Pricing Contingent Capital Bonds: Incentives Matter

Charles P. Himmelberg

Goldman Sachs & Co

Sergey Tsyplakov

University of South Carolina*

Classification Codes: G12, G13, G32

Key words: contingent capital, structural pricing model, credit spreads, market imperfections, money burning.

*The views in this paper are those of the authors and do not necessarily reflect the views of Goldman Sachs.Charlie Himmelberg is Managing Director at Goldman Sachs, 200 West Street, NY, NY 10282; email: Charles.Himmelberg@gs.com; phone: 917-343-3218. Sergey Tsyplakov is Associate Professor of Finance at the Moore School of Business, University of South Carolina in Columbia, SC 29208; e-mail: sergey@moore.sc.edu; phone: 803-466-2197. We thank Charles Calomiris, Amanda Hindlian, Sandra Lawson, Louise Pitt, Steve Strongin, Zhenyu Wang and seminar participants at Federal Reserve Bank of New York for helpful discussions and comments.

Pricing Contingent Capital Bonds: Incentives Matter

Abstract

This paper presents a dynamic pricing model that quantifies and prices the incentive effects of Contingent Capital (CoCo) bonds (that is, debt instruments that automatically convert to equity when the issuing bank's capital falls below a pre-defined trigger). Our model has important implications for the optimal design of CoCo bonds. If properly designed, CoCo bonds in the capital structure encourage banks to maintain high capital ratios. In particular, our model suggests that firms would preemptively raise equity to avoid the dilutive consequences of automatic conversion. Assuming this pre-commitment to conservative capital structure management is priced by the market at time of issuance, it benefits both issuers and investors. By contrast, CoCo's that simply write down bond principal without diluting shareholders create perverse incentives to pursue higher leverage, thus increasing both the expected social cost of financial distress as well as the cost of capital for issuers. These results highlight the fundamental importance of incentive considerations in the design of contingent capital bonds.

1 Introduction

Contingent Capital bonds – also known as contingent convertibles (or CoCos) – are bonds that can be automatically written-down or converted to common equity when the issuer's financial health deteriorates to a pre-defined threshold or trigger. The most natural issuers of contingent capital are often assumed to be banks, but results we discuss have general implications for capital structure research.

There are two primary advantages of CoCos, we argue. The first is that they can automatically recapitalize a bank when it is overleveraged and likely in distress, but still has significant enterprise value. When conversion is triggered, the CoCo turns into equity or is partially written-off. No new external funds are injected into the bank, and no government funds are needed for bailouts. For this reason, CoCos are currently viewed as a potentially valuable tool for helping bank regulators address the "too big to fail" problem.

The second advantage – one that is the focus of this paper, but has been largely ignored by researchers and policymakers – is that CoCos can create a strong incentive for a bank to issue equity in response to future capital losses thereby committing the bank to a conservative capital structure. They would similarly commit bank to lower leverage during periods of rising investment opportunities ("credit booms"). By reducing the risk of future excess leverage as well reducing the probability of future conversion this "incentive dividend" can increase valuation, reduce the costs of bank capital, and reduce the social welfare costs of financial distress.

The devil, however, is in the details. If the CoCo bond is poorly designed, its incentive effects on dynamic capital structure decisions are no better than straight debt, and may even be worse for a range of design parameters. It is well-understood that a bank financed with straight debt and managed on behalf of shareholders has no incentive to issue equity during financial distress because the cost of dilution exceeds the benefit of lower financial distress with the most of the benefits going to bondholders. This ex-post conflict of interest is the well-known "debt overhang problem."[1] Among other phenomena, this logic can explain why banks that are overlevered and vulnerable to distress are often reluctant to issue equity, and may help explain why bank managements often view equity as a relatively expensive form of external financing.[2]

Debt overhang also generates an incentive to pursue high-risk but low-return investments, which may also help explain what often seems to be the rapid deterioration of over-levered financials. The fact that banks with plain vanilla debt structures choose not to raise capital during economic downturns generates not only excess leverage but also negative externalities to the rest of the financial system. Regulatory capital requirements are designed to mitigate such risks. The current Basel III proposal, for example, requires capital-qualifying debt to be loss-absorbing, meaning debt that is either written off or converted into common equity upon the occurrence of a severe financial distress or other "trigger event." But so far these efforts (including Basel-III) do not appear to recognize that loss-absorbing securities can have powerful incentive effects on bank behavior. This lack of understanding has potentially serious consequences for the future behavior of banks in the next financial crisis.

Some bonds already in existence feature conversion terms that writedown principal for bondholders and thus transfer wealth to equityholders.[3] For example, in March of 2010, the Rabobank issued a loss-absorbing bond that assumes a 75% principal write-down if the bank's capital ratio falls below 7% trigger. The dynamic incentive effects captured by our model

[1] See Myers (1984) and Myers and Majluf (1984).

[2] In addition to shareholders dilution, when the bank issues equity under duress, it can be viewed by the market as a negative signal that the bank is in even worse shape than previously thought. This problem is related to an adverse selection problem suggested by Akerlof (1970) and Leland and Pyle (1977). Due to this adverse selection problem, the bank can only sell equity at a discount relative to what the existing shareholders would prefer.

[3] European banks have already issued more than $10 billion worth of loss-absorbing bonds over the last two years (Pitt, Hindlian, Lawson and Himmelberg (2011)).

suggests this contract has potentially perverse incentives. If the bank becomes distressed and its capital ratio is just above the trigger, the bank may find it optimal to ”burn money” (or, equivalently, ”burn capital”) to push its capital ratio below the trigger. By burning assets, the bank can breach the trigger, write-down 75% of principal and guarantee a windfall for shareholders.[4] This write-down feature of the bond is thus obviously objectionable, since it gives banks the incentive to pursue value-destroying strategies and thus accelerate the spiral into financial distress. In other words, these incentives only exacerbate ”debt overhang” problems by magnifying incentives for risk shifting and/or underinvestment.[5] These incentives are highly undesirable because it is exactly during the crisis that is important to have mechanisms in place that discourage banks from exercising value-destroying strategies.

But CoCos can have powerful positive incentives effects if the CoCo dilutes shareholders rather than CoCo investors. If the CoCo conversion is properly dilutive for equity, the bank has strong incentives to avoid triggering conversion by preemptively de-leveraging and raising equity capital well before it becomes financially distressed.[6] The bank's future decision to issue equity, recapitalize, and move away from the conversion trigger will be driven by the trade-off between diluting equity on the bank's terms vs. accepting punitive terms dictated by the CoCo contract. As such, a tranche of CoCo bonds in the bank's capital structure acts like an ex-ante "commitment mechanism" that provides the bank with incentives to maintain a more conservative leverage ratio.

These incentives effects, whether value-creating or value destroying, have not yet been carefully analyzed or quantified by existing models in the literature on CoCos, and this paper

[4] Financial literature discusses how ”money burning” can mitigate adverse selection costs in asymmetric models that analyze dividend choices or over/underpricing of equity issuance (See for exmaple, John and Williams (1985), Ambarish, John, and Williams (1987), Allen and Faulhaber (1989), Grinblatt and Hwang (1989)). In these models, the equilibrium is characterized by money burning or other type of costly signaling.

[5] See Berg and Kaserery (2011).

[6] In the Appendix 1 we use a simple one-period numerical example to illustrate incentives that the CoCo tranche can create for the bank with respect to issuing equity or burning money.

is intended to fill this gap.[7] The model in this paper builds on the existing literature by introducing dynamic equity issuance (and money burning) and quantify its impact. The results show that if the contractual terms of the CoCo do not dilute shareholders upon conversion, the bank will have no incentive to recapitalize by issuing equity preemptively. Moreover, we show that CoCo contracts designed to allow large write-offs (like the loss-absorbing bond recently issued by Rabobank) can generate incentives for bank managers to "burn money".

We explore a number of important extensions of the model intended to accommodate practical design considerations. First, we allow for negative "jumps" in asset values (i.e., bank runs). Second, we quantify and price conflicts of interest that arise between contingent capital holders, equity holders, and various classes of bond investors. We also consider the consequence of transaction costs for equity issuance. Finally, we consider the case of the so-called "soft trigger" in which the conversion does not necessarily occur at the first instant when the trigger is breached. In this extension, we assume that, when below the trigger, the CoCo conversion is probabilistic so that the bank can default without CoCo being converted.

We start with a model in which there are no frictions or jumps except proportional default costs. In the model, the bank issues both a CoCo tranche and straight debt. Both CoCo debt and straight debt pay coupons and have the same maturity. Conversion is triggered when the equity ratio drops below a contractually specified threshold. The CoCo bond converts for a fixed dollar amount of shares. As an alternative to conversion, the bank can choose to issue equity at any time prior to breaching the trigger, thereby raising its capital ratio and moving away from the trigger. The bank can also choose to "burn money" by choosing to dispose assets, thereby reducing its capital ratio and triggering conversion. The

[7] The literature that present models for CoCo bond pricing includes McDonald (2009), Albul, Jaffee, and Tchistyi (2010) Pennacchi (2010), Andersen and Buffum (2002), and Sundaresan and Wang (2011).

bank's decisions on equity issuance, money burning and default are chosen to maximize the existing shareholders' wealth, i.e., to maximize share price. We estimate the range of CoCo contract terms over which the bank will be willing to issue equity to avoid the conversion, or conversely, for the case of "anti-dilutive" terms, to burn money and accelerate conversion.

Our numerical calculations suggest that desirable incentives can be created by CoCos that are a relatively small fraction of debt outstanding, but the terms of the conversion have to be fairly dilutive for the existing shareholders. To reduce default costs of by one percentage point, a CoCo bond should convert to about \$1.20 per dollar of par value of market value of equity and the CoCo size should be about 5% of total assets. On the other hand, loss-absorbing contracts with principal write-downs on the order of 75% may perversely cause the bank to "burn" as much as 3% of assets to accelerate conversion, especially as the bond approaches maturity.

We extend the model to allow for negative jumps in the diffusion process that describes the value of a bank's assets. This is to capture the fact that US financial system is periodically hit by large crises, producing unexpected extreme losses in bank assets that can quickly deplete banks' capital ratios. Comparative statics on the model suggest that while CoCo bonds retain their importance as a source of contingent capital per se, such jumps weaken the incentive for banks to issue equity preemptively. This is because equity values can drop all the way through the width of the CoCo tranche, thus increasing the probability of conversion and default despite the bank's effort to hold excess capital. We also introduce transaction costs of issuing equity. There costs are assumed to have both fixed and variable components, and are meant to capture informational frictions.[8] When the bank faces fixed costs, it will issue equity infrequently and by discrete amounts. All else equal, such costs obviously reduce

[8]For example, the drop in stock prices can be driven by a negative market perception of earnings per share dilution or by adverse selection considerations. See Loughran and Ritter (1995).

the incentives for the bank to issue equity. Similarly, in the economy with the "soft trigger", the bank's incentive to issue equity and manage its leverage weakens.

Just as plain-vanilla debt in capital structures introduces conflicts of interests between debt- and equity-holders, so too does CoCo debt introduce additional layers of conflict. The ex-post issuance of dilutive CoCo debt on top of the pre-existing debt entails a wealth transfer from the shareholders to the existing debtholders. Holders of existing straight debt benefit because the incentive created by the CoCo debt will cause the bank to decrease its leverage ratio during distressed times and lower overall default risk. Therefore, existing shareholders will be reluctant to issue CoCo bonds with dilutive terms on top of the pre-existing debt in its capital structure; bank regulators would have to mandate such issuances.[9] If issued simultaneously with senior debt, however, bank's shareholders will benefit because the CoCo tranche will increase the value of the newly issued straight debt and thus reduce borrowing costs.

2 Model

2.1 Time Line of the Model

We consider a dynamic structural model of the bank in the economy with no taxes. At time 0, the bank issues both the CoCo debt and straight debt in its capital structure. Both CoCo bond and straight bond assume coupons payments, and have the same maturity. At maturity, par values are paid to their claimants respectively, assuming the CoCo has not been converted in prior periods. Bank's assets are stochastic and can be subject to jumps. Bank's assets continuously generate cash flow that is proportional to assets size. The bank uses its

[9] During distress periods, when the conversion is likely, there could be also a potential conflict of interests between holders of the CoCo bonds and the holders of the existing debt. In such cases, debtholders of the straight debt may pressure the regulators to trigger the CoCo conversion earlier because the conversion will make remaining straight debt less risky. On the other hand, if the CoCO conversion assumes some principal write-off, the holders of CoCo will try to pressure the regulators to postpone the conversion.

cash flow to make coupon payments and pays residual as dividends to the equityholders. If the bank's assets value declines significantly and its capital ratio (measured as the book equity divided by assets) reaches some predetermined trigger or falls below it, the CoCo bond will automatically convert to equity for a fixed dollar amount of shares.[10] At conversion, the equity value to preexisting shareholders will depend on the value of the bank's assets and the conversion terms. After conversion, the bank continues servicing remaining straight debt and making coupon payments. If the asset value falls further, the bank can choose to default on its straight debt. At any time, the bank can issue equity to increase its capital ratio and move away from the conversion trigger. When the bank issues equity, it may incur transaction costs that have fixed and variable components.

We also assume that the bank can burn money and reduce the size of its assets. By burning money the bank can reduce its capital ratio and force the conversion. Equity issuance, money burning and default are endogenous and chosen to maximize the existing shareholders' wealth, i.e., current share price. Figure 1 illustrates the time line of the model by depicting the sample paths of bank assets, total debt and the CoCo. Along this sample trajectory, the bank issues equity at time t_1, and conversion happens at time t_2. After time t_2, the asset value remains above the default boundary on the sample paths 1 and 2 and the bank pays off its straight debt at maturity T. On sample path 3, asset values decline below the default boundary and the bank defaults.

[10] The model can incorporate the conversion to a fixed number of shares. McDonald (2010) discusses the advantages and disadvantages of these two cases.

2.2 Continuous-Time Pricing Model

2.2.1 Value of the bank capital

We assume that during periods when there is no injection of capital (or money burning) and no arrival of jumps, the value of the bank's assets follows a log-normal stochastic process:

$$\frac{dV}{V} = (r - \alpha)dt + \sigma dW_V, \tag{1}$$

where r is a short-term risk-free rate which is assumed to be constant; α is the payout rate; W_V is a Weiner process under the risk-neutral measure; and σ is the instantaneous volatility coefficient. Given payout rate of α, bank's assets generate continuous cash flows of $\alpha \cdot V$. Equityholders can choose to increase (decrease) the size of the bank's capital by issuing equity (burning money) of any size. If the bank decides to issue equity (burn money) and raise (reduce) capital by $\triangle V_t$, the value of the bank's assets at time $t+$ will be:

$$V_{t+} = V_t + \triangle V_t, \tag{2}$$

where $\triangle V_t > 0$ is the size of newly raised equity ($\triangle V_t < 0$, if the bank burns money). We assume that when the bank issues equity and raises capital, it incurs transaction costs TC, which have both a fixed component (proportional to the level of its current assets) and a variable component (proportional to the size of newly raised assets $\triangle V > 0$). As such, if the bank issues equity and raises the size of its assets by $\triangle V$, total transaction costs $TC = e \cdot (V_t + \triangle V)$, and e is a positive constant. These transaction costs are paid by the equityholders of the bank. For simplicity, we assume that there are no transaction costs if the bank burns money.

We also assume that the bank's asset value can be affected by the infrequent jumps due to the arrival of "big" events that have more than a marginal effect on the bank's asset value. We assume that the arrival of jumps are independent from each other. Following

Merton (1990), jumps are described by a Poisson-arrival process. The arrival time and size of the jump Y are independent. Specifically, the probability that the jump arrives during time interval Δt ($\Delta t \to 0$) is $\lambda \cdot \Delta t$, where λ is risk-neutral arrival intensity that describes expected number of jumps per year. If at time t, a jump of size Y_t arrives , the bank assets will change from V_t to $V_{t+} = Y_t \cdot V_t$ in the next instant $t+$, where Y_t ($\geqslant 0$) is realization of a random variable Y at t that describes the percentage change in bank's assets. Variable Y_t is independently and identically distributed. The expected change in bank's assets due to jumps is $\lambda \cdot k$, where k is the expected percentage change in the bank's capital if the jump occurs, i.e., $k = \mathbb{E}^Q(Y - 1)$, where $\mathbb{E}^Q$ is the expectation under the risk neutral measure Q. With jumps incorporated, the value of bank's assets is described by the following equation:

$$\frac{dV}{V} = (r - \alpha - \lambda \cdot k)dt + \sigma dW_V + dq, \tag{3}$$

where $dq = \lambda \cdot (Y_t - 1) \cdot dt$, which describes fluctuations in bank assets due to jumps. Obviously, if $\lambda = 0$, then the term $dq = 0$, and the assets process will be the same as in (1).

Jump events allow the bank asset values to decline by a lumpy random amount so that its capital ratio can decline far enough to cause CoCo conversion or even default. Jumps are asymmetric and can lead to large negative random fluctuations in bank's assets, but not instant upsurges. As such, the expected percentage change in the bank's capital if the jump occurs is $k < 0$. As shown in (3), the diffusion drift of the jump-diffusion process, is adjusted by term "$-\lambda \cdot k$".

Figure 3 provides an illustration of the two sample paths: the ordinary diffusion process and the jump-diffusion process with asymmetric jumps. Note that both processes have the same long-run drift.

2.2.2 Straight Bond and Contingent Capital

The bank is assumed to have both straight bond and contingent capital bond in its capital structure. Straight debt and contingent capital bond require continuous coupon payments at the rate of f and c per unit of time, respectively, before maturity, and principal payments of F and C, at maturity T. Given a payout rate of α from bank's assets, the bank generates continuous dividends net of interest payments of $\alpha \cdot V - f - c$.[11] In the frictionless model, if at any time before maturity the bank's capital ratio, $\frac{(V-F-C)}{V}$, falls below the predetermined threshold θ (conversion trigger), the CoCo bond automatically converts to a fixed dollar amount of shares. The terms of conversion pre-specify that each \$1 of CoCo bond converts to $(\pi) \times \$1$ of equity, where π is a conversion ratio.[12] Note, if the parameter $\pi > 1$, the conversion transaction will dilute the value of the existing equityholders because the transaction is associated with a wealth transfer from equityholders to the holders of CoCo bonds. In terms of total payments, at the conversion instant, the existing equityholders collectively issue shares valued at $(\pi) \cdot C$ to CoCo holders and the bank's debt size declines by C from $F + C$ to F. After the conversion, the bank will have to continue servicing the remaining debt by paying coupon f, so that its continuous dividends will be $\alpha \cdot V - f$. At maturity, the bank has to make a par payment F. The bank can endogenously choose to default any time before maturity when its equity value declines to zero. At default, the equityholders' value is zero, and debtholders recover the bank's assets V_t minus proportional default costs $(1 - DC) \cdot V_t$, where DC is a positive constant, $0 < DC < 1$.

With jumps in the asset value process, bank's assets value can decline instantly to some

[11] When the bank operates with negative dividends, i.e., $\alpha \cdot V - f - c < 0$, it corresponds to the case where it continuously injects equity to make interest payment.

[12] These two assumptions differ from assumptions in Sundaresan and Wang (2011). In their model, they assume a market-based trigger where both the conversion trigger and conversion price themselves depend on the market prices of the bank's shares. They show that this inter-dependence may result in multiple equilibria or even in an absence of equilibrium price of the bank's equity.

level V' well below the conversion trigger so that $\frac{V'-F-C}{V'} < \theta$, but not low enough, so that the bank still has positive enterprise value, $V' > F$. In such cases, the conversion price π has to be modified. Specifically, if π is large, the portion of bank's equity that CoCo holders are entitled to receive after the conversion, $(\pi) \cdot C$, could be larger than the total market value of bank's equity after the conversion, i.e., $E(V', F, t) < (\pi) \cdot C$, where $E(V', F, t)$ is the total market value of equity which is a function of its asset value V'; its principal F; and time t $(\leq T)$. As such, if the conversion were to occur according at price π, the equity value of pre-existing shareholders would become negative. Without the modification, negative equity values would imply that pre-existing shareholders have to default at the conversion point. To avoid this default, we introduce a simple renegotiation of the conversion price. According to this renegotiation, the CoCo conversion price is reduced from π to $\pi'(\pi > \pi' > 0)$ such that the value for the existing shareholders will remain small but positive ε, where π' satisfies the following equation $E(V', F, t) - (\pi') \cdot C = \varepsilon$. This renegotiation corresponds to the case in which the value of existing shareholders is almost wiped out (i.e., they receive ε) but the bank remains operational and is taken over by the CoCo holders. This renegotiation procedure can be viewed as a “technical default” or a ”prepackaged default”.

We also consider the "soft trigger" case in which the conversion does not necessarily occur at the first instant when the trigger is breached. Instead, we assume that given that the bank's capital ratio is below the trigger, the conversion event is probabilistic that is described by the Poisson-arrival process. Specifically, if the bank's ratio declines and remains below the trigger during any (infinitesimally short $dt \to 0$) time interval between t and $t + dt$, the probability that the trigger occurs is $\mu \cdot dt$, and the probability that the trigger does not occur is $1 - \mu \cdot dt$, where μ is Poisson-arrival intensity that describes a frequency of the conversion event per unit of time. In this setting, the bank's capital ratio can cross the triggering boundary multiple times and stay below the trigger for some period of time without triggering, assuming the conversion event did not arrive in prior periods. As such, the average length of time the bank stays below the trigger ratio without conversion is $\frac{1}{\mu}$. Note, in the frictionless model, the conversion takes place at the first instant the trigger is crossed which corresponds to the case of $\mu = \infty$.

Finally, we consider the case where the bank faces covenants that restrict dividend payments to its shareholders. This covenant specifies that after the conversion, the bank cannot pay out its earnings as dividends for the remaining maturity, and will have to retain them to increase its capital base. In the model analytics, the asset payout parameter α is reduced to α^*, $(\alpha > \alpha^*)$, where $\alpha^*(V)$ is a function of asset value V such that $\alpha^*(V) = \min\{\alpha, \frac{f}{V}\}$, and continuous dividends will be zero or negative, i.e., $\alpha^*(V) \cdot V - f = 0$ or < 0.

2.3 Valuation and Debt and Equity and Overview of the Numerical Algorithm

We assume complete markets for the bank's assets, so that the CoCo bond, straight debt and equity can be regarded as tradeable financial claims for which the usual pricing conditions must hold. Effectively, the model assumes that the information about the bank's assets and the equity issuance (money burning) strategy of the bank is publicly available. The market values of the bank's equity, the CoCo bond and the straight debt are a function of three variables: the size of the bank's asset value V; its principal $F+C$, if no conversion took place in prior periods, or F, if the CoCo bond was converted before; and the time t $(\leq T)$. Equity value can be determined by solving stochastic control problems with fixed and free boundary conditions, where the control variables are the equity issuance decisions (or money burning decision) as well as default strategy. These valuation problems are described in detail in Appendix 2.

The numerical algorithm used to compute the values of equity and debt is based on the finite-difference method augmented by a "policy iteration". Specifically, we are using dynamic programming on the discretized grid of the state space $(V, \{F,\ F+C\}, t)$ with a discrete time step Δt. At each node on the grid, the partial derivatives are computed according to Euler's method. The backward induction procedure starts at the terminal date T, at which the values for straight debt, the CoCo bond and equity are known. We run backward recursion using time steps Δt and take into account the bank's optimal default decision and its optimal strategy to raise capital (or burn assets). For more detail of the numerical algorithm see Titman and Tsyplakov (2005) or Tsyplakov (2007).

3 Model-Generated Spreads and Expected Default Costs for the Base Model

In our calculations, for simplicity, we assume the principal value of the straight bond and the CoCo bond paid at maturity are equal to the value of perpetual debt with periodic coupon payment discounted at the risk-free rate, i.e., $F = \frac{f}{r}$ and $C = \frac{c}{r}$, respectively. This implies that, since the straight debt is generally risky, it is sold at a discount relative to its face value, i.e., $D < F$, where D is the market values of the straight bond. The CoCo bond can be sold above or below par, i.e., it can be $Z < C$, or $Z > C$, where Z is the market value of the CoCo bond.[13] Given this simplification, the credit spread at origination is calculated from the following simple equation: $D = \frac{f}{r+spread_D}$, and $Z = \frac{c}{r+spread_{CoCo}}$. In order to measure the bank's average cost of debt, we calculate its *weighted-average* (WA) *spread*. The WA spread is calculated from the following expression $D + Z = \frac{f+c}{r+spread_{WA}}$. We also calculate the expected value of the default costs by subtracting the total value of debt plus equity from the value of the bank's underlevered assets: $default\ costs = V - E - (D + Z)$. We measure the size of default costs as percentage of total assets. By analyzing the size of these costs

[13] Such an approach simplifies the numerical calculation. Otherwise, we would need extra numerical iterations to find the par coupon rate.

as a function of the CoCo contract terms we can gauge its efficiency in reducing these costs. Finally, we calculate the expected value of future net equity issuances as well as the expected value of burnt money both measured ex-ante at time zero.[14] We will report default costs and expected equity issuance (and the value of burnt money) as a percentage of the bank's initial assets. In our analysis, we will show the range of CoCo terms that will incentivize the bank to preemptively issue enough equity so that the bank will always keep its capital ratio above the trigger making it default-free.

3.1 The Trade-offs that the Bank Faces

Punitive conversion terms generate incentives for bank's shareholders to avoid dilution, and encourage the bank to issue equity capital and maintain higher capital ratios. Theoretically, the bank's optimal equity issuance strategy is determined by the choice between managing shareholder dilution through raising equity capital while avoiding the CoCo conversion or accepting dilution through conversion and, at the same time, regaining the option to declare bankruptcy on straight debt and avoid paying coupons, whenever this becomes too costly (that is, when asset value is low). Note, when the bank chooses to preemptively recapitalize and avoid conversion, it effectively forfeits the default option. In the model, the magnitude of the dilution is determined by two parameters: 1) the conversion price π, and 2) the relative size of the CoCo bond. The value of the option to default increases with the size of the straight debt. Thus, if the dilution associated with the CoCo conversion is not too high and the default option is valuable, the bank will find it optimal to let the conversion take place and continue to operate with a straight debt. Conversely, if this dilution is significant and the default option is not too valuable, then the bank will preemptively raise capital and avoid conversion, thereby eliminating debt overhang problem.

[14] The calculation of expected value of future equity issuance (and the expected size of burnt money) requires the solution of partial differential equations similar to the one that describe the value of equity.

The incentive to burn money emerges only when the trigger assumes a large write-down of the CoCo's principal. The value of the option to burn money is driven by the following trade-offs between the benefits and costs. The benefits stem from the fact that the bank can immediately get rid of CoCo coupon payments by paying a fraction of the CoCo face value. After the principal is written-down, the bank gains the option to declare bankruptcy on remaining straight debt. The cost of burning money is that shareholders give away the optionality of profiting from upside return on the assets. When the remaining maturity is short, this optionality has low value, implying that the expected amount the shareholders can burn will increase as remaining maturity shortens. Theoretically, an instant before the bond matures, shareholders can optimally choose to burn as much as the size of entire amount of the principal write-down.

3.2 Base case parameter values

The parameter values are chosen to roughly match empirical observations for a bank that is initially well-capitalized. The volatility of banks' assets is set at $\sigma = 5\%$. The risk free rate is assumed to be $r = 5\%$. The asset payout yield α is set at 5%, which means that the risk neutral diffusion drift of assets is 0% per year. We set the proportional default costs parameter DC at 50%. For the base case, the bank's initial leverage ratio at time 0 is $\frac{F+C}{V_0} = 92\%$, and its initial capital ratio is $\frac{V_0-F-C}{V_0} = 8\%$.

The parameters of the contingent capital bond are as follows: the CoCo's bond face value is set at 5% of initial assets, i.e., $\frac{C}{V_0} = 5\%$. A conversion trigger is set at $\theta = 6\%$, so the asset value has to fall to the level of V', so that $\frac{V'-F-C}{V'} \leq 6\%$ to trigger the conversion. Thus, the bank's initial balance sheet composition is the following: 8% is shareholders equity, 5% is CoCo bond, and 87% is straight debt. The conversion price π of CoCo is varied from 1.0 to a dilutive price of 1.4. In the section in which we quantify the effect of the incentive to burn money, we consider the loss-absorbing CoCo bond with parameter π set at 0.25 (i.e., 75% write-down of principal).

In the base case, the transaction costs of raising equity e are assumed to be 0%. We will present the comparative statics for transaction costs of 0.25% and 0.5%. If jumps are introduced in the calculations, we assume that the jump size Y_t is uniformly distributed between 1 and 0. This means that the average jump leads to a 50% loss in asset value (parameter $k = -0.5$). Depending of the size of realized jump, it can potentially lead to an immediate trigger or default. For the base case, the probability of jump is zero, $\lambda = 0$. In comparative statics, the jump probability is varied between λ =1% and λ =2% per year. With non-zero jump probability, the risk neutral diffusion drift of the value process is adjusted upward by $\lambda \cdot k = 0.5\%$ and 1% respectively. For the base case model, the CoCo automatically converts at the first instant when the conversion ratio it breached. For the case with "soft trigger", the conversion event is probabilistic that is described by the Poisson-arrival process with the arrival intensity, μ. In comparative statics, the arrival intensity μ is varied between 0 (CoCo never converts) and 4.0 (i.e., average wait time is one quarter year).

3.3 Comparative Statics

3.3.1 Bank's equity issuance strategy in frictionless model

We start the analysis with a model in which there are no frictions with the exception of proportional default costs. In this setting, bank's assets are described by a standard continuous lognormal process without jumps. In a frictionless model, the CoCo bond is default-free (not risk-free), because the conversion always takes place at the first instant when the bank's capital ratio crosses the trigger which is set above default boundary. The remaining straight debt is not necessarily default-free because the bank can default on it after the CoCo bond is converted. In the following sections, we will pay particular attention to the question of how the CoCo tranche can affect the bank's leverage policy and incentives to issue equity. In the later section, we will concentrate on the incentive to burn money and will report the parameter range for which this incentive has a material impact on the pricing of CoCo bond and the size of default costs.

Financing with CoCo bond relative to financing with the all straight debt Figure 4 provides the comparison analysis of the scenario where the CoCo bond replaces part of the straight debt and the scenario when the bank is financed with all straight debt. In both scenarios, the bank has the same total leverage of 92%. The comparison is presented as a function of the CoCo conversion price π. Figure 4 shows that the weighted-average credit spread (of CoCo and the straight bond) declines as the conversion price increases. When the conversion price $\pi = 1.0$ (i.e., the terms are neither dilutive nor loss-absorbing), the bank's weighted-average credit spread declines to less than 41bp from 80bp, relative when the bank is financed with all straight debt. The default costs decline from 10% to 5.9%. These results confirm earlier studies that CoCo bonds can reduce overall default risk even if the incentive affects are not a factor.

CoCo structure and incentives to issue equity for the base case The model demonstrates that the CoCo bond with even moderately dilutive conversion terms will create incentives for the bank to issue equity, de-lever and reduce the probability of triggering the conversion. As plotted in Figure 4, if the size of the CoCo is 5% of bank's assets (base case), the present value of future equity issuance measured over the entire maturity period is more than 5% (as percentage of the initial size of bank's assets) for the conversion price $\pi = 1.1$ and above. If the conversion price increases gradually from $\pi = 1.0$ to $\pi = 1.32$ and higher, the expected equity issuance rate gradually increases to the steady state level of

35.8%. For moderately dilutive conversion prices below $\pi = 1.32$, the bank is expected to issue some equity in the future, but its size will not be enough to completely avoid the CoCo conversion. For this range, the weighted-average spread and the default costs are still above zero. If the CoCo's conversion price is very dilutive ($\pi = 1.32$ and higher), the bank will always avoid the conversion trigger and default by preemptively raising just enough equity to keep its leverage above the conversion trigger at all times. The spreads depicted in Figure 4 reflects the equity issuance strategy of the bank and shows that the CoCo spread exhibit a non-monotonic relation with conversion rate. For moderately dilutive conversion terms $1.0 < \pi < 1.32$, the CoCo bond has negative credit spreads. This result reflects the fact that in the frictionless model, the CoCo bond is default-free and there is a non-zero probability that the CoCo will convert at the price above its par value implying negative credit spreads. Within the range of $1.0 < \pi < 1.25$, the CoCo spread declines as the conversion price increases because an increase in the conversion price benefits the holders of the CoCo debt but the probability of the conversion does not change significantly. As the conversion price increases from $\pi = 1.28$ to 1.33, the bank is expected to issue relatively large amount of equity so that the probability that its capital ratio will cross the trigger will decline. As a result, for this range, the CoCo spread will increase from negative values to zero. The weighted-average credit spread, the spread of straight debt, and default costs all decline to zero as conversion price increases suggesting that incentives created by the CoCo bond can significantly reduce the overall risk of the system. Note that for the conversion price $\pi > 1.33$, all value do not change because a further increase in conversion rate above this level will not change the expected size of equity issuance and the bank's debt will remain risk free.

The affect of issuing equity In order to quantify the effect of issuing equity we also consider the scenario in which the bank cannot issue equity. As shown in Figure 5, if the option to issue equity is ignored then the weighted-average credit spread (and of CoCo and the straight bond) and the default costs will be overestimated, especially when the conversion price is dilutive. Given the base case parameter values, if the conversion price is $\pi = 1.3$ or higher, the option to issue equity will reduce the bank's weighted-average credit spreads and default costs by more than one-half when compared to the scenario when the bank cannot issue equity. For the conversion price higher than $\pi = 1.32$, the bank is expected to issue enough equity to avoid conversion and default thereby reducing credit spreads and default costs to zero and making the bank default-free. The model that ignores the possibility that the bank will preemptively raise capital will be unable to predict this incremental reduction in default risk.

Will the bank voluntarily issue CoCo bonds? In this section, we explore whether the bank will be willing to issue a new CoCo debt on top of the pre-existing straight debt. For the base case, the bank's initial capital structure has 5% in CoCo bonds and 87% in straight bonds. Without the CoCo tranche, the straight debt is risky and its credit spread is 43 bp above the risk-free rate. With the issuance of a CoCo bond, existing holders of straight debt may benefit because the CoCo tranche can incentivize the bank to recapitalize and reduce its leverage ratio during distressed times thereby reducing or even eliminating the default probability.[15] As such, the CoCo tranche can provide a protection to holders of existing bonds at the shareholders' expense. To assess this potential wealth transfer, we calculate the change in equity value of the existing shareholders and the change in yield of the preexisting straight bond at the instant when the bank issues the CoCo bond when the

[15] Albul, Jaffee, and Tchistyi, (2010) analyze a similar case when the CoCo bond is issued on top on the pre-existing straight debt. However, they don't consider the case when the bank can recapitalize.

straight debt is already in place. Figure 5 shows that at the issuance of the CoCo tranche with the highly dilutive conversion price ($\pi > 1.32$), the yield of the straight debt will decline to zero, while the existing equityholders will lose as much as 7.3% of their stock value. In contrast, if the CoCo debt has a non-dilutive CoCo conversion term ($\pi = 1.0$), there will be no value transfer because the bank will not be issuing equity and de-lever. Due to a possibility of wealth transfer, the bank will be reluctant to issue a CoCo bond with dilutive terms on top of its pre-existing non-callable debt in its capital structure. These concerns are mitigated should the bank issue CoCo debt at the same time it issues straight bond. If issued simultaneously, the bank's shareholders will benefit from such transaction because the CoCo debt will increase the value of the newly issued straight debt resulting in lower overall borrowing costs.

The the size of the CoCo bond and the trigger ratio Table 6 reports comparative statics as a function of the CoCo size (measured as a percentage of initial assets, $\frac{C}{V_0}$). In these tests, values are calculated given that both the size of straight debt and CoCo bond are varied while maintaining the total size of debt at the base case level of 92%. Results testify that when the bank has a larger size CoCo tranche, it will have more incentive to preemptively recapitalize to avoid dilution. This is because a larger size CoCo trance can lead to a larger shareholder dilution at conversion.

With higher capital ratio at which the CoCo conversion is triggered, there will be a higher likelihood of breaching the trigger. If the bank wants to keep its capital ratio above the higher set trigger, the bank has to start issuing equity at a higher levels of capital. As such, the expected value of equity issuance rate should be higher and expected default costs should be lower. Figure 7 confirms this intuition. Expected default costs and weighted average spreads are both lower for higher conversion trigger ratios. For example, if the conversion price is held at π =1.26, and the trigger ratio increases from 5% to 7%, the expected size of equity issuance increases from about 10% to 35%. If the trigger is at 5%, the bank will always avoid conversion at π =1.26, making the system risk-free. These results suggest that at the higher trigger, the commitment to preemptively issue equity and avoid conversion will be stronger even for less dilutive conversion prices.

Volatility of the drift process Figure 8 demonstrates that for lower assets volatility (parameter σ), the bank will issue larger amount of equity while holding the CoCo terms constant. If the asset volatility is 4%, the bank's debt will become default free at the conversion price of π =1.2, because the bank will always issue equity to avoid breaching the trigger and eliminate default. In comparison, if volatility in creases to 5% (base case), the conversion price has to be as high as \$1.32 in order to create incentive for the bank to maintain its capital ratio above the trigger at all times. The reason behind this result relates to the previously discussed trade-offs that drive the bank's decision to issue equity. A reduction in asset volatility will reduce the value of the default option and increase the relative value of the option to issue equity, producing stronger incentives to issue equity and avoid triggering the conversion.

Asset payout ratio In this section we vary the payout ratio of the bank's assets described by parameter α. Note that a higher payout ratio implies lower risk-neutral drift in the asset value (calculated as $r - \alpha$) and higher dividend yield. Figure 9 depicts the relation between credit spreads and CoCo terms for payout ratios of 4%, 5% (base case) and 6%. The credit spreads and expected default costs both are lower for the bank that pays out less. Lower asset payout ratio implies that the bank will be willing to issue more equity for a the same level of dilution terms. For example, if the bank's asset payout rate is 4%, the bank will be incentivized to issue 9% given the CoCo's conversion price is π =$1.12. In comparison, if the asset payout rate is set at 5%, the bank's expected equity issuance rate declines to 6%. The intuition for this result is similar to the trade-offs described in the previous section and it is that lowering payout ratio decreases the value of default option and generates more incentives to issue equity and avoid the trigger. As a policy suggestion: if regulators mandate banks to hold CoCo bonds, their terms should be more dilutive (punitive) for banks that hold riskier assets and the banks that have higher dividend payout yield.

The effect of maturity Figure 10 demonstrates that the expected size of future equity issuance increases as (remaining) maturity of CoCo bond shortens. Consequently, default costs and weighted-average spreads decline too. If the initial capital ratio is held at the base case (8%), the bank is expected to maintain its capital ratio above the trigger and eliminate default if the (remaining) maturity is shorter than 3.5 years given that the conversion price is at π =$1.1. For this case it means the following: if the remaining maturity is shorter than 3.5 years and the CoCo has not converted in the prior periods, the bank's debt will become default free for the remaining 3.5 years of maturity. As the conversion price becomes more dilutive (π =$1.3), the length of the remaining maturity within which bank is default-free is longer. The intuition of this result is that for a shorter maturity, the bank is expected to issue more equity because the value of default option declines as remaining maturity shortens.

3.3.2 Incentives to burn money in the frictionless model

The effect of maturity The bank will have incentive to burn more money and accelerate the conversion only if at conversion, the bond's principal will be written down. This write-down of principal will generate windfall gains for the bank's shareholders at the expense of CoCo investors. The model implies that if the write-down is moderate and its maturity is long as in the base case (T = 10 years), the option to burn burn money has small impact on the initial bond price. As we argued earlier, the option to burn money will be deeper in-the-money if the remaining maturity is short.

To quantify the impact of these incentives use example of the Rabobank's loss-absorbing bond that assumes 75% write-down of principal.[16] In this model, an analog of Rabobank's bond is the CoCo bond with the conversion price of $\pi = 0.25,$ which implies that \$1 of CoCo will convert to 25 cents of equity. In the comparative statics, we vary the maturity of the bond and keep remaining parameter values at the base case level including the the bank's initial capital ratio at 8%. In order to disentangle the incremental impact of "money burning" incentives we also calculate spreads and deadweight costs for the case where the bank is not allowed to burn money.[17] The model predicts, the money burning strategy becomes more profitable just before CoCo bond becomes due. Figure 11 shows that when maturity is very short, the bank is expected to burn as much as 2% of its assets to trigger the conversion. The intuition of this result is very simple: an instant before maturity, the shareholders face two choices: to pay a full amount of CoCo principal or to burn money (capital) and pay only 25% of CoCo's principal if the conversion occurs. By burning money at the amount of 2%, the bank can drop its capital ratio from 8% to 6% in order to cross the trigger and capture gains coming from 75% reduction of principal. The 75% write-down of principal provides gains to the shareholders valued at 3.75% when measured as fraction of the banks assets. These gains are calculated as the product of $75\% \cdot 5\% = 3.75\%$, where the last number reflects the fact that CoCo bond is 5% of assets. These gains exceed the amount that the bank needs to burn (2%). An increase in deadweight costs up to two percentage points for a short maturity reflects the expected amount of burnt money.

[16] Rabobank's Contingent Notes issued in early 2010 include a write down of 75% of the principal of the notes if the conversion trigger of is crossed with the remaining 25% of the notes to be repaid in cash.

[17] The expected deadweight costs are calculated as the difference of the bank's underlevered assets minus total market value of debt plus equity.

Overall, this argument implies that, if at matuirty, the bank needs to burn less than 3.75% to breach the trigger, it will find it optimal to do that. When the remaining maturity is long, this tradeoff is more complex and bank managers will be less willing to burn money because burning money eliminates a valuable optionality that its assets can increase in value and generate positive net income for the bank's shareholders. This optionality can also explain why credit spreads and deadweight costs exhibit non-monotonic relation with maturity.[18]

Bank's capital ratio Figure 12 plots the relation between bank's capital ratio and the expected value of the burnt money for maturities of one-day ($\frac{1}{365}$year) one quarter, and a half year. In these tests, we vary the value of the bank's asset V and keep the size of CoCo and straight debt constant at the base case values. The expected amount of money burning exhibits the inverse U-shaped relation with the capital ratio. Specifically, the expected size of burnt money increases for the capital ratio in the range between 9% and 15% and declines for capital ratio of 15% and larger. The explanation of this non-monotonic relation is the following: When the bank's assets lose value and its capital ratio declines to the level very close to the conversion trigger, there is a high probability that the trigger will be breached simply because value of bank's assets can decline even more. In such cases, the bank has a lesser need to accelerate the trigger by burning money, or if needed, it does not have to burn large amount. Conversely, if the bank operates at a capital ratio that is significantly higher than the conversion trigger of 6%, the option to burn money will be farther out of the money, because it is unlikely that the bank's capital ratio will drift close to the conversion trigger. In such cases, the bank would need to burn a significant amount of money if it wants to force the conversion trigger which will exceed expected gains. Therefore, at the capital ratio well above the conversion trigger, the bank is not expected to burn much of money. In

[18] Unreported results indicate that larger CoCo tranche and larger write-downs imply that shareholders will be willing to burn more money in order to force a larger amount of write-down.

the middle range, at the capital ratio around 9%, the banks is expected to burn the largest amount of money. The expected deadweight costs exhibits similar relation.

Discussion on loss-absorbing feature of CoCos As the model implies, the option to "burn money" is likely be in-the-money in times of crisis, when the bank is in financial distress. This write-down feature of the loss-absorbing bonds is thus highly undesirable, because it is exactly during the crisis that is important to have incentive structures in place that discourage banks from exercising value-destroying strategies. The write-down feature included in CoCo bonds will also exacerbate other inefficiencies and value-reducing incentives associated with a typical debt overhang problem. For example, the incentive to accelerate the trigger will encourage the bank to increase risk even more, especially its downside risk. Also, the bank will be more incentivized to underinvest in positive NPV projects. These value-reducing incentives attributable to the presence of the loss-absorbing CoCo bond will be worse than those generated by ordinary straight debt. Finally, due to the impact of money burning incentives, the bank's stock price and the price of CoCo bond will likely experience higher uncertainty near maturity date. This uncertainty can create liquidity problems for the bank because near maturity, it will likely be looking to recapitalize and raise funds. As a policy recommendation, our results suggest that the design of the CoCo that include large principal write-downs are not necessarily the best tool for maintaining the stability of banks.

3.3.3 The effect of frictions and other considerations

In the following sections we will consider a number of market frictions and variations on the model which allow us to evaluate additional issues that may affect the pricing of CoCo bonds and the bank's capital structure strategy.

The effect of jumps When asymmetric jumps are incorporated in the asset value process, the analysis becomes more complicated. Jumps introduce skewness in the asset return distribution and a "fat" tail for negative returns. With jumps, there is always a non-zero probability that the trigger is breached and/or the default occurs even if the bank issues equity and tries to avoid the conversion. With the arrival of jump, the CoCo bond can default without being triggered. Since the CoCo bond is junior, at default, only holders of straight debt are paid in full, so the CoCo investors they will receive zero value. Even if the bank is committed to issuing equity, it can eliminate the diffusion risk and reduce the impact of jump risk but cannot completely eliminate it. In the comparative statics, we vary the probability of jumps between 0% (base case) and 2% (i.e., on average, a large sudden losses happens every 50 years), holding the distribution of the jump size unchanged. As we mentioned earlier the jump size is uniformly distributed between 1 and 0, leading to an average 50% loss in asset value.

We first consider the extent to which jumps can affect bank's incentives to issue equity and recapitalize. As shown in Figure 13, for most of highly dilutive conversion prices, the bank's will be less committed to issue equity and recapitalize if there is a higher probability of jumps. For example, given the conversion rate of 1.3 or higher and the probability of jumps of 2% per year, the bank's expected equity issuance rate reaches the plateau at the level 6.5%. In comparison, without jumps, the bank is expected to issue as much as 30%. This is because in the frictionless case, the bank can increase its equity issuance, recapitalize and keep the bank's capital ratio above the trigger at all times. In contrast, with jump risk, a further increase in equity issuance has a diminishing marginal impact on the equityholders' value and credit spreads. For example, as Figure 13 shows, an increase in equity issuance driven by an increase in conversion price from $\pi = 1.0$ to $\pi = 1.4$ will reduce expected default costs from 6 to about 2.5 percentage point when the jump probability is 2%. The weighted-average spread will decline by about 12 bp but still will be above 50bp, even if the bank is expected to issue equity. In comparison, in a frictionless economy, default costs will decline by more than 6% and credit spreads will decline to zero. These results suggest that in the economic environment in which changes are predominantly driven by sudden negative changes (i.e., financial crisis) and less by a continuous diffusion, the effectiveness of CoCo as a self-disciplining mechanism will weakened, and changes in CoCo terms have a diminished impact on the bank's incentives to issue equity and manage its capital structure.

We also consider the impact of jumps on the bank's incentives to burn money for the cases where CoCo contracts allow large write–downs. Untabulated results show that the expected value of money burning decreases insignificantly when the jump probability increases from zero to 2%.

Transaction costs of issuing equity In this section we explore how changes in transaction costs of issuing equity impact the effectiveness of CoCo bonds. If there are fixed and variable costs associated with issuing equity, the bank will be raising equity infrequently and in lumpy amounts. If these costs are too high, the bank will not choose to issue equity at all. We consider 3 cases: zero costs of issuing equity (base case); 0.1%; and 0.25% costs of issuing equity. Figure 14 presents the comparative statics. As expected, with larger transaction costs, the bank will be willing to issue equity and avoid conversion only if the CoCo bond has more dilutive terms when compared with the base case. For example, if the bank faces 0.5% transaction costs, it is expected to issue equity at the amount of about 3% if the conversion price is 1.3. Similar to our arguments in the previous section: imperfections related to costs of issuing equity reduce the ex-ante commitment of a bank to maintain higher capital ratios.

"Soft trigger" In the frictionless model, the CoCo converts at the first instant when the conversion ratio it tripped. In practice, there is number of frictions that allow the bank to temporarily operate without triggering the conversion even if its capital ratio drops below the contractually prespecified trigger. First, in reality, the bank's capital ratio can only be verified quarterly when it releases its quarterly accounting reports. Thus, in the period between reports, the bank's capital ratio can drop below the trigger. Second, when the bank is close to be in distress, it will likely have more incentives to misrepresent its true financial condition and overstate its capital ratio in order to delay or avoid the conversion. Because of these reasons, there may be a situation when the bank's actual capital ratio declines below the trigger, but the conversion does not occur for some period of time. During such period, the bank's financial ratio can recover and its capital ratio can increase back above the trigger, or, conversely, the capital ratio can decline further and the bank can default without converting its CoCo bond.

In order to capture the effect of these frictions, we modify the model by introducing the so-called "soft trigger ratio". With the "soft trigger", the conversion will not necessarily occur at the instant the triggering threshold is breached. Instead, we assume that given that the bank's capital ratio is below the trigger, the conversion event can happen with some probability described by a Poisson-arrival process. Specifically, if the bank's ratio declines and remains below the trigger during any short time interval between t and $t + dt$ ($dt \to 0$), the probability that the conversion occurs during this period is $\mu \cdot dt$, and the probability that the trigger does not occur is $1 - \mu \cdot dt$, where μ is the arrival intensity of the conversion event per unit of time. In this setting, the bank's capital ratio can cross the triggering boundary multiple times and stay below the trigger for some period of time without conversion. With the assumption of the Poisson-arrival process, the probability that the conversion occurs next period does not depend on the duration of time the bank's capital ratio have already stayed below the trigger in the prior periods. The average length of time the bank stays below the trigger ratio without converting is $\frac{1}{\mu}$. Note, in the frictionless model, the conversion arrival intensity is $\mu = \infty$.

Figure 15 shows that if the arrival intensity of the conversion event declines, the bank's incentive to issue equity and manage its leverage weakens because the likelihood of delutive conversion will be lower. For example, if the conversion intensity is μ =0.5 and the bank has the CoCo tranche that specifies the dilutive conversion price of π=\$1.3, the bank is expected to issue equity at the amount of 8%. In comparison, for the base case, the bank's expected equity issuance is above 10%. For any CoCo terms, credit spreads and default costs increase with the decline in the conversion intensity. For example, let us look at the case where the conversion intensity is μ =0.5, which implies that on average the wait time after the bank's ratio declines below the trigger is one year ($\frac{1}{\mu}$ =2 year). For this case, the weighted-average spread widens by about 20 bp, and default costs increase by about 1.5 percentage points relative to the base case in which the conversion occurs instantly ($\mu = \infty$) at the trigger. The intuition of this result is that the longer the bank stays below the trigger ratio without automatic recapitalization, the higher will be the default probability, and the lower will be the positive impact of the CoCo tranche on the credit risk of the bank. Figure 15 also demonstrates that the spread of the CoCo bond is very sensitive to changes in conversion intensity ratio, because with the "soft trigger", the CoCo bond is not default-free anymore. Since CoCo holders have a junior claim on the bank's assets in default, any increase in default probability will have a significant larger impact on the value of CoCos than on the value of the bank's straight debt.

Restricting dividend payouts The discussion among bank regulators evolves around the question of whether to require banks to hold convertible debt tranche in the capital structure at all times. If this mandate is implemented, banks will have to reissue a new convertible bond every time the existing CoCo tranche is converted or paid off. In practice, however, there can be a delay period before bank will be able to reissue and sell new CoCo bond to replace an old tranche. To provide incentives for the bank to promptly reissue a new CoCo, bank regulators may require banks to retain earnings, restrict payouts and build capital during these periods. Alternatively, such restrictions can be included in debt covenants that protect bondhoders by preventing distribution of capital to bank's shareholders if the bank operates without CoCo in the capital structure.

We can use our model to assess the impact of such covenant. Ideally, in order to fully implement this setup in our model, we have to assume that the bank can choose the size of the new convertible bond that replaces the outgoing tranche. Technically, such setup will be difficult to implement within the framework of the model because of the need to price multiple CoCos.[19] Instead, we make the following simplifying assumption: as in the base model, we assume that the bank issues a CoCo bond at time zero along with a straight debt. After the CoCo conversion is triggered, the bank will face the restriction that it cannot pay out its earnings as dividends until maturity, and will have to retain them to increase its capital base. Before the conversion, the bank is unconstrained to pay out its earnings as in the base case. After the conversion, the bank still can default on its existing straight debt if bank's capital declines to the default boundary.

In Figure 16, we compare this case with the base case in which the bank is unconstrained. The payout restriction has an impact on the pricing of CoCo bond because such restriction will create incremental incentive to issue more equity and raise capital. With such restriction, the bank is expected to issue more equity for same level of dilution when compared to the

[19]Such extension of the model will increase the dimensionality of the problem and will make the stochastic control problem significantly more complex.

base case, and the default risk will be reduced even more. Specifically, model predicts that at the conversion price of as low as π =1.25, a payout restriction can incentivize the bank to raise enough capital to become risk-free. Without restrictions, the conversion price has to be as high as π=1.32. The explanation of this result is that a payout restriction reduces the bank's default option (after the conversion) thereby creating more incentives to issue equity preemptively and avoid the conversion trigger. From the regulatory perspective, the model implies that the restriction imposed on capital distributions can strengthen incentive effects of CoCo bond, and, as such the can be viewed as complimentary tool that can help to reduce risk of the financial sector.

4 Conclusion

The model presented in the paper demonstrates that optimally structured contingent capital can be an economically attractive financial tool that can enhance the stability of the bank. The model quantifies the market conditions for which banks will opt for contingent capital if they face the choice between equity, straight debt, or contingent capital. The model implies that, in general, regulators should promote the CoCo debt that contains potentially dilutive terms for the existing shareholders, and avoid CoCos that include principal write-downs.

The model can be extended to incorporate a number of important issues. The model can quantify a possibility that the conversion is triggered by regulatory authorities at their own discretion. A reasonable assumption is to link the probability of regulatory-initiated triggers with the leverage ratio of the bank or the risk conditions of the overall banking industry. For such cases, a bank will likely be less motivated to issue equity capital and keep its leverage at a moderate level .

The model can incorporate some additional monetary losses that the bank's shareholders suffer when the conversion trigger takes place. For example, markets can view the conversion as a bad signal about the prospects of the bank. Alternatively, if upon conversion, a large stake of equity is transferred to CoCo holders, the bank managers will likely suffer personally because of the possibility of losing their job or a reduction in compensation. Depending on the managerial compensation contract, managers may respond to these threats by lowering the bank's leverage and its risk even below the level preferred by shareholders.

We also can introduce so-called "inconvenience costs" that CoCo investors may incur when the conversion takes place. This assumption can capture the fact that there could be some financial losses for CoCo investors because of the need to rebalance their investment portfolio when they become holders of a bank's equity. Alternatively, CoCo investors may face additional costs resulting from capital gains taxes. As such, CoCo investors will require ex-ante compensation and higher yields.

Finally, the model can offer a platform for analyzing the case when CoCos convert to a preferred stock or a security that has a property of a preferred stock. There are several reasons why conversion to preferred stock can be more attractive to typical fixed-income investors when compared to conversion to common stock. First, a preferred stock carries less risk and behaves more like a bond than a typical common stock. Second, preferred stock has a higher priority than common stock. Third, when the CoCo converts to common stock, the bank's stock price can be subject to manipulation during the periods when the bank's capital ratio is near the trigger, which can potentially lead to "death spiral." The conversion to preferred stock mitigates such risk. Finally, fixed income investors may be reluctant to become holders of the common stock because, by mandate, these investors specialize in fixed income securities.

References

[1] Admati, Anat, Peter DeMarzo, Martin Hellwig, and Paul Peiderer, 2010, Fallacies, Irrelevant Facts, and Myths in the Discussion of Capital Regulation: Why Bank Equity is Not Expensive," Stanford University.

[2] Allen, F. and Faulhaber, G. R., 1989. Signaling by underpricing in the ipo market. Journal of Financial Economics, 23:2, 303-323.

[3] Akerlof, George. 1970. "The Market for 'Lemons': Quality Uncertainty and the Market Mechanism." Quarterly Journal of Economics 84: 488–500.

[4] Albul, B., Jaffee, D. M. and Tchistyi, A., 2010, Contingent convertible bonds and capital structure decisions. University of California Berkeley Coleman Fung Risk Management Research Center Working Paper 2010-01.

[5] Ambarish, R., John, K., and Williams, J., 1987, Efficient signalling with dividends and investments. Journal of Finance, 42:2, 321-343.

[6] Andersen, Leif, and Dan Buffum, 2002, Calibration and Implementation of Convertible Bond Models, Banc of America Securities.

[7] Berg, Tobias and Christoph Kaserery, 2011, Does contingent capital induce excessive risk-taking and prevent an efficient recapitalization of banks? Humboldt Universität zu Berlin and Technische Universität München (TUM).

[8] Castillo, Frederick R., 2011, Coconundrum, Harvard Business Law Review Online.

[9] Flannery, M. J., 2009a, MarketValued Triggers Will Work for Contingent Capital Instruments," University of Florida, Working paper.

[10] Flannery, M. J., 2009b, Stabilizing Large Financial Institutions with Contingent Capital Certificates," University of Florida, Working Paper.

[11] Grinblatt, M. and Hwang, C. Y., 1989. Signalling and the pricing of new issues. Journal of Finance, 44:2, 393-420.

[12] John, K. and Williams, J., 1985. Dividends, dilution and taxes: A signalling equilibrium, Journal of Finance, 40, 1053-1070.

[13] Harvard Law School Forum on Corporate Governance and Financial Regulation "The Loss Absorbency Requirement and "Contingent Capital" Under Basel III, April 24, 2011.

[14] Leland, Hayne, and David Pyle. 1977. "Informational Asymmetries, Financial Structure, and Financial Intermediation." Journal of Finance 32: 371–387.

[15] Loughran, T., and Ritter, J., 1995, The new issues puzzle, Journal of Finance 50, 23-51.

[16] McDonald, Robert. 2010. Contingent Capital With a Dual Price Trigger. Kellogg School of Management.

[17] Merton, R. C., 1974, On the Pricing of Corporate Debt: The Risk Structure of Interest Rates," Journal of Finance, 29(2), 449-470.

[18] Myers, S. C. and Majluf, N. S., 1984, Corporate Financing and Investment Decisions when Firms Have Information That Investors do not Have, Journal of Financial Economics 13, 187-221.

[19] Myers, Stewart C. 1984. "The Capital Structure Puzzle." Journal of Finance 39: 575–592.

[20] Myers, Stewart C., and Nicolas Majluf, 1984, “Corporate Finance and Investment Decisions When Firms Have Information That Investors Do Not Have,” Journal of Financial Economics 5, 187-221

[21] Pennacchi, George, 2010, “A Structural Model of Contingent Bank Capital,” Working Paper, University of Illinois College of Business.

[22] Pitt, Louise, Amanda Hindlian, Sandra Lawson, and Charles P. Himmelberg, 2011, Global Markets Institute: Contingent capital: possibilities, problems and opportunities, Credit Research, Goldnam Sachs

[23] Pitt, Louise and Ron Perrotta, 2011, The Banks and Finance Pitt Stop: Contingent capital back in focus; bank capital remains attractive, Credit Research, Goldman Sachs.

[24] Sundaresan, S and Z. Wang, 2011, On the Design of Contingent Capital with Market Trigger, Federal Reserve Bank of New York, Staff Reports.

[25] Titman, S., and S. Tsyplakov, 2005, A Dynamic Model of Optimal Capital Structure, Review of Finance 11, 401-451.

[26] Tsyplakov, S., 2007, Investment Frictions and Leverage Dynamics, Journal of Financial Economics 89, 423-443.

5 Appendix 1 : A Simple One-Period Model

In this section, we use a simple one-period numerical example in order to illustrate incentives that the CoCo tranche can create for the bank with respect to issuing equity (burning money). In the example, we consider the bank that is financed with a tranche of CoCo bond and a straight debt. The CoCo bond and the straight bond have principal values of $C = \$20$ and $F = \$60$ respectively, and both mature next period at $t = T$. As shown on Figure 2, at the current time $t = T - 1$, the bank's assets are valued at $V_{T-1} = \$87$, but next period they are expected to drift to either \$88 or \$86 with equal probabilities. For simplicity, we assume that the risk-free rate is zero and investors are risk-neutral. The conversion trigger is breached when the bank's capital ratio (measured as the ratio of $\frac{V-F-C}{V}$) declines below 8%. If the conversion trigger is breached at $t = T$, the CoCo bond will convert into the predefined value of the bank's equity. Otherwise, the equityholders have to make principal payments to the CoCo holders and the holders of straight debt or default.

We consider two cases of CoCo design. In the first case, the conversion price will be dilutive to the existing shareholders of the bank, where the conversion price equals \$1.1. This conversion price means that \$1 of CoCo bond will convert to \$1.1 of equity. In the other case, we assume that the CoCo bond is loss-absorbing with the conversion price of \$0.75 (corresponding to a 25% write-down of principal). At the current asset value of \$87, the banks capital ratio is above the conversion trigger, i.e., $\frac{V-F-C}{V} = \frac{\$87-\$20-\$60}{\$87} = 8.05\%$, which exceeds 8% trigger. If the asset values drifts down to the value of \$86 at time T, however, the CoCo will convert because the trigger will be breached, i.e., $\frac{\$86-\$20-\$60}{\$86} = 6.9\% < 8\%$.

Assume for now that bank's shareholders cannot affect the value of its assets. For the dilutive CoCo, the value of the equity will be either \$88-\$20-\$60=\$8 at $t = T$, if the asset value increases to \$88; or $\$86 - 1.1 \cdot \$20 - \$60 = \$4$, if asset value drifts down to \$86 and the

CoCo converts. Thus, at time $T-1$, the bank's equity is valued at $\$6.0 = 0.5 \cdot (\$8 + \$4)$. For the CoCo with the loss-absorbing conversion, the equity value at $t = T$ will be either \$8, if the asset value increases to \$88; or $\$11(=\$86 - 0.75 \cdot \$20 - \$60)$, if asset value declines to \$86. Thus, at time $T-1$, the bank's equity is valued at $\$9.5 = 0.5 \cdot (\$8 + \$11)$.

Now consider how the CoCo tranche can affect the shareholders' incentive to issue equity or burn money. For simplicity, assume that at time $T-1$, the shareholders of the bank can choose to either issue \$2 of equity, burn \$2 of assets, or do nothing. It is straightforward to see that if the conversion price is dilutive to the shareholders, they will choose to issue \$2 of equity at time $T-1$ in order to avoid breaching the trigger at $t = T$. Particularly, with \$2 of new equity issued at time $T-1$, the size of the bank's assets increases to \$89, and the value of the firm's assets will drift to either \$90 or \$88 at time T. At these values, there will be no breaching the trigger and the total value of the equity at time T will be either \$10 or \$8, respectively. As such, the equity value of the existing shareholders at time $T-1$ is $\$7 = 0.5 \cdot (\$10 + \$8) - \$2$, where the last term accounts for \$2 of newly issued equity. This equity value is higher than for the case when the bank do not issue equity.

Conversely, for the CoCo bond that assumes 25% write-off, the bank's shareholders will be better off by burning \$2 worth of bank's assets at time $T-1$ and forcing the conversion at time T regardless of whether the asset value increases or decreases next period. As depicted in Figure 2, the total value of bank's equity at time T will be either \$11 ($= \$86 - 0.75 \cdot \$20 - \$60$) or \$9 ($= \$84 - 0.75 \cdot \$20 - \$60$), if the assets drift from \$85 (\$2 of assets are burnt) to \$86 and \$84, respectively. Thus at time $T-1$, the equity value of the existing shareholders is $\$10 = 0.5 \cdot (\$11 + \$9)$, which is a higher than the value of equity in the case where the bank cannot burn its assets.

6 Appendix 2

6.1 Valuation of the Bank's Equity

In this section, we present valuation of the bank's equity for the case before $E(V, F + C, t)$, and after the conversion $E(V, F, t)$. The value of equity is a function of its asset value V; its principal $F + C$, if no conversion took place in prior periods, or F, if the CoCo bond was converted before; and time t ($\leq T$). The CoCo bond is described by parameters π, θ and the size of CoCo bond C. First, we describe the valuation of equity for the case in which the conversion already have taken place in prior periods. For this case, the bank continues paying interest of its straight bond f and its par value F at maturity. At maturity date T, the value of the the bank's equity is

$$E(V, F, T) = \max(0,\ V - F). \tag{4}$$

Any time prior to maturity, using standard arbitrage arguments, the value of the equity $E(V, F, t)$ is given by the solution to the following PDE:

$$\frac{\sigma^2 V^2}{2} E_{VV} + (r - \alpha - \lambda \cdot k) E_V + E_t + \alpha V - f - rE - \lambda \cdot\ \mathbb{E}^Q\{E(Y \cdot V, F, t) - E(V, F, t)\} = 0, E \geq 0,$$

where subscripts t and V denote partial derivatives and $\mathbb{E}^Q$ is the expectation operator under the risk neutral measure Q. In this equation, αV is the cash flow generated by the bank's assets and $\alpha V - f$ is the dividend payout to the shareholders; and the last term represents the expected change in equity value due to jumps. Note that after the CoCo bond converts, the bank will not choose to issue equity because such transaction would dilute the shareholders' value and will benefit holders of the straight debt at the shareholders' expense. Also, without CoCo bond the bank will not choose to burn its assets.

Now, consider the valuation of the bank's equity for the case if the CoCo was not converted in the prior periods. At maturity $t = T$:

$$\begin{cases} E(V, F+C, T) = V - C - F, \ \frac{V-F-C}{V} > \theta, \text{ no conversion,} \\ E(V, F+C, T) = \max\{0, (V - F - \ (\pi) \cdot C\}, \text{ if } \frac{V-F-C}{V} < \theta \text{ and } V \geq F, \\ E(V, F+C, T) = 0, \text{ otherwise.} \end{cases}$$

Before maturity, $t < T$, the bank makes its equity issuance as well as default choices. These choices maximize the market value of the bank's equity, which is the present value of net cash flow to the equityholders. The solution involves determining the free boundary conditions that divide the state space $(V, \{F + C \text{ or } F\}, t)$ into the three regions that characterize the bank's choices: the *no equity issuance/no money burning* region, the *equity issuance* region, the *money burning* region, the conversion region, and the *default* region.[20]

In the *no equity issuance/* no *money burning* region, it is not optimal for the bank to issue equity capital (or burn money). In this region, the equity value $E(V, F + C, t)$ equals the instantaneous cash flow net of coupon payment plus the expected value of the equity at time $t + \Delta t$ calculated under the risk neutral measure Q:

$$E(V, F+C, t) = [\alpha V - f - c]dt + e^{-rdt}\mathbb{E}^Q\{E(V_{t+dt}, F+C, t+dt)\}, \ t + dt \leq T. \qquad (6)$$

In this region, the equityholders will not choose to issue equity (or burn money) and the following inequalities should hold for any ΔV:

[20] For brevity, we omit the discussion of smooth pasting conditions.

$$\begin{cases} [\alpha V - f - c]dt + e^{-rdt}\mathbb{E}^Q(E(V_{t+dt} + \triangle V, F + C, t + dt)\ ) - \triangle V - TC < \\ [\alpha V - f - c]dt + e^{-rdt}\mathbb{E}^Q(E(V_{t+dt}, F + C, t + dt)), \\ \triangle V > 0 \text{ , i.e., equity issuance is not profitable.} \\ E(V - \triangle V, F, t) - \ (\pi) \cdot C < [\alpha V - f - c]dt + e^{-rdt}\mathbb{E}^Q(E(V_{t+dt}, F + C, t + dt)) \\ \text{s.t., } \frac{V - \Delta V - F - C}{V - \Delta V} < \theta,\ \triangle V > 0 \text{ , i.e., "money burning" that leads to an immediate} \\ \text{conversion is not profitable.} \end{cases} \tag{7}$$

where TC are transaction costs of raising equity, $TC = e \cdot (V + \triangle V)$. The value of the equity $E(V, F + C, t)$ in the *no equity issuance/no money burning* region is given by the solution to the following PDE:

$$\begin{aligned} &\tfrac{\sigma^2 V^2}{2} E_{VV} + (r - \alpha - \lambda \cdot k)E_V + E_t + \alpha V - f - c - rE- \\ &\lambda \cdot \mathbb{E}^Q\{E(Y \cdot V, F, t) - E(V, F + C, t)\} = 0, \text{ for any } \tfrac{V - F - C}{V} > \theta,\ E > 0.^{21} \end{aligned} \tag{8}$$

In the equity issuance region, the value of the bank's equity $E(V, F + C, t)$, $t < T$, can be determined by maximizing the expected value of equity, over all equity issuance policies $\triangle V$:

$$E(V, F{+}C, t) = [\alpha V{-}f{-}c]dt{+}e^{-rdt}\mathbb{E}^Q(\max_{\triangle V > 0}[E(V_{t+dt}{+}\triangle V, F{+}C, t{+}dt)\ {-}\triangle V{-}TC]), \text{ for } t < T. \tag{9}$$

The bank will raise new capital if the net benefit of raising capital exceeds the transaction costs TC. This condition characterizes a region where equityholders raise equity and increase the size of the banks's capital. In the money burning region, there has to be some non-zero amount of assets $\triangle V^* > 0$ to be burn that satisfies the following condition:

$$E(V-\triangle V^{*},F,t)-\ (\pi)\cdot C > E(V,F+C,t),\ \text{s.t.},\ \frac{V-\Delta V^{*}-F-C}{V-\Delta V^{*}}=\theta,\ \triangle V^{*}>0\ ,\ \text{for}\ t<T.^{22} \tag{10}$$

In this region, the equity value is set at: $E(V, F+C, t) = E(V-\triangle V^{*}, F, t)-\ (\pi)\cdot C$. In the *CoCo conversion* region, the banks capital ratio is below the trigger θ, and the following condition is held $\frac{V-F-C}{V} \leq \theta$. In this region, the CoCo converts to equity at the dollar amount of shares $(\pi)\cdot C$. In this region, the equity value for the existing stockholders is set:

$$E(V,F+C,t)=E(V,F,t)-(\pi)\cdot C, \qquad \text{if } V>F,\ \text{and } E(V,F,t)-(\pi)\cdot C>0. \tag{11}$$

where $(\pi)\cdot C$ is the equity value that the CoCo holders take over after the conversion. The remaining fraction of equity value $E(V,F,t)-(\pi)\cdot C$ belongs to pre-existing shareholders of the bank. The first inequality reflects the condition that the CoCo conversion can take place only if the bank's assets are larger than the size of its debt. As we pointed out earlier, a large negative jump can instantly reduce the value of bank's assets to some level V' well below the conversion trigger so that $\frac{V'-F-C}{V'} < \theta$, but above the default level $V' > F$. If the conversion price π is high, the conversion price is reduced from π to $\pi'(>0)$ such that the value of the existing shareholders remains small but positive at $\varepsilon > 0$, where π' satisfies $E(V', F+C, t) = E(V', F, t) - (\pi')\cdot C = \varepsilon$. In this "renegotiation" region, the value of the CoCo holders will be $E(V', F, t)$. In default region, the equity value $E = 0$, if

$$E(V,F,t)=[\alpha V-f]dt+e^{-rdt}\mathbb{E}^{Q}\{E(V_{t+dt},F,t+dt)\}<0,\ \ t<T. \tag{12}$$

6.2 Valuation of the Straight Bond

In this description, we assume that the straight debt is senior to CoCo. To calculate the value of the debt, we need to consider the bank's default strategy and the shareholders'

[22] Value ΔV^{*} is the minimum amount of "money burning" needed to breach the conversion trigger.

optimal strategy to raise capital. At maturity date T, the value of the bank's debt is

$$\begin{cases} D(V,F,T)=F, \text{ if } V\geq F \text{ and } D(V,F,T)=(1-DC)\cdot V, \text{ otherwise,} \\ \text{CoCo bond was converted in prior periods.} \\ D(V,F+C,T)=F, \text{ if } V\geq F \text{ and } D(V,F+C,T)=(1-DC)\cdot V, \text{ otherwise,} \\ \text{no prior CoCo conversion.} \end{cases} \tag{13}$$

where DC represents proportional default costs ($0 \leq DC \leq 1$). Any time prior to maturity $t < T$, the value of debt satisfies:

$$\begin{cases} \frac{\sigma^2 V^2}{2} D_{VV} + (r-\alpha-\lambda\cdot k)D_V + D_t + f - rD - \lambda\cdot \mathbb{E}_t^Q\{D(Y\cdot V, F+C,t) - D(V,F+C,t)\} = 0, \\ \text{if } E(V,F+C,t)\geq 0, \text{ no prior CoCo conversion.} \\ \frac{\sigma^2 V^2}{2} D_{VV} + (r-\alpha-\lambda\cdot k)D_V + D_t + f - rD - \lambda\cdot \mathbb{E}_t^Q\{D(Y\cdot V, F,t) - D(V,F,t)\} = 0, \\ \text{if } E(V,F,t)\geq 0, \text{ the CoCo converted in prior periods.} \end{cases} \tag{14}$$

In the conversion region, the value of the straight debt is

$$D(V,F+C,t) = D(V,F,t), \ if \quad \frac{V-F-C}{V} \leq \theta. \tag{15}$$

In the region where the bank issues equity and increases its capital by $\triangle V$, the value of straight debt satisfies:

$$D(V,F+C,t_+) = D(V+\triangle V, F+C, t). \tag{16}$$

In the region where the bank "burns" assets by the amount of $\triangle V^*$ and forces the conversion, the value of straight debt satisfies:

$$D(V,F+C,t_+) = D(V-\triangle V^*, F, t), \text{ s.t. } \frac{V-\triangle V^* - F - C}{V - \triangle V^*} = \theta. \tag{17}$$

In default region, the assets are transferred to the holders of the straight debt:

$$D(V,F,t) = (1-DC)\cdot V, \text{ if } E(V,F,t) = 0. \tag{18}$$

6.3 Valuation of the CoCo Bond

Because of lower priority, at default, the CoCo holders recover zero value. To calculate the value of CoCo debt, we need to consider the bank's default strategy and strategy to raise capital as well the possibility of conversion and renegotiation. At the debt maturity date T, if conversion did not take place in prior periods, the value of the CoCo bond, $Z(V,F+C,T)$ is

$$\begin{cases} Z(V,F+C,T) = C, \ \frac{V-F-C}{V} > \theta, \\ Z(V,F+C,T) = \min\{(V-F), \ (\pi)\cdot C\}, \text{ if } \frac{V-F-C}{V} < \theta \text{ and } V \geq F, \\ Z(V,F+C,T) = 0, \text{ otherwise.} \end{cases} \tag{19}$$

Any time prior to maturity $t < T$, the value of CoCo bond satisfies:

$$\frac{\sigma^2 V^2}{2} Z_{VV} + (r-\alpha-\lambda\cdot k)Z_V + Z_t + c - rZ - \lambda\cdot\mathbb{E}_t^Q\{Z(Y_t\cdot V,F+C,t) - Z(V,F+C,t)\} = 0, \\ \text{if } E(V,F+C,t) \geq 0. \tag{20}$$

At the conversion boundary, the value of the CoCo bond is

$$Z(V,F+C,t) = \min\{E(V,F,t),(\pi)\cdot C\} \text{ if } \frac{V-F-C}{V} \leq \theta. \tag{21}$$

In the renegotiation region, (i.e., for V', s.t., $\frac{V'-F-C}{V'} < \theta$, $V' > F$). The bank remains operational and is taken over by the holders of the CoCo bond. For such values of assets, the value of the CoCo holders will be $Z(V',F+C,t) = E(V',F,t)$. In the region where the bank burns money by the amount of $\triangle V' > 0$ and forces the conversion, the value of CoCo debt satisfies:

$$Z(V, F+C, t_+) = Z(V-\triangle V', F+C, t_+) = (\pi)\cdot C, \text{ s.t.}, \frac{V-\triangle V'-F-C}{V-\triangle V'} = \theta. \quad (22)$$

In the region where the bank issues equity and increases its capital by $\triangle V$ at t, the value of CoCo bond satisfies:

$$Z(V, F+C, t_+) = Z(V+\triangle V, F+C, t). \quad (23)$$

In default region:

$$Z(V, F+C, t_+) = 0. \quad (24)$$

Figure 1: Sample Path for Bank's Assets and CoCo Debt

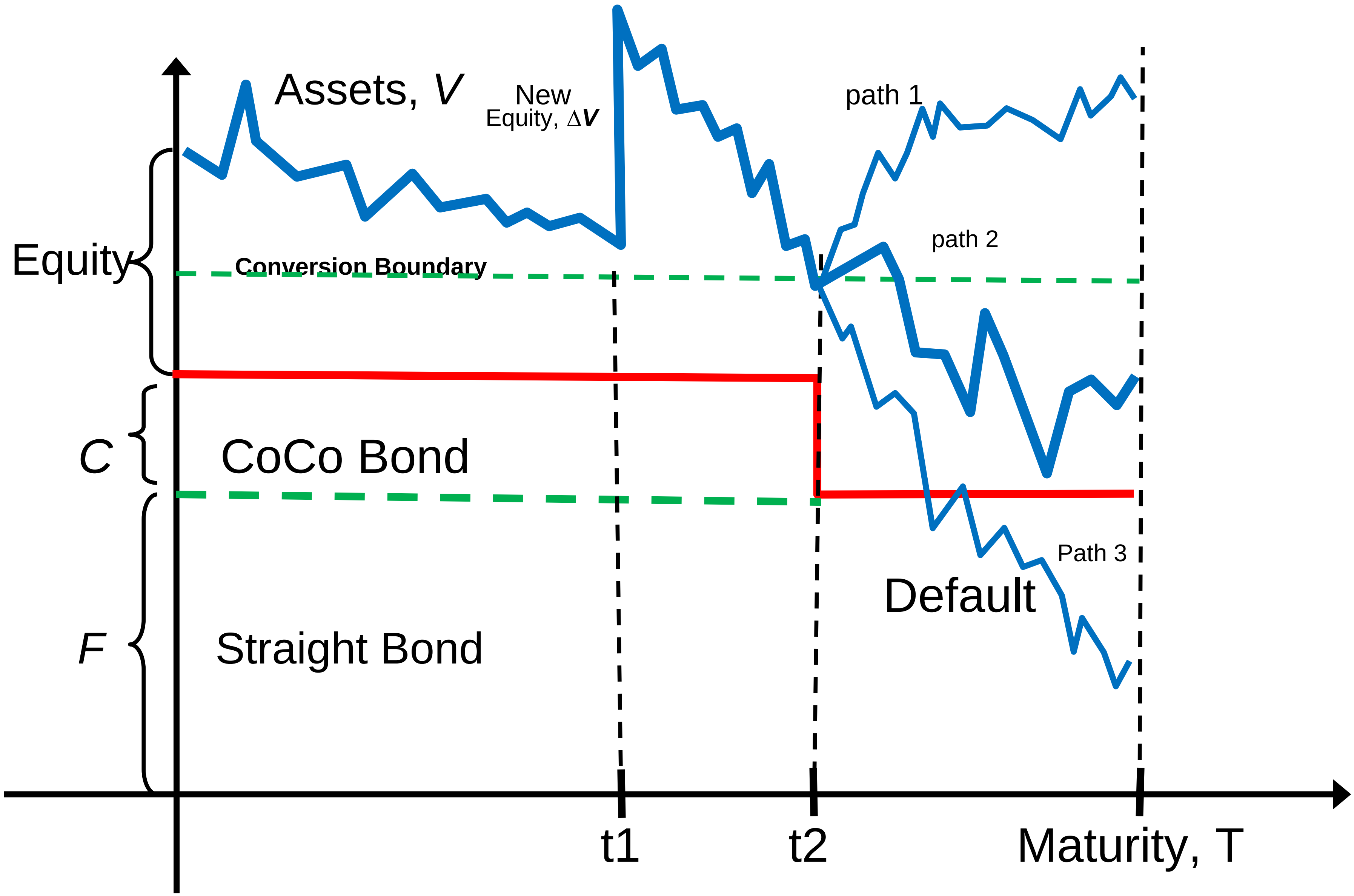

Figure 2: A one-period numerical example

These graphs depicts assets value and the equity value of the bank at time T-1 and T. The bank is financed with the tranche of CoCo bond and a straight debt. The CoCo bond and the straight bond have principal values of \$20 and \$60 respectively, and both mature at period T. The CoCo conversion trigger is assumed to be 8%. If the conversion trigger is breached at t=T, CoCo bond will convert into the predetermined value of the bank's equity. Otherwise, the equity holders have to make principal payments the CoCo holders and the holders of straight debt. We consider two cases of CoCo design: 1) The CoCo assumes dilutive conversion price of \$1.1, i.e., \$1 of CoCo debt converts into \$1.1 of equity; 2) the CoCo bond has a loss-absorbing feature with the conversion price of \$0.75, i.e., 25% of the CoCo bond is written-down as soon as the conversion boundary is breached. At time t=T-1, the bank's assets are valued at \$87, but next period they expected to drift to either the level of \$88 or \$86 with equal probabilities. Discount rate is zero and investors are risk-neutral. We consider two cases for decision that the bank can take at t-1: 1) bank's shareholders cannot affect the value of its assets; 2) the bank can choose to either issue \$2 of equity, "burn money" in the amount of \$2 and reduce its capital ratio, or do nothing .

time, t=T-1 **t=T**

Assets **\$87**

prob=0.5 **\$88** Cap. Ratio= 9.09 (No Conversion)

prob=0.5 **\$86** Cap ratio=6.97% (CoCo converts)

Dilutive Conversion price: \$1 of CoCo converts to \$1.1 of equity

Equity value if the bank cannot issue new equity

t=T-1 **t=T**

\$6

\$8 =\$88-20-60 (No Conversion)

\$4 =\$86-(1.1)20-60 (CoCo converts)

Assets process after bank issues \$2 of equity

87 **+2** **\$89**

\$90 No Conversion

\$88 No Conversion

Equity value for existing shareholders after issuing \$2 of equity

\$7

\$8 =\$90-20-60-2 (No Conversion)

\$6 =\$88-20-60-2 (No Conversion)

Loss-Absorbing CoCo: 25% of CoCo is written off, i.e., \$1 of CoCo converts to \$0.75 of equity

Equity value if the bank cannot "burn money"

\$9.5

\$8 =\$88-20-60 (No Conversion)

\$11 =\$86-(0.75)20-60 (CoCo converts)

Assets after bank "burns money " and reduces the size of its assets by \$2

\$87 **-2** **\$85**

\$86 Cap. Ratio=6.97% (CoCo converts)

\$84 Cap. Ratio=4.76% (CoCo converts)

Equity value for existing shareholders after they burn \$2 of assets

\$10

\$11 =\$86-(0.75)20-60 (CoCo converts)

\$9 =\$84-(0.75)20-60 (CoCo converts)

Figure 3: Sample paths for bank's assets for the processes with and without jumps

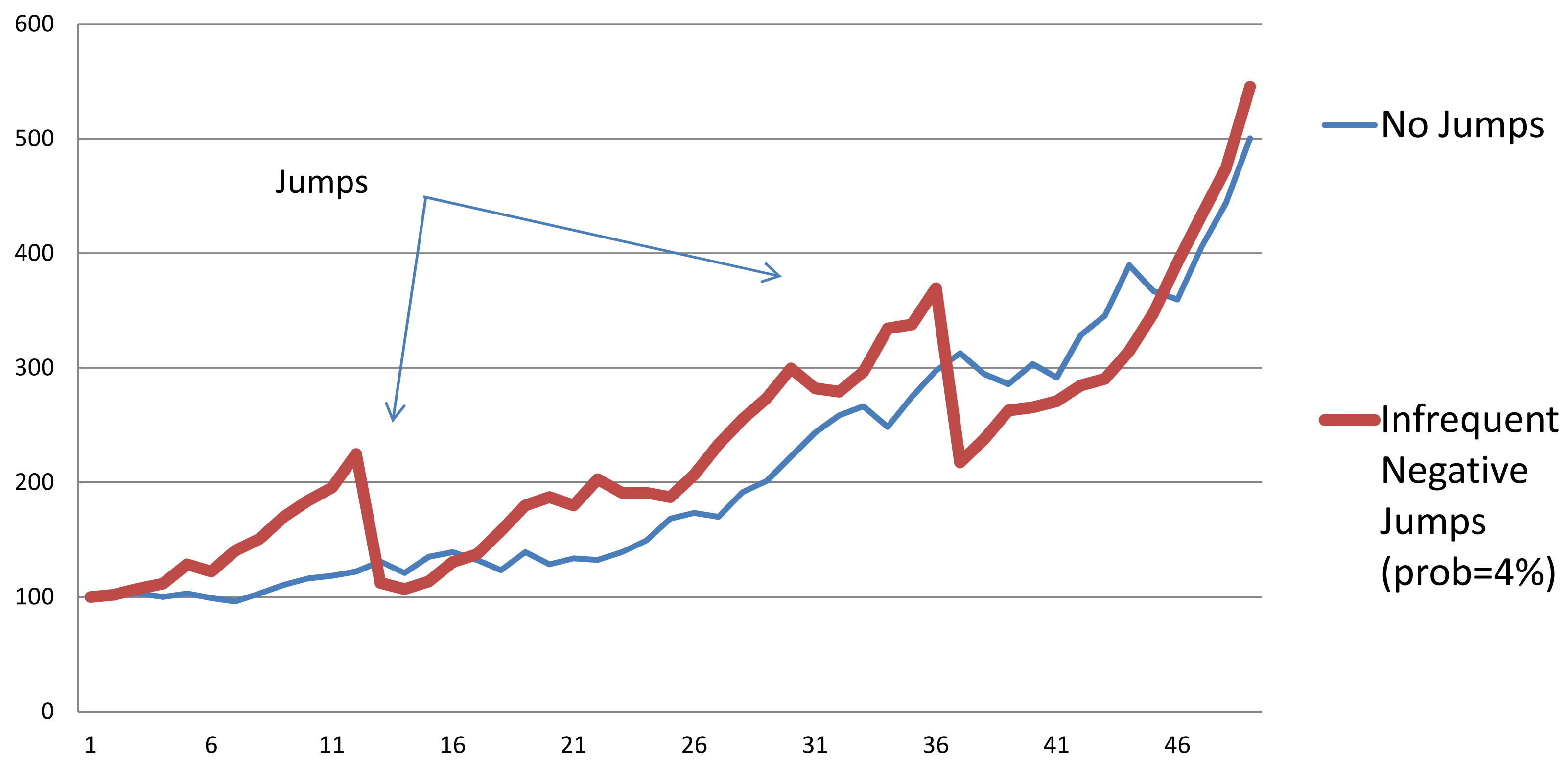

Figure 4: Base case terms of CoCo bond and the case with a straight debt only

These figures show changes in credit spreads and other values as a function of the CoCo conversion price of the bank that has CoCo and straight debt in its capital structure. The conversion price $X is the market value of equity to which $1 of CoCo debt will convert when the conversion trigger is breached. The size of CoCo bond is 5% of the bank's initial assets and straight debt is 87%. The bank's initial leverage ratio is 92%, i.e., its initial capital ratio is 8%. The CoCo conversion is triggered when the bank's capital ratio declines to 6% or below. The CoCo contract terms are as in the base case. For comparison, the table also presents the scenario when the bank is financed with straight debt of the same total size of 92% of total assets. The credit spread is the difference between the yield of the bond and the risk free rate. Expected default costs are measured as the difference of the bank's unlevered assets minus total market value of debt and equity expressed as % of the bank's unlevered assets. The expected value of future net equity issuances is measured

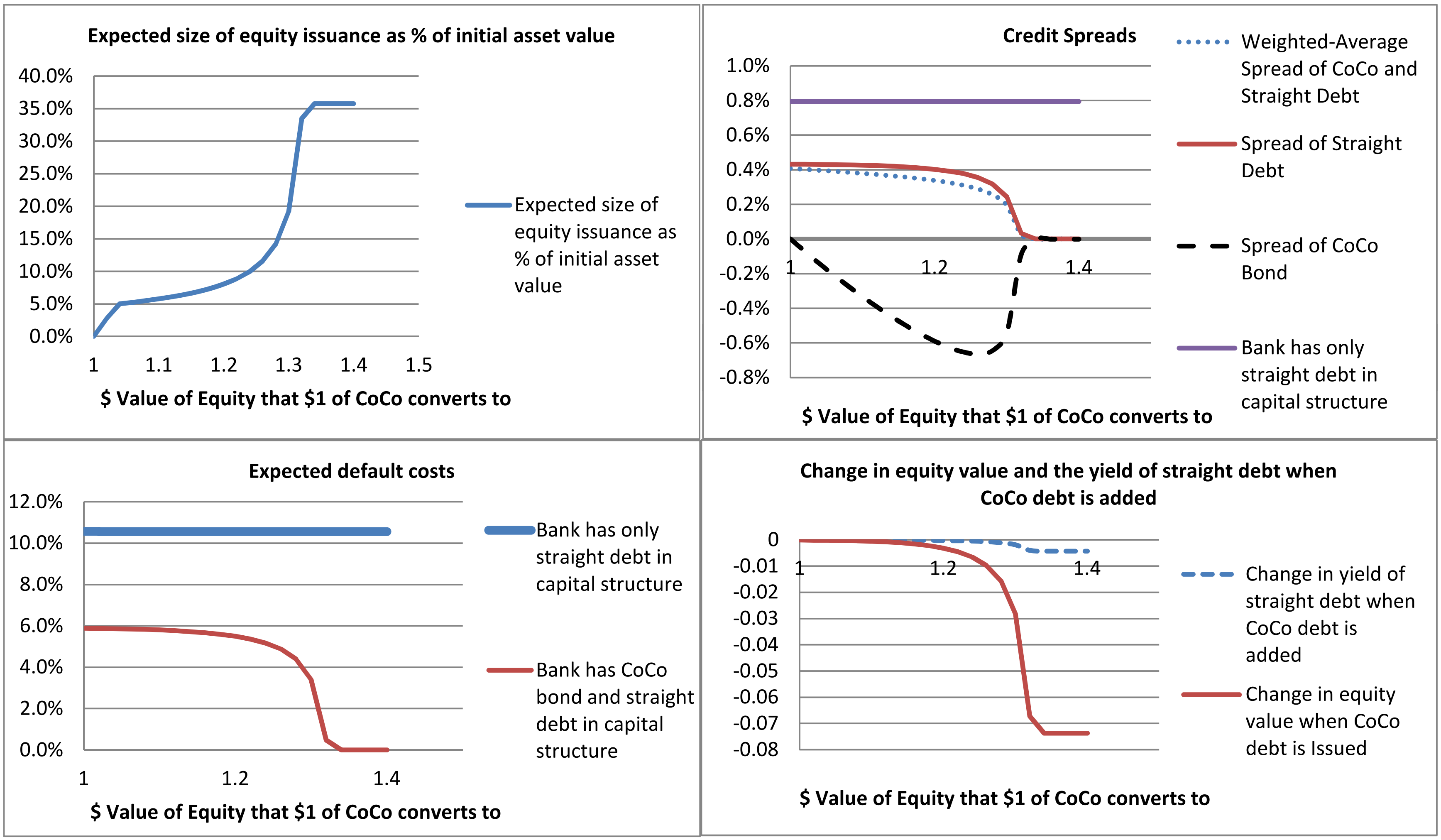

Figure 5: Comparison to the case where the bank cannot issue equity

These figures show changes in credit spreads and other values as a function of the CoCo conversion price: for 2 cases where the bank can costlessly issue equity (base case) and the case where the bank cannot issue equity. The conversion price $X is the market value of equity to which $1 of CoCo debt will convert when the conversion trigger is breached. If the conversion price is above $1, then the conversion will be dilutive (punitive) to existing shareholders . The size of CoCo bond is 5% of the bank's initial assets. The CoCo conversion is triggered when the bank's capital ratio declines to 6% or below. The bank's initial leverage ratio is 92%, i.e., its initial capital ratio is 8%. The CoCo contract terms are as in the base case. The spread is the difference between the yield of the bond and the risk free rate. Expected default costs are measured as the difference of the bank's unlevered assets minus total market value of debt and equity as % of the bank's unlevered assets. The expected value of future net equity issuances is measured ex-ante at time zero as % of the bank's initial assets.

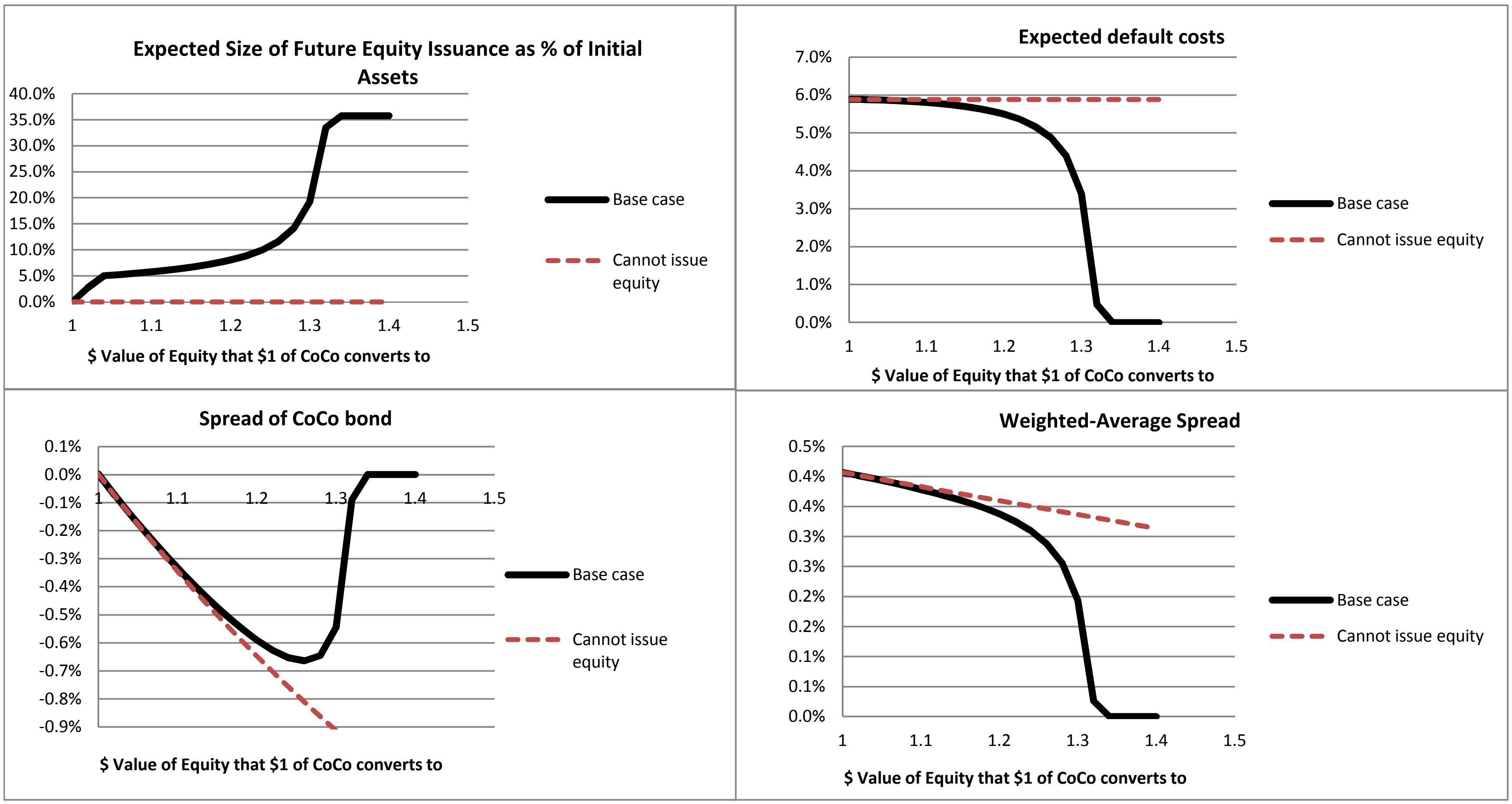

Figure 6: Size of the CoCo

These figures show changes in credit spreads and other values as a function of the CoCo conversion price for different sizes of the CoCo tranche . The value are calculated given that the size of straight debt and CoCo bond are both varied so that the initial leverage of the bank is held at the base case level of 92% . The initial bank's capital is at 8%. The conversion price $X is the market value of equity to which $1 of CoCo debt will convert when the conversion trigger is breached. If the conversion price is above $1, then the conversion will be dilutive (punitive) to existing shareholders . The CoCo conversion trigger ratio is the bank's capital ratio of 6%. The spread is the difference between the yield of the bond and the risk free rate. Expected default costs are measured as the difference of the bank's unlevered assets minus total market value of debt and equity as % of the bank's unlevered assets. The expected value of future net equity issuances is measured ex-ante at time zero as % of the bank's initial assets.

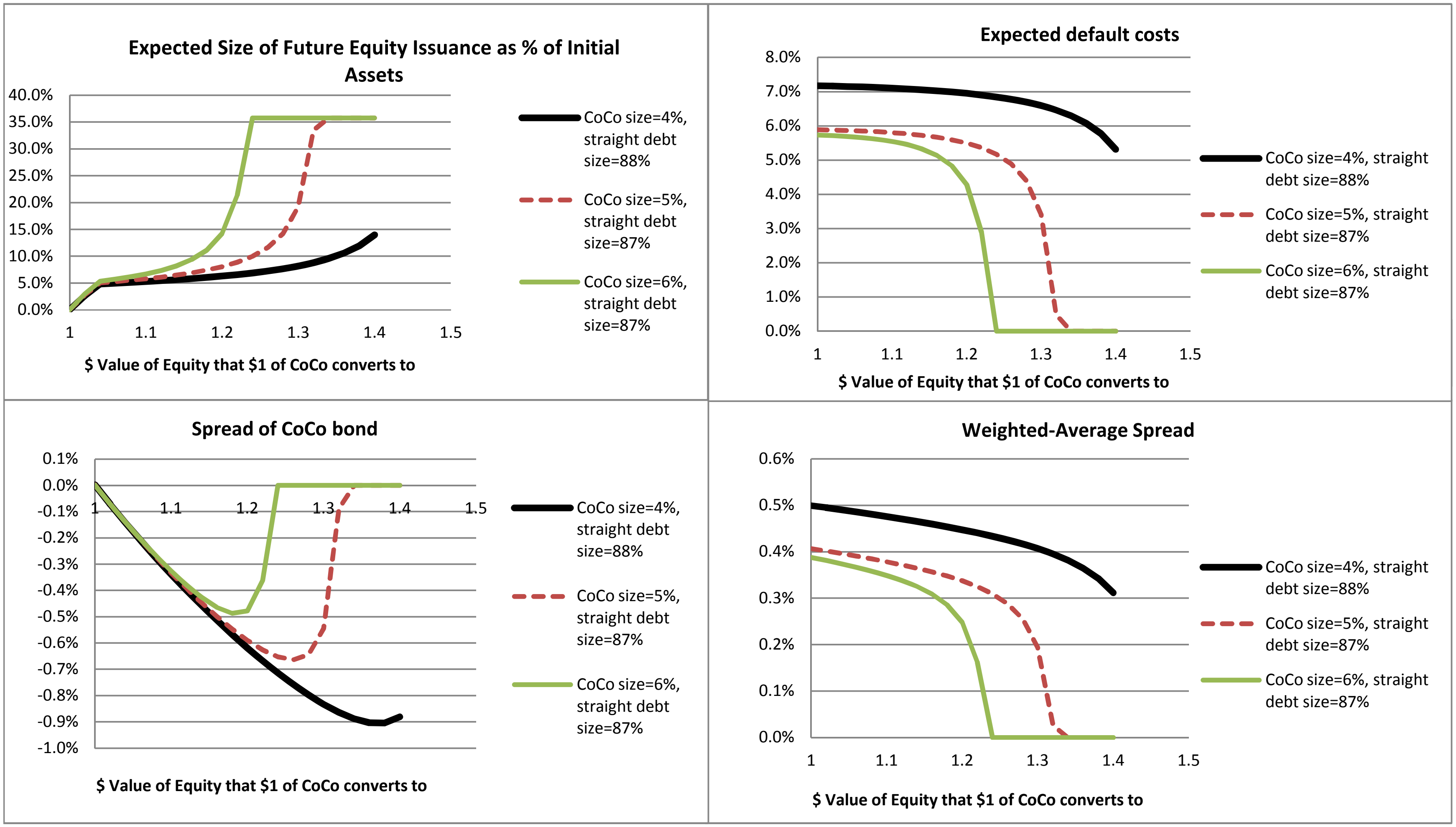

Figure 7: Conversion trigger ratio

These figures show changes in credit spreads and other values as a function of the CoCo conversion price for different values of the conversion trigger ratio. The conversion price $X is the market value of equity to which $1 of CoCo debt will convert when the conversion trigger is breached. If the conversion price is above $1, then the conversion will be dilutive (punitive) to existing shareholders . The size of CoCo bond is 5% of the bank's initial assets. The bank's initial leverage ratio is 92%, i.e., its initial capital ratio is 8%. The CoCo contract terms are as in the base case. The spread is the difference between the yield of the bond and the risk free rate. Expected default costs are measured as the difference of the bank's unlevered assets minus total market value of debt and equity as % of the bank's unlevered assets. The expected value of future net equity issuances is measured ex-ante at time zero as % of the bank's initial assets.

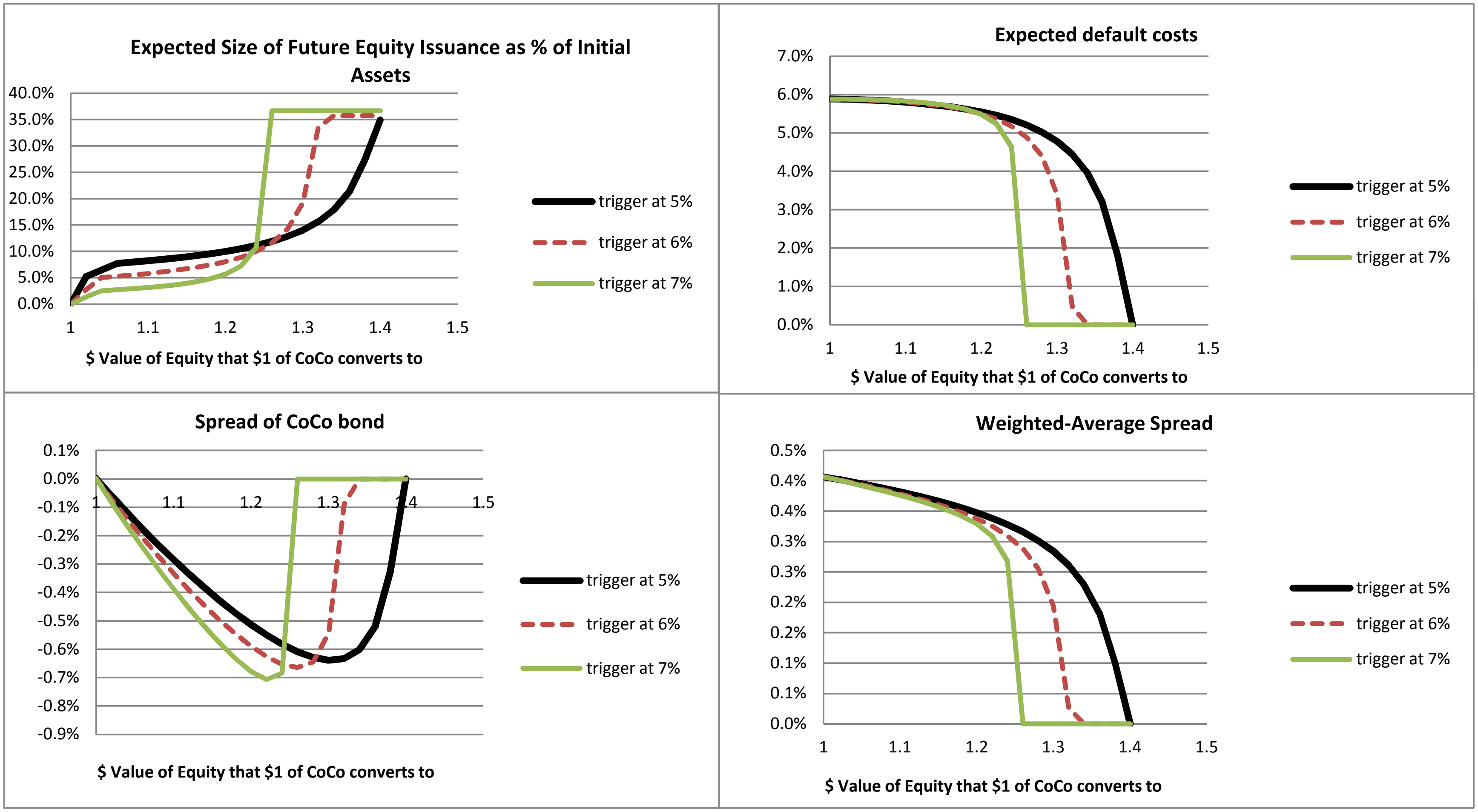

Figure 8: Volatility of the drift process

These figures show changes in credit spreads and other values as a function of the CoCo conversion price for different values of the drift volatility (Vol). The conversion price $X is the market value of equity to which $1 of CoCo debt will convert when the conversion trigger is breached. If the conversion price is above $1, then the conversion will be dilutive (punitive) to existing shareholders . The size of CoCo bond is 5% of the bank's initial assets. The CoCo conversion trigger ratio is the bank's capital ratio of 6%. The bank's initial leverage ratio is 92%, i.e., its initial capital ratio is 8%. The CoCo contract terms are as in the base case. The spread is the difference between the yield of the bond and the risk free rate. Expected default costs are measured as the difference of the bank's unlevered assets minus total market value of debt and equity as % of the bank's unlevered assets. The expected value of future net equity issuances is measured ex-ante at time zero as % of the bank's initial assets.

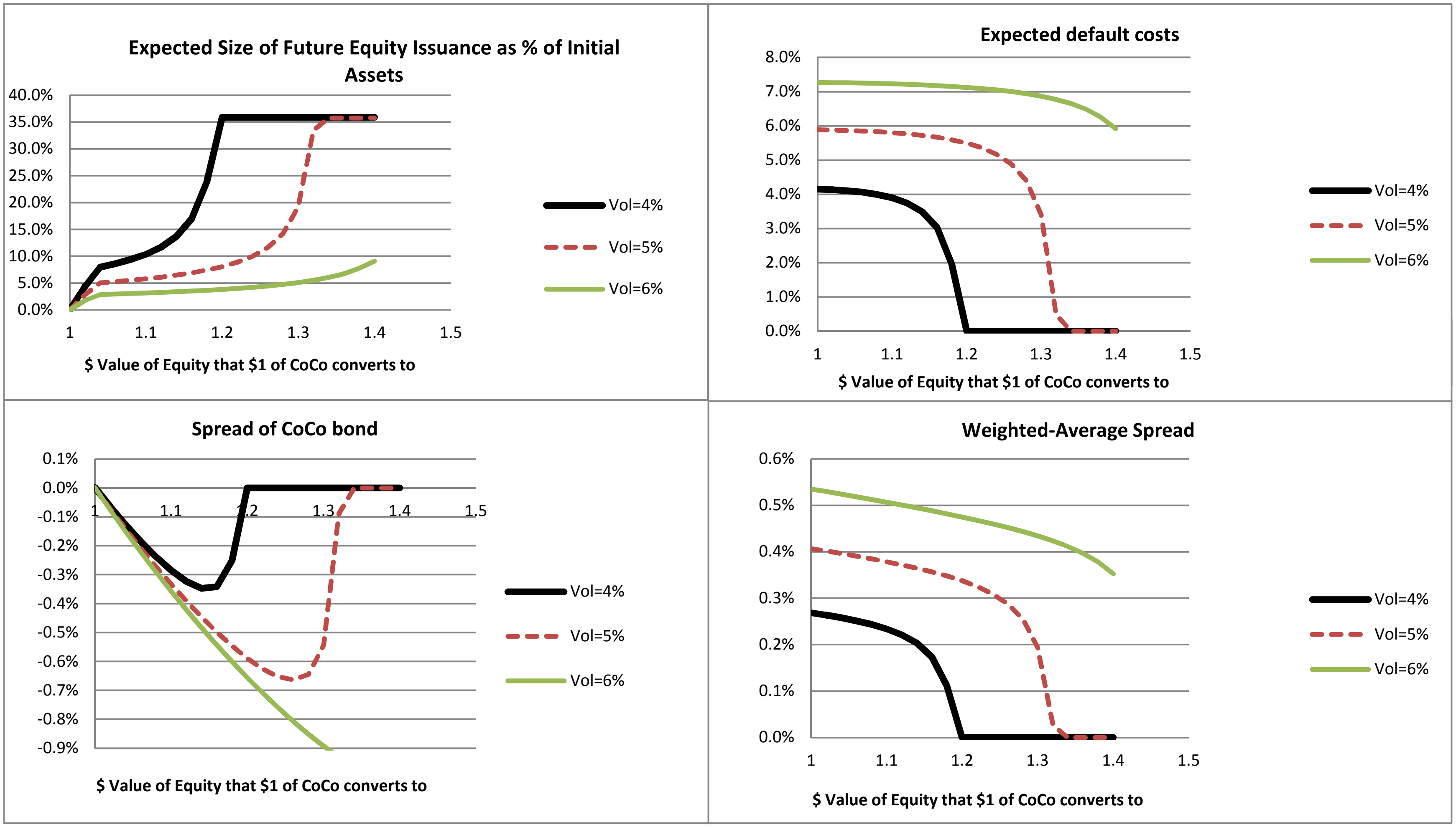

Figure 9: Asset payout rate

These figures show changes in credit spreads and other values as a function of the CoCo conversion price for different asset payout rates. The conversion price $X is the market value of equity to which $1 of CoCo debt will convert when the conversion trigger is breached. If the conversion price is above $1, then the conversion will be dilutive (punitive) to existing shareholders . The size of CoCo bond is 5% of the bank's initial assets. The CoCo conversion trigger ratio is the bank's capital ratio of 6%. The bank's initial leverage ratio is 92%, i.e., its initial capital ratio is 8%. The CoCo contract terms are as in the base case. The spread is the difference between the yield of the bond and the risk free rate. Expected default costs are measured as the difference of the bank's unlevered assets minus total market value of debt and equity as % of the bank's unlevered assets. The expected value of future net equity issuances is measured ex-ante at time zero as % of the bank's initial assets.

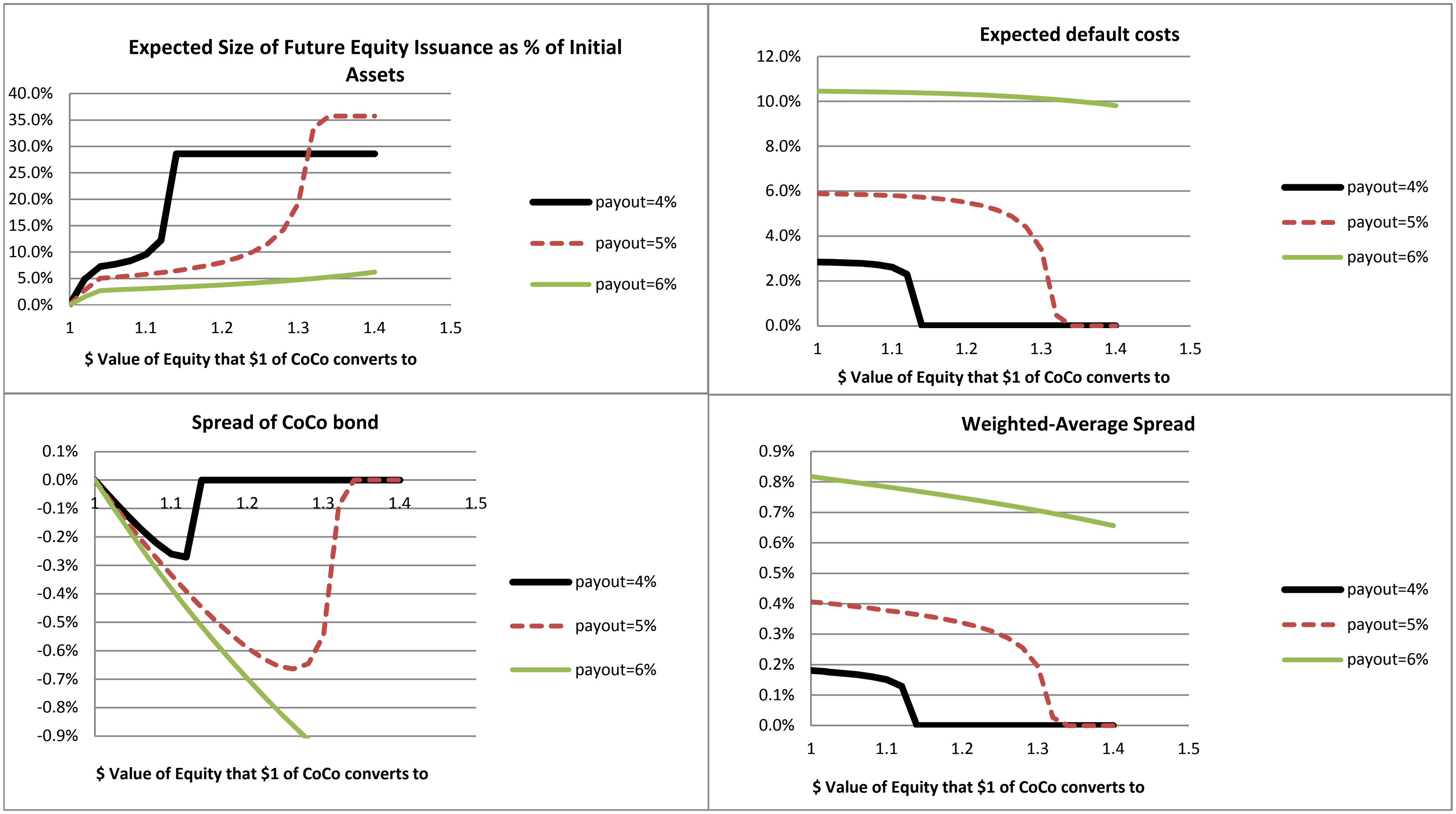

Figure 10: Maturity

These figures show credit spreads and other values as a function of maturity for three cases of CoCo conversion price. The conversion price $X is the market value of equity to which $1 of CoCo debt will convert when the conversion trigger is breached. The size of CoCo bond is 5% of the bank's initial assets. The CoCo conversion is triggered when the bank's capital ratio declines to 6% or below. The bank's initial leverage ratio is 92%, i.e., its initial capital ratio is 8%. The CoCo contract terms are as in the base case. The spread is the difference between the yield of the bond and the risk free rate. Expected default costs are measured as the difference of the bank's unlevered assets minus total market value of debt and equity as % of the bank's unlevered assets. The expected value of future net equity issuances is measured ex-ante at time zero as % of the bank's initial assets.

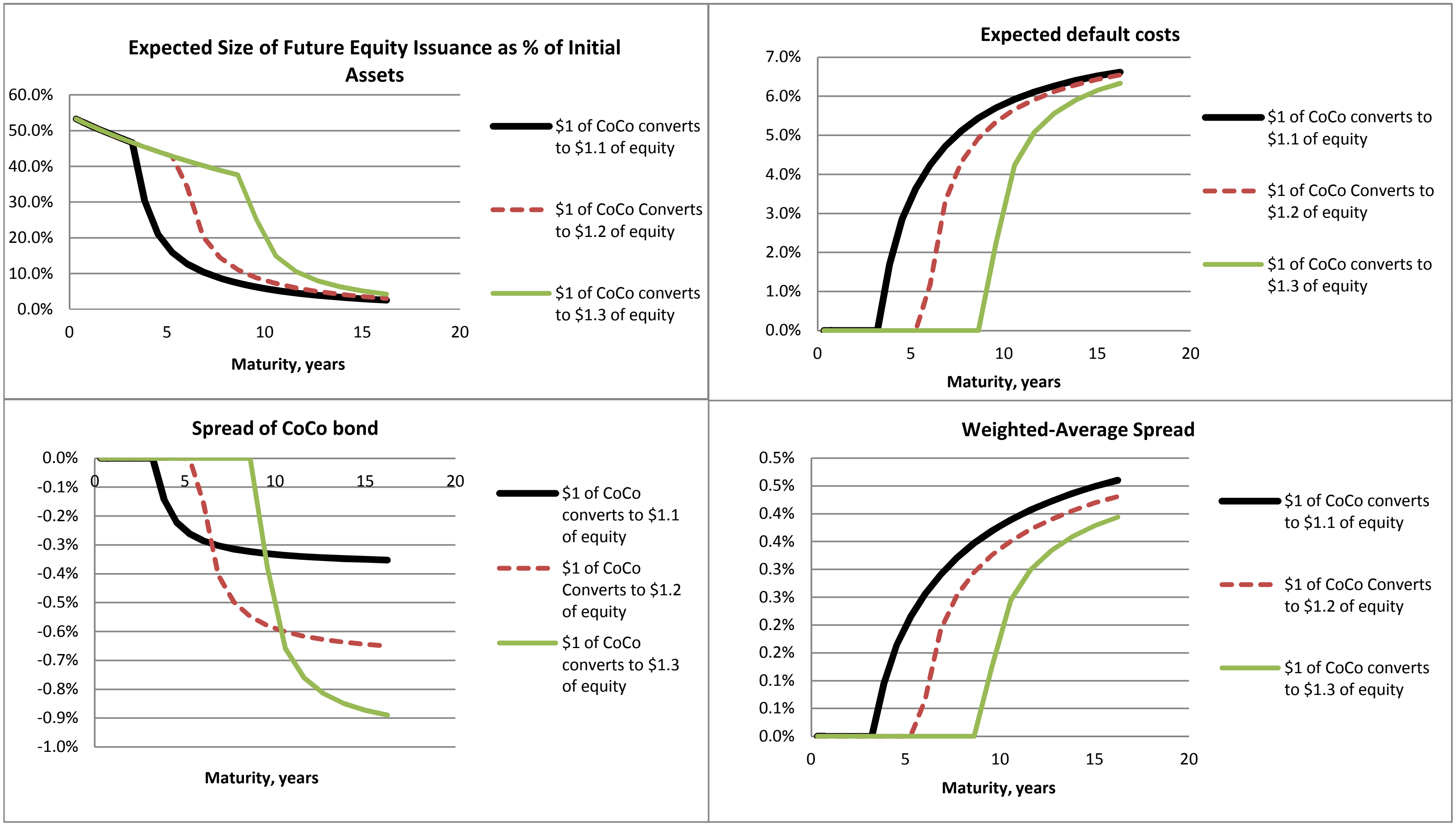

Figure 11: Expected size of money burning as a function of CoCo maturity
These figures depict the credit spreads and expected value of "money burning" that the bank will undertake in order to force the CoCo conversion as a function of the CoCo bond maturity. For comparison, we also plot these values for the case in which the bank is not allowed to burn money. The CoCo contract assumes the conversion price of 0.25, implying the 75% write-down of bond's principal at the conversion trigger. The size of CoCo bond is 5% of the bank's initial assets. The CoCo conversion is triggered when the bank's capital ratio declines to 6% or below. The bank's initial leverage ratio is 92%, i.e., its initial capital ratio is 8%. The remaining contract terms are as in the base case. The spread is the difference between the yield of the bond and the risk free rate. Expected default costs are measured as the difference of the bank's unlevered assets minus total market value of debt and equity as % of the bank's unlevered assets. The expected value of burnt money is measured ex-ante as % of the bank's unlevered assets.

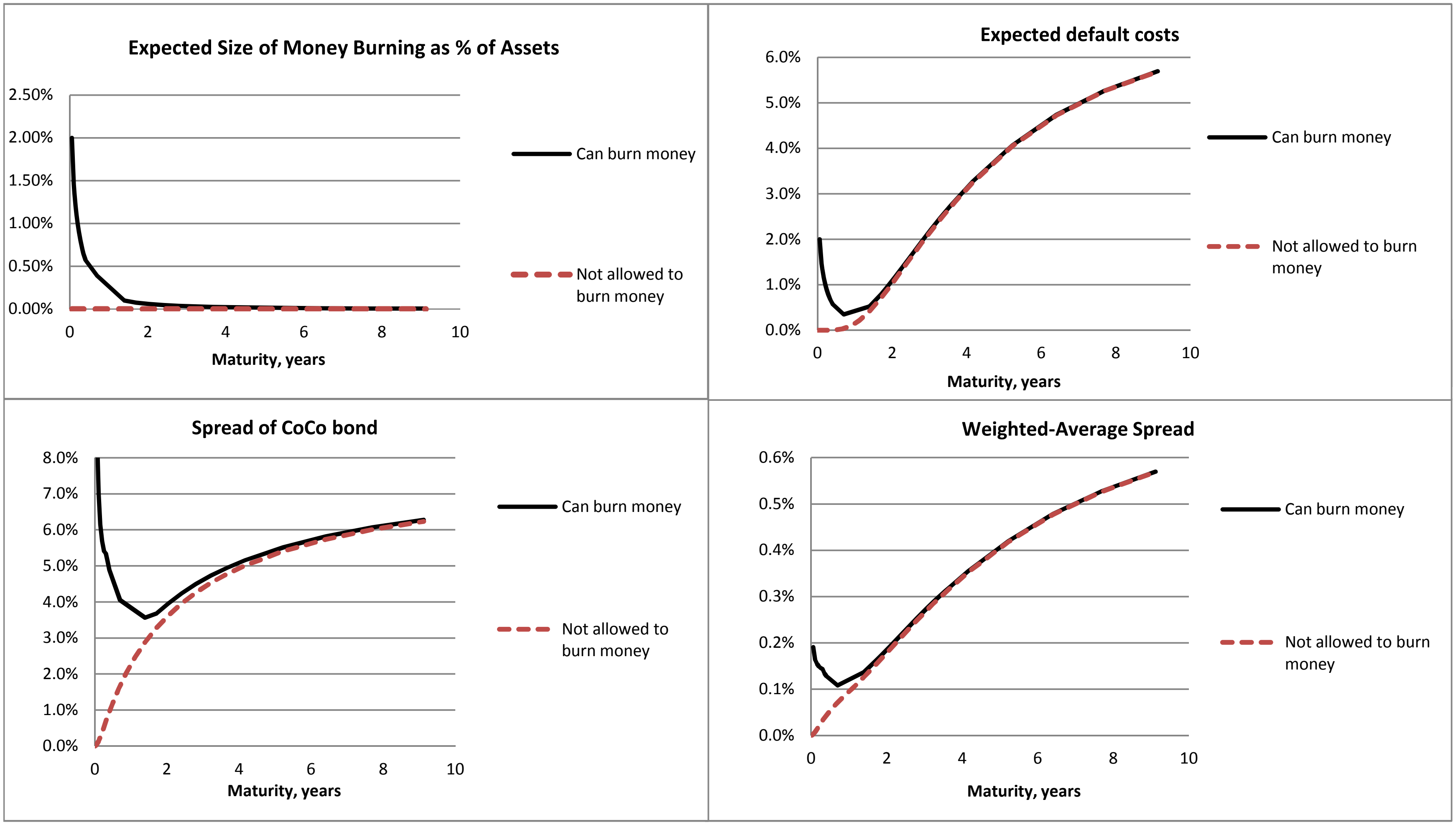

Figure 12: Expected size of money burning as a function of the bank's capital ratio
These figures depict the credit spreads and expected value of "money burning" as a function of the bank's capital ratio (measured as *(V-C-F)/V* while keeping the maturity at 1/365 years, 0.25 year and 0.5 years In this graphs, we vary the value of the bank's asset *V* and keep the size of CoCo and straight debt constant as in the base case. The CoCo contract assumes the conversion price of 0.25 which corresponds to 75% write-down of the principal at the triggering boundary. The CoCo conversion is triggered when the bank's capital ratio declines to 6% or below. The remaining contract terms are as in the base case. The spread is the difference between the yield of the bond and the risk-free rate. Expected default and deadweight costs are measured as the difference of the bank's unlevered assets minus total market value of debt and equity reported as % of the bank's unlevered assets. The expected value of burnt money is measured ex-ante as % of the bank's unlevered assets.

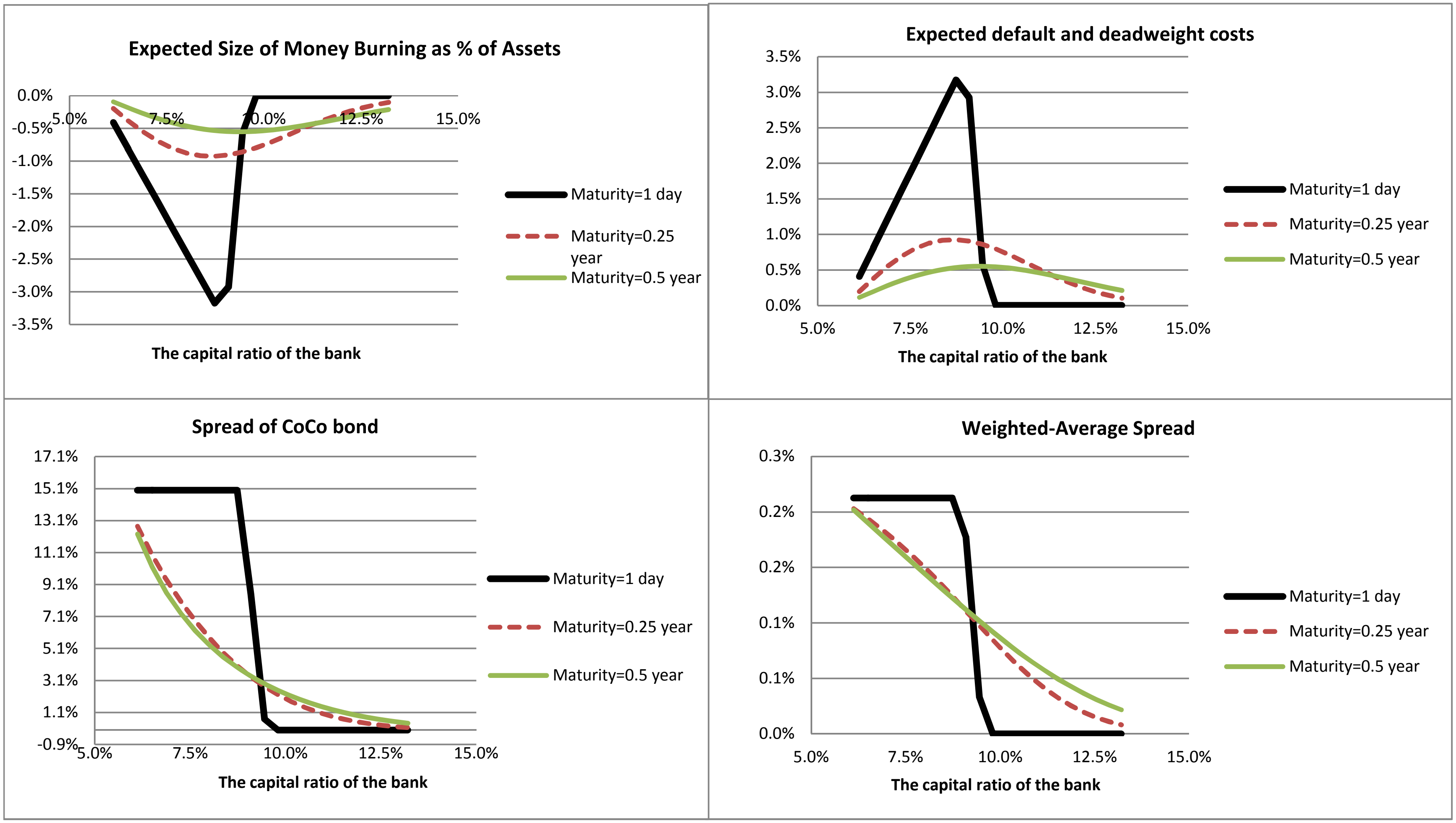

Figure 13: Effect of Jumps

These figures show changes in credit spreads and other values as a function of the CoCo conversion price. The conversion price $X is the market value of equity to which $1 of CoCo debt will convert when the conversion trigger is breached. If the conversion price is above $1, then the conversion will be dilutive (punitive) to existing shareholders . The size of CoCo bond is 5% of the bank's initial assets. The CoCo conversion trigger ratio is the bank's capital ratio of 6%. The bank's initial leverage ratio is 92%, i.e., its initial capital ratio is 8%. The CoCo contract terms are as in the base case. The spread is the difference between the yield of the bond and the risk free rate. Expected default costs are measured as the difference of the bank's unlevered assets minus total market value of debt and equity as % of the bank's unlevered assets. The expected value of future net equity issuances is measured ex-ante at time zero as % of the bank's initial assets.

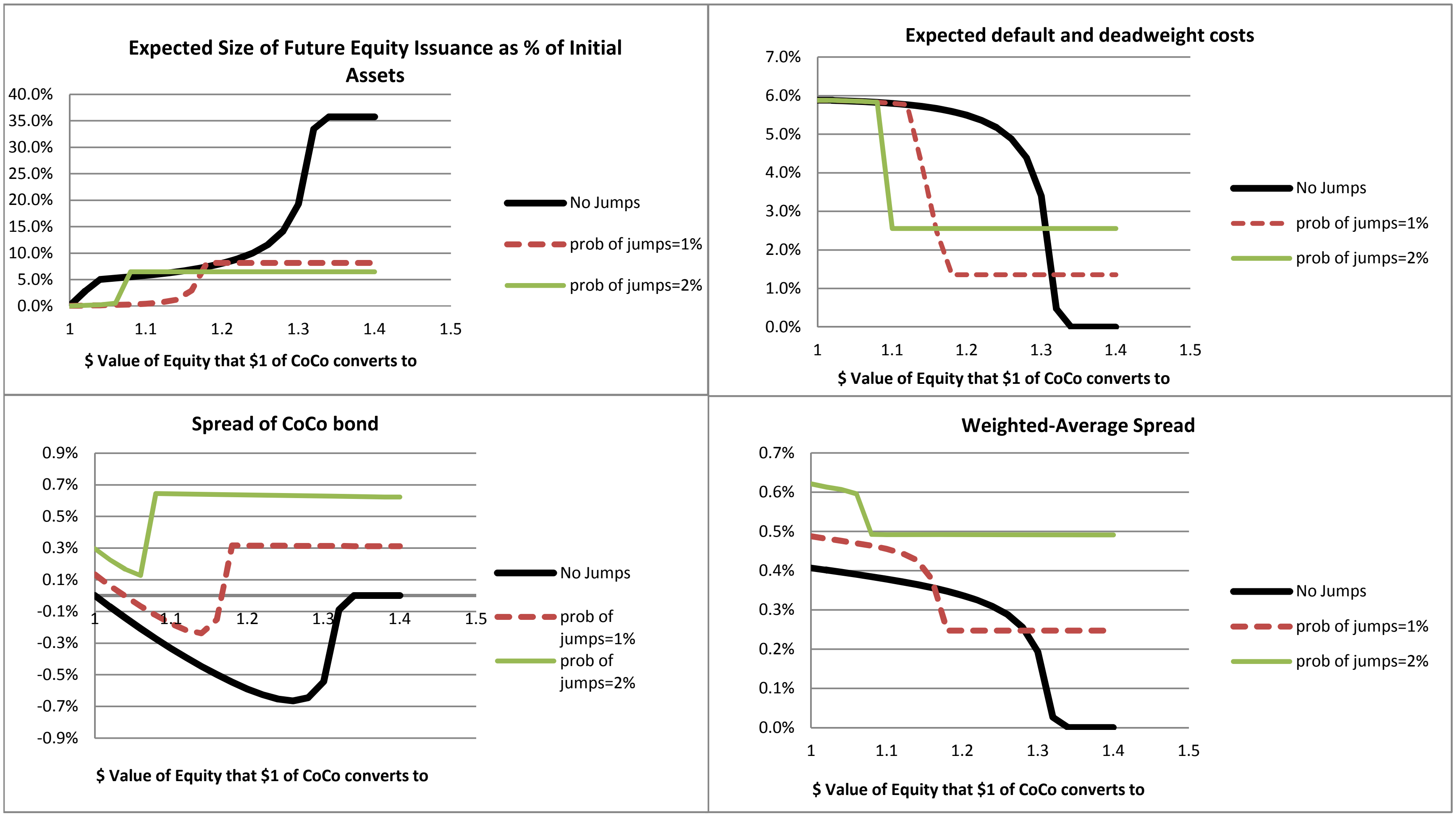

Figure 14: The effect of transaction costs of issuing equity

These figures show changes in credit spreads and other values as a function of the CoCo conversion price: for 3 cases of transaction costs of issuing equity: zero costs (base case), 0.1% costs, and 0.25%. The conversion price $X is the market value of equity to which $1 of CoCo debt will convert when the conversion trigger is breached. If the conversion price is above $1, then the conversion will be dilutive (punitive) to existing shareholders . The size of CoCo bond is 5% of the bank's initial assets. The CoCo conversion is triggered when the bank's capital ratio declines to 6% or below. The bank's initial leverage ratio is 92%, i.e., its initial capital ratio is 8%. The CoCo contract terms are as in the base case. The spread is the difference between the yield of the bond and the risk free rate. Expected default costs are measured as the difference of the bank's unlevered assets minus total market value of debt and equity as % of the bank's unlevered assets. The expected value of future net equity issuances is measured ex-ante at time zero as % of the bank's initial assets.

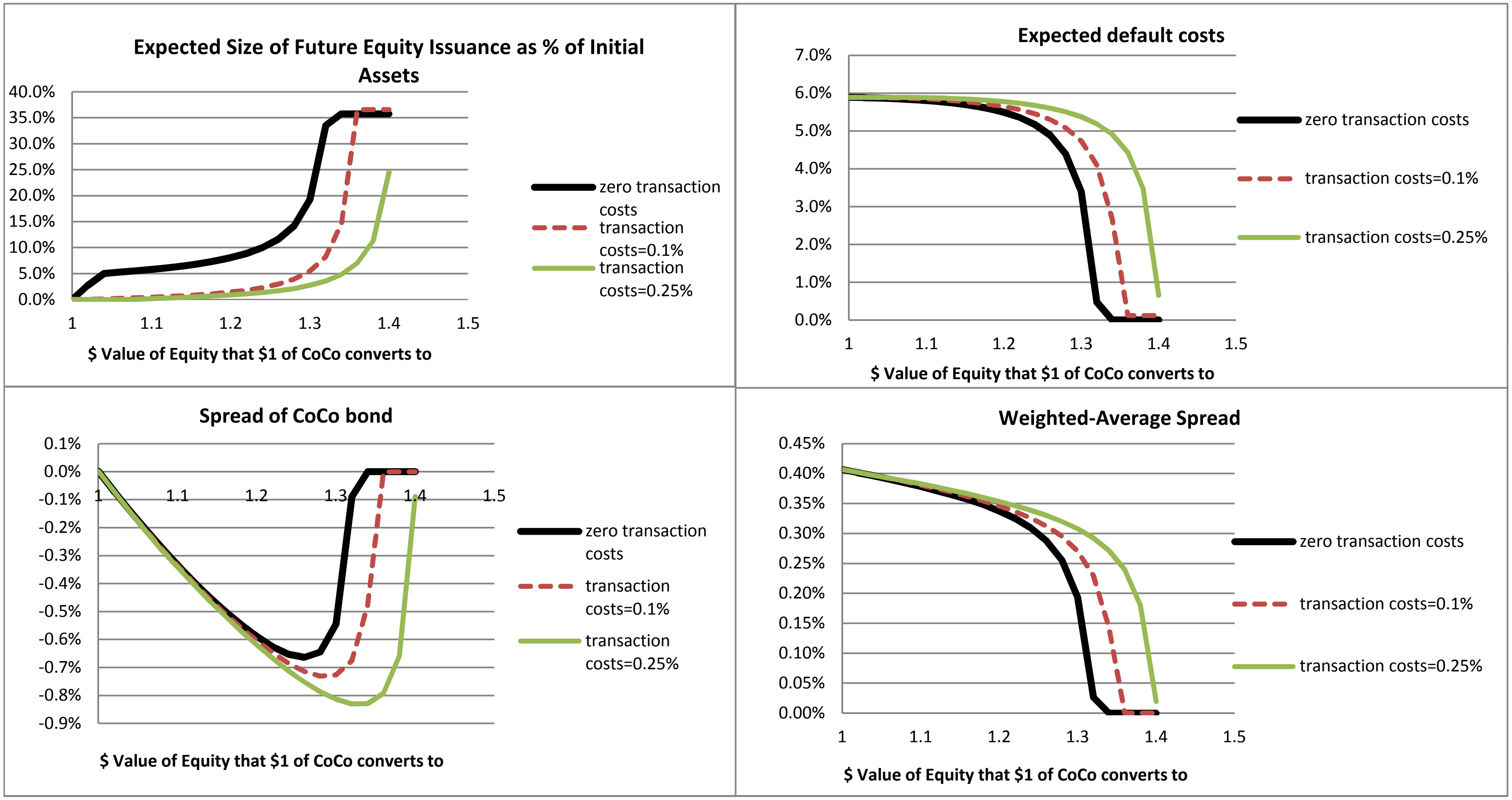

Figure 15: "Soft trigger" ratio

These figures show changes in credit spreads and other values as a function of the arrival intensity of the conversion event , μ , given that the bank's capital ratio is below the trigger of 6%. These values are given for three cases of CoCo conversion price. The conversion price $X is the market value of equity to which $1 of CoCo debt will convert to, given that the conversion event arrives. The remaining CoCo contract terms are as in the base case. The spread is the difference between the yield of the bond and the risk free rate. Expected default costs are measured as the difference of the bank's unlevered assets minus total market value of debt and equity as % of the bank's unlevered assets. The expected value of future net equity issuances is measured ex-ante at time zero as % of the bank's initial assets.

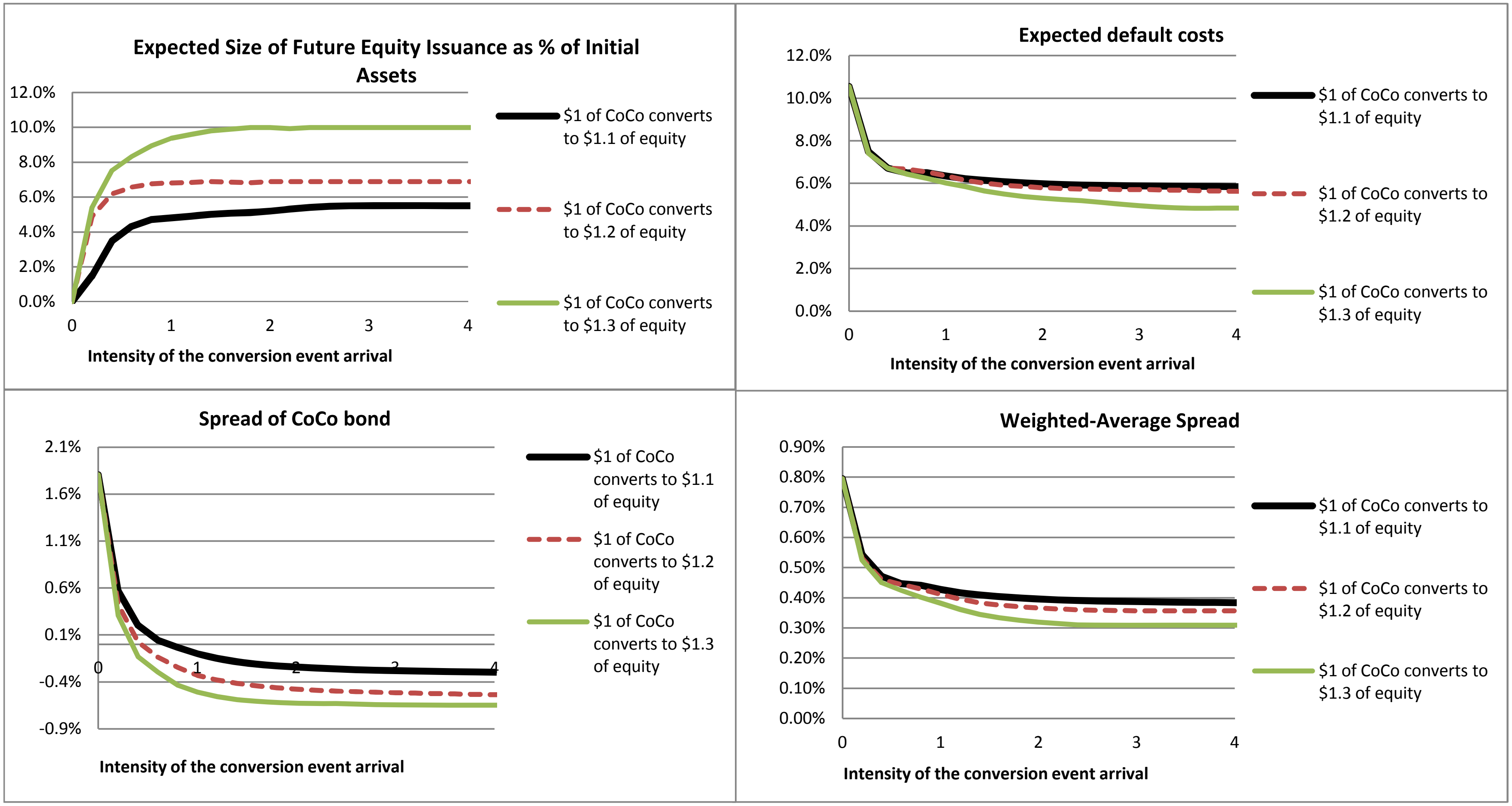

Figure 16: Covenant that restricts the bank's divident payouts for the remaining maturity after the CoCo conversion is triggered
These figures show changes in credit spreads and other values as a function of the CoCo conversion price for two cases: 1) the base case, and 2) the case where the bank, after the CoCo conversion is triggered, is required to retain earnings to rebuild capital and pay no dividends until maturity. The conversion price $X is the market value of equity to which $1 of CoCo debt will convert when the conversion trigger is breached. If the conversion price is above $1, then the conversion will be dilutive (punitive) to existing shareholders . The size of CoCo bond is 5% of the bank's initial assets. The CoCo conversion is triggered when the bank's capital ratio declines to 6% or below. The bank's initial leverage ratio is 92%, i.e., its initial capital ratio is 8%. The CoCo contract terms are as in the base case. The spread is the difference between the yield of the bond and the risk free rate. Expected default costs are measured as the difference of the bank's unlevered assets minus total market value of debt and equity as % of the bank's unlevered assets. The expected value of future net equity issuances is measured ex-ante at time zero as % of the bank's initial assets.