Volatility Downside Risk*

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Abstract

This paper derives and tests the cross-sectional predictions of an intertemporal equilibrium asset pricing model with generalized disappointment aversion and time-varying macroeconomic uncertainty. To the contrary of the existing literature, disappointment may result not only from a fall in the market index, but also from a rise in a volatility index. Theoretically, we show that besides the market return and changes in market volatility, three two-asset option-like payoffs, contingent to the disappointing event, are also priced factors: a long binary cash-or-nothing option, a short put on the market index and a long call on the volatility index. Implied measures of market and volatility downside risks similar to those considered in the literature explicitly express as linear combinations of exposures to these options and their underlying instruments. Empirically, we find that the cross-section of stock returns reflects a premium for bearing undesirable exposures to these options. The signs of the estimated risk premia are consistent with theory, their economic magnitudes show that a long/short strategy on exposure to each of these options pays on average more than 5% per annum, and these rewards are not explained by coskewness, size, value, and momentum factors.

Keywords: Generalized Disappointment Aversion, Option Payoffs, Downside Risks, Cross-Section

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1 Introduction

Downside risk is currently the focus of a large and growing body of literature in financial economics. It corresponds to the financial risk of a security, a portfolio or any other type of investment, borne by an investor in case of an adverse economic or financial scenario. The analogue if the scenario is favourable, is called upside uncertainty. The asymmetric treatment of downside risk versus upside uncertainty has long been well-accepted among practitioners and academic researchers (Roy 1952; Markowitz 1959), and led to new developments in asset pricing and financial risk management, such as the concept of the value-at-risk and the expected shortfall, as well as axiomatic approaches to preferences that allow investors to place greater weights on adverse market conditions in their utility functions. These developments include the lower-partial moment framework of Bawa and Lindenberg (1977), the prospect theory of choice of Kahneman and Tversky (1979), the theory of disappointment aversion of Gul (1991), recently generalized by Routledge and Zin (2010) who embed them in the recursive utility framework of Epstein and Zin (1989). These new theories suggest priced downside risks in the capital market equilibrium.

In this article, we explicitly derive and test the cross-sectional predictions of an intertemporal equilibrium asset pricing model, where the representative investor has generalized disappointment aversion (GDA) preferences and macroeconomic uncertainty is time-varying. In particular, if the investor's risk aversion parameter is larger than one and his elasticity of intertemporal substitution is finite, as generally agreed in the asset pricing literature, then the disappointing event ($\mathcal{D}$) may be triggered not only by a fall in the market return, but also by a rise in market volatility, to the contrary of existing asset pricing studies on downside risks (see for example Bawa and Lindenberg 1977; Ang, Chen and Xing 2006; Post et al. 2010; Brownlees and Engle 2011 among others). The investor is disappointed if the return of holding a long position in the market index combined with a short position in the volatility index falls below a constant threshold. This threshold as well as the ratio of the short versus the long position depend on the investor's preference parameters.

The GDA investor exhibits both risk aversion (i.e. aversion to regular betas on market return and on changes in market volatility) and disappointment aversion (i.e. aversion to expected downside losses). We refer to the combination of both risk and disappointment as the effective risk. We explicitly disentangle the components of the asset premium that are due to risk exclusively, from those that are due to disappointment exclusively, and from those that are due to the interaction between risk and disappointment. An investor with expected utility (henceforth EU) preferences requires two premiums to invest in a risky asset. These two premiums are compensations for covariations with the market return, $Cov\left(R_i^e, r_W\right)$, and with changes in market volatility, $Cov\left(R_i^e, \Delta\sigma_W^2\right)$, and are exclusively due to risk aversion, since they are the only premiums required by a risk averse but disappointment neutral investor.

In comparison to the EU investor, the GDA investor requires three additional premia that compensate for covariation with three two-asset option-like payoffs, contingent to the disappointing event. The first premium is a compensation for the covariance with a long binary cash-or-nothing option, $Cov\left(R_i^e, I\left(\mathcal{D}\right)\right)$, where $I\left(\cdot\right)$ is the indicator function that takes the value 1 if the condition is met and 0 otherwise. We show that this premium is exclusively due to disappointment aversion, since it is the only premium required by a risk neutral but disappointment averse investor. The second premium is a compensation for the covariance of the asset returns with a short put option on the market index, $Cov\left(R_i^e, r_W I\left(\mathcal{D}\right)\right)$, and the third premium is a compensation for the covariance with a long call option on the volatility index, $Cov\left(R_i^e, \Delta\sigma_W^2 I\left(\mathcal{D}\right)\right)$. These latter compensations are not exclusively due to either risk aversion or disappointment aversion, as they are required if and only if the investor is both risk averse and disappointment averse. If the investor's elasticity of intertemporal substitution is infinite, then changes in market volatility and the call option on the volatility index are not priced. Besides, only a fall in the market return may cause disappointment, consistent with measures of downside risks considered in the literature.

We explore the cross-sectional predictions of the model using all common stocks traded on the NYSE, AMEX and NASDAQ markets covering the period from July 1963 to December 2010. The main results of the paper relate to the cross-sectional pricing of the three option-like payoffs on market and volatility indexes. Our empirical methodology uses portfolio sorts on individual stock exposures to these options, as well as cross-sectional regressions of Fama and MacBeth (1973) to estimate these factor risk premia. Our main finding is that options on market and volatility indexes are highly significant factors in the cross-section of stock returns. Across individual stocks, we see a wide dispersion in sensitivities to options, which generates cross-sectional variation in the risk premia attributed to these factors. The signs of estimated factor risk premia are all consistent with the theory, and their economic magnitudes show that a long/short strategy on exposure to each of these options pays on average more than 5% per annum, and these rewards are not explained by coskewness, size, value, and momentum factors.

We complement, in several ways, the existing theoretical and empirical asset pricing literature on how asset prices are affected by downside risks. Downside risks may be measured through the market downside beta, empirically examined in the cross-section of stock returns by Ang, Chen and Xing (2006), the semi-variance beta due to Bawa and Lindenberg (1977) and empirically examined in the cross-section of stock returns by Post et al. (2010), or other measures such as the marginal expected shortfall, estimated and empirically examined for the regulation of systemic risk in US financial firms by Brownlees and Engle (2011). In all these studies, the downside event is a sufficient decline in the market index, corresponding to a special case of the GDA model with infinite elasticity of intertemporal substitution. The literature does not exactly tell what factors the downside beta, the semi-variance beta, and the marginal expected shortfall measure exposures to. We show that these measures are explicit linear combinations of the same three multivariate betas (on the market return, on the long binary cash-or-nothing and the short put options on the market index) and provide the associated coefficients. Their analogue in the general GDA model where changes in market volatility and the long call option on the volatility index are also priced, include betas on these two latter factors in the linear combinations.

While little or nothing has been said about volatility downside risk, we demonstrate that a dynamic equilibrium asset pricing model with generalized disappointment aversion preferences and time-varying macroeconomic uncertainty provides a convenient theoretical setup for examining the empirical evidence that volatility downside risk is priced. Ultimately, we provide a unified theoretical framework that can explain the empirical findings that asset sensitivities to the market return and to changes in market volatility are priced (Ang, Hodrick, Xing and Zhang 2006; Adrian and Rosenberg 2008), that the market downside beta, the semi-variance beta and the marginal expected shortfall are priced (Ang, Chen and Xing 2006; Post et al. 2010; Brownlees and Engle

2011), and that the volatility downside beta and the relative downside potential of an asset are priced. Again, there is little or no empirical evidence regarding the two latter measures, and we view this as an important contribution to the literature. Furthermore, being motivated by dynamic consumption-based equilibrium asset pricing and behaviorial decision theory, our setup attempts to extend research on systemic financial risk onto many of the directions advocated by Brunnermeier et al. (2010).

We also examine the empirical performance of our cross-sectional model on standard sets of sorted portfolios: size, book-to-market, momentum, long-term reversal and industry portfolios. Our results still compare to those obtain on individual stocks. In terms of the pricing errors, our five-factor model with market beta, volatility beta and exposures to the three options provides a significant improvement over the standard CAPM model. It is comparable to the four-factor model of Carhart (1997), but in contrast, it has the benefit of being motivated by dynamic consumption-based equilibrium asset pricing and behaviorial decision theories. We decompose the portfolio premia into parts attributable to each of the five factors from the model. We find that the three options account for significant parts of the total premium required to invest in stocks, and that they are relevant for interpreting differences in risk compensation across size, book-to-market and momentum portfolios. We finally show that our results are robust to different data subsamples, to alternative measures of market volatility and to alternative specifications of the disappointment region.

The balance of the paper is organized as follows. In Section 2, we present and develop the theoretical setup from which we derive the implied cross-sectional model and discuss the option interpretation of the new factors. Section 3 derives the multivariate cross-sectional linear beta pricing model and show how different measures of market and volatility downside risks related to exposures to option payoffs and their underlying instruments. Section 4 contains an extensive empirical assessment of the model. Section 5 concludes. An external appendix, available from authors' web pages[1], contains additional material and proofs.

[1]e.g. http://www.adamfarago.com/research

2 Theoretical setup

2.1 Assumptions on investors' preferences

We consider a representative investor with generalized disappointment aversion preferences (GDA) of Routledge and Zin (2010). Following Epstein and Zin (1989) and Weil (1989), such an investor derives utility from consumption, recursively as follows:

$$
\begin{aligned}
V_t &= \left\{(1-\delta)\, C_t^{1-\frac{1}{\psi}} + \delta \left[\mathcal{R}_t\left(V_{t+1}\right)\right]^{1-\frac{1}{\psi}}\right\}^{\frac{1}{1-\frac{1}{\psi}}} \quad \text{if } \psi > 0 \ \text{ and } \ \psi \neq 1 \\
&= C_t^{1-\delta} \left[\mathcal{R}_t\left(V_{t+1}\right)\right]^{\delta} \quad \text{if } \psi = 1.
\end{aligned} \tag{2}
$$

The investor then maximizes this lifetime utility subject to the budget constraint

$$
W_{t+1} = \left(W_t - C_t\right) R_{W,t+1}, \tag{3}
$$

where W is the total wealth and R_W is the simple gross return to the claim on aggregate consumption C, which we refer to as the market return.

Equation (2) states that the current period lifetime utility V_t is a combination of current consumption C_t, and $\mathcal{R}_t\left(V_{t+1}\right)$, the certainty equivalent of next period lifetime utility, implicity defined by:

$$
U\left(\mathcal{R}\right) = E\left[U\left(V\right)\right] - \ell E\left[\left(U\left(\kappa\mathcal{R}\right) - U\left(V\right)\right) I\left(V < \kappa\mathcal{R}\right)\right] \tag{4}
$$

where

$$
\begin{aligned}
U\left(X\right) &= \frac{X^{1-\gamma} - 1}{1-\gamma} \quad \text{if } \gamma > 0 \ \text{ and } \ \gamma \neq 1 \\
&= \ln X \quad \text{if } \gamma = 1,
\end{aligned} \tag{5}
$$

and where the parameter $\ell \geq 0$ modulates the importance of disappointment versus satisfaction, while the parameter $0 < \kappa \leq 1$ modulates both the amplitude and the frequency of disappointment. When ℓ is equal to zero, $\mathcal{R}$ reduces to expected utility (EU) preferences, while V_t represents the Epstein and Zin (1989) recursive utility. When $\ell > 0$, outcomes lower than $\kappa\mathcal{R}$ receive an extra weight, decreasing the certainty equivalent. Thus, the parameter ℓ is interpreted as a measure of disappointment or loss aversion, while the parameter κ is the percentage of the certainty equivalent such that outcomes below it are considered disappointing. The special case $\kappa = 1$ corresponds the original disappointment aversion preferences of Gul (1991).

With EU preferences, Hansen et al. (2008) derive the stochastic discount factor in terms of the continuation value of utility of consumption, as follows:

$$M^*_{t,t+1} = \delta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left(\frac{V_{t+1}}{\mathcal{R}_t \left(V_{t+1} \right)} \right)^{\frac{1}{\psi} - \gamma} = \delta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \left(\frac{V_{t+1}/C_{t+1}}{\mathcal{R}_t \left(V_{t+1} \right) / C_t} \right)^{\frac{1}{\psi} - \gamma}. \tag{6}$$

If $\gamma = 1/\psi$, equation (6) corresponds to the stochastic discount factor of an investor with time-separable utility and constant relative risk aversion, where only changes in the level of consumption determines an asset premium. Otherwise, there is an additional premium to compensate for changes in the welfare valuation ratio.

Following Hansen et al. (2007) and Routledge and Zin (2010), the intertemporal marginal rate of substitution of an investor with GDA preferences is given by:

$$M_{t,t+1} = M^*_{t,t+1} \left(\frac{1 + \ell I \left(\mathcal{D}_{t+1} \right)}{1 + \ell \kappa^{1-\gamma} E_t \left[I \left(\mathcal{D}_{t+1} \right) \right]} \right), \tag{7}$$

where $I(\cdot)$ is an indicator function that takes the value 1 if the condition is met and 0 otherwise, and $\mathcal{D}_{t+1}$ denotes the disappointing event $V_{t+1} < \kappa \mathcal{R}_t \left(V_{t+1} \right)$. Equation (7) shows that, compared to the EU stochastic discount factor, the GDA stochastic discount factor includes an additional term that will add a premium for expected losses conditional upon disappointment.

2.2 Substituting out consumption

Using the second equality in (6), the logarithm of $M^*_{t,t+1}$ can be written as

$$m^*_{t,t+1} = \ln \delta - \gamma \Delta c_{t+1} - \left(\gamma - \frac{1}{\psi} \right) \Delta z_{V,t+1} \tag{8}$$

where the processes in the right-hand side of equation (8) are defined by

$$\Delta c_{t+1} \equiv \ln\left(\frac{C_{t+1}}{C_t}\right) = \ln C_{t+1} - \ln C_t \quad \text{and} \quad \Delta z_{V,t+1} \equiv \ln\left(\frac{V_{t+1}}{C_{t+1}}\right) - \ln\left(\frac{\mathcal{R}_t\left(V_{t+1}\right)}{C_t}\right) \tag{9}$$

and represent respectively the change in the log consumption level (consumption growth), and the change in the log welfare valuation ratio (welfare valuation ratio growth). Notice that the stochastic discount factor depends directly on current consumption growth, and indirectly on future consumption growths through the welfare valuation ratio growth. It turns out that the disappointing event is equivalent to $\Delta c_{t+1} + \Delta z_{V,t+1} < \ln \kappa$.

Following Epstein and Zin (1989), the log return on wealth is related to consumption growth and the welfare valuation ratio growth through

$$r_{W,t+1} = -\ln\delta + \Delta c_{t+1} + \left(1 - \frac{1}{\psi}\right)\Delta z_{V,t+1}. \tag{10}$$

If consumption growth is substituted out from the log SDF using the above relationship, we get

$$m^*_{t,t+1} = (1-\gamma)\ln\delta - \gamma r_{W,t+1} - \left(\frac{\gamma - 1}{\psi}\right)\Delta z_{V,t+1}, \tag{11}$$

and the disappointing event is equivalent to $r_{W,t+1} + (1/\psi)\,\Delta z_{V,t+1} < \ln\left(\kappa/\delta\right)$.

Note that the return $r_{W,t}$ on the wealth portfolio is not directly observed by the econometrician. The return to a stock market index is sometimes used to proxy for this return as in Epstein and Zin (1991). Also, the welfare valuation ratios $z_{V,t} \equiv \ln\left(V_t/C_t\right)$ and $z_{\mathcal{R},t} \equiv \ln\left(\mathcal{R}_t\left(V_{t+1}\right)/C_t\right)$ are unobservable. Following Hansen et al. (2008) and Bonomo et al. (2011), we can exploit the dynamics of aggregate consumption growth and the recursion (2) in addition to the definition of the certainty equivalent (4) to solve for the unobserved welfare valuation ratios.

From equation (10), it follows that stochastic volatility of aggregate consumption growth is a sufficient condition for stochastic volatility of the market return. In that case, market volatility measures time-varying macroeconomic uncertainty. In all what follows, this additional assumption is coupled with our assumption on investors' preferences. More specifically, assume for example that the logarithm of consumption follows a heteroscedastic random walk as in Bonomo et al. (2011), were the stochastic volatility of consumption growth is an AR(1) process that can be well-approximated in population by a two-state Markov chain as shown in Garcia et al. (2008). Then, it can be shown that the welfare valuation ratios satisfy

$$z_{V,t} = \varphi_{V0} + \varphi_{V\sigma}\sigma^2_{W,t} \quad \text{and} \quad z_{\mathcal{R},t} = \varphi_{\mathcal{R}0} + \varphi_{\mathcal{R}\sigma}\sigma^2_{W,t} \tag{12}$$

were $\sigma^2_{W,t} \equiv Var_t\left[r_{W,t+1}\right]$ is the conditional variance of the market return, and were the drift coefficients φ_{V0} and $\varphi_{\mathcal{R}0}$ and the loadings $\varphi_{V\sigma}$ and $\varphi_{\mathcal{R}\sigma}$ depend on investor's preference parameters and on parameters of the dynamics of consumption volatility. In this case, equation (11) becomes

$$m^*_{t,t+1} = (1-\gamma)\ln\delta^* - \gamma r_{W,t+1} - \left(\frac{\gamma-1}{\psi}\right)\varphi_{V\sigma}\Delta\sigma^2_{W,t+1}, \tag{13}$$

and the disappointing event is equivalent to $r_{W,t+1} + (1/\psi)\,\varphi_{V\sigma}\Delta\sigma^2_{W,t+1} < \ln\left(\kappa/\delta^*\right)$, where

$$\Delta\sigma^2_{W,t+1} \equiv \sigma^2_{W,t+1} - \varphi\sigma^2_{W,t}, \quad \ln\delta^* = \ln\delta + \frac{1}{\psi}\left(\varphi_{V0} - \varphi_{\mathcal{R}0}\right) \quad \text{and} \quad \varphi = \frac{\varphi_{\mathcal{R}\sigma}}{\varphi_{V\sigma}}.$$

Our definitions and notations for $\Delta z_{V,t+1}$ and $\Delta\sigma^2_{W,t+1}$ presume that $z_{\mathcal{R},t} \approx z_{V,t}$, meaning that $\varphi_{\mathcal{R}\sigma} \approx \varphi_{V\sigma}$, and consequently $\varphi \approx 1$.[2] This shows that changes in the welfare valuation ratio can empirically be proxied by changes in a stock market volatility index, where volatility can be estimated using a generalized autoregressive conditional heteroscedasticity (GARCH) model, can be computed from high-frequency index returns (realized volatility), or can be measured by the option implied volatility (VIX).

Disappointment may occur due to a fall in the market return. It may also occur following a sharp rise in market volatility. This means that the loading coefficient $\varphi_{V\sigma}$ of the welfare valuation ratio onto the market volatility must be negative. In all what follows, we take as given that $\varphi_{V\sigma} < 0$ and refer the reader to the external appendix where we show in a calibration assessment that this important theoretical implication of the model holds for a broad range of reasonable values of

[2] We show in the external appendix that this indeed is the case for reasonably calibrated values of preference parameters and consumption dynamics.

preference parameters and endowment dynamics.

2.3 Cross-sectional implications of GDA preferences

For every asset i in the economy, optimal consumption and portfolio choice by the representative investor induces a restriction on its simple gross return, $R_{i,t+1}$, that is implied by the Euler condition:

$$E_t\left[H^*_{t,t+1}\left(1+\ell I\left(\mathcal{D}_{t+1}\right)\right)R^e_{i,t+1}\right]=0 \tag{14}$$

where $R_{f,t+1}$ denotes the risk-free simple gross return, $R^e_{i,t+1}=R_{i,t+1}-R_{f,t+1}$ denotes the excess return of asset i over the risk-free return, and $H^*_{t,t+1}$ denotes the risk-adjusted density defined by

$$H^*_{t,t+1}=\frac{M^*_{t,t+1}}{E_t\left[M^*_{t,t+1}\right]}\approx 1+\theta^*_t\left(m^*_{t,t+1}-E_t\left[m^*_{t,t+1}\right]\right), \tag{15}$$

where θ^*_t is a positive coefficient that ensures that the volatility of $M^*_{t,t+1}$ remains unchanged under the approximation, or that the mean squared approximation error is minimal.

The asset premium, after some algebraic manipulation, can be written as

$$E_t\left[R^e_{i,t+1}\right]=\frac{1}{1+\ell\pi^{\mathbb{H}}_t}\left[Cov_t\left(R^e_{i,t+1},-H^*_{t,t+1}\right)+\ell Cov_t\left(R^e_{i,t+1},-H^*_{t,t+1}I\left(\mathcal{D}_{t+1}\right)\right)\right] \tag{16}$$

where $\pi^{\mathbb{H}}_t=E^{\mathbb{H}}_t\left[I\left(\mathcal{D}_{t+1}\right)\right]$ is the risk-adjusted disappointment probability, and where $E^{\mathbb{H}}_t\left[\cdot\right]$ denotes the expectation under the risk-adjusted density $H^*_{t,t+1}$.

Equation (16) shows that the asset effective risk premium is determined by two covariances. The first covariance is the compensation for regular risks, while the second covariance reveals compensation for downside risks and expected downside losses conditional upon disappointment. If the investor is simply risk averse and disappointment neutral ($\ell=0$), effective risk premium is solely a compensation for regular risks, the covariance between the asset excess return and the regular risk-adjustment density $H^*_{t,t+1}$. An investor who is particularly sensitive to downside losses ($\ell>0$) requires an additional compensation. The required compensation is proportional to the covariance between the asset excess return and the product $H^*_{t,t+1}I\left(\mathcal{D}_{t+1}\right)$, the coefficient of proportionality being determined by the investor's disappointment aversion parameter ℓ.

Using the approximation (15) in the pricing relation (16), we show that the cross-sectional risk-return tradeoff may be written in linear covariance form as

$$E_t\left[R^e_{i,t+1}\right] = p_{W,t}\sigma_{iW,t} + p_{X,t}\sigma_{iX,t} + p_{\mathcal{D},t}\sigma_{i\mathcal{D},t} + p_{W\mathcal{D},t}\sigma_{iW\mathcal{D},t} + p_{X\mathcal{D},t}\sigma_{iX\mathcal{D},t} \tag{17}$$

where $\sigma_{iW,t} \equiv Cov_t\left(R^e_{i,t+1}, r_{W,t+1}\right)$ denotes the covariance between the asset excess returns and the market return, $\sigma_{iX,t} \equiv Cov_t\left(R^e_{i,t+1}, \Delta\sigma^2_{W,t+1}\right)$ denotes the covariance between the asset excess returns and changes in market volatility, and where $\sigma_{iW\mathcal{D},t} \equiv Cov_t\left(R^e_{i,t+1}, r_{W,t+1}I\left(\mathcal{D}_{t+1}\right)\right)$ and $\sigma_{iX\mathcal{D},t} \equiv Cov_t\left(R^e_{i,t+1}, \Delta\sigma^2_{W,t+1}I\left(\mathcal{D}_{t+1}\right)\right)$ and $\sigma_{i\mathcal{D},t} \equiv Cov_t\left(R^e_{i,t+1}, I\left(\mathcal{D}_{t+1}\right)\right)$ denote covariances between the asset excess returns and outcomes that are all contingent to the disappointing event.

The risk prices associated to these covariance risk measures are given by:

$$\begin{aligned} p_{W,t} &= \frac{\theta^*_t}{1+\ell\pi^{\mathbb{H}}_t}\gamma \text{ and } p_{X,t} = \frac{\theta^*_t}{1+\ell\pi^{\mathbb{H}}_t}\left(\frac{\gamma-1}{\psi}\right)\varphi_{V\sigma}, \\ p_{\mathcal{D},t} &= -\frac{\ell}{1+\ell\pi^{\mathbb{H}}_t}\left(1+\gamma\theta^*_t\mu_{W,t} + \left(\frac{\gamma-1}{\psi}\right)\varphi_{V\sigma}\theta^*_t\mu_{X,t}\right), \\ p_{W\mathcal{D},t} &= \frac{\theta^*_t}{1+\ell\pi^{\mathbb{H}}_t}\ell\gamma \text{ and } p_{X\mathcal{D},t} = \frac{\theta^*_t}{1+\ell\pi^{\mathbb{H}}_t}\ell\left(\frac{\gamma-1}{\psi}\right)\varphi_{V\sigma}, \end{aligned} \tag{18}$$

where $\mu_{W,t} \equiv E_t\left[r_{W,t+1}\right]$ and $\mu_{X,t} \equiv E_t\left[\Delta\sigma^2_{W,t+1}\right]$ represent the means of the market return and changes in market volatility, respectively.

2.4 Interpreting the new factors

Equation (17) corresponds to a linear multifactor representation of expected excess returns in the cross-section. The unrestricted model is a five-factor model which we refer to as GDA5 throughout the rest of the paper. It states that, in addition to the market return and changes in market volatility, three additional factors command a risk premium. These factors are all payoffs that are contingent to the disappointing event, making them interpretable as options. To illustrate the option interpretation of the new factors in more detail, consider first the special case $\psi = \infty$. This restriction implies that $p_{X,t} = p_{X\mathcal{D},t} = 0$, so the cross-sectional model (17) reduces to a three-factor model where $\sigma_{iW,t}$, $\sigma_{i\mathcal{D},t}$, and $\sigma_{iW\mathcal{D},t}$ are the only priced risks, henceforth GDA3. Additionally, the disappointing event reduces to $r_{W,t+1} < \ln(\kappa/\delta)$, that is, the investor is disappointed if the market return falls below a constant threshold determined by investor's preference parameters. This enables us to give a straightforward interpretation of the priced factors in the GDA3 model.

The GDA3 disappointment indicator can be written as

$$I(\mathcal{D}_{t+1}) = I\left(W_{t+1} < \frac{\kappa P_t}{\delta}\right), \tag{19}$$

where P_t denotes the price of the claim to aggregate consumption. This is the payoff of a long position in a binary cash-or-nothing put option on aggregate wealth, with a strike price of $\kappa P_t/\delta$ and maturing in one period. In the empirical section, $r_{W,t+1}$ is measured by the stock market index return, so $I(\mathcal{D}_{t+1})$ is the payoff of a regular binary cash-or-nothing put option on the stock market index. If $\kappa = \delta$, the option is at-the-money, while for $\kappa < \delta$ the option is out-of-the-money.

Likewise, we show that the factor $r_{W,t+1} I(\mathcal{D}_{t+1})$ is approximately equivalent to

$$r_{W,t+1} I(\mathcal{D}_{t+1}) = -\frac{1}{P_t} \max\left(\frac{\kappa P_t}{\delta} - W_{t+1}, 0\right) + \left(\frac{\kappa}{\delta} - 1\right) I\left(W_{t+1} < \frac{\kappa P_t}{\delta}\right), \tag{20}$$

and represents the payoff of a short position in a European put option on aggregate wealth (henceforth, the stock market index), with a strike price of $\kappa P_t/\delta$ and maturing in one period, together with either a long (if $\kappa > \delta$) or a short (if $\kappa < \delta$) position in the binary cash-or-nothing put option. If κ is close to δ, what will be the base case for our empirical investigation, the second term in (20) can be ignored, so $r_{W,t+1} I(\mathcal{D}_{t+1})$ can be interpreted as the payoff of a short position in a regular European put option on the stock market index.

Now, let us consider the more general case of the GDA5 model. The disappointing event $\mathcal{D}_{t+1}$ may be expressed as

$$r_{W,t+1} - a(\sigma_W/\sigma_X)\Delta\sigma^2_{W,t+1} < b \quad \text{where} \quad a = -(1/\psi)\varphi_{V\sigma}(\sigma_X/\sigma_W) \quad \text{and} \quad b = \ln(\kappa/\delta^*), \tag{21}$$

where $\sigma_W = Std[r_{W,t+1}]$ and $\sigma_X = Std\left[\Delta\sigma^2_{W,t+1}\right]$ are the respective unconditional volatilities of the market return and changes in market volatility. Recalling that $\varphi_{V\sigma} < 0$, notice this implies

$a > 0$ and that both the coefficients a and b depend on investor's preference parameters. The GDA5 disappointment indicator $I(\mathcal{D}_{t+1})$ is again the payoff of a long position in a binary cash-or-nothing option, but the interpretation of the contingent event now warrants some care. The term $(\sigma_W/\sigma_X)\,\Delta\sigma^2_{W,t+1}$ may be viewed as the return on a volatility index with same standard deviation as the market return. In this case, disappointment occurs if the return of a long position in the market index combined with a short position in the volatility index falls below the constant threshold b, the short position in the volatility index being a times the long position in the market index.

In the GDA5 model, the three option factors are two-asset options as their payoffs do not depend on a single instrument but on both market and volatility indexes. The disappointing event may occur (and then options mature in-the-money) due to a fall in the market index or an increase in the volatility index, or both. The factor $r_{W,t+1}I(\mathcal{D}_{t+1})$ can still be interpretable as the payoff for shorting a put option on the market index, as it is negative if disappointment occurs due to a fall in the market index. Similarly, the factor $\Delta\sigma^2_{W,t+1}I(\mathcal{D}_{t+1})$ is interpretable as the payoff for longing a call option on the volatility index, as it is positive if disappointment occurs due to an increase in the volatility index. Likewise, the factor $I(\mathcal{D}_{t+1})$ can be seen as either a binary put option on the market index or a binary call option on the volatility index. In particular, if the coefficient a is equal to one, the long position in the market index is exactly balanced by the short position in the volatility index in determining disappointment. As a decreases from one towards zero, the options are more likely to mature in-the-money due to a fall in the market index rather than an increase in the volatility index. The opposite happens as a increases from one towards infinity. In our empirical investigation, we motivate our base case values of a and b and provide robustness of our results to departures from the base case.

It is important to determine what characteristic of investors' behavior is responsible for a command of a premium related to a specific factor at the market place. As it is revealed by the prices of risk in equation (18), a combination of three preference parameters determines whether a given factor is priced in the cross-section (i.e. has a nonzero price of risk). These parameters are γ (the parameter governing regular risk aversion), ψ (the elasticity of intertemporal substitution), and

ℓ (the measure of disappointment aversion). In particular, equation (18) reveals that $p_{W,t} \neq 0$ if and only if $\gamma \neq 0$, regardless of the disappointment aversion parameter ℓ. This shows that compensation for the covariance with the market return is exclusively due to investors' risk aversion. Note also that $\gamma > 0$ implies $p_{W,t} > 0$. Thus, investors require a premium for a security that has a low return when the market return is low ($\sigma_{iW,t} > 0$).

The asset pricing literature generally agrees on investors' risk aversion parameter $\gamma > 1$. Assuming that $\gamma \neq 1$, it follows from equation (18) that $p_{X,t} \neq 0$ if and only if $\psi \neq \infty$, regardless of the disappointment aversion parameter ℓ. Thus, we can argue that compensation for the covariance with changes in market volatility is exclusively due to imperfect intertemporal substitution of consumption. Investor's risk aversion $\gamma > 1$ and imperfect intertemporal substitution of consumption $\psi < \infty$ together imply that $p_{X,t} < 0$. Thus, consistent with the existing theoretical and empirical literature (see for example Ang, Hodrick, Xing and Zhang 2006; Adrian and Rosenberg 2008), investors are willing to pay a premium for a security that tend to pay off when changes in market volatility are high ($\sigma_{iX,t} > 0$).

Our next observation is that $p_{\mathcal{D},t} \neq 0$ if and only if $\ell \neq 0$, regardless of other preference parameters. This shows that compensation for the covariance with the cash-or-nothing option is exclusively due to disappointment aversion. This model-implied premium $p_{\mathcal{D},t} < 0$ when $\ell > 0$ shows that disappointment averse investors are willing to pay a premium for securities that tend to move upward when the disappointing event occurs ($\sigma_{i\mathcal{D},t} > 0$). We invite the reader to observe that $Cov_t\left(R^e_{i,t+1}, I\left(\mathcal{D}_{t+1}\right)\right) = E_t\left[R^e_{i,t+1} \mid \mathcal{D}_{t+1}\right] - E_t\left[R^e_{i,t+1}\right]$, meaning that $\sigma_{i\mathcal{D},t}$ is also interpretable as the relative downside potential of the asset. Thus, assets with $\sigma_{i\mathcal{D},t} < 0$ are undesirable because they have lower expected payoffs than usual when disappointment sets in.

We also observe that, $p_{W\mathcal{D},t} \neq 0$ if and only if both $\gamma \neq 0$ and $\ell \neq 0$. This shows that neither risk aversion alone, nor disappointment aversion alone suffices to explain the requirement by investors to be compensated for the covariance with the put option on the market index. Investor's risk aversion $\gamma > 1$ and disappointment aversion $\ell > 0$ together imply that $p_{W\mathcal{D},t} > 0$. Investors require a premium for a security that tend to move downward when a low market return in a disappointing state further decreases ($\sigma_{iW\mathcal{D},t} > 0$). Presuming again that $\gamma > 1$, $p_{X\mathcal{D},t} \neq 0$ if and only if both

$\psi \neq \infty$ and $\ell \neq 0$. It turns out that neither imperfect intertemporal substitution of consumption alone, nor disappointment aversion alone suffices to explain the requirement by investors to be compensated for the covariance with the call option on the volatility index. Investor's risk aversion $\gamma > 1$, imperfect intertemporal substitution of consumption $\psi < \infty$ and disappointment aversion $\ell > 0$ altogether imply $p_{X\mathcal{D},t} < 0$. Investors are willing to pay a premium for a security that tend to move upward when large changes in market volatility in a disappointing state further increase ($\sigma_{iX\mathcal{D},t} > 0$).

3 Beta pricing and other downside risk measures

The cross-sectional risk-return relation (17) may ultimately be expressed as a multivariate linear beta pricing model:

$$E_t\left[R^e_{i,t+1}\right] = \lambda^\top_{F,t}\beta_{iF,t} \tag{22}$$

where $\beta_{iF,t}$ is the vector containing the multivariate regression coefficients of asset excess returns onto the factors, and $\lambda_{F,t}$ is the vector of factor risk premiums, respectively given by

$$\beta_{iF,t} = \Sigma^{-1}_{F,t}\sigma_{iF,t} \text{ and } \lambda_{F,t} = \Sigma_{F,t}p_{F,t}. \tag{23}$$

The vector $\sigma_{iF,t}$ contains the covariances of the asset excess returns with the priced factors, the vector $p_{F,t}$ contains the associated factor risk prices, and $\Sigma_{F,t}$ is the factor covariance matrix. It is important to note that if the covariance between the market return and changes in market volatility is negative ($Cov_t\left(r_{W,t+1}, \Delta\sigma^2_{W,t+1}\right) < 0$), consistent with the leverage effect as postulated by Black (1976) and documented by Christie (1982) and others, then the signs of the elements of $\lambda_{F,t}$ are the same as of the corresponding elements of $p_{F,t}$. This beta representation nests both the three-factor model GDA3 ($\psi = \infty$) and the five-factor model GDA5 ($\psi \neq \infty$).

In this section, we argue and show that exposures of asset payoffs to the three option factors provide a rational interpretation of downside risks as studied in the literature. To achieve this, we show how our multivariate betas from equation (23) are related to a number of different measures put forward in previous empirical research to capture the market downside risk of an asset.

One of the most popular measures of the market downside risk is the market downside beta empirically examined by Ang, Chen and Xing (2006), and defined as

$$\beta_{i,t}^{DM} \equiv \frac{Cov_t\left(R_{i,t+1}^e, r_{W,t+1} \mid \mathcal{D}_{t+1}\right)}{Var_t\left[r_{W,t+1} \mid \mathcal{D}_{t+1}\right]}. \tag{24}$$

Post et al. (2010) advocate to use the semi-variance (SV) beta to measure the market downside risk. They study how realized market downside risk measures are related to future returns, and argue that the SV beta captures downside market risk better than the market downside beta. The SV beta emerges from the lower partial moment framework of Bawa and Lindenberg (1977), and is defined by

$$\beta_{i,t}^{SV} \equiv \frac{E_t\left[R_{i,t+1}^e r_{W,t+1} \mid \mathcal{D}_{t+1}\right]}{E_t\left[r_{W,t+1}^2 \mid \mathcal{D}_{t+1}\right]}. \tag{25}$$

Acharya et al. (2010) and Brownlees and Engle (2011) use the Marginal Expected Shortfall (MES) to measure the systemic risk of financial institutions during a financial crisis. They show that the MES, together with the leverage of the institution, are able to predict emerging risks during a financial crisis. We believe the pricing of this systemic risk measure in the cross-section of stock returns is an important topic, and then it is worth showing how the MES expresses in terms of exposures of financial institutions to theoretically motivated factors that are priced at the market place. The MES of an asset is defined by

$$MES_{i,t} \equiv E_t\left[-R_{i,t+1}^e \mid \mathcal{D}_{t+1}\right]. \tag{26}$$

In case of MES, we also emphasize that, to be considered as a measure for systemic risk, that is a more severe and unfrequent downside risk (for example 5% worst days for market return or volatility), the GDA preference parameter κ that modulates both the amplitude and the frequency of disappointment must be sufficiently lower than one.

When used in previous literature, the above measures define the downside event $\mathcal{D}_{t+1}$ as the market return falling below a certain threshold. This case corresponds to our GDA3 model. Our discussion in Section 2, on the other hand, suggests that volatility downside risks may also be priced in the cross-section of stock returns, that it should be distinguished from market downside risk, and that the tradeoff between the two sorts of downside risk should also be emphasised. To this end, we also introduce a measure of volatility downside risk that is analogue to the market downside beta:

$$\beta_{i,t}^{DV} \equiv \frac{Cov_t\left(R_{i,t+1}^e, \Delta\sigma_{W,t+1}^2 \mid \mathcal{D}_{t+1}\right)}{Var_t\left[\Delta\sigma_{W,t+1}^2 \mid \mathcal{D}_{t+1}\right]}. \tag{27}$$

We show[3] that each of these four measures can be written in the following form:

$$a_{W,t}\beta_{iW,t} + a_{W\mathcal{D},t}\beta_{iW\mathcal{D},t} + a_{\mathcal{D},t}\beta_{i\mathcal{D},t} + a_{X,t}\beta_{iX,t} + a_{X\mathcal{D},t}\beta_{iX\mathcal{D},t} + a_{R,t}E_t\left[R_{i,t+1}^e\right], \tag{28}$$

i.e. as a linear combination of our multivariate betas and the mean return of the asset. The $a_{f,t}$ coefficients for each measure are presented in Table 1. Note, that these coefficients arise when the GDA5 model is used. If we start from the GDA3 model, which more closely corresponds to the previous literature on market downside risk, the measures[4] can be written as

$$a_{W,t}\beta_{iW,t} + a_{W\mathcal{D},t}\beta_{iW\mathcal{D},t} + a_{\mathcal{D},t}\beta_{i\mathcal{D},t} + a_{R,t}E_t\left[R_{i,t+1}^e\right], \tag{29}$$

where the $a_{f,t}$ coefficients are exactly the ones reported in Table 1. There are a couple of observations we would like to make regarding theses measures of downside risk.

First, observe that the SV beta and the MES not only vary because of multivariate betas on GDA factors but also because of expected returns of the asset. For this reason, we argue that they should not be employed when empirically analyzing a contemporaneous relationship with expected returns. The empirical analysis of whether they predict future expected returns (e.g. see Post et al. 2010 for the SV beta), may also be puzzled by a momentum or reversal effect already incorporated in the measure. Given this observation, in empirical studies on downside risks and expected returns that examine the SV beta or the MES, we advocate using the relative SV beta and the relative

[3]To save space, we refer the reader to the external appendix for complete derivations of these relations.
[4]Note, that the volatility downside beta, $\beta_{i,t}^{DV}$ is theoretically valid only under the GDA5 model.

MES which we define by

$$\beta_{i,t}^{RSV} \equiv \beta_{i,t}^{SV} - a_{R,t}^{SV} E_t\left[R_{i,t+1}^e\right] = \frac{E_t\left[R_{i,t+1}^e r_{W,t+1} \mid \mathcal{D}_{t+1}\right]}{E_t\left[r_{W,t+1}^2 \mid \mathcal{D}_{t+1}\right]} - \frac{E_t\left[r_{W,t+1} \mid \mathcal{D}_{t+1}\right]}{E_t\left[r_{W,t+1}^2 \mid \mathcal{D}_{t+1}\right]} E_t\left[R_{i,t+1}^e\right]$$
$$RMES_{i,t} \equiv MES_{i,t} - a_{R,t}^{MES} E_t\left[R_{i,t+1}^e\right] = E_t\left[-R_{i,t+1}^e \mid \mathcal{D}_{t+1}\right] - E_t\left[-R_{i,t+1}^e\right]. \quad (30)$$

We invite the reader to observe that the RMES of an asset is simply equivalent to the opposite of its relative downside potential as introduced in the previous section.

Second, note that the coefficients in Table 1 only vary through time but do not vary in the cross-section. So, variations of $\beta_{i,t}^{DM}$, $\beta_{i,t}^{DV}$, $\beta_{i,t}^{RSV}$ and $RMES_{i,t}$ across stocks result from variations in GDA factor risk exposures. Ultimately this means that, investigating the cross-sectional pricing of all existing downside risk measures reduces to investigating whether GDA risk factors are priced in the cross-section of stock returns. To this end, the GDA model provides a unified theoretical framework that can explain existing empirical findings on the pricing of these downside risk measures, as themselves are all linear combinations of the same GDA risk factors. An extensive empirical analysis of the cross-sectional risk-return relation (17) and the multivariate beta pricing model (22) will by carried out in subsequent sections, with the novelty of emphasizing on the pricing of volatility downside risk and the trade-off between market and volatility downside risks, an area the cross-sectional asset pricing literature has been silent in.

Finally, observe from Table 1 that $a_{W,t}$, $a_{W\mathcal{D},t}$ and $a_{\mathcal{D},t}$ are positive while $a_{X,t}$ and $a_{X\mathcal{D},t}$ are negative. This shows that, both the relative SV beta and the relative MES increase with the betas on the market return, the put option and the cash-or-nothing option, and decrease with the betas on changes in market volatility and the call option. Also note that while exposure to the cash-or-nothing option influences the relative SV beta and the relative MES, it plays no role in determining the market downside beta and the volatility downside beta. An empirical investigation of how the $a_{i,t}$ coefficients for the different measures vary through time and how they weight the different components of downside risks through the business cycle is left out for future research.

4 Empirical assessment

The cross-sectional risk-return relation (17) and its multivariate beta representation (22) are the basis for our empirical assessment. We empirically investigate both the GDA5 and the GDA3 models. Notice that the cross-sectional GDA3 model is not nested in the cross-sectional GDA5 model. As shown previously, the disappointing event implied by these two models are different and we must define in each case a disappointment region that is consistent with its own theoretical implication. We recall that, in general, the disappointing event is given by $r_{W,t+1} - a\,(\sigma_W/\sigma_X)\,\Delta\sigma^2_{W,t+1} < b$ where $a > 0$ for the GDA5 model and $a = 0$ for the GDA3 model. Our base case specification uses $b = -0.005$ for both models and $a = 3$ for the GDA5 model[5]. We later investigate in Section 4.3.2 how sensitive are our main results to alternative values of the coefficients a and b.

4.1 Data

Following common practice in the literature, we test our model using all common stocks (CRSP share codes 10 and 11) traded on the NYSE, AMEX and NASDAQ markets. The source of the data is the Center for Research in Security Prices (CRSP) and the analysis covers the period between July, 1963 and December, 2010. The market return is the value-weighted average return on all NYSE, AMEX, and NASDAQ stocks from CRSP, while the risk free rate is the one-month US Treasury bill rate from Ibbotson Associates. Both time series are obtained from Kenneth R. French's data library[6].

Testing the GDA5 model necessitates a measure for market volatility. Several approaches have been used for measuring market volatility in cross-sectional asset pricing studies. For example, Ang, Hodrick, Xing and Zhang (2006) use the option-implied volatility (VIX) index, Adrian and Rosenberg (2008) estimate market volatility from a GARCH model, while Bandi et al. (2006) use realized volatility computed from high-frequency index returns. We chose to use the GARCH model-based estimate of market volatility in our main specification. The most important advantage

[5]These values of a and b match those implied by a calibrated Markov-switching endowment economy, similar to Bonomo et al. (2011), that reproduces the aggregate stock market behavior. Further details can be found in the external appendix.

[6]http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

of this approach is to use the entire sample period[7]. We obtain our measure of market volatility by fitting the Exponential GARCH model of Nelson (1991) to the daily market return series using the whole sample period. The exact model specification and the coefficient estimates are presented in Table 2. In Section 4.3.2, we examine the robustness of our results to alternative measures of market volatility.

When presenting our results, we compare the performance of the GDA model specifications to that of the familiar CAPM and the model of Carhart (1997), henceforth four-factor model. Daily return series on these factors, as well as on portfolios used as test assets in Section 4.3.4, are also collected from the Kenneth R. French's data library.

4.2 Portfolio sorts

We sort individual stocks based on the covariances between their excess returns and the cross-sectional factors from equation (17). Then we form five portfolios based on quintiles of each factor exposures, and examine whether average excess returns of these portfolios display monotonic patterns that are consistent with economic intuition as described by the signs of the prices of risk implied by theory. Our methodology closely follows Ang, Chen and Xing (2006). For every month $t \geq 12$ throughout the whole sample period, we calculate conditional realized covariances from equation (17) using twelve-month of daily data from month $t-11$ to month t. For each stock, we also calculate the conditional average monthly excess return over the same twelve-month period. Risk and reward are thus contemporaneously measured. Stocks are then sorted into five quintile based on their realized covariances, and the average excess returns on these quintile portfolios are calculated. Finally, we take the time-series average of the portfolio excess returns. As pointed out by Ang, Chen and Xing (2006) this use of overlapping information is more efficient, but induces moving average effects, which can be accounted for by reporting robust t-statistics that are adjusted using 12 Newey and West (1987) lags.

Table 3 presents annualized average excess returns of portfolios created by sorting stocks based on their realized covariances with the factors. Note that this is numerically equivalent to sorting

[7]We can obtain daily VIX and realized volatility data starting from 1986 only.

on univariate factor betas. We focus on equal-weighted portfolios (Panel A) when analyzing the results, but the patterns are similar if considering value-weighted portfolios (Panel B). The first column shows the results for sorting on the CAPM beta. We find a monotonically increasing pattern between realized average returns and realized beta. Quintile Low (High) of σ_{iW} has an average excess return of 5.10% (17.99%) per annum, and the spread in average excess returns between quintile portfolios Low and High is 12.89% per annum. The pattern and the magnitude of the premia are in line with the existing literature (see for example Ang, Chen and Xing 2006; Ruenzi and Weigert 2011). In the second column, stocks are sorted into portfolios based on their univariate volatility beta, σ_{iX}. In line with previous empirical findings (see for example Adrian and Rosenberg 2008; Ang, Hodrick, Xing and Zhang 2006), we find that stocks with higher covariance with changes in market volatility pay lower returns on average. Stocks in the quintile with the lowest (highest) σ_{iX} earn 15.82% (6.19%) per annum in excess of the risk-free rate. The average difference between quintile portfolios Low and High is -9.63% per annum.

The third and the fourth columns of Table 3 correspond to the results for sorting on covariances with two payoffs on regular options on the market index as implied by the GDA3 model. The options mature in-the-money if $r_{W,t+1} < -0.005$. Column 3 shows the results for sorting on the covariance with a long binary cash-or-nothing put option. We find that average excess returns are monotonically decreasing with that covariance. Stocks in the quintile with the lowest (highest) $\sigma_{i\mathcal{D}}$ earn 19.27% (4.60%) per annum in excess of the risk-free rate. The average difference between quintile portfolios Low and High is -14.67% per annum, which is statistically significant at the 1% level. Column 4 shows a monotonically increasing pattern between realized average returns and realized covariance with a short put option on the market index, $\sigma_{iW\mathcal{D}}$. Quintile Low (High) has an average excess return of 4.35% (20.01%) per annum, and the spread in average excess returns between quintile portfolios Low and High is 15.66% per annum, which is statistically significant at the 1% level. These results are consistent with investors requiring premiums to invest in stocks with low downside potential and in stocks that tend to have low payoffs in a down and further declining market, such as the GDA representative investor described in Section 2.1.

The last three columns of Table 3 correspond to the results for sorting on covariances with three payoffs on two-asset options on market and volatility indexes as implied by the GDA5 model. The options now mature in-the-money if $r_{W,t+1} - 3\,(\sigma_W/\sigma_X)\,\Delta\sigma^2_{W,t+1} < -0.005$. This allows for volatility downside risk pricing, the focus of this paper, in addition to market downside risk, as a rise in the volatility index may also trigger disappointment. The results for sorting on $\sigma_{i\mathcal{D}}$ and $\sigma_{iW\mathcal{D}}$ are shown in columns 5 and 6, and are very similar to those obtained in columns 3 and 4, although the disappointing event is now more likely to occur due to a rising market volatility rather than a falling market return, as discussed in Section 2.4. This in part points to the fact that our results are robust to alternative definitions of the disappointing event. The last column shows that average excess returns are monotonically decreasing with realized covariance with a long call on the volatility index. Stocks in the quintile with the lowest (highest) $\sigma_{iX\mathcal{D}}$ earn 16.47% (5.63%) per annum in excess of the risk-free rate. The average difference between quintile portfolios Low and High is -10.84% per annum. This latter result adds to the existing literature. It is also consistent with investors requiring a premium to invest in stocks that tend to have low payoffs in an up and further increasing volatility state, such as the GDA representative investor described in Section 2.1.

Finally, all five risk measures of the GDA5 model generate monotonic patterns in the average returns of beta-sorted portfolios with statistically significant differences between the lowest and the highest quintile portfolios. Moreover, these patterns are in line with the signs of the prices of risk suggested by theory, as shown in equation (18). However, observe that these univariate betas may be highly correlated, making it difficult to disentangle the marginal effect of the different factors. The upper left corner of Table 4 shows the average cross-sectional correlation between the univariate exposures over our sample period. Interestingly, market (downside) risk is not correlated to volatility (downside) risk. The correlation between σ_{iW} ($\sigma_{iW\mathcal{D}}$) and σ_{iX} ($\sigma_{iX\mathcal{D}}$) is -0.06 (-0.01), pointing to the fact that volatility downside risk is a separate component of overall risk, that warrants the same special treatment that has been given to market downside risk throughout the literature.

The upper left corner of Table 4 also evidence a couple of high correlation values between exposures to option payoffs and exposures to their underlying instruments. For example, there is a 0.92 correlation between σ_{iX} and $\sigma_{iX\mathcal{D}}$. The correlation of 0.81 between σ_{iW} and $\sigma_{iW\mathcal{D}}$ is in line with Ang, Chen and Xing (2006) and Post et al. (2010) who find that the regular CAPM beta and their univariate measures for market downside risk are highly correlated. One possible solution to this problem is to calculate factor exposures from a single multivariate regression implied by our multifactor model, instead of calculating them from univariate regressions. We follow the multivariate approach for the remaining of the paper.

4.3 Fama-MacBeth regressions

We now focus on the empirical evaluation of equation (22). Using the two-pass cross-sectional regression method of Fama and MacBeth (1973, henceforth FM), we estimate the factor risk premia and examine if option-like payoffs derived from the theoretical model are important in explaining the cross-section of stock returns, and if they command a significant and fairly large portion of the total asset risk premium. To compute conditional multivariate betas, we follow Lewellen and Nagel (2006) and instead of trying to determine the appropriate set of conditioning variables, we use short-window regressions to calculate the factor loadings. For every month $t \geq 12$, we use twelve months of daily data from month $t-11$ to month t to run the following time-series regression for each asset i in the first stage of the FM procedure:

$$R^e_{i,\tau} = \alpha^\beta_{i,t} + \beta_{iW,t} r_{W,\tau} + \beta_{iW\mathcal{D},t} r_{W,\tau} I\left(\mathcal{D}_\tau\right) + \beta_{i\mathcal{D},t} I\left(\mathcal{D}_\tau\right) + \beta_{iX,t} \Delta\sigma^2_{W,\tau} + \beta_{iX\mathcal{D},t} \Delta\sigma^2_{W,\tau} I\left(\mathcal{D}_\tau\right) + \varepsilon^i_\tau. \quad (31)$$

Again, this approach induces overlapping information when calculating the conditional factor loadings and we account for this by reporting Newey and West (1987) adjusted standard errors in all our tests.

The lower right corner of Table 4 shows that the average cross-sectional correlations between the multivariate betas over our sample period are considerably lower than those between the univariate betas used for portfolio sorts in the previous section. So using these multivariate betas in the cross-sectional regressions of the FM procedure reduces the problem of multicollinearity. The second stage of the FM procedure corresponds to estimating the cross-sectional regressions

$$\mu_{i,t} = \alpha^\lambda_t + \beta_{iW,t}\lambda_{W,t} + \beta_{iW\mathcal{D},t}\lambda_{W\mathcal{D},t} + \beta_{i\mathcal{D},t}\lambda_{\mathcal{D},t} + \beta_{iX,t}\lambda_{X,t} + \beta_{iX\mathcal{D},t}\lambda_{X\mathcal{D},t} + \eta^i_t, \quad (32)$$

where the conditional average excess returns for each month t is the average monthly excess return from month $t-11$ to t, so that risk and reward are contemporaneously measured. Factor risk premia obtain by averaging the lambdas over the sample period ($\hat{E}[\lambda_{F,t}]$).

Ang, Hodrick, Xing and Zhang (2006) argue that in order to have a factor risk explanation, there should be contemporaneous patterns between factor loadings and average returns. Several cross-sectional asset pricing studies focus on this contemporaneous relationship (e.g. Ang, Chen and Xing (2006), Cremers et al. (2011), Fama and MacBeth (1973), Lewellen and Nagel (2006) and Ruenzi and Weigert (2011), among others). We follow this common approach to derive our main results and, in Section 4.3.3 we also report results from cross-sectional regressions of future average excess returns on current betas.

4.3.1 Individual stocks and contemporaneous returns

Following Black et al. (1972), as a standard method for handling the errors-in-variable problem induced by the two-pass cross-sectional regression method, the majority of cross-sectional asset pricing studies use portfolios as test assets. However, Ang et al. (2010) have recently argued that creating portfolios destroys important information and leads to larger standard errors. They show that using individual stocks permits more efficient tests of whether factors are priced, and there should be no reason to create portfolios. Cremers et al. (2011), Lewellen (2011) and Ruenzi and Weigert (2011) are recent examples focusing on individual stocks as base assets in FM regressions. Our main results are based on individual stocks from the CRSP universe as base assets for the FM regressions. Nevertheless, we report results with portfolios as base assets in Section 4.3.4 where we decompose asset premia and measure the parts that can be attributed to different factors.

Results from analyzing the contemporaneous relationship between factor loadings and average returns using individual stocks as base assets are presented in Tables 5 and 6. Theory implies no constant in the cross-sectional regressions. However, since there is no consensus in the empirical literature whether to include a constant or not, we report results both with constant in Table 5, and without in Table 6. The top panels of the tables show estimates of factor risk premia for the listed models.

The bottom panels report, for every factor f, the annualized spread $\hat{E}\left[\left(\beta_{75^{th}f,t} - \beta_{25^{th}f,t}\right)\lambda_{f,t}\right]$ between two hypothetical portfolios with different betas on the factor f, everything else being equal. We refer to this number as the interquartile spread (IQS) of the factor. The first portfolio's beta is the 75th percentile while the second's is the 25th percentile of the cross-sectional distribution of individual stock betas on factor f. The IQS thus represents a premium for shorting low beta stocks and longing high beta stocks. It is worth noting that it would actually be hard to create portfolios that differ only in one of the multivariate betas, everything else being equal. So we look at the IQS only as an indicative number to help interpret the economic magnitude of the risk premia reported in the FM regressions. In Section 4.3.5 we quantify the premium attributable to each factor on actual portfolios.

Focusing on Table 5, the first column corresponds to the basic CAPM, with a significant positive market price of risk, together with a significant constant term at the 10% level, consistent with similar results in Ang, Chen and Xing (2006). The second model in column 2 includes both the market return and changes in market volatility. Both risk premia are statistically significant and the signs are consistent with the results of Ang, Hodrick, Xing and Zhang (2006) and Adrian and Rosenberg (2008). Column 3 presents the result for the GDA3 model where investors only dislike downside risks in falling market returns. All factor risk premia are significantly estimated at the 1% level, and the estimated constant is no longer significant. The signs of the estimates are in line with the predictions of the theoretical model as discussed in Section 2.4. In economic terms the IQS of the binary cash-or-nothing option on the market index is -6.59%, and is comparable to the IQS of the market return of 6.19%, while both are smaller than the IQS of the put option on the market return of 9.5%. These results confirm the portfolio sorts of Table 3.

Let us now examine the results from the GDA5 model where investors dislike downside risks in both falling market returns and rising market volatility, presented in column 4. All factor risk premia are significantly estimated, while the estimated constant is not significant and is considerably lower than that of the CAPM and GDA3 models. Regarding the economic magnitudes of the estimated factor risk premia, IQS of the binary cash-or-nothing and the put options on the market index have decreased compared to the GDA3 model. This shows that not being indifferent to volatility downside risk reduces the marginal effect of market downside risk. However, IQS of these two factors are still non-negligible, -4.59% and 4.63% respectively. Changes in market volatility have an IQS of -5.84%, and the call option on the volatility index has an IQS of -7.35%. The large IQS of the call option on the volatility index relative to the put option on the market index may be due to the fact that our base case disappointing event favors rising market volatility relative to falling market returns in triggering disappointment. We emphasize that this empirical analysis of volatility downside risk is novel to the cross-sectional asset pricing literature. Regarding overall fit, the GDA5 model provides further improvement over the GDA3 model, as measured by the cross-sectional R^2, from 5.08% to 6.43%.

The remaining columns of Table 5 shows estimation results of cross-sectional asset pricing models featuring factor risks that are not motivated by the theory of disappointment aversion as discussed in this article, or by any theory at all. These risks are the coskewness risk studied by Harvey and Siddique (2000), and betas on the size, value and momentum factors examined by Carhart (1997). In columns 5 and 6 we estimate the coskewness model and control for coskewness risk in the GDA5 model. We measure coskewness risk as the coefficient on the squared market return from the bivariate regression of the asset's excess return on the market return and the squared market return. We denote the coskewness risk premium by λ_{W^2} in cross-sectional models. Column 5 shows that coskewness has a statistically significant negative risk premium, confirming the findings of Harvey and Siddique (2000).

When coskewness is added to the GDA5 model in column 6, the statistical and economic magnitudes and significance of GDA5 factor risk premia barely change. The largest change actually occurs for the coskewness risk premium estimate compared to the model in column 5; it decreases by half in magnitude. Also, adding coskewness to the GDA5 model does not improve the fit of the model considerably, as measured by the cross-sectional R^2, from 6.43% to 6.98%. The IQS of coskewness is significantly smaller that of any of the GDA5 factor risk, and falls from -3.64% to -2.43% when controlling for the GDA5 factors. All in all, coskewness does not seem to drive out any of the GDA5 factors, if anything, it is the other way around.

Column 7 shows estimation results for the four-factor model. The size and momentum factors are positive and significant. The IQS of the momentum factor is particularly big. The value premium is insignificant and has a negative sign. While this result seems to be puzzling at first, Ang et al. (2010) points out that when the estimation uses individual stocks, the value premium is negative. They argue that the book-to-market effect is a characteristic effect rather than a reward for bearing the HML factor loading risk. If the book-to-market ratio is included in cross-sectional regressions instead of the HML factor, the coefficient on the book-to-market ratio is positive and significant. They also argue that when book-to-market sorted portfolios are used as base assets in the FM regressions, the HML factor loadings are induced to have a positive coefficient through forcing the portfolio breakpoints to be based on book-to-market characteristics. This is confirmed in our results of Section 4.3.4.

The last column of Table 5 presents the specification where both the GDA5 factor betas and the Carhart (1997) factor betas are included in the cross-sectional model. The important observation here is that the sign, and significance of the GDA5 factor risk premia estimates do not change considerably compared to column 4. The statistical magnitudes of the GDA5 factor risk premia decrease slightly, but their economic magnitudes are still important. Moreover, the GDA5 factor betas provide improvement in overall fit when added to the Carhart (1997) cross-sectional model, as measured by the cross-sectional R^2, from 9.85% to 11.14%. The IQS of the momentum factor falls when the GDA5 factors are controlled for (from 9.17% to 7.57%), while the IQS of the size factor (3.31%) is comparable to that of the binary cash-or-nothing and the put option on the market index (-3.12% and 2.90%), but smaller that of the call option on the volatility index (-4.62%).

Table 6 repeats the same analysis of Table 5 restricting the constant term in the cross-sectional regressions to zero. The conclusions that can be drawn from the table are virtually the same as those drawn from Table 5. While the economic magnitudes are somewhat bigger than those in previous the table, the patterns are very similar. Also, the statistical significance of the volatility-related factor risk premia in the GDA5 model (λ_X and $\lambda_{X\mathcal{D}}$) is weaker than in Table 5, but their economic magnitudes remain important and unaffected. This is probably due to the high cross-sectional correlation of -0.74 between β_{iX} and $\beta_{iX\mathcal{D}}$ as shown in Table 4. As we have already pointed it out, this high correlation makes it hard to disentangle the effect of the two risk measures.

To conclude this subsection, FM regressions analyzing the contemporaneous relationship between expected returns and factor exposures show that options on market and volatility indexes as implied by generalized disappointment aversion preferences are priced in the cross-section of stock returns. The associated factor risk premia are both statistically and economically significant, and their signs are consistent with the theoretical predictions. In the following subsection we assess the robustness of these results.

4.3.2 Robustness checks

Alternative disappointment regions

We recall that the disappointment region implied by the theoretical setup as discussed in Section 2.4 is given by $r_{W,t+1} - a\left(\sigma_W/\sigma_X\right)\Delta\sigma^2_{W,t+1} < b$ where $a > 0$ for the GDA5 model and $a = 0$ for the GDA3 model. Our main empirical results in the previous section assume $b = -0.005$ for both models and $a = 3$ for the GDA5 model. In this section we focus on the GDA5 model and examine the changes to our results as we vary the coefficients a and b. Results are reported in Table 7. Our baseline specification ($a = 3$ and $b = -0.005$) is reported in column 6 for comparisons. We vary the coefficient a across the values 0, 1, 3 and 5, and the cutoff point b across the values 0, -0.005 and -0.007 so as to maintain a reasonable frequency of the disappointing event.

The top panel of the table shows that by decreasing the threshold b, anything else equal, the frequency of disappointment decreases. For example, the frequency of disappointment decreases from 45.67% to 21.36% as b falls from 0 to -0.005, keeping $a = 0$. The numbers are respectively from 41.36% to 24.93%, with $a = 1$, and from 35.76% to 26.67%, with $a = 3$. We also observed that with $b = 0$, increasing the coefficient a reduces the frequency of disappointment, while it is the contrary with a sufficiently negative b. The middle panel of the table shows that estimates of the factor risk premia are remarkably robust across alternative disappointment regions; there is barely any change in them. Also, the overall fit of the model, as measured by the cross-sectional R^2, is very similar across the different disappointing regions. Interestingly, what changes is the economic magnitude of the factors, then attributable to changes in beta estimates from the first stage FM regressions.

When $a = 0$, thus rising volatility cannot trigger disappointment, and when $b = 0$ as in column 1, the market return has the largest IQS of 8.02%, followed by the put option on the market index with an IQS of 7.24%, while the binary cash-or-nothing option has the smallest IQS, -2.82%. As b decreases to -0.005 in column 2, focusing on more severe disappointing outcomes, everything else equal, the IQS of the binary cash-or-nothing option jumps to -6.74%, that of the put option on the market index increases to 9.76%, while that of the volatility-based factors are almost unaffected. To the contrary, positive values of a rises the IQS of the volatility factors and decreases that of the market-based factors relative to $a = 0$. For example, comparing column 1 and 5, the IQS of changes in market volatility rises from -3.86% to -6.40%, while that of the call option on the volatility index rises from -4.36% to -7.68%. At the same time, the IQS of the market return falls from 8.02% to 6.68%, while that of the put option on the market index falls from 7.24% to 4.86%, corroborating the tradeoff between market and volatility downside risks in this model.

Finally, when the market return and changes in market volatility are equally likely to trigger disappointment, meaning that $a = 1$, columns 3 and 4 show that the economic magnitudes of the put option on the market index and the call option on the volatility index are comparable. Their respective IQS are 6.26% and -6.56% respectively in column 3 when $b = 0$, and 6.69% and -6.33% respectively in colum 4 when $b = -0.005$. Ultimately, alternative disappointing events simply rearrange the economic significance of the GDA factor risks, without affecting the factor risk premia which remain statistically significant and carry the signs predicted theory.

Alternative measures of market volatility

In this subsection we explore how the GDA5 model results change if different measures of market volatility are considered. Our main results of Section 4.3.1 uses a daily market volatility estimated by fitting an Exponential GARCH model to the daily market return series. Alternative approaches include using the option-implied volatility (VIX) index, calculating daily realized variance from intra-daily market returns, or fitting a different GARCH model. For a detailed description of the estimation of these alternative measures, we refer the reader to Appendix A. The corresponding results are presented in Table 8.

Panel A presents the results for the whole sample period, from January 1963. Since the VIX and the intra-daily market return data are available from 1986, only results for alternative GARCH models are presented. The results are very robust, with only minor changes across the different GARCH specifications. Accounting for leverage effect in GARCH modelling increases the cross-sectional fit of the GDA5 model, and improves the IQS of the call option on the volatility index. The standard GARCH model has an R^2 (IQS) of 5.65% (-5.36%) while that of the EGARCH and the GJR-GARCH are 6.43% (-7.35%) and 6.32% (-7.64%) respectively.

Panel B presents the results for the subsample starting from 1986, when data are available for all the volatility measures. The R^2, the signs and the statistical significance of the factor risk premia estimates are very similar across all volatility specifications. The statistical significance of the volatility-related factors is lost for this shorter sample period, but their economic magnitudes are still important. As we have already discussed, this may probably be due to the high correlation between β_{iX} and $\beta_{iX\mathcal{D}}$. Also, observed that, accounting for leverage effect in GARCH modelling improves the IQS of the call option on the volatility index with respect to nonparametric volatility measures. The VIX and the realized volatility have IQS of -4.58% and -4.54%, while that of the EGARCH and the GJR-GARCH are -7.18% and -6.82% respectively.

4.3.3 Individual stocks and future returns

In this subsection, we check if current realized multivariate betas on the GDA factors predict high future returns over the next months, similar to the contemporaneous relationship between multivariate betas and realized average returns from the previous subsection. Lewellen (2011) is a recent example to analyze predictive FM regressions. We carry out the same exercise as in Section 4.3.1, measuring the realized multivariate betas from equation (31), but now the left-hand side of the cross-sectional regression (32) is $(1/h) \sum_{j=1}^{h} R^e_{i,t+j}$, the average monthly excess returns over the next months. We consider three different predictability horizons: one month ($h = 1$), three months ($h = 3$) and six months ($h = 6$).

The cross-sectional predictive regression results are shown in Table 9. The top panel displays results for the GDA3 model, while the bottom panel displays results for the GDA5 model. The conclusions are very similar to those obtained by analyzing the contemporaneous relationship between betas and returns in Section 4.3.1. GDA factor risks predict future returns at conventional levels of significance. Positive betas on the market return and the short put on the market index predict higher future returns, while it is the contrary for positive betas on the long binary cash-or-nothing option, changes in market volatility and the long call on the volatility index, everything else equal. The signs of the predictability coefficients, the GDA factor risk premia, are in line with the theoretical implications of the model, and controlling for coskewness, and for size, value and momentum factor betas does not affect these predictability patterns. In particular, coskewness does not predict future returns beyond the predictability of the GDA multivariate betas. Also, notice that an increase in the HML factor beta significantly predicts higher future returns, although the contemporaneous relation between expected returns and the HML factor beta is insignificant as shown in Section 4.3.1.

4.3.4 Portfolios as test assets

Although Ang et al. (2010) argue that it is more efficient to use individual stocks in cross-sectional asset pricing tests than portfolios, most of the literature uses portfolios as base assets. Therefore, we repeat the analysis of Section 4.3.1, to examine the empirical performance of our GDA models using portfolios as test assets, with otherwise unchanged methodology.

We use value-weighted return series of five different sets of portfolios: (i) 25 (5×5) portfolios formed on size and book-to-market, (ii) 25 (5×5) portfolios formed on size and momentum, (iii) 25 (5×5) portfolios formed on size and long-term reversal, (iv) 30 industry portfolios, and (v) 30 portfolios consisting of 10 size, 10 book-to-market, 10 momentum. Table 10 shows the results of FM regressions for four different cross-sectional models (CAPM, four-factor, GDA3, and GDA5). Consistent with the results on individual stocks, the signs of the GDA factor risk premia estimates validate the theoretical predictions, and this is true for all sets of test portfolios. These estimates are statistically significant at conventional levels across the different sets of portfolios, except for the industry portfolios where the premium of the long call on the volatility index appears insignificant. The fit of the models, as measured by the sum of squared pricing errors (labelled with “SSE”) shows that the GDA3 model is a considerable improvement compared to the standard CAPM, while the GDA5 model further improves upon the GDA3. The best fit (lowest SSE) is provided by the four-factor model for all sets of test portfolios, but the fit of the GDA5 model is quite comparable. Overall, our results on the pricing of downside risks through option exposures and especially of volatility downside risks are robust to alternative test assets.

Figure 1 shows scatter plots of actual versus predicted returns for the different models on 10 size (S1 to S10), 10 book-to-market (B1 to B10), 10 momentum (M1 to M10) portfolios. It gives a visual impression of our findings: fitted returns of the GDA models line up along the 45-degree line in a manner that is remarkably similar to the four-factor model, and the contrast with the CAPM is stark. We invite the reader to bear in mind that the GDA factors are motivated by dynamic asset pricing and behavioral decision theories, while a theoretical approach to size, value and momentum factors is rather nonexistent. Intuitively, exposures to options on the market and volatility indexes improve the fit of the CAPM and a model with market return and changes in market volatility, because some stocks are more highly correlated with these factors in bad times, when disappointment sets in and options expire in-the-money, than they are in good times, when the market is up and the level of volatility is satisfying.

4.3.5 Decomposing portfolio risk premia

We already assess the economic importance of the GDA factors in section 4.3.1 by comparing the premium difference, termed IQS, in two hypothetical portfolios that differ in their exposures to one of the factors, everything else being equal. However, these hypothetical portfolios are hard to create in real life. In the current subsection we decompose the actual expected excess returns of the 10 size, 10 book-to-market, 10 momentum portfolios into parts attributable to the different factors. For every asset i and factor f, we compute the annualized $\hat{E}\left[\beta_{if,t}\lambda_{f,t}\right]$ where the $\beta_{if,t}$ and the $\lambda_{f,t}$ are estimated from the first and second stages of the FM procedure, respectively. This exercise can be carried out for any set of portfolios. We chose the 10 size, 10 book-to-market, 10 momentum portfolios because this set provides the most illustrative example. Figure 2 shows the decomposition of all portfolio premia for three different models (CAPM, four-factor and GDA5), while Table 11 quantifies the results for the top and bottom portfolios in each category.

Focusing first on the 10 size portfolios, observe from the table that actual average excess returns decrease from small firms (8.01%) to big firms (4.07%), a positive small-minus-big spread of 3.9% that illustrates the well-documented size premium (see for example Fama and French 1992). The failure of the standard CAPM in pricing size portfolio is apparent. It predicts an increase in average excess returns from small (4.23%) to big (6.66%) firms, a negative spread of -2.44% totally off the actual value. The four-factor model provides a much improved fit compared to the standard CAPM. The predicted average excess returns show the same patterns as the actual: they decrease from 6.67% for small to 4.59% for big firms, a spread of 2.09%, mostly dominated, and not surprisingly, by its size factor component, accounting for 1.75% out of the 2.09% spread. The GDA3 model also predicts that average excess returns decrease from small (7.02%) to big firms (4.99%) firms, a spread of 2.03%, comparable to that of the four-factor model, and mostly driven by its put option component (6.58%). Notice that, the GDA5 model provides the best fit for the size portfolios, with a predicted small-minus-big difference of 2.67%, dominated by the put and call option components, 2.81% and 4.04% respectively. These option components of the small-minus-big spread as predicted by the GDA models are large enough to compensate for the negative spreads on other factors.

In the case of the 10 book-to-market portfolios, realized average excess returns increase from growth stocks (3.75%) to value stocks (10.22%), a positive value premium of 6.50% as documented in the cross-sectional asset pricing literature (see for example Fama and French 1992). Figure 2 shows that the excess returns predicted by the CAPM are rather flat or slightly concave across these portfolios, generating a negative value premium of -0.50%, and corroborating the inconsistency of the CAPM with this empirical regularity of the data. The four-factor model provides the best prediction of the value premium (5.59%), less than 1% off the actual value. A large part of this spread, 3.48% out of 5.59%, represents the value factor component. The GDA3 and GDA5 models are not as successful as the four-factor model, but they provide much improvement over the CAPM. The GDA5 predicts a 3.67% value premium, mostly dominated by its put and call components, 2.27% and 2.10%, respectively.

The last set of portfolios to look are the 10 momentum. The actual excess returns increase from looser (-2.02%) to winner (12.42) portfolios, a positive winner-minus-looser spread of 14.5%, corroborating the well-documented momentum premium (see for example Jegadeesh and Titman 1993). The CAPM predicts momentum premium of only 4.08%, more than three times smaller than the actual premium. The four-factor model again performs the best in this dimension, with a predicted spread of 10.51%, almost exclusively due its momentum factor component, accounting for 9.04% out of the 10.51% spread. The GDA3 model predicts a momentum premium of 7.39%, half the actual value, while the GDA5 predicts a momentum premium of 9.05%, close to the value predicted by the four-factor model. The predicted momentum premium by the GDA5 model is equally distributed between its components from the three option factors (4.39%) and their underlying instruments (4.66%).

Ultimately, both the GDA3 and the GDA5 models provide considerable improvement on the standard CAPM, while the performance of the GDA5 model is comparable to that of the four-factor model. However, one important observation is at stake. The improved fit of the four-factor model on size portfolios comes mostly from the size (SMB) factor, its improved fit on the book-to-market portfolios comes mostly from the value (HML) factor, and the improved fit on the momentum portfolios is almost exclusively due to the momentum (WML) factor. This observation shows how each factor is tailor-made to explain its respective anomaly. For the GDA5 model, on the other hand, the improved fit for all three sets of portfolios mainly comes from contributions from two sources: the premium associated with the short put option on the market index and the premium associated with the long call option on changes in market volatility. Both factors arise due to investors' disappointment aversion and time-varying macroeconomic uncertainty.

5 Conclusion and Future Work

This paper provides an empirical analysis of downside risks in asset prices. The approach is consistent with general equilibrium implications for asset returns in the cross-section when investors have totally rational and axiomatized asymmetric preferences. The theoretical setup explicitly disentangles the components of an asset premium that are due to the different characteristics of investors' behavior, and shows that asymmetric preferences lead to option pricing in the cross-section of stock returns. These options provide a straightforward way for investors to act on their views of two of the most closely followed market variables, the market return and changes in market volatility. Empirical results show that the cross-section of stock returns reflects a premium for bearing undesirable exposures to these options, and that the new cross-sectional model significantly improves over nested specifications without the option factors.

The paper also derives explicit cross-sectional relations between existing downside risk measures and betas on the market return, changes in market volatility and option factors. The weights associated to these relations and how they vary through time and in relation with the business cycle may constitute an interesting avenue for future research.

Apppendix

A Different measures of market volatility

A.1 VIX

The daily value of the VIX index is obtained from CBOE through the WRDS service. The variance of the market is calculated as $(VIX/100)^2$. Since the VIX measures 30-day expected volatility of the S&P500 Index, we divide this value by 30 to get the daily variance of the market. So, the change in the daily market variance is calculated as

$$\Delta\sigma_{W,t}^{2,VIX} = \frac{(VIX_t/100)^2 - (VIX_{t-1}/100)^2}{30} \tag{B.1}$$

A.2 Realized Volatility

To calculate daily realized volatility, we use intra-daily return series of the S&P 500. The data comes from Olsen Financial Technologies and covers the period between February 1986 and September 2010. Daily realized market variance is calculated as

$$\sigma_{W,t}^{2,RV} = \sum_{j=1}^{N_t} r_{j,t}^2 \ , \tag{B.2}$$

where $r_{j,t}$ denotes the 10-minute log return series with length N_t, on the trading day t. Following Bandi et al. (2006) we correct the variance estimates for the lack of overnight returns by multiplying them with a constant factor

$$\xi = \frac{\frac{1}{T}\sum\limits_{t=1}^{T} r_{W,t}^2}{\frac{1}{T}\sum\limits_{t=1}^{T} \sigma_{W,t}^{2,RV}},$$

where $r_{W,t}$ denotes daily log returns on the market. The change in the daily market variance is calculated as

$$\Delta\sigma_{W,t}^{2,RV} = \xi\left(\sigma_{W,t}^{2,RV} - \sigma_{W,t-1}^{2,RV}\right) \tag{B.3}$$

A.3 GARCH-type models

In this approach, we fit a model with conditional heteroskedasticity to the daily log market return series $r_{W,t}$ (the value-weighted average return on all NYSE, AMEX, and NASDAQ stocks from CRSP). We consider three different models: the standard GARCH(1,1), the EGARCH(1,1,1) by Nelson (1991) and the GJR-GARCH(1,1,1) by Glosten et al. (1993). The models are given as (the difference is in the variance equation):

$$\begin{aligned} r_{W,t+1} &= \mu + \sigma_{W,t}\varepsilon_{t+1} \ , \quad \text{with} \ \ \varepsilon_{\mathrm{t+1}} \overset{\text{iid}}{\sim} \mathcal{N}\left(0,1\right) \\ GARCH: \ \sigma^2_{W,t+1} &= \omega + \nu\sigma^2_{W,t}\varepsilon^2_{t+1} + \phi\sigma^2_{W,t} \\ EGARCH: \ \ln\left(\sigma^2_{W,t+1}\right) &= \omega + \nu\left(\left|\varepsilon_{t+1}\right| - \sqrt{2/\pi}\right) + \theta\varepsilon_{t+1} + \phi\ln\left(\sigma^2_{W,t}\right) \\ GJR-GARCH: \ \sigma^2_{W,t+1} &= \omega + \left(\nu + \theta I\left(\epsilon_{t+1} < 0\right)\right)\sigma^2_{W,t}\varepsilon^2_{t+1} + \phi\sigma^2_{W,t} \end{aligned} \tag{B.4}$$

The change in the daily market variance is calculated as

$$\Delta\sigma^{2,model}_{W,t} = \hat{\sigma}^{2,model}_{W,t} - \hat{\sigma}^{2,model}_{W,t-1} \tag{B.5}$$

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Table 1: Coefficients for measures of downside risk

	$\beta_{i,t}^{DM}$	$\beta_{i,t}^{DV}$	$\beta_{i,t}^{SV}$	$MES_{i,t}$
$a_{W,t}$	1	$\beta_{W,t}^{DV}$	$1-\beta_{\mathcal{D},t}^{SV}E_t\left[r_{W,t+1}\right]$	$-\left(E_t\left[r_{W,t+1} \mid \mathcal{D}_{t+1}\right]-E_t\left[r_{W,t+1}\right]\right)$
$a_{\mathcal{D},t}$	0	0	$\left(1-\pi_t\right)\beta_{\mathcal{D},t}^{SV}$	$-\left(1-\pi_t\right)$
$a_{W\mathcal{D},t}$	1	$\beta_{W,t}^{DV}$	$1-\beta_{\mathcal{D},t}^{SV}\pi_t E_t\left[r_{W,t+1} \mid \mathcal{D}_{t+1}\right]$	$-\left(1-\pi_t\right)E_t\left[r_{W,t+1} \mid \mathcal{D}_{t+1}\right]$
$a_{X,t}$	$\beta_{X,t}^{DM}$	1	$\beta_{X,t}^{SV}-\beta_{\mathcal{D},t}^{SV}E_t\left[\Delta\sigma_{W,t+1}^2\right]$	$-\left(E_t\left[\Delta\sigma_{W,t+1}^2 \mid \mathcal{D}_{t+1}\right]-E_t\left[\Delta\sigma_{W,t+1}^2\right]\right)$
$a_{X\mathcal{D},t}$	$\beta_{X,t}^{DM}$	1	$\beta_{X,t}^{SV}-\beta_{\mathcal{D},t}^{SV}\pi_t E_t\left[\Delta\sigma_{W,t+1}^2 \mid \mathcal{D}_{t+1}\right]$	$-\left(1-\pi_t\right)E_t\left[\Delta\sigma_{W,t}^2 \mid \mathcal{D}_t\right]$
$a_{R,t}$	0	0	$\beta_{\mathcal{D},t}^{SV}$	-1

The entries of the table are expressions of the $a_{f,t}$ coefficients in the following relation:

$$Measure_{i,t} = a_{W,t}\beta_{iW,t} + a_{W\mathcal{D},t}\beta_{iW\mathcal{D},t} + a_{\mathcal{D},t}\beta_{i\mathcal{D},t} + a_{X,t}\beta_{iX,t} + a_{X\mathcal{D},t}\beta_{iX\mathcal{D},t} + a_{R,t}E_t\left[R_{i,t+1}^e\right],$$

where $Measure_{i,t}$ denotes different measures for downside risk ($\beta_{i,t}^{DM}$, $\beta_{i,t}^{DV}$, $\beta_{i,t}^{SV}$, and $MES_{i,t}$), and within the table, the following notations are used:

$$\beta_{X,t}^{DM} \equiv \frac{Cov_t\left(\Delta\sigma_{W,t+1}^2, r_{W,t+1} \mid \mathcal{D}_{t+1}\right)}{Var_t\left[r_{W,t+1} \mid \mathcal{D}_{t+1}\right]}, \quad \beta_{W,t}^{DV} \equiv \frac{Cov_t\left(\Delta\sigma_{W,t+1}^2, r_{W,t+1} \mid \mathcal{D}_{t+1}\right)}{Var_t\left[\Delta\sigma_{W,t+1}^2 \mid \mathcal{D}_{t+1}\right]},$$

$$\beta_{X,t}^{SV} \equiv \frac{E_t\left[r_{W,t+1}\Delta\sigma_{W,t+1}^2 \mid \mathcal{D}_{t+1}\right]}{E_t\left[r_{W,t+1}^2 \mid \mathcal{D}_{t+1}\right]}, \quad \beta_{\mathcal{D},t}^{SV} \equiv \frac{E_t\left[r_{W,t+1} \mid \mathcal{D}_{t+1}\right]}{E_t\left[r_{W,t+1}^2 \mid \mathcal{D}_{t+1}\right]} \quad \text{and} \quad \pi_t \equiv Prob_t\left(\mathcal{D}_{t+1}\right).$$

Table 2: Estimation Results of the EGARCH model

	μ	ω	ν	θ	ϕ
Coeff	4.13E-4	-0.141	0.150	-0.074	0.985
s.e.	(0.0001)	(0.0098)	(0.0050)	(0.0031)	(0.0010)

The entries of the table are the coefficient estimates of the following Exponential GARCH model specification:

$$
\begin{aligned}
r_{W,t+1} =& \mu + \sigma_{W,t}\varepsilon_{t+1} \\
\ln\left(\sigma^2_{W,t+1}\right) =& \omega + \nu\left(|\varepsilon_{t+1}| - \sqrt{2/\pi}\right) + \theta\varepsilon_{t+1} + \phi\ln\left(\sigma^2_{W,t}\right) \\
\varepsilon_{t+1} \overset{iid}{\sim}& \mathcal{N}\left(0,1\right)
\end{aligned}
$$

using daily index return data from January 1963 to December 2010. Robust standard errors of the coefficient estimates are given in parenthesis.

Table 3: Average returns of portfolios sorted on different measures of risk

Panel A: equal-weighted portfolios

			$\mathcal{D}_1$		$\mathcal{D}_2$		
	σ_{iW}	σ_{iX}	$\sigma_{i\mathcal{D}}$	$\sigma_{iW\mathcal{D}}$	$\sigma_{i\mathcal{D}}$	$\sigma_{iW\mathcal{D}}$	$\sigma_{iX\mathcal{D}}$
Low	5.10	15.82	19.27	4.35	18.71	4.62	16.47
2	7.60	11.97	11.24	7.15	11.88	7.11	12.13
3	9.47	9.54	9.24	8.85	9.23	8.93	9.54
4	11.33	7.81	7.25	11.31	6.57	11.45	7.62
High	17.99	6.19	4.60	20.01	5.16	19.52	5.63
H-L	**12.89**	**-9.63**	**-14.67**	**15.66**	**-13.55**	**14.90**	**-10.84**
t-stat	3.70	-5.11	-4.14	4.22	-4.47	4.49	-5.60

Panel B: value-weighted portfolios

			$\mathcal{D}_1$		$\mathcal{D}_2$		
	σ_{iW}	σ_{iX}	$\sigma_{i\mathcal{D}}$	$\sigma_{iW\mathcal{D}}$	$\sigma_{i\mathcal{D}}$	$\sigma_{iW\mathcal{D}}$	$\sigma_{iX\mathcal{D}}$
Low	6.22	16.44	14.65	6.11	16.88	6.12	16.10
2	6.60	12.31	8.61	6.28	11.93	5.83	12.90
3	7.34	9.92	7.24	7.18	8.73	6.82	9.95
4	8.06	7.90	6.36	9.10	6.14	9.67	8.31
High	12.73	4.64	6.03	15.37	4.57	16.05	5.28
H-L	**6.50**	**-11.81**	**-8.62**	**9.26**	**-12.31**	**9.93**	**-10.82**
t-stat	1.88	-5.72	-2.42	2.44	-3.36	2.84	-4.78

The table shows the equal-weighted (Panel A) and value-weighted (Panel B) average returns of stocks sorted by realized covariances. For each month, σ-s are calculated using daily simple excess returns over the previous 12 months (including the given month). For each month and each risk measure, we rank stocks into 5 portfolios, and the average monthly excess returns (over the previous 12 months) of these portfolios are calculated. The table reports the annualized average return of these portfolios over the whole sample period (July, 1963 to December, 2010). The row labelled "H-L" reports the difference between the returns of portfolio 5 and portfolio 1. The row labelled "t-stat" is the t-statistics computed using Newey-West (1987) standard errors with 12 lags for the H-L difference.
$\mathcal{D}_1$ corresponds to the disappointing region $r_{W,t} < -0.005$, while $\mathcal{D}_2$ corresponds to $r_{W,t+1} - 3\,(\sigma_W/\sigma_X)\,\Delta\sigma^2_{W,t+1} < -0.005$.

Table 4: Correlations between measures of risk

	σ_{iW}	$\sigma_{i\mathcal{D}}$	$\sigma_{iW\mathcal{D}}$	σ_{iX}	$\sigma_{iX\mathcal{D}}$	β_{iW}	$\beta_{i\mathcal{D}}$	$\beta_{iW\mathcal{D}}$	β_{iX}	$\beta_{iX\mathcal{D}}$
σ_{iW}	1.00									
$\sigma_{i\mathcal{D}}$	-0.43	1.00								
$\sigma_{iW\mathcal{D}}$	0.84	-0.43	1.00							
σ_{iX}	-0.06	0.71	-0.06	1.00						
$\sigma_{iX\mathcal{D}}$	-0.04	0.68	-0.01	0.92	1.00					
β_{iW}	0.91	-0.29	0.57	-0.03	-0.01	1.00				
$\beta_{i\mathcal{D}}$	0.05	0.50	0.05	-0.01	-0.00	0.11	1.00			
$\beta_{iW\mathcal{D}}$	0.02	0.00	0.51	0.02	0.03	-0.33	0.24	1.00		
β_{iX}	-0.03	0.04	-0.01	0.41	0.07	-0.09	-0.35	0.07	1.00	
$\beta_{iX\mathcal{D}}$	0.07	0.02	0.05	0.06	0.42	0.12	-0.14	-0.17	-0.74	1.00

The table shows the correlation matrix of several measures of risk connected to our analysis. At every month $t \geq 12$, we calculate the cross-sectional correlations between the estimated risk measures using daily data from month $t-11$ to t. The reported values are the time-series averages of these cross-sectional correlations over the sample. The sample period is from July, 1963 to December, 2010.

Table 5: Fama-Macbeth regressions on contemporaneous returns

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Cons	0.0037*	0.0028	0.0027	0.0018	0.0027	0.0014	0.0028*	0.0019
	(0.0019)	(0.0018)	(0.0019)	(0.0018)	(0.0019)	(0.0018)	(0.0016)	(0.0016)
λ_W	0.0061***	0.0063***	0.0062***	0.0064***	0.0063***	0.0065***	0.0048***	0.0056***
	(0.0017)	(0.0017)	(0.0016)	(0.0017)	(0.0017)	(0.0017)	(0.0012)	(0.0012)
$\lambda_{\mathcal{D}}$			-0.4338***	-0.4729***		-0.4202***		-0.3283***
			(0.0776)	(0.0847)		(0.0787)		(0.0637)
$\lambda_{W\mathcal{D}}$			0.0064***	0.0055***		0.0043***		0.0034***
			(0.0013)	(0.0009)		(0.0008)		(0.0007)
λ_X		-7.6E-6***		-6.5E-6**		-6.1E-6**		-4.1E-6*
		(2.8E-6)		(2.7E-6)		(2.8E-6)		(2.4E-6)
$\lambda_{X\mathcal{D}}$				-6.6E-6***		-6.2E-6***		-4.8E-6**
				(2.3E-6)		(2.3E-6)		(2.1E-6)
λ_{W^2}					-1.7E-4***	-8.9E-5***		
					(3.6E-5)	(2.9E-5)		
λ_{SMB}							0.0029***	0.0027***
							(0.0011)	(0.0010)
λ_{HML}							-0.0007	-0.0001
							(0.0010)	(0.0006)
λ_{WML}							0.0111***	0.0092***
							(0.0013)	(0.0010)
adj R^2	0.0384	0.0472	0.0508	0.0643	0.0495	0.0698	0.0985	0.1114
Economic magnitudes (annualized %)								
λ_W	5.49	5.74	6.19	6.11	5.78	6.24	4.97	5.35
$\lambda_{\mathcal{D}}$			-6.59	-4.59		-4.10		-3.12
$\lambda_{W\mathcal{D}}$			9.48	4.63		3.62		2.90
λ_X		-2.90		-5.84		-5.61		-3.27
$\lambda_{X\mathcal{D}}$				-7.35		-6.90		-4.62
λ_{W^2}					-3.64	-2.43		
λ_{SMB}							3.51	3.31
λ_{HML}							-1.18	-0.15
λ_{WML}							9.17	7.57

The Table presents results of Fama-MacBeth (1973) regressions. For each month $t \geq 12$ the β-s are calculated using daily data over the previous 12 months (months $t-11$ to t). The dependent variable in the cross-sectional regression for each month t is the average monthly excess return from month $t-11$ to t. The standard errors (in parenthesis) are corrected for 12 Newey-West (1987) lags. Adjusted R^2 of the model is also reported. The sample period is from July, 1963 to December, 2010.

In the lower panel we report the annualized spread $\hat{E}\left[\left(\beta_{75^{th}f,t} - \beta_{25^{th}f,t}\right)\lambda_{f,t}\right]$ between two hypothetical portfolios with different betas on the given factor f, everything else being equal. The first portfolio's beta is the 75th percentile while that of the second is the 25th percentile of the cross-sectional distribution of individual stock betas on factor f.

Table 6: Fama-Macbeth regressions on contemporaneous returns

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
λ_W	0.0087***	0.0082***	0.0081***	0.0076***	0.0082***	0.0075***	0.0064***	0.0066***
	(0.0023)	(0.0021)	(0.0022)	(0.0020)	(0.0022)	(0.0020)	(0.0015)	(0.0015)
$\lambda_{\mathcal{D}}$			-0.5147***	-0.5113***		-0.4494***		-0.3610***
			(0.0986)	(0.1129)		(0.1064)		(0.0771)
$\lambda_{W\mathcal{D}}$			0.0069***	0.0057***		0.0045***		0.0035***
			(0.0015)	(0.0010)		(0.0009)		(0.0007)
λ_X		-7.4E-6**		-5.9E-6*		-5.5E-6		-3.9E-6
		(3.7E-6)		(3.5E-6)		(3.5E-6)		(3.0E-6)
$\lambda_{X\mathcal{D}}$				-5.8E-6*		-5.3E-6*		-4.2E-6
				(3.0E-6)		(2.9E-6)		(2.6E-6)
λ_{W^2}					-1.2E-4***	-7.3E-5**		
					(4.1E-5)	(3.3E-5)		
λ_{SMB}							0.0033***	0.0031***
							(0.0012)	(0.0012)
λ_{HML}							-0.0007	0.0001
							(0.0010)	(0.0007)
λ_{WML}							0.0113***	0.0095***
							(0.0013)	(0.0010)
Economic magnitudes (annualized %)								
λ_W	8.24	7.77	8.14	7.36	7.71	7.21	6.81	6.48
$\lambda_{\mathcal{D}}$			-8.27	-5.01		-4.43		-3.46
$\lambda_{W\mathcal{D}}$			11.19	5.03		3.97		3.10
λ_X		-3.17		-6.36		-6.05		-3.71
$\lambda_{X\mathcal{D}}$				-7.95		-7.40		-5.09
λ_{W^2}					-3.45	-2.37		
λ_{SMB}							4.01	3.83
λ_{HML}							-1.12	0.03
λ_{WML}							9.43	7.79

The Table presents results of Fama-MacBeth (1973) regressions. For each month $t \geq 12$ the β-s are calculated using daily data over the previous 12 months (months $t-11$ to t). The dependent variable in the cross-sectional regression for each month t is the average monthly excess return from month $t-11$ to t. The standard errors (in parenthesis) are corrected for 12 Newey-West (1987) lags. The sample period is from July, 1963 to December, 2010.

In the lower panel we report the annualized spread $\hat{E}\left[\left(\beta_{75^{th} f,t} - \beta_{25^{th} f,t}\right)\lambda_{f,t}\right]$ between two hypothetical portfolios with different betas on the given factor f, everything else being equal. The first portfolio's beta is the 75th percentile while that of the second is the 25th percentile of the cross-sectional distribution of individual stock betas on factor f.

Table 7: Fama-Macbeth regressions with different definitions for the disappointing event

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
a	0	0	1	1	3	3	3	5	5	5
b	0	-0.005	0	-0.005	0	-0.005	-0.007	0	-0.005	-0.007
% of $\mathcal{D}_t$	45.67	21.36	41.36	24.93	35.76	26.67	24.29	34.12	28.13	26.42
Cons	0.0018	0.0019	0.0016	0.0017	0.0018	0.0018	0.0018	0.0018	0.0018	0.0019
	(0.0018)	(0.0018)	(0.0018)	(0.0018)	(0.0018)	(0.0018)	(0.0018)	(0.0018)	(0.0018)	(0.0018)
λ_W	0.0064***	0.0064***	0.0064***	0.0064***	0.0064***	0.0064***	0.0064***	0.0064***	0.0064***	0.0064***
	(0.0016)	(0.0016)	(0.0017)	(0.0017)	(0.0017)	(0.0017)	(0.0017)	(0.0017)	(0.0017)	(0.0017)
$\lambda_{\mathcal{D}}$	-0.3306***	-0.4404***	-0.4812***	-0.5435***	-0.4451***	-0.4729***	-0.4629***	-0.3946***	-0.4139***	-0.4076***
	(0.0745)	(0.0778)	(0.0921)	(0.0947)	(0.0797)	(0.0847)	(0.0812)	(0.0697)	(0.0765)	(0.0768)
$\lambda_{W\mathcal{D}}$	0.0063***	0.0064***	0.0064***	0.0064***	0.0059***	0.0055***	0.0054***	0.0055***	0.0050***	0.0048***
	(0.0012)	(0.0012)	(0.0012)	(0.0011)	(0.0010)	(0.0009)	(0.0009)	(0.0009)	(0.0008)	(0.0008)
λ_X	-7.2E-6***	-7.2E-6***	-6.7E-6**	-6.8E-6**	-6.5E-6**	-6.5E-6**	-6.7E-6**	-6.7E-6**	-6.6E-6**	-6.6E-6**
	(2.8E-6)	(2.7E-6)	(2.8E-6)	(2.8E-6)	(2.7E-6)	(2.7E-6)	(2.7E-6)	(2.7E-6)	(2.7E-6)	(2.7E-6)
$\lambda_{X\mathcal{D}}$	-6.0E-6***	-4.1E-6***	-6.2E-6***	-6.2E-6***	-6.5E-6***	-6.6E-6***	-6.7E-6***	-6.5E-6***	-6.7E-6***	-6.8E-6***
	(1.3E-6)	(8.9E-7)	(2.2E-6)	(2.2E-6)	(2.3E-6)	(2.3E-6)	(2.3E-6)	(2.4E-6)	(2.4E-6)	(2.4E-6)
adj R^2	0.0628	0.0607	0.0645	0.0631	0.0644	0.0643	0.0636	0.0630	0.0630	0.0629
Economic magnitudes (annualized %)										
λ_W	8.02	6.88	7.23	6.69	6.68	6.49	6.46	6.52	6.43	6.38
$\lambda_{\mathcal{D}}$	-2.82	-6.74	-4.13	-6.32	-3.82	-4.69	-4.94	-3.38	-3.87	-3.98
$\lambda_{W\mathcal{D}}$	7.24	9.76	6.26	6.69	4.86	4.62	4.61	4.29	4.04	3.92
λ_X	-3.86	-3.17	-5.93	-4.68	-6.40	-5.59	-5.56	-6.70	-6.03	-5.79
$\lambda_{X\mathcal{D}}$	-4.36	-3.60	-6.56	-6.33	-7.68	-7.55	-7.63	-7.84	-7.80	-7.86

The Table presents results of Fama-MacBeth (1973) regressions. For each month t the realized β-s are calculated using daily data over the previous 12 months (months $t-11$ to t). The dependent variable in the cross-sectional regression for each month t is the average monthly excess return over the same period (previous 12 months: $t-11$ to t). The standard errors (in parenthesis) are corrected for 12 Newey-West (1987) lags. The last row reports adjusted R^2 of given the model. The sample period is from July, 1963 to December, 2010.
Each column uses a different definition for the disappointing event $I(\mathcal{D}_t)$. The disappointing region is defined as

$$r_{W,t+1} - a\,(\sigma_W/\sigma_X)\,\Delta\sigma^2_{W,t+1} < b.$$

The values of a and b vary throughout the different specifications. "% of $\mathcal{D}_t$" denotes the average percentage of days when the disappointing event occurs in a one-year period.

Table 8: Fama-Macbeth regressions with different measures of market volatility

Panel A: 1964/07-2010/12					
	VIX	RV	GARCH	EGARCH	GJR-GARCH
Cons			0.0024	0.0018	0.0018
			(0.0019)	(0.0018)	(0.0018)
λ_W			0.0064***	0.0064***	0.0065***
			(0.0017)	(0.0017)	(0.0017)
$\lambda_{\mathcal{D}}$			-0.4803***	-0.4729***	-0.4951***
			(0.0642)	(0.0847)	(0.0772)
$\lambda_{W\mathcal{D}}$			0.0051***	0.0055***	0.0054***
			(0.0009)	(0.0009)	(0.0009)
λ_X			-6.6E-6***	-6.5E-6**	-7.1E-6***
			(1.6E-6)	(2.7E-6)	(2.6E-6)
$\lambda_{X\mathcal{D}}$			-4.9E-6***	-6.6E-6***	-6.4E-6***
			(1.3E-6)	(2.3E-6)	(2.3E-6)
adj R^2			0.0565	0.0643	0.0632
Economic magnitudes (annualized %)					
λ_W			6.02	6.11	6.13
$\lambda_{\mathcal{D}}$			-4.47	-4.59	-4.89
$\lambda_{W\mathcal{D}}$			4.35	4.63	4.65
λ_X			-5.67	-5.84	-6.96
$\lambda_{X\mathcal{D}}$			-5.36	-7.35	-7.64
Panel B: 1987/01-2010/09					
	VIX	RV	GARCH	EGARCH	GJR-GARCH
Cons	0.0033	0.0035	0.0035	0.0027	0.0028
	(0.0027)	(0.0028)	(0.0027)	(0.0027)	(0.0027)
λ_W	0.0066***	0.0066**	0.0068***	0.0068***	0.0068***
	(0.0025)	(0.0026)	(0.0026)	(0.0025)	(0.0025)
$\lambda_{\mathcal{D}}$	-0.2061***	-0.2769***	-0.3058***	-0.4819***	-0.4716***
	(0.0719)	(0.0836)	(0.0686)	(0.1097)	(0.0986)
$\lambda_{W\mathcal{D}}$	0.0061***	0.0049***	0.0052***	0.0055***	0.0055***
	(0.0019)	(0.0016)	(0.0013)	(0.0014)	(0.0014)
λ_X	-5.1E-5	-2.6E-5	-5.2E-6*	-6.7E-6	-7.3E-6
	(3.5E-5)	(2.5E-5)	(2.9E-6)	(5.0E-6)	(4.8E-6)
$\lambda_{X\mathcal{D}}$	-6.8E-5*	-3.2E-5	-3.4E-6	-6.8E-6	-6.1E-6
	(4.0E-5)	(2.2E-5)	(2.2E-6)	(4.2E-6)	(4.2E-6)
adj R^2	0.0424	0.0374	0.0389	0.0474	0.0462
Economic magnitudes (annualized %)					
λ_W	7.27	6.45	5.96	6.13	6.08
$\lambda_{\mathcal{D}}$	-2.10	-2.82	-3.48	-5.30	-5.30
$\lambda_{W\mathcal{D}}$	7.21	4.06	4.39	4.56	4.68
λ_X	-2.07	-2.89	-3.74	-5.90	-6.27
$\lambda_{X\mathcal{D}}$	-4.58	-4.54	-3.17	-7.18	-6.82

The Table presents results of Fama-MacBeth (1973) regressions using different approaches to measure market volatility. Appendix A describes the different approaches. For each month t the realized β-s are calculated using daily data over the previous 12 months (months $t-11$ to t). The dependent variable in the cross-sectional regression for each month t is the average monthly excess return over the same period (previous 12 months: $t-11$ to t). The standard errors (in parenthesis) are corrected for 12 Newey-West (1987) lags.

Table 9: Fama-Macbeth regressions using future returns

	R_{t+1}			$R_{t+1,t+3}$			$R_{t+1,t+6}$		
λ_W	0.0054*	0.0050*	0.0051*	0.0053**	0.0050**	0.0050**	0.0053**	0.0050**	0.0050**
	(0.0028)	(0.0027)	(0.0026)	(0.0025)	(0.0024)	(0.0024)	(0.0023)	(0.0022)	(0.0022)
$\lambda_{\mathcal{D}}$		-0.1845			-0.2018*			-0.2147**	
		(0.1171)			(0.1109)			(0.1047)	
$\lambda_{W\mathcal{D}}$		0.0042**			0.0041**			0.0042**	
		(0.0020)			(0.0019)			(0.0018)	
λ_X			-1.4E-5***			-1.4E-5***			-1.4E-5***
			(4.7E-6)			(5.0E-6)			(5.2E-6)
λ_W	0.0047*	0.0046*	0.0037*	0.0046**	0.0045**	0.0033*	0.0046**	0.0045**	0.0031*
	(0.0025)	(0.0025)	(0.0021)	(0.0023)	(0.0023)	(0.0018)	(0.0021)	(0.0021)	(0.0017)
$\lambda_{\mathcal{D}}$	-0.3528***	-0.3265***	-0.2221***	-0.3804***	-0.3598***	-0.2152***	-0.3905***	-0.3718***	-0.2043***
	(0.1265)	(0.1183)	(0.0836)	(0.1198)	(0.1097)	(0.0792)	(0.1186)	(0.1065)	(0.0769)
$\lambda_{W\mathcal{D}}$	0.0025*	0.0023**	0.0021**	0.0025**	0.0024**	0.0019**	0.0026**	0.0024***	0.0018**
	(0.0013)	(0.0011)	(0.0010)	(0.0012)	(0.0010)	(0.0009)	(0.0010)	(0.0008)	(0.0007)
λ_X	-1.2E-5***	-1.2E-5***	-7.2E-6**	-1.3E-5***	-1.3E-5***	-7.4E-6**	-1.3E-5**	-1.3E-5**	-7.7E-6**
	(4.3E-6)	(4.4E-6)	(3.1E-6)	(4.7E-6)	(4.8E-6)	(3.4E-6)	(5.0E-6)	(5.1E-6)	(3.9E-6)
$\lambda_{X\mathcal{D}}$	-1.3E-5***	-1.3E-5***	-9.3E-6***	-1.3E-5***	-1.3E-5***	-8.7E-6***	-1.4E-5***	-1.3E-5***	-9.2E-6**
	(4.1E-6)	(3.9E-6)	(2.9E-6)	(4.6E-6)	(4.4E-6)	(3.4E-6)	(4.9E-6)	(4.7E-6)	(4.0E-6)
λ_{W^2}		-5.8E-5			-5.3E-5			-8.3E-5	
		(5.9E-5)			(6.2E-5)			(6.2E-5)	
λ_{SMB}			0.0013			0.0018**			0.0021**
			(0.0010)			(0.0009)			(0.0009)
λ_{HML}			0.0018***			0.0019***			0.0019***
			(0.0005)			(0.0005)			(0.0004)
λ_{WML}			-0.0016			-0.0019**			-0.0022***
			(0.0010)			(0.0009)			(0.0008)

The Table presents results of Fama-MacBeth regressions. For each month t the realized β-s are calculated using daily data over the previous 12 months (months $t-11$ to t). The dependent variable in the cross-sectional regression for each month t is the average monthly excess return over the **next month** ($t+1$, labelled "R_{t+1}"), **next 3 months** ($t+1$ to $t+3$, labelled "$R_{t+1,t+3}$"), and **next 6 months** ($t+1$ to $t+6$, labelled "$R_{t+1,t+6}$"). The standard errors (in parenthesis) are corrected for 12 Newey-West lags.

Table 10: Fama-Macbeth regressions on portfolios

		25 Size - B/M	25 Size - Mom	25 Size - LRev	30 Industry	10 Size, 10 B/M, 10 Mom
CAPM	λ_W	0.0064***	0.0068***	0.0075***	0.0054***	0.0052***
		(0.0019)	(0.0019)	(0.0019)	(0.0017)	(0.0017)
	SSE	0.0017	0.0022	0.0017	0.0041	0.0015
GDA3	λ_W	0.0052***	0.0053***	0.0062***	0.0052***	0.0047***
		(0.0017)	(0.0017)	(0.0018)	(0.0017)	(0.0017)
	$\lambda_{\mathcal{D}}$	-0.7783**	-0.9482***	-1.0619***	-0.5945**	-0.8828***
		(0.3218)	(0.3034)	(0.2975)	(0.2438)	(0.2409)
	$\lambda_{W\mathcal{D}}$	0.0120***	0.0172***	0.0156***	0.0085***	0.0101***
		(0.0037)	(0.0039)	(0.0035)	(0.0022)	(0.0036)
	SSE	0.0010	0.0013	0.0010	0.0033	0.0009
GDA5	λ_W	0.0047***	0.0048***	0.0054***	0.0049***	0.0046***
		(0.0016)	(0.0017)	(0.0017)	(0.0016)	(0.0016)
	$\lambda_{\mathcal{D}}$	-0.7255**	-1.1974***	-0.8148***	-0.7842***	-0.6165**
		(0.3358)	(0.3146)	(0.2832)	(0.2435)	(0.2690)
	$\lambda_{W\mathcal{D}}$	0.0072**	0.0108***	0.0084***	0.0088***	0.0089***
		(0.0034)	(0.0035)	(0.0032)	(0.0024)	(0.0028)
	λ_X	-1.1E-5	-3.0E-5***	-2.1E-5***	-1.8E-5***	-1.6E-5*
		(8.2E-6)	(1.1E-5)	(7.7E-6)	(6.6E-6)	(9.5E-6)
	$\lambda_{X\mathcal{D}}$	-1.0E-5**	-2.1E-5***	-1.9E-5***	-3.6E-6	-9.1E-6*
		(5.1E-6)	(7.4E-6)	(6.2E-6)	(6.5E-6)	(5.4E-6)
	SSE	0.0007	0.0009	0.0007	0.0027	0.0007
Four-factor	λ_W	0.0043***	0.0051***	0.0051***	0.0053***	0.0046***
		(0.0016)	(0.0017)	(0.0017)	(0.0016)	(0.0016)
	λ_{SMB}	0.0020	0.0025*	0.0033***	0.0007	0.0012
		(0.0013)	(0.0013)	(0.0012)	(0.0013)	(0.0013)
	λ_{HML}	0.0037***	0.0030	0.0032**	0.0002	0.0020*
		(0.0012)	(0.0019)	(0.0016)	(0.0010)	(0.0011)
	λ_{WML}	0.0131***	0.0064***	0.0080***	0.0189***	0.0050***
		(0.0022)	(0.0017)	(0.0018)	(0.0014)	(0.0015)
	SSE	0.0005	0.0007	0.0006	0.0021	0.0005

The Table presents results of Fama-MacBeth regressions. **The base assets are portfolios.** For each month t the realized β-s are calculated using daily data over the previous 12 months (months $t-11$ to t). The dependent variable in the cross-sectional regression for each month t is the average monthly excess return over the same period (previous 12 months - $t-11$ to t). The standard errors (in parenthesis) are corrected for 12 Newey-West (1987) lags. The row labelled "SSE" presents the average sum of squared pricing errors for the given model. The sample period is from July, 1963 to December, 2010.

Table 11: Decomposing the excess return of portfolios

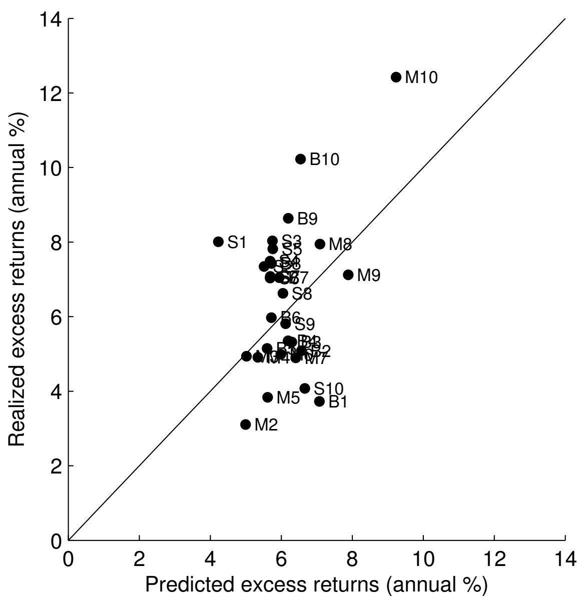
		Size			Book-to-Market			Momentum		
		S1	S10	S10-S1	B1	B10	B10-B1	M1	M10	M10-M1
	Actual Return	8.01	4.07	**-3.93**	3.73	10.22	**6.50**	-2.02	12.42	**14.45**
CAPM	λ_W	4.23	6.66	**2.44**	7.07	6.54	**-0.53**	5.15	9.24	**4.08**
	predicted	4.23	6.66	**2.44**	7.07	6.54	**-0.53**	5.15	9.24	**4.08**
	unexplained	3.78	-2.59		-3.35	3.68		-7.18	3.19	
Four-factor	λ_W	4.58	5.34	**0.76**	5.47	6.40	**0.92**	6.66	6.21	**-0.44**
	λ_{SMB}	1.51	-0.24	**-1.75**	-0.23	1.10	**1.33**	-0.71	0.99	**1.70**
	λ_{HML}	0.66	-0.29	**-0.94**	-1.33	2.15	**3.48**	-0.35	-0.14	**0.21**
	λ_{WML}	-0.08	-0.23	**-0.16**	0.09	-0.06	**-0.15**	-4.77	4.27	**9.04**
	predicted	6.67	4.59	**-2.09**	4.00	9.59	**5.59**	0.84	11.34	**10.51**
	unexplained	1.33	-0.51		-0.27	0.63		-2.86	1.08	
GDA3	λ_W	2.86	6.31	**3.45**	6.50	5.64	**-0.87**	4.22	8.02	**3.81**
	$\lambda_{\mathcal{D}}$	-0.84	0.26	**1.10**	0.60	0.18	**-0.43**	-0.28	0.19	**0.48**
	$\lambda_{W\mathcal{D}}$	5.00	-1.58	**-6.58**	-1.94	1.35	**3.28**	-1.00	2.11	**3.10**
	predicted	7.02	4.99	**-2.03**	5.17	7.16	**1.99**	2.94	10.32	**7.39**
	unexplained	0.98	-0.92		-1.44	3.06		-4.96	2.10	
GDA5	λ_W	2.98	6.12	**3.14**	6.31	5.40	**-0.91**	4.04	7.88	**3.84**
	$\lambda_{\mathcal{D}}$	0.20	0.08	**-0.13**	-0.41	0.35	**0.76**	-0.10	-0.04	**0.07**
	$\lambda_{W\mathcal{D}}$	2.07	-0.74	**-2.81**	-0.89	1.37	**2.27**	-1.23	1.78	**3.00**
	λ_X	-0.68	0.49	**1.17**	0.91	0.36	**-0.55**	-0.58	0.24	**0.82**
	$\lambda_{X\mathcal{D}}$	2.79	-1.25	**-4.04**	-1.14	0.96	**2.10**	-0.23	1.10	**1.32**
	predicted	7.37	4.69	**-2.67**	4.76	8.43	**3.67**	1.90	10.95	**9.05**
	unexplained	0.64	-0.62		-1.04	1.79		-3.93	1.47	

The Table shows the actual average excess returns of size (S1 and S10), book-to-market (B1 and B10) and momentum (M1 and M10) portfolios, as well as their parts that are predicted and unexplained by the CAPM, the four-factor model and the GDA models, as well as the decomposition of the predicted premium into parts attributable to the different factors.

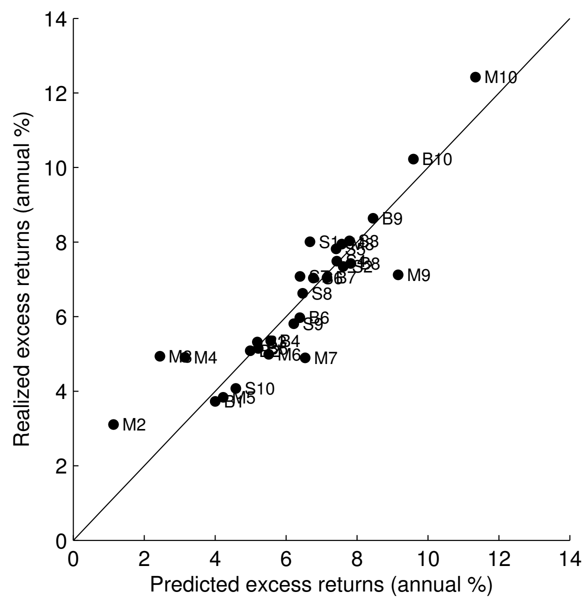
Figure 1: Actual versus predicted returns of portfolios

10 Size, 10 Book-to-Market, and 10 Momentum portfolios

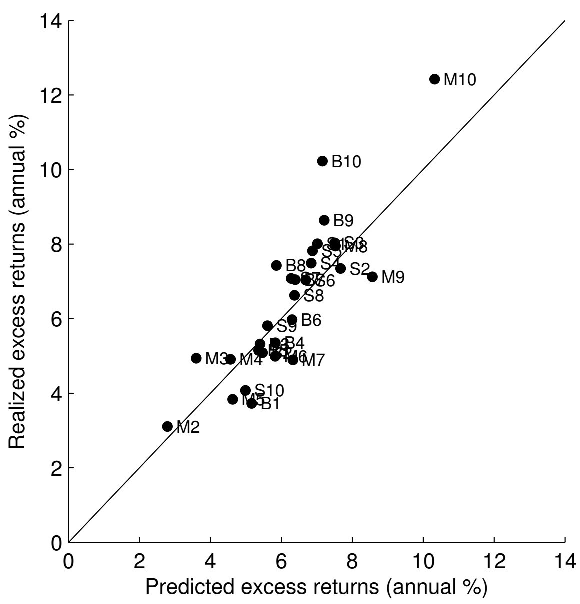
(a) CAPM



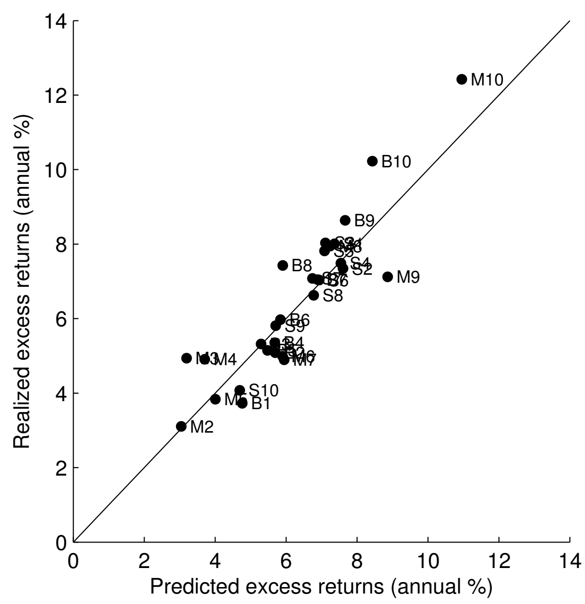
(b) Four-factor



(c) GDA3

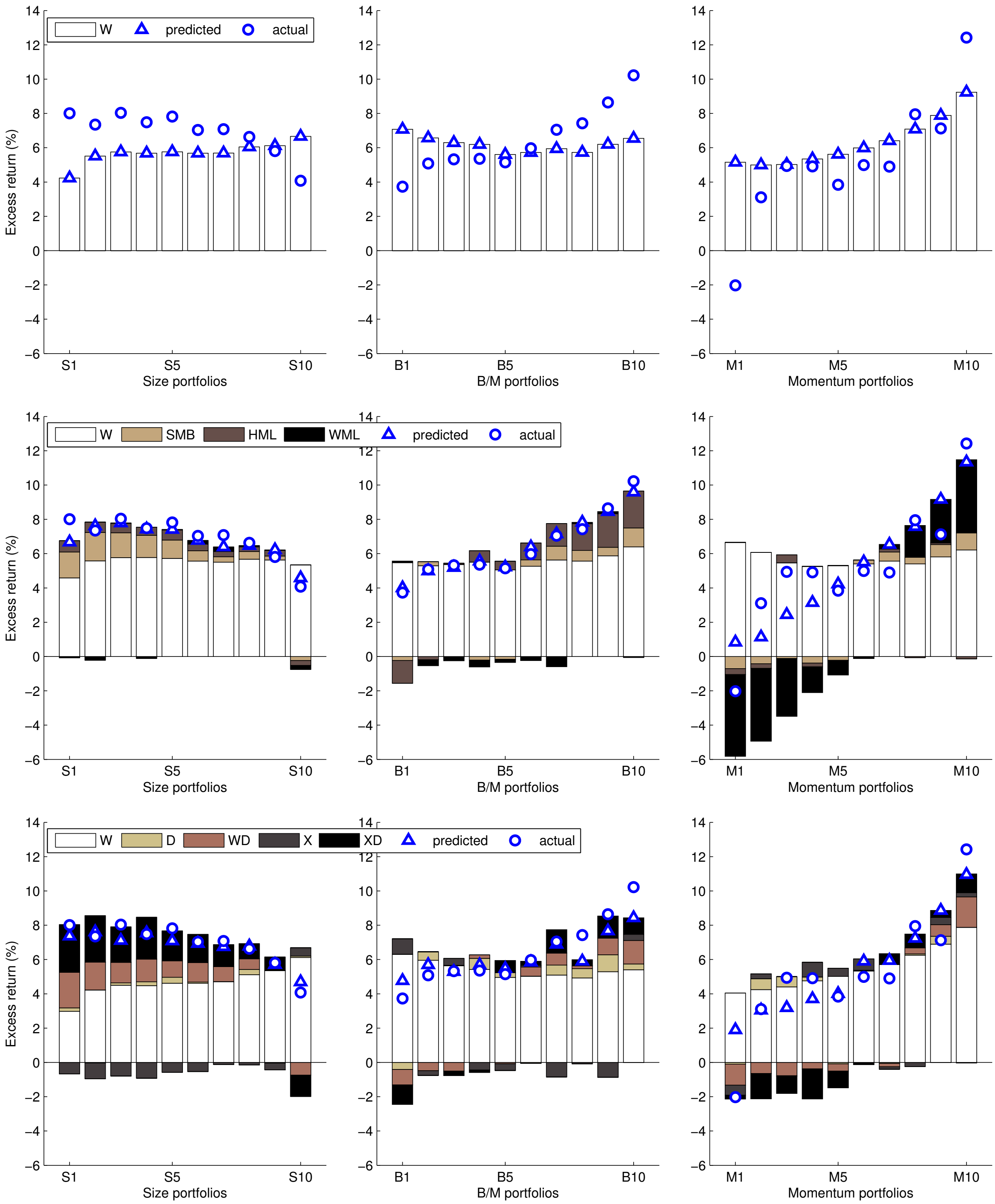


(d) GDA5



This figure shows the realized average excess returns for the 10 size (S1 to S10), 10 book-to-market (B1 to B10), 10 momentum (M1 to M10) portfolios, against the predicted average excess returns from models reported in Table 10.

Figure 2: Returns on the 10 Size, 10 Book-to-Market and 10 Momentum portfolios



This figure shows the decomposition of the predicted average excess return of 10 Size (left column), 10 B/M (middle column), and 10 momentum (right column) portfolios. Each part represents $E\left[\beta_{if,t}\lambda_{f,t}\right]$ connected to factor f from the standard CAPM in the top row, the four-factor model in the middle row, and the GDA5 model bottom row. The symbol $\triangle$ represents predicted average excess return (sum of the parts), while $\circ$ represents the actual average excess return of the portfolios.