Unlocking Value: Equity Carve-outs as Strategic Real Options *

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Equity carve-outs appear to be transitory arrangements, resolved by either a complete sale or a buy-back. Why do firms perform expensive listings just to reverse them shortly thereafter? We interpret carve-outs as strategic options to sell out or to buy back a unit, depending on the evolution of strategic synergies. The separate listing reduces the exposure to negative synergies, generates valuable information and may be reversed if synergies again turn valuable. We compute the optimal stake sold and the optimal timing for the final sale or buy back decision. The model explains the temporary nature of carve-outs as well as why in highly uncertain sectors and in more transparent markets, carve-outs are preferred over spin-off, and buy-backs are more common relative to sales.

JEL classification: G34, G13, G32.

Key words: Equity carve-out; Real option; Spin-off; Buy-back; Sell-out.
1 Introduction

In an equity carve-out (ECO), a parent company sells a portion of a subsidiary’s common stock through an initial public offering, creating an independently listed unit. ECOs constitute a significant fraction of IPOs in the US. In the 1990s, over 10% of the IPOs were ECOs; in 1993, five of the six largest IPOs in the US capital market history were ECOs. ECOs are common in industries with high value uncertainty, high sales growth and considerable investments in R&D and marketing (Allen and McConnell 1998). The selling firm is usually a large company and the subsidiary represents a small fraction of its parent activities.

There is evidence on a value-enhancing effect of ECOs. In a survey of managers involved in ECOs by Schipper and Smith (1986), the ECO decision is justified as either a shift in corporate focus, a need for greater autonomy of the subsidiary, or increasing the stock market’s awareness of its activities. A public listing produces new information for investors as well as for the parent company (Nanda (1991) and Slovin, Sushka, and Ferraro (1995)) and better information allows market-based incentives for its management (Holmström and Tirole 1993).

ECOs appear to be temporary. Within a few years, most ECOs are either sold off or reacquired by a parent. CBS Corporation, a major TV network

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1Carved-out firms appear to have higher growth potential as indicated by their high price/earnings (Schipper and Smith 1986), market-to-book ratios and R&D expenses of relative to the parent (Powers 2003).

<table>
<thead>
<tr>
<th>Second event</th>
<th>Frequency</th>
<th>Number of years between carve-out and second event</th>
</tr>
</thead>
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<tr>
<td></td>
<td></td>
<td>Average</td>
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<tr>
<td>Re-acquired by parent</td>
<td>22</td>
<td>5.7</td>
</tr>
<tr>
<td>Completely divested</td>
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<tr>
<td>Acquired by another firm</td>
<td>13</td>
<td>6.5</td>
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<tr>
<td>Spun-off</td>
<td>4</td>
<td>1.5</td>
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<tr>
<td>Exchange offer or cash sale to Subsidiary</td>
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<td>9.5</td>
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<tr>
<td>Declared bankruptcy</td>
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<td>1.7</td>
</tr>
<tr>
<td>Liquidation</td>
<td>2</td>
<td>3.5</td>
</tr>
<tr>
<td>Offer to re-acquire pending</td>
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<td>2.5</td>
</tr>
<tr>
<td>Offer to divest pending</td>
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<td>19</td>
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</tr>
<tr>
<td>Total</td>
<td>68</td>
<td></td>
</tr>
</tbody>
</table>

Source: Schipper and Smith (1986)

Table 1: Evidence on subsequent events after a ECO
facing declining profitability, carved out in 1999 its Infinity Broadcasting
division, active in radio stations, arguing that the market did not recognize
its prospects. Yet two years later CBS reacquired the stake sold, citing loss
of synergies across divisions (Wall Street Journal (2000)). Typically, most
ECO ceased to exist within 2-6 years, as a result of a so-called “second
stage” event. Schipper and Smith (1986) found that 44 out of 73 carved out
subsidiaries are reacquired by the parent, completely divested, spun-off, or
liquidated within a few years (see Table 1). Klein, Rosenfeld, and Beranek
(1991) found that 56% of all carve-outs are reacquired, while 38% are followed
by a complete sell-off (see Table 2). Hand and Skantz (1999b) find that 42.7%
of the carved-out subsidiaries are sold, 17.4% are reacquired, and 13.2% are
spun off (Table 3). Similar results are reported by Miles, Woolridge, and
Tocchet (1999) and Boone (2002). A study by McKinsey indicates that after
five years, just 8% of carved out firms remains public under the control of
the parent (Annema, Fallon, and Goedhart 2002).

Yet it is unclear why corporations implement partial demergers via rela-
tively expensive public listings, if they anticipate reversing them later.3 If a
subsidiary is no longer critical for the corporate strategy, a spin-off (namely,
a distribution of shares to existing shareholders) would be a quick and in-
expensive way to dispose of it (Michaely and Shaw 1995). In the model
of Myers and Majluf (1984), a spin-off would resolve the adverse selection
problem due caused by a managerial preference for existing shareholders.4

We propose a theoretical model to explain the life cycle of carved out sub-

3Nanda (1991) argues that a parent firm chooses to carve out a subsidiary instead of
issuing own shares when it perceives the unit to be undervalued.

4Benveniste, Huijing Fu, and Yu (2000) find an average of 9.5% first day return for
between 1981-1988 carving out was 3 times more expensive than spinning-off.
<table>
<thead>
<tr>
<th>Elapsed time</th>
<th>Number of reacquisition</th>
<th>Number of sell-offs</th>
<th>Total frequency</th>
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</thead>
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<tr>
<td>&lt; 1 year</td>
<td>0</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>1-2 years</td>
<td>2</td>
<td>7</td>
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<td>6</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3-4 years</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>4-5 years</td>
<td>5</td>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>&gt; 5 years</td>
<td>10</td>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>No second event</td>
<td>0</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>Total</td>
<td>25</td>
<td>19</td>
<td>44</td>
</tr>
</tbody>
</table>

**Median Elapsed Time**  
- 4.5 years  
- 1.33 years  
- 3.17 years  

Source: Klein, Rosenfeld, and Beranek (1991)

Table 2: Timing of subsequent second-event

<table>
<thead>
<tr>
<th>Type of second event</th>
<th>Frequency</th>
<th>Elapsed time</th>
<th>Number of sell-offs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spin-off or split-off</td>
<td>38</td>
<td>&lt; 1 year</td>
<td>37</td>
</tr>
<tr>
<td>Sell-off</td>
<td>122</td>
<td>1-2 years</td>
<td>23</td>
</tr>
<tr>
<td>Re-acquisition</td>
<td>50</td>
<td>2-3 years</td>
<td>19</td>
</tr>
<tr>
<td>Bankruptcy, liquidation, or delisting</td>
<td>11</td>
<td>3-4 years</td>
<td>13</td>
</tr>
<tr>
<td>None</td>
<td>66</td>
<td>4-5 years</td>
<td>19</td>
</tr>
<tr>
<td>No information</td>
<td>5</td>
<td>&gt; 5 years</td>
<td>11</td>
</tr>
</tbody>
</table>

Source: Hand and Skantz (1999a)

Table 3: Frequency of second stage events
sidiaries. It interprets ECOs as a strategic decision to improve the management of the interaction between the parent and the subsidiary, while retaining a “call option to reacquire”, as well as a “put option to sell”. The ECO is then a temporary phase in a dynamic strategy which recognizes learning over time on the potential synergies between the subsidiary and the parent. The final decision to buy-back or to sell-out is thus left to a future date, while the parent benefits from the information flow generated by the listing to reduce the cost of an eventual sale.

We show that as in a classic real option, the value of an ECO strategy depends on the flexibility gained, and on the uncertainty over the evolution of strategic synergies. Co-ordination between two units creates operating, marketing and financial synergies, which may at times turn negative. Lack of focus may create conflicting business interests; less direct incentives can discourage initiative. An independent listing may contain “negative synergies”, e.g. if it reduces internal conflicts. Moreover, a market listing produces new information useful for managerial compensation, as well as on strategic opportunities useful to better manage the synergistic interaction. Gilson, Healy, Noe, and Palepu (1998) and Fu (2002) offer evidence that

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6 A separate listing may reduce also internal power conflicts (Rajan and Zingales 2001).

7 Barnes and Noble justified the carve-out of their dotcom unit as required by the need for greater autonomy of the subsidiary, which was competing with the parent company in seeking to catch up with the technical lead gained by Amazon (Frack, Nodine, and Schechter 2002).

8 Several authors have shown that outside investors can produce information useful to manage the firm (see Khanna, Sleza, and Bradley (1994), Subrahmanyam and Titman (1999), Dow and Rahi (2003) and Habib, Johnsen, and Naik (1997)).
carve-outs result in higher analyst coverage and lower information asymmetry.

Floating a stake in a subsidiary thus can help reduce negative synergies. Clearly, the parent firm could simply sell or spin off the subsidiary. Yet this implies an irreversible loss of control, which is potentially very costly. If changes in technology, regulation or demand cause synergies to turn positive again, the parent firm may not be able to reacquire the subsidiary, as it may have fallen under the control of competitors. Alternatively the firm might be restructured in a way that makes a repurchase unattractive, or the management may not be willing to lose its independence. A partial sale where control is retained may thus be optimal to retain the option to reacquire.

We use a real option pricing approach in continuous time in order to determine the optimal timing to perform the carve out as well as the subsequent optimal timing to exercise the sell-out or buy-back options. This requires also endogenizing the optimal stake to be carved out as a function of IPO costs and the degree of market transparency. The optimal retained stake trades off the desire to reduce the impact of negative synergies, against the desire to reduce underpricing. It also rises with market transparency, and falls with the uncertainty of synergies. When a listing is expensive and offers little informative value (because of low transparency, or low volatility of synergies), a spin-off becomes the preferred alternative. Intuitively, when the underpricing cost is high, for low uncertainty the option value to buy back has minimal value; hence the parent wants to divest immediately the subsidiary and the spin-off becomes the preferred alternative.

We obtain several interesting empirical implications. The model is clearly

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9For an introduction on real option methodology see Dixit and Pindyck (1994). For applications of real options to corporate issues see Lambrecht (2001, 2005).
consistent with the temporary nature of the ECOs and the frequency of sell-outs and reacquisitions. Additionally, it can explain specific empirical findings in the ECO literature, on how subsequent events are correlated with both the percentage retained by the parent as well as the time elapsed (Klein, Rosenfeld, and Beranek (1991) and Boone (2002)). Consistently with our argument that firms would perform ECOs when synergies are negative, ECOs are more likely to be sold rather than bought back. Moreover, as the elapsed time from the ECO increases, the likelihood of a reacquisition increases.

Our model also predicts that in more transparent markets carve-outs should be preferred to spin-offs and higher stakes should be retained. Various studies find a much lower frequency of carve-outs in Europe than in the US. Comparing US to German data, Elsas and Loffler (2001) show that retained stakes are significantly lower than in US (57% on average against the 69% of Allen and McConnell (1998)). Additional implications, so far not explored empirically, are that carve-outs should be more prevalent in industries where synergies are more uncertain, as the flexibility offered by the carve-out makes it preferable to a spin-off. On average, fewer shares should be retained, and the buy-back becomes a more likely second event action.

In conclusion, we view ECOs as a strategic, if temporary choice to delay a decision on control over a subsidiary. By incurring some issue cost, the firm obtains additional information to manage the interaction, while acquainting the market with the subsidiary. The final decision on either relinquishing or regaining full control depends on strategic consideration driven by the evolution of the fundamental value of the cooperation between parent and the carved-out unit.

In the next section we present the basic model. Section 3 discusses the comparative statics and the empirical implications. A final section summa-
rizes the results and concludes.

2 The model

2.1 The Time Structure

We distinguish two components of a fully-owned subsidiary value: its value as an independent unit, given by the discounted net cash flow, and the synergy component, given by the discounted value of the synergy flow, $s_t$. For simplicity, we assume the net cash flow is non stochastic and is normalized to 1. It follows that the subsidiary value as independent unit is given by $\frac{1}{r}$, where $r$ is the instantaneous discount rate.

The value of the synergies is uncertain due to changes in technology, regulation or demand. Thus, we assume that this synergy flow, $s_t$, evolves over time following an Arithmetic Brownian Motion without drift:\(^{10}\)

$$ds = \sigma dz \quad (1)$$

where:

$\sigma$ is the variance parameter;

$dz$ is the increment of a Wiener process.

We assume that synergies reflect external effects with the parent firm and are not captured by other investors in the subsidiary. Shares in the subsidiary will therefore be priced based only on its cash flow.\(^{11}\) Upon a spin-off, all

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\(^{10}\)An arithmetic Brownian Motion process is appropriate since synergies can take negative values. The assumption of no drift allows more tractable solutions and is consistent with our concept of synergies.

\(^{11}\)Allowing outside investors to bargain over a fraction of the synergies do not change the results substantially.
internal synergies (positive or negative) are lost. We assume that in a partial sale, they are reduced proportionally.

Taking a firm public implies some costs, including the possible underpricing of the initial stake sold. We treat initially this cost as exogenously determined and independent on the amount of shares sold. Later we assume, in line with most of the literature, that underpricing increases in the stake sold.

Figure 1 illustrates the decision tree facing the parent firm. At the beginning, the parent firm holds full control of the subsidiary, it can at each instant either spin-off the subsidiary, perform an ECO, or postpone the decision, retaining it as a fully owned subsidiary. Under an ECO, the parent receives a fraction of the value.

An alternative to the ECO is an equity spin-off. In this case the subsidiary becomes a totally independent entity and its shares are distributed to the parent’s shareholders as pro rata stock dividends. This will stop the flow of synergies, $s_t$, and establish the stand-alone value for the subsidiary assets at $\frac{1}{r}$. Yet, a complete spin-off eliminates the possibility to regain control of the subsidiary if it were to become again profitable to do so; it thus represents a loss of a strategic option. Even if it were possible to reacquire the former subsidiary, the purchase would now require a significant premium since the controlling shareholders could demand the full increase of value that the subsidiary brings to its former parent. Therefore, we assume that there cannot be any subsequent net gain after spin-off. At the same time, the spin-off transfers ownership to existing shareholders. As this avoids any conflict of interests associated with a sale, we assume there is no underpricing in this case.

When the firm is carved out, the parent retains a fraction $\alpha$ of the total
Figure 1: The Decision Tree
subsidiary, and incurs a cost which is proportional to the stake sold. We define the underpricing percentage as $\delta$, so the net cost is given by $(1 - \alpha)\delta$. We assume that at a later date it is possible to sell the rest of the shares without further discount, since market trading will over time remove any asymmetric information, the classic cause of underpricing.

After a carve-out, the parent has still the option, at each instant, to buy back the firm, to sell out the remaining shares, or to wait. Buying back all the shares recaptures the full synergy flow, $s_t$. In contrast, after a sell-out the subsidiary becomes an independent unit and there are no more synergies with the parent. We restrict the final sell-out or buy-back to be made at a single date.

Intuitively, if $s_t$ keeps deteriorating after the ECO, at some point the best strategy is to exercise the option to sell out, in which case the parent receives the fair value of the shares it owns, $(1 - \alpha)\frac{s_t}{s_t}$. If instead $s_t$ improves over time, it will at some point become profitable to again integrate with the subsidiary, buying back all shares at $\frac{s_t}{s_t}$. This would recapture for the parent the full expected value of synergies $\frac{s_t}{s_t}$.$^{12}$

An alternative to the spin-off and the carve-out is the full retention of the subsidiary, in which case the company keeps receiving the synergies in their entirety, $s_t$. The parent firm could “wait and see” how the synergies evolve over time, keeping the option to spin off or carve-out at a later date.

$^{12}$Note that it is critical for the parent firm to retain control over the ECO to avoid having to pay the full premium reflecting all synergy gains. While here the firm pays none of it, results would not be much affected if they had to pay a fraction.
2.2 Optimal timing for the carve out

The section studies the decision to carve out, taking into account the opportunity to subsequently sell out or buy back. The comparison between the initial decision to perform either a carve out or a spin-off is analyzed in the next section.

The parent firm can at each instant carve out or keep retaining full control of the subsidiary. As it is intuitive, we will show that when the synergies turn sufficiently negative, the parent will perform a carve-out incurring some costs. Subsequently, depending on the evolution of synergies, the parent firm will either sell out or reacquire the firm. The solution identifies three threshold synergy flow levels, for which it is optimal for the parent to exercise the option to carve out, \( s^*_C \), to sell out, \( s^*_S \), and to buy back, \( s^*_B \). Since the stochastic process for the synergy flow is a stable Markov process, the optimal thresholds are time invariant.

**Proposition 1** When the underpricing costs is not too high, namely

\[
\delta \leq \frac{-1 + \sqrt{-1 + 2\alpha + s^*_B^2 (-1 + \alpha)^2 - (1 - \alpha) (\beta s^*_S + \ln (1 + s^*_B))}}{1 - \alpha} \tag{2}
\]

the optimal thresholds to carve out, to buy back and sell out are given by the solution of the following system:

\[
\beta s^*_B = \beta s^*_S + \ln \frac{-1 + \alpha - \sqrt{1 - 2\alpha + \alpha^2 \beta^2 s^*_S^2}}{\alpha (1 + \beta s^*_S)} \tag{3}
\]

\[
\beta s^*_S = \beta s^*_B + \ln \frac{-\alpha + \sqrt{-1 + 2\alpha + (1 - \alpha)^2 \beta^2 s^*_B^2}}{(1 - \alpha) (1 + \beta s^*_B)} \tag{4}
\]

\[
\beta s^*_C = -1 - \beta \delta - W \left[ - (1 + \beta s^*_B) \exp^{-1-\beta s^*_B-\beta \delta} \right] \tag{5}
\]

where \( \beta = \frac{\sqrt{2\alpha}}{\sigma} \) and \( W[z] \) is the principal solution of \( z = W \exp^W \).
When the above condition is not satisfied, then

$$\beta s^*_C = \beta s^*_S = -1 - \beta \delta - W \left[ - (1 + \beta s^*_B) \exp^{-1 - \beta s^*_B - \beta \delta} \right]$$

**(Proof.** See Appendix A)**

While these optimal thresholds cannot be derived analytically, we can easily study their comparative static. Later, we derive the optimal share retained that maximizes the value of the subsidiary for the parent.

Intuitively, when the underpricing cost is prohibitively high, full retention or a spin off are better options than a carve out. We solve later for the optimal choice.

**Sell-out**

**Proposition 2** There always exists an optimal sell-out threshold, $s^*_S < 0$. Moreover, $s^*_S$ is increasing in the fraction $\alpha$ of shares retained, $\lim_{\alpha \to 0} s^*_S = -\infty$ and $\lim_{\alpha \to 1} s^*_S = -\frac{1}{\beta}$.

**(Proof.** See Appendix B)**

These results imply that when the synergies are sufficiently negative, it is optimal for the parent to sell out. The trade off is between stopping to incur negative synergies and keeping the option to buy back the unit if synergies turn positive again. The threshold depends on how many shares have been retained at the carve-out stage. The more shares have been retained, the larger is the effect of negative synergies. The parent is therefore induced to sell out earlier, i.e. at higher (less negative) synergies (see Fig. 2).

The existence of the option to buy back lowers the threshold to sell out (see Appendix C): intuitively, when both the option to sell out and buy back exists, the firm prefers to wait longer to retain these strategic options.
Buy back

**Proposition 3** There always exists an optimal buy back threshold, \( s_B^* > 0 \).
Moreover, \( s_B^* \) is increasing in the fraction \( \alpha \) of shares retained, \( \lim_{\alpha \to 0} s_B^* = \frac{1}{\beta} \)
and \( \lim_{\alpha \to 1} s_B^* = \infty \).

**Proof.** See Appendix B  ■

The features of the optimal buy back strategy are similar to the sell-out case. The parent firm buys back the ECO only when the synergies are positive and sufficiently large, since maintaining a separate listing disseminates information which leads to a loss of competitive opportunities to competitors. The trade off balances the full capture of positive synergies against losing the option to sell out if the synergies turn negative. This threshold also depends on the stake retained. The lower the amount of shares retained, the higher the synergies the parent foregoes. Hence, it prefers to buy back earlier (i.e. for lower synergies) (Fig. 2).
It can be shown that the opportunity to sell out induces the parent to wait longer to perform a buy back than otherwise (see Appendix C). Intuitively, this follows from the fact that when both options exist the firm prefers to learn more on the future synergies before losing the strategic sale option.

**Carve-out**

**Proposition 4** There always exists an optimal carve-out threshold, \( s^*_C < 0 \), which decreases in a concave fashion in the stake retained. Moreover, \( \lim_{\alpha \rightarrow 0} s^*_C = \frac{1}{\beta} - \delta - \frac{1}{\beta} \sqrt{2 \exp^{-2\delta^2}} \) and \( \lim_{\alpha \rightarrow 1} s^*_C = \frac{1}{\beta} - 1 \).

**Proof.** See Appendix D.

From equation (5), the parent performs the carve-out when synergies become sufficiently negative. At this threshold, the losses due to negative synergies are larger than underpricing cost and the loss of the option to wait (see Fig. 2).

The threshold to carve out would be higher if no option to buy back existed. On the contrary, the existence of the option to sell out induces the parent to carve out earlier, i.e. for higher (less negative) synergies.

If there were no underpricing, the higher is the amount of shares retained, the later the timing of the ECO. A large stake retained leads to a smaller reduction in negative synergies, so the option to carve out is exercised later. Thus the optimal synergy threshold to carve out is monotonically decreasing in the retained share.

The underpricing affects the choice of the stake retained, as the parent firm faces a trade off between selling shares and incurring high underpricing, or experiencing more negative synergies. Higher underpricing delays the carveout decision, i.e. the carveout threshold, irrespective of the amount of
Range | Reacquisition | Sell-offs | No secondary event | Total |
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<td>1%-50%</td>
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<tr>
<td>Total</td>
<td>25</td>
<td>19</td>
<td>8</td>
<td>52</td>
</tr>
</tbody>
</table>

Source: Klein, Rosenfeld, and Beranek (1991)

Table 4: Percentage of subsidiary shares retained by parent

So far the fraction of shares retained has been treated as exogenous; in this section we are going to address this issue.

Empirical evidence indicates clearly that in an ECO the stake retained is usually considerable: in 3 carveouts out of 4 the parent firms retain the majority of the shares (see Table 4).

The choice of the fraction of shares to float on the market is affected by several considerations. While accounting and tax issues are important factors, the main cost of going public comes from underpricing the offering.\(^{13}\)

It is well known that IPOs are underpriced, suggesting that a firm has to sell its subsidiary at a discount when doing an ECO.\(^{14}\) Moreover, many

\(^{13}\)For more detail on accounting and tax issues see Willens and Zhu (1999). Other considerations may involve control, voting rights and financial constraints.

studies report that when pre-issue shareholders sell a smaller fraction in the IPO, the market valuations of the floated companies tend to be higher. This suggests that in practice, selling a larger fraction of the firm in an ECO, leads to a larger discount at which the firm is sold (Downes and Heinkel (1982), Ritter (1984), Kim, Krinsky, and Lee (1995), Klein (1996) and Van der Goot (1997) among the others).\textsuperscript{15}

Theoretically, there are two main mechanisms that can account for the positive relation between shares retained and pricing. Firstly, under asymmetric information about firm value, a larger fraction of shares sold may signal lower firm value (Grinblatt and Hwang 1989). Alternatively, Ritter (1984) suggests that the relationship between pricing and shares retained arises from a managerial moral hazard problem. According to this argument, selling a larger fraction of the firm worsens the agency problem with the manager, which reduces firm value and therefore the issue price.

In order to understand the optimal fraction of shares retained, we apply the empirical finding to our setting without modeling it explicitly. We therefore assume that when the firm sells a larger fraction of the subsidiary’s shares, it has to do so at a larger discount. Given that the firm value is normalized to 1, the fraction of shares retained is inversely related to the percentage of underpricing. Hence, the total cost of the ECO for the parent firm is a convex decreasing function of the fraction of shares retained. More formally, we assume that underpricing is a decreasing function of shares retained: $\frac{\partial \delta}{\partial \alpha} < 0$. This allows us to formulate the following proposition.

\textsuperscript{15}Schipper and Smith (1986), Prezas, Tarimcilar, and Vasudevan (2000), Benveniste, Huijing Fu, and Yu (2000), Na and Amaman (2002) and Hogan and Olson (2002) analyze underpricing during ECOs. These studies do not focus on the relationship between shares retained and firm value.
Proposition 5 When the level of underpricing is inversely related to the amount of shares retained, there exists an optimal level of shares retained which ranges between 0 and 1.

Proof. See Appendix E. ■

The negative relation between shares retained and underpricing creates a trade off for the parent. On the one hand the parent wants to sell as many shares as possible to reduce negative synergies. On the other hand, however, the larger the fraction of shares sold the higher is the underpricing. The total cost of the carve-out for the parent therefore increases more than proportionally. The combination of these two forces determines the optimal amount of shares retained. It depends on how severe the underpricing is, compared to the gains from reducing negative synergies. The higher the underpricing, the more the parent accepts to incur losses related to negative synergies and tenders fewer shares. One might therefore expect that at times or in markets where underpricing is more severe, parent firms sell smaller fractions of the subsidiary in an ECO. Moreover, given that underpricing is widely believed to be the result of an asymmetric information problem between issuer and the purchasing public, one might expect to see less severe underpricing in more transparent markets. Our model therefore predicts that more transparency should lead to ECOs in which larger fractions of shares are sold. This is consistent with the evidence provided by Elsas and Loffler (2001)(see also our discussion in the Introduction).
3 Comparative Statics

3.1 Timing of the carve-out and spin off decision

Interpreting the carve-out as a temporary strategy to retain flexibility allows us to explain some intriguing empirical facts. A first observation is that sell-outs are the most common final event following a carve-out. In addition, sell-outs are most common in the first years, while as time goes by, buy backs become more frequent (Klein, Rosenfeld, and Beranek (1991) and Boone (2002)). This evidence is consistent with our model, as soon as a non-negligible stake is retained. As it can be seen is Fig. 2, the threshold to sell out is closer to the threshold to carve out than the one to buy back. This implies that the sell-out threshold will be reached on average both sooner and more frequently.

The main alternative to the initial decision to perform a carve-out for a unit generating negative synergies is a spin-off. In this case, all shares in the subsidiary are distributed to current shareholders and no underpricing is incurred. However, the option to buy back is lost.

Proposition 6 The parent decides to spin off as soon as \( s_{SO}^* = -\frac{1}{\beta} \).

**Proof.** The spin-off is a special case of the carve-out without option to buy back and to sell out, where \( \alpha = 1 \). Adapting the results of Appendix A, we obtain:

\[
  s_{SO}^* = -\frac{1}{\beta}
\]  

(7)

Thus a carve-out would be preferred to a spin-off even though it implies an underpricing cost, as it reduces the negative synergies while retaining the
opportunity to buy back the subsidiary. However, when the underpricing cost is large and synergies not very volatile, the spin-off becomes the preferred strategy, since the option to buy back is not valuable.

3.2 The effect of uncertainty

Uncertainty has a major impact on the optimal timing to carve out, buy back and sell out, as it is in the case with any strategic or financial option.

**Proposition 7** The higher is uncertainty over future synergies, the higher the optimal threshold to buy back, $s^*_S$; and the lower the optimal threshold to sell out, $s^*_B$, and to carve out, $s^*_C$.

**Proof.** From equations (3) and (4), $\beta s^*_B$ and $\beta s^*_S$ are constant with respect to uncertainty. Hence, the higher the uncertainty, the lower $\beta$, the higher $s^*_B$ and the lower $s^*_S$ (as it is negative).

Solving equation (14) using equations (20), (21) and (22), it is easy to show that also $s^*_C$ is lower, the higher is uncertainty. ■

This result is consistent with traditional results in real option theory. The higher is uncertainty, the later it is optimal to exercise any option. In the case of the sell-out option, higher uncertainty implies that negative synergies are more likely to become positive in the nearer future. Thus the parent is willing to suffer longer from negative synergies rather than to sell out and lose the option to buy back. Of course, higher uncertainty increases the probability that synergies turn negative again. Hence the buy back threshold will also be higher, as the parent prefers to wait longer and keep alive the option to buy back and sell out.

The effect of uncertainty on the optimal threshold to carve out is less clear, as the total effect is the result of two opposing effects. As before, for
more negative synergies values a higher uncertainty induces the parent to wait longer to carve out. However, uncertainty affects the relative weight of the costs and benefits of carving out. The underpricing cost becomes less relevant with respect to the potential gain from repurchasing the unit were synergies to become positive again, and hence carving out becomes a more attractive option. The total effect at moderate levels of uncertainty is that an increase in uncertainty leads to a lower threshold to carve out. However, as uncertainty increases without bound, the carve-out threshold approaches the buy-back threshold, and further, also the sell-out threshold. Thus the higher is uncertainty, the more likely is the buy-back, even though the sell-out remains the most likely outcome. In conclusion, the higher is the uncertainty over synergies, the higher is the value of the flexibility that the carve-out offers relative to the spin-off.

While it may be argued that underpricing should be increasing in the underlying volatility, in our approach synergies are not priced in the subsidiary level. Moreover, more volatile synergies can often be positive for firm value, as they are more likely to become positive, and may be contained if they become negative by the option to spin off or sell off.

The effect of higher uncertainty on the likelihood of a buy back is also affected by its effect on the optimal stake retained. On one hand, the higher is uncertainty, the shorter is the expected time of remaining in the carve out status and so the higher is the opportunity costs of underpricing (as underpricing is paid in order to enjoy the flexibility status of carve out). At the same time, the higher is uncertainty, the more negative are the synergies at which the carve out is performed and hence the smaller is the stake that the parent firm wants to retain.
The total effect of uncertainty on the optimal stake retained is not monotonic and depends on which of these two effects prevails. For low level of uncertainty, the effect on the period as carve out prevails, so that higher uncertainty increases the stake retained. When instead uncertainty is very high, the synergy component prevails and hence for higher uncertainty the parent firm sells more shares.

4 Concluding Remarks

Equity carve-outs are popular among large corporations as a strategy to refocus their businesses without relinquishing strategic control over the carved-out units. This provides the company with a high degree of flexibility concerning future corporate strategy. At the same time, a separate listing can offer immediate benefits on operational performance generating information which helps to manage the interaction between units. In addition, better information may reduce financing constraints and improve managerial performance. This may well account for the popularity of ECOs in high growth and high uncertain sectors.

We adopt a strategic real option approach to explain why parents might prefer an expensive listing to a spin-off. The approach describes the potential benefits of equity carve-outs as the acquisition of future strategic opportunities of either capturing positive synergies or avoiding conglomeration costs, while at the same time allowing the market to generate information on the potential of its subsidiary.\footnote{More specifically, it is equivalent to a strategic growth option such as described by Kulatilaka and Perotti (1998), since it enables the firm to take advantage of future growth opportunities better than its rivals when conditions change favourably.}

We conclude that the choice of an equity carve-out is just the first part of
a sequential contingent strategy to either sell-off the subsidiary in a phased manner or re-acquire it if favourable information on its internal strategic value emerges. Applying a strategic real option approach gives a comprehensible interpretation for this strategy, and has potential to explain the empirical results on its timing.

An ECO appears to be an optimal intermediate step even when the likely outcome is a sell-out. Selling the subsidiary in a phased manner allows the market to generate new information on its value, and on related opportunities. This improves the management of the unit and facilitates any subsequent sell out (Nanda 1991). At the same time, when synergies are highly valuable, the option to buy back will have some value. When the benefits of greater public information gathering on the subsidiary no longer outweigh the costs of a separate listing and the loss of some opportunities, the parent company will reacquire the minority share from the market.

The key feature of the option acquired with a ECO is therefore that the final decision on buying back or selling out does not need to be pre-planned by the parent, but can be state-contingent at a future date, when more information has become available. Accordingly, these options have considerable value for firms facing large uncertainty over strategic synergies. Finally, we have produced novel empirical implications for the frequency of ECOs, buy-backs and sell-outs, across industry characteristics, measures of strategic uncertainty and financial markets transparency.

More generally, an ECO may be seen as a two-way improvement in information disclosure, allowing both investors to review the subsidiary activities, as well as the parent to profit from the information generated on the market. It gives the opportunity to mitigate negative synergies, by selling out part of the share, while retaining the flexibility to subsequently reacquire or to sell
out the subsidiary depending on how synergies evolve over time. We intend to elaborate on this dynamic learning process in future research.

References


ELSAS, R., AND Y. LOFFLER (2001): “Equity Carve-Outs and Corporate Control and Equity Carve-outs in Germany,” Unpublished manuscript.


A Proof Proposition 1

In order to find the optimal threshold values, we derive the equation that equals the expected returns of the parent firm for each possible status to the expected rate of capital appreciation, that is, we derive the Bellman equation for each possible situation: full control, carve out, buy back and sell-out. This allows valuing the options. Subsequently, we set the boundary conditions to uniquely identify the optimal threshold value.

**Full control.** When the parent firm has full control, the Bellman equation requires that over any interval of time $dt$ the expected returns together with the compound option to carve out has to be equal to the expected rate of capital appreciation.

As this expected return depends on the value of the synergies through Ito’s lemma, the Bellman equation can be expressed as:

$$\frac{1}{2} \sigma^2 V''[s] - r V[s] + (1 + s) = 0$$  \hspace{1cm} (8)

The general solution of this differential equation is given by:

$$V_F = A_1 \exp^{\beta s} + A_2 \exp^{-\beta s} + \frac{1 + s}{r}$$  \hspace{1cm} (9)

where $A_1$ and $A_2$ are the differential constant that have to be found and $\pm \beta$ are the two solutions of the quadratic equation derived from the differential
equation (8) where:

$$\beta = \frac{\sqrt{2r}}{\sigma}$$  \hspace{1cm} (10)

**Carve-out.** Following the same steps, the conditions for the subsidiary value when the parent has performed a carve-out is equal to:

$$\frac{1}{2}\sigma^2 V''[s] - rV'[s] + \alpha (1 + s) = 0$$  \hspace{1cm} (11)

and the valuation is equal to:

$$V_C = B_1 \exp^{\beta s} + B_2 \exp^{-\beta s} + \frac{\alpha (1 + s)}{r}$$  \hspace{1cm} (12)

**Buy back and sell-off.** In these cases no options are embedded, so the subsidiary value corresponds to the traditional NPV value. They are respectively:

$$V_B = \frac{1 + s}{r}, \quad V_S = 0$$  \hspace{1cm} (13)

**Optimal timing** In order to solve for the optimal exercise thresholds and define the three constants in the option valuations, we impose the usual value matching and smooth pasting conditions. These two conditions together imply that at the optimal exercise synergy level, the pay-offs of exercising or not are identical (see Appendix C of Chapter 4 of Dixit and Pindyck (1994)).

When the parent holds full control, the optimal threshold to carve out has to satisfy the following conditions:

$$V_F (s_C^*) = V_C (s_C^*) + (1 - \alpha) \left( \frac{1}{r} - \frac{\delta}{r} \right)$$  \hspace{1cm} (14)

$$V_F' (s_C^*) = V_C' (s_C^*)$$  \hspace{1cm} (15)
An additional limit condition is that when synergies to infinity, the option to exercise a carve-out is almost worthless (as the parent firm never exercises it). This implies that $A_1 = 0$.

When the subsidiary has been carved-out, the optimal threshold to buy back has to meet the following conditions:

$$V_C (s_B^*) = V_B (s_B^*) - (1 - \alpha) \frac{1}{r}$$

(16)

$$V'_C (s_B^*) = V'_B (s_B^*)$$

(17)

In the case of a switch from carve-out to sell-out, at the optimal threshold the following condition have to be verified:

$$V_C (s_S^*) = V_S (s_S^*) + \frac{\alpha}{r}$$

(18)

$$V'_C (s_S^*) = V'_S (s_S^*)$$

(19)

This system of six equations determines the three thresholds and the three remaining constants. The system cannot be solved entirely explicitly. The constant can be solved analytically and are given by:

$$B_1 = \frac{\exp^{\beta s_B} (1 - \alpha) + \alpha \exp^{\beta s_S}}{(\exp^{2\beta s_B} - \exp^{2\beta s_S}) \beta r}$$

(20)

$$B_2 = \exp^{2\beta s_S} B_1 + \frac{\alpha}{r \beta} \exp^{\beta s_S}$$

(21)

$$A_2 = - \exp^{2\beta s_C} B_1 + B_2 + \frac{1 - \alpha}{r \beta} \exp^{\beta s_C}$$

(22)

Note that in order to have a positive option values $s_B^* > s_S^*$.

Taking this into consideration, the threshold points are given by the so-
olution of the following system of equations:

\[ \beta s^*_B = \beta s^*_S + \ln \frac{-1 + \alpha - \sqrt{1 - 2\alpha + \alpha^2 \beta^2 s^*_S}}{\alpha (1 + \beta s^*_S)} \]  
(23)

\[ \beta s^*_S = \beta s^*_B + \ln \frac{-\alpha + \sqrt{-1 + 2\alpha + (1 - \alpha)^2 \beta^2 s^*_B}}{(1 - \alpha) (1 + \beta s^*_B)} \]  
(24)

\[ \beta s^*_C = -1 - \beta \delta - W \left[ - (1 + \beta s^*_B) \exp^{-1-\beta(\delta-s^*_B)} \right] \]  
(25)

where \( W[z] \) is the principal solution of \( z = W \exp^W \).\(^{17}\)

For the existence of the logarithm \( s^*_S < -\frac{1}{\beta} \) and \( s^*_B > \frac{1}{\beta} \).

To have economic sense\( s^*_C \geq s^*_S \). This occurs when:

\[ \delta \leq \frac{-1 + \sqrt{-1 + 2\alpha + s^*_B^2 (-1 + \alpha)^2 - (1 - \alpha) (\beta s^*_S + \ln (1 + s^*_B))}}{1 - \alpha} \]  
(26)

When this condition is verified the optimal threshold are those above. When the condition is not verified \( s^*_C < s^*_S \) the trade off is different: the parent sells the subsidiary out immediately after the carve out. In this case the carve-out and sell-out occur at:

\[ \hat{s}_C = \hat{s}_S = \frac{-1}{\beta} - (1 - \alpha) \delta \]  
(27)

and the subsidiary value for the parent at the carve-out is:

\[ 1 + (1 - \alpha) \delta \]  
(28)

\section*{B Proof Proposition 2}

At the optimal switching time, \( s^*_S \), condition (18) with equations (20), (21) and (22) becomes:

\[ \frac{(1 - \alpha) 2 \exp^{\beta(s^*_B+s^*_S)} + \alpha (1 - \beta s^*_S) \exp^{2\beta s^*_S} + \alpha (1 + \beta s^*_S) \exp^{2\beta s^*_S}}{(\exp^{2\beta s^*_B} - \exp^{2\beta s^*_S}) \beta r} = 0 \]  
(29)

\(^{17}\)We eliminate two other possible solutions as they are minimums and they would determine negative option values.
Equation (29) is an increasing monotonic function in respect of $s^*_S$. It follows that there is a unique value of $s^*_S$ for which this function equals zero.

At the optimal switching time, $s^*_B$, condition (16) has to be satisfied. Inserting equations (20), (21) and (22), it becomes:

$$2\alpha \exp^{\beta (s^*_S + s^*_B)} - (1 - \alpha) \exp^{2\beta s^*_B} (\beta s^*_B - 1) + (1 - \alpha) \exp^{2\beta s^*_S} (\beta s^*_S + 1) \frac{(\exp^{2\beta s^*_B} - \exp^{2\beta s^*_S}) \beta r}{(\exp^{2\beta s^*_B} - \exp^{2\beta s^*_S}) \beta r} = 0$$

(30)

Equation (30) is a decreasing monotonic function in respect of $s^*_B$. It follows that there is a unique value of $s^*_B$ for which this function equals zero.

As $\alpha$ tends to 0, equations (29) and (30) become respectively:

$$2 \exp^{(s^*_S + s^*_B)\beta} = 0$$

(31)

$$\frac{\exp^{2s^*_B\beta} (1 - s^*_B\beta) + \exp^{2s^*_S\beta} (1 + s^*_B\beta)}{(\exp^{2s^*_B\beta} - \exp^{2s^*_S\beta}) \beta r} = 0$$

(32)

The only solution of this system of equations that gives positive option values, is $s^*_S \to -\infty$ and $s^*_B \to \frac{1}{\beta}$.

As $\alpha$ tends to 1, equations (29) and (30) become respectively:

$$\frac{\exp^{2s^*_B\beta} (1 - s^*_S\beta) + \exp^{2s^*_B\beta} (1 + s^*_S\beta)}{(\exp^{2s^*_B\beta} - \exp^{2s^*_S\beta}) \beta r} = 0$$

(33)

$$\frac{2 \exp^{(s^*_S + s^*_B)\beta}}{(\exp^{2s^*_B\beta} - \exp^{2s^*_S\beta}) \beta r} = 0$$

(34)

The only solution of this system of equations that gives positive option values, is $s^*_S \to -\frac{1}{\beta}$ and $s^*_B \to \infty$.

In order to determine the how $s^*_S$ and $s^*_B$ are varying in respect of the shares retained, we study the analyse the derivatives of the conditions (29) and (30)

In equilibrium condition (29), that we indicate as $f$, has to be satisfied
and hence its derivative in respect of $\alpha$ has to be equal to 0:

$$\frac{\partial f}{\partial \alpha} + \frac{\partial f}{\partial s_B^*} \frac{\partial s_B^*}{\partial \alpha} + \frac{\partial f}{\partial s_S^*} \frac{\partial s_S^*}{\partial \alpha} = 0 \quad (35)$$

where:

$$\frac{\partial f}{\partial \alpha} = \frac{\alpha^{-1} \exp^{3s_B^*} (\beta s_B^* - 1) + \exp^{3s_B^*} (\beta s_B^* + 1)}{\exp^{3s_B^*} + \exp^{3s_S^*}} < 0 \quad (36)$$

$$\frac{\partial f}{\partial s_B^*} = -\frac{2 \exp^\beta(s_B^* + s_S^*) \left((1 - \alpha) \left(\exp^{2\beta s_B^*} + \exp^{2\beta s_S^*}\right) + 2 \alpha \exp^\beta(s_B^* + s_S^*)\right)}{(\exp^{2\beta s_B^*} - \exp^{2\beta s_S^*})^2 r} < 0 \quad (37)$$

$$\frac{\partial f}{\partial s_S^*} = \exp^{2\beta s_B^*} + \exp^{2\beta s_S^*} \left(a \exp^{2\beta s_B^*} + \exp^{2\beta s_S^*}\right) + 2 \exp^\beta(s_B^* + s_S^*) (1 - \alpha) > 0 \quad (38)$$

At the same time the derivative of equation (30), which we indicate with $g$, in respect of $\alpha$ is given by:

$$\frac{\partial g}{\partial \alpha} + \frac{\partial g}{\partial s_S^*} \frac{\partial s_S^*}{\partial \alpha} + \frac{\partial g}{\partial s_B^*} \frac{\partial s_B^*}{\partial \alpha} = 0 \quad (39)$$

where:

$$\frac{\partial g}{\partial \alpha} = \frac{\alpha^{-1} (\beta s_B^* - 1) \exp^{2\beta s_B^*} + (\beta s_B^* + 1) \exp^{3\beta s_B^*}}{\exp^{2\beta s_B^*} + \exp^{2\beta s_S^*}} > 0 \quad (40)$$

$$\frac{\partial g}{\partial s_S^*} = 2 \exp(s_B^* + s_S^*) \left(\exp^{2\beta s_B^*} + \exp^{2\beta s_S^*}\right) + \alpha (\exp^{2\beta s_B^*} + \exp^{2\beta s_S^*}) > 0 \quad (41)$$

$$\frac{\partial g}{\partial s_B^*} = -\frac{\exp^{2\beta s_B^*} + \exp^{2\beta s_S^*}}{(\exp^{2\beta s_B^*} - \exp^{2\beta s_S^*})^2 r} \left(2 \alpha \exp^\beta(s_B^* + s_S^*) + (\exp^{2\beta s_B^*} + \exp^{2\beta s_S^*}) (1 - \alpha)\right) < 0 \quad (42)$$

From equations (35), $\frac{\partial s_B^*}{\partial \alpha} = \frac{\partial f}{\partial \alpha} \frac{\partial s_B^*}{\partial f} - \frac{\partial f}{\partial s_B^*} \frac{\partial s_B^*}{\partial \alpha}$, and inserting it in equation (39) we have that:

$$\frac{\partial g}{\partial \alpha} + \frac{\partial g}{\partial s_S^*} \frac{\partial s_S^*}{\partial \alpha} + \frac{\partial g}{\partial s_B^*} \frac{\partial s_B^*}{\partial \alpha} = 0 \quad (43)$$
Plugging in the above partial derivatives it results that:

\[
\frac{\partial s_B^*}{\partial \alpha} = \frac{1}{\alpha \beta} \frac{2 \beta s_B^* \exp((s_B^* + s_S^*)^\beta) + (1 - \beta s_B^*) \exp^{2 \beta s_S^*} - (1 + s_B^*) \exp^{2 \beta s_B^*}}{\exp^{2 \beta s_B^*} + \exp^{2 \beta s_S^*} - 2 (1 - \alpha) \exp((s_B^* + s_S^*)^\beta)} > 0
\]  

(44)

\[
\frac{\partial s_B^*}{\partial \alpha} = \frac{\alpha^{-1} - 2 \beta s_S^* \exp((s_B^* + s_S^*)^\beta) - (1 - \beta s_B^*) \exp^{2 \beta s_S^*} + (1 + s_B^*) \exp^{2 \beta s_B^*}}{(1 - \alpha) \exp^{2 \beta s_B^*} + (1 - \alpha) \exp^{2 \beta s_S^*} - 2 \alpha \exp((s_B^* + s_S^*)^\beta)} > 0
\]  

(45)

C Properties of entry benchmarks

Option to buy back and no option to sell out. When the firm is in the carve-out status as there is no option to sell out, the constant \( B_2 \) is equal to 0: when the value of the synergies tends to minus infinity the value of remaining in the carve-out status is equal only to the expected profits as the option to buy back is worthless as the probability to exercise it is zero. Hence in this case:

\[
V_C = B_1 \exp^{\beta s} + \frac{\alpha}{r} + \frac{s}{r}
\]  

(46)

It follows that the system of equations that has to be solved constituted by the different value matching and smooth pasting conditions are given by equations (14), (15), (16), (17) and the solutions are given by:

\[
A_{1s}^b = \frac{(1 - \alpha) \left( \exp^{\beta s_{CB}} - \exp^{-1 + 2 \beta s_{CB}} \right)}{\beta r}
\]  

(47)

\[
B_1^b = \frac{(1 - \alpha)}{\beta \exp r}
\]  

(48)

\[
s_B^b = \frac{1}{\beta}
\]  

(49)

\[
s_{CB}^b = -\frac{1}{\beta} - \delta - \frac{1}{\beta} W \left[ -2 \exp^{-2 - \beta \delta} \right]
\]  

(50)
In particular if there is no underpricing:

\[ s_{CB}^b = -\frac{1}{\beta} \]  (51)

**Option to sell out and no option to buy back.** When the firm is in the carve-out status and there is no option to buy back, the constant \( B_1 = 0 \), because when the value of the synergies tends to infinity the value of being in the carve-out status is equal to the expected profits: the option to sell out is worthless as the probability to exercise it, is zero. Hence:

\[ V_C = B_2 \exp^{-\beta s} + \alpha \frac{\pi + s}{r} \]  (52)

It follows that the system of equations that has to be solved constituted by the different value matching smooth pasting conditions are given by equations (14), (15), (18), (19) and the solutions are given by:

\[ B_2^b = \frac{\alpha}{\exp \beta r} \]  (53)

\[ A_{1C}^b = \frac{\alpha}{\exp \beta r} + \frac{1 - \alpha}{r \beta} \exp^{-1 - \beta \delta} \]  (54)

\[ s_{SS}^b = -\frac{1}{\beta} \]  (55)

\[ s_{CS}^b = -\frac{1}{\beta} - \delta \]  (56)

**No option to buy back and no option to sell out.** When the firm is in the carve-out status and there are no options to buy back and to sell out, the value matching and the smooth pasting between the full control and the carve-out status change. The pay-off of the carve-out status is equal to the expected profits without any option value added, that is \( B_1 = B_2 = 0 \).

The system of equations is given by equations (14) and (15) and the
solutions are given by:

\[ A^b_{sC} = \frac{1 - \alpha^n}{\bar{r}\beta} \exp^{-1-\beta \delta} \quad (57) \]

\[ s^b_{CC} = -\frac{1}{\beta} - \delta \quad (58) \]

**D Proof Proposition 4**

The derivative of the optimal switching time, \( s^*_C \), in respect of the retained shares is given by:

\[ \frac{ds^*_C}{d\alpha} = \frac{\partial s^*_C}{\partial s^*_B} \frac{\partial s^*_B}{d\alpha} + \frac{\partial s^*_C}{d\alpha} \quad (59) \]

where:

\[ \frac{\partial s^*_C}{\partial s^*_B} = \frac{\beta s^*_B W [- (1 + \beta s^*_B) \exp [-1 - \beta s^*_B + \beta \delta]]}{(1 + \beta s^*_B) (1 + W [- (1 + \beta s^*_B) \exp^{-1-\beta s^*_B+\beta \delta}])} < 0 \quad (60) \]

\[ \frac{\partial s^*_C}{d\alpha} = \frac{\beta \delta (1 - \alpha^{-1})}{(1 - \alpha)^2 (1 + W [- (1 + \beta s^*_B) \exp^{-1-\beta s^*_B+\beta \delta}])} > 0 \quad (61) \]

\[ \frac{\partial s^*_B}{d\alpha} > 0 \quad (62) \]

When \( \delta = 0 \), \( \frac{\partial s^*_C}{d\alpha} = 0 \) and hence \( \frac{ds^*_C}{d\alpha} < 0 \). More precisely \( s^*_C \) is a decreasing function that goes from \(-\frac{0.6}{\beta}\) for \( \alpha \) that tends to 0, to \(-\frac{1}{\beta} - 1\) for \( \alpha \) that tends to 1.

When \( \delta > 0 \), as \( \alpha \) tends to 0 \( s^*_C \) tends to \(-\frac{1}{\beta} - \delta - \frac{1}{\beta} W [-2 \exp^{-2-\delta \beta}] \) and as \( \alpha \) tends to 1, \( s^*_C \) tends to \(-\frac{1}{\beta} - \delta \). Concerning the derivatives, as \( \alpha \) tends to 0, \( \frac{\partial s^*_C}{\partial s^*_B} \frac{\partial s^*_B}{d\alpha} \) tends to 0 while \( \frac{\partial s^*_C}{d\alpha} > 0 \), hence \( s^*_C \) is initially increasing in respect of \( \alpha \). As \( \alpha \) tends to 1, \( \frac{\partial s^*_C}{d\alpha} \frac{\partial s^*_B}{d\alpha} \) tends to \(-\infty\) and \( \frac{\partial s^*_C}{d\alpha} \) tends to 0. Hence given the monotonicity of the partial derivatives, for low values of \( \alpha \), \( s^*_C \) is an increasing function, it reaches a maximum and afterwards the negative component of the derivative is prevailing and \( s^*_C \) decreases in \( \alpha \).

As \( \frac{\partial s^*_C}{d\alpha} > 0 \), it follows that the maximum occurs for higher levels of \( \alpha \) the higher the level of underpricing.
E Proof Proposition 5

By assumption $\delta$ is a decreasing function of $\alpha$ and the optimal threshold to carve out can be written as:

$$\beta s_C^* = -1 - \beta \delta (\alpha) - W \left[ - (1 + \beta s_B^*) \exp^{-1 - \beta s_B^* - \beta \delta (\alpha)} \right]$$  \hspace{1cm} (63)

It follows that the optimal level of shares retained is given by the highest threshold to carve out. The optimal threshold to carve out is determined by two elements: $-\beta \delta (\alpha)$, which is increasing in $\alpha$ and the last part of equation (63) is decreasing in $\alpha$. Hence there must be an optimal $\alpha$, that maximizes (5). This maximum lies between 0 and 1 as the derivative is always positive for $\alpha = 0$ and tends to $-\infty$ when $\alpha = 1$.

The analysis of the effect of uncertainty goes similarly. The first part of (63) increases as uncertainty. The last part of (63) increases as uncertainty increases. However, when uncertainty increases the effect is higher and tends to zero as uncertainty tends to 0. Hence when uncertainty is low an increase in uncertainty increases the optimal shares retained increases. For higher uncertainty level the higher uncertainty, the lower is the optimal retained shares.