FINANCIAL CRISIS, VALUE-AT-RISK FORECASTS AND THE PUZZLE OF DEPENDENCY MODELING

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Abstract

Forecasting Value-at-Risk (VaR) for financial portfolios is a staggering task in financial risk management. The turmoil in financial markets as observed since September 2008 called for more complex VaR models, as "standard" VaR approaches failed to anticipate the collective market movements faced during the financial crisis. Hence, recent research on portfolio management mainly focussed on modeling return interdependencies via dynamic conditional correlations (DCC, Engle (2002)) volatility spillover (e.g. the BEKK model, named after Baba, Engle, Kraft and Kroner, (1995)) or copulas (Embrechts et al. (2002)).
In this paper, we analyze VaR estimates based on extreme value theory (EVT) models combined with parametric copulas. Tails of the return distributions are modeled via Generalized Pareto Distribution (GPD) approaches applied to GARCH filtered residuals to capture excess returns. Copula models are used to account for tail dependence. Drawing on this EVT-GARCH-Copula approach, we evaluate portfolios consisting of German Stocks, market indices and FX-rates, with a data sample covering both calm and turmoil market phases.
Moreover, models accounting for variable and invariant dependency schemes are evaluated using statistical backtesting and Basel II criteria . The results strongly support the EVT-GARCH-Copula approach, as 99% VaR forecasts clearly outperform estimates stemming from alternative models accounting for dynamic conditional correlations and volatility spillover for all asset classes in turmoil market times.

1. Introduction

Even though financial portfolio research focussed on risk measurement and risk management during the past decade, the recent financial crisis made evident that there is still a lack of reliable indicators for financial risk. In this paper, we address the accuracy of Value-at-Risk (VaR) predictions.

Drawing on daily return data of stocks, stock indices and foreign exchange rates, in a period covering both the run-up and the outbreak of the actual financial crisis, we base VaR models on the assumptions of normally, t and GPD distributed returns. We explore the use of accounting for volatility spillover (BEKK, Engle and Kroner (1995)), dynamic conditional correlations (DCC, Engle (2002), (2009)) and joint probability functions (copulas, Sklar (1959) and Embrechts et al. (2002)) in the prediction of daily VaR.

The DCC and BEKK model represent widely accepted approaches when it comes to the modeling of conditional correlations and covariances respectively. According to Caporin and McAleer (2010), DCC and BEKK do co-exist, whereas the question of distinctions has not been clarified so far.

DCC models separate variance modeling from correlation modeling. In this two step procedure, univariate methods can be used in the first step (variance modeling), and estimation becomes feasible even for large portfolios. Hence, following the seminal paper by Engle (2002), DCC models became rather popular in empirical analyses of financial portfolios.

In contradiction to the two-step DCC approach, BEKK models represent a direct generalization of the univariate ARCH models (Bollerslev (1986)), whereas the estimated parameters become infeasible high when the portfolio size gets bigger. Due to the fact, that all parameters are estimated in a one-step procedure, the model allows for direct interaction between lagged volatilities of all assets. As a consequence, known as the curse of dimensionality, this model is mainly applied in either theoretical or bivariate empirical analyses regarding volatility spillover.

Anyhow, both approaches are able to capture dynamic properties of financial interdependencies, especially in turmoil market times. Due to the fact that the dependence modeled via DCC and BEKK is varies with nonlinear monotonic transformations[1], we classify both models as a "variable" dependence structure.

On the other hand, however, the copula approach separates the dependence structure from the choice of margins. Henceforth, it is classified as an "invariant" scheme of dependence. Albeit copula methodology is widely known(Sklar (1959)), its usage for modeling dependence structure of financial assets has started no more than a few years ago (e.g. Embrechts et al. (2002)).

So far, there is no explicit answer to the question of how to choose an optimal copula for financial timeseries and its choice is still based on empirical analysis. Concerning elliptical copulas, Malevergne and Sornette (2003) conclude for bivariate portfolios that Gaussian copulas can not be rejected against t copulas for currencies and stocks. In contradiction, Kole et al. (2007) underline the quality of t copula performance compared to Gaussian copula for stocks and bonds.

Up to now, however, only few applications of DCC, BEKK and copulas to VaR estimation and VaR prediction are reported. Hakim et al. (2007), (2009) compare DCC and BEKK for stocks, bonds and FX-rates[2]. Palaro and Hotta (2006) use a

[1]The linear correlation coefficient varies varies with the marginal distributions.

[2]Australia and New Zealand.

copula approach to model VaR for a portfolio consisting of Nasdaq and S&P 500 indices. Ozun and Cifter (2007) investigate Latin American markets[3] and Aloui et al. (2011) investigate several bivariate portfolios consisting of emerging and the US markets[4].

Due to the fact that these studies, differ in terms of asset classes, do cover different time horizons and address either multivariate variable or bivariate invariant dependency, the results are hardly comparable. Therefore, the contribution of this paper lies in analyzing all approaches in turmoil market times for different asset classes for higher order portfolios. Consequently, we provide a broad comparison of DCC, BEKK and copulas.

Further, we also relax the classical comparison of Gaussian and t copula by combining Gaussian copulas with t margins and vice versa. This allows for analysing the isolated impact stemming from the individual copulas. Additionally, we make use of the copula as an invariant dependence measure and analyse the application of Extreme Value Theory (EVT) for the margins.

Hence, this investigation, expands the research on the estimation of VaR by explicitly analysing variable and invariant dependency measurements for several asset classes throughout calm and turmoil market times.

2. Methodology

Forecasting. The scope of this paper is twofold. On the one handside, in order to scrutinize the variable modeling of dependency, this contribution investigates conditional correlation and covariance forecasts based on DCC and BEKK models with the assumption of normally and t(5) distributed returns.

On the other handside, however, invariant dependency structure, namely the modeling of joint probability functions via copulas is also analysed. Owing to the separation of dependency from the margins, copulas allow for applying EVT to the univariate margins.

A rolling window approach is applied to forecast the one-day ahead VaR thresholds. The window size is at 500 observations for all data sets and the portfolio weights are assumed to be all equal and do not change over time. Due to the relevance of the Basel II document the 99% VaR forecasts are presented[5].

DCC. The DCC model of Engle (2002) belongs to the class of multivariate GARCH models. The approach separates variance modeling from correlation modeling. Let the Nx1 vector r_t be a set of N asset log returns at time t. Volatilities are calculated in order to construct volatility adjusted residuals ϵ_t. For our research, we assume that each return follows a univariate GARCH(1,1) process. The correlations are estimated based on the standardized residuals. Let R_t denote the correlation matrix and D_t the diagonal matrix with conditional standard deviations at time t.

[3]Bovespa and IPC Mexico.

[4]Brazil, Russia, India, China and the US are investigated.

[5]The results regarding 95% VaR forecasts are available upon request.

The full DCC setup is given by:

$$y_t|\Im_{t-1} \sim N(0, D_t R_t D_t),$$

$$D_t^2 = diag\left\{H_t\right\},\ H_t = V_{t-1} y_t,$$

$$H_{i,i,t} = w_i + \alpha_i y_{i,t-1}^2 + \beta_i H_{i,i,t-1},$$

$$\epsilon_t = D_t^{-1} y_t,$$

$$R_t = diag\left\{Q_t^{1/2}\right\} Q_t diag\left\{Q_t^{1/2}\right\},$$

$$Q_t = \Omega + \alpha \epsilon_{t-1} \epsilon_{t-1}' + \beta Q_{t-1},$$

whereas $\Omega = (1-\alpha-\beta)\bar{R}$ and α and β are always positive and their sum is less than one.

In order to analyze a potential benefit of VaR estimation with incorporated dynamic correlations, we also compared the model with the CCC model[6]. In contradiction to the setup of the DCC model, which can be seen as a generalisation of the CCC approach, the CCC model defines each pair of correlations between assets to be time invariant:

$$H_t = D_t R D_t.$$

Furthermore, the log likelihood is defined:

$$L = -\frac{1}{2} \sum_t (n\ log\ (2\pi) + 2\ log\left|D_t\right| + y_t' D_t^2 y_t - \epsilon_t' \epsilon_t + log\left|R_t\right| + \epsilon_t' R_t^{-1} \epsilon_t).$$

BEKK. Engle and Kroner (1995) introduced the BEKK approach which represents a parametrization of the multivariate GARCH process and allows for direct interaction between lagged volatilities of all assets. The conditional covariance of BEKK(1,1,K) model is given by :

$$H_t = C`C + \sum_{k=1}^{K} A_k \epsilon_{t-1} \epsilon`_{t-1} A_k + \sum_{k=1}^{K} G`_k H_{t-1} G_k.$$

Where C,A_k and G_k are NxN matrices but C is upper triangular and the summation limit K determines the generality of the process. Due to the proposed parametrization, the parameters to be estimated reduce to $N(5N+1)/2$ for the applied BEKK(1,1,1).

[6]Results are available upon request.

Copula. The copula approach is based on Sklar's Theorem (1959):
Let $X_1, ...X_n$ be random variables, $F_1, ..., F_n$ the corresponding marginal distributions and H the joint distribution, then there exists a copula C: $[0,1]^n \rightarrow [0,1]$ such that:

$$H(x_1, ..., x_n) = C(F_1(x_1), ...F_n(x_n)).$$

Conversely if C is a copula and $F_1, ..., F_n$ are distribution functions, then H (as defined above) is a joint distribution with margins $F_1, ...F_n$.

The Gaussian and t copula belong to the family of elliptical copulas and are derived from the multivariate normal and t distribution respectively.

The setup of the Gaussian copula is given by:

$$C^{Ga}(x_1, ..., x_n) = \Phi_\rho(\Phi^{-1}(x_1), ..., \Phi^{-1}(x_n)),$$

$$= \int_{-\infty}^{\Phi^{-1}(x_1)} ... \int_{-\infty}^{\Phi^{-1}(x_n)} \frac{1}{2(\pi)^{\frac{n}{2}} |\rho|^{\frac{1}{2}}} exp\left(-\frac{1}{2} z^T \rho^{-1} z\right) dz_1...dz_n$$

whereas Φ_ρ stands for the multivariate normal distribution with correlation matrix ρ and Φ^{-1} symbolizes the inverse of univariate normal distribution.

Along the lines of the Gaussian copula, the t copula is given by:

$$C^t(x_1, ..., x_n) = t_{\rho,v}(t_v^{-1}(x_1), ..., , t_v^{-1}(x_n)),$$

$$= \int_{-\infty}^{t^{-1}(x_1)} ... \int_{-\infty}^{t^{-1}(x_n)} \frac{\Gamma\left(\frac{v+n}{2}\right)}{\Gamma\left(\frac{v}{2}\right)(v\pi)^{\frac{n}{2}} |\rho|^{\frac{1}{2}}} \left(1 + \frac{1}{v} z^T \rho^{-1} z\right)^{-\frac{v+n}{2}} dz_1...dz_n,$$

in this setup $t_{\rho,v}$ stands for the multivariate t distribution with correlation matrix ρ and v degrees of freedom (d.o.f.). t_v^{-1} stands for the inverse of the univariate t distribution and v influences tail dependency. For $v \rightarrow \infty$ the t distribution approximates a Gaussian.

Due to the fact that estimating parameters for higher order copulas might be computationally cumbersome, all parameters are estimated in a two step maximum likelihood method given by Joe and Xu (1996)[7]. The two steps divide the log likelihood into one term incorporating all parameters concerning univariate margins and into one term involving the parameters of the chosen copula.

VaR. VaR is defined as the quantile at level α of the distribution of portfolio returns:

$$VaR_\alpha = F^{-1}(\alpha) = \int_{-\infty}^{VaR_\alpha} f(r)dr = P(r \leq VaR_\alpha).$$

Given the parametric approach, quantiles are direct functions of the variances and we can directly translate the quantiles of the estimated portfolio variances into VaR. Let α be the quantile, VaR at time t is given by: $VaR_t = -\alpha\sqrt{H_t}$ for both normal and t distributions.[8]

[7]This approach is also known as inference for the margins (IFM).

[8]E.g. the 99% VaR of PF return y_t represents the empirical 1% quantile of the variance.

In the context of copulas, the estimated VaR at time $t+1$ is simply the empirical quantile of the vector of simulated portfolio returns based on the information available at time t.

A rolling window approach is applied to forecast the one-day ahead VaR thresholds based on the given dependence. The rolling window size is at 500 observations for all data sets and 10.000 scenarios are simulated for each day.

Backtesting. In order to evaluate the different forecasting techniques we apply regulatory Basel II criteria[9] as well as statistical backtesting. More concrete, in addition to the absolute amount of misspecifications, unconditional coverage (UC), independence (IND) and conditional coverage (CC) are applied. The unconditional coverage test, proposed by Kupiec (1995) checks if the expected failure rate of a VaR model is statistically different from its realized failure rate. Therefore Kupiec proposed the following setup:

$$LR_{UC} = -2ln[(1-p)^{T-N}p^N] + 2ln[(1-(N/T))T - N(N/T)^N],$$

where p stands for the percent left tail level, T for the total days and N for the number of misspecifications. LR_{UC} follows a $\chi^2(1)$ distribution. Due to the fact that the UC method exclusively tests the equality between VaR violations and the chosen confidence level, Christoffersen (1998) developed a likelihood ratio statistic to test whether the VaR misspecifications are correlated in time. Let T_{ij} be the number of observed values i followed by j. Whereas 1 represents a misspecification and 0 a correct estimation. π represents the probability of observing an exception and π_i the probability of observing an exception conditional on state i. The likelihood ratio is defined as:

$$LR_{IND} = -2ln[(1-\pi)^{(T_{00}+T_{01})}\pi^{(T_{01}+T_{11})}] + 2ln[(1-\pi_0)^{T_{00}}\pi_0^{T_{01}}(1-\pi_1)^{T_{10}}\pi_1^{T_{11}}].$$

Thus, this approach rejects a model that either creates too many or too few clustered VaR violations.

The CC test combines both test statistics with the following likelihood ratio statistic:

$$LR_{CC} = LR_{UC} + LR_{IND}$$

Each statistic is $\chi^2(1)$ distributed whereas their sum is distributed as $\chi^2(2)$. With 95 % confidence level, the critical value of $\chi^2(1)$ for UC and IND is 3,84 and of $\chi^2(2)$ for CC 5,99 respectively.

3. Data

Four different portfolios are investigated: Two portfolios comprising national stock indices, one currency portfolio and one portfolio of individual German stocks.

[9]For 99% VaR forecasts, the Basel II document allows for 4 misspecifications within 250 days. If a model results in 5-9 misspecifications a penalty multiplicator is used to compensate the lack of quality. Consequently, more than 10 misspecifications lead to a rejection of the model. The numbers are simply extrapolated in order to backtest the investigated samples consisting of 1000 observations.

The examined data of Portfolio I and II are the daily closing prices of five international stock indices, that is to say, CAC, DAX, IBEX, MIB and PSI[10] and AEX, DAX, ISEQ, MIB and Nikkei[11] respectively. Portfolio III contains five German stocks listed in the DAX: BASF, Daimler, Deutsche Bank, EON and Lufthansa (each asset represents another business sector). Portfolio IV includes foreign exchange rates and consists of CHF, CZK, GBP, NOK and USD[12], whereas all currencies are denominated against the EURO.

Data on all portfolios cover the 1500 day period from the beginning of 2004 until the end 2009. Using a rolling window of 500 observations leads to a set of 1000 successive daily (one step) VaR forecasts for every portfolio in the crisis period. For illustration purposes, we also present Portfolio I in the pre-crisis period covering 2001 - 2005 (1000 observations)[13].

4. Results

In times of financial crises, applying heavy-tailed return distributions for VaR-estimation can be assumed to lead to more reliable results than, say, normally distributed return models. We corroborate this assumption by a preliminary analysis of the portfolios in the "previous" sample covering the calm period 2001 to 2005.

VaR	**Distribution**	**Model**	**Basel**	LR_{CC}
VaR 99%	Normal	DCC	12	**0,39**
VaR 99%	Normal	BEKK	10	**2,97**
VaR 99%	Normal	G-Cop	13	**0,83**
VaR 99%	Normal	t-Cop	12	**0,39**
VaR 99%	t dist	DCC	2	9,65
VaR 99%	t dist	BEKK	1	13,40
VaR 99%	t dist	G-Cop	2	9,65
VaR 99%	t dist	t-Cop	2	9,65
VaR 99%	EVT	G-Cop	7	**5,39**
VaR 99%	EVT	t-Cop	6	6,89

Table 1. Backtesting: PF I 2001-2005 (1000 observations).

Table 1 reports the backtesting results of the 99% VaR estimates for PF I in calm market times with the assumption of normally, t(5) and GPD distributed returns. For sake of clarity, simply the absolute amount of misspecifications (Basel) and the statistics of conditional coverage (LR_{CC})[14] are presented.

[10]CAC = France, DAX = Germany, IBEX = Spain, MIB = Italy and PSI = Portugal.

[11]AEX = Netherlands, DAX = Germany, ISEQ = Ireland, MIB = Italy and Nikkei = Japan.

[12]CHF = Swiss franc, CZK = Czech koruna, GBP = Pound sterling, NOK = Norwegian krone and USD = US dollar.

[13]Results for Porfolios II, III and IV covering 2001-2005 are available upon request

[14]The conditional coverage includes the unconditional coverage and independence teststatistic

Given the forecasting range from 2001 to 2005, 99% VaR estimates with the assumption of normally distributed returns outperform the VaR estimates with t(5) distributed returns. GPD distributed returns do also lead to an adequate number of absolute misspecification whereas the test statistics, caused by two clustered misspecifications, are close to be rejected.

With focus on the dependence structure, we find that the differences between the variable and invariant approaches are marginal regarding the total number of VaR misspecifications and consequently do not differ in terms of Basel II backtesting. Basically DCC, BEKK and copulas show similar results concerning CC statistics. However, the assumption of normally distributed returns seems to be adequate for the analyzed portfolio for the given calm period throughout all models.

The time from 2005-2009 represents the balance point of our investigation, since it is characterized by a higher return volatility than the time from 2001-2005.

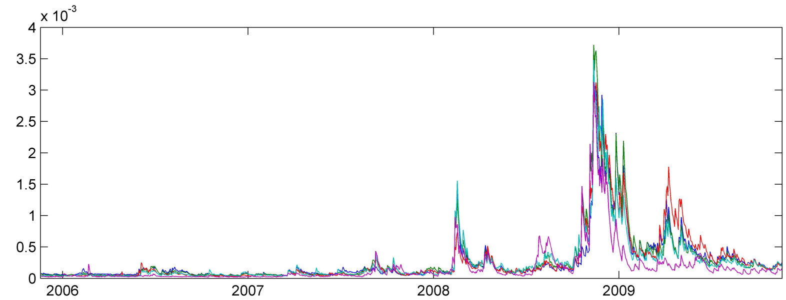

Figure 1. PF I: Estimated GARCH volatility (2005-2009).

Figure 1 plots the estimated GARCH variance for the five indices included in the analysed portfolio I [15]. The graphs show that the volatility of the portfolio returns significantly increased after 15th September 2008.

VaR	**Distribution**	**Model**	**Basel**	LR_{CC}
VaR 99%	Normal	DCC	25	16,24
VaR 99%	Normal	BEKK	25	16,24
VaR 99%	Normal	G-Cop	24	14,49
VaR 99%	Normal	t-Cop	24	14,49
VaR 99%	t dist	DCC	**5**	**3,09**
VaR 99%	t dist	BEKK	**4**	**4,72**
VaR 99%	t dist	G-Cop	**5**	**3,09**
VaR 99%	t dist	t-Cop	**4**	**4,72**
VaR 99%	EVT	G-Cop	**14**	**1,44**
VaR 99%	EVT	t-Cop	**14**	**1,44**

Table 2. Backtesting: PF I 2005-2009 (1000 observations).

[15] Graphs for PF II, III and IV can be found in the Appendix.

Table 2 reports the backtesting results of the 99% VaR estimations for PF I in the time from 2005-2009. Unlike the results for 2001-2005, the VaR estimators for the subsample covering financial turmoil, based on the assumption of t(5) distributed returns outperform the forecasts based on normal distribution. In the period from 2005-2009, which is characterized by higher return volatility than in the period from 2001-2005, all VaR estimators based on the normal distribution fail the statistical backtesting criteria due to too many VaR violations.

On account of the fatter tails of the t distribution, the VaR estimations result in less misspecifications. Moreover, total amounts of misspecifications are in line with the Basel II backtesting criteria. In addition to the comparison of elliptical distributions, copulas in combination with GPD distributed returns do also pass the applied backtesting criteria. The different dependence structures, however, do not contribute to better VaR forecasts.

The same result for the time of 2005-2009 is observed for portfolio II (See ***Table 3***). The t and EVT distribution outperform the normal distribution. Due to its fatter tails the t distribution results in less VaR misspecifications and hence in more consistent backtesting results.

VaR	**Distribution**	**Model**	**Basel**	LR_{CC}
VaR 99%	Normal	DCC	23	12,84
VaR 99%	Normal	BEKK	25	16,24
VaR 99%	Normal	G-Cop	27	20,03
VaR 99%	Normal	t-Cop	23	12,84
VaR 99%	t dist	DCC	4	**4,71**
VaR 99%	t dist	BEKK	3	6,83
VaR 99%	t dist	G-Cop	6	**1,87**
VaR 99%	t dist	t-Cop	5	8,93
VaR 99%	EVT	G-Cop	13	**0,83**
VaR 99%	EVT	t-Cop	11	**0,09**

Table 3. Backtesting: PF II 2005-2009 (1000 observations).

The total number of VaR misspecifications differs vaguely between variable and invariant dependencies. All VaR estimates based on the normal distribution get rejected by CC for all scrutinies. In contrast to Gaussian copula and DCC, the BEKK and t-copula based on elliptical t distribution fail the CC backtesting for the 99% VaR forecasts. Anyhow, the absolute amount of misspecifications differs only slightly and therefore seems to be of less importance regarding Basel II backtesting.

Table 4 gives the VaR backtesting results regarding stocks, namely PF III. Simply comparing the absolute amount of VaR misspecifications, forecasts based on the assumption of GPD show the smallest deviations from the expected amount of misspecifications. This is also indicated by the applied teststatistics. The Gaussian distribution delivers too many VaR misspecifications whereas, however, the t distribution leads to too conservative estimators for all dependency models. Despite of the BEKK model, which results in slightly different misspecifications, all other models are accepted by statistical backtesting. In contrast to the results regarding PF I and PF II, there is no distribution which categorically outperforms

VaR	Distribution	Model	Basel	LR_{CC}
VaR 99%	Normal	DCC	18	**4,49**
VaR 99%	Normal	BEKK	20	8,64
VaR 99%	Normal	G-Cop	18	**5,22**
VaR 99%	Normal	t-Cop	17	**4,09**
VaR 99%	t dist	DCC	4	**4,71**
VaR 99%	t dist	BEKK	3	6,83
VaR 99%	t dist	G-Cop	4	**4,71**
VaR 99%	t dist	t-Cop	4	**4,71**
VaR 99%	EVT	G-Cop	14	**1,44**
VaR 99%	EVT	t-Cop	12	**0,38**

TABLE 4. Backtesting: PF III 2005-2009 (1000 observations).

the other one in terms of statistical backtesting results. From a Basel II perspective, the normal distribution does not breach the maximum amount of allowed VaR violations but results in a compensation via penalty multiplicator and is therefore outperformed by the t distribution.

Table 5 shows the results of portfolio IV. In contradiction to the results of the other portfolios, the normal distribution outperforms the t distribution in terms of backtesting quality. As to the portfolio consisting of currencies, the assumption of t distributed returns results in immoderately conservative VaR estimators and therefore inadequately less misspecifications. The total amounts of misspecifications based on assumed normal distributed returns are more than twice as high as the misspecifications based on t distributed returns and consequently the t distribution gets rejected by CC. Unaffected by the characteristics of FX-rates, the GPD distribution also passes the backtesting criteria for the 99% VaR. According to the results of the other portfolios, the total amount of misspecifications based on the DCC, BEKK and copula approach differs slightly.

VaR	Distribution	Model	Basel	LR_{CC}
VaR 99%	Normal	DCC	12	**2,67**
VaR 99%	Normal	BEKK	12	**2,67**
VaR 99%	Normal	G-Cop	12	**2,67**
VaR 99%	Normal	t-Cop	10	**2,97**
VaR 99%	t dist	DCC	2	9,63
VaR 99%	t dist	BEKK	4	11,55
VaR 99%	t dist	G-Cop	3	6,83
VaR 99%	t dist	t-Cop	1	13,48
VaR 99%	EVT	G-Cop	11	**0,09**
VaR 99%	EVT	t-Cop	10	**0,00**

TABLE 5. Backtesting: PF IV 2005-2009 (1000 observations).

To sum it up, our findings indicate that the incorporation of dynamic conditional correlations and spillover might only be of little help for VaR estimations. The dynamic correlation coefficients do not outperform the volatility spillover with respect to the chosen regulatory and statistical VaR backtesting criteria.

Moreover, based on elliptical distributions and within the given VaR backtesting setup, the variable and invariant approach appear to perform in the same way. Within the investigated subsamples the total amount of VaR misspecifications stemming from either BEKK or DCC are equal or faintly different than the number of misspecifications stemming from elliptical copulas. However, the differences are marginal and the bilateral rejection rate based on both the regulatory and statistical backtesting is similar.

Due to this result, it gives the impression that the adequate choice of univariate margins is more crucial for estimating VaR than the incorporation of dynamic conditional correlations or volatility spillover. This result is underlined by the backtesting performance of GPD margins.

Even though, the invariant approach based on elliptical margins does not outperform dynamic conditional correlations and spillover, the facility for using elliptical copulas with univariate extreme value distributions leads to a setup which outperforms DCC and BEKK models, since it results in an adequate 99% VaR performance throughout all asset classes in turmoil market times .

5. Conclusion

We illustrate that both variable and invariant dependency structures lead to acceptable VaR results for higher dimensional portfolios. Consequently, copulas represent an interesting alternative to the explicit parametric modeling of the dynamics of conditional correlations and volatility spillover in the context of financial risk measurement.

As to the estimation of VaR in volatile market times, we come to the result that assumptions concerning the distribution of returns prove to be crucial. VaR forecasts based on EVT distributions outperformed normal and t distributed returns since only this approach exclusively leads to adequate VaR estimates throughout all analysed asset classes (German stocks, market indices and FX-rates).

Regarding the modeling of invariant interdependencies, our results contradict the findings of Huang et al. (2009), as the backtesting performance of conditional Gaussian and t copulas based on t-GARCH tends to result in a similar amount of VaR misspecifications. Whereas the results related to variable dependence of Hakim et al. (2007) can be confirmed, the usage of DCC and also BEKK model seems to be of limited use for VaR estimations.

The current findings add substantially to our understanding of modeling VaR for multivariate portfolios. As a conclusion, the choice of the univariate margin has stronger impact on the quality of VaR forecasts than the choice of dependence structure modeled by dynamic conditional correlations, volatility spillover or elliptical copulas.

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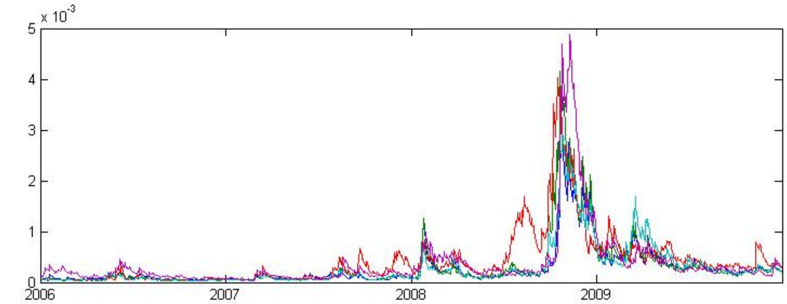

Figure 2. PF II: Estimated GARCH volatility (2005-2009).

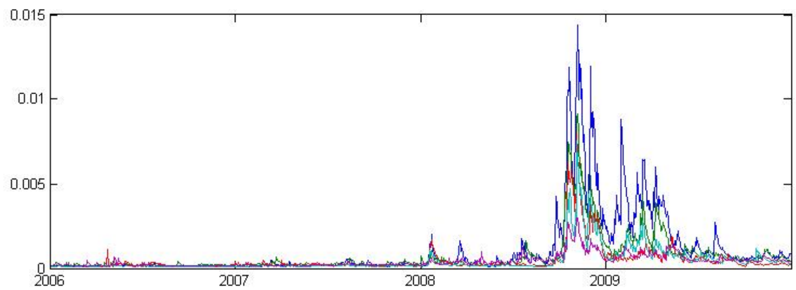

Figure 3. PF III: Estimated GARCH volatility (2005-2009).

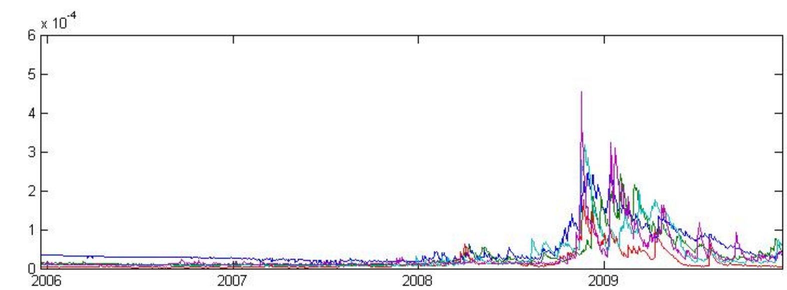

Figure 4. PF IV: Estimated GARCH volatility (2005-2009).