Modelling and Forecasting Dynamic VaR Thresholds for Risk Management and Regulation*

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Abstract

The paper presents methods of estimating Value-at-Risk (VaR) thresholds utilising two calibrated models and three conditional volatility or GARCH models. These are used to estimate and forecast the VaR thresholds of an equally-weighted portfolio, comprising: the S&P500, CAC40, FTSE100 a Swiss market index (SMI). On the basis of the number of (non-)violations of the Basel Accord thresholds, the best performing model is PS-GARCH, followed by VARMA-AGARCH, then Portfolio-GARCH and the Riskmetrics^{TM} –EWMA models, both of which would attract a penalty of 0.5. The worst forecasts are obtained from the standard normal method based on historical variances.

Keywords: Value at Risk (VaR) modelling, Forecasting risk thresholds, Portfolio Spillover-Garch, Risk management and Regulation

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1. Overview

When asked what he thought the markets would do, J.P. Morgan replied: “Stock markets will fluctuate.”

The 1980’s and 1990’s were characterized by a series of financial disasters, many of which could be attributed, entirely or in part, to poor risk management. The high levels of integration in modern financial markets do not permit a “laissez-faire” approach to the regulation of financial institutions, as systemic risk could lead to serious financial problems in the financial system. The groundbreaking Basel Capital Accord, originally signed by the Group of Ten (G10) countries in 1988, but since largely adopted by over 100 countries, requires Authorised Deposit-taking Institutions (ADI’s) to hold sufficient capital to provide a cushion against unexpected losses. Value-at-Risk (VaR) is a procedure designed to forecast the maximum expected loss over a target horizon, given a (statistical) confidence limit. Initially, the Basel Accord stipulated a standardized approach which all institutions were required to adopt in calculating their VaR thresholds. This approach suffered from several deficiencies, the most notable of which were its conservatism (or lost opportunities) and its failure to reward institutions with superior risk management expertise.

Following much industry criticism, the Basel Accord was amended in April 1995 to allow institutions to use internal models to determine their VaR and the required capital charges. However, institutions wishing to use their own models are required to have the internal models evaluated by the regulators using the backtesting procedure. The Basel Accord penalises institutions which use models with a greater number of violations than would be expected, given the statistical 1% level of confidence, this procedure is known as backtesting. The penalties that are applicable for violations are given in Table 1.

The Basel Accord (BA) was adopted by the Australian government in 1988, with the Australian Prudential Regulatory Authority (APRA) as the national regulator of financial markets. According to APRA, Australia is now fully compliant with 11 BA principles, largely compliant with 12, and materially non-compliant with 2. Importantly, Australia is compliant with Principle 12, which states that:

> *"Banking supervisors must be satisfied that banks have in place systems that accurately measure, monitor and adequately control market risk; supervisors should have the powers to impose specific limits and/or a specific capital charge on market risk exposures, if warranted."*

Although the use of VaR is a statutory requirement for Australian ADIs, it is of immense value to any entity wishing to manage their risk exposure. Unlike other measures of risk exposure, such as the 'Greeks' (namely, well-known parametric risk measures), convexity and duration, which are only applicable to a small class of assets, VaR is a general procedure that is widely applicable in any situation.

The plan of the remainder of the paper is as follows. Section 2 describes alternative methods of estimating VaR thresholds based on two calibrated models and three conditional volatility or GARCH models. The five models of volatility discussed in Section 3 are used to estimate and forecast the VaR thresholds of an equally-weighted portfolio, comprising four financial stock indexes. Some concluding comments are given in Section 4.

2. Description of VaR Threshold Models

A comparison of alternative univariate and multivariate conditional and stochastic volatility models is given in McAleer (2005). This section will discuss a range of conditional volatility or GARCH models. The advantages and disadvantages of each model, which are presented from the most basic to the most sophisticated, are described in Table 2 (for further details, see McAleer and Veiga (2004)).

(i) Standard Normal (SN)

The Standard Normal (SN) approach forecasts the conditional variance at time t as the historical variance over the previous 250 business days. It is extremely simple and easy to implement. However, as it is not a statistical model, it is difficult to calibrate (such as choosing critical values), and can also lead to excessive violations of the Basel Accord thresholds.

(ii) Riskmetrics^{TM}

Riskmetrics^{TM} (1996) developed a model which estimates the conditional variances and covariances based on the exponentially weighted moving average (EWMA) method, which is a special case of the ARCH(∞) model of Engle (1982). This approach forecasts the conditional variance at time t as a linear combination of the conditional variance and the squared unconditional shock at time t-1. It is simple to estimate and is computationally straightforward for a given portfolio with fixed weights. However, as it is not a statistical model, it is difficult to calibrate (such as choosing critical values), and can also lead to excessive violations of the Basel Accord thresholds. Moreover, if the forecasts are for a fixed portfolio, the portfolio weights cannot be varied but, if the portfolio weights are not fixed, estimation is more complicated.

(iii) Portfolio-GARCH

This approach applies the GARCH(1,1) model to the aggregated returns on the portfolio when it is treated as a single asset. It is simple to estimate and is computationally straightforward. However, as the forecasts are for a given (fixed) portfolio, the portfolio weights cannot be varied, and it can lead to excessive violations of the Basel Accord thresholds.

(iv) VARMA-AGARCH

This approach models each conditional variance and conditional covariance series using the VARMA-AGARCH model of Hoti et al. (2002), which is an extension of the VARMA-GARCH model of Ling and McAleer (2003), and uses the approach of Bollerslev (1990) to calculate the constant conditional correlations. These forecasted conditional correlations and variances are then used to produce the portfolio variance, which is the essential ingredient in calculating VaR thresholds. The model has well established structural and statistical properties, accommodates spillovers, captures asymmetries, can have variable weights for forecasting, fits and forecasts the data very well, and satisfies the Basel Accord thresholds. However, the model can be computationally demanding for a large number of assets.

(v) Portfolio Spillover-GARCH (PS-GARCH)

This approach models each series using the Portfolio Spillover-GARCH (PS-GARCH) model of McAleer and Veiga (2004), and calculates constant conditional correlations. These forecasted conditional correlations and variances are then used to produce the portfolio variance to calculate the VaR thresholds. The model has well established structural and statistical properties, is computationally straightforward, works well for a large number of assets, accommodates spillovers, can capture asymmetries, can have variable weights for forecasting, fits and forecasts the data very well, and satisfies the Basel Accord thresholds.

3. Empirical Example

The five models of volatility discussed in the previous section will be used to estimate and forecast the VaR thresholds of an equally-weighted portfolio, comprising four financial stock indexes. The alternative models used are SN, RiskMetrics^{TM} (1996) exponentially weighted moving average (EWMA) model, Portfolio-GARCH, VARMA-AGARCH, and Portfolio Spillover-GARCH (PS-GARCH). For daily data, Riskmetrics^{TM} sets the decay parameter at 0.94 and the number of lagged observations at 74, thereby using a restricted MA(74) process. In the empirical example, the weights in the portfolio are taken as given.

As discussed in the previous section, one of the major differences in the various approaches is that SN, Riskmetrics^{TM} and Portfolio-GARCH model the portfolio directly, while VARMA-AGARCH and PS-GARCH model each individual asset separately then aggregate them into a portfolio. Hence, the VARMA-AGARCH and PS-GARCH procedures are able to model portfolios where the weights change over time. This is an important consideration in optimal portfolio modelling.

The data used are daily prices measured at 16:00 (London time) for four international stock market indices (henceforth referred to as synchronous data), namely the S&P 500 (US), CAC 40 (France), FTSE 100 (UK) and a Swiss market index (SMI). All prices are expressed in local currencies. The data were obtained from DataStream for the period 3 August 1990 to 30 March 2004 as this was the longest series available at the time of collecting the data. Figure 1 gives the histogram and descriptive statistics for the portfolio returns. Kurtosis for the series is 6.9, which indicates that the distribution is highly leptokurtic. Furthermore, the Jarque-Bera Lagrange multiplier test of normality indicates that the distribution is highly non-normal.

Rolling windows are used to forecast the 1-day ahead conditional returns, conditional correlations and conditional variances. These estimates are used to produce the 1-day ahead rolling VaR forecasts. In order to strike a balance between efficiency in estimation and a viable number of rolling regressions, the rolling window size is set at 3000 for all four data sets. This leads to a forecasting period from 2 May 2002 to 30 March 2004, giving 562 forecasts.

As the penalties under the Basel Accord are determined on the basis of the number of violations over the previous 250 business days, Table 3 shows the number of violations of the 562 forecasts standardized according to 250 business days. The realized returns on the portfolio and threshold forecasts for each model are given in Figure 2.

On the basis of the results in Table 3 and Figure 2, it is clear that the rolling thresholds are highly correlated across the five forecasts from the various models and methods. The correlations of the VaR threshold forecasts are reported in Table 4. The two highest correlations are between the pairs (Riskmetrics^{TM} –EWMA, Portfolio-GARCH) and (Riskmetrics^{TM} –EWMA, PS-GARCH), while the pairs (Riskmetrics^{TM} –EWMA, VARMA-AGARCH) and (VARMA-GARCH, PS-GARCH) also have high correlations. Not surprisingly, SN has relatively low pairwise correlations with Riskmetrics^{TM} –EWMA, Portfolio-GARCH, VARMA-AGARCH and PS-GARCH.

On the basis of the number of (non-)violations of the Basel Accord thresholds, the best performing model is PS-GARCH, followed closely by VARMA-AGARCH, neither of which would lead to the imposition of any penalties. The next best performing threshold forecasts are given by the Portfolio-GARCH and Riskmetrics^{TM} –EWMA models, both of which would have a penalty of 0.5. Not surprisingly, the worst forecasts are obtained from the SN method.

4. Concluding Remarks

The paper described alternative methods of estimating Value-at-Risk (VaR) thresholds based on two calibrated models and three conditional volatility or GARCH models. The five models of volatility were used to estimate and forecast the VaR thresholds of an equally-weighted portfolio, comprising four financial stock indexes, namely S&P500, CAC40, FTSE100 a Swiss market index (SMI). On the basis of the number of (non-)violations of the Basel Accord thresholds, the best performing model was PS-GARCH, followed closely by VARMA-AGARCH, neither of which led to the imposition of any penalties. The next best performing threshold forecasts were given by the Portfolio-GARCH and Riskmetrics™ –EWMA models, both of which had a penalty of 0.5. Not surprisingly, the worst forecasts were obtained from the standard normal method based on historical variances.

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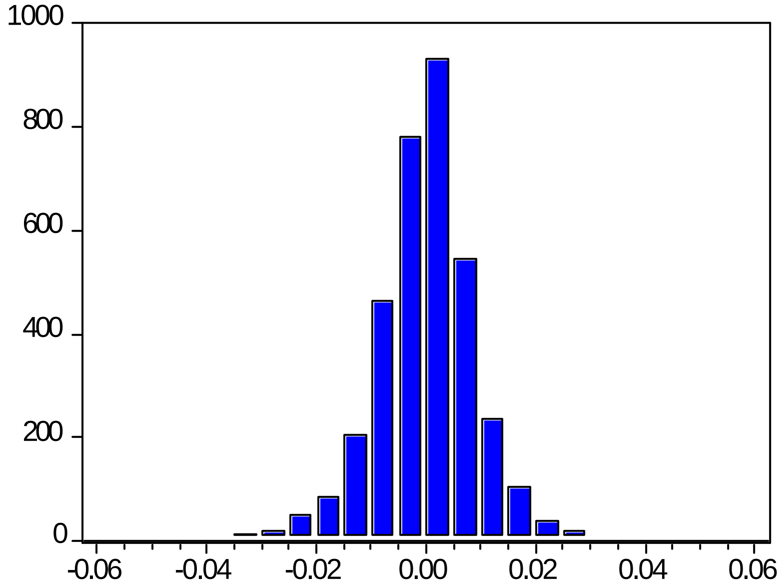
Figure 1: Histogram and Descriptive Statistics of Portfolio Returns



Series: PF	
Sample 8/06/1990 3/30/2004	
Observations 3562	
Mean	0.000259
Median	0.000348
Maximum	0.055092
Minimum	-0.058298
Std. Dev.	0.010112
Skewness	-0.124846
Kurtosis	6.905647
Jarque-Bera	2273.213
Probability	0.000000

Figure 2: Realized Portfolio Returns and VaR Threshold Forecasts

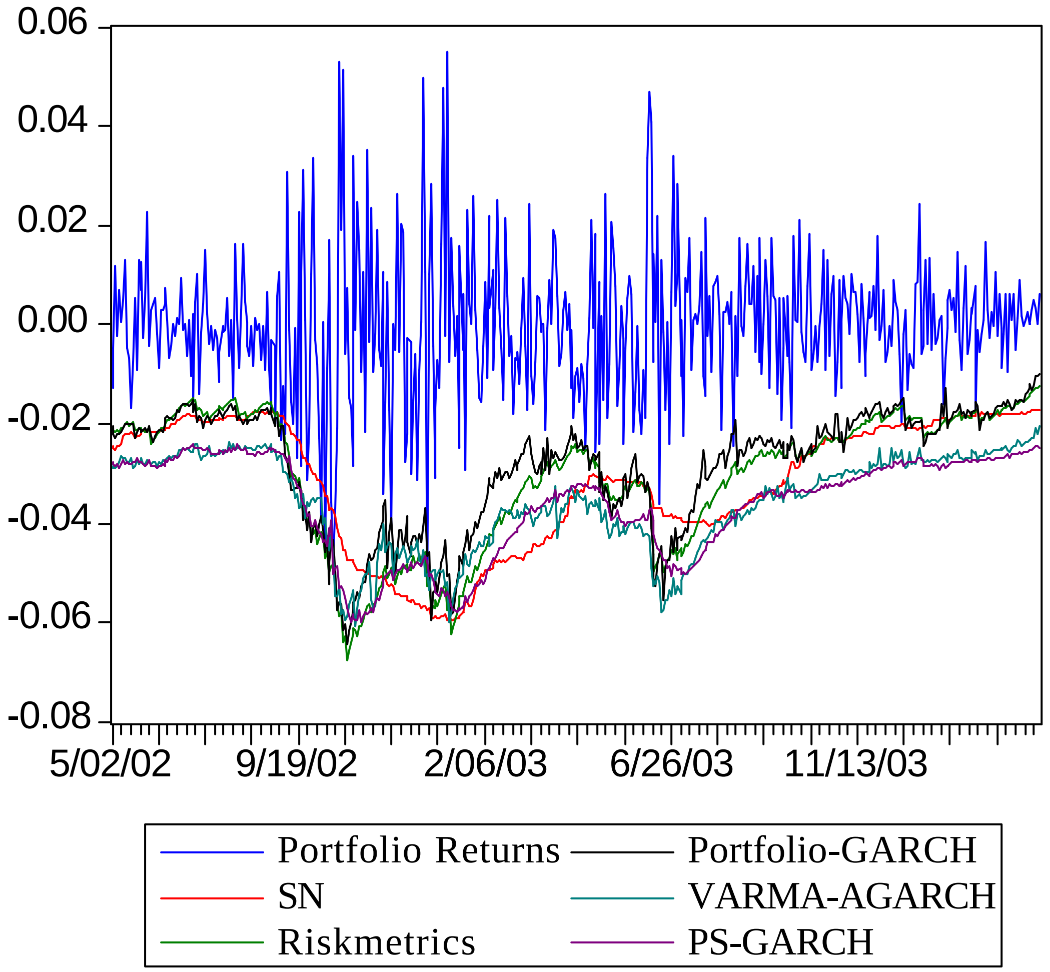


Table 1: Basel Accord Penalty Zones

Zone	Number of Violations	Increase in *k*
Green	0 to 4	0.00
Yellow	5	0.40
	6	0.50
	7	0.65
	8	0.75
	9	0.85
Red	10+	1.00

Note: The number of violations is calculated on the basis of 250 business days.

Table 2: Alternative VaR Threshold Models		
Model	Advantages	Disadvantages
SN	1) Extreme simplicity (use historical averages and standard deviations); 2) Ease of implementation.	1) Extreme simplicity; 2) Not a statistical model, so it is difficult to calibrate (such as choosing critical values); 3) Can lead to excessive violations of the Basel Accord thresholds.
Riskmetrics–EWMA	1) Simple to estimate; 2) Computationally straightforward for a given portfolio with fixed weights.	1) Not a statistical model, so it is difficult to calibrate (such as choosing critical values); 2) If forecasts are for a given (fixed) portfolio, the portfolio weights cannot be varied; 3) If the portfolio weights are not fixed, estimation is more complicated; 4) Can lead to excessive violations of the Basel Accord thresholds;
Portfolio-GARCH	1) Simple to estimate; 2) Computationally straightforward.	1) Must have a given fixed portfolio; 2) Forecasts are for a given (fixed) portfolio, so the portfolio weights cannot be varied; 3) Can lead to excessive violations of the Basel Accord thresholds.
VARMA-AGARCH	1) Structural ands statistical properties of the model have been established; 2) Accommodates spillovers; 3) Captures asymmetries; 4) Portfolio weights can be varied for forecasting; 5) Fits and forecasts the data very well; 6) Satisfies the Basel Accord thresholds.	1) Can be computationally demanding for a large number of assets.
PS-GARCH	1) Structural ands statistical properties of the model have been established; 2) Computationally straightforward; 3) Works well for a large number of assets; 4) Accommodates spillovers; 5) Can capture asymmetries; 6) Portfolio weights can be varied for forecasting; 7) Fits and forecasts the data very well; 8) Satisfies the Basel Accord thresholds.	

Table 3: Number of Violations of the Basel Accord Thresholds

Model	Number of Violations	Penalty
SN	7	0.65
Riskmetrics^{TM} -EWMA	6	0.50
Portfolio-GARCH	6	0.50
VARMA-AGARCH	2	0
PS-GARCH	1	0

Note: The number of violations is standardized according to 250 business days.

Table 4: Correlations of the VaR Threshold Forecasts

Model	SN	Riskmetrics	Portfolio-GARCH	VARMA-AGARCH	PS-GARCH
SN	1.000	0.890	0.804	0.868	0.919
Riskmetrics		1.000	0.973	0.948	0.972
Portfolio Garch			1.000	0.926	0.908
VARMA-AGARCH				1.000	0.945
PS-GARCH					1.000

References

Bollerslev, T. (1986), "Generalized Autoregressive Conditional Heteroscedasticity", *Journal of Econometrics*, 31, 307-327.

Bollerslev, T. (1990), "Modelling the Coherence in Short-run Nominal Exchange Rates: A Multivariate Generalized ARCH Approach", *Review of Economics and Statistics*, 72, 498-505.

Engle, R.F. (1982), "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation", *Econometrica*, 50, 987-1007.

Hoti, S., F. Chan and M. McAleer (2002), "Structure and Asymptotic Theory for a Multivariate Asymmetric Volatility: Empirical Evidence for Country Risk Ratings", Invited paper to the Australasian Meeting of the Econometric Society, Brisbane, Australia, July 2002.

Jorion, P. (2000), "Value at Risk: The New Benchmark for Managing Financial Risk", McGraw-Hill, New York.

Ling, S. and M. McAleer (2003), "Asymptotic Theory for a Vector ARMA-GARCH Model", *Econometric Theory*, 19, 278-308.

McAleer, M. (2005), "Automated Inference and Learning in Modelling Financial Volatility", *Econometric Theory*, 21, 232-261.

McAleer, M. and B. Veiga (2004), "Modelling Portfolio Spillovers for VaR Thresholds", unpublished paper, School of Economics and Commerce, University of Western Australia.

Riskmetrics^{TM} (1996), J.P. Morgan, Technical Document, Fourth edition.