{later}} + \beta_{\text{turn}} + \beta_{\text{size}} + \beta_{\text{focus}} + \beta_{\text{US}} + \varepsilon
\]

- Early: dummy variable; 1 for early stage investments; 0 otherwise
- Epan: dummy variable; 1 for expansion investments; 0 otherwise
- Later: dummy variable; 1 for later stage investments; 0 otherwise
- Turn: dummy variable; 1 for turnaround investments; 0 otherwise
- Size: logarithm of total investment amount in terms of 2004 USD
- Focus: dummy variable; 1 for funds with industry focus; 0 otherwise
- U.S.: dummy variable; 1 for investments in U.S. companies; 0 otherwise

\(^67\) The explanatory power of the early and later stage variables increase even further, while the standardized discriminant coefficients vary only marginally. As these figures only complement our analysis we do not include them in the paper. The results are available upon request.

Due to non-normally distributed PE returns, we rather analyze the logarithm of excess returns ($\ln(Excess)$) then absolute values.\(^6^9\) The Nasdaq Composite Index again serves as public benchmark. We run regression models for PE investments corresponding with the 10%, 20% and 30% worst performing Nasdaq investments (clusters 8, 9, 10). First we only include stage variables and then rerun the regression models with all variables.

Table 14 shows the results of our analysis. The high F-statistics of all six models indicate there overall significance and adequacy. Explanatory power of the models in terms of $R^2$ decreases with number of clusters included (from 17.7% for cluster 10 to 8.3% for clusters 8-10). The worse the PM performance, the larger the share of deviation of PM’s excess returns that can be explained by stage variables. We check for multi-collinearity and find no significant results.\(^7^0\)

The signs of stage variables’ regression coefficients are constant over all models: early stage investments are negatively related with excess returns over PM, while remaining stage variables are all positively related with excess returns. The coefficients of the expansion and later stage investments are highly significant in all models, while t-statistics of early stage investments increase with number of included cases. For models III to IV (20% and 30% worst performing Nasdaq investments) they are highly significant. As our sample of turnaround investments is rather small we are not surprised to observe low t-statistics for three models. The results provide further evidence for the adequacy of our hypotheses I, II and III. Stage information is indeed an important determinant of PE’s “hedging” capabilities (hypothesis I). Early stage PE investments have weaker than average “hedging” capabilities (hypothesis II). Later stage PE investments in contrast are better than average “hedges” against bearish PM conditions (Hypothesis III)

For the remaining variables, regression results are less consistent with our theory. Size exhibits a negative not significant coefficient in all models. In contrast to our

\(^6^9\) See Ick (2005) pp. 13-16 for a discussion of the non-normality of excess returns. We calculate the dependent variable as follows: $\ln(Excess) = \ln(\text{IRR} + 1.2) - \ln(\text{IRR}_b + 1.2)$ Before taking the logarithm, we add 1.2 to the IRR figures to ensure that all cases are defined (not negative).

\(^7^0\) Results of this test are available upon request.
discriminant analysis this indicates a negative relationship between size and hedging capabilities and contradicts our hypothesis IV. Coefficients of the “focus” variable are very unstable, even changing signs between regression models. Combined with low t-statistics, the explanatory power of this variable for PE’s excess returns during low PM returns seems to be low. We use the “U.S.” variable to test for the relevance of geographical diversification and in fact find a negative relationship between U.S. investments and hedging capabilities. But again t-statistics are very low, neglecting the statistical significance of this relation.

To check robustness, we rerun our regression models using the S&P 500 as reference benchmark. The results shown in table 15 are in line with that for the Nasdaq Composite Index. All stage variables are statistically significant in all models, with the exception of the turnaround variable. Results for our control variables improve marginally. In model VI the size and “focus” variable are significant but still contradicting our expectations.

5.6 Conclusion

Confirmation of hypothesis I: Stage information is an important factor for PE’s “hedging” capabilities. Our empirical analysis entirely underpins the importance of the stage variable for distinguishing PE projects that generated excess returns over low PM returns from other that fail to do so.

Confirmation of hypothesis II: Early stage investments are negatively related with “hedging” capabilities. The empirical results consistently support this hypothesis. While the negative relation between the early stage variable and the probability of belonging to the “hedge” group is highly significant (discriminant model), significance decreases for some of our regression models. As the direction of the relation is stable and coefficients are significant in most regression models, we are confident to confirm this hypothesis. The diversification potential of early stage (VC) investments is limited due to close to market entry valuations, dependence on IPOs and active timing of divestments.

Confirmation of hypothesis III: Later stage investments are positively related with “hedging” capabilities. Our empirical results fully support this hypothesis. Later stage PE investments, typically pursued by BO funds, consistently generate excess returns over PM investments even during times of bearish stock markets. Investors can indeed benefit
from diversification effects for their PM portfolios. Acquisition prices can be based on fundamental analysis rather than to extensively rely on market valuations. With trade sales and secondary buyouts, functioning alternative exit channels further limit the dependence on vibrant PM market.

**Rejection of hypothesis IV:** Investment size is positively related with “hedging” capabilities. In this case we observe deviating empirical results. The discriminant analysis provides evidence for our hypotheses, showing that the probability of belonging to the “hedge” group increases with size. But we observe the opposite relation in our regression models. Size is negatively related with excess returns in times of low PM returns. Despite these coefficients not being statistically significant in all cases we do not see sufficient support for our hypothesis and reject it.

### 6 Summary and Implications

In this paper we show how cross-sectional correlation factors between PE and PM returns can be approximated despite the lack of time series market quotes for PE investments. Based on a sample comprising 2,380 realized PE investments, we observe low cross-sectional correlations between PE and stock index returns. Results are relatively stable over broad, industry specific and local benchmark indices. Selecting a public benchmark for relative performance appraisal is therefore somehow arbitrary with respect to its explanatory power. The loose relationship between PE and PM returns promises attractive diversification potential for pure PM investors.

We sort our PE investments in descending order of simultaneous PM investments’ returns. In times of low PM returns (worst 10%), we observe even lower PE returns. Despite low correlations, PE investments thus fail to generate excess returns over PM during bearish stock market conditions.

We point out that distinct valuation approaches and exit channels are likely to trigger different results for early and later stage investments. As financial statements become more relevant for valuation purposes as firms mature, early stage investors are more likely to follow “comparable” market valuation levels when pricing investment opportunities. In addition, IPOs are the most important exit channel of VC investors and
timing of public offerings to match valuation peaks in public markets is a critical success factor. Return maximization and offering a hedge opportunity are thus conflicting goals for VC funds. Later stage investments are likely to be valued based on fundamental historic data and can easier be exited via alternative channels, e.g. trade sales or secondary buyouts. Size, geographical diversification and focused investment strategy are other characteristics potentially explaining PE’s hedging capabilities.

For our sample, later stage investment indeed generate excess returns in times of low PM returns, while early stage investments even perform worse. Discriminant and regression analyses provide significant statistical evidence for the influence of investment stage on “hedging” capabilities. The other variables are not significant.

Investors seeking diversification benefits should in case of PE not solely rely on correlation coefficients. Instead, care needs to be taken with respect to PE fund’s investment strategy. The conflict of objectives between active market timing and smoothing of PM returns, is resolved in favor of active market timing and return maximization. BO funds are less dependent on PM’s return developments and are thus the better complement for PM portfolios.
References


The table summarizes the sample’s distribution regarding investment stage and industry classification of the PE projects. For early stage investments the share of high-tech driven industries (IT, Internet & Media, Healthcare and Telecommunication) is relatively large. On the other hand low-tech industries are investment targets of later stage and turnaround investors. In the sample this is the case for the Consumer Discretionary, Industrial Production and Services industry clusters.
**Table 2**

<table>
<thead>
<tr>
<th>Industry</th>
<th>Investment Horizon</th>
<th>short</th>
<th>medium</th>
<th>long</th>
<th>total</th>
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</thead>
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<tr>
<td></td>
<td>frequency row [%]</td>
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<td>IT</td>
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<td></td>
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<td>healthcare</td>
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<td></td>
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<td>other industry</td>
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<tr>
<td>Total</td>
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<td>56.47</td>
<td>19.75</td>
<td>100.00</td>
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Investment horizons are considered to be short if they are up to 24 months long. Medium investment horizons lie between 25 and 72 months. Long investments horizons exceed 72 month. There are only small differences regarding the share of short, medium and long term investments between industry clusters.
<table>
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<tr>
<th>Stage</th>
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<th></th>
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<th></th>
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<td></td>
<td>small</td>
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<td>large</td>
<td>Total</td>
<td></td>
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<tr>
<td>early stage</td>
<td>frequency</td>
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<tr>
<td></td>
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<td>59.62</td>
<td>11.97</td>
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<tr>
<td>expansion</td>
<td>frequency</td>
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<td>128</td>
<td>121</td>
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<td></td>
<td>row [%]</td>
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<td>52.09</td>
<td>28.68</td>
<td>100.00</td>
</tr>
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<td></td>
<td>row [%]</td>
<td>16.30</td>
<td>53.87</td>
<td>29.83</td>
<td>100.00</td>
</tr>
</tbody>
</table>

Investment amounts are in terms of 2004 US Dollar. Investment amounts are considered small if they are below USD 1,000,000. Medium investment amounts are between ZSD 1,000,000 and USD 10,000,000. Large investment amounts exceed USD 10,000,000. The arithmetic mean is USD 11.7 m. The large standard deviation of USD 23.3 m. indicates the wide dispersion of investment amounts. Early stage investments have a larger share (28%) of small investments, while the other stages are pretty similar regarding their amount distribution. This is in line with Venture Capital and Buyout Funds general investment strategy.
In this table the IRR figures of our sample are described. Return figures are provided for the full sample (all investments) and alternative perspectives (industry, stage, investment amount). Overall the PE sample generated mean IRR of 50.5% with a median of 18.1% and a standard deviation of 354.2%.

High-tech industries (IT, Telecommunications, Internet & Media) exhibit higher mean IRRs with larger dispersion levels than Non-high-tech industries (Consumer Discretionary, Industrial Production, Services). Regarding stage perspective, early stage investments generated the lowest mean IRR with the median being negative. Return distributions of later stage and turnaround investments have higher means and medians, but with larger standard deviations. The investment amount perspective reveals descending mean returns with increasing size. We will analyze in more detail whether this relation is significant and impacts hedging capabilities of PE investments.
To analyze correlations between PE and PM returns we follow a cross sectional approach. We calculate correlations based on comparable IRR figures generated with simulation approach illustrated in appendix A.

In general PE returns exhibit low correlations with public stock indices. Of the broad indices, the Nasdaq Composite Index (0.18) has the highest and the S&P 500 (0.05) the lowest correlation factor with our full PE sample, with all factors being significant at the 99% level. For industry clusters, correlation with Nasdaq Composite range from 0.01 (services) to 0.44 (telecommunication) and not all coefficients are significantly different from zero. Correlation factors of the stage clusters range more narrowly around 0.17 for Nasdaq Composite. Turnaround investments are the exception (0.07), but as this sample is small, cross-sectional correlation results are unstable as indicated by high p-value (58%). Size seems to have little impact on cross-sectional correlation factors. While specific indices add explanatory power for some clusters (consumer discretionary, industrial production, healthcare, UK investments) they fail to do so for others (IT, telecommunications, German investments). As correlations are consistently on a low level, there seem to be little value added of selecting industry specific benchmarks to explain PE investment returns. Based on our results PE seems to offer substantial diversification potential for pure PM portfolios.

| Table 5 |  |
|---|---|---|---|---|---|---|---|---|---|
| Correlation factors | obs | Nasdaq Comp. | p-value | S&P 500 | p-value | MSCI World | p-value | Specific benchmark | p-value |
| IRR based | 2380 | 0.18 | 0.00 | 0.05 | 0.01 | 0.10 | 0.00 |  |
| Industry | 361 | 0.28 | 0.00 | 0.05 | 0.31 | 0.12 | 0.02 | S&P IT | 0.19 | 0.00 |
| | 273 | 0.05 | 0.39 | 0.03 | 0.61 | 0.01 | 0.92 | S&P Consumers | 0.19 | 0.04 |
| | 322 | 0.02 | 0.69 | -0.01 | 0.91 | 0.01 | 0.83 | S&P Industrials | 0.07 | 0.20 |
| | 175 | 0.44 | 0.00 | 0.24 | 0.00 | 0.33 | 0.00 | Nasdaq Telecom. | 0.38 | 0.00 |
| | 172 | 0.01 | 0.86 | 0.23 | 0.00 | 0.57 | 0.03 |  |
| | 288 | 0.20 | 0.00 | -0.05 | 0.42 | 0.03 | 0.64 | S&P Healthcare | 0.08 | 0.35 |
| | 233 | 0.15 | 0.02 | 0.09 | 0.17 | 0.14 | 0.03 |  |
| | 580 | 0.11 | 0.01 | 0.06 | 0.12 | 0.10 | 0.01 |  |
| Stage | 426 | 0.18 | 0.00 | 0.19 | 0.00 | 0.20 | 0.00 |  |
| | 263 | 0.13 | 0.04 | -0.01 | 0.83 | 0.02 | 0.72 |  |
| | 449 | 0.20 | 0.00 | 0.04 | 0.45 | 0.04 | 0.08 | Russell 2000 | 0.10 | 0.03 |
| | 68 | 0.07 | 0.58 | 0.15 | 0.22 | 0.08 | 0.51 |  |
| | 1,175 | 0.20 | 0.00 | 0.02 | 0.49 | 0.09 | 0.00 |  |
| Investment Amount | 373 | 0.20 | 0.00 | 0.03 | 0.53 | 0.19 | 0.05 |  |
| | 1,300 | 0.22 | 0.00 | 0.07 | 0.01 | 0.12 | 0.00 |  |
| | 707 | 0.18 | 0.00 | 0.07 | 0.08 | 0.08 | 0.05 |  |
| Geography | Germany | 104 | 0.47 | 0.00 | 0.07 | 0.45 | 0.16 | 0.10 | OAX | 0.29 | 0.00 |
| | UK | 342 | 0.03 | 0.62 | 0.08 | 0.13 | 0.08 | 0.14 | FTSE UK 100 | 0.10 | 0.06 |
This table summarizes the results of our sensitivity check for cross-sectional correlations with respect to sample size. We use four bootstrap models (refer to appendix B for a detailed description) to randomly draw 100 portfolios each comprising 50, 100, 200, or 500 PE investments. We construct corresponding PM portfolios with matching investment timing and amount. Based on these PE and PM portfolios, we estimate 100 correlation coefficients and sort them in ascending order. The reference benchmark is the Nasdaq Composite Index. The correlation of its returns with our PE sample (0.18) serves as reference case for the sensitivity check. In the table, cumulative densities (1%, 25%, 50%, 75%, 100%) describe the distributions of correlation coefficients the models generate. We see that dispersion substantially declines with sample size. Correlation factors for 50 randomly drawn investment pairs (model I) range from -0.28 to 0.62. Large estimation errors due to sample selection become unlikely if one draws 500 investment pairs (model IV). Here correlation factors range from 0.06 to 0.38 and center much closer around the reference value of 0.18. Looking at the 25% and 75% cumulative density levels one can see that for model III (200 investments) the estimate ranges from 0.13 to 0.24 and thus narrowly around the reference value. Sample size (relative to total population) indeed substantially impacts reliability of results. Small samples (n < 200) result in cross-sectional estimates of questionable quality.

<table>
<thead>
<tr>
<th>Cross-sectional correlation</th>
<th>Bootstrap models</th>
<th>Model I</th>
<th>Model II</th>
<th>Model III</th>
<th>Model IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nasdaq Composite Index</td>
<td></td>
<td># 50 investments</td>
<td>#100 investments</td>
<td>#200 investments</td>
<td>#500 investments</td>
</tr>
<tr>
<td>Cumulative densities</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1%</td>
<td>-0.28</td>
<td>-0.03</td>
<td>0.02</td>
<td>0.06</td>
<td></td>
</tr>
<tr>
<td>25%</td>
<td>0.11</td>
<td>0.15</td>
<td>0.13</td>
<td>0.16</td>
<td></td>
</tr>
<tr>
<td>50%</td>
<td>0.23</td>
<td>0.24</td>
<td>0.20</td>
<td>0.19</td>
<td></td>
</tr>
<tr>
<td>75%</td>
<td>0.35</td>
<td>0.29</td>
<td>0.24</td>
<td>0.23</td>
<td></td>
</tr>
<tr>
<td>100%</td>
<td>0.62</td>
<td>0.60</td>
<td>0.43</td>
<td>0.38</td>
<td></td>
</tr>
</tbody>
</table>
Table 7

<table>
<thead>
<tr>
<th>Performance clusters</th>
<th>Mean IRR Nasdaq Comp. [%]</th>
<th>Mean IRR Full PE sample [%]</th>
<th>Mean IRR S&amp;P 500 [%]</th>
<th>Mean IRR Full PE sample [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st - best</td>
<td>58.64</td>
<td>257.05</td>
<td>30.22</td>
<td>74.44</td>
</tr>
<tr>
<td>2nd</td>
<td>35.99</td>
<td>30.79</td>
<td>24.55</td>
<td>43.69</td>
</tr>
<tr>
<td>3rd</td>
<td>29.68</td>
<td>46.57</td>
<td>21.74</td>
<td>37.18</td>
</tr>
<tr>
<td>4th</td>
<td>25.70</td>
<td>65.04</td>
<td>18.69</td>
<td>65.65</td>
</tr>
<tr>
<td>5th</td>
<td>22.13</td>
<td>18.70</td>
<td>16.01</td>
<td>71.19</td>
</tr>
<tr>
<td>6th</td>
<td>19.32</td>
<td>31.50</td>
<td>13.54</td>
<td>42.57</td>
</tr>
<tr>
<td>7th</td>
<td>16.92</td>
<td>59.33</td>
<td>11.21</td>
<td>28.71</td>
</tr>
<tr>
<td>8th</td>
<td>13.85</td>
<td>24.50</td>
<td>8.58</td>
<td>107.00</td>
</tr>
<tr>
<td>9th</td>
<td>7.11</td>
<td>3.57</td>
<td>4.67</td>
<td>53.66</td>
</tr>
<tr>
<td>10th - worst</td>
<td>-22.01</td>
<td>-31.64</td>
<td>-10.29</td>
<td>-18.61</td>
</tr>
</tbody>
</table>

In this table we analyze the hedging properties of PE investments in times of bad performing public stock markets. To do so, we form ten clusters according to their PM performance and compare mean (IRR) returns with that of corresponding PE investments. We are interested to see whether PE investments in our sample on average still generated positive returns in times of low public stock market returns. The analysis includes our full PE sample (2380 investments). We form 10 clusters of 238 investments each with descending PM returns, with the 1st cluster containing the best and the 10th cluster including the worst performing. The mean returns of our simulated investments in the Nasdaq Composite Index gradually decrease from 58% to -22%. But despite an overall low correlation between the return pairs, PE returns for the worst performing cluster (10) are surprisingly low. Thus, in times of bearish public markets PE investments were not able to generate positive returns and even performed worse. Columns three and four summarize the same analysis relative to S&P 500. Again mean IRR figures of the PE investments for the 10th cluster are below that of the PM.
Table 8

<table>
<thead>
<tr>
<th>Performance clusters</th>
<th>Mean IRR - early stage Nasdaq Comp. [%]</th>
<th>Mean IRR - later stage Nasdaq Comp. [%]</th>
<th>PE investments PE sample [%]</th>
<th>PE investments PE sample [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st - best</td>
<td>62.42</td>
<td>64.34</td>
<td>63.62</td>
<td>47.07</td>
</tr>
<tr>
<td>2nd</td>
<td>37.77</td>
<td>59.83</td>
<td>46.38</td>
<td>59.45</td>
</tr>
<tr>
<td>3rd</td>
<td>32.05</td>
<td>59.45</td>
<td>42.61</td>
<td>59.45</td>
</tr>
<tr>
<td>4th</td>
<td>28.06</td>
<td>53.09</td>
<td>38.72</td>
<td>53.09</td>
</tr>
<tr>
<td>5th</td>
<td>24.38</td>
<td>92.43</td>
<td>29.82</td>
<td>92.43</td>
</tr>
<tr>
<td>6th</td>
<td>19.52</td>
<td>47.18</td>
<td>103.52</td>
<td>47.18</td>
</tr>
<tr>
<td>7th</td>
<td>15.36</td>
<td>23.69</td>
<td>-2.28</td>
<td>23.69</td>
</tr>
<tr>
<td>8th</td>
<td>7.33</td>
<td>33.10</td>
<td>-34.35</td>
<td>33.10</td>
</tr>
<tr>
<td>9th</td>
<td>-14.68</td>
<td>7.08</td>
<td>-47.21</td>
<td>7.08</td>
</tr>
<tr>
<td>10th - worst</td>
<td>-34.16</td>
<td>44.76</td>
<td>-72.81</td>
<td>44.76</td>
</tr>
</tbody>
</table>

In this table we analyze and compare hedging properties of early and later stage PE investments in times of bad performing public stock markets. To do so, we form ten clusters according to PM performance and compare mean (IRR) returns with that of corresponding PE investments. Our early stage sample comprises 426 investments and thus each cluster contains 42 or 43 investments. The later stage sample comprises 448 investment cluster size varies between 44 and 45. We select the Nasdaq Composite Index as public benchmark for both analyses. PM performance differs between the early and later stage investments due to different timing of the PE investments. As every PM investment matches the timing of a PE investment, Nasdaq returns differ. The clusters descend with respect to PM returns, with the 1st cluster being the best and the 10th cluster being the worst performing. Our focus is on worst performing clusters. For the early stage investments we observe underperformance relative to the Nasdaq for clusters 7 to 10. Thus, these PE investments weren’t capable to generate positive returns in times of low PM returns. In contrast, later stage returns are positive for all clusters. These PE investments preserve their positive returns despite low Nasdaq returns. These results support our hypothesis that later stage PE investments are the better hedge against PM downturns than early stage PE investments.
Table 9 contains the group statistics of our discriminant model. The grouping variable is outperformance of PE over Nasdaq Composite Index. We analyze the PE returns for the 10% worst performing Nasdaq investments (N=238). If PE outperformed the NasdaqComp.the grouping variable is equal to one (hedge) otherwise it is zero (no hedge).

The table reveals that the mean values for the first three stage variables (early, expansion, later) differ substantially between the non-hedge group (0) and the hedge group (1). On the other hand the relatively high within group standard deviation decrease their explanatory power. But overall significant results for these variables seem feasible. Regarding the two remaining stage variables, the group statistics indicate lower relevance for the classification. Turnaround and other stage both vary marginally in their mean values, and especially the other stage variable exhibits an extraordinary high within group standard deviation.

Looking at the variables further describing PE project’s characteristics, little explanatory power can be expected. While the size variable differs substantially in its group means (m$ 6.8 vs. m$ 1.4) its within group standard deviation are enormous. All other variables are relatively close regarding the group means.
## Table 10

**Discriminant Analysis**  
*Tests of Equality of Group Means*

<table>
<thead>
<tr>
<th>Discriminant</th>
<th>Wilks’ Lambda</th>
<th>F</th>
<th>df1</th>
<th>df2</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>earlystage</td>
<td>0.91</td>
<td>23.06</td>
<td>1</td>
<td>230</td>
<td>0.00</td>
</tr>
<tr>
<td>expansion</td>
<td>0.95</td>
<td>13.08</td>
<td>1</td>
<td>230</td>
<td>0.00</td>
</tr>
<tr>
<td>later</td>
<td>0.90</td>
<td>26.12</td>
<td>1</td>
<td>230</td>
<td>0.00</td>
</tr>
<tr>
<td>turnaround</td>
<td>0.99</td>
<td>2.08</td>
<td>1</td>
<td>230</td>
<td>0.15</td>
</tr>
<tr>
<td>OTHERSTG</td>
<td>0.98</td>
<td>3.63</td>
<td>1</td>
<td>230</td>
<td>0.06</td>
</tr>
<tr>
<td>size</td>
<td>0.96</td>
<td>10.72</td>
<td>1</td>
<td>230</td>
<td>0.00</td>
</tr>
<tr>
<td>usa</td>
<td>1.00</td>
<td>0.25</td>
<td>1</td>
<td>230</td>
<td>0.62</td>
</tr>
<tr>
<td>focus_fund</td>
<td>0.99</td>
<td>2.52</td>
<td>1</td>
<td>230</td>
<td>0.11</td>
</tr>
</tbody>
</table>

This table summarizes the test of equality means for the two different groups. Highly significant differences indicate high explanatory power of variables. The very high significance levels for the first three stage variable underpin their relevance for the grouping model. Furthermore, the test for the size variable is highly significant. But with its high within group standard deviation its explanatory power remains doubtful. For all other variables the significance levels exceed 5%. Based on the significance figures, the relevance of the U.S. (38%) variable can already at this point be neglected.

## Table 11

**Discriminant Analysis**  
*Goodness of fit Measures*

<table>
<thead>
<tr>
<th>Eigenvalues</th>
<th>Eigenvalue</th>
<th>% of Variance</th>
<th>Cumulative %</th>
<th>Canonical Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.28</td>
<td>100</td>
<td>100</td>
<td>0.47</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Wilks’ Lambda Test of Function(s)</th>
<th>Wilks’ Lambda</th>
<th>Chi-square</th>
<th>df</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.78</td>
<td>55.67</td>
<td>8</td>
<td>0.00</td>
<td></td>
</tr>
</tbody>
</table>

We look at eigenvalue, canonical correlation, wilk’s lambda and significance of the chi-square test to assess the adequacy of the discriminant model. The eigenvalue of approximately 0.28 indicates that the between group deviation is only 28% of the within group deviation. As there is a discrepancy between the groups, the model has explanatory power, but as expected not all the discrepancy can be explained by the model. This is also reflected by the canonical correlation of 0.47 and the wilk’s lambda of 0.78. The very high significance level of the chi-square test underpins the overall adequacy of our model.
Table 12

<table>
<thead>
<tr>
<th>Discriminant Analysis Structure Matrix</th>
<th>Function = 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>later</td>
<td>0.64</td>
</tr>
<tr>
<td>earlystage</td>
<td>-0.60</td>
</tr>
<tr>
<td>expansion</td>
<td>0.45</td>
</tr>
<tr>
<td>size</td>
<td>0.41</td>
</tr>
<tr>
<td>focus_fund</td>
<td>0.20</td>
</tr>
<tr>
<td>turnaround</td>
<td>0.18</td>
</tr>
<tr>
<td>usa</td>
<td>-0.06</td>
</tr>
</tbody>
</table>

The structure matrix ranks the variable according to their explanatory power for the hedging capabilities. This table reveals that indeed later stage and early stage variables have the highest correlation with the dependent variable and are thus most important for classification. Not only the expansion stage, but also the real investment amount variable has a correlation coefficient exceeding 0.4. As already expected from the group statistics, remaining variables (focus, turnaround, U.S.) contribute only marginally to the explanatory power of the discriminant model.

Table 13

<table>
<thead>
<tr>
<th>Discriminant Analysis Classification Results</th>
<th>Outperformance (yes = 1 no = 0)</th>
<th>Predicted Group Membership</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cases Selected</td>
<td>Count</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>135</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>28</td>
<td>43</td>
</tr>
<tr>
<td></td>
<td>%</td>
<td>83.9</td>
<td>16.1</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>39.4</td>
<td>60.6</td>
</tr>
</tbody>
</table>

|                                             | 1                               | 100.0                       |

a 76.7% of selected original grouped cases correctly classified.

Of the 161 PE investments that are no successful hedges, 26 are incorrectly classified as such by the model. Of the 71 successful PE hedges, 28 are incorrectly classified as not being a hedge. Overall approximately 77% of the cases are classified correctly by the discriminant model. As the PE projects and its returns are very heterogeneous in nature, it is not surprising that the model is not perfect. Although this limits the predictability power of the model, it is no downturn for the cross sectional difference we carve out.
Table 14
OLS Regression
Excess Return over Nasdaq Comp.

<table>
<thead>
<tr>
<th></th>
<th>I (N=238)</th>
<th>II (N=238)</th>
<th>III (N=476)</th>
<th>IV (N=476)</th>
<th>V (N=714)</th>
<th>VI (N=714)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cluster 10 10% worst perf.</td>
<td>-0.74***  [-7.768]</td>
<td>-0.06 [-0.078]</td>
<td>-0.53*** [-8.834]</td>
<td>-0.44 [-0.898]</td>
<td>-0.349*** [-7.885]</td>
<td>-0.43 [-1.154]</td>
</tr>
<tr>
<td>Cluster 10 20% worst perf.</td>
<td>-0.10 [-0.726]</td>
<td>-0.06 [-0.438]</td>
<td>-0.30*** [-3.016]</td>
<td>-0.24** [-2.339]</td>
<td>-0.40*** [-5.030]</td>
<td>-0.34*** [-3.991]</td>
</tr>
<tr>
<td>Clusters 9-10 20% worst perf.</td>
<td>0.81*** [3.969]</td>
<td>0.93*** [4.224]</td>
<td>0.54*** [3.976]</td>
<td>0.61*** [4.141]</td>
<td>0.35*** [3.192]</td>
<td>0.39*** [3.420]</td>
</tr>
<tr>
<td>Clusters 8-10 30% worst perf.</td>
<td>0.747*** [4.691]</td>
<td>0.86*** [4.741]</td>
<td>0.45*** [4.138]</td>
<td>0.49*** [3.923]</td>
<td>0.24*** [2.736]</td>
<td>0.28*** [2.689]</td>
</tr>
<tr>
<td>Nasdaq Comp.</td>
<td>-0.07 [-1.457]</td>
<td>-0.01 [-0.44]</td>
<td>0.42* [1.247]</td>
<td>0.50** [1.650]</td>
<td>0.27 [1.601]</td>
<td>0.33* [1.858]</td>
</tr>
<tr>
<td>early stage</td>
<td>-0.05 [-0.534]</td>
<td>-0.05 [-0.595]</td>
<td>-0.01 [-0.143]</td>
<td>-0.01 [-0.143]</td>
<td>-0.01 [-0.143]</td>
<td>-0.01 [-0.143]</td>
</tr>
<tr>
<td>expansion stage</td>
<td>0.14 [0.511]</td>
<td>-0.10 [-0.575]</td>
<td>-0.10 [-0.575]</td>
<td>-0.10 [-0.575]</td>
<td>-0.10 [-0.575]</td>
<td>-0.10 [-0.575]</td>
</tr>
<tr>
<td>later stage</td>
<td>0.38 [0.981]</td>
<td>0.51 [1.247]</td>
<td>0.42* [1.650]</td>
<td>0.50** [2.107]</td>
<td>0.27 [1.601]</td>
<td>0.33* [1.858]</td>
</tr>
<tr>
<td>turnaround stage</td>
<td>-0.07 [-1.457]</td>
<td>-0.01 [-0.44]</td>
<td>0.42* [1.247]</td>
<td>0.50** [1.650]</td>
<td>0.27 [1.601]</td>
<td>0.33* [1.858]</td>
</tr>
<tr>
<td>size</td>
<td>-0.07 [-1.457]</td>
<td>-0.01 [-0.44]</td>
<td>0.42* [1.247]</td>
<td>0.50** [1.650]</td>
<td>0.27 [1.601]</td>
<td>0.33* [1.858]</td>
</tr>
<tr>
<td>focus</td>
<td>0.14 [0.511]</td>
<td>-0.10 [-0.575]</td>
<td>-0.10 [-0.575]</td>
<td>-0.10 [-0.575]</td>
<td>-0.10 [-0.575]</td>
<td>-0.10 [-0.575]</td>
</tr>
<tr>
<td>U.S.</td>
<td>-0.05 [-0.534]</td>
<td>-0.05 [-0.595]</td>
<td>-0.01 [-0.143]</td>
<td>-0.01 [-0.143]</td>
<td>-0.01 [-0.143]</td>
<td>-0.01 [-0.143]</td>
</tr>
<tr>
<td>R²</td>
<td>0.161</td>
<td>0.177</td>
<td>0.111</td>
<td>0.115</td>
<td>0.079</td>
<td>0.083</td>
</tr>
<tr>
<td>adj. R²</td>
<td>0.147</td>
<td>0.151</td>
<td>0.103</td>
<td>0.101</td>
<td>0.074</td>
<td>0.073</td>
</tr>
<tr>
<td>F statistics</td>
<td>11.21***</td>
<td>6.86***</td>
<td>14.66***</td>
<td>8.44***</td>
<td>15.16***</td>
<td>8.758***</td>
</tr>
</tbody>
</table>

The table presents the results of our OLS regression. The dependent variable is the logarithm of the PE’s excess return over Nasdaq Composite Index. To explore PE’s hedging capabilities we only include the investment corresponding with the worst performing PM (Nasdaq) returns. The first two regression models include the 10% worst performing cases while the remaining models cover the 20% and 30% worst performing cases respectively. The first column describes the independent variables. Models I, III and IV only comprise stage variables, while II, IV and VI include all variables. The last three rows present the model diagnostics. The t-statistics are provided [ ] underneath every coefficient. The asterisks indicate significance at following levels: * 10%, ** 5%, *** 1%.
The table presents the results of our OLS regression. The dependent variable is the logarithm of the PE’s excess return over S&P 500. To explore PE’s hedging capabilities we only include the investment corresponding with the worst performing PM (S&P 500) returns. The first two regression models include the 10% worst performing cases while the remaining models cover the 20% and 30% worst performing cases respectively. The first column describes the independent variables. Models I, III and IV only comprise stage variables, while II, IV and VI include all variables. The last three rows present the model diagnostics. The t-statistics are provided [] underneath every coefficient. The asterisks indicate significance at following levels: * 10%, ** 5%, *** 1%.

<table>
<thead>
<tr>
<th>OLS Regression</th>
<th>I Cluster 10 10% worst perf. (N=238)</th>
<th>II Cluster 10 10% worst perf. (N=238)</th>
<th>III Clusters 9-10 20% worst perf. (N=476)</th>
<th>IV Clusters 9-10 20% worst perf. (N=476)</th>
<th>V Clusters 8-10 30% worst perf. (N=714)</th>
<th>VI Clusters 8-10 30% worst perf. (N=714)</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>-0.67*** [-6.304]</td>
<td>-0.05 [-0.067]</td>
<td>-0.40*** [-5.960]</td>
<td>0.48 [0.906]</td>
<td>-0.18*** [-3.579]</td>
<td>0.75** [1.899]</td>
</tr>
<tr>
<td>early stage</td>
<td>-0.37** [-2.578]</td>
<td>-0.38** [-2.420]</td>
<td>-0.46*** [-4.303]</td>
<td>-0.40*** [-3.484]</td>
<td>-0.63*** [-6.913]</td>
<td>-0.50*** [-5.055]</td>
</tr>
<tr>
<td>expansion stage</td>
<td>0.76*** [3.646]</td>
<td>0.85*** [3.751]</td>
<td>0.48*** [3.055]</td>
<td>0.62*** [3.643]</td>
<td>0.32* [2.515]</td>
<td>0.50*** [3.694]</td>
</tr>
<tr>
<td>later stage</td>
<td>0.43** [2.561]</td>
<td>0.46** [2.462]</td>
<td>0.33*** [2.736]</td>
<td>0.42*** [3.071]</td>
<td>0.16* [1.661]</td>
<td>0.33*** [3.094]</td>
</tr>
<tr>
<td>turnaround stage</td>
<td>0.40 [1.053]</td>
<td>0.51 [1.291]</td>
<td>0.30* [1.208]</td>
<td>0.43* [1.711]</td>
<td>0.01 [0.426]</td>
<td>0.27 [1.202]</td>
</tr>
<tr>
<td>size</td>
<td>-0.06 [-1.275]</td>
<td>-0.05 [-1.620]</td>
<td>-0.05 [-1.620]</td>
<td>-0.04 [-0.953]</td>
<td>-0.04 [-1.735]</td>
<td>-0.04 [-1.735]</td>
</tr>
<tr>
<td>focus</td>
<td>0.34 [1.135]</td>
<td>-0.09 [-0.555]</td>
<td>0.34 [1.135]</td>
<td>-0.09 [-0.555]</td>
<td>-0.407*** [-3.401]</td>
<td>-0.407*** [-3.401]</td>
</tr>
<tr>
<td>U.S.</td>
<td>-0.04 [-0.299]</td>
<td>-0.09 [-0.953]</td>
<td>-0.04 [-0.299]</td>
<td>-0.09 [-0.953]</td>
<td>-0.01 [-0.101]</td>
<td>-0.01 [-0.101]</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.159</td>
<td>0.171</td>
<td>0.103</td>
<td>0.112</td>
<td>0.095</td>
<td>0.114</td>
</tr>
<tr>
<td>adj. $R^2$</td>
<td>0.145</td>
<td>0.145</td>
<td>0.096</td>
<td>0.098</td>
<td>0.090</td>
<td>0.105</td>
</tr>
<tr>
<td>F statistics</td>
<td>11.09***</td>
<td>6.59***</td>
<td>13.55***</td>
<td>8.13***</td>
<td>18.71***</td>
<td>12.52***</td>
</tr>
</tbody>
</table>
Appendix A – Simulation Model

We strive to generate cash flow patterns for public market investments that mimic our private equity investments. As we do have detailed cash flow information for the PE investments, a purchase (sale) of shares in the benchmark is simulated when a negative (positive) cash flow occurs for the underlying PE investment. Doing so we achieve pairs of public and private equity returns that are matched and thus comparable. To illustrate our approach we use sample investment $i$ of a private equity fund.

The monthly cash flow information forms the starting point for our model. In a first step we look at the shares we need to purchase in the public benchmark investment $b$ in period $t$. When cash flows into the private equity investment we simulate an investment in the public benchmark by purchasing an equivalent number of shares:

$$ SP_{tb} = \frac{CFI_{ti}}{P_{tb}} $$

with:

- $SP_{tb}$ = shares purchased of public benchmark investment $b$ in period $t$
- $CFI_{ti}$ = cash inflow of investment $i$ in period $t$
- $P_{tb}$ = price per share of benchmark investment $b$ in period $t$

The total number of shares bought $SP_{b}$ over the entire holding period $T$ then is:

$$ SP_{b} = \sum_{t=0}^{T} SP_{tb} $$

We now add the sale of shares to our simulation. A sale occurs with a cash outflow (distribution) from the private equity investment. To determine the number of shares to be sold in period $t$ we use the ratio of cash outflows in period $t$ and total cash outflows over $T$.

$$ SS_{tb} = \frac{CFO_{ti}}{CFO_{t}} \cdot SP_{b} $$

with:

- $SS_{tb}$ = shares sold of public benchmark investment $b$ in period $t$
Extreme cash flow patterns can trigger a simulated sale of more shares than have been purchased previously. If for example a substantial cash outflow is followed by cash inflows and a write-off, all shares would be sold when the cash outflow occurs and latter “re-purchased” with the cash inflow. To avoid these misleading short sales we introduce the following boundaries for the number of shares sold in period $t$:

$$SS_{ib} = \frac{CFO_{ib}}{CFO_i} \cdot SP_b \quad \forall t : S_{t-1,b} \geq \frac{CFO_{ib}}{CFO_i} \cdot SP_b$$

$$SS_{ib} = S_{t-1,b} \quad \forall t : S_{t-1,b} < \frac{CFO_{ib}}{CFO_i} \cdot SP_b$$

with:

$$S_{t-1,p} = \text{shares held in benchmark investment } b \text{ at the end of period } t-1$$

In addition, a sale needs to be simulated if the private equity investment is completely written off. We are not interested in partial write-offs during the holding period, but solely in final write-offs marking the termination of investment $i$. After this point in time no further cash in- or outflows occur. To appropriately reflect this fact, we simulate a sale of all remaining shares of $b$ in the period after the write-off.:

$$SS_{ib} = S_{t-1,b} \quad \forall t : CF_{ib} = WO_i$$

with:

$$WO_i = \text{final and complete write-off of investment } i$$

We can now track the number of shares held at the end of each period $t$. As we analyze net cash flows, timing of share purchases and sales within a period does not matter. The number of shares held at the end of period $t$ is:

$$S_{tb} = S_{t-1,b} + SP_{ib} - SS_{ib}$$
The last step is to transform the time series of shares held into a cash flow stream. To do so we multiply the change of shares held with the share price for each period $t$:

$CF_{ib} = (S_{t-1,b} - S_{tb}) \cdot P_{ib}$

with:

$CF_{ib} = \text{simulated cash flow of mimicked investment in benchmark } b \text{ for private equity investment } i \text{ in period } t$

We have now generated the cash flow of an investment in a public benchmark that mimics the corresponding investment in a private equity project. These cash flow pairs enable us to perform cross sectional analysis between public and private equity. The model is flexible with regard to both key parameters, the public benchmark and the source of the cash flow stream. Besides cash flows of private equity projects, other alternative asset classes can be used to enable a cross sectional analysis.
Appendix B – Bootstrap Approach

We use the following setup:\(^1\) Sample \((\text{IRR}_{i1}, \ldots, \text{IRR}_{in}), (\text{IRR}_{b1}, \ldots, \text{IRR}_{bn})\); for \(n = 2380\), each from empirical distribution \(\hat{F}\), describing individual private or public equity investments.

with:

\[
\begin{align*}
\text{IRR}_i &= \text{IRR of Private Equity investment } i \\
\text{IRR}_b &= \text{IRR of Public Equity investment } b \text{ corresponding to PE investment } i \\
n &= \text{number of PE and PM investments}
\end{align*}
\]

Simulation of private equity portfolios:

We randomly select \(B = 100\) independent bootstrap samples \(x^1_m, x^2_m, \ldots, x^B_m\) each comprising \(m = 50, 200, 500\) data values, describing a PE project’s return. Thus, \(x_m\) is a vector of \(m\) PE investment returns. Each vector \(\{x^i, \ldots, x^B\}\) is a random sample from \(\{\text{IRR}_{i1}, \ldots, \text{IRR}_{in}\}\). The samples are drawn with replacement from our empirical distribution \(\hat{F}\). Our bootstrap approach generates 100 portfolios each consisting of \(m\) PE investments. For each of the portfolios we empirically estimate distribution mean and standard deviation as follows:

\[
\begin{align*}
\mu_s^m &= \frac{\sum_{j=1}^{m} x^m_j}{m} \\
\sigma_s^m &= \sqrt{\frac{\sum_{j=1}^{m} (x^m_j - \mu_s^m)^2}{m-1}}
\end{align*}
\]

\(^1\) Our bootstrap model is based on the work of Efron / Tibshirani (1993)
**Simulation of public market (stock index) portfolios:**

In line with the methodology for the PE portfolio simulation we select $B = 100$ samples $y_m^1, y_m^2, \ldots, y_m^n$ each $y_m$ corresponding with the selected $x_m$. We thus ensure that the PM bootstrap portfolios match the PE bootstrap portfolios with respect to investment timing and amount. Distribution mean and standard deviation are estimated likewise for the PE investments.

**Calculation of cross-sectional correlation factors:**

We now have 100 PE and 100 PM portfolios each comprising $m$ investments. As $x_m$ and $y_m$ are matched we can calculate the covariance between PE and PM returns as follows:

$$
\text{cov}(X,Y) = \frac{1}{m} \sum_{j=1}^{m} (x_j - \mu_x)(y_j - \mu_y)
$$

The cross-sectional correlation factor between PE (X) and PM (Y) can then be calculated as:

$$
\rho(X,Y) = \frac{\text{cov}(X,Y)}{\sigma_x \sigma_y}
$$