Identifying the Fundamental Economic Trend of Commercial Real-Estate in UK: with Applications to Pricing Derivatives on IPD Index

Radu Tunaru*

December 19, 2012

Abstract

In this article I investigate empirically what determines the dynamics of the IPD index that is representative for the commercial real-estate in UK. The macroeconomic and interest rate variables identified in this context can reproduce a proxy fundamental economic component underpinning the commercial real-estate price returns in UK. The analysis covers the period January 1987 to December 2011 and it is conducted at monthly and quarterly frequency. The motivation for this research is to provide a tool for pricing IPD property derivatives and other investment applications based on these financial products. The IPD derivatives pricing is developed employing the conditional Esscher transform, suitable for incomplete markets such as property. The model can also be used for risk management purposes and for trading strategies based on signalling of market disillusion.
JEL: G13, G02, C32, C54

*University of Kent, Business School, Park Wood, CT2 7PE, ++441227 824608, r.tunaru@kent.ac.uk

1 Introduction

Property derivatives have a wide range of applications. These financial contracts can be used as outright investment vehicles, for hedging exposure to real-estate risk, for designing acquisition offers and selling the financial economics gains associated with a particular building or portfolio of buildings. They can be used also for diversifying portfolios and generating alpha, or as components of new type of financial contracts such as index-linked mortgages. Real-estate as an asset class resembles some characteristics of commodities markets. For commercial property though there is a natural association with investment type of assets such as equity shares. However, commercial properties are lacking fungibility, they have high entry costs since one single commercial property may cost more than GBP 100 million, there are high transaction costs (stamp duty, legal fees etc) and usually it takes months to complete a single trade. Black (1986) suggested that for a smooth functionality of derivatives, a homogeneous underlying asset would facilitate standardization and an easy understanding of associated risks. These characteristics impact on the liquidity of this asset class, particularly for short-term and it emphasizes the importance of idiosyncratic risks in this market. Shiller (2008) pointed out also the psychological barriers faced by property market participants. Developing successful real-estate derivatives market is not straightforward since liquidity is difficult to establish and because there is predictability in the returns of the underlying asset or index. Derivatives are experiencing a buoying activity when there is large uncertainty regarding the future underlying prices. Carlton (1984) argued that predictable market price changes cannot be perceived as risky. Thus, for long periods of time the implied market views tend to be unidirectional, which makes it difficult to find counterparties willing to trade against the trend.

In spite of eloquent arguments put forward by Robert Shiller and his co-authors, see Shiller (1993), Case et al. (1993), Shiller and Weiss (1999), Shiller (2008), for developing derivatives markets to help out hedge real-estate risk, there is a clear discrepancy between the vast size of the cash real-estate market and the corresponding size of property derivatives markets. There could be many explanations for this. A survey conducted by the MIT Real Estate Research Center, see Geltner and Fisher (2007) and Geltner and Pollakowski (2006), indicated that one important reason why investors do not engage in property derivatives trading is a lack of understandable, useful, and flexible pricing models. On the other hand a survey of institutions, investment managers, property companies and investment banks undertaken by Hermes in May 2006 revealed that the most significant obstacles to trading property derivatives were the following: the fact that they had to require investment committee approval (38%), lack of liquidity (27%), insufficient systems and controls (5%) and lack of product and modelling understanding (5%). I believe that the lack of liquidity and commitment to trade property derivatives comes mainly from a lack of knowledge of the analytical aspects related to this asset class. Providing a reliable model for benchmarking will help investors getting a better insight on how these products can be valued and consequently increase liquidity of property derivatives markets.

Eurex started on 4th February 2009 offering futures on U.K. Investment Property Database index (IPD). Total returns swaps were traded OTC in US on U.S. National Council of Real Estate Investment Fiduciaries (NCREIF) National Index (NPI) and also in UK on IPD. Fisher (2005) described the market of NCREIF-based swap products and pointed to some applications. An excellent introduction to these real estate derivatives markets can be found in Geltner and Fisher (2007), Syz (2008) and Fabozzi et al. (2009, 2010), respectively.

In this paper a great empirical analysis is undertaken in order to determine a plausible fundamental economic term (FET) of returns for the commercial real-estate in UK with a view to price IPD derivatives. This research brings in two new developments. By studying the determinants of commercial property markets, multivariate regression models are identified for the fundamental economic term of commercial property. Hence, macroeconomic variables and interest rate variables can be used now to construct the latent benchmark towards which the observed process of IPD returns should revert to over time. I argue that while there is clear predictability in the property prices evolution, this is more evident when the prices are on a sustained bull run. Moreover, when the observed index return departs from the FET too much and for too long then the fall in the property prices becomes inevitable[1]. It is advocated here that the difference between the observed property prices and the FET is due to market sentiment and when this difference becomes too large inevitably it leads to price crashes. Hence, this research provides a theoretical motivation for the need of property derivatives

[1]It is also true that the difference can also be negative so that the price correction can be upward. However, historical evidence suggests that in property markets the evolution is almost always over-optimistic rather than over-pessimistic

markets. Investors with long positions in real-estate markets should always be wary of the possible sudden market crash. The second novelty added here is the pricing of property derivatives under an incomplete market paradigm using the conditional Esscher transform and a GARCH model for the market sentiment. While the methodology in this paper is centred on commercial real-estate derivatives, I argue that it can be applied to residential derivatives too.

The article is organised as follows. Section 2 is devoted to a thorough literature review of property derivatives methods with a focus on commercial property. Section 3 describes the modelling approach based on the split of returns into a fundamental economic term given by a multiple linear regression and a market sentiment term generated by a GARCH in mean model. One aspect that is often neglected when pricing property derivatives is the incomplete character of associated derivatives. This is discussed in Section 4. Data and methodology is presented in Section 5. The main part of the empirical work is described in Section 6 which is is then used for studying the implications for futures pricing. In Section 7 I present some important applications in investment and risk management that follow directly following the line of this research. Last section contains a discussion of the findings and recommendations for investors.

2 Literature Review

Fisher et al. (1994) classified real-estate indices in two main categories: (1) transactions-based, taking into account the actual transaction-prices over the period, and (2) valuation or appraisal based, derived from valuation-models and continuously updated property-characteristics. In a traditional appraisal-based index, all of the properties in the index population are appraised regularly, and the index returns calculated from a simple aggregation of those appraised values each period. One example is the NPI, on which many U.S. fund managers are benchmarked wholly or partly, and the IPD family of indices used in the UK and continental Europe.

In general there are two classes of models proposed in the literature. In one class there are equilibrium models such as Geltner and Fisher (2007, 2008) who proposed a methodology for pricing real estate forward and swap contracts with an equilibrium argument. With their method, the forward price equals the expectation at time zero of the time T value of the index, discounted at the adjusted rate equal to the market equilibrium required ex ante risk premium in the index total return going forward, where the premium is over the riskless rate. This formula can be used for an appraisal index if the lag or momentum effect in the index is taken into consideration when calculating the expectation of the future value of the index and other smoothing effects that may diminish the risk premium associated with the index. A more recent equilibrium model is described by Lizieri et al. (2011).

In the second category, there are models using a no-arbitrage or costless replication approach. One of earliest attempts to use derivatives pricing in relation to property prices has been provided by Titman and Torous (1989) who developed a model for pricing commercial mortgages. Buttimer et al. (1997) were the first to propose pricing real-estate derivatives such as total return swaps under a Black-Scholes framework. Later on Bjork and Clapham (2002) used also a Black-Scholes model for pricing total return swaps, while Pierangelo and Gheno (2008) applied a similar model for pricing European and American options on property. Patel and Pereira. (1996) were the first to look at the effect of counterparty default risk when pricing a property derivative. They argued that the total return swap price is no longer equal to zero in this case because a compensation for the additional risk is required. Otaka and Kawaguchi (2002) proposed a model for pricing commercial real-estate under an incomplete market set-up, based on (1) a security market where stocks, bonds, currencies, and derivative securities are traded without friction, (2) a space market with the rents of buildings, and (3) a property market where the prices of real properties are determined. Another model in this category is due to Baran et al. (2008), employing the well-known Schwartz and Smith (2000) factor model for pricing real-estate derivatives under a martingale measure. Here the emphasis is related to the constrained maximum likelihood method combined with Kalman filter applied for parameter estimation.

There is also a third category of models that do not submit to either of the two approaches. A model based on the expected value of the future rents has been proposed for the US commercial estate by Ghysels et al. (2007). Syz (2008) and Syz and Vanini (2009) looked at the impact of market frictions (like transaction costs, transaction time, and short sale constraints) on the real estate swap market. Van Bragt et al. (2009) considered a transaction-based house price index with autocorrelation and as in Jokivuolle (1998), first modelled the (unobserved) underlying market fluctuations with a random walk process with drift. Then the observed index is reconstructed with an updating rule going back to Blundelll and Ward (1987) but improved by adding multiple lags and using the accrued value of past observations. Based on this methodology they derive closed-form pricing solutions for forwards, swaps and European put and call options within a risk-neutral set-up. Although they were accounting for the serial correlation present in the dynamics of the index they do not explain how they deal with the incomplete character of the market. Recently, in a series of papers Fabozzi et al. (2009, 2010, 2011) a solution to accommodate serial correlation and the incomplete character of property markets has been offered. The solutions are analytical and based on mean-reverting processes with constant and linear in time long-run means.

3 Modelling Issues

In order to have a good model for pricing property derivatives one must focus first and foremost on a model for the underlying index that captures the characteristics of this index.

3.1 Characteristics of commercial real-estate markets

Geltner et al. (2003) emphasized that autocorrelation is a main characteristic for appraisal-based indices because appraisers update previous prices slowly and they do not react quickly to new market information. Transaction-based indices may also be subject to serial correlation because of lack of informational efficiency of real estate markets by comparison to public securities markets. Another important point is that the underlying index cannot be traded fractionally, that is there is a lack of divisibility of the asset due to the intrinsic nature of real-estate markets. Our model is tested on the IPD index because the vast majority of commercial real-estate derivatives in UK and Europe are written on the IPD family of indices. In conjunction to this observation it is important to recognize the incomplete markets feature of the IPD index. Last but not least, on the cash market it is difficult if not impossible to short the property. The immediate implication is that market sentiment is most of the time unilateral and the property prices may exhibit inertia.

A good model for a commercial property index such as IPD should satisfy several important requirements. First the model should be mean-reverting since property prices cannot in theory increase forever. Property prices are known to be sensitive to levels and changes in interest rates and to inflation as well, both known to have mean-reverting characteristics. Secondly, it should pass backtesting tests and it should be easy to understand and apply by market practitioners. Property markets, commercial and residential alike, inherently have a slow business clock. Therefore, a discrete time model may be more appropriate than a continuous time model. Consequently, from a modelling perspective special care is needed for the calibration to historical data.

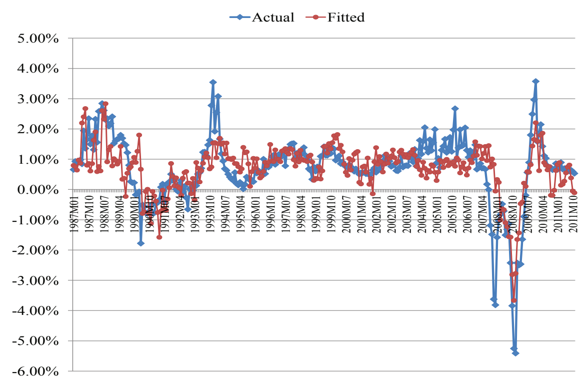
3.2 An Improved Model for Derivatives on IPD

Let S_t be the IPD index level at time t and consider the transformation $S_t = S_{t-1}e^{Y_t}$. Then Y_t is the logarithmic return for the period $[t-1, t]$.

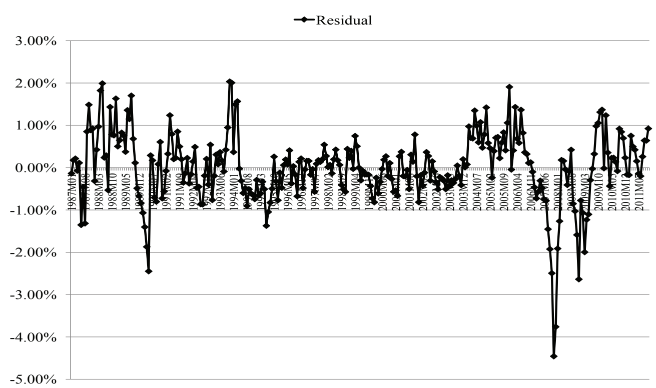
The graph illustrated in Figure 1 shows the *observed* time series of IPD index logarithmic returns between January 1987 and December 2011 and a *proxy* time series for the *unobservable but fundamental* IPD index logarithmic return. These two series are plotted in parallel in Figure 2(a) while the residual or noise corresponding series is illustrated in Figures 2(b). The noise time series is most of the time between -2% and 2%, the exceptions being associated with the crisis of 2007-08, which although related to real-estate it was more of a systemic crisis for commercial real-estate in UK[2].

[2]Quite remarkably the IPD real estate for office in London was increasing through the crisis.

Figure 1: Multiple regression model for IPD UK index logarithmic return for the period January 1987 to December 2011, monthly. The "Actual" represents the observed IPD return while "Fitted" represents the fundamental IPD return spanned by GDP growth, change in inflation, change in GLC inflation and the change in BBR.



(a) Logarithmic return of IPD and fundamental series



(b) Series of residuals

From a modelling point of view the core idea is that the observed property index is the sum of the *FET* and of a market sentiment term. Thus, we are in agreement with other published research[3] that there is an underlying

[3]See Fabozzi et al. (2009, 2010, 2011), MacKinnon and Zaman (2009),Van Bragt et al.

unobservable term that is acting as a benchmark. The *FET* in the modelling approach presented here is evolving in time according to a multivariate linear regression model spanned by macroeconomic and interest rate variables. The difference between the observed IPD index logarithmic return series and the associated proxy fundamental series represents the *market sentiment value* and is modelled with a GARCH(p,q) model.

Assumption 3.1. *The* observed *return Y_t is the sum of FET and an error term.*

For modelling purposes we specify that

$$Y_t = X_t'\beta + Z_t \tag{1}$$

where

$$X_t'\beta = X_t^{(1)}\beta_1 + X_t^{(2)}\beta_2 + \dots X_t^{(d)}\beta_d \tag{2}$$

represents the *FET* given by macroeconomics and real-estate related covariate information. Hence

$$Z_t = Y_t - X_t'\beta \tag{3}$$

can be interpreted as the additional term due to market sentiment.

4 IPD Derivatives pricing

For derivatives pricing I am going to employ the conditional Esscher transform approach detailed in Siu et al. (2004) that offers an elegant solution to the incomplete market problem.

Consider $\mathcal{T}$ a set index of time underpinning the observable data. For the IPD UK index $\mathcal{T}$ can be monthly, quarterly ar annually. The model is a discrete time model specified by the following equations under the physical measure $\mathbb{P}$

$$B_t = B_{t-1}e^r \tag{4}$$

where r is the risk-free per period for the money market account

and

$$\begin{cases} S_t = S_{t-1}e^{Y_t}, \\ Y_t = X_t'\beta + Z_t, \\ Z_t = \lambda\sqrt{h_t} - \frac{1}{2}h_t + \xi_t \end{cases} \tag{5}$$

(2009).

X_t' is a vector of values of explanatory variables corresponding to the period $[t-1, t]$, and the noise $\{Z_t\}_{t\in\mathcal{T}}$ is driven by a GARCH(p,q) model

$$\begin{cases} \xi_t|\mathcal{F}_{t-1} \sim N(0, h_t), & \text{under } \mathbb{P}; \\ h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \xi_{t-i}^2 + \sum_{j=1}^{q} \mu_j h_{t-j}, & \text{for each } t \in \mathcal{T}\backslash\{0\}. \end{cases} \tag{6}$$

with $p \geq 0, q \geq 1, \alpha_0 > 0, \alpha_i \geq 0,\ \forall i$ and $\mu_j \geq 0,\ \ \forall j$. For ensuring covariance stationarity of the GARCH(p,q) model the condition

$$\sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \mu_j < 1 \tag{7}$$

must be satisfied. The model described above says that the logarithmic return of the IPD index is the sum of a *FET* value $X_t'\beta$ linearly spanned by macroeconomic and interest rate variables and a market sentiment factor evolving under a GARCH specification with a mean driven by a risk preference parameter λ.

It follows directly that under physical measure $\mathbb{P}$

$$Y_t|\mathcal{F}_{t-1} \sim N\left(X_t'\beta + \lambda\sqrt{h_t} - \frac{1}{2}h_t, h_t\right) \tag{8}$$

The conditional Esscher transform is defined by the series of parameters $\{\theta_t\}_{t\in\mathcal{T}\backslash\{0\}}$ such that θ_t is known given the information $\mathcal{F}_{t-1}$. These parameters, whose value will be identified later on, are used to define the Esscher transforms[4] $\{\Lambda_t\}_{t\in\mathcal{T}}$, with $\Lambda_0 = 1$ and

$$\Lambda_t = \prod_{k=1}^{t} \frac{e^{\theta_k Y_k}}{M_{Y_k|\mathcal{F}_{k-1}}(\theta_k)},\ \ t \in \mathcal{T}\backslash\{0\}. \tag{9}$$

where $M_{Y_k|\mathcal{F}_{k-1}}(\cdot)$ is the conditional moment generating function under $\mathbb{P}$. Since $\{\Lambda_t\}_{t\in\mathcal{T}}$ is a $\mathbb{P}$-martingale, as in Siu et al. (2004) the probability measures defined by the conditional Esscher transforms given by[5]

$$\mathbb{P}_{t,\Lambda_t}(\{Y_t \in B\}|\mathcal{F}_{t-1}) = \mathbb{E}_{\mathbb{P}_t}\left(1_{\{Y_t\in B\}} \frac{e^{\theta_t Y_t}}{\mathbb{E}_{\mathbb{P}_t}(e^{\theta_t Y_t}|\mathcal{F}_{t-1})}|\mathcal{F}_{t-1}\right) \tag{10}$$

[4]See Bühlmann et al. (1996) for a theoretical motivation.

[5]The probability measure $\mathbb{P}_t$ is the restriction of $\mathbb{P}$ to the filtration information set $\mathcal{F}_t$.

for any Borel set B.

If $F(u;\theta_t|\mathcal{F}_{t-1})$ is the cdf of $Y_t|\mathcal{F}_{t-1}$ under $\mathbb{P}_{t,\Lambda_t}$ then $M_{Y_t|\mathcal{F}_{t-1}}(u;\theta_t)$ is the associated mgf. It is easy to very that

$$M_{Y_t|\mathcal{F}_{t-1}}(u;\theta_t) = \frac{M_{Y_t|\mathcal{F}_{t-1}}(u+\theta_t)}{M_{Y_t|\mathcal{F}_{t-1}}(\theta_t)}. \tag{11}$$

The martingale condition $\mathbb{E}_{\mathbb{Q}}(e^{-r}S_t|\mathcal{F}_{t-1}) = S_{t-1}, \quad \forall t \in \mathcal{T}\backslash\{0\}$ is equivalent to the identifying condition for the conditional Esscher parameters θ_t^q

$$r = \ln\{M_{Y_t|\mathcal{F}_{t-1}}(1:\theta_t^q)\}, \quad \forall t \tag{12}$$

The parameters $\{\theta_t^q\}_{t\in\mathcal{T}-\{0\}}$ are the risk-neutral parameters. If this condition is satisfied then $\{e^{-rt}S_t\}_{t\in\mathcal{T}}$ is a $\mathbb{Q}$-martingale where $\mathbb{Q} \equiv P_{T,\Lambda_T^q}$.

Solving the equation (12) gives the solution in closed form

$$\theta_t^q = \frac{r - X_t'\beta - \lambda\sqrt{h_t}}{h_t}. \tag{13}$$

Using (11) one can show that

$$M_{Y_t|\mathcal{F}_{t-1}}(u;\theta_t) = \exp\left\{u[r + X_t'\beta - \frac{1}{2}h_t] + \frac{1}{2}u^2 h_t\right\} \tag{14}$$

which implies that, under the conditional Esscher equivalent martingale measure $\mathbb{Q}$,

$$Y_t|\mathcal{F}_{t-1} \sim N\left(r + X_t'\beta - \frac{1}{2}h_t, h_t\right). \tag{15}$$

Then, one can determine directly the distribution of ξ under $\mathbb{Q}$

$$\xi_t|\mathcal{F}_{t-1} \sim N(r - \lambda\sqrt{h_t}, h_t).$$

Denoting $\varepsilon_t = \xi_t - r + \lambda\sqrt{h_t}$ we have the following relationships, under $\mathbb{Q}$

$$\begin{cases} \varepsilon_t|\mathcal{F}_{t-1} \sim N(0,h_t), \\ Y_t = r + X_t'\beta - \frac{1}{2}h_t + \varepsilon_t, \\ h_t = \alpha_0 + \sum_{i=1}^p \alpha_i[\varepsilon_{t-i} + r - \lambda\sqrt{h_{t-1}}]^2 + \sum_{j=1}^q \mu_j h_{t-j} \end{cases}. \tag{16}$$

4.1 Futures Pricing

If $\mathbb{Q}$ is an equivalent martingale measure then the futures price at time 0 for maturity T is

$$F_{t,T} = \mathbb{E}_{\mathbb{Q}}[S_T|\mathcal{F}_t] \tag{17}$$

Being in an incomplete market[6] a mechanism is required to identify a suitable pricing measure $\mathbb{Q}$. The conditional Esscher measure presented above is fixing the pricing measure.

One can see that by using the tower property and the fact that all returns are normally distributed as in (15), we can derive recursively

$$\begin{aligned}
F_{t,T} &= \mathbb{E}_{\mathbb{Q}}(S_T|\mathcal{F}_t) \\
&= \mathbb{E}_{\mathbb{Q}}(\frac{S_T}{S_{T-1}}S_{T-1}|\mathcal{F}_t) \\
&= \mathbb{E}_{\mathbb{Q}}(e^{Y_t}S_{T-1}|\mathcal{F}_t) \\
&= \mathbb{E}_{\mathbb{Q}}(\mathbb{E}_{\mathbb{Q}}(e^{Y_t}S_{T-1}|\mathcal{F}_{T-1})|\mathcal{F}_t) \\
&= \mathbb{E}_{\mathbb{Q}}[\exp(r + X'_T\beta)S_{T-1}|\mathcal{F}_t] \\
&= \exp(r + X'_T\beta)\mathbb{E}_{\mathbb{Q}}[S_{T-1}|\mathcal{F}_t] \\
&\ldots \\
&= S_t \exp\left[((X'_T + X'_{T-1} + \ldots X'_{t+1})\beta) + (T-t)r)\right]
\end{aligned}$$

Hence the formula for the futures on IPD UK index is

$$F_{t,T} = S_t \exp\left[((X'_T + X'_{T-1} + \ldots X'_{t+1})\beta) + (T-t)r)\right]. \tag{18}$$

REMARK: Due to the quotation system of Eurex futures we have that

$$F_{t,T}^{Eurex} = \mathbb{E}_{\mathbb{Q}}\left[\frac{S_T}{S_t}\right] \times 100 = \frac{100}{S_t}F_{t,T}$$

So, if working with Eurex prices 120 for a view that $\frac{S_T}{S_t} = 120$ then we use

$$F_{t,T} = \frac{S_t}{100}F_{t,T}^{Eurex} \tag{19}$$

[6]This is due to the fact that it is not possible to trade in the IPD index per se or the portfolio it represents.

whereas if working with Eurex prices 0.85 for a view that $\frac{S_T}{S_t} = 0.85$ then

$$F_{t,T} = S_t F_{t,T}^{Eurex} \tag{20}$$

The formula (18) shows that pricing futures is very different from pricing futures on equity indexes. The difference is given by the sum of the future FET realisations over all periods from t until maturity T.

Once the pricing measure is fixed, an investor can use this measure to price other derivatives in IPD such as TRSs, European call and put options, and other structured products such as cliquet-type products.

5 Data and Methodology

5.1 Data

The fundamental value is given by a combination of the determinants of the IPD index. Hence, in this section we are going to identify the determinants of the IPD index. The variables investigated are selected based on previous studies focused on linking commercial real-estate prices to macroeconomic and interest rate type variables, see Dobson and Goddard (1992); McCue and Kling (1994); Ling and Naranjo (1997); Wit and Dijk (2003); Fisher et al. (2004); Clayton et al. (2009).

The list of variables, considered for generating the fundamental value of the IPD index, are presented in Table 1. Data has been downloaded from IPD website, from Bank of England website and from Datastream. Moreover, I use Eurex futures settlement prices on IPD UK All Return index with the five yearly maturities ending in December. The Eurex futures daily settlement prices were available but for simplicity I have worked with the monthly average settlement price.

The IPD UK Annual All Property Index futures contract is traded on Eurex for five annual maturities relevant to the March cycle, covering total returns to end of December in the preceding year. Each futures contract is for GBP 50000 and a par value of 100, cash settled. The futures price is given as a percentage with two decimals, expressed as 100 plus the percentage total return in the year to the end of December. The minimum tick is 0.05 points, equivalent to GBP 25. Daily settlement prices for each of the five maturities are calculated from volume-weighted average of the prices of all transactions (minimum five) during the minute before 17:30. If there are no

Table 1: List of variables analysed for determining the fundamental value of the IPD index.

Variable	Description
IPDRETURNS	IPD Index monthly returns (in percentages)
GOLDPRICEGROWTH	Percentage Monthly Growth in Gold Price in Sterling
EXRATERETURNS	The Returns on the Spot Exchange Rate USD into Sterling
CHANGEUKUNEMPLOY	Monthly Change in UK Unemployment rate
CHANGEINF	Monthly Changes in UK Inflation Rate
CHANGEUKLIBOR3M	Monthly Change in UK LIBOR 3m
CHANGEUKLIBOR6M	Monthly Change in UK LIBOR 6m
CHANGEUKLIBOR12M	Monthly Change in UK LIBOR 12m
CHANGEUKTBILL3M	Monthly Change in UK Treasury Bill Tender 3M Middle Rate
CHANGEGLCINF	Monthly Change in GLC UK implied inflation spot curve
CHANGEGLCREAL	Monthly Change in GLC UK implied real spot curve
CHANGEBOERATE	Monthly Change in Official Bank of England Rate
INDPRODGROWTH	The Growth Rate of Industrial Production UK (Index of Productio
FTSE100RETURNS	The Returns of FTSE 100 Price Index
CHANGEFTSE100DY	Monthly Change in FTSE 100 Dividend Yield
CHANGEGBPSWAP5Y	Monthly Change in GBP 5 year Swap Rate
CHANGEGBPSWAP10Y	Monthly Change in GBP 10 year Swap Rate

trades available then settlement prices are given by Eurex based on other sources of market data.

5.2 Methodology

In this research we consider more precisely the fundamental level underpinning our model and provide a mechanism to understand the market crashes and also provide a procedure to forecast them. The idea goes back to Black(1988) who suggested that in a market that is characterised by mean-reversion in returns, various investors may develop a mis-perception about the true speed of reversion which ultimately may lead to the crash of the market. If the investors believe that the speed of reversion is high then, upon observing a positive return, the investors will liquidate a long position and keep a short position. Likewise, if the speed is slow then the investors will hold onto their long positions and will try to cover quick short positions. The main problem in testing this theory is that the expectations of mean-reversions are not observable and one must derive them from the flux of trades. For example, high volume after positive returns points out to a fast expected speed reversion. Hillebrand (2003) applied Black's theory to the Black Monday crash of 1987 and revealed that the mean-reversion after the crash was significantly higher than its value before the crash. Hence, following the bull period 1982-1986 the investors were caught into an illusion about the true mean-reversion speed.

We draw on the ideas presented in Hillebrand (2003) and explain the property market crash as a mean-reversion disillusion correction. In other words, the wrong expectation of mean-reversion speed by a larger and larger number of investors is causing a strong departure from the fundamental property returns series. As depicted in Figure 2(a) the larger the departure between the observed property returns and the fundamental property returns the higher the probability of a crash. MacKinnon and Zaman (2009) support the idea that there is a long-term level of a real-estate index towards which the observable index will revert.

5.2.1 GARCH Parameter Estimation

The model (5) based on a GARCH in mean specification and the estimation of parameters will be done with maximum likelihood. For simplicity we assume a GARCH(1,1) for the volatility of the market sentiment component.

Table 2: GARCH(1,1) in mean maximum likelihood estimation for the IPD UK All monthly index for the period January 1987 to January 2009.

parameter	α_0	α_1	μ_1	λ
	Period January 1987 to December 2011			
MLE estimate	1.31e-05	0.7384	0.1265	-0.0166
	Period January 1987 to January 2007			
MLE estimate	7.57e-06	0.4564	0.4087	-0.00424

In Table 2 the results of estimation from the monthly data for the period January 1987 to January 2009 are presented.

It may come as a surprise the negative estimate of the parameter λ indicative of the risk preferences of the representative market investor. The persistent negative performance of the index during 2007 and 2008 is indicative of a systemic crisis. Therefore, "expecting" a negative return for holding the index justifies the negative risk premium.

Another conclusion from the analysis of the estimation of the GARCH(1,1) model for the two sample periods, up to January 2007 and up to December 2011, indicates that the parameter μ_1[7] has changed post-crisis. The decrease in the estimated value of μ_1 shows that there was a clear change in the dynamics of market sentiment for IPD index. This also points out to the importance of parameter estimation uncertainty and the sample used for inference.

6 The Determinants of IPD Index

One key part of our modelling approach is the determination of the *fundamental* value of the IPD index. This is not equal to the observed level of the IPD index. The fundamental value of the commercial property is spanned by important macroeconomic variables and other important variables for real-estate space such as interest rates. Our period of investigation is December 1986 until December 2011. In this area of research it is difficult to get covariate information more frequent than monthly. At the same time the business

[7] In general for financial time series this parameter would be the larger parameter normally, indicating a persistent volatility dependence.

trading time in real-estate is quite slow, with the average transaction time in the region of three months due to frictional legal costs and procedures.

Two separate similar studies have been conducted, one using monthly data and one using quarterly data. I envisage that, as futures contracts on IPD mature, there will be a convergence of quarterly tenor for marking to market purposes. The same procedure is followed for monthly and quarterly data. In order to improve the estimation results we employ the Newey-West estimation procedure in Eviews.

6.1 Monthly Calibration

The majority of the variables investigated for constructing a FET proxy model are non-stationary (see Appendix) and therefore the variables have been transformed in order to achieve stationarity. The results of the stationarity tests (Augmented Dickey-Fuller) are presented in Table 3 and they show that all variables are stationary in this format.

Moreover, the following pairs of variables have more than 90% sample correlation value: CHANGEGBPSWAP10Y - CHANGEGBPSWAP5Y, CHANGEUKLIBOR6M - CHANGEUKLIBOR12M, CHANGEUKLIBOR6M - CHANGEUKLIBOR3M. In order to avoid problems with multicollinearity CHANGEGBPSWAP10Y, CHANGEUKLIBOR6M and CHANGEUKLIBOR12M were left out of the analysis.

In addition, the variable GDP for UK, that is available only at quarterly frequency, has been augmented at monthly frequency by linear interpolation as showed in Appendix 3. This is a very important macroeconomic variable and, as it will seen below, it is significant in constructing the fundamental component of the IPD index. This process is illustrated in Appendix C.

Thus, a regression model is selected by fitting the IPD returns series on the set of covariate variables from the list above which remained after the preliminary round. The model selection routine that is employed is general to specific, that is I start with all variables in the model and then I eliminate one by one those variables with the highest p-values of the t-test statistic, larger than the significance level. This procedure is easily run in Eviews in few minutes.

For the period January 1987 to December 2011 the multiple linear regression model selected following the backward elimination selection algorithm leads to the model in Table 4. This model has an R^2 equal to 53.3% but unfortunately the Jarque-Bera test is equal to 10.32 with a $p-$value of 0.0057,

Table 3: Augmented Dickey-Fuller tests for stationarity for all variables in the study, for monthly data between January 1987 and December 2011. Notes: The optimum number of lags used in the ADF test equation is based on AIC. *, **, and *** denote significance at the 10%, 5% and 1% level respectively.

Variable	Test result
IPDRETURNS	-4.226815***
GOLDPRICEGROWTH	-9.055336***
EXRATERETURNS	-15.30349***
CHANGEUKUNEMPLOY	-2.926745**
CHANGEINF	-4.424668***
CHANGEUKLIBOR3M	-5.138848***
CHANGEUKLIBOR6M	-5.104176***
CHANGEUKLIBOR12M	-5.363347***
CHANGEUKTBILL3M	-6.533523***
CHANGEGLCINF	-14.45073***
CHANGEGLCREAL	-15.65426***
CHANGEBOERATE	-5.563567***
INDPRODGROWTH	-3.626369***
FTSE100RETURNS	-13.42140***
CHANGEFTSE100DY	-8.104989***
CHANGEGBPSWAP5Y	-7.2981***
CHANGEGBPSWAP10Y	-14.186***

Table 4: Multiple regression model for IPD index logarithmic return for the period January 1987 to December 2011.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.3611	0.1039	3.47	0.0006
GDPGROWTH	2.2653	0.3368	6.72	0.0000
CHANGEINF	0.5615	0.2043	2.75	0.0064
CHANGEGLCINF	0.3834	0.2078	1.85	0.0662
CHANGEBOERATE	0.4672	0.1595	2.93	0.0037
R^2	53%			

Table 5: Multiple regression model for IPD index logarithmic return for the period January 1987 to December 2011 selected by backward stepwise elimination.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.3461	0.0965	3.58	0.0004
GDPGROWTH	2.2202	0.3245	6.84	0.0000
CHANGEINF	0.4113	0.1186	3.47	0.0006
CHANGEGLCINF	0.4189	0.2005	2.09	0.0376
CHANGEBOERATE	0.6105	0.1143	5.34	0.0000
R^2	60%			

indicating a lack of normality for regression errors.

The residual analysis identifies 11 data points that are outliers and that lead to a failure of Jarque-Bera test of normality. The normality is restored (Jarque-Bera test equal to 1.51 with p-value 0.47) after removing these 11 outlier data from the sample of 287 observations in the study period. Refitting the final model to the slightly reduced sample gives a very good model, with an improved R^2 equal to 60% and other estimates described in Table 5. The variables spanning the fundamental level of commercial property in UK are GDP growth, change in inflation, GLC UK implied inflation spot curve and change in BBR. The goodness-of-fit analysis depicted in Figure 3(a) and the residual analysis in Figures 3(a) and 3(b) of the model (1) indicate a very good fitting overall. However, visual inspection of the residual time series seems to point out to three distinctive periods in the historical evolution

Table 6: Multiple regression model for IPD index logarithmic return for the period January 1987 to December 2011. Chow test for structural breaks in January 1991 and January 2008.

Test	statistic	Type of distribution for test	p-value
F-statistic	7.1515	Prob. F(10,261)	0.0000
Log likelihood ratio	66.8376	Prob. Chi-Square(10)	0.0000

of IPD. This possibility can be tested with a Chow test.

6.1.1 Chow test

The next step of the analysis is to test for structural breaks in December 1991 and December 2007 and if the parameters are not constant across over time then refit models for all three subperiods.

The results of the Chow test for structural breaks in January 1991 and January 2008 are presented in Table 6. The p-values clearly indicate that the parameters are significantly different across the subsample periods marked by 1991 and 2008.

Investigating the models that determine the fundamental component of the IPD index over the three periods of time helps to understand *ex post* what happened with the evolution of commercial real-estate prices in UK. The only lesson to learn towards IPD derivatives pricing is that parameters estimates may change following a property crash or price re-alignment.

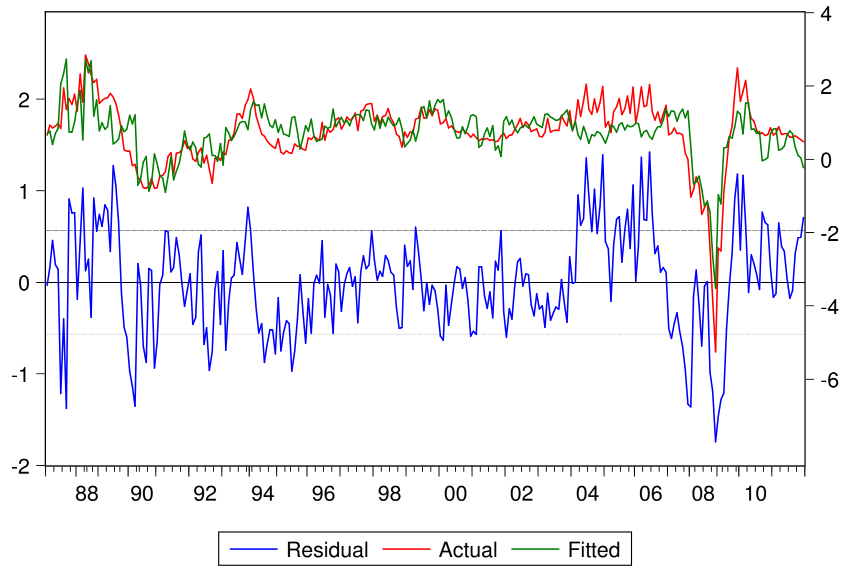
6.2 Analysis of models for each subperiod

In Table 7 there are presented the models selected by backward elimination with the data for each of the three subperiods delineated by the breakup points tested with the Chow test. For the period January 1987 to December 1990, the GDP growth and change in BBR are the variables giving the fundamental for the monthly series of returns for IPD.

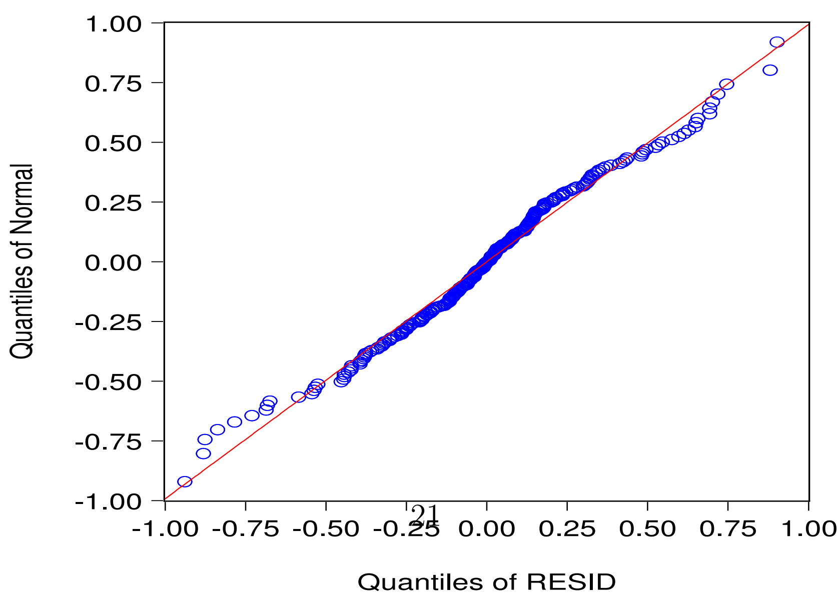
The period January 1991 to December 2007 was characterised by a sustained period of positive returns. For this period of time, the monthly series of returns for IPD has a fundamental given by GDP growth, change in inflation, GLC UK implied inflation spot curve[8] and change in BBR. Although

[8]This variable is given by appealing to the Fisher relationship; the implied inflation

Figure 2: Multiple regression model for IPD UK index logarithmic return for the period January 1987 to December 2011, monthly. The "Actual" represents the observed IPD return while "Fitted" represents the fundamental IPD return spanned by GDP growth, change in inflation, change in GLC inflation and the change in BBR.



(a) Residuals of the time series on the left axis and the observed vs fitted on the right axis



(b) QQ plot

Table 7: Multiple regression models selected for IPD index logarithmic return for each subperiod between January 1987 to December 1990.

(a) The period January 1987 to December 1990

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.5030	0.2140	2.35	0.0242
GDPGROWTH	2.1298	0.5095	4.17	0.0002
CHANGEBOERATE	0.5355	0.1343	3.98	0.0003
R^2	53%			

(b) The period January 1991 to December 2007

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.6395	0.1449	4.41	0.0000
GDPGROWTH	0.9220	0.4386	2.10	0.0369
CHANGEINF	0.3335	0.1068	3.12	0.0021
CHANGEGLCINF	0.3761	0.1581	2.37	0.0184
CHANGEBOERATE	0.6081	0.1790	3.39	0.0008
R^2	29%			

(c) The period January 2008 to December 2011

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.5074	0.1067	4.75	0.0000
GDPGROWTH	2.9260	0.3168	9.23	0.0000
CHANGEINF	0.4274	0.1665	2.56	0.0144
CHANGEBOERATE	2.2464	0.3446	6.51	0.0000
R^2	89%			

the R^2 is not very high for this period, the residual analysis illustrated in Figures 4(b) and 4(e) indicate a very good fit of this model. Moreover, during the period 2004-2006 the observed IPD index return series was persistently above the fundamental level given by the macroeconomic factors. Therefore the sharp decline in IPD return observed late 2007 can be interpreted as resulting from a discovery of the fundamental economic value. In other words the period of disillusion stopped in 2007.

term structure is calculated as the difference of instantaneous nominal forward rates and instantaneous real rates. This yield curve represents the GLC UK implied inflation spot curve with maturity years 5.

Table 8: Multiple regression model including one lag for IPD index logarithmic return for the period January 2008 to December 2011. The R-square is equal to 85%.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.0723	0.0424	1.70	0.0894
GDPGROWTH	0.6056	0.1661	3.64	0.0003
CHANGEGLCINF	0.5292	0.2242	2.35	0.0190
LOGRETURNIPD(-1)	0.7590	0.0493	15.37	0.0000

The model for the period January 2008 to December 2011 shows that GDP growth, change in inflation and change in BBR were the significant variables. The goodness-of-fit as measured by R^2 is excellent. The residual analysis confirms this conclusion, which is quite remarkable given that the period covered included the subprime-liquidity crisis. The very close fit can be interpreted as due to the fact that market participants paid a closer look to the macroeconomic variables.

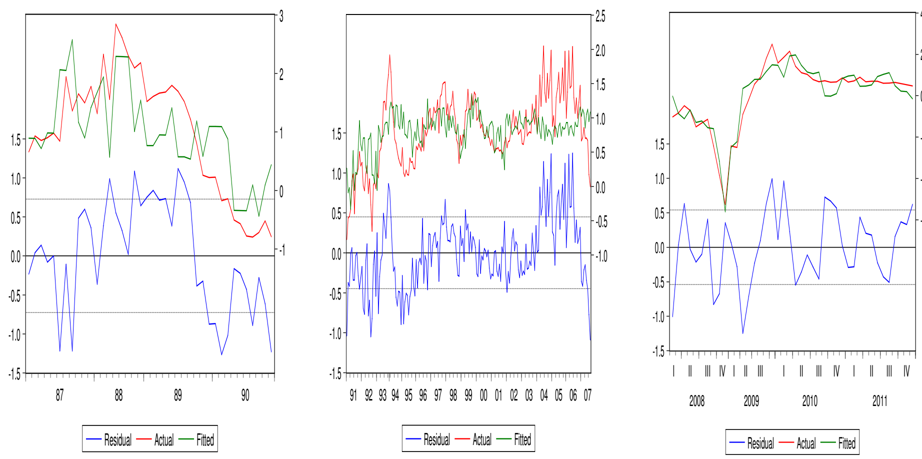
Looking at the results for all three periods in Figure 3 it is evident that the important variables to explain the dynamics of monthly IPD logarithmic return series are the GDP growth and change in BBR. Hence, in an indirect manner, the monetary committee plays a very important role in real-estate markets through the setting up of the BBR.

6.3 Model with one lag

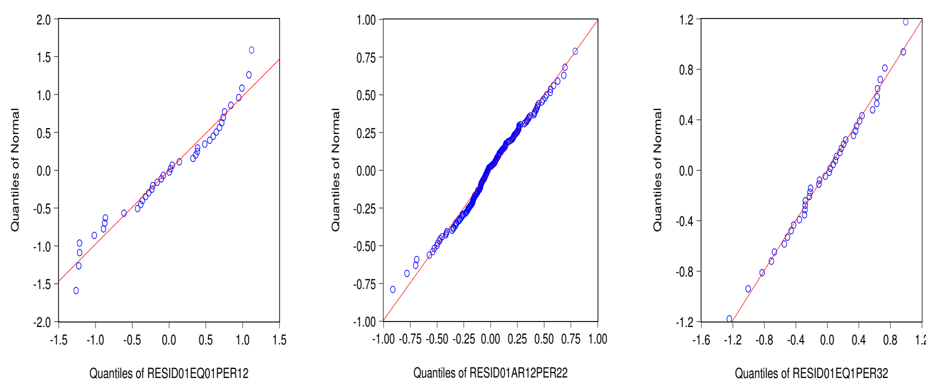
The goodness of fit as measured by the R^2 measure varied in magnitude for the models in the three subperiods presented above. The question that any investor would then ask is whether it is possible to have a model fitting very well, that is with a very high R^2 over the entire period.

Since we know from literature that property markets return price series exhibit serial correlation, it is worth investigating what happens if lags of the IPD series are included in the fundamental construction. The model fitting results of this model are presented in Table 8. The R^2 has improved substantially to 85%. The goodness-of-fit revealed in Figure 4 shows that the fit is extremely good.

Figure 3: Final model selection of the fundamental economic term for IPD UK index logarithmic return for the three periods January 1987 to December 1990, January 1991 to December 2007 and January 2008 to December 2011. The "Actual" represents the observed IPD return while "Fitted" represents the fundamental IPD return spanned by GDP growth, change in GLC inflation and the returns from the previous two periods.

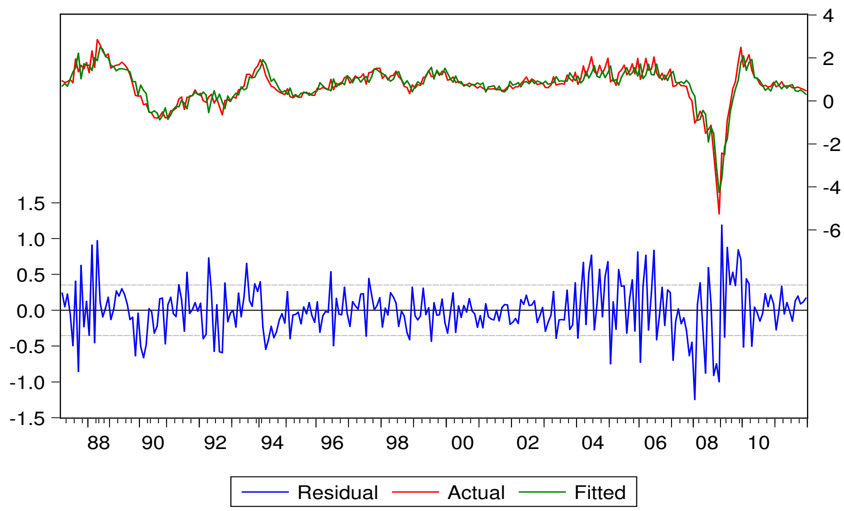


(a) January 1987 to December 199 (b) January 1991 to December 2007 (c) January 2008 to December 2011

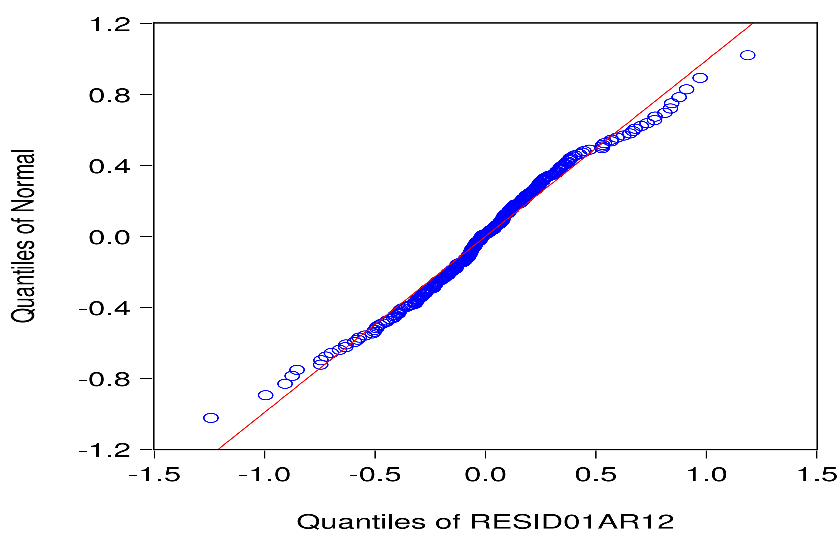


(d) QQ plot January 1987 to December 1990 (e) QQ plot January 1991 to December 2007 (f) QQ plot January 2008 to December 2011

Figure 4: Multiple regression model with one lag for the fundamental economic underlying of the IPD UK index logarithmic return, monthly, for the period January 1987 to December 2011. The "Actual" represents the observed IPD return while "Fitted" represents the fundamental IPD return spanned by GDP growth, change in GLC inflation and the returns from the previous period.



(a) actual levels



(b) log scale

Table 9: Model with one lag for IPD index logarithmic return for the period January 1987 to December 2011. Chow test for structural breaks in January 1991 and January 2008.

Test	statistic	Type of distribution for test	p-value
F-statistic	6.641165	Prob. F(8,263)	0.0000
Log likelihood ratio	50.59927	Prob. Chi-Square(8)	0.0000

6.3.1 Chow test for Model with one lag

It is interesting to see whether the structural breaks identified vis-a-vis the multiple linear regression model without lag are significant for this new model that includes one lag of the IPD index. The results presented in Table 9 shows that the null hypothesis that parameters are constant across the three subperiods is rejected at 1% level of significance. Thus, it may be useful to refit the model with one lag for each subsample period. The model selection results are depicted in Table 10 and the goodness-of-fit performance can be gauged from Figure 5.

Table 10(a) shows the results of model selection when including also one added lag, fitted for the period January 1987 to December 1990. The only two significant variables are the GDP growth and the previous return on IPD index. As it can be seen from Figure 6(a) the fit is very good but visual inspection suggests that there could be a one month shift between the observed and fitted series, with the fundamental economic series leading the observed. In other words, for this period and under this model, the observed IPD series follows the FET series with one month lag.

For the second period January 1991 to December 2007, somehow surprisingly, introducing one lag of the IPD return series seems to make all other explanatory variables redundant. The other variable that may be considered is the GLC UK implied inflation spot curve, which had a p-value equal to 0.1303. The model fitting results are illustrated in Table 10(b).

The model selected for the second period January 1991 to December 2007 fits exceptionally well as it can be seen from the residual analysis as well, Figures 6(b) and 6(e). Recalling that post 1993 the returns on IPD were all positive, this market behaviour that can be explained by the fact that investors disregarded the macroeconomic and interest rate variables, and they only considered the property prices evolution per se, almost in isolation.

Table 10: Multiple regression models selected by stepwise backward elimination for IPD index logarithmic return for the period January 1987 to December 1990. One lag of IPD return included in explanatory variables.

(a) Model with one lag January 1987 to December 1990

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.0087	0.0654	-0.1330	0.8949
GDPGROWTH	0.7636	0.2668	2.8624	0.0070
LOGRETURNIPD(-1)	0.8229	0.0638	12.8853	0.0000
R^2	86%			

(b) Model with one lag January 1991 to December 2007

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.1419	0.0336	4.2222	0.0000
LOGRETURNIPD(-1)	0.8298	0.0328	25.2849	0.0000
R^2	71%			

(c) Model with one lag January 2008 to December 2011

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.2045	0.0735	2.7835	0.0083
GDPGROWTH	1.4226	0.2773	5.1287	0.0000
CHANGEGLCINF	0.8775	0.1926	4.5547	0.0001
LOGRETURNIPD(-1)	0.5988	0.0469	12.7675	0.0000
R^2	93%			

There is no surprise then that the market sentiment being unilateral led to the self-fulfilling prophecy effect.

The results for the period January 2008 to December 2011, that is after the subprime crisis, are different. The results shown in Table 10(c) indicate that property investors should have considered GDP growth, GLC UK implied inflation spot curve and the previous month return on the IPD index.

The fit of the model with one lag for the period January 2008 to December 2011 is exceptionally good, see Figures 6(c) and 6(f) and this is quite remarkable given that this is a turbulent period generally speaking.

The model for the FET can be improved from the point of view of the *fitting in the sample* by including a second lag. The results are presented in the Appendix B.1.

6.4 Quarterly Calibration

We start from the same set of variables that were used in building up the fundamental component of the IPD UK index return series at monthly frequency.

Table 11 provides the stationarity tests for all variables under investigation. Since we prefer the general to specific approach we need to make sure that all variables involved are stationary. The majority of them are in the first differences.

As with monthly analysis, before proceeding to the model selection we eliminate variables having more than 90 % correlation with other variables in the study. These are CHANGEBOERATE, CHANGEUKLIBOR6M, CHANGEUKLIBOR12M, CHANGEUKTBILL3M AND CHANGEGBPSWAP10Y.

In order to achieve normality of regression errors we eliminate three outliers Q4 2007, Q1 2008 and Q2 2009. The results of the first selected model are presented in Table 12. The significant variables are GDP growth, the UK Industrial Production growth and the change in inflation.

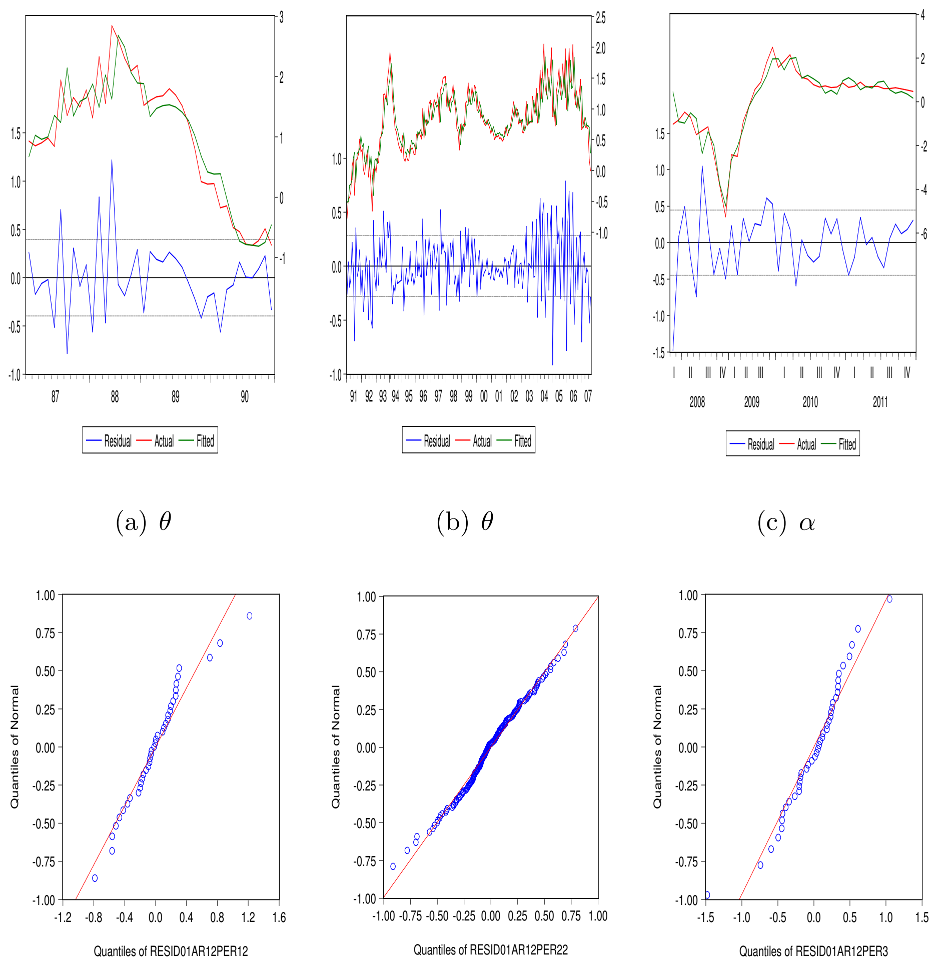
The goodness-of-fit analysis described in Figures 7(a) and 7(b) shows a very good fit.

Next we look at the Chow test for the two possible structural breaks.

6.4.1 Chow test

The results summarised in Table 13 reveal that the parameters are not constant across these three sub-periods.

Figure 5: Fundamental final model selection for IPD UK index logarithmic return for the three periods January 1987 to December 1990, January 1991 to December 2007 and January 2008 to December 2011. The "Actual" represents the observed IPD return while "Fitted" represents the fundamental IPD return spanned by GDP growth, change in GLC inflation and the returns from the previous period.



(a) θ (b) θ (c) α

(d) β (e) β (f) σ

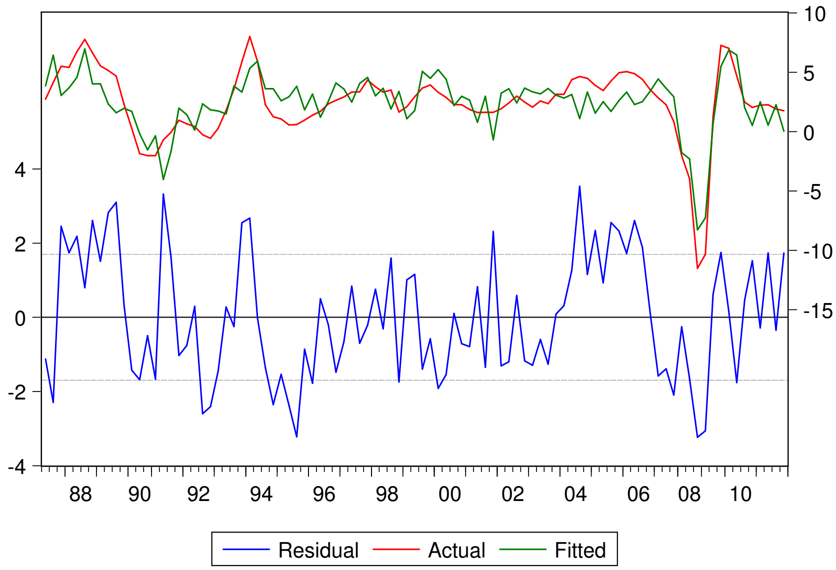
Table 11: Augmented Dickey-Fuller tests for stationarity for all variables in the study, for quarterly data between Q1 1987 to Q4 2011. Notes: The optimum number of lags used in the ADF test equation is based on AIC. *, **, and *** denote significance at the 10%, 5% and 1% level respectively.

Variable	Test result
LOGRETURNIPD	-3.9043***
GOLDPRICEGROWTH	-9.0678***
EXRATERETURNS	-7.8883***
CHANGEUKUNEMPLOY	-3.2287**
GDPGROWTH	-4.0782***
CHANGEINF	-3.2797**
CHANGEUKLIBOR3M	-9.4252***
CHANGEUKLIBOR6M	-9.5399***
CHANGEUKLIBOR12M	-9.4698***
CHANGEUKTBILL3M	-9.3855***
CHANGEGLCINF	-13.136***
CHANGEGLCREAL	-7.6713***
CHANGEBOERATE	-8.7275***
INDPRODGROWTH	-5.2774***
LABCOSTGROWTH	-2.6662*
LABPRODGROWTH	-5.8001***
FTSE100RETURNS	-10.361***
CHANGEFTSE100DY	-10.171***
CHANGEGBPSWAP5Y	-5.0930***
CHANGEGBPSWAP10Y	-4.9685***

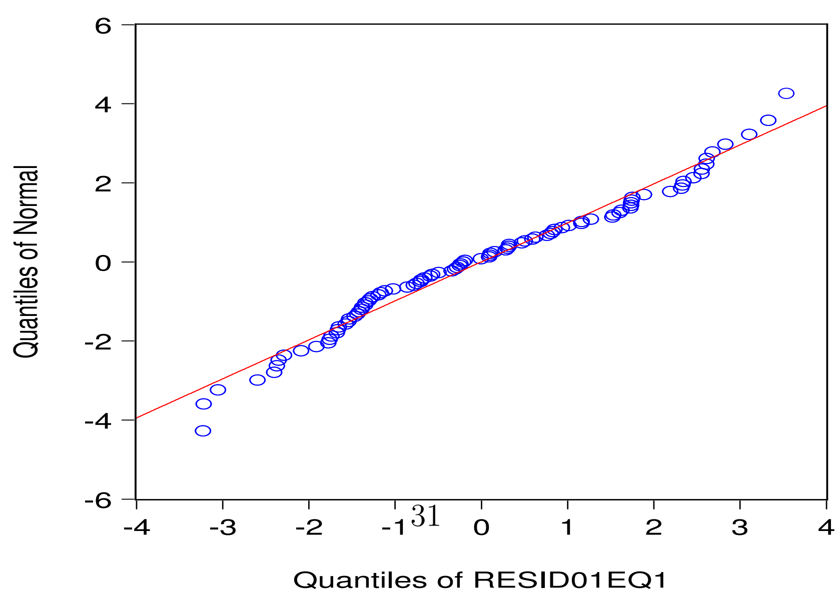
Table 12: Multiple regression model for IPD index logarithmic return for the period Q1 1987 to Q4 2011.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.3356	0.3585	3.7256	0.0003
GDPGROWTH	1.8899	0.3820	4.9464	0.0000
INDPRODGROWTH	0.5476	0.1856	2.9494	0.0040
CHANGEINF	1.3052	0.2636	4.95085	0.0000
R^2	68%			

Figure 6: Multiple regression model for IPD UK index logarithmic return for the period January 1987 to December 2011, quarterly data. The "Actual" represents the observed IPD return while "Fitted" represents the fundamental IPD return spanned by GDP growth, change in inflation, change in GLC inflation and the change in BBR.



(a) Residuals of the time series on the left axis and the observed vs fitted on the right axis



(b) QQ plot

Table 13: Multiple regression model for IPD index logarithmic return for the period January 1987 to December 2011, quarterly data. Chow test for structural breaks in January 1991 and January 2008.

Test	statistic	Type of distribution for test	p-value
F-statistic	3.0633	Prob. F(8,83)	0.0045
Log likelihood ratio	24.5775	Prob. Chi-Square(8)	0.0018

The refitted model over the first period Q2 1987 to Q4 1990 gives the results in Table 14(a) showing a very good fit.

The model re-estimated over the second period Q1 1991 to Q4 2007 gives the results in Table 14(b) indicating a good fit. As in the monthly case, post 1993 all IPD quarterly returns were positive. The fit of the model for the third period is described in Table 14(c). In this third period post subprime crisis the fit is remarkably good. The GDP growth and change in inflation are the determinant variables influencing the evolution of the IPD index returns. As with the monthly analysis, it is clear that the investors paid less attention to these important economic variables during the boom period of 1991 to 2007, but the realignment of commercial property prices following the subprime-liquidity crisis forced investors to link their appraisal valuations to changes in economy related to GDP and inflation.

6.5 Model with one lag

Here we shall consider the model with one lag for quarterly data. The estimated results for the entire period are displayed in Table 15. The fit looks excellent but there is a very high residual in 2009, as it can be observed in Figure 8.

The results of the Chow test with breakpoints in Q1 1991 and Q1 2008 are summarised in Table 16 and they confirm that the parameters are not constant across these three sub-periods.

The inference results for each of the three subperiods are presented in Appendix B.2.

Table 14: Multiple regression model for IPD index logarithmic return for the period three periods: Q2 1987 to Q4 1990, Q1 1991 to Q4 2007, and Q1 2008 to Q4 2011.

(a) The period Q2 1987 to Q4 1990.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	2.1140	0.7749	2.7279	0.0213
GDPGROWTH	1.6026	0.8126	1.9720	0.0769
INDPRODGROWTH	1.1342	0.3671	3.0892	0.0115
CHANGEINF	1.1125	0.4083	2.7246	0.0214
R^2	69%			

(b) The period Q1 1991 to Q4 2007.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.7771	0.5652	3.1440	0.0025
GDPGROWTH	1.2155	0.5675	2.1418	0.0360
CHANGEINF	0.9938	0.3001	3.3108	0.0015
R^2	30%			

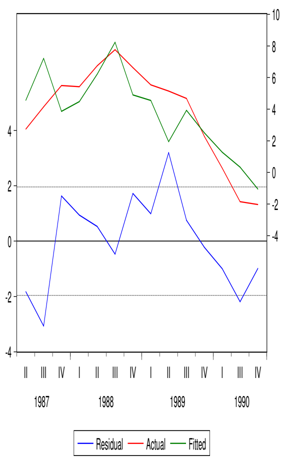
(c) The period Q1 2008 to Q4 2011.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.9366	0.3599	2.6020	0.0246
GDPGROWTH	3.1863	0.3084	10.3304	0.0000
CHANGEINF	1.7695	0.2767	6.3932	0.0001
R^2	93%			

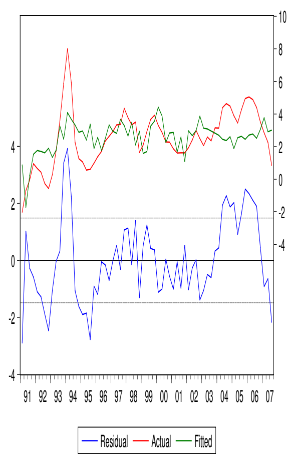
Table 15: Multiple regression model with one lag for IPD index logarithmic return for the period Q3 1987 to Q4 2011.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.4134	0.3377	1.2244	0.2240
GDPGROWTH	1.1865	0.4704	2.5223	0.0134
INDPRODGROWTH	0.4316	0.2041	2.1144	0.0372
LOGRETURNIPD(-1)	0.546566	0.162361	3.366361	0.0011
R^2	75%			

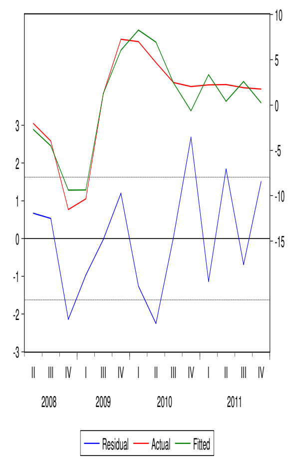
Figure 7: Fundamental final model selection for IPD UK index logarithmic return for the three periods January 1987 to December 1990, January 1991 to December 2007 and January 2008 to December 2011, with quarterly data. The "Actual" represents the observed IPD return while "Fitted" represents the fundamental IPD return spanned by GDP growth, change in GLC inflation and the returns from the previous two periods.



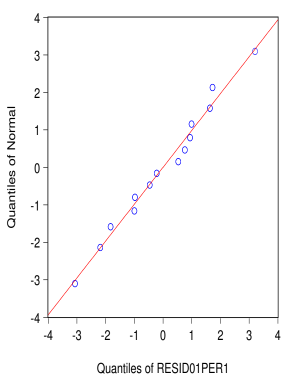
(a) Q1 1987 to Q4 1990



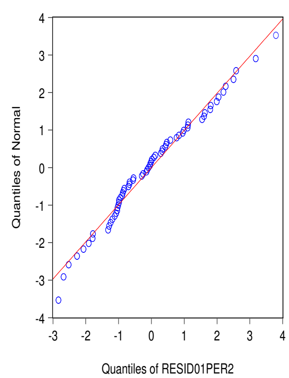
(b) Q1 1991 to Q4 2007



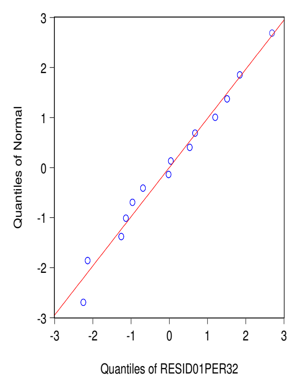
(c) Q1 2008 to Q4 2011



(d) Q1 1987 to Q4 1990

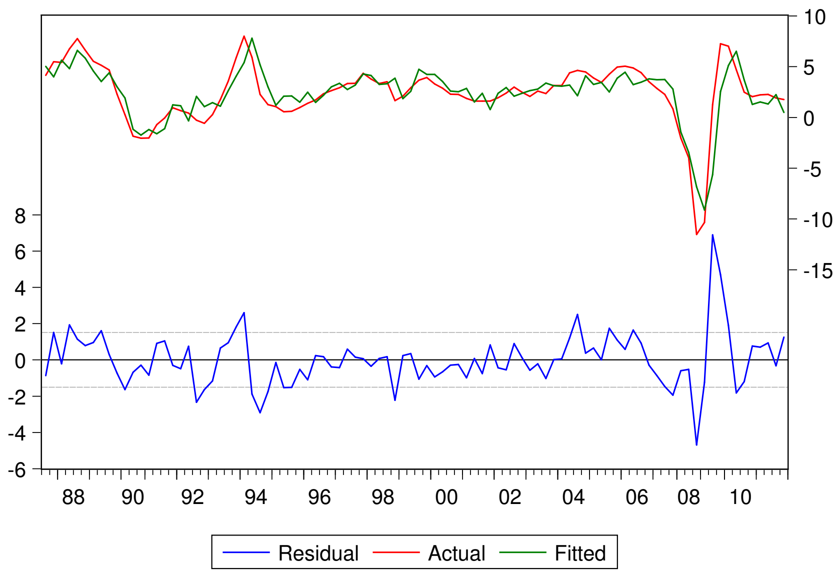


(e) Q1 1991 to Q4 2007

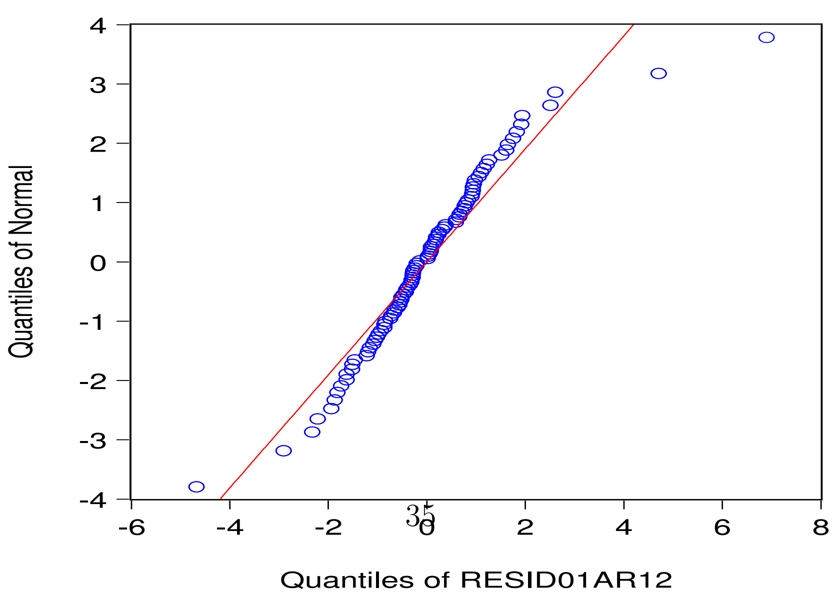


(f) Q1 2008 to Q4 2011

Figure 8: Multiple regression model including one lag for IPD UK index logarithmic return for the period January 1987 to December 2011, quarterly data. The "Actual" represents the observed IPD return while "Fitted" represents the fundamental IPD return spanned by GDP growth, change in inflation, change in GLC inflation and the change in BBR.



(a) Residuals of the time series on the left axis and the observed vs fitted on the right axis



(b) QQ plot

Table 16: Multiple regression model with one lag for IPD index logarithmic return for the period January 1987 to December 2011. Chow test for structural breaks in January 1991 and January 2008.

Test	statistic	Type of distribution for test	p-value
F-statistic	9.7996	Prob. F(8,82)	0.0000
Log likelihood ratio	63.0680	Prob. Chi-Square(8)	0.0000

7 The Implied Fundamental Economic Trend

In this section we shall use the observed Eurex futures prices on IPD UK index to infer the investor market view on the outlook ahead for the fundamental economic level of commercial real-estate in UK.

The idea is to employ formula (18) to reverse engineer the expression

$$(X'_T + X'_{T-1} + \dots X'_{t+1})\beta \tag{21}$$

where β is the one that has been calibrated on the historical data.

It follows then that

$$(X'_T + X'_{T-1} + \dots X'_{t+1})\beta = \ln\left(\frac{F^{Eurex}_{t,T}}{100}\right) - (T-t) \times r \tag{22}$$

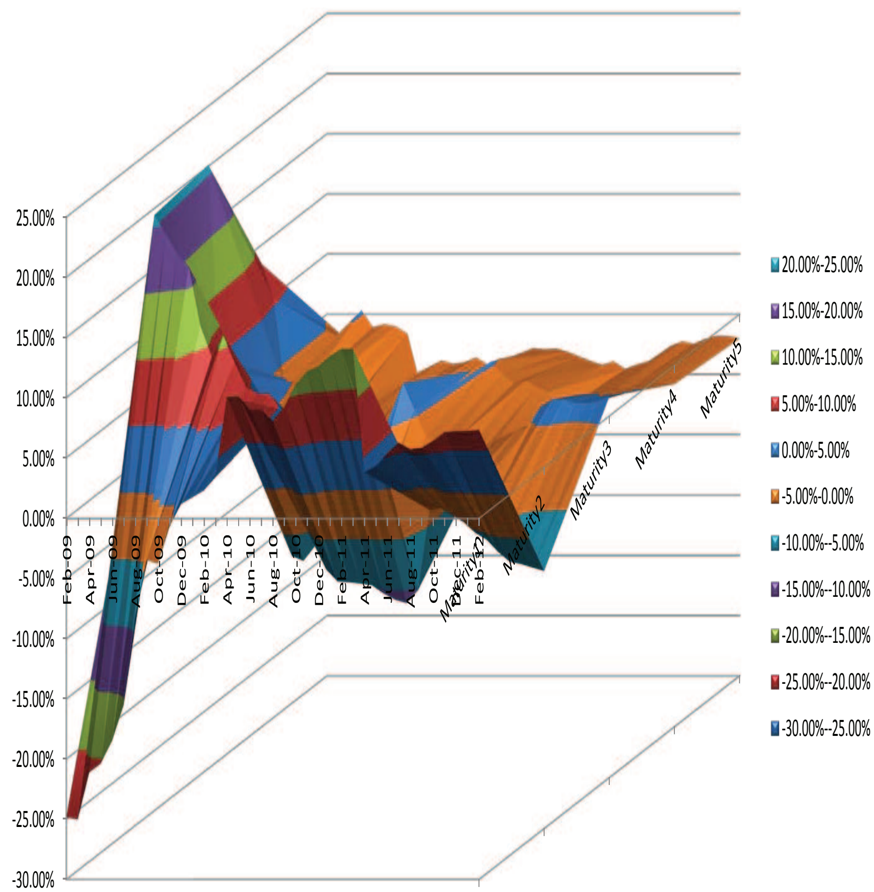
and this identity can be used to bootstrap the FET for various horizons T.

Figure 9 shows the FET implied from average monthly Eurex futures prices between February 2009 and February 2012. The period March 2009 to March 2010 can be characterised as a period of optimism for the first two maturities (March 2010 and March 2011) where high levels of FET was envisaged, jointly with pessimistic views towards the last three maturities (March 2012, 2013 and 2014). Second period, March 2010 to March 2011 continued almost the same, with high implied fundamental for the first maturity (March 2011) and very pessimistic outlook for March 2012 and March 2014, and an almost zero value for March 2015. The evolution of Eurex IPD futures prices during the last period, March 2011 to February 2012 indicates a negative outlook for March 2013 between -10% and - 5% followed by a recovery in the next year to March 2014 to positive values no larger than 5%

and a longer view for March 2015 and March 2015 closer to zero, which can be interpreted as a sign of economic stagnation.

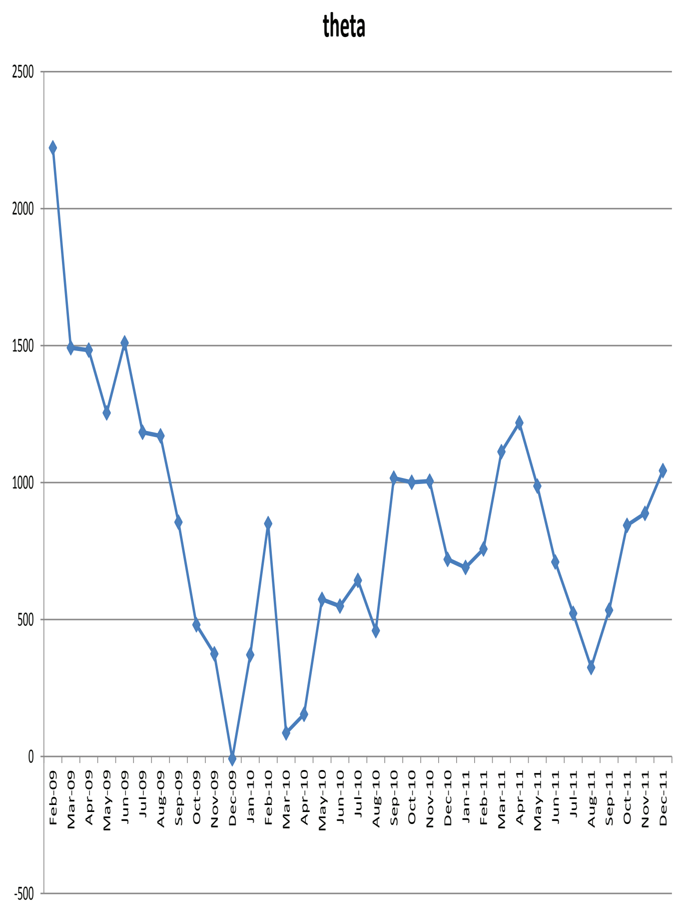
The market representative investor appears to be concerned with two type of maturities, short term, represented by the one year and two year contracts and a longer term view given by the fourth year and fifth year maturities. One can argue that the term structure of futures on IPD index could be made much longer, similar to swap curves and credit default swap curves. A longer term structure of futures contracts will also help improve the liquidity of shorter term contracts since the latter can be used as hedges for the former.

Figure 9: The implied fundamental economic values for all five Eurex futures maturities for the period February 2009 to February 2012.



This methodology assumes that the investor "knows" the riskfree rate r that applies to the required maturities.

Figure 10: The implied Girsanov risk-neutral parameter for the IPD index for the period February 2009 to December 2011.



Another important quantity that can be analysed is the Girsanov parameter θ_t^q that determines the change from physical measure to the conditional

Esscher pricing measure. This parameter can be calculated using the formula (13) only when we know the values for the corresponding FET in the future. Here we have used the estimated FET values from the first multiple regression model. In a real-market environment the parameters θ_t^q can be constructed on values for FET implied from futures markets or based on in-house analysis for the future values of macroeconomic and interest rate variables.

Using the estimates of the GARCH(1,1) model, the monthly averaged risk free rates from the market and the fundamental trend estimated with the model in Table 5, we obtain the monthly evolution depicted in Figure 10. The Girsanov coefficient provides also a measure of risk preference. This coefficient was very high in the first half of 2009 but then dropped in value in the second half and became more stationary in 2010.

8 Investment and Risks Management Applications

In this section I shall explore some applications from an investment point of view as well as some risk management applications.

8.1 Taking advantage of market signalling

The modelling approach presented in this paper allows investors to benefit from strategies based on market signalling. When the market sentiment is pushing IPD index returns (prices) far away from the corresponding FET value, the empirical evidence shows that there is a high probability that the difference will disappear or change sign in the near future. For example, over the period 2004-2007 there was a persistent signal that the market may experience a drastic correction. The longer the successive series of large differences between the observed IPD index return and the associated FET , the higher the probability of a change or market crash to occur.

The IPD futures traded on Eurex allow investors to take synthetic positions for outright directional speculation or for harvesting alpha. Going short the long end of the futures curve at any time between 2005 and 2007 would have led to significant gains. Unfortunately there were no futures contracts traded at the time so it would not have been possible to take advantage of this trading strategy.

However, investors in commercial real-estate could have taken advantage of the direction signalled by the model presented in this paper. I have performed the following ex post exercise on the excess returns calculated as the difference between the returns on the observable IPD index and the returns on the FET for the IPD index. Using the monthly data series and the model for FET spanned by the model discussed in Section 6.1 with estimates in Table 5, I associate a yellow signal for months ending a series of nine consecutive negative excess returns and a red signal for months ending a series of nine consecutive positive excess returns. Hence, yellow indicates that the market has been over-pessimistic and it is highly likely to see an increase in IPD index, while red indicates that the market has been over-optimistic and it is highly likely to see a drop in the IPD index.

For the period January 1987 to December 2011, first there was a red period for all months between May 1989 to October 1989, then a yellow period for all months between December 1994 to September 1995, followed by another yellow period for all months between February 2000 to June 2000, and another yellow period for all months between May 2003 and August 2003. Then a red period appeared between August 2004 and February 2005, and another red period signalled in October 2006 and November 2006. The last signalled period was yellow between August 2007 and February 2008.

Table 17: The monthly return is the logarithmic return of IPD index in excess of the return on the benchmark *FET* return. The cumulative excess return is annualised and it is calculated for a series of previous eight months plus the current one.

RED months $\downdownarrows$			Yellow months $\upuparrows$		
Month	excess return	cumulative excess return	Month	excess return	cumulative excess return
May-89	0.37%	99.50%			
Jun-89	1.36%	98.44%			
Jul-89	1.15%	103.41%			
Aug-89	1.70%	116.49%			
Sep-89	0.68%	103.34%			
Oct-89	0.11%	98.44%			
			Dec-94	-0.74%	-56.81%
			Jan-95	-0.29%	-60.00%
			Feb-95	-0.66%	-63.99%
			Mar-95	-0.61%	-64.07%
			Apr-95	-0.33%	-62.29%
			May-95	-0.51%	-57.84%
			Jun-95	-1.37%	-67.28%
			Jul-95	-1.05%	-72.14%
			Aug-95	-0.83%	-72.20%
			Sep-95	-0.50%	-71.39%
			Feb-00	-0.82%	-33.61%
			Mar-00	-0.24%	-36.48%
			Apr-00	-0.61%	-40.04%
			May-00	-0.33%	-43.11%
			Jun-00	-0.03%	-41.55%
Aug-04	0.60%	71.49%			
Sep-04	1.07%	82.93%			
Oct-04	0.47%	88.72%			
Nov-04	0.78%	97.74%			
Dec-04	1.42%	103.97%			
Jan-05	0.58%	102.39%			
Feb-05	0.46%	99.38%			
Oct-06	0.11%	78.01%			
Nov-06	0.12%	74.48%			
			Aug-07	-1.45%	-65.60%
			Sep-07	-1.93%	-85.45%
			Oct-07	-2.49%	-106.39%
			Nov-07	-4.46%	-140.47%
			Dec-07	-3.76%	-171.05%
			Jan-08	-1.91%	-188.29%
			Feb-08	-1.27%	-196.90%

The evolution of the commercial real-estate in UK before the subprime crisis suggests that there is an asymmetry between upward outlook and downward outlook, the periods of overoptimism or positive illusion being marked by much higher returns than the periods of pessimism or negative illusion. However, the analysis presented in Table 17 suggests that extreme negative excess returns may also occur. In other words it is possible for the market to experience absolutely high negative returns.

8.2 VaR and ES calculations

In this section I shall show how to calculate two of the most important risk measures for derivatives on IPD, taking advantage of the methodology highlighted in this paper.

The return per period is normally distributed with the Gaussian distribution given in (8). It is therefore straightforward to calculate VaR at the end of period $[t, t+1]$ for the critical level α.

$$VaR_{\alpha} = \sqrt{h_{t+1}}\Phi^{-1}(\alpha) + X'_{t+1}\beta + \lambda\sqrt{h_{t+1}} - \frac{1}{2}h_{t+1} \tag{23}$$

In order for this formula to be fully operational, the value of the FET for IPD index return should be known for the period $[t, t+1]$. The tradeoff for this flexible methodology is the requirement to forecast the FET value to the required horizon.

The same formula stands when calibration is done quarterly, with different model parameter estimates and different vector of covariates X. When the horizon is longer the model (5) can be used to determine the Gaussian distribution of return over that length of time.

Given our model calibrations presented earlier, I can show here how to calculate the value-at-risk for January 2012 for a portfolio represented by the IPD index. In order to apply the formula given in (23) the estimate of λ is taken from Table 2, the value for h_{t+1} calculated under the physical measure from the model (5) for the next period and this is equal to 0.0017%. In addition, it has been assumed that the FET for January 2012 would have the same value as the FET in December 2012.

In Table we present the VaR for the IPD index portfolio at various levels of confidence.

Table 18: Value at risk calculated for January 2012 at various critical levels α for a portfolio with the IPD index composition. Calculations based on model calibrated on monthly data between January 1987 to December 2011.

α	VaR
1%	1.1024%
5%	1.1243%
10%	0.9761%

Table 19: Reduction and increasing factors for the futures prices on IPD when covariates stay the same but there are changes of the riskfree rate.

Maturity in months	3	6	12	24	36	48	60
- 25 bps	0.9950	0.9875	0.9728	0.9441	0.9162	0.8891	0.8628
+ 25 bps	1.0050	1.0125	1.0278	1.0591	1.0914	1.1246	1.1589
- 50 bps	0.9900	0.9753	0.9464	0.8913	0.8395	0.7905	0.7445
+ 50 bps	1.0100	1.0253	1.0565	1.1218	1.1912	1.2649	1.3431

8.3 Marking to Model

Perhaps the most straightforward application of the modelling approach presented in this paper is related to the calculation of P&L positions on IPD derivatives. For accounting or risk management purposes, investors may need to mark to model on a daily or weekly basis.

A sudden change in GDP growth or inflation may impact immediately on the property derivatives. Using the model presented here it is easy to imply the new theoretical value of various property derivatives.

For example, immediately after the March roll-over consider that a reduction with 25 bps of the riskfree interest rate is imminent. Everything else staying equal, it is easy to see the impact of this change on futures prices. Using formula (18) we see that the futures price $F_{t,T}$ will go down by a factor equal to $e^{-0.25\times(T-t)}$. A similar calculation can be done for an increase of 25 bps or for any other value. In Table 19 I present the adjustment factors for futures prices when increasing or decreasing the interest rate by 25 bps or 50 bps.

9 Summary Discussion

There are several important conclusions coming out of this research.

- The investors can fit a so-called "fundamental economic term" level of the commercial property prices based on macroeconomic and interest rate variables.
- The variables spanning the *FET* at monthly and quarterly frequencies can be quite different. This can be explained by the different flow of information and business clock grafted on monthly and quarterly or annual IPD index
- The large departures from *FET* signal market corrections or crashes. Hence, our model can be used not only for pricing IPD derivatives but also as a basis for trading strategies.
- Statistical analysis points out to
  1. Between January 1993 and July 2007, all IPD index logarithmic returns were positive.
  2. The fitting of various econometric models based on macro and interest rate variables was exceptionally good in the period January 2008 to December 2011, indicating that investors in commercial property space in UK paid a lot more attention to these variables than in the previous period.
  3. The model fitting best the data for the period January 1991 to December 2007 was based solely on IPD return lag, indicating a lot of momentum, inertia and investor's exuberance
- The Eurex futures contracts are the underlying that should be used to calibrate all derivatives on IPD index.
- The model presented in this paper can be used for risk measures calculations in analytical format taking advantage of the normal distribution, for trading strategies based on disillusion effect, and for marking to model when important variables for real-estate asset class such as interest rate are changed.

- It seems that having more maturities on IPD futures will complete the futures curve and will allow trades to be put on the shape of the curve. In this way the liquidity of shorter maturities may be improved greatly.

- It is easy to calculate risk measures such as VaR once the model is calibrated and a view is taken on the future values of FET .

- Since the modelling framework is based on the multi-period conditional Esscher pricing, it is easy to adapt this methodology for pricing total return swaps and structured products such as cliquets.

- Preliminary research indicates that this methodology is also applicable to other markets such as US and other real-estate indexes.

References

Baran, L., R. Buttimer, and S. Clark (2008). Calibration of a commodity price model with unobserved factors: The case of real estate index futures. *Review of Futures Markets 16*(4).

Bjork, T. and E. Clapham (2002). On the pricing of real estate index linked swaps. *Journal of Housing Economics 11*, 418–432.

Black, D. (1986). Success and failure of futures contracts: Theory and empirical evidence. Monograph, New York University. Monograph Series in Finance and Economics.

Blundelll, G. and C. Ward (1987). Property portfolio allocation: A multi-factor model. *Journal of Property Research 4*, 145–156.

Bühlmann, H., F. Delbaen, P. Embrechts, and A. Shiryaev (1996). No-arbitrage, change of measure and conditional Esscher transforms. *CWI Quarterly 9*(4), 291–317.

Buttimer, R. J., J. Kau, and C. Slawson (1997). A model for pricing securities dependent upon a real estate index. *Journal of Housing Economics 6*, 16–30.

Carlton, D. W. (1984). Futures markets: Their purpose, their history, their growth, their success and failures. *Journal of Futures Markets 4*, 237–271.

Case, K. E., R. J. Shiller, and A. N. Weiss (1993). Index-based futures and options trading in real estate. *Journal of Portfolio Management 19*(2), 83–92.

Clayton, J., D. C. Ling, and A. Naranjo (2009). Commercial real estate valuation: Fundamentals versus investor sentiment. *Journal Real Estate Financial Economy 38*, 5–37.

Dobson and Goddard (1992). The determinants of commercial property prices and rents. *Bulletin of Economic Research 44*(4), 301–321.

Fabozzi, F. J., R. J. Shiller, and R. S. Tunaru (2009). Hedging real estate risk. *Journal of Portfolio Management 35*(5), 92–103.

Fabozzi, F. J., R. J. Shiller, and R. S. Tunaru (2010). Property derivatives for managing European real-estate risk. *European Financial Management 16*(1), 8–26.

Fabozzi, F. J., R. J. Shiller, and R. S. Tunaru (2011). A pricing framework for real-estate derivatives. *European Financial Management 18*(1).

Fisher, J., D. Gatzlaff, D. Geltner, and D. Haurin (2004). An analysis of the determinants of transaction frequency of institutional commercial real estate investment property. *Real Estate Economics 32*(2), 239–264.

Fisher, J., D. Geltner, and B. R. Webb (1994). Value indexes of commercial real estate: A comparison of index construction methods. *Journal of Real Estate Finance and Economics 9*, 137–164.

Geltner, D. and J. Fisher (2007). Pricing and index considerations in commercial real estate derivatives. *Journal of Portfolio Management 33*, 99–117.

Geltner, D. and J. Fisher (2008). *Handbook of Finance*, Chapter Pricing commercial real estate derivatives. New Jersey: John Wiley and Sons.

Geltner, D., B. D. MacGregor, and G. M. Schwann (2003). Appraisal smoothing and price discovery in real estate markets. *Urban Studies 40*(5-6), 1047–1064.

Geltner, D. and H. Pollakowski (2006). A set of indexes for trading commercial real estate based on the real capital analytics database. Technical report, MIT Center for Real Estate.

Ghysels, E., A. Plazzi, and R. Valkanov (2007). Valuation in US commercial real estate. *European Financial Management 13*, 472–497.

Hillebrand, E. (2003). A mean-reversion theory of stock-market crashes. working paper.

Jokivuolle, E. (1998). Pricing European options on autocorrelated indices. *Journal of Derivatives 6*, 39–52.

Ling, D. C. and A. Naranjo (1997). Economic risk factors and commercial real estate returns. *Journal of Real Estate Finance and Economics 14*(3), 283–307.

Lizieri, C., G. Marcato, P. Ogden, and A. Baum (2011). Pricing inefficiencies in private real estate markets using total return swaps. *Journal of Real Estate Finance and Economics*. DOI: 10.1007/s11146-010-9268-x.

MacKinnon, G. H. and A. A. Zaman (2009). Real estate for the long term: The effect of return predictability on long-horizon allocations. *Real Estate Economics 37*, 117–153.

McCue, T. E. and J. L. Kling (1994). Real estate returns and the macroeconomy: Some empirical evidence from real estate investment trust data 1972-1991. *The Journal of Real Estate Research 9*(3), 277–287.

Otaka, M. and Y. Kawaguchi (2002). Hedging and pricing of real estate securities under market incompleteness. Technical report, MTB Investment Technology Institute Co. Ltd., Tokyo.

Patel, K. and R. Pereira. (1996). Pricing property index linked swaps with counterparty default risk. *Journal of Real Estate Finance and Economics 36*(1), 5–21.

Pierangelo, C. and A. Gheno (2008). A model for pricing real estate derivatives with stochastic interest rates. Munich Personal RePEc Archive 9924, Munich.

Schwartz, E. S. and J. Smith (2000). Short term-variations and long-term dynamics in commodity prices. *Management Science 46*, 893–911.

Shiller, R. J. (1993). Measuring asset value for cash settlement in derivative markets: Hedonic repeated measures indices and perpetual futures. *Journal of Finance 68*, 911–931.

Shiller, R. J. (2008). Derivatives markets for home prices. Discussion paper 1648, Cowles Foundation.

Shiller, R. J. and A. N. Weiss (1999). Home equity insurance. *Journal of Real Estate Finance and Economics 19*, 21–47.

Siu, T., H. Tong, and H. Yang (2004). On pricing derivatives under Garch models: A dynamic Gerber-Shiu approach. *North American Actuarial Journal 8*.

Syz, J. M. (2008). *Property Derivatives*. Chichester: Wiley.

Syz, J. M. and P. Vanini (2009). Arbitrage free price bounds for property derivatives. *Journal of Real Estate Finance and Economics*.

Titman, S. and W. Torous (1989). Valuing commercial mortgages: an empirical investigation of the contingent-claim approach to pricing risky debt. *Journal of Finance 44*, 345–373.

Van Bragt, D., M. Francke, B. Kramer, and A. Pelsser (2009). Risk-neutral valuation of real estate derivatives. OFRC working paper No. 2009-02, University of Amsterdam Ortec Finance Research Center, Amsterdam.

Wit, I. D. and R. V. Dijk (2003). The global determinants of direct office real estate returns. *Journal of Real Estate Finance and Economics 26*(1), 27–45.

A Econometric Output Determinants

A.1 Monthly Data

Table 20: List of outliers from regression model

	Outliers
1	1987M10
2	1988M04
3	1988M05
4	1988M09
5	1988M12
6	1989M08
7	1990M04
8	1990M05
9	2008M01
10	2009M11
11	2009M12

B Modelling the fundamental component of IPD index return

B.1 Monthly Modelling

B.1.1 Model with two lags

Here we show the results obtained when considering also a second lag of the IPD index return variable. The model with fitting results is presented in Table 21.

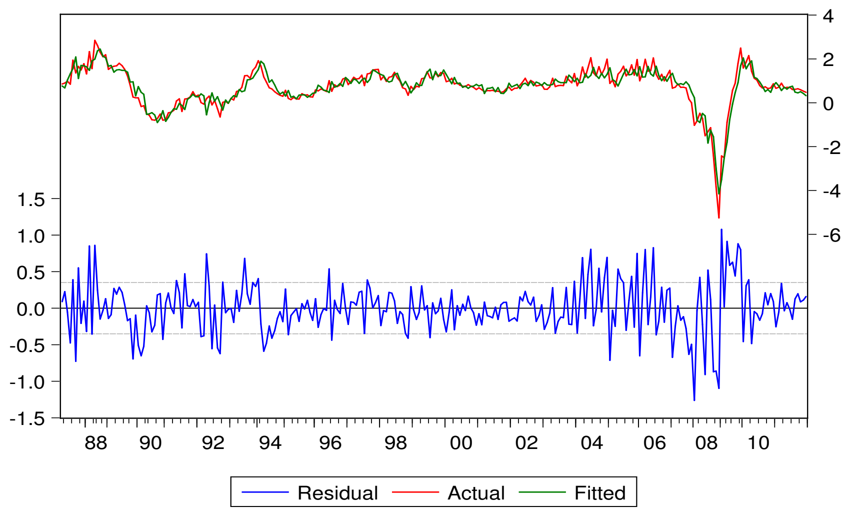
As can be seen from Figure 11 the fit is excellent.

B.1.2 Chow test for Model with two lags

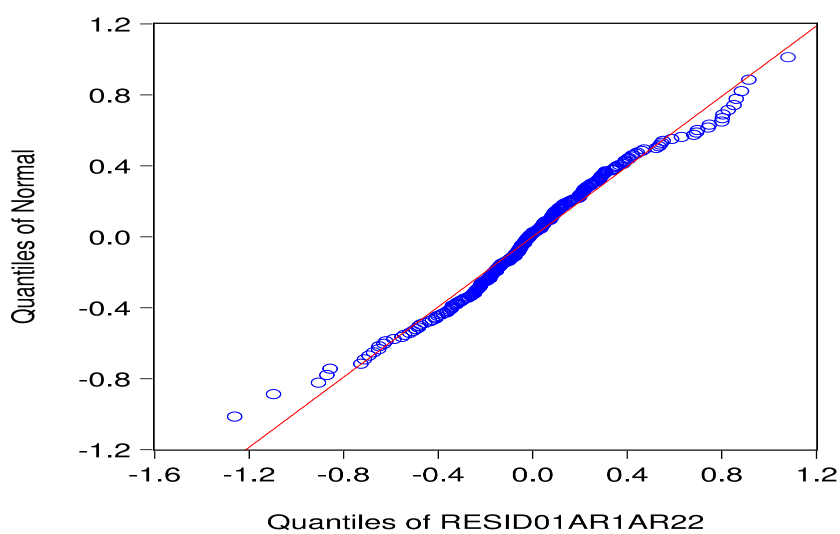
It is interesting to see whether the structural breaks identified vis-a-vis the multiple linear regression model without lag are significant for this new model that includes one lag of the IPD index. The results presented in Table 22 shows that the null hypothesis that parameters are constant across the three subperiods is rejected at 1% level of significance.

As done above, it is useful to refit the model with one lag for each subsample period.

Figure 11: Multiple regression model for IPD UK index logarithmic return, monthly, for the period January 1987 to December 2011. The "Actual" represents the observed IPD return while "Fitted" represents the fundamental IPD return spanned by GDP growth, change in GLC inflation and the returns from the previous two periods.



(a) actual levels



(b) log scale

Table 21: Multiple regression model including one lag for IPD index logarithmic return for the period January 2008 to December 2011. The R-square is equal to 85%.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.0638	0.0482	1.3229	0.1870
GDPGROWTH	0.5804	0.1667	3.4809	0.0006
CHANGEGLCINF	0.5419	0.2316	2.3389	0.0201
LOGRETURNIPD(-1)	0.6500	0.0684	9.4921	0.0000
LOGRETURNIPD(-2)	0.1260	0.0769	1.6371	0.1028

Table 22: Model with one lag for IPD index logarithmic return for the period January 1987 to December 2011. Chow test for structural breaks in January 1991 and January 2008.

Test	statistic	Type of distribution for test	p-value
F-statistic	8.0140	Prob. F(10,259)	0.0000
Log likelihood ratio	73.8668	Prob. Chi-Square(10)	0.0000

Table 23(a) shows the results of model selection when including also one added lag fitted for the period January 1987 to December 1990. The only two significant variables are the GDP growth and the previous return on IPD index. The model fitting results for the second period are illustrated in Table 23(b). Only the two lags of the returns of IPD index are significant, in line with the results obtained for the model with one lag.

Ffor the period January 2008 to December 2011, that is after the subprime crisis, the results are different. The regression estimation and testing results shown in Table 23(c) indicate that property investors should have considered GDP growth, GLC UK implied inflation spot curve and the previous month return on the IPD index. The second lag of IPD was not significant over this period. This can be interpreted as the commercial property market in the UK has lost some of its serial correlation (memory, inertia) starting in 2008. This behaviour can be attributed to the series of problems affecting financial markets over that period.

The fit is exceptionally good and this is again remarkable given that this is a turbulent period generally speaking. As can be seen from Figure 12 the

Table 23: Multiple regression model with two lags for IPD index logarithmic return for each period, January 1987 to December 1990, January 1001 to December 2007 and January 2008 to December 2011.

(a) January 1987 to December 1990

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.0986	0.0816	-1.2089	0.2350
GDPGROWTH	0.7748	0.2028	3.8201	0.0005
LOGRETURNIPD(-1)	0.3789	0.1583	2.3937	0.0223
LOGRETURNIPD(-2)	0.5058	0.1544	3.2755	0.0024
R^2	91%			

(b) January 1001 to December 2007

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.1099	0.0294	3.7286	0.0003
LOGRETURNIPD(-1)	0.5817	0.0669	8.6967	0.0000
LOGRETURNIPD(-2)	0.2889	0.0634	4.5569	0.0000
R^2	74%			

(c) January 2008 to December 2011

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.2054	0.0796	2.5808	0.0140
GDPGROWTH	1.4197	0.2911	4.8763	0.0000
CHANGEGLCINF	0.8978	0.2047	4.3846	0.0001
LOGRETURNIPD(-1)	0.5519	0.0823	6.7039	0.0000
LOGRETURNIPD(-2)	0.0518	0.0664	0.7811	0.4397
R^2	93%			

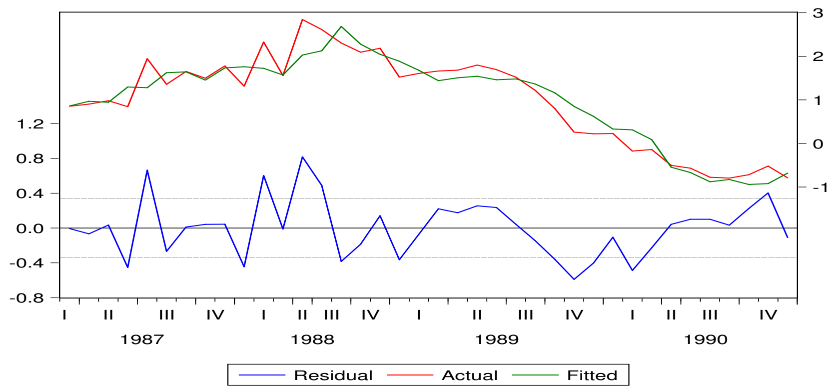
fit is very good for all three subperiods. Including the second lag had the effect to pull the observed and the fundamental series closer together.

B.2 Quarterly Modelling

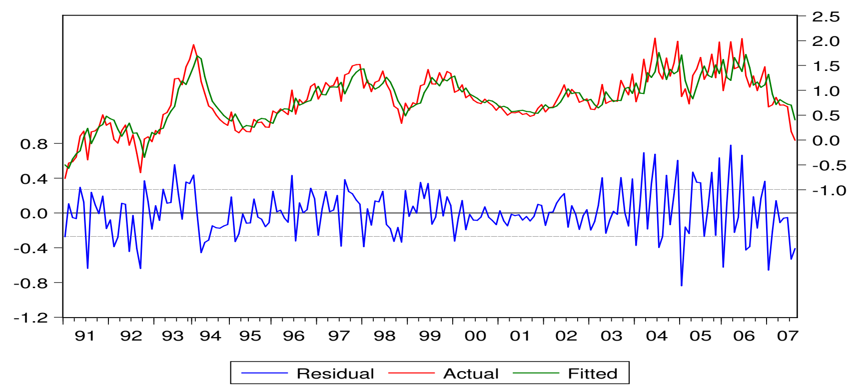
In Table 24 we describe the results for each subperiod of the model selected for the fundamental of IPD. The first period prior to 1991 could be easily characterised by the UK GDP growth, UK industrial production growth and the previous quarter IPD return, as can be seen from results in Table 24(a). The results for the second period between 1991 and 2008 are similarly good. It can be observed in Table 24(b) that for this period only the lagged return was significant, similar to the monthly analysis. Last but not least, the third period following the subprime crisis was determined by the GDP growth and UK industrial production growth plus the return on IPD index over the previous period. The results are depicted in Table 24(c).

The results for the three subperiods presented in Figure 13 reveal that it is possible to estimate the fundamental level to a close degree. Remark, once again, that the fit illustrated in Figure 14(b), is based solely on the previous index return, there is no connection to the macroeconomic variables or the interest rates.

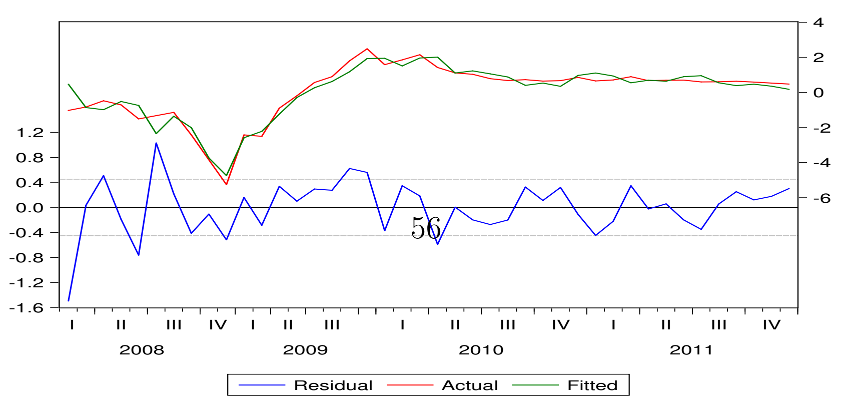
Figure 12: Multiple regression model including one lag for IPD UK index logarithmic return, for the three periods January 1987 to December 1990, January 1991 to December 2007 and January 2008 to December 2011. The "Actual" represents the observed IPD return while "Fitted" represents the fundamental IPD return spanned by GDP growth, change in GLC inflation and the returns from the previous two periods.



(a) actual levels



(b) log scale



(c) log scale

Table 24: Multiple regression model with one lag for IPD index logarithmic return, for quarterly data, for all three subperiods, Q3 1987 to Q4 1990,Q1 1991 to Q4 2007 and Q1 2008 to Q4 2011.

(a) The period Q3 1987 to Q4 1990

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.1853	0.4309	-0.4300	0.6773
GDPGROWTH	0.8383	0.1977	4.2406	0.0022
INDPRODGROWTH	0.5956	0.3312	1.7980	0.1057
LOGRETURNIPD(-1)	0.7933	0.0838	9.4591	0.0000
R^2	93%			

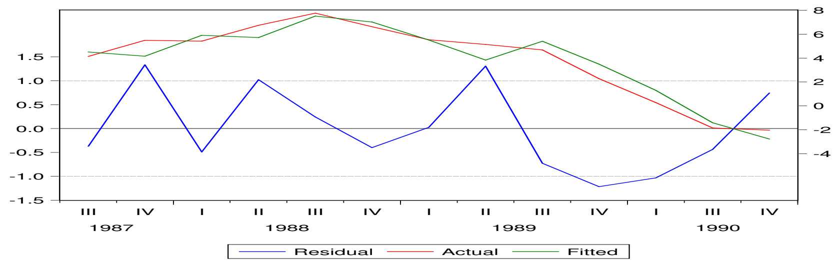
(b) The period Q1 1991 to Q4 2007.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.4737	0.1522	3.1124	0.0028
LOGRETURNIPD(-1)	0.8292	0.0622	13.3234	0.0000
R^2	75%			

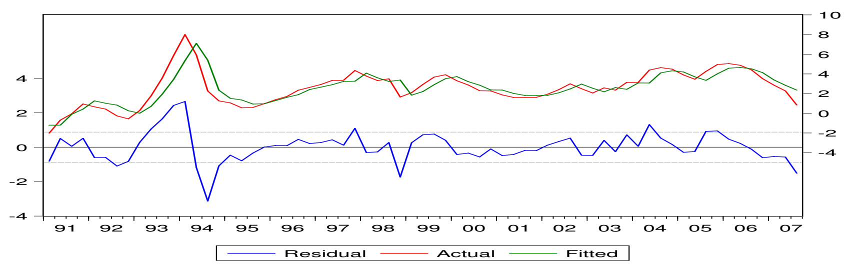
(c) The period Q1 2008 to Q4 2011.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.9924	0.6651	2.9953	0.0134
GDPGROWTH	2.4648	0.6483	3.8019	0.0035
INDPRODGROWTH	1.1609	0.2616	4.4375	0.0013
LOGRETURNIPD(-1)	0.1677	0.0867	1.9343	0.0818
R^2	89%			

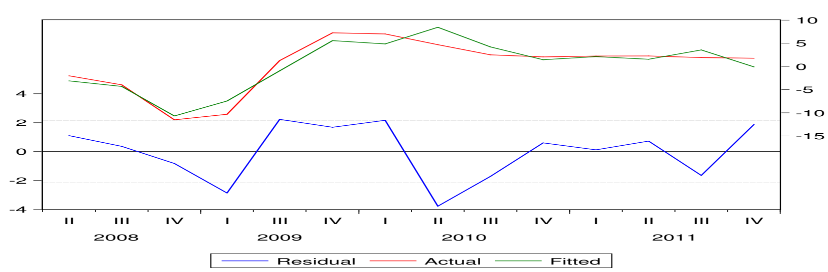
Figure 13: Multiple regression model including one lag for IPD UK index logarithmic return, quarterly, for the period January 1987 to December 2011. The "Actual" represents the observed IPD return while "Fitted" represents the fundamental IPD return spanned by various variables.



(a) fundamental given by GDP growth, industrial production growth and IPD one lag



(b) fundamental given by IPD one lag



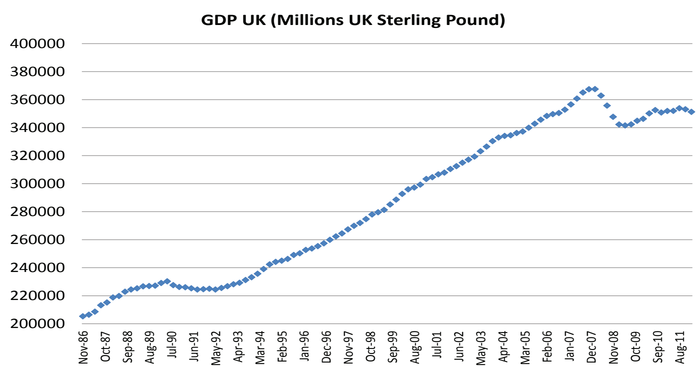
(c) fundamental given by GDP growth, industrial production growth and IPD one lag

C GDP monthly interpolation

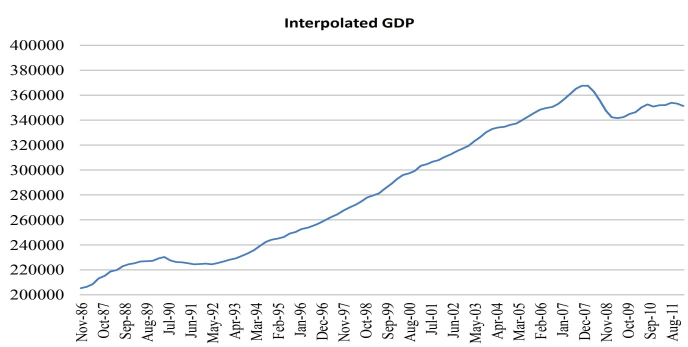
One problem in constructing the fundamental component for the IPD commercial index is the lack of data at monthly frequency since this is reported at quarterly tenors.

Therefore, we have used linear interpolation to fill in the monthly GDP levels for the other two months in the quarter. The graphs with quarterly and monthly GDP levels are illustrated here in Figure 15(a) and 15(b).

Figure 14: IPD Annual UK historical trend for the period 1980-2009.



(a) actual levels



(b) log scale

D Calibration GARCH model

$$Z_t = \lambda\sqrt{h_t} - \frac{1}{2}h_t + \xi_t \tag{24}$$

$$\xi_t \sim N(0, h_t) \tag{25}$$

$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \xi_{t-i}^2 + \sum_{j=1}^{q} \beta_j h_{t-j} \tag{26}$$

The calibration is done with maximum likelihood. For convenience we have used a GARCH(1,1) process but other GARCH processes such as NGARCH with a leverage effect may be considered.