Joint affine term structure models: Conditioning information in international bond portfolios

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Keywords: government bonds, fixed income portfolios, portfolio optimization, joint affine term structure models, international bond investment

JEL–class.: G11, G12, G15

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I am grateful for comments from the participants of the brown bag series at Monash University and Jörg Laitenberger. All remaining errors are mine, of course.

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1. Introduction

It is well known that investors benefit from diversifying internationally.[1] Therefore, institutional investors will invest in government bonds that are issued by different countries rather than solely investing in their home country. In the challenge of managing such an international government bond portfolio it is essential to have a model that can cope with the cross section and time series of yields of *two* countries.

This paper proposes a model for conditional bond investment in international markets. For matching the variation of treasury yields in two economies, we rely on a joint affine term structure model (ATSM).[2] We identify an optimal government bond portfolio for two countries conditional on the information in both term structures of interest rates in discrete time. In an empirical study, the model is tested with treasury yield data from the US and UK in the period of 1983-2012. An interpretation of the factors as 'level', 'slope' and 'spread' is given and we identify common and local factors that drive the term structure of yields of both countries.[3] Thereafter, we calculate the optimal conditional portfolio proposed in the model and identify the common risk factors that drive the portfolio weights. In this manner we can study the interaction of the time series of treasury yields and the variation of portfolio weights. In our empirical study we can show that common risk factors in international bond returns are not only important for modelling the joint dynamics of interest rates, but can also explain the investment choice of an international bond investor.

One of the most puzzling phenomenons in international finance is the forward premium anomaly documented by Fama (1984). Since the variation in international term structure of interest rates and exchange rates depend on each other it is intuitive to look for risk driving factors that model the variation of treasury yields and exchange rates

[1] Grubel (1968), Solnik (1974), De Santis and Gerard (1997) et al. report significant benefits from diversifying internationally.

[2] See Backus et al. (2001), Bansal (1997), Dewachter and Maes (2001) and Egorov et al. (2011).

[3] In the sense of Litterman and Scheinkman (1991) and DeJong (2000) who give a factor interpretation for single term structure models.

jointly. Backus et al. (2001), Bansal (1997) and Hodrick and Vassalou (2002) were the first to study the risk driving factors in the context of two-currency term structure models. A problem that arises, when modelling interest rates and exchange rates jointly, is to capture the different volatility characteristics of both (see Inci and Lu, 2004). Together with the former, Dewachter and Maes (2001) model exchange rate movements with an additional risk driving factor.

There is a stream in literature that argues that the difference in variation of interest rates and exchange rates is better explained by including time variation in the risk premium (Brennan and Xia (2006), Sarno et al. (2012) and Graveline and Joslin (2011)). In these papers an extended affine formulation of the risk premium is used to explain that time variation.[4] However, Feldhütter et al. (2012) argue that investors prefer simple models (completely affine) to more complex models (essientially affine) even if they know that the simple model is wrong.[5] Since the focus of the paper lies on a risk minimizing international bond portfolio and not on exploiting the exchange rate risk premium, we follow the argumentation of Feldhütter et al. (2012) and stick to the completely affine term structure model. For completely affine ATSM's Egorov et al. (2011) provide a classification similar to the one of Dai and Singleton (2000) for single ATSM's.

Portfolio decisions are usually not made with continuous adjustments. Papers that model the joint dynamics of interest rate term structures for international bond portfolio application have focused on continuous time investment (see Dewachter and Maes, 2001). However, empirical studies on the forward discount puzzle draw a different picture. Eichenbaum and Evans (1995) first documented that after an interest rate increase, a currency continuous to appreciate for another 8 to 12 quarters before it starts to depreciate. One would expect this to happen if the investor makes discrete rather than continuous time portfolio adjustments. Therefore, we follow Bacchetta and van Wincoop

[4] Duffee (2002) and Dai and Singleton (2002) proposed an extended affine model to bring more flexibility in the risk premium for single term structure models.

[5] That is if they face parameter uncertainty, which is modelled with Marcov Chain Monte Carlo (MCMC).

(2010), who propose infrequent portfolio decisions for international bond investment, in fixing a discrete time investment horizon. Korn and Koziol (2006) propose a model for domestic bond portfolio optimization. For international bond portfolios Hunter and Simon (2004) can find diversification benefits. Driessen et al. (2003) and Juneja (2012) study the common risk factors of international bond portfolios but they do not propose a proper model to explain the variation in treasury yields.

The contribution of the present paper is twofold. Although there is growing literature concerned with the joint modelling of term structures of interest rates, surprisingly little is known about the implications for bond portfolio managers. Firstly, we propose a simple model for international bond investment for a discrete time investment horizon conditional on the joint term structure of interest rates. Building on the rich literature on joint ATSM's, we propose a portfolio model. Therefore, we extend the single ATSM of Korn and Koziol (2006) for joint ATSM's.[6] Secondly, we give an empirical study of our model for US and UK treasury yields in the period of 1983 to 2012. To our best knowledge, we are the first to give an interpretation of the common risk driving factors for international bond portfolios.[7] Building on our model we can show that the common risk driving factors in the two economies drive the variation in international bond portfolios, too. As a consequence we can show that joint ATSM's are capable for modeling an optimal portfolio conditional on the information in the cross section and time series of *two* countries. The remainder of the paper is organized as follows. Section '2 The Model' sets up the modeling framework, and specifies the optimal portfolio strategy. Section '3 Empirical Study' discusses the data, the estimation procedure and the portfolio strategy. In section 4 the results are presented. Section 5 concludes and technical details, figures and tables are shifted to the appendix.

[6] Korn and Koziol (2006) use an ATSM with uncorrelated Gaussian factors in the Ar representation. For using the model in the joint term structure model framework we have to extend the formulation to correlated factors in the conanical representation (see Dai and Singleton, 2000).

[7] Driessen (2005) and Juneja (2012) present a pure empirical study of common risk factors for bond returns without proposing a model for the joint term structure.

2. The Model

Great improvements have been made in modelling single term structures for pricing bonds, interest rate derivatives and bond portfolios.[8] Fulfilling the needs of an institutional investor, who wants to diversify internationally, is a little more precarious. Two country models are a significant extension of single country models in jointly modelling the dynamics of the term structures of interest rates and exchange rates. In the subsection 2.1 the model for matching the variation in cross section and time series of two term structures is presented and in subsection 2.2 we propose an optimal portfolio model that makes advances in joint ATSM's available for international bond investors.

2.1. Model of the Joint Term Structure

We follow Dai and Singleton (2000) and Egorov et al. (2011) in defining the price of a zero coupon bond. Let two economies be described by the probability space $(\Omega, \mathcal{F}, P)$ where P denotes the physical measure. Q and Q^* shall be the equivalent martingale measures for the US and the UK, respectively.[9] In the absence of arbitrage, the time-t prices of a US and a UK zero-coupon bond, that mature at $t+\tau$, $P(t,\tau)$ and $P^*(t,\tau)$ are given by

$$P(t,\tau) = E_t^{\mathbb{Q}}\left[\exp(-\int_t^{t+\tau} r(u)du)\right] \text{ and } \tag{1}$$

$$P^*(t,\tau) = E_t^{\mathbb{Q}^*}\left[\exp(-\int_t^{t+\tau} r^*(u)du)\right], \tag{2}$$

where $E^{\mathbb{Q}}$ and $E^{\mathbb{Q}^*}$ denote $\mathcal{F}_t$ conditional expectations under Q and Q^*, respectively. A joint ATSM is obtained under the assumption that the instantaneous short rates $r(t)$ and

[8] See Chapman and Pearson (2001) and Dai and Singleton (2003) for literature reviews.
[9] In the following a * shall indicate the foreign economy.

$r^*(t)$ are affine functions of a vector of latent state variables $X(t) = [X_1(t), X_2(t), \ldots, X_N(t)]'$,

$$r(t) = \delta_0 + \delta' X(t) \text{ and } r^*(t) = \delta_0^* + \delta^{*'} X(t) \quad (3)$$

where δ_0 and δ_0^* are scalars and δ' and $\delta^{*'}$ are $N \times 1$ vectors. $X(t)$ nests the local, as well as, the common factors that drive both economies. Common factors enter both expressions of $r(t)$ and $r^*(t)$ through non zero δ's and δ^*'s. Further more, the weighting of the factors for the specific country is expressed in the value of δ. If δ_i for the common factor tends to zero the dynamics of the short rate are (almost) exclusively driven by the local factor. If, in contrast, δ is equally weighted for both countries their exists a common risk factor that drives the dynamics of both economies. That has important implications for the international bond portfolio. If the short rate is (almost) mutually independent to the other economy there is diversification potential per se without emphasizing much effort in the risk management. If, on the other hand side, the bonds depend on common risk factors it is important *which* bonds of each country are in the portfolio to benefit from diversification. The local factors are forced to be mutually independent since they would not be local otherwise. They may depend on each other through the correlated common factors, though. This one joint ATSM can be decomposed in two single ATSM's if the local factors are mutually independent (see Egorov et al., 2011). The joint dynamics of $X(t)$ follow an affine diffusion of the form:

$$dX(t) = K[\vartheta - X(t)]dt + \Sigma S_t dW(t). \quad (4)$$

$W(t)$ is an N-dimensional independent standard Brownian motion under P, K and Σ are $N \times N$ parameter matrices and ϑ is an $N \times 1$ parameter vector. The definition of $X(t)$ given in equation (4) is a general formulation.[10] We further determine the characteristics

[10]Equation (4) nests famous stochastic processes in finance such as the Vasicek (1977) and Cox et al. (1985) model.

of the affine diffusion by specifying S_t. The matrix S_t is diagonal with (i,i)-th elements:

$$S_{t(ii)} = \sqrt{\alpha_i + \beta_i' X(t)},\ i = 1, \ldots, N, \tag{5}$$

where α_i is a scalar parameter and β_i is an $N \times 1$ parameter vector.

Papers that are mainly concerned with the exchange rate risk premia try to explain the different variation of interest rates and exchange rates with time varying risk premia (see Brennan and Xia (2006), Sarno et al. (2012) and Graveline and Joslin (2011)). However, in using constant risk premia we follow Feldhütter et al. (2012) who argue that investors prefer simple models (completely affine) to more complex models (essientially affine) even if they know that the simple model is wrong. The domestic risk premium for US bonds is defined as $\Lambda = \alpha \cdot \lambda_t$, where λ_t is an $N \times 1$ parameter vector. The risk premium is country specific and independent from the foreign risk premium, i.e. the risk premium parameter of the foreign factor is zero. Likewise, the UK risk premium is defined as $\Lambda^* = \alpha \cdot \lambda_t^*$, where λ_t^* is an $N \times 1$ parameter vector.

So far the model describes the joint dynamics of both term structures of interest rates. We can as well model the country specific dynamics of each country, separately. Under the risk neutral measure Q the affine diffusion X of the US reads:

$$dX(t) = K^Q[\vartheta^Q - X(t)] + \Sigma \cdot \alpha \cdot dW^Q(t) \tag{6}$$

where $dW^Q(t) = dW(t) - \Lambda$, ϑ^Q and K^Q represent the risk neutral measure. In the same way we define the diffusion for the UK under the risk neutral measure Q^*:

$$dX(t) = K^{Q*}[\vartheta^{Q*} - X(t)] + \Sigma \cdot \alpha \cdot dW^{Q*}(t) \tag{7}$$

where $dW^{Q*}(t) = dW(t) - \Lambda^*$, ϑ^{Q*} and K^{Q*} represent the risk neutral measure. Having outlined the short rates r and r^* and the underlying diffusion processes $X(t)$, we can now turn to the zero bond prices. Under the risk-neutral measure Q the price of an US zero-coupon bond reads

$$P(t,\tau) = \exp(-A(\tau) - B(\tau)'X(t)), \tag{8}$$

where $A(\cdot)$ and $B(\cdot)$ satisfy the ordinary differential equations (ODEs) (see Dai and Singleton, 2000):

$$\frac{dA(\tau)}{\tau} = \vartheta'\kappa B(\tau) - \frac{1}{2}\sum_{i=1}^{N}[\Sigma' B(\tau)]_i^2\alpha_i - \delta_0 \text{ and} \tag{9}$$

$$\frac{dB(\tau)}{\tau} = -\kappa B(\tau) - \frac{1}{2}\sum_{i=1}^{N}[\Sigma' B(\tau)]_i^2\beta_i + \delta'. \tag{10}$$

Under the risk-neutral measure Q^* the price of a US zero-coupon bond follows the same dynamics

$$P^*(t,\tau) = \exp(-A^*(\tau) - B^*(\tau)'X(t)), \tag{11}$$

where $A^*(\cdot)$ and $B^*(\cdot)$ satisfy ordinary differential equations(ODEs) similar to the ones in equation(9). With the initial conditions $A(0) = A^*(0) = 0_{N\times 1}$ and $B(0) = B^*(0) = 0_{N\times 1}$ and $r(t)$ and $r^*(t)$ defined above, these ODEs are completely specified. For the process $X(t)$ the solutions to these ODEs are available in closed form. Kim and Orphanides (2005) give a very handy closed form solution of the bond prices in vector notation. The yield of a zero-coupon bond maturying at τ is given by:

$$y_{t,\tau} = A_\tau + B_\tau \cdot X_t \tag{12}$$

with

$$
\begin{aligned}
A_\tau =& -\frac{1}{\tau}[(K\mu)'(m_{1,\tau} - \tau I)K^{-1'}\delta + \\
& \frac{1}{2}\delta' K^{-1}(m_{2,\tau - \Sigma\Sigma' m_{1,\tau}} - m_{1,\tau}\Sigma\Sigma' + \tau\Sigma\Sigma')K^{-1'}\delta - \tau\delta_0] \qquad (13) \\
B_\tau =& \frac{1}{\tau} m_{1,\tau}\delta \qquad (14)
\end{aligned}
$$

where

$$
\begin{aligned}
m_{1,\tau} =& -K^{-1'}(exp(-K'\tau) - I) \qquad (15) \\
m_{2,\tau} =& -vec^{-1}((K \otimes I) + (I \otimes K))^{-1} vec(exp(-K\tau)\Sigma\Sigma' exp(-K'\tau - \Sigma\Sigma')). \qquad (16)
\end{aligned}
$$

A, B are functions of $K, \mu, \Sigma, \delta, \delta_0$ and the risk neutral and physical parameters correspond in the following way:

$$
\begin{aligned}
K =& K^Q - \Sigma\Lambda \qquad (17) \\
\mu =& K^{-1}(K^Q\mu^Q - \Sigma\Lambda) \qquad (18)
\end{aligned}
$$

To avoid over identification we follow the restrictions of Dai and Singleton (2000). Therefore, K is lower triangle and Σ is the identity matrix. To completely charactize the joint dynamics of the term structures of interest rates, we need to model the Dollar and Pound exchange rate. If we assume complete markets and the absence of arbitrage we know from Backus et al. (2001) and Ahn (2004) that the exchange rate S is:

$$
\frac{S(t+\tau)}{S(t)} = \frac{M^*(t+\tau)}{M(t+\tau)} \qquad (19)
$$

where M and M^* are the pricing kernels for the US and UK market, respectively. Following Egorov et al. (2011) we assume that the pricing kernels follow the form:

$$\frac{dM(t)}{M(t)} = -\, r(t)dt - \Lambda' dW(t) \tag{20}$$

$$\frac{dM^*(t)}{M^*(t)} = -\, r^*(t)dt - \Lambda^{*'} dW(t) \tag{21}$$

Having outlined the affine diffusion that underlies the short rate process and the pricing kernels of the two economies, we can obtain the change in the exchange rate in logarithmic form (see Backus et al. (2001) and Inci and Lu (2004)):

$$ds(t) = (r(t) - r^*(t))dt + \frac{1}{2}\sum_{i=1}^{N}(\lambda_i^2 - \lambda_i^{*2})dt + \sum_{i=1}^{N}(\lambda_i - \lambda_i^*)dW \tag{22}$$

Formulas for the expected exchange rate $E[s]$ and variance of the exchange rate $Var[s]$ are given in the appendix. For our empirical study we use a four factor model for the joint dynamics of both countries. With S defined in equation (5) we have zero factors govern the instantaneous variance of the short rate. This is a $A_0(4)$ model in the Dai and Singleton (2000) sense. In our model the first two factors are common factors and model the joint dynamics (see Egorov et al., 2011). The US and UK term structure of interest rates is modeled by one local factor each. The local factors are mutually independent. Under the physical measure the diffusion process is given by:

$$d\begin{bmatrix} X_{1t} \\ X_{2t} \\ X_{3t} \\ X_{4t} \end{bmatrix} = \begin{bmatrix} \kappa_{11} & 0 & 0 & 0 \\ \kappa_{21} & \kappa_{22} & 0 & 0 \\ \kappa_{31} & \kappa_{32} & \kappa_{33} & 0 \\ \kappa_{41} & \kappa_{42} & 0 & \kappa_{44} \end{bmatrix} \begin{bmatrix} -X_{1t} \\ -X_{2t} \\ -X_{3t} \\ -X_{4t} \end{bmatrix} dt + d\begin{bmatrix} W_{1t} \\ W_{2t} \\ W_{3t} \\ W_{4t} \end{bmatrix}. \tag{23}$$

Without loss of generality X_1 and X_2 are assumed to be the common factors. X_3 is the US local factor and X_4 is the UK local factor. As both local factors are required to be mutually independent $\kappa_{43} = 0$.

2.2. The Optimal Portfolio

In the model above the joint interest rates of both countries and the resulting exchange rate were defined. Since institutional investors will not only hold a single bond but rather manage a whole portfolio, it is intuitive to ask for applications for portfolio management. The short rate dynamics in the joint ATSM are modelled in a continuous time setting. That might be the reason why the few papers that are concerned with portfolio management, model the optimal portfolio in continuous time (see Dewachter and Maes, 2001). However, empirical studies of the forward premium puzzle indicate that modeling the investor's portfolio decision in discrete time might help to explain the phenomenon better (see Bacchetta and van Wincoop, 2010).[11]

With the joint ATSM presented above we have an expectation of the future short rate drift and volatility and its bond prices. Therefore, we have all the information to calculate the expected returns and covariances of these bonds conditional on the information in the term structure of interest rates. In doing so, we extend the model of Korn and Koziol (2006) for international bonds with an underlying joint ATSM. Firstly, for modelling an international bond portfolio we extend the original model from single to joint term structure modelling. Secondly, as Dai and Singleton (2000) state, it is important to incorporate correlation between the factors in a single country model. Therefore, it is even more important to allow for correlations between common and local factors in a joint ATSM. Thirdly, we use the canonical representation in the Dai and Singleton (2000) sense. For modelling two term structures jointly it is essential to allow the short rate to depend on the factors with distinct weight (see equation (3)). Otherwise, the

[11] The focus of Bacchetta and van Wincoop (2010) is more on international macro. They study the question whether a model in which the investor makes his portfolio decision infrequently can explain the forward premium puzzle better. They do not help the investor in *making* a portfolio and linking it the conditional information in both term structures of interest rates.

short rate is forced to depend on the common factors, even if there is no justification in the data.[12]

The investor can choose a combination of zero coupon from two countries and different time to maturity $\hat{T} = T_0 < T_1 < \cdots < T_{\bar{T}}$ for each country. This leads to $2\bar{N}$ bonds to invest in. $\mu \in \mathbb{R}^{2\bar{N}}$ is the vector of expected returns and $\Sigma \in \mathbb{R}^{2\bar{N}\times 2\bar{N}}$ is the matrix of covariances:

$$\mu = \begin{bmatrix} \mu(T_1) \\ \vdots \\ \mu(T_N) \\ \mu^*(T_1) \\ \vdots \\ \mu^*(T_N) \end{bmatrix}, \Sigma = \begin{bmatrix} \sigma(T_1) & \cdots & \sigma^{,}(T_1,T_N) & \sigma^{,*}(T_1,T_1) & \cdots & \sigma^{,*}(T_1,T_N) \\ \vdots & \ddots & & & & \vdots \\ \sigma^{,}(T_N,T_1) & & & & & \sigma^{,*}(T_N,T_N) \\ \sigma^{*,}(T_1,T_1) & & & & & \sigma^{*,*}(T_1,T_N) \\ \vdots & & & & \ddots & \vdots \\ \sigma^{*,}(T_N,T_1) & \cdots & \sigma^{*,}(T_N,T_N) & \sigma^{*,*}(T_N,T_1) & \cdots & \sigma^{*}(T_N) \end{bmatrix}.$$

μ and σ can be evaluated as follows:[13]

$$\begin{aligned} \mu\,(T_i) &= \frac{e^{M_{(1)}(T_i)+\frac{1}{2}S_{(1)}(T_i)^2}}{P\,(0,T_i)} - 1, \qquad (24)\\ \mu^*(T_i) &= \frac{e^{M^*_{(1)}(T_i)+\frac{1}{2}S^*_{(1)}(T_i)^2}}{P^*(0,T_i)} - 1,\\ \sigma\,(T_i) &= \frac{e^{2\cdot M_{(1)}(T_i)+S_{(1)}(T_i)^2}\cdot(e^{S_{(1)}(T_i)^2}-1)}{P\,(0,T_i)},\\ \sigma^*(T_i) &= \frac{e^{2\cdot M^*_{(1)}(T_i)+S^*_{(1)}(T_i)^2}\cdot(e^{S^*_{(1)}(T_i)^2}-1)}{P^*(0,T_i)},\\ \sigma^{*,}(T_i,T_j) &= \frac{e^{M^{*,}_{(2)}(T_i,T_j)+\frac{1}{2}S^{*,}_{(2)}(T_i,T_j)^2}}{P^*(0,T_i)\cdot P\,(0,T_j)} - \frac{e^{M^*_{(1)}(T_i)+M_{(1)}(T_j)+\frac{1}{2}(S^*_{(1)}(T_i)^2+S_{(1)}(T_j)^2)}}{P^*(0,T_i)\cdot P\,(0,T_j)} \end{aligned}$$

[12]Korn and Koziol (2006) propose an optimal portfolio model for a domestic german bond portfolio with an uncorrelated Gaussian single TSM in the A_r representation (see Babbs and Nowman, 1999).

[13]One uses the fact that the state variables X_i are normally distributed. Hence e^X is log-normally distributed and it is $E\left[e^X\right] = e^{E[X]+\frac{1}{2}Var[X]}$ and $Var\left[e^X\right] = E\left[e^X\right]^2(e^{Var[X]}-1)$. In contrast to Korn and Koziol (2006), the Brownian motions are correlated. As Law (2007) shows however, only the calculations of the covariances change: $Cov\left[e^{X_i},e^{X_j}\right] = (e^{Cov\left[X_i,X_j\right]}-1)e^{E[X_i]+E\left[X_j\right]+\frac{Var[X_i]+Var\left[X_j\right]}{2}}$.

where

$$
\begin{aligned}
M^{*}_{(1)}(T_i) =& A^{*}(T_i) + [B(T_i)]^T \cdot E^{\mathbb{P}}_0 [X_n(t)] , \\
M_{(1)}(T_i) =& A^{*}(T_i) + [B(T_i)]^T \cdot E^{\mathbb{P}}_0 [X_n(t)] , \\
S^{*}_{(1)}(T_i)^2 =& [B(T_i)]^T \cdot [Var^{\mathbb{P}}_0 [X_n(t)]] \cdot [B(T_i)] + s^2(\epsilon), \\
M^{,}_{(2)}(T_i, T_j) =& A\ (T_i) + A\ (T_j) + [B(T_i) + B(T_j)]^T \cdot E^{\mathbb{P}}_0 [X_n(t)] , \\
M^{*,*}_{(2)}(T_i, T_j) =& A^{*}(T_i) + A^{*}(T_j) + [B(T_i) + B(T_j)]^T \cdot E^{\mathbb{P}}_0 [X_n(t)] , \\
M^{*,}_{(2)}(T_i, T_j) =& A^{*}(T_i) + A\ (T_j) + [B(T_i) + B(T_j)]^T \cdot E^{\mathbb{P}}_0 [X_n(t)] \text{ and} \\
S^{*,}_{(2)}(T_i, T_j)^2 =& [B(T_i) + B(T_j)]^T \cdot [Var^{\mathbb{P}}_0 [X_n(t)]] \cdot [B(T_i) + B(T_j)] + s^2(\epsilon).
\end{aligned}
$$

As equation (24) shows, the expected returns and covariances are available in closed form. That makes the model very easy to implement for risk management of an international bond portfolio. Having calculated 'μ' and 'σ' for the bond portfolio, the theoretic model is fully specified and we can bring the model to the data. That shall bring some clarity on how the cross section and time series of treasury yields is related to the portfolio weights evolution.

3. Empirical Study

In the former section we have fully specified a theoretic model that enables the investor to sep up an optimal international bond portfolio. To see whether the model of the present paper can explain the variation in the time series and cross section of treasury yields of *two* countries, we test the model empirically with government bond data. The question is whether that variation can explain the evolution of the portfolio weights of an international bond portfolio. We follow Driessen et al. (2003) who give an interpretation of common factors in international bond portfolios in an empirical study.[14] In the following section we describe the data, formulate the state space model to cope with the panel data and present the portfolio strategy.

3.1. Data

We aim to study an international bond portfolio consisting of US and UK treasury bonds. For the sake of simplicity, we use zero coupon government bonds and leave coupon payments aside. The data is provided by the US Federal Reserve and the Bank of England (see Gurkaynak et al., 2006). The period of 1979 to 1982 is known to be econometrically precarious because of the so called US Federal Reserve's experiment (see Chapman and Pearson, 2001). Therefore, we exclude that period. Since the optimal portfolio is predicted for the period ahead, we can not allow for a brake in the time series. Hence, we do not take treasury yields before 1979 into account. That is the reason why we investigate the period from January 1983 to July 2012. We follow Egorov et al. (2011) and use daily observations of 6 months, 2-, 5- and 10-Years treasury yields from the US and the UK.

[insert Figure 1 about here.]

In figure 1 the US and UK treasury yields are reported. The time series of the US treasury yields is shown in the first row. In the majoirty of the period the US term structure is normal (upward sloping). Further more, the short end of the term structure is more volatile than the long end and the interest rate level decreases consistently over time. Roughly the same picture can be drawn for the time series of UK treasury yields from the second row of figure 1. Most of the period the UK term structure is

[14]Driessen et al. (2003) study the common factors in international bond portfolios for hedged and unhedged portfolios in the Litterman and Scheinkman (1991) sense. However, they do not give a theoretic model that can explain the variation in the cross section and time series of international bonds.

normal although there are more periods of an inverse term structure than there are for the US. The short end of the yield curve is even more volatile than the US equivalent is. Decreasing interest rate levels can be found for the UK, too. Having described the time series of the US and UK term structures, the question to be answered here is, how the yield movement over time does effect international bond portfolio management. Therefor, we will present the parameter estimation procedure in the following subsection.

3.2. State Space Model

Term structure models have the positive feature that they capture the time series dynamics in the factors and the cross section is a resulting function of these factors and the time to maturity (see DeJong, 2000). Dealing with such panel data, it is a natural approach to define a state space model. The factors can be found in the transition equation and the treasury yields of different maturities are in the measurement equation. Several techniques have been proposed for estimating the model parameters. The efficient method of moments is applied to term structure models by Dai and Singleton (2000). Eraker (2001) and Feldhütter et al. (2012) use Markov-Chain Monte Carlo methods for estimating term structure models. However, these methods are computationally very intensive. Since our model relies on a completely affine Gaussian setup, we follow Babbs and Nowman (1999) in using Kalman Filtering with a very straight forward direct maximum likelihood estimation.

At time t let there be observed zero-coupon bond yields with maturity τ_1 trough τ_k in vector y_t. Vector $A(\tau_i)$ and $B(\tau_i)$ are as defined equation 9:

$$y_t = \begin{bmatrix} Y_t^{US}(\tau_1) \\ \vdots \\ Y_t^{US}(\tau_k) \\ Y_t^{UK}(\tau_1) \\ \vdots \\ Y_t^{UK}(\tau_k) \end{bmatrix}, A = \begin{bmatrix} A(\tau_1) \\ \vdots \\ A(\tau_k) \\ A^*(\tau_1) \\ \vdots \\ A^*(\tau_k) \end{bmatrix}, B = \begin{bmatrix} B(\tau_1)' \\ \vdots \\ B(\tau_k)' \\ B^*(\tau_1)' \\ \vdots \\ B^*(\tau_k)' \end{bmatrix}. \tag{25}$$

The state space model form reads:[15]

$$y_t = A + BX_t + \epsilon \tag{26}$$

$$X_{t+h} = e^{-Kh}X_t + (I - e^{-Kh}\mu + \upsilon_{t+h}) \tag{27}$$

where h is the time between two observations. The measurement equation is the first equation in the system and is a function of the parameters $(\delta_0, \delta_0^*, \delta_{Xi}, \delta_{Xi}^*, K, \lambda, \lambda^*)$ and an error term ϵ. DeJong (2000) stated that it is important to observe more treasury yields than model factors to identify the risk premium, accurately. We follow Duan and Simonato (1999) and Geyer and Pichler (1997) in observing all maturities with a certain error ϵ. This error is assumed to be serially and cross sectionally uncorralleted. The second equation is the transition equation with the conditional mean and variance of the factors with an error term $\upsilon_{t+h} \sim N(0, \Omega_n)$ where Ω_n is :(see Kim and Orphanides, 2005)

$$vec(\Omega_n) = -((K \otimes I) + (I \otimes K))^{-1} vec(e-(K + K')h - I). \tag{28}$$

[15] The notation e^X, where X is a square matrix, denotes the matrix exponential $e^X = I + X + X^2/2 + X^3/6 + \cdots$ (see Kim and Orphanides, 2005).

3.3. Portfolio Strategy

An international bond portfolio has two main risk driving factors - interest rate and exchange rate risk. The problem in modeling the joint dynamics of interest rates and exchange rates is the different variation of the two (see Dewachter and Maes (2001), Graveline and Joslin (2011) et al.). That difference is even more important in portfolio management, because it changes the risk characteristics of the portfolio considerably. In the present paper, we want to focus on the portfolio risk caused by the variation of the term structure rather than the variation of the exchange rates.[16] Therefore, we follow Hunter and Simon (2004) and study a hedged international bond portfolio. Yet, as Driessen et al. (2003) point out, since the difference between forward currency rates and current spot exchange rates are usually close to zero, the *returns* of a hedged and an unhedged portfolio will not distinct significantly. Therefore, hedged bond returns are primarily driven by the variation in the underlying term structure of interest rates.

According to Morey and Simpson (2001) there are three different currency hedging strategies. In their classification of (a) unhedged, (b) always hedged and (c) selectively hedged, we follow strategy (b) and always hedge the exchange rate risk exposure with forward contracts. From formula (1) we get the expected amount of foreign currency at the end of the investment horizon and buy a 12 month forward contract of that amount.[17] Morey and Simpson (2001) identify the transaction costs of a 12 months forward rate contract between the US and UK to be 0.001093 of the contract value.

The investment set consists of zero coupon bonds from the US and UK. For each country there are bonds with 1, 2, 5 and 10 years time to maturity. In step (1) we take the parameter set estimated with maximum likelihood in the former subsection. That set of parameters will determine the optimal portfolios of the whole empirical study conditional

[16]For a treatment of exchange rate risk premia see Brennan and Xia (2006).

[17]Since formula (1) gives us only the expected value of the held amount at the end of the period this amount is by definition risky. Every unexpected bond price change will cause a non hedged risk exposure. However, Driessen et al. (2003) finds that this unhedged and therefore risky exposure is negligible.

on the information in the term structure of interest rates. The conditioning information is brought into the empirical study through the factors which reflect the information currently available at this point in time. In step (2a) we take the common (X_{c1} and X_{c2}) and local factors (X_{US} and X_{UK}) on the 2^{nd} of January 1983($[X_{c1}, X_{c2}, X_{US}, X_{UK}]_{02/01/1983}$). Taking the parameters and factors from subsection 3.2 we can calculate the optimal portfolio with the model in subsection 2.2. In the μ-σ sense we calculate the optimal portfolio for an investment horizon of 1 Year and a fixed volatility of $\sigma = 10\%$. Short selling is permitted. Then we roll over one day. In step (2b) we take the factors $[X_{c1}, X_{c2}, X_{US}, X_{UK}]_{03/01/1983}$ and repeat the procedure. Since we study an investemnt horizon of 260 trading days we keep rolling this window till the 31^{st} of July 2011.

4. Results

4.1. Parameter Estimates

For matching the time series and cross section of treasury yields of the two countries the estimated parameters are reported in Table 1. Each standard error is given in parenthesis. The first panel reports the factor independent short rate constants. The middle panel shows the factor depended parameters. $i = 1, 2$ are the common factors, $i = 3$ is the local factor US and $i = 4$ is the local factors UK. Note that the local parameters US (UK) are set to zero with no standard error for $i = 4$ ($i = 3$). The model estimates rely equally on the common factors with values from $0.0097 (= \delta^*_{X1})$ to $0.0133 (= \delta_{X2})$ and the local factors with values from $0.0103 (= \delta_{X3})$ to $0.0136 (= \delta^*_{X4})$. These parameters are precisely estimated with a small standard error. K defines the factor dependence structure of the joint ATSM and each parameter κ is the same for both countries. Yet, the local Factors $i = 3, 4$ are mutually independent and the factor dependence is set to zero ($\kappa_{43} = 0$). In line with Feldhütter et al. (2012), the risk premium parameters λ_{Xi} λ^*_{Xi} are estimated with great standard error. In the last panel the standard deviation of observational error is reported. In line with Egorov et al. (2011) the model matches the data well. We obtain the biggest observational error for 10year UK treasury yields with $\epsilon^* = 0.0031$.

[insert Table 1 about here.]

In formulating the state space model, it became clear that the time series of treasury yields is represented by the evolution of the factors. We aim to get a better understanding of how the time series of yields influences the conditional portfolio weights later on. Hence we need to relate the common and local factor to the corresponding yields, first. For a standard three factor ATSM there is consensus in literature to interpret the first three factors as 'level' 'slope' and 'curvature' (see DeJong (2000), Litterman and Scheinkman (1991), Babbs and Nowman (1999) et al.). However, the case is a little more precarious in the present multi country model. The fitted common and local factors and treasury yields are reported in Figure 2. The first panel of the graph shows the common factors of the model. The first common factor is fitted and with a correlation of 0.9848 highly correlated to the 'level' of the US treasury yields. The second common factor is fitted to the spread of US and UK 5 year treasury yields in the second graph of the first row. With 0.7175 the correlation of the factor and the data is still high but lower than for the first factor. These two common factors shall explain the common movements of both economies. The variation in the yiels that can not be explained by the common factors will be explained by the local factors in the second row of the figure. On the left hand side the local factor US is fitted to the slope the US data which is the spread of the 10year treasury yield minus the 6m treasury yield. With 0.9382 the local factor US is highly correlated to the slope in the US data which could not be captured the common factors. The last graph of the figure fits the local factor UK to the shortrate UK. This is the variation in the UK data that can not be matched by the common factors and has a high correlation of 0.9460. These findings are line with Juneja (2012) and Driessen et al.

(2003). Driessen et al. (2003) find that the common risk driving factors in international bond returns are the 'level' and 'slope' of the underlying terms structure of yields and the spread between the different yield curves.

[insert Figure 2 about here.]

4.2. Conditional Portfolio Weights

Having estimated the parameters of our joint ATSM in subsection 4.1 we can realize the optimal portfolio of the model presented in section 2. The beauty of term structure models is that the variation in the time series dimension of interest rates is captured in the latent factors. That enables us to set up on optimal bond portfolio *conditional* on the present state of the economy. Precisely speaking, on the present state of two economies since the common and local factors capture the variation of both countries. We calculate the portfolio weights as presented in subsection 3.3.

The evolution of portfolio weights is reported in figure 3.[18] The first graph in the upper left position of the figure shows the fitted 'level' of US and UK treasury yields and the evolution of the long end portfolio weights. The long end portfolio weights are defined as the amount invested in US and UK bonds with 10 years to maturity. The correlations of -0.8573 (US) and -0.8617 (U.K) are highly negative indicating that an increase of interest rate levels leads to a decrease of the amount invested in long maturity bonds. This is caused by the decreasing relative attractiveness of long bonds. When the overall level of interest rates rises, the return of short bonds also increases. In that case long bonds have relatively lower excess returns in comparison to short bonds with considerable more risk. Therefore, long bonds become less attractive and the amount invested is reduced. The second graph in the first row fits the slope of US and UK treasury yield curve and the short end portfolio weights.[19] The short end portfolio weights are the sum of 1

[18] The same results are available for fitted factors and portfolio weights to see that the model and software work properly. For the sake of brevity, we do not report these results here. The factor and portfolio weight evolution is available from the authors upon request.

[19] As in the former section the slope defined as the spread of the 10year and the 6months maturity bond.

and 2 year bonds for the US and UK. The correlations of -0.6821(US) and -0.7139(UK) indicate that if the slope of the data increases the amount invested in the short end decreases. That is in the same line of argumentation as in the previous graph. The long end bonds are more attractive, the higher the spread between long and short end is. On the other hand side if the yield curve becomes inverse (negatively sloped) the portfolio weights shift to the short end.

[insert Figure 3 about here.]

The first graph of the second row shows the fitted short ends of the US and UK yield curves and the Portfolio slope. The short end of the yield curve is simply the 6 months bond. The Portfolio slope is defined as the 10 year bond minus the sum of the 1 and 2 year bonds of the US and the UK portfolio weights. The correlations of -0.7461 (US) and -0.7753 (UK) indicate that if the short end yields increase the money is invested in short maturity bonds and the amount of long maturity bonds is reduced. The last graph in the second row fits the short end of the treasury yield curve to the portfolio duration. The portfolio duration is measured as the amount invested in the bonds(w) times the bond's maturity ($[w_1, \cdots w_{10}, w_1^*, \cdots, w_{10}^*]' \cdot [1, 2, 5, 10, 1, 2, 5, 10]$). The correlations of the short end bonds and the portfolio duration are -0.7686 (US) and -0.8077 (UK). That is what you would expect a risk averse investor to do since he will decrease the portfolio duration when the short rate increase and vice versa. Our results are in line with Juneja (2012) and Driessen et al. (2003) who report that the common factors relate mostly to the 'level' and 'slope' of the US treasury yields. Our empirical study shows that common factors in international bond returns are not only an empirical phenomenon. The empirical findings can be supported by the proposed model and the model can link the investor's decision conditional on the common factors in international bond returns.

5. Conclusion

In this paper, we proposed a simple model for international bond investment. The investor can optimize a government bond portfolio in a discrete time investment horizon conditional on the information in the term structure of two countries. The variation in the cross section and time series of treasury yields is captured by a joint ATSM. Easy closed form solutions for the return and variance were given. The model is tested in an empirical study of US and UK government bonds in the period of 1983 to 2012. An interpretation of the factors as 'level', 'slope' and 'spread' not only helped to interpret the cross section and time series of the treasury yields but also to understand the evolution of our conditional portfolio weights.

We showed that the 'level' of treasury yields is highly correlated with a decrease in the long end portfolio weights. Put it differently, when the overall 'level' of interest rates increases the long maturity bonds get unattractive since the short maturity bonds pay relatively high returns with far lower risk. The 'slope' of yields is highly correlated with a decrease in the short bond portfolio weights. That is rational as well. When the spread between long and short end becomes larger the short end has a significant lower return and should therefore be under weighted. Finally, the short end of treasury yields is highly correlated with the slope of the portfolio and a reduction in portfolio duration. A risk averse investor wants to be compensated with a significant higher return if the investment bears more risk. When the short rate increases their is to earn a high return with relatively low risk associated with the short time to maturity. The long end portfolio weights become less attractive in terms of return and bear the same high risk. Therefore, the slope of the portfolio increases and the duration decreases. These findings are line with Juneja (2012) and Driessen et al. (2003). Driessen et al. (2003) find that the common risk driving factors in international bond returns are the 'level' and 'slope' of the underlying term structure of yields and the spread between the different yield curves.

Our empirical study showed that common factors in international bond returns are not only an empirical phenomenon. The empirical findings can be supported by the proposed model and the model can link the investor decision conditional on the common factors in international bond returns.

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A. Appendix: Exchange Rate Calculations

We follow Dewachter and Maes (2001) and derive the expected discrete-time exchange rate. We take the conditional expectation of the exchange rate dynamics from equation (22):

$$E_t[ds(t)] = [\delta_0 + \delta' X(t) - (\delta_0^* + \delta^{*'} X(t)) + \frac{1}{2}[\lambda^2 - \lambda^{*2}]' \cdot I]dt \tag{29}$$

By definition it is,

$$\begin{aligned} E_t[s(t+\tau) - s(t)] &= \int_t^{t+\tau} E_t[ds(u)] \\ &= \int_t^{t+\tau} E_t[\delta_0 + \delta' X(u) - (\delta_0^* + \delta^{*'} X(u))d(u)] + \frac{1}{2}[\lambda^2 - \lambda^{2*}]' \cdot [I \cdot \tau] \end{aligned} \tag{30}$$

With the Expectation of the diffusion X,

$$E_t[X(t+\tau)] = e^{-K\tau} X(t) + (I - e^{-K\tau}\mu) \tag{31}$$

and evaluating the integral gives:

$$\begin{aligned} E_t[s(t+\tau) - s(t)] =& \tau(\delta_0 - \delta_0^*) + [\delta - \delta^*]' \cdot K^{-1} \cdot [(-e^{-K\cdot\tau} + I) \cdot X(t) + \mu \cdot (K \cdot \tau + e^{-K\cdot\tau} + I)] \\ & + \tfrac{1}{2}[\lambda^2 - \lambda^{2*}]' \cdot [I \cdot \tau] \end{aligned} \tag{32}$$

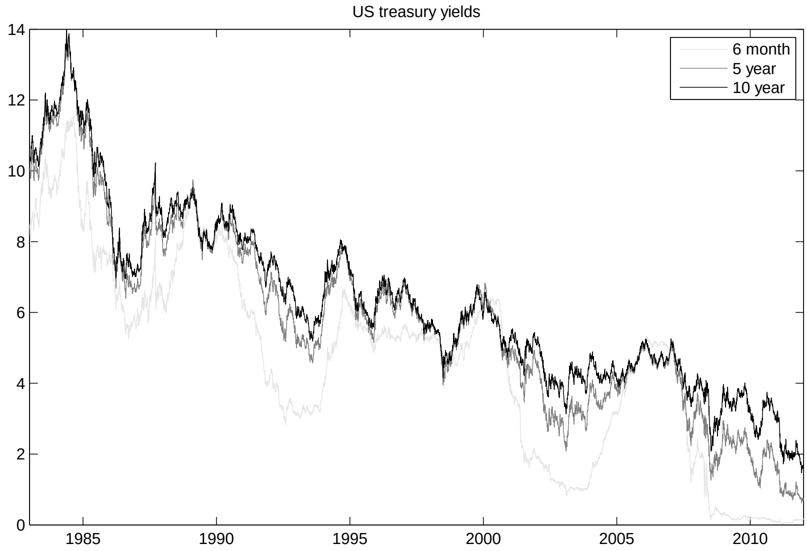
Similar calculations yield to the variance of the exchange rate change:

$$\begin{aligned} Var_t[s(t+\tau) - s(t)] =& [\delta - \delta^*]' \cdot vec^{-1}[-(K \otimes I + I \otimes K)^{-1} \cdot vec((K + K')^{-1} \cdot e^{-(K+K')\tau} - (I\tau + I))] \cdot [\delta - \delta^*] \\ & + [\lambda - \lambda^*]' \cdot I\tau \end{aligned} \tag{33}$$

Table 1: **4-factor joint TSM parameter estimates**

δ_0		0.1188		
		(0.0456)		
δ_0^*		0.2500		
		(0.0604)		
	i=1	i=2	i=3	i=4
δ_{Xi}	0.0127	0.0133	0.0103	0
	(0.0069)	(0.0068)	(0.0076)	-
δ_{Xi}^*	0.0097	0.0131	0	0.0136
	(0.0060)	(0.0079)	-	(0.0061)
κ_{1i}	0.3314	0	0	0
	(0.5168)	-	-	-
κ_{2i}	-0.1719	0.1000	0	0
	(0.1396)	(0.6555)	-	-
κ_{3i}	-0.1977	0.0841	0.1000	0
	(0.1295)	(0.2149)	(0.6982)	-
κ_{4i}	0.1717	-0.0405	0	0.8161
	(0.1956)	(0.1199)	-	(0.4397)
λ_{Xi}	0.0055	-1.5654	0.6850	0
	(1.5463)	(0.5936)	(0.7471)	-
λ_{Xi}^*	-0.1609	-0.9873	0	-1.6639
	(1.0456)	(1.8319)	-	(1.2250)
	6m	2y	5y	10y
ϵ	0.0000	0.0015	0.0000	0.0017
	(0.0000)	(0.010)	(0.0000)	(0.0011)
ϵ^*	0.0000	0.0015	0.0001	0.0031
	(0.0000)	(0.0011)	(0.0001)	(0.0018)

The Table reports the estimation results from the four-factor joint ATSM. The estimation is done using daily US and UK treasury yield data from January 1983 to July 2012. We report the parameter estimates and the standard errors in parentheses. A * indicates parameters for the UK market. ϵ is the standard deviation of observational error associated with the 6 months, 2-, 5- and 10-Years treasury yields from the US and the UK. All other coefficients for the model are described in the text.



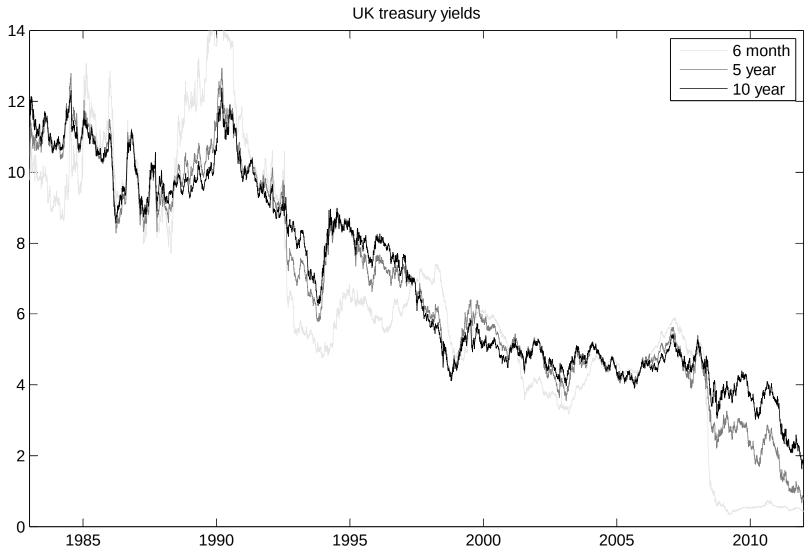


Figure 1: **Time-series of 6 months, 2-, 5- and 10-year zero coupon US and UK Treasury yields.**
The maximum likelihood estimataion is based on panel data of daily US and UK Treasury yields from the 2nd of January 1983 to the 31st of July 2012.

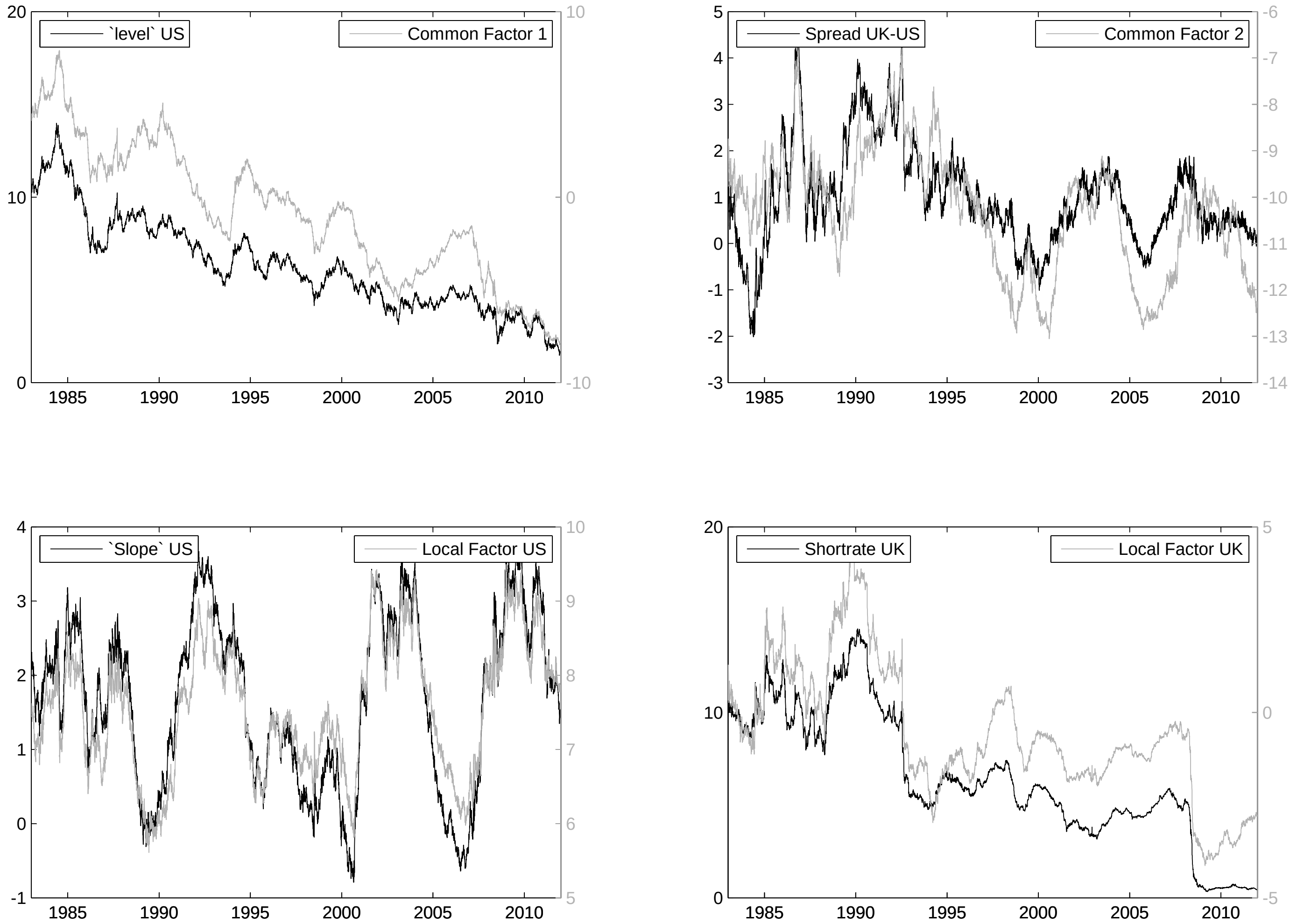


Figure 2: **Fitted factors in the four-factor joint ATSM and US and UK Treasury yields.**
The figure shows the local and common factors of the estimated joint ATSM. Each factor is plotted with its corresponding treasury yield. In the upper row the first common factor is fitted to the 'level' of US treasury yields(10 year US treasury bond). The second common factor and the spread between the 5 year US and UK treasury yields are plotted in the upper right graph. The bottom row reports the two local factors. The left graph shows the local factor US and the slope of the US treasury yields (10year -6months). The right graph shows the local factor UK and the shortrate UK(6months). Factors and treasury yields run from the 2nd of January 1983 to the 31st of July 2012.

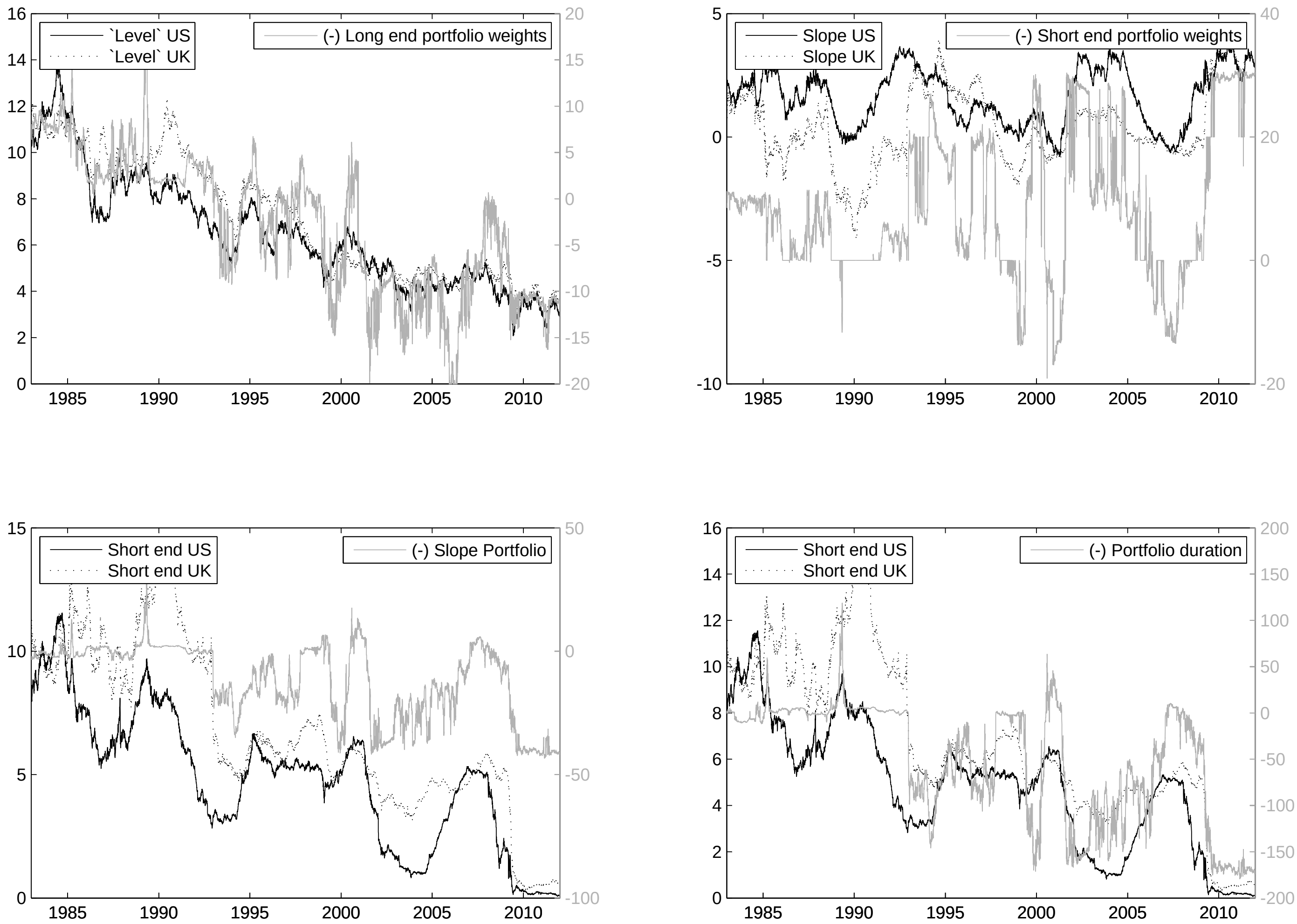


Figure 3: **Fitted Portfolio weights and US and UK Treasury yield data.** The figure shows the Portfolio weights of the the conditional bond portfolio. The Portfolio weights are fitted to their corresponding US and UK Treasury yield data. The portfolio weights are calculated as the sum of both countries and the following yields: long end = 10 years; short end = 6 months + 2 years; slope = 10 years - (6 months + 2 years); duration = $(w_1, \cdots w_{10}, w_1^*, \cdots, w_{10}^*)' \cdot (1, 2, 5, 10, 1, 2, 5, 10)$. The description of the data is given in the text. The portfolio weights and treasury yield data run from the 2^{nd} of January 1983 to the 29^{th} of July 2011.