

## **Roll-Over Parameters and Option Pricing**

Sol Kim\*

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For S&P 500 options, we examine the empirical performance of several options pricing models with respect to the roll-over strategies of parameters. We compare pricing and hedging performances of several models using the traditional roll-over strategy of the parameters, which is the nearest-to-next approach, with those using the new roll-over strategy, the next-to-next approach. It is found that the next-to-next strategy decreases pricing and hedging errors of all options pricing models compared to the nearest-to-next approach. Pricing errors of AHBS-type models are decreased larger than the mathematically complicated models (the stochastic volatility and the stochastic volatility with jumps models) by the next-to-next strategy. Hence, next-to-next strategy can mitigate over-fitting problems of AHBS-type models. However, there is not much improvement of the next-to-next strategy for mathematically complicated models.

EFM Classification Codes: 410

Keywords: Option Pricing; Volatility Smiles; Black and Scholes; Traders' rules; Stochastic Volatility; Jumps

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For S&P 500 options, we examine the empirical performance of several options pricing models with respect to the roll-over strategies of parameters. We compare pricing and hedging performances of several models using the traditional roll-over strategy of the parameters, which is the nearest-to-next approach, with those using the new roll-over strategy, the next-to-next approach. It is found that the next-to-next strategy decreases pricing and hedging errors of all options pricing models compared to the nearest-to-next approach. Pricing errors of AHBS-type models are decreased larger than the mathematically complicated models (the stochastic volatility and the stochastic volatility with jumps models) by the next-to-next strategy. Hence, next-to-next strategy can mitigate over-fitting problems of AHBS-type models. However, there is not much improvement of the next-to-next strategy for mathematically complicated models.

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## I. Introduction

Ever since Black and Scholes (1973) published their seminal article on option pricing in 1973, various theoretical and empirical researches have been conducted on option pricing. One important direction in which the Black and Scholes (1973) model can be modified is to generalize the geometric Brownian motion, which is used as a process for the dynamics of log stock prices. For example, Merton(1976) and Naik and Lee (1990) propose a jump-diffusion model. Hull and White (1987), Johnson and Shanno (1987), Scott (1987), Wiggins (1987) and Heston (1993) suggest a stochastic volatility model. Naik (1993) considers a regime-switching model that assumes jumps of the volatility. Duan (1995) and Heston and Nandi (2000) develop an option pricing framework based on the GARCH process. Madan, Carr and Chang (1998) use the Variance Gamma process as an alternative model for capturing the dynamics of log stock prices.

Bakshi, Cao and Chen (1997, 2000) and Kim (2005) have conducted a comprehensive empirical study on the relative merits of competing option pricing models. They have discovered that taking the stochastic volatility into account is of the first order in importance for improving the Black and Sholes (1973) model. However, among the striking empirical findings, Dumas, Fleming and Whaley (1998), Jackwerth and Rubinstein (2001) and Li and Pearson (2007) and Kim (2009) examine the performance of a number of these mathematically sophisticated models and find that they predict option prices less well than a pair of ad hoc approaches sometimes used by option traders. Ad hoc approaches can be an alternative to the complicated models for pricing options; they are known as ad hoc Black and Scholes models (henceforth AHBS).

There are two versions of the AHBS models. In the “relative smile” approach, the implied volatility skew is treated as a fixed function of moneyness,  $S/K$ , whereas the implied volatility for a fixed strike  $K$  varies as the stock index  $S$  varies. This is also known as the “sticky volatility”

method. In the “absolute smile” approach, the implied volatility is treated as a fixed function of the strike price  $K$ , and the implied volatility for a fixed strike does not vary with  $S$ . This is also known as the “sticky delta” method. Dumas, Fleming and Whaley (1998), Jackwerth and Rubinstein (2001) and Li and Pearson (2007) adopt the “absolute smile” approach. On the other hand, Kirgiz (2001) and Kim and Kim (2004) adopt the “relative smile” approach. That is, the type of the AHBS model seems to be important for pricing and hedging options. Jackwerth and Rubinstein (2001), Li and Pearson (2007), Kim (2009), and Choi and Ok (2011) have found that the “absolute smile” approach shows better performance than the “relative smile” approach for pricing options. Further, a simpler model among the AHBS models shows better performance than the other models. That is, the presence of more parameters in AHBS models actually cause over-fitting.

When the options are priced and hedged, we need to estimate the parameters that are needed to plug into each model. For one day ahead pricing and hedging, the parameters are estimated using the previous days’ options data. For one week ahead pricing and hedging, the options data before seven days are used. In general trading dates, there are no complicated problems. However, it is standard to eliminate the nearest option contracts with expiries less than 7 days, as well as to use the next-to-nearest option contracts with expiries less than 7 days plus 1 month for the empirical study. When forecasting the parameters for the next-to-nearest option contracts with expiry less than 7 days plus 1 month, we have the problem of the roll-over strategies of the parameter. One can use either the nearest contracts with expiry greater than 6 days (the nearest-to-next roll-over strategy) or the next-to-nearest contracts with expiry greater than 6 days plus 1 month (the next-to-next roll-over strategy). Recently, Choi and Ok (2011) have shown that the next-to-next roll-over strategy can mitigate the over-fitting problems of AHBS models and can make AHBS models with more parameters to become the better model than the AHBS model with less parameters. As a result, the next-to-next roll-over strategy of the parameters can be useful for the AHBS type models.

However, is the next-to-next roll-over strategy functioned only with the AHBS models? Or can this strategy also be functioned with mathematically complicated models, stochastic volatility (henceforth SV) and stochastic volatility with jump (henceforth SVJ) models? In this paper, we examine the empirical performance of several options pricing models with respect to the roll-over strategies of parameters. Not only the traders' rules, that is, AHBS-type models, but also the SV model and the SVJ model are considered for a horse race competition. We compare the pricing and hedging performances of several option pricing models using the traditional roll-over strategy of the parameters, which is the nearest-to-next approach, with those using the new roll-over strategy, the next-to-next approach. We examine whether the new roll-over strategy of the parameters can be functioned not only for the AHBS-type models but also for the mathematically complicated models, the SV and SVJ models. After considering the new roll-over strategy of the parameters, we try to find out the best options pricing model.

We fill the gaps that have not been resolved in previous researches. First, when the roll-over strategies of the parameters are examined, Choi and Ok (2011) and Choi, Jordan, and Ok (2012) do not consider the mathematically complicated models that are shown to be competitive options pricing models. In this paper, we examine whether the new roll-over strategy of the parameters for the SV and the SVJ models is functioned. Second, in previous researches, the new roll-over strategy of the parameters, or the next-to-next strategy, is not considered for hedging performance. When we try to find out the best options pricing model, both pricing and hedging performance must be considered. Pricing performance of the options pricing models measures the ability to forecast the level of options price; however, hedging performance measures the ability to forecast the variability of options prices. If a specific model shows better performance than the other models for both performance measures, that model can truly be the best options pricing model. Third, Choi and Ok (2011) and Choi, Jordan and Ok (2012) consider the sample period with a span of just two years. For two years, the dates that require the roll-over of the parameters are twenty four days when we examine the one day ahead out-of-sample

pricing and hedging performance. The effect of the roll-over strategy can be exaggerated due to a small sample. In this paper, we examine the roll-over strategies using sample dates with a span of 13 years. If the new roll-over strategy works well, even for a long sample period, we can conjecture that there is a structural change of the parameters when the maturity of options is rolled-over. Fourth, recent researches that examine the performance of AHBS-type models have considered KOSPI 200 options, which is one of the emerging markets. Although KOSPI 200 options are the biggest derivatives products in the world, in terms of trading volume, these products are traded in the emerging market. We use S&P 500 (SPX) option prices for our empirical work. S&P 500 options have been the focus of many existing investigations including, among others, Bakshi, Cao and Chan (1997), Bates (1996), Dumas, Fleming and Whaley (1995). Also, the roll-over strategies of the parameter are not compared in the SPX options market. If the new roll-over strategy is well functioned for SPX options, we can conjecture that it is not only fit to the emerging markets, but can also be generally applied to the advanced options markets.

It is found that when we use the traditional method, the nearest-to-next strategy, the SVJ and SV models show better performance compared to the AHBS-type models for pricing options. Among AHBS-type models, a simpler model with less parameters shows better performance than other models. That is, for AHBS-type models, the presence of more parameters actually cause over-fitting, whereas it does not cause such over-fitting problem for mathematically complicated models. For hedging performance, AHBS-type models show better performance compared to the mathematically complicated models; however, the differences among the models are not significant. When we use the new method, the next-to-next strategy, the results differ. The next-to-next strategy decreases the pricing and hedging errors of all options pricing models. Pricing errors of AHBS-type models are decreased largely by the next-to-next strategy. AHBS-type models show better performance than the mathematically complicated models for pricing options. That is, the next-to-next strategy can mitigate over-fitting problem of AHBS-

type models. On the other hand, the improvement of mathematically complicated models using the next-to-next strategy is not much. For hedging performance, the next-to-next strategy also decreases the errors of all options pricing models; yet, the difference between the results using the nearest-to-next strategy and those using the next-to-next strategy are not much. The next-to-next strategy can mitigate over-fitting problems of AHBS-type models. However, there is no drastic improvement of the next-to-next strategy for mathematically complicated models.

The outline of this paper is as follows. The AHBS models, the stochastic volatility with jumps model and the roll-over strategies of the parameters are reviewed in Section 2. The data used for the analysis are described in Section 3. Section 4 describes the parameter estimates of each model and evaluates pricing and hedging performances of alternative models. Section 5 concludes our study by summarizing the results.

## II. Options Pricing Models

### 1. Ad Hoc Black-Scholes Model

Despite its significant pricing and hedging biases, the Black and Scholes (1973) model (henceforth the BS model) continues to be widely used by market practitioners. However, when practitioners apply the BS model, they commonly allow the volatility parameter to vary across strike prices of options as well as to fit the volatility to the observed smile pattern. As Dumas, Fleming and Whaley (1998) show, this procedure can avoid some of the biases associated with the BS model's constant volatility assumption.

We have to construct the AHBS model in which each option has its own implied volatility depending on a strike price and the time to maturity. Specifically, the spot volatility of the asset that enters the BS model is a function of the strike price and the time to maturity or a combination of both. However, we only consider the function of the strike price because the liquidity of the index options market is concentrated in the nearest expiration contract. Dumas,

Fleming and Whaley (1998) show that the specification that includes a time parameter performs worst of all, indicating that the time variable is an important cause of the over-fitting problem at the estimation stage.

There are two versions of the ad hoc approach. In the “relative smile” approach, the implied volatility skew is treated as a fixed function of moneyness,  $S/K$ , and the implied volatility for a fixed strike  $K$  varies as the stock index  $S$  varies. This is also known as the “sticky volatility” method. In the “absolute smile” approach, the implied volatility is treated as a fixed function of the strike price  $K$ , and the implied volatility for a fixed strike does not vary with  $S$ . This is also known as the “sticky delta” method. These models are so called the ad hoc Black-Scholes model (henceforth AHBS). Dumas, Fleming and Whaley (1998), Jackwerth and Rubinstein (2001) and Li and Pearson (2007), Kim (2009) and Choi and Ok (2011), Choi, Jordan and Ok (2012), who report that the AHBS model outperforms other options pricing models, adopt the “absolute smile” approach. On the other hand, Kirgiz (2001) and Kim and Kim (2004), who report the AHBS model does not outperform others, adopt the “relative smile” approach. That is, the specific type of the AHBS model seems to be important for pricing and hedging performances.

Specifically, we adopt the following six specifications for the BS implied volatilities:

$$R1: \sigma_i = \beta_1 + \beta_2 \cdot (S/K_i) \quad (1)$$

$$R2: \sigma_i = \beta_1 + \beta_2 \cdot (S/K_i) + \beta_3 \cdot (S/K_i)^2 \quad (2)$$

$$R3: \sigma_i = \beta_1 + \beta_2 \cdot (S/K_i) + \beta_3 \cdot (S/K_i)^2 + \beta_4 \cdot (S/K_i)^3 \quad (3)$$

$$A1: \sigma_i = \beta_1 + \beta_2 \cdot K_i \quad (4)$$

$$A2: \sigma_i = \beta_1 + \beta_2 \cdot K_i + \beta_3 \cdot K_i^2 \quad (5)$$

$$A3: \sigma_i = \beta_1 + \beta_2 \cdot K_i + \beta_3 \cdot K_i^2 + \beta_4 \cdot K_i^3 \quad (6)$$

where  $\sigma_i$  is the implied volatility for an  $i$ th option of strike  $K_i$  and spot price  $S$ .

R1, R2 and R3 models are the “relative smile” approaches using the “relative” moneyness as the independent variables. A1, A2 and A3 models are the “absolute smile” approaches using the “absolute” strike prices as the independent variables. R1 is the ad hoc Black-Scholes model that considers the intercept and the moneyness as the independent variables. R2 is the ad hoc Black-Scholes model that considers the intercept, the moneyness and the square of the moneyness as the independent variables. R3 is the ad hoc Black-Scholes model that considers the intercept, the moneyness, the square and the third power of the moneyness as the independent variables. A1 is the ad hoc Black-Scholes model that considers the intercept and the strike price as the independent variables. A2 is the ad hoc Black-Scholes model that considers the intercept, the strike price and the square of the strike price as the independent variables. A3 is the ad hoc Black-Scholes model that considers the intercept, the strike price, and the square and the third power of the strike price as the independent variables. Up to now, previous studies do not consider the third power of the moneyness and the strike price. In this paper, the performances of the AHBS models with higher degrees are examined.

For implementation, we follow a four-step procedure. First, we abstract the BS implied volatilities from each option. Second, we set up the implied volatilities as the dependent variable and the moneyness or the strike price as the independent variables. We also estimate the  $\beta_i (i = 1, 2, 3, 4)$  by ordinary least squares. Third, using the estimated parameters from the second step, we plug each option’s moneyness or the strike price into the equation, and obtain the model-implied volatility for each option. Finally, we use volatility estimates computed in the third step in order to price options with the following BS formula.

$$C_{t,\tau,K} = S_t N(d_1) - K e^{-r\tau} N(d_2) \quad (7)$$

$$P_{t,\tau,K} = K e^{-r\tau} N(-d_2) - S_t N(-d_1) \quad (8)$$

$$d_1 = \frac{\ln[S_t / K] + (r + \sigma^2 / 2)\tau}{\sigma\sqrt{\tau}}, \quad d_2 = d_1 - \sigma\sqrt{\tau} \quad (9)$$

where  $N(\cdot)$  is the cumulative standard normal density. The AHBS model, although theoretically inconsistent, can be a more challenging benchmark than the simple BS model for any competing options valuation model.

## 2. Stochastic Volatility with Jumps Model

Bakshi, Cao and Chan (1997) derived a closed-form option pricing model that incorporates both stochastic volatility and random jumps. Under the risk neutral measure, the underlying non-dividend-paying stock price  $S_t$  and its components for any time  $t$  are given by

$$\frac{dS_t}{S_t} = [R_t - \lambda\mu_J]dt + \sqrt{V_t}dZ_{S,t} + J_t dq_t \quad (10)$$

$$dV_t = [\theta_v - K_v V_t]dt + \sigma_v \sqrt{V_t} dZ_{v,t} \quad (11)$$

$$\ln[1 + J_t] \sim N(\ln[1 + \mu_J] - 1/2\sigma_J^2, \sigma_J^2) \quad (12)$$

where  $R_t$  is the instantaneous spot interest rate at time  $t$ ,  $\lambda$  is the frequency of jumps per year and  $V_t$  is the diffusion component of return variance (conditional on no jump occurring).

$Z_{S,t}$  and  $Z_{v,t}$  are standard Brownian motions, with  $Cov_t[dZ_{S,t}, dZ_{v,t}] = \rho dt$ .  $J_t$  is the percentage jump size (conditional on a jump occurring) that is lognormally, identically and independently distributed over time, with unconditional mean,  $\mu_J$ . The standard deviation of  $\ln[1 + J_t]$  is  $\sigma_J$ .  $q_t$  is a Poisson jump counter with intensity  $\lambda$ , that is,  $\Pr[dq_t = 1] = \lambda dt$  and  $\Pr[dq(t) = 0] = 1 - \lambda dt$ .  $\kappa_v$ ,  $\theta_v / \kappa_v$  and  $\sigma_v$  are the speed of adjustment, long-run mean and variation coefficient of the diffusion volatility  $V_t$ , respectively.  $q_t$  and  $J_t$  are uncorrelated with each other or with  $Z_{S,t}$  and  $Z_{v,t}$ .

For a European call option written on the stock with strike price  $K$  and time to maturity  $\tau$ , the closed form formula for price  $C_{t,\tau}$  at time  $t$  is as follows.

$$C_{t,\tau} = S_t P_1(t, \tau; S, R, V) - K e^{-R\tau} P_2(t, \tau; S, R, V) \quad (13)$$

where the risk neutral probabilities,  $P_1$  and  $P_2$ , are computed from inverting the respective characteristic functions of the following:

$$\begin{aligned} & P_j(t, \tau; S_t, R_t, V_t) \\ &= \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \text{Re} \left[ \frac{\exp(-i\phi \ln K) f_j(t, \tau, S_t, R_t, V_t; \phi)}{i\phi} \right] \quad (j=1,2) \end{aligned} \quad (14)$$

The characteristic functions,  $f_j$ , are given in equations (A-1) and (A-2) of the Appendix. The price of a European put on the same stock can be determined from the put-call parity.

The option valuation model in equation (13) and (14) contains the most existing models as special cases. For example, we obtain (i) the BS model by setting  $\lambda = 0$  and  $\theta_v = \kappa_v = \sigma_v = 0$ ; we obtain (ii) the SV model by setting  $\lambda = 0$ , where in order to derive each special case from equation (14), one may need to apply L'Hopital's rule.

In applying the option pricing models, one always encounters the difficulty where spot volatilities and structural parameters are unobservable. As estimated in standard practice, we estimate the parameters of each model for every sample day. Since closed-form solutions are available for an option price, a natural candidate for the estimation of parameters in the formula is a non-linear least squares procedure, involving a minimization of the sum of percentage squared errors between the model and the market prices.<sup>1</sup> Estimating the parameters from the

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<sup>1</sup> Conventionally, the objective function to minimize the sum of squared errors is used.

asset returns can be an alternative method; however, historical data reflect only what happened in the past. Furthermore, the procedure using historical data is not capable of identifying risk premiums, which must be estimated from the options data conditional on the estimates of other parameters. The important advantage of using option prices to estimate parameters is to allow one to use the forward-looking information contained in the option prices.

Let  $O_{i,t}^*$  denote the model price of option  $i$  on day  $t$ , and  $O_{i,t}$  denote the market price of option  $i$  on day  $t$ . To estimate parameters for each model, we minimize the sum of percentage squared errors between the model and the market prices:

$$\min_{\phi_t} \sum_{i=1}^N \left[ \frac{O_{i,t}^* - O_{i,t}}{O_{i,t}} \right]^2 \quad (t = 1, \dots, T) \quad (15)$$

where  $N$  denotes the number of options on day  $t$ , and  $T$  denotes the number of days in the sample.

### 3. Rollover Strategies

When options are priced and hedged, we need to estimate the parameters that are needed to plug into each model. For one day ahead pricing and hedging performance, the parameters are estimated using the previous days' options data. For one week ahead pricing and hedging performance, the options data seven days ago are used. In general trading dates, there is no complicated problems to implement this methodology. However, it is standard to eliminate the nearest option contracts with expiries less than 7 days, and use the next-to-nearest option contracts with expiries less than 7 days plus 1 month for the empirical study due to the liquidity

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However, we adopt the above function since the conventional method that gives more weight to relatively expensive in-the-money options makes the worse fit for out-of-the-money options. In our sample, there was no large difference between the results using the sum of squared errors and those using the sum of squared percentage errors.

problems of options contract. When forecasting the parameters for the next-to-nearest option contracts with expiry less than 7 days plus 1 month, one can use either the nearest contracts with expiry greater than 6 days or the next-to-nearest contracts with expiry greater than 6 days plus 1 month. The information content of these two contracts may differ and thus, the rollover procedure may be important to the accuracy of the parameter forecasting. Choi and Ok (2011) consider two rollover strategies from then on the day of the rollover from the nearest contract to the next contract: the nearest-to-next strategy and the next-to-next strategy. Figure 1 shows the difference between the nearest-to-next strategy and the next-to-next strategy. Specifically, when the nearest contract's expiry is less than 7 days, the next-to-next strategy uses the next-to-nearest contracts on the previous day(s), whereas the nearest-to-next one uses the nearest-term contracts. These two strategies are different only on the day(s) when the expiry of the nearest-term option contracts is less than 7 days. In this paper, the performances of the nearest-to-next strategy and the next-to-next strategy are compared. If the next-to-next roll-over strategy shows better performance than the nearest-to-next one, we can conjecture that there is a structural change when the nearest-to-next contracts are changed into the nearest contract.

### III. Data

The S&P 500 index (hence SPX) option data used in this paper come from Option Metrics LLC. The data include the end-of day bid and ask quotes, implied volatilities, open interest and daily trading volume for the SPX options traded on the Chicago Board Options Exchange from January 4, 1996 through December 31, 2008. The data also include daily index values and estimates of dividend yields, as well as the term structures of zero-coupon interest rates constructed from LIBOR quotes and Eurodollar futures prices. We use the bid-ask average as our measure of the option price.

The following rules are applied in order to filter the data needed for the empirical test. We

use out-of-the-money options for calls and puts. First of all, since there is only a very thin trading volume for the in-the-money (henceforth ITM) option, the credibility of price information is not entirely satisfactory. Therefore, we use the price data with regards to both put and call options that are near-the-money and out-of-the-money (henceforth OTM). Second, if both call and put option prices are used, ITM calls and OTM puts, which are equivalent according to the put-call parity, are used to estimate the parameters. Third, as Huang and Wu (2004) mention, “the Black-Scholes model has been known to systematically misprice equity index options, especially those that are out-of-the-money (OTM).” We recognize the need for an alternative option pricing model in order to mitigate this effect. As options with less than 7 days to expiration may induce biases due to low prices and bid-ask spreads, they are excluded from the sample. Because the liquidity is concentrated in the nearest expiration contract, we only consider options with the nearest maturity. To mitigate the impact of price discreteness on option valuation, prices lower than 0.4 are not included. Prices not satisfying the arbitrage restriction are excluded.

We divide the option data into several categories, according to the moneyness,  $S/K$ . Table 1 describes certain sample properties of the SPX option prices used in this study. Summary statistics are reported for the option price as well as for the total number of observations, according to each moneyness-option type category. Note that there are 42,396 call- and 64,316 put-option observations, with deep OTM<sup>2</sup> options, respectively, taking up 24% for calls and 49% for puts. Table 2 presents the “volatility smiles” effects for 26 consecutive six-month sub-periods. We employ six fixed intervals for the degree of moneyness, and compute the mean over alternative sub-periods of the implied volatility. SPX options market seems to be “sneer” independent of the sub-periods employed in the estimation. As the  $S/K$  increase, the implied volatilities decrease to near-the-money; however, after that, they increase to out-of-the-money put options. The implied volatility of deep out-of-the-money puts is larger than that of deep

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<sup>2</sup> For the call option, deep OTM options are options in  $S/K < 0.94$ . For the put option, deep OTM options are options in  $1.00 < S/K < 1.03$ .

out-of-the-money calls. That is, a volatility smile is skewed towards one side. The skewed volatility smile is sometimes called a 'volatility sneer' because it looks more like a sardonic smile than a sincere smile. In the equity options market, the volatility sneer is often negatively skewed, where lower strike prices for out-of-the money puts have higher implied volatilities and, thus, higher valuations.<sup>3</sup> This is consistent with Rubinstein (1994), Derman (1999), Bakshi, Kapadia and Madan (2001), and Dennis and Mayhew (2002). As the smile evidence is indicative of negatively-skewed implicit return distribution with excess kurtosis, a better model must be based on a distributional assumption that allows for negative skewness and excess kurtosis.

#### IV. Empirical Results

In this section, we examine the empirical performances of each model with respect to in-sample pricing, out-of-sample pricing and hedging performance. The analysis is based on two measures: mean absolute percentage errors (henceforth MAPE) and root mean squared errors, (henceforth RMSE) as follows.

$$\text{MAPE} = \frac{1}{T} \sum_{t=1}^T \frac{1}{N} \sum_{i=1}^N \left| \frac{O_{i,t} - O_{i,t}^*}{O_{i,t}} \right| \quad (16)$$

$$\text{RMSE} = \sqrt{\frac{1}{T} \sum_{t=1}^T \frac{1}{N} \sum_{i=1}^N [O_{i,t} - O_{i,t}^*]^2} \quad (17)$$

where  $O_{i,t}^*$  denotes the model price of option  $i$  on day  $t$ , and  $O_{i,t}$  denotes the market price of option  $i$  on day  $t$ .  $N$  denotes the number of options on day  $t$ , and  $T$  denotes the number of days in the sample. MAPE measures the magnitude of pricing errors, while RMSE measures the volatility of errors. If some options pricing model has the lowest values for both

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<sup>3</sup> See Rubinstein(1994) and Bakshi, Cao and Chan(1997).

MAPE and RMSE, that model is the best.

### 1. In-sample Pricing Performance

Table 3 reports the mean and the standard error of the parameter estimates for each model.  $R^2$  values for each AHBS-type model are also reported. For AHBS-type models, each parameter is estimated by the ordinary least squares every day. For the BS, SV and SVJ models, each parameter is estimated by minimizing the sum of the percentage squared errors between the model and the market option prices every day. First, the daily estimates of each model's parameters have excessive standard errors. However, such estimation will be valuable for the following reasons. The estimated parameters can be generated by indicating market sentiment on a daily basis, and the estimated parameters may suggest future specification of more complicated dynamic models. Because these ad hoc Black-Scholes models are based on not theoretical backgrounds, but on the traders' rule, it is not a fatal problem. Second, as expected, the R3 and A3 models that have four independent variables show higher  $R^2$  values compared to other models. Therefore, it is necessary to check for the over-fitting problem by examining the out-of-sample pricing performance. Third, the implied correlation of the SV and SVJ models has negative values. The negative estimate indicates that the implied volatility and the index returns are negatively correlated and the implied distribution recognized by option traders is negatively skewed. This is consistent with the volatility sneer pattern shown in table 2.

We evaluate the in-sample pricing performance of each model by comparing the market prices with the model's prices computed by using the parameter estimates from the current day. Table 4 reports in-sample valuation errors for the alternative models computed over the whole sample of options. The SVJ model shows the best performance, closely followed by the A3 model for MAPE; the A3 model outperforms other models for RMSE. Roughly, the SVJ and the A3, the complex models, are the best models for in-sample pricing. This is a rather obvious result when the use of larger number of the parameters in the SVJ and A3 model is considered.

Surprisingly, although the SV model has five parameters, the SV model does not show better performance than the A3 and the R3 model with four parameters. The in-sample pricing performance is not simply contingent on the number of free parameters. Lastly, all models show moneyness-based valuation errors. The models exhibit the worst fit for the out-of-the-money options. The fit, as measured by MAPE, steadily improves as we move from out-of-the-money to near-the-money options. Overall, all AHBS-type models and mathematically complicated models demonstrate better performance than the BS model. Also, the traders' rule can explain the current market price in the options market although it is not rooted in rigorous theory.

## 2. Out-of-sample Pricing Performance

In-sample pricing performance can be perverted due to the dormant problem of over-fitting to the data. A good in-sample fit might be a result of having an increasingly larger number of parameters. To reduce the effect of this connection to inferences, we turn to examining the model's out-of-sample pricing performance. In the out-of-sample pricing, the presence of more parameters may actually cause over-fitting and thus, have the model penalized if the extra parameters do not improve its structural fitting. This analysis also has the purpose of evaluating the stability of each model's parameter over time. To control the parameters' stability over alternative time periods, we analyze out-of-sample valuation errors for one day or one week. We use the current day's estimated parameters in order to price options for the following day (or week).

Table 5 and table 6, respectively, report one-day and one-week ahead out-of-sample pricing errors for alternative models computed over the whole sample of options. First of all, we examine the out-of-sample pricing performance using the nearest-to-next roll-over strategy. Panel A of table 5 and table 6 represents the results using the nearest-to-next roll-over strategy. For one day ahead out-of-sample pricing, the SVJ model shows the best performance, closely

followed by the SV model. The SVJ and the SV models also exhibit better fit for the one week ahead out-of-sample pricing. For the in-sample pricing performance, the AHBS-type models are competitive. However, for the out-of-sample pricing performance, the mathematically complicated models show better performance than the AHBS-type models. That is, the presence of more parameters of the SVJ and SV models actually does not cause the problem of overfitting. Contrary to Jackwerth and Rubinstein (2001), Li and Pearson (2007) and Kim (2009), the traders' rules do not mathematically dominate more sophisticated models although the traders' rules is not far behind. With respect to moneyness-based errors, similar to the results of the in-sample pricing performance, MAPE steadily decreases as we move from deep out-of-the-money to near-the-money options for all models. Generally, the SVJ model outperforms all other models.

Pricing errors increase from in-sample to out-of-sample pricing. The average of MAPE of all models is 0.1334 for in-sample pricing, and increases to 0.4022 for one-day ahead out-of-sample pricing. One-week ahead out-of-sample pricing errors grow to 0.6953, which is almost five times as much as the in-sample pricing errors. The relative margin of pricing performance is significantly changed when compared to that of the in-sample pricing results. The difference of the BS and the best model, the SVJ model, becomes smaller in the out-of-sample pricing. The ratio of the BS model to the SVJ model for MAPE is 8.2279 for in-sample pricing errors. The ratio of the BS model to the SVJ model decreases to 2.5267 and to 1.4218 for one-day ahead and one-week ahead out-of-sample errors, respectively. As the term of the out-of-sample pricing becomes longer, the difference between the BS model and the SVJ model becomes smaller. The pricing performance of the SVJ model, which is the best model for in-sample pricing, is maintained as the term of out-of-sample pricing gets longer, implying that the presence of more parameters actually does not cause over-fitting. However, for the AHBS-type models, A3 and R3, the best models among them for in-sample pricing do not remain their position for one day and one week ahead out-of-sample pricing. For out-of-sample pricing, the A3 and the R3

models are changed into the very last, implying that the presence of more parameters actually cause over-fitting. This result is consistent with the result of Jackwerth and Rubinstein (2001), Li and Pearson (2007) and Kim (2009). As a result, the mathematically complicated options pricing models do not have over-fitting problems, whereas the AHBS-type models have those problems when the nearest-to-next roll-over strategy is used. To mitigate these problems, we need to consider the new roll-over strategy, the next-to-next strategy.

Second, we examine the pricing performance for the next-to-next roll-over strategy, suggested by Choi and Ok (2011). Panel B of table 5 and table 6 represents the results using the next-to-next roll-over strategy. Above all, the next-to-next roll-over strategy decreases the pricing errors of all options pricing models. After using the next-to-next strategy, the averages of the MAPEs of all options pricing models are decreased from 0.4022 (0.6953) to 0.2557 (0.3485) for one day (one week) ahead out-of-sample pricing. Panel A and panel B of Figure 1 represent the MAPE of each options pricing model for both the nearest-to-next and the next-to-next roll-over strategies, respectively. The pricing errors of the AHBS-type models are decreased largely by the next-to-next strategy. Among them, the models with more parameters, R3 and A3, are favored the most. On the other hand, the improvement of the mathematically complicated models, including the BS model, is not great. Using the next-to-next strategy, the averages of the MAPEs of all AHBS-type models are decreased from 0.4426 (0.7968) to 0.2321 (0.2321) however, those of the mathematically complicated models are decreased from 0.2176 (0.4480) to 0.1924 (0.3342) for one day (one week) ahead out-of-sample pricing, respectively. For one day ahead out-of-sample pricing, the A2 model generally shows the best performance. The A3 models exhibit better fit for the one week ahead out-of-sample pricing. When the nearest-to-next roll-over strategy is applied, the SVJ model shows better performance compared to other models. However, using the next-to-next strategy, AHBS-type models show better performance than the mathematically complicated models. When the next-to-next roll-over strategy is applied, the A2 or A3 model outperforms all other models.

Finally, we examine the relative strength of the absolute and relative smile approaches for pricing options. For the in-sample pricing performance, the averages of the MAPEs of relative smile and absolute smile approaches are 0.0969 and 0.1005, respectively. Using the nearest-to-next strategy, for one day (one week) ahead out-of-sample pricing, the average MAPEs of alternative relative smile and absolute smile approaches are 0.4724 (0.8628) and 0.4128 (0.7308), respectively. Using the next-to-next strategy, for one day (one week) ahead out-of-sample pricing, the averages of the MAPEs of the alternative relative smile and absolute smile approaches are 0.2462 (0.3518) and 0.2180 (0.2836), respectively. Specifically, the effects of the reduction of pricing errors for the absolute smile approach are much better compared to those for the relative smile approach. This result is consistent with that of Jackwerth and Rubinstein (2001), Li and Pearson (2007), Kim (2009) and Choi and Ok (2011), who report that the “absolute smile” model beats the “relative smile” model for predicting prices. The result can be explained by the fact that the absolute smile model implicitly adjusts for the negative correlation between the index return and the level of the volatilities. Because the absolute model treats the skew as a fixed function of the strike instead of the moneyness  $S/K$ , it creates a smaller implied volatility than the relative smile model when there is an increase in the stock price.

### 3. Hedging Performance

Hedging performance is an important tool for gauging the forecasting power of the volatility of the underlying assets. We examine hedges in which only a single instrument (i.e., the underlying asset) can be employed. In practice, option traders usually focus on the risk due to the underlying asset price volatility alone, and carry out a delta-neutral hedge, employing only the underlying asset as the hedging instrument.

We implement hedging with the following method. Consider hedging as a short position in an option,  $O_{t,\tau;K}$ , with  $\tau$  periods to maturity and strike price of  $K$ . Let  $\Delta_{S,t}$  be the number of shares of the underlying asset to be purchased, and  $\Delta_0 (= O_{t,\tau;K} - \Delta_{S,t} S_t)$  be the residual

cash positions. We consider the delta hedging strategy of  $\Delta_{S,t} = \partial O_{t,\tau;K} / \partial S_t$  and  $\Delta_{0,t}$ .

To examine the hedging performance, we use the following steps. First, on current day  $t$ , we short an option, and construct a hedging portfolio by buying  $\Delta_{S,t}$  shares of the underlying asset<sup>4</sup>, and investing  $\Delta_{0,t}$  in a risk-free asset. To compute  $\Delta_{S,t}$ , we use the estimated parameters from the previous trading day and the current day's asset price. For the SV and SVJ models, we use the estimated instantaneous volatility from the previous day. For the AHBS model, the volatility parameter necessary to compute the delta position is obtained by plugging the option specific strike price into the regression equation along with the previous day's parameter estimates. Second, we liquidate the position after the next trading day (one day ahead hedging) or the next week (one week ahead hedging). Then we compute the hedging error as the difference between the value of the replicating portfolio and the option price at the time of liquidation:

$$\varepsilon_t = \Delta_{S,t} \cdot S_{t+\Delta t} + \Delta_{0,t} e^{r\Delta t} - O_{t+\Delta t, \tau-\Delta t; K}. \quad (18)$$

Table 7 and table 8 present one day and one week hedging errors over alternative moneyness categories, respectively. First, using the nearest-to-next roll-over strategy, the A1 model has the best hedging performance for one day and one week. With the exception of the BS model, the SV or the SVJ model is the worst performer. For hedging performance, AHBS-type models show better performance than the other models. However, the difference among the models is not so large. This result is consistent with Kim (2009). The ratios of the BS model to the A1 model, which is the best performer, are 1.1724 and 1.0967 for one-day ahead and one-week ahead hedging errors, respectively. In each moneyness category, the hedging errors are highest for ATM options and become smaller as we move to OTM options. These patterns are observed

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<sup>4</sup> The delta, for a put option, is negative, implying that a short position in the put options should be hedged with a short position in the underlying stock.

for all models and for each rebalancing frequency. Second, we examine hedging performances using the next-to-next roll-over strategy. In Panel A and Panel B of Figure 2, the errors of all models are decreased and the AHBS-type models are favored the most, similar to the pricing results. A3 is the best performer for both one day and one week ahead hedging errors. Similar to the results of the out-of-sample pricing, the next-to-next strategy can mitigate the over-fitting problem of AHBS-type models. AHBS-type models that have more parameters shows better hedging performance compared to those with less parameters. However, the next-to-next strategy does not make extreme decreases of the hedging errors. The impact of the next-to-next roll-over strategy for hedging performance is not drastic.

## V. Conclusion

For S&P 500 options, we implement a horse race competition among several options pricing models. We consider the traders' rules to predict future implied volatilities by applying simple ad hoc rules to the observed current implied volatility patterns as well as to the mathematically complicated options pricing models, SV and SVJ models, for pricing and hedging options. The roll-over strategies of the parameters for each options pricing model are also compared. In the nearest-to-next strategy, the options data of the nearest term contract on day  $t - k$  is used to estimate the parameters of the next-to-nearest contract on day  $t$ , whereas in the next-to-next roll-over strategy, the next-to-nearest contract on day  $t - k$  is used to estimate the parameter of the next-to-nearest contract on day  $t$ .

It is found that when we use the traditional roll-over method, the nearest-to-next strategy, the SVJ and SV models show better performance than the AHBS-type models for pricing options. Among AHBS-type models, a simpler model with less parameter shows better performance compared to other models. That is, for AHBS-type models, the presence of more parameters actually cause the problem of over-fitting; however, it does not cause an over-fitting problem

for mathematically complicated models. For hedging performance, AHBS-type models show better performance than the mathematically complicated models; yet, the differences among the models are not significant. When we use the new roll-over method, the next-to-next strategy decreases the pricing and hedging errors of all options pricing models. The pricing errors of the AHBS-type models are decreased largely by the next-to-next strategy. Moreover, AHBS-type models show better performance than the mathematically complicated options pricing models for pricing options. That is, the next-to-next strategy can mitigate the over-fitting problem of AHBS-type models. On the other hand, the improvement of mathematically complicated models using the next-to-next roll-over strategy is not much. For hedging performance, the next-to-next strategy also decreases the errors of all options pricing models; however, the difference between the results using the nearest-to-next strategy and those using the next-to-next strategy are not large.

As a result, when the next-to-next strategy is considered, the AHBS-type model has the advantage of simplicity and can be a competitive model for pricing and hedging S&P 500 index options. The next-to-next strategy can mitigate the over-fitting problems of AHBS-type models. However, the improvement of the next-to-next strategy for the mathematically complicated models is not much.

## Appendix

The characteristic functions  $\hat{f}_j$  for the SVJ model are respectively given by

$$\begin{aligned}\hat{f}_1 = & \exp \left[ -i\phi \ln[B(t, \tau)] - \frac{\theta_v}{\sigma_v^2} \left[ 2 \ln \left( 1 - \frac{[\xi_v - \kappa_v + (1+i\phi)\rho\sigma_v][1-e^{-\xi_v\tau}]}{2\xi_v} \right) \right] \right. \\ & - \frac{\theta_v}{\sigma_v^2} [\xi_v - \kappa_v + (1+i\phi)\rho\sigma_v]\tau + i\phi \ln[S_t] \\ & + \lambda(1+\mu_J)\tau [(1+\mu_J)^{i\phi} e^{(i\phi/2)(1+i\phi)\sigma_J^2} - 1] - \lambda i\phi\mu_J\tau \\ & \left. + \frac{i\phi(i\phi+1)(1-e^{-\xi_v\tau})}{2\xi_v - [\xi_v - \kappa_v + (1+i\phi)\rho\sigma_v][1-e^{-\xi_v\tau}]} V_t \right],\end{aligned}\quad (\text{A-1})$$

and

$$\begin{aligned}\hat{f}_2 = & \exp \left[ -i\phi \ln[B_{t,\tau}] - \frac{\theta_v}{\sigma_v^2} \left[ 2 \ln \left( 1 - \frac{[\xi_v^* - \kappa_v + i\phi\rho\sigma_v][1-e^{-\xi_v^*\tau}]}{2\xi_v} \right) \right] \right. \\ & - \frac{\theta_v}{\sigma_v^2} [\xi_v^* - \kappa_v + i\phi\rho\sigma_v]\tau + i\phi \ln[S_t] \\ & + \lambda(1+\mu_J)\tau [(1+\mu_J)^{i\phi} e^{(i\phi/2)(i\phi-1)\sigma_J^2} - 1] - \lambda i\phi\mu_J\tau \\ & \left. + \frac{i\phi(i\phi-1)(1-e^{-\xi_v^*\tau})}{2\xi_v^* - [\xi_v^* - \kappa_v + i\phi\rho\sigma_v][1-e^{-\xi_v^*\tau}]} V_t \right],\end{aligned}\quad (\text{A-2})$$

where

$$\begin{aligned}\xi_v &= \sqrt{[\kappa_v - (1+i\phi)\rho\sigma_v]^2 - i\phi(1+i\phi)\sigma_v^2} \\ \xi_v^* &= \sqrt{[\kappa_v - i\phi\rho\sigma_v]^2 - i\phi(i\phi-1)\sigma_v^2}\end{aligned}$$

The characteristic functions for the SV model can be obtained by setting  $\lambda = 0$  in (A-1) and (A-2).

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**Table 1: S&P 500 Options Data**

This table reports the average option price and the number of options, which are shown in parentheses, for each moneyness and options type (call or put) category. The sample period is from January 4, 1996 to December 31, 2008. The last bid-ask average of each option contract is used to obtain the summary statistics. Moneyness of an option is defined as  $S/K$ , where  $S$  denotes the spot price and  $K$  denotes the strike price.

Call Options			Put Options		
Moneyness	Price	Number	Moneyness	Price	Number
$S/K < 0.94$	2.7833	10,033	$1.00 < S/K < 1.03$	13.0104	17,966
$0.94 < S/K < 0.96$	4.8054	13,580	$1.03 < S/K < 1.06$	6.5072	15,149
$0.96 < S/K < 1.00$	12.2916	18,783	$S/K > 1.06$	3.0967	31,201
Total	7.6435	42,396	Total	6.6693	64,316

**Table 2: Implied Volatilities Sneer**

This table reports the implied volatilities calculated by inverting the Black-Scholes (1973) model separately for each moneyness category. The implied volatilities of individual options are then averaged within each moneyness category and across 26 consecutive six-month sub-periods. Moneyness is defined as  $S/K$ , where  $S$  denotes the spot price and  $K$  denotes the strike price. 1996 01-06 is the period from January, 1996 to June, 1996.

	$S/K < 0.94$	$0.94 < S/K < 0.97$	$0.97 < S/K < 1.00$	$1.00 < S/K < 1.03$	$1.03 < S/K < 1.06$	$S/K > 1.06$
1996 01-06	0.1308	0.1204	0.1207	0.1553	0.1841	0.2249
1996 07-12	0.1413	0.1243	0.1289	0.1606	0.1884	0.2332
1997 01-06	0.1626	0.1579	0.1636	0.1895	0.2108	0.2516
1997 07-12	0.1920	0.1867	0.1957	0.2223	0.2477	0.3092
1998 01-06	0.1517	0.1491	0.1588	0.1917	0.2226	0.2872
1998 07-12	0.2364	0.2037	0.2139	0.2406	0.2709	0.3492
1999 01-06	0.1831	0.1851	0.2011	0.2267	0.2511	0.3132
1999 07-12	0.1626	0.1658	0.1774	0.2035	0.2267	0.2887
2000 01-06	0.1922	0.1840	0.1969	0.2238	0.2417	0.3025
2000 07-12	0.2095	0.1844	0.1891	0.2070	0.2259	0.2841
2001 01-06	0.2120	0.1962	0.2040	0.2269	0.2391	0.2996
2001 07-12	0.2159	0.1977	0.2115	0.2401	0.2638	0.3525
2002 01-06	0.1781	0.1694	0.1740	0.1993	0.2256	0.2889
2002 07-12	0.2711	0.2683	0.2787	0.3070	0.3270	0.3850
2003 01-06	0.2427	0.2248	0.2208	0.2325	0.2504	0.2951
2003 07-12	0.1563	0.1459	0.1446	0.1689	0.1911	0.2444
2004 01-06	0.1464	0.1262	0.1243	0.1504	0.1768	0.2281
2004 07-12	0.1227	0.1127	0.1121	0.1354	0.1589	0.2036
2005 01-06	0.1197	0.1025	0.0986	0.1233	0.1510	0.1994
2005 07-12	0.1628	0.0902	0.0926	0.1205	0.1472	0.1958

2006 01-06	0.1205	0.0968	0.0981	0.1289	0.1562	0.2131
2006 07-12	0.2184	0.0875	0.0903	0.1201	0.1480	0.1954
2007 01-06	0.1666	0.0945	0.0955	0.1301	0.1618	0.2236
2007 07-12	0.1809	0.1548	0.1713	0.2124	0.2386	0.2917
2008 01-06	0.1902	0.1771	0.1942	0.2284	0.2471	0.2893
2008 07-12	0.3975	0.3296	0.3435	0.3754	0.4017	0.4722

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**Table 3: Parameters**

This table reports the mean and the standard error of the parameter estimates for each model. The mean and the standard deviation of R<sup>2</sup>s for each AHBS-type model are reported. For the AHBS-type models, each parameter is estimated by the ordinary least squares every day. For the BS, SV and SVJ models, each parameter is estimated by minimizing the sum of the percentage squared errors between the model and the market option prices every day. R1 is the ad hoc Black-Scholes model that considers the intercept and the moneyness as the independent variables. R2 is the ad hoc Black-Scholes model that considers the intercept, the moneyness and the square of the moneyness as the independent variables. R3 is the ad hoc Black-Scholes model that considers the intercept, the moneyness, the square and the third power of the moneyness as the independent variables. A1 is the ad hoc Black-Scholes model that considers the intercept and the strike price as the independent variables. A2 is the ad hoc Black-Scholes model that considers the intercept, the strike price and the square of the strike price as the independent variables. A3 is the ad hoc Black-Scholes model that considers the intercept, the strike price, and the square and the third power of the strike price as the independent variables. BS is the Black-Scholes (1973) option pricing model. SV is the option pricing model considering the continuous-time stochastic volatility. SVJ is the option pricing model considering the continuous-time stochastic volatility and the jumps.

Panel A: AHBS-type Models					
	$\beta_0$	$\beta_1$	$\beta_2$	$\beta_3$	$R^2$
R1	-0.6097 (0.0052)	0.8006 (0.0046)			0.9326 (0.1124)
R2	1.7984 (0.0631)	-3.9174 (0.1255)	2.3059 (0.0621)		0.9720 (0.0427)
R3	26.9431 (1.0340)	-77.5085 (3.0511)	73.9906 (2.9996)	-23.2409 (0.9828)	0.9833 (0.0289)
A1	1.0360 (0.0048)	-0.0008 (0.0000)			0.9152 (0.1202)
A2	4.3900 (0.0656)	-0.0071 (0.0001)	0.0000 (0.0000)		0.9761 (0.0377)
A3	-17.1892 (1.0140)	0.0559 (0.0030)	-0.0001 (0.0000)	0.0000 (0.0000)	0.9847 (0.0285)

Panel B: Other Models									
	$\sigma$	$\lambda$	$\mu_J$	$\sigma_J$	$\kappa$	$\theta$	$\sigma_v$	$\rho$	$v_t$
BS	0.1699 (0.0012)								
SV					5492.2986 (5200.8240)	6.3328 (0.4005)	140.5703 (127.4397)	-0.5426 (0.0023)	6.5856 (6.3503)
SVJ	2.4129 (0.1533)	0.0084 (0.0268)	2.2860 (04780)		5814.9122 (3853.0067)	0.4964 (0.0287)	2.2812 (0.3920)	-0.2469 (0.0119)	25.9249 (23.5741)

**Table 4: In-Sample Pricing Errors**

This table reports in-sample pricing errors with respect to moneyness. The in-sample pricing performance of each model is evaluated by comparing the market prices with the model's prices computed by using the parameter estimates from the current day. S/K is defined as moneyness, where S denotes the asset price and K denotes the strike price. MAPE denotes mean absolute percentage errors and RMSE denotes root mean squared errors. R1 is the ad hoc Black-Scholes model that considers the intercept and the moneyness as the independent variables. R2 is the ad hoc Black-Scholes model that considers the intercept, the moneyness and the square of the moneyness as the independent variables. R3 is the ad hoc Black-Scholes model that considers the intercept, the moneyness, the square and the third power of the moneyness as the independent variables. A1 is the ad hoc Black-Scholes model that considers the intercept and the strike price as the independent variables. A2 is the ad hoc Black-Scholes model that considers the intercept, the strike price and the square of the strike price as the independent variables. A3 is the ad hoc Black-Scholes model that considers the intercept, the strike price and the square and the third power of the strike price as the independent variables. BS is the Black-Scholes (1973) option pricing model. SV is the option pricing model considering the continuous-time stochastic volatility. SVJ is the option pricing model considering the continuous-time stochastic volatility and jumps.

Moneyness	BS	R1	R2	R3	A1	A2	A3	SV	SVJ	
MAPE	S/K<0.94	0.2780	0.3521	0.2537	0.1337	0.4723	0.2116	0.1089	0.1186	0.0984
	0.94<S/K<0.96	0.2145	0.2268	0.1546	0.0915	0.2555	0.1431	0.0769	0.0561	0.0467
	0.96<S/K<1.00	0.1328	0.1090	0.0854	0.0615	0.1169	0.0800	0.0565	0.0561	0.0442
	1.00<S/K<1.03	0.2987	0.0655	0.0425	0.0480	0.0720	0.0436	0.0471	0.0586	0.0516
	1.03<S/K<1.06	0.6453	0.0749	0.0469	0.0384	0.0879	0.0461	0.0357	0.0566	0.0448
	S/K>1.06	0.9065	0.0957	0.0739	0.0590	0.1148	0.0691	0.0566	0.0702	0.0712
Total		0.4838	0.1308	0.0940	0.0658	0.1557	0.0863	0.0595	0.0666	0.0588
RMSE	S/K<0.94	0.8478	0.6683	0.4764	0.2383	2.8438	0.3796	0.2173	0.3040	0.4919
	0.94<S/K<0.96	1.3162	0.9581	0.6833	0.3747	1.0699	0.7589	0.3370	0.5302	0.6625
	0.96<S/K<1.00	2.3751	1.3847	1.1295	0.8592	1.5330	1.0672	0.8222	0.9734	1.1253
	1.00<S/K<1.03	4.7693	1.0109	0.8813	0.9327	1.0172	0.9080	0.8887	1.1914	1.5422
	1.03<S/K<1.06	4.7025	0.5752	0.5852	0.3648	0.5569	0.5734	0.3191	0.5246	1.3276

S/K>1.06	3.6577	0.3225	0.3329	0.2059	0.3643	0.3000	0.1828	0.2238	1.2709
Total	3.4873	0.8635	0.7191	0.5752	1.2557	0.7065	0.5432	0.7100	1.1934

**Table 5: One Day Ahead Out-of-Sample Pricing Errors**

This table reports one day ahead out-of-sample pricing errors with respect to moneyness. Each model is estimated every day during the sample period; one day ahead out-of-sample pricing errors are computed using the estimated parameters from the previous trading day. Panel A reports one day ahead out-of-sample pricing errors using the nearest-to-next roll over strategy. Panel B reports one day ahead out-of-sample pricing errors using the next-to-next roll over strategy. S/K is defined as moneyness, where S denotes the asset price and K denotes the strike price. MAPE denotes mean absolute percentage errors and RMSE denotes root mean squared errors. R1 is the ad hoc Black-Scholes model that considers the intercept and the moneyness as the independent variables. R2 is the ad hoc Black-Scholes model that considers the intercept, the moneyness and the square of the moneyness as the independent variables. R3 is the ad hoc Black-Scholes model that considers the intercept, the moneyness, the square and the third power of the moneyness as the independent variables. A1 is the ad hoc Black-Scholes model that considers the intercept and the strike price as the independent variables. A2 is the ad hoc Black-Scholes model that considers the intercept, the strike price and the square of the strike price as the independent variables. A3 is the ad hoc Black-Scholes model that considers the intercept, the strike price, and the square and the third power of the strike price as the independent variables. BS is the Black-Scholes (1973) option pricing model. SV is the option pricing model considering the continuous-time stochastic volatility. SVJ is the option pricing model considering the continuous-time stochastic volatility and jumps.

Panel A: Nearest-to-Next											
	Moneyness	BS	R1	R2	R3	A1	A2	A3	SV	SVJ	
MAPE	S/K<0.94	0.5341	1.5407	0.9417	1.5318	1.9915	0.9234	1.6504	0.5778	0.4373	
	0.94<S/K<0.96	0.3247	0.3937	0.3218	0.3152	0.4314	0.2639	0.2966	0.2566	0.2623	
	0.96<S/K<1.00	0.1802	0.1765	0.1610	0.1461	0.1558	0.1315	0.1182	0.1412	0.1474	
	1.00<S/K<1.03	0.3000	0.1205	0.1059	0.1072	0.1048	0.0868	0.0888	0.1241	0.1177	
	1.03<S/K<1.06	0.6398	0.1673	0.1545	0.1574	0.1455	0.1297	0.1288	0.1831	0.1701	
	S/K>1.06	0.9048	0.2719	0.6004	1.5192	0.2345	0.4619	1.0370	0.2295	0.2221	
		Total	0.5291	0.3496	0.3731	0.6945	0.3765	0.3116	0.5502	0.2258	0.2094
RMSE	S/K<0.94	3.1253	9.8866	6.9483	11.2266	12.1547	7.4598	11.6676	10.2350	1.8681	
	0.94<S/K<0.96	2.2157	2.5736	2.2606	2.2050	2.8509	1.9258	1.8610	4.5890	2.0238	
	0.96<S/K<1.00	2.9984	2.6426	2.3948	2.3005	2.3494	1.9160	1.7958	4.1612	2.5945	
	1.00<S/K<1.03	4.9140	2.2118	2.1536	2.1869	1.8059	1.7111	1.7057	4.1129	2.6332	

1.03<S/K<1.06	4.7668	1.7302	1.7318	1.8537	1.3526	1.3127	1.2769	3.6266	2.1428
S/K>1.06	3.7063	1.6942	3.1487	12.5589	1.3521	2.3913	9.4232	4.0781	1.7147
Total	3.7990	3.6539	3.2101	7.7982	4.1522	2.9598	6.3635	5.0155	2.1709

Panel B: Next-to-Next										
	Moneyness	BS	R1	R2	R3	A1	A2	A3	SV	SVJ
MAPE	S/K<0.94	0.4918	0.9766	0.6284	1.0934	1.0709	0.5456	0.9756	0.4143	0.4069
	0.94<S/K<0.96	0.3204	0.3412	0.3045	0.2646	0.3045	0.2433	0.2133	0.2340	0.2397
	0.96<S/K<1.00	0.1795	0.1688	0.1563	0.1415	0.1469	0.1260	0.1123	0.1322	0.1381
	1.00<S/K<1.03	0.2992	0.1163	0.1008	0.1021	0.0999	0.0811	0.0831	0.1121	0.1091
	1.03<S/K<1.06	0.6390	0.1612	0.1447	0.1450	0.1392	0.1213	0.1202	0.1661	0.1592
	S/K>1.06	0.9042	0.2016	0.1865	0.1906	0.1865	0.1573	0.1591	0.1954	0.2018
	Total	0.5241	0.2664	0.2174	0.2549	0.2564	0.1813	0.2162	0.1916	0.1931
RMSE	S/K<0.94	2.0566	6.5118	4.4744	9.9655	7.7868	4.2148	10.1036	2.0878	1.9821
	0.94<S/K<0.96	2.0071	1.9145	1.8257	1.7539	1.5357	1.3176	1.2002	1.8637	1.7718
	0.96<S/K<1.00	2.9282	2.3118	2.1726	2.0654	1.9846	1.6376	1.4830	2.1965	2.1697
	1.00<S/K<1.03	4.9063	1.9392	1.8888	1.9098	1.4814	1.3808	1.3534	2.0800	2.2010
	1.03<S/K<1.06	4.7669	1.4731	1.4869	1.4545	1.0535	1.0306	0.9614	1.5444	1.8706
	S/K>1.06	3.6934	1.1429	1.1481	1.1399	0.8229	0.7548	0.7765	1.1771	1.6165
	Total	3.6995	2.5918	2.1065	3.4315	2.7239	1.7325	3.2841	1.7813	1.9167

**Table 6: One Week Ahead Out-of-Sample Pricing Errors**

This table reports one week ahead out-of-sample pricing with respect to moneyness. Each model is estimated every day during the sample period; one week ahead out-of-sample pricing errors are computed using estimated parameters from one week ago. Panel A reports one week ahead out-of-sample pricing errors using the nearest-to-next roll over strategy. Panel B reports one week ahead out-of-sample pricing errors using the next-to-next roll over strategy. S/K is defined as moneyness, where S denotes the asset price and K denotes the strike price. MAPE denotes mean absolute percentage errors and RMSE denotes root mean squared errors. R1 is the ad hoc Black-Scholes model that considers the intercept and the moneyness as the independent variables. R2 is the ad hoc Black-Scholes model that considers the intercept, the moneyness and the square of the moneyness as the independent variables. R3 is the ad hoc Black-Scholes model that considers the intercept, the moneyness, the square and the third power of the moneyness as the independent variables. A1 is the ad hoc Black-Scholes model that considers the intercept and the strike price as the independent variables. A2 is the ad hoc Black-Scholes model that considers the intercept, the strike price and the square of the strike price as the independent variables. A3 is the ad hoc Black-Scholes model that considers the intercept, the strike price, and the square and the third power of the strike price as the independent variables. BS is the Black-Scholes (1973) option pricing model. SV is the option pricing model considering the continuous-time stochastic volatility. SVJ is the option pricing model considering the continuous-time stochastic volatility and jumps.

Panel A: Nearest-to-Next											
	Moneyness	BS	R1	R2	R3	A1	A2	A3	SV	SVJ	
MAPE	S/K<0.94	0.8617	1.9879	2.0170	3.0647	2.6419	2.0196	3.6512	1.3913	0.7470	
	0.94<S/K<0.96	0.4497	0.5460	0.4920	0.4991	0.5954	0.4316	0.5740	0.5209	0.5237	
	0.96<S/K<1.00	0.2190	0.2455	0.2304	0.2148	0.2103	0.1795	0.1709	0.2549	0.2709	
	1.00<S/K<1.03	0.3069	0.1766	0.1677	0.1667	0.1433	0.1316	0.1333	0.2247	0.2172	
	1.03<S/K<1.06	0.6341	0.2672	0.2738	0.2784	0.2119	0.2234	0.2277	0.3665	0.3355	
	S/K>1.06	0.8970	0.5984	1.4879	2.7130	0.4559	1.1021	1.6741	0.5318	0.4778	
		Total	0.5808	0.5424	0.7952	1.2507	0.5488	0.6527	0.9910	0.4874	0.4085
RMSE	S/K<0.94	4.2860	11.7077	12.6689	15.3663	14.5579	11.5353	15.0424	12.9565	2.7126	
	0.94<S/K<0.96	2.9367	3.4075	3.4598	3.3580	3.6598	2.6120	3.1233	6.5186	3.1778	
	0.96<S/K<1.00	3.6061	3.6611	3.4406	3.3757	3.0477	2.5708	2.5055	6.3450	4.1884	
	1.00<S/K<1.03	5.1224	3.3300	3.3049	3.3273	2.5613	2.4069	2.4213	6.3463	4.2508	

1.03<S/K<1.06	4.8038	2.7604	2.7448	2.8641	2.1222	2.0400	2.2054	5.7462	3.3905
S/K>1.06	3.6253	2.5126	4.1344	15.3676	2.0307	3.4903	11.2947	5.6007	2.6057
Total	4.0802	4.6395	5.1587	9.8868	5.1212	4.4360	7.9132	6.9945	3.4238

Panel B: Next-to-Next										
	Moneyness	BS	R1	R2	R3	A1	A2	A3	SV	SVJ
MAPE	S/K<0.94	0.6769	1.0920	1.1717	0.8539	0.9536	0.9828	0.6170	0.5046	0.7284
	0.94<S/K<0.96	0.4182	0.4678	0.4482	0.4071	0.3642	0.3345	0.3078	0.3604	0.4830
	0.96<S/K<1.00	0.2159	0.2419	0.2340	0.2157	0.1933	0.1758	0.1600	0.2083	0.2569
	1.00<S/K<1.03	0.3104	0.1707	0.1601	0.1581	0.1306	0.1155	0.1169	0.1727	0.1984
	1.03<S/K<1.06	0.6392	0.2405	0.2361	0.2382	0.1869	0.1863	0.1881	0.2892	0.3231
	S/K>1.06	0.9025	0.3294	0.3274	0.3285	0.2926	0.2784	0.2793	0.3076	0.4234
	Total	0.5618	0.3640	0.3647	0.3266	0.3042	0.2932	0.2534	0.2901	0.3783
RMSE	S/K<0.94	2.7911	7.3373	8.4343	5.7274	7.4183	8.2250	3.0896	2.2932	2.8595
	0.94<S/K<0.96	2.7126	2.7285	2.6666	2.5978	2.0137	1.8453	1.7212	2.3140	2.7989
	0.96<S/K<1.00	3.6344	3.2995	3.2114	3.1088	2.5312	2.2361	2.0776	3.0648	3.4921
	1.00<S/K<1.03	5.3199	3.0122	2.9683	2.9811	2.0871	1.9427	1.9221	3.0605	3.3834
	1.03<S/K<1.06	5.0098	2.4232	2.4145	2.4438	1.6461	1.5466	1.5552	2.3768	2.8596
	S/K>1.06	3.8287	1.8045	1.8136	1.8225	1.3018	1.1693	1.2655	1.5868	2.2649
	Total	4.0749	3.3527	3.5615	2.9933	2.9041	3.0086	1.8623	2.4379	2.9154

**Table 7: One Day Ahead Hedging Errors**

This table reports one day ahead hedging error with respect to moneyness. Only the underlying asset is used as the hedging instrument. Parameters and spot volatility implied by all options of the previous day are used to establish the current day's hedge portfolio, which is then liquidated the following day. For each option, its hedging error is the difference between the replicating portfolio value and its market price. Panel A reports one day ahead hedging errors using the nearest-to-next roll over strategy. Panel B reports one day ahead hedging errors using the next-to-next roll over strategy. MAPE denotes mean absolute percentage errors and RMSE denotes root mean squared errors. R1 is the ad hoc Black-Scholes model that considers the intercept and the moneyness as the independent variables. R2 is the ad hoc Black-Scholes model that considers the intercept, the moneyness and the square of the moneyness as the independent variables. R3 is the ad hoc Black-Scholes model that considers the intercept, the moneyness, the square and the third power of the moneyness as the independent variables. A1 is the ad hoc Black-Scholes model that considers the intercept and the strike price as the independent variables. A2 is the ad hoc Black-Scholes model that considers the intercept, the strike price and the square of the strike price as the independent variables. A3 is the ad hoc Black-Scholes model that considers the intercept, the strike price, and the square and the third power of the strike price as the independent variables. BS is the Black-Scholes (1973) option pricing model. SV is the option pricing model considering the continuous-time stochastic volatility. SVJ is the option pricing model considering the continuous-time stochastic volatility and jumps.

Panel A: Nearest-to-Next											
	Moneyness	BS	R1	R2	R3	A1	A2	A3	SV	SVJ	
MAPE	S/K<0.94	0.6677	0.6652	0.6888	0.6885	0.6857	0.6876	0.6910	0.6703	0.6762	
	0.94<S/K<0.96	0.3542	0.3337	0.3376	0.3420	0.3281	0.3357	0.3414	0.3632	0.3901	
	0.96<S/K<1.00	0.1526	0.1463	0.1484	0.1475	0.1461	0.1484	0.1473	0.1623	0.1723	
	1.00<S/K<1.03	0.1270	0.1170	0.1160	0.1158	0.1165	0.1155	0.1153	0.1383	0.1335	
	1.03<S/K<1.06	0.2082	0.1751	0.1717	0.1725	0.1740	0.1696	0.1699	0.1883	0.1869	
	S/K>1.06	0.3019	0.2052	0.2205	0.2485	0.2003	0.2135	0.2358	0.2171	0.2196	
		Total	0.2748	0.2351	0.2420	0.2509	0.2344	0.2391	0.2467	0.2511	0.2566
RMSE	S/K<0.94	2.5132	2.7778	2.7379	2.6767	2.8555	2.7129	2.6856	3.6700	2.7955	
	0.94<S/K<0.96	1.5283	1.6582	1.6308	1.5964	1.6561	1.6085	1.5817	2.1158	2.0952	

0.96<S/K<1.00	1.4958	1.5212	1.5138	1.5077	1.5224	1.5094	1.5050	1.8682	1.8283
1.00<S/K<1.03	2.0369	1.6609	1.6735	1.6580	1.6522	1.6677	1.6542	2.1256	2.1287
1.03<S/K<1.06	1.6514	1.2474	1.2429	1.2336	1.2429	1.2330	1.2238	1.5762	1.6345
S/K>1.06	1.3234	0.9074	0.9160	1.0574	0.9071	0.9038	1.0349	1.1785	1.2405
Total	1.6874	1.5366	1.5282	1.5363	1.5472	1.5159	1.5288	1.9725	1.8503

Panel B: Next-to-Next

		Moneyness	BS	R1	R2	R3	A1	A2	A3	SV	SVJ
MAPE	S/K<0.94	0.6500	0.6197	0.6313	0.6179	0.6068	0.6257	0.6127	0.6547	0.6756	
	0.94<S/K<0.96	0.3539	0.3294	0.3366	0.3337	0.3209	0.3337	0.3301	0.3611	0.3903	
	0.96<S/K<1.00	0.1525	0.1459	0.1483	0.1475	0.1456	0.1483	0.1472	0.1622	0.1723	
	1.00<S/K<1.03	0.1270	0.1170	0.1160	0.1158	0.1165	0.1155	0.1153	0.1383	0.1329	
	1.03<S/K<1.06	0.2082	0.1750	0.1714	0.1718	0.1739	0.1694	0.1695	0.1877	0.1859	
	S/K>1.06	0.3024	0.2007	0.2005	0.2010	0.1973	0.1963	0.1971	0.2133	0.2163	
	Total	0.2733	0.2290	0.2306	0.2291	0.2254	0.2281	0.2265	0.2481	0.2553	
RMSE	S/K<0.94	2.3741	2.6121	2.5715	2.5004	2.6184	2.5213	2.4721	3.3134	2.9120	
	0.94<S/K<0.96	1.5172	1.6290	1.6141	1.5843	1.6206	1.5948	1.5668	2.1139	2.0934	
	0.96<S/K<1.00	1.4937	1.5120	1.5106	1.5061	1.5131	1.5073	1.5030	1.8547	1.8303	
	1.00<S/K<1.03	2.0454	1.6653	1.6774	1.6626	1.6578	1.6728	1.6594	2.1289	2.1032	
	1.03<S/K<1.06	1.6639	1.2526	1.2494	1.2366	1.2496	1.2411	1.2311	1.5756	1.5971	
	S/K>1.06	1.3398	0.9067	0.8959	0.8978	0.9081	0.8881	0.8921	1.1397	1.2071	
	Total	1.6751	1.5059	1.4970	1.4772	1.5044	1.4827	1.4676	1.9060	1.8502	

**Table 8: One Week Ahead Hedging Errors**

This table reports one week ahead hedging error with respect to moneyness. Only the underlying asset is used as the hedging instrument. Parameters and spot volatility implied by all options of the previous day are used to establish the current day's hedge portfolio, which is then liquidated the next week. For each option, its hedging error is the difference between the replicating portfolio value and its market price. Panel A reports one week ahead hedging errors using the nearest-to-next roll over strategy. Panel B reports one week ahead hedging errors using the next-to-next roll over strategy. MAPE denotes mean absolute percentage errors and RMSE denotes root mean squared errors. R1 is the ad hoc Black-Scholes model that considers the intercept and the moneyness as the independent variables. R2 is the ad hoc Black-Scholes model that considers the intercept, the moneyness and the square of the moneyness as the independent variables. R3 is the ad hoc Black-Scholes model that considers the intercept, the moneyness, the square and the third power of the moneyness as the independent variables. A1 is the ad hoc Black-Scholes model that considers the intercept and the strike price as the independent variables. A2 is the ad hoc Black-Scholes model that considers the intercept, the strike price and the square of the strike price as the independent variables. A3 is the ad hoc Black-Scholes model that considers the intercept, the strike price, and the square and the third power of the strike price as the independent variables. BS is the Black-Scholes (1973) option pricing model. SV is the option pricing model considering the continuous-time stochastic volatility. SVJ is the option pricing model considering the continuous-time stochastic volatility and jumps.

**Panel A: Nearest-to-Next**

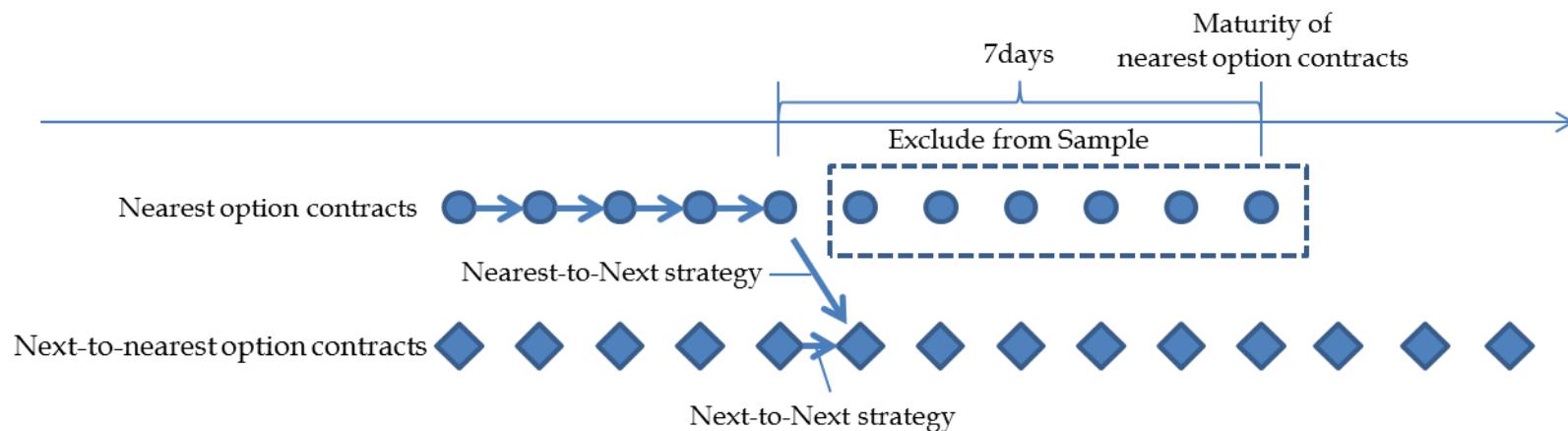
	Moneyness	BS	R1	R2	R3	A1	A2	A3	SV	SVJ
MAPE	S/K<0.94	2.8491	2.9664	2.9921	2.9648	2.9794	3.0041	2.9787	3.5156	3.3768
	0.94<S/K<0.96	1.1475	1.1192	1.1244	1.1293	1.1165	1.1214	1.1266	1.3321	1.3710
	0.96<S/K<1.00	0.4419	0.4281	0.4313	0.4308	0.4231	0.4293	0.4295	0.4546	0.4716
	1.00<S/K<1.03	0.3703	0.3721	0.3696	0.3701	0.3719	0.3684	0.3689	0.4033	0.3946
	1.03<S/K<1.06	0.6822	0.6543	0.6489	0.6538	0.6519	0.6440	0.6464	0.6920	0.6786
	S/K>1.06	1.0605	0.8051	0.8375	0.8674	0.7921	0.8211	0.8445	0.8481	0.8426
	Total	0.9597	0.8797	0.8919	0.9003	0.8751	0.8861	0.8924	0.9823	0.9730
RMSE	S/K<0.94	6.6513	7.0792	7.0946	7.0745	7.1525	7.0756	6.9844	8.8376	8.2344
	0.94<S/K<0.96	3.1401	3.2563	3.2330	3.2005	3.2709	3.2093	3.1837	4.1987	4.1980
	0.96<S/K<1.00	3.1357	3.2304	3.2024	3.1794	3.2309	3.1813	3.1667	3.5489	3.5308

1.00<S/K<1.03	4.1265	3.7116	3.6913	3.6902	3.7169	3.6885	3.6935	3.9930	4.0198
1.03<S/K<1.06	3.0727	2.8233	2.8127	2.8464	2.8169	2.8002	2.8106	3.1969	3.2028
S/K>1.06	2.5903	2.2104	2.2283	2.3903	2.1970	2.2066	2.3509	2.4710	2.4540
Total	3.5724	3.4869	3.4816	3.5086	3.4987	3.4659	3.4768	4.1056	3.9970

Panel B: Next-to-Next										
	Moneyness	BS	R1	R2	R3	A1	A2	A3	SV	SVJ
MAPE	S/K<0.94	2.8162	2.9226	2.9128	2.8485	2.8982	2.9104	2.8508	3.4766	3.4192
	0.94<S/K<0.96	1.1432	1.1174	1.1244	1.1180	1.0969	1.1168	1.1090	1.3261	1.3721
	0.96<S/K<1.00	0.4414	0.4271	0.4314	0.4303	0.4212	0.4291	0.4284	0.4544	0.4713
	1.00<S/K<1.03	0.3703	0.3720	0.3695	0.3699	0.3717	0.3682	0.3687	0.4036	0.3942
	1.03<S/K<1.06	0.6822	0.6535	0.6470	0.6498	0.6513	0.6424	0.6444	0.6923	0.6771
	S/K>1.06	1.0629	0.7870	0.7852	0.7859	0.7783	0.7766	0.7786	0.8341	0.8305
	Total	0.9570	0.8698	0.8685	0.8628	0.8610	0.8633	0.8582	0.9739	0.9726
RMSE	S/K<0.94	6.6353	7.0186	6.9674	6.8980	7.0307	6.9493	6.9083	8.8121	8.2492
	0.94<S/K<0.96	3.1257	3.2232	3.2097	3.1744	3.2071	3.1841	3.1525	4.1951	4.2015
	0.96<S/K<1.00	3.1262	3.2123	3.1997	3.1761	3.2087	3.1809	3.1627	3.5385	3.5450
	1.00<S/K<1.03	4.1231	3.7120	3.6924	3.6906	3.7175	3.6898	3.6943	3.9845	4.0072
	1.03<S/K<1.06	3.0734	2.8179	2.8049	2.8145	2.8116	2.7934	2.8023	3.1838	3.1673
	S/K>1.06	2.5988	2.1864	2.1765	2.1772	2.1762	2.1613	2.1668	2.4002	2.4054
	Total	3.5684	3.4644	3.4454	3.4270	3.4620	3.4314	3.4210	4.0830	3.9863

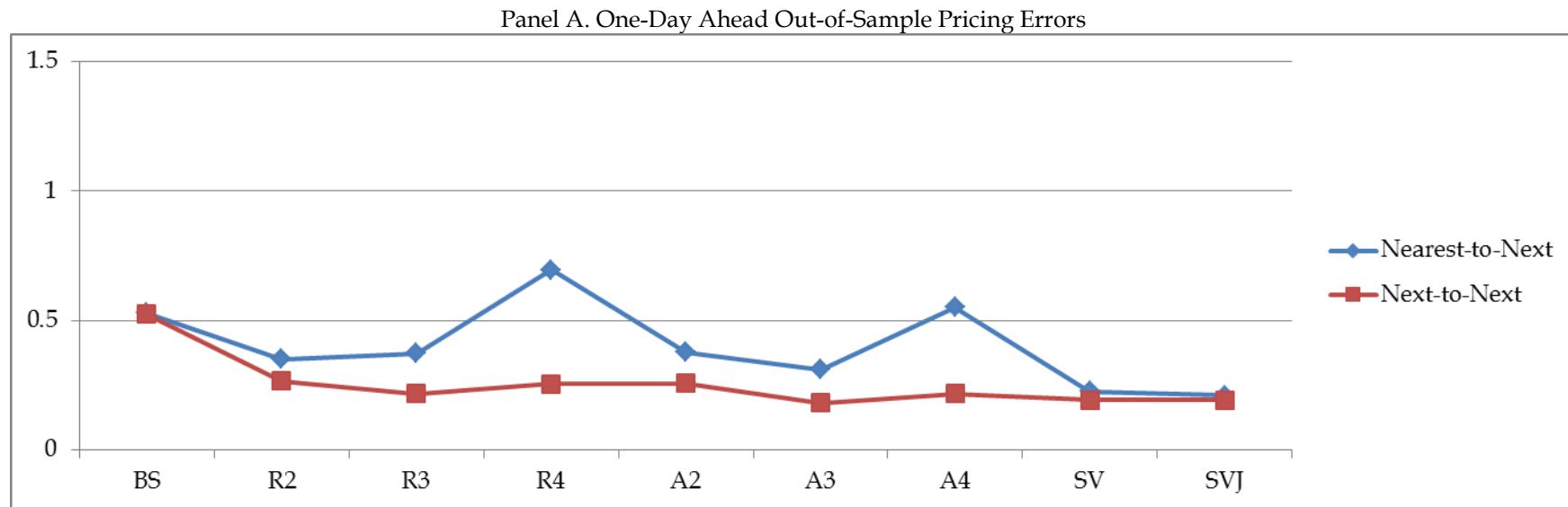
**Figure 1: Roll-over Strategies**

This figure shows the difference between the nearest-to-next roll-over strategy and the next-to-next roll-over strategy. This figure represents the example for one day ahead pricing and hedging performance. The circle represents the options data from the nearest option contract. The diamond represents the options data from the next-to-nearest option contract. It is standard to eliminate the nearest option contracts with expiries less than 7 days, and use the next-to-nearest option contracts with expiries less than 7 days plus 1 month for the empirical study due to liquidity problems of the options contract. When forecasting the parameters for the next-to-nearest option contracts with expiry less than 7 days plus 1 month, one can use either the nearest options contracts (circle) with expiry greater than 6 days or the next-to-nearest options contracts (diamond) with expiry greater than 6 days plus 1 month. When the nearest contract's expiry is less than 7 days, the next-to-next strategy uses the next-to-nearest contracts on the previous day(s), whereas the nearest-to-next one uses the nearest-term contracts. These two strategies are different only on the day(s) when the expiry of nearest-term option contracts is less than seven days.

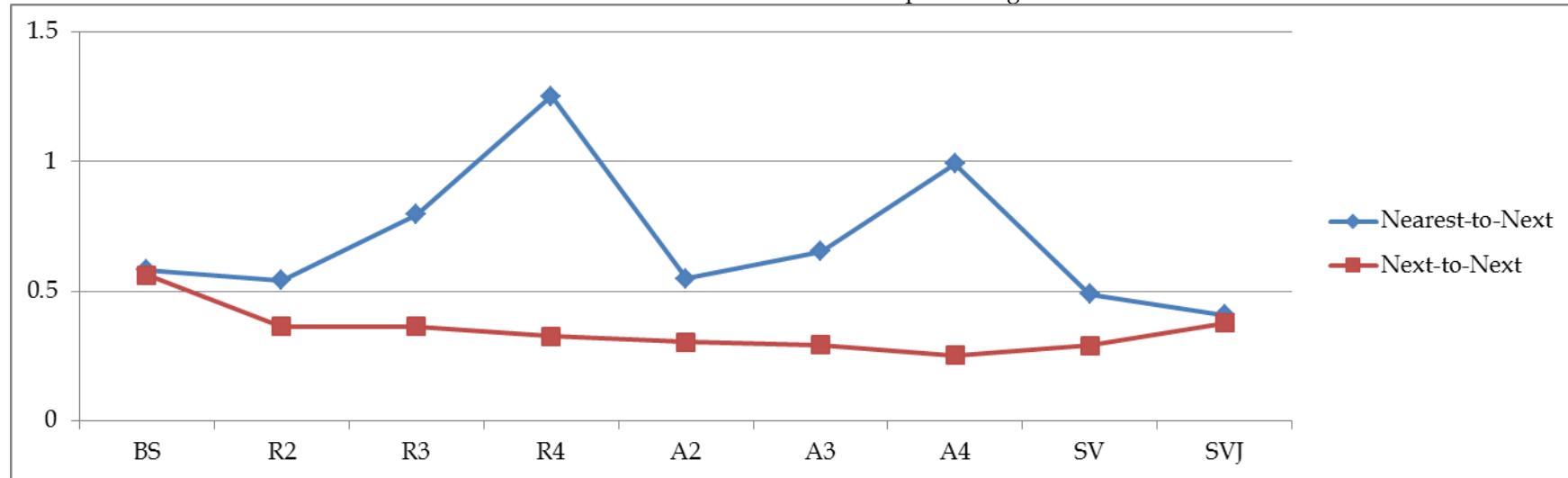


**Figure 2: Out-of-Sample Pricing Errors**

This figure shows the mean absolute percentage errors (MAPE) of out-of-sample pricing for each option pricing models with respect to the roll-over strategies. Panel A represents one-day ahead out-of-sample pricing errors and Panel B represents one-week ahead out-of-sample pricing errors. R1 is the ad hoc Black-Scholes model that considers the intercept and the moneyness as the independent variable. R2 is the ad hoc Black-Scholes model that considers the intercept, the moneyness and the square of the moneyness as the independent variables. R3 is the ad hoc Black-Scholes model that considers the intercept, the moneyness, the square and the third power of the moneyness as the independent variables. A1 is the ad hoc Black-Scholes model that considers the intercept and the strike price as the independent variables. A2 is the ad hoc Black-Scholes model that considers the intercept, the strike price and the square of the strike price as the independent variables. A3 is the ad hoc Black-Scholes model that considers the intercept, the strike price, and the square and the third power of the strike price as the independent variables. BS is the Black-Scholes (1973) option pricing model. SV is the option pricing model considering the continuous-time stochastic volatility. SVJ is the option pricing model considering the continuous-time stochastic volatility and the jumps.

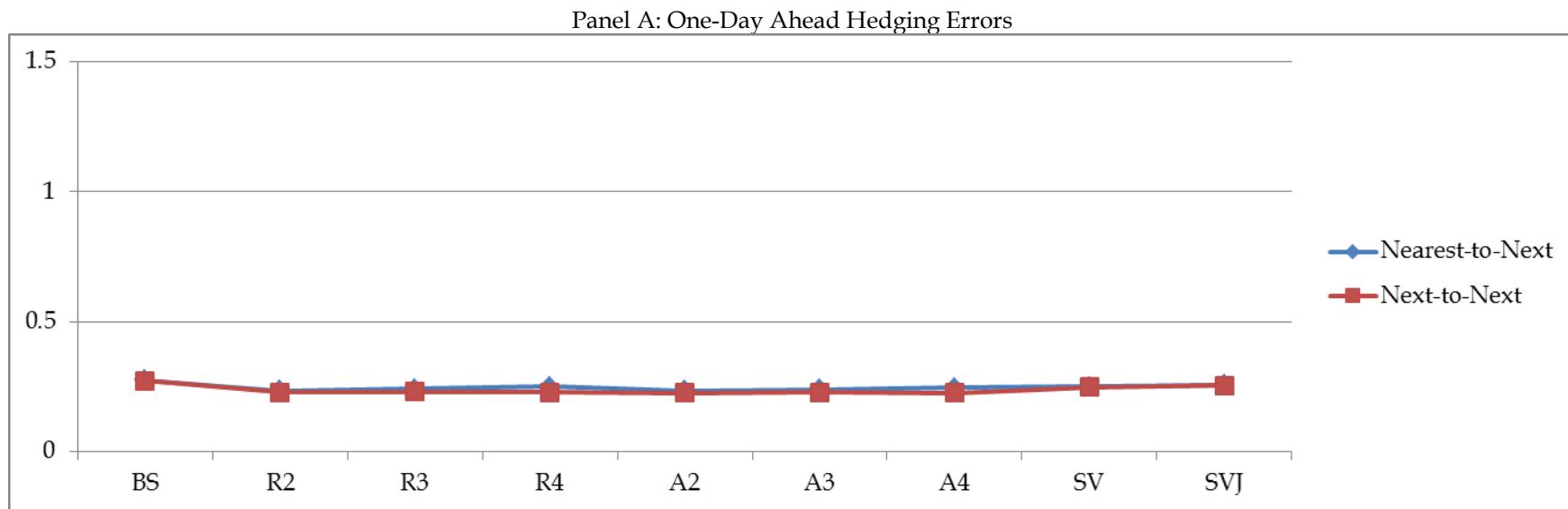


Panel B. One-Week Ahead Out-of-Sample Pricing Errors



**Figure 3: Hedging Errors**

This figure shows the mean absolute percentage errors (MAPE) of hedging for each option pricing models with respect to the roll-over strategies. Panel A represents one-day ahead hedging errors and Panel B represents one-week ahead hedging errors. R1 is the ad hoc Black-Scholes model that considers the intercept and the moneyness as the independent variables. R2 is the ad hoc Black-Scholes model that considers the intercept, the moneyness and the square of the moneyness as the independent variables. R3 is the ad hoc Black-Scholes model that considers the intercept, the moneyness, the square and the third power of the moneyness as the independent variables. A1 is the ad hoc Black-Scholes model that considers the intercept and the strike price as the independent variable. A2 is the ad hoc Black-Scholes model that considers the intercept, the strike price and the square of the strike price as the independent variables. A3 is the ad hoc Black-Scholes model that considers the intercept, the strike price, and the square and the third power of the strike price as the independent variables. BS is the Black-Scholes (1973) option pricing model. SV is the option pricing model considering the continuous-time stochastic volatility. SVJ is the option pricing model considering the continuous-time stochastic volatility and the jumps.



Panel B: One-Week Ahead Hedging Errors

