# On the Pricing of Performance Sensitive Debt*

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**Abstract**

Performance sensitive debt (PSD) contracts link a loan's interest rate to the borrower's measure of credit relevant firm performance, e.g., if the borrower becomes less credit worthy, the interest rate increases according to a predetermined schedule. PSD provisions are included in approximately 35% of all U.S. and Canadian corporate loans (1993 - 2010, Thomson Reuters Dealscan). Based on financial valuation theory and observed contractual specifications, we derive and empirically test a new pricing model for PSD contracts where the underlying credit performance measure is the ratio between cashflow and debt. The model includes instantaneous cashflow risk and leverage risk represented by jumps. Our sample consists of 3,052 PSD loans. We separately consider

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the subsamples of *interest increasing* and *interest decreasing* PSD contracts. According to our model, the market prices interest decreasing contracts as if no leverage risk is present. Interest increasing contracts are priced consistent with a substantial leverage risk (risk-adjusted jump intensity of 0.4). Borrowers using interest increasing contracts are more profitable, have lower cashflow volatility, lower initial distance-to-default, and lower leverage, compared to the interest decreasing subsample. Thus, borrowers in the two subsamples are of distinctly different credit quality and have different capacities of future increased leverage, supporting our results. These results also support and refine the signaling hypothesis in Manso, Strulovici, & Tchistyi (2010), by suggesting that high quality borrowers signal quality through the use of interest increasing PSD loans. Analyzing pricing errors, we confirm the findings of Eom, Helwege & Huang (2004) that continuous structural debt models tend to overprice debt issued by less risky borrowers, a problem which is reduced if the underlying process of the state variable includes jumps.

JEL Codes: G12, G13, G32

# 1 Introduction

Performance sensitive debt (PSD) contracts link the interest rate paid on a firm's loan to a measure of its credit relevant performance over time. The two most common categories of firm credit performance measures are cash flow ratios and credit ratings. Since the mid 1990's performance sensitive provisions in both private and public corporate loans are common. Using Thomson Reuter's Dealscan database for the years 1993 - 2010 we find that PSD loans constitute 11.2% of the total number of loans in the database and 35.1% of loans granted in the U.S. and Canada. Market participants indicate that more than half of recently issued syndicated bank loans in Europe include such provisions. Based on financial valuation theory, using results from Mjøs & Persson (2010) and observed contractual specifications, we propose a valuation model to price PSD contracts with a cash flow and leverage based performance measure. Cash flows are assumed lognormal, whereas changes in debt are modeled using jumps. In the special case of no jumps in the performance measure, we derive a closed form valuation formula. We also compare the theoretical market price of a PSD contract with the par value of the loan at time of issue[1], and analyze model prices for alternative jump intensity specifications. We, in particular, study two important subclasses of PSD contracts; *interest increasing* and *interest decreasing* PSD contracts. The first category includes loan contracts where the borrower initially pays the lowest contractual interest rate, and where the interest rate increases if the borrower's performance measure deteriorates. The second category includes loan contracts where the borrower pays the highest contractual interest rate initially, and where the interest rate decreases if the borrower's performance measure improves.

[1]Our analysis does not take into account potential administrative and regulatory related costs incurred by the lender and, thus, assumes that the initial market value equals par value of the loan.

Our paper is related to the article by Manso, Strulovici, & Tchistyi (2010). Using the framework of Leland (1994), they develop a general pricing model for a broad class of performance sensitive debt. They derive closed form solutions for the market value of infinite horizon, linear, asset-based PSD contracts and step-up PSD contracts, respectively. A linear PSD contract has interest payments of the form $C(x) = \beta_0 - \beta_1 x$, where $x$ is some credit relevant performance measure, $\beta_0$ and $\beta_1$ are constants, and $\beta_0 > 0$.

The essential property of a step-up/-down PSD contract, compared to a linear PSD contract, is that the interest rate is constant within a certain range of the credit performance measure. As such, step-up/step-down loans are easier to implement, and actually observed in markets, in contrast to linear PSD contracts, which imply continuously changing interest rates for a continuously changing performance measures.

Manso, Strulovici, & Tchistyi (2010) show that with no other market imperfections than bankruptcy costs and tax benefits of debt, the use of PSD contracts leads to earlier default and lower equity value compared to comparable fixed-rate debt, and therefore find the use of these contracts not optimal. Consequently, they develop a screening model where the company can choose to issue either performance sensitive debt or fixed rate debt. They find that the existence of PSD contracts can be explained by the contracts' ability to mitigate adverse selection problems for the borrower. This conclusion is supported by an empirical analysis which shows that firms using performance sensitive debt are more likely to get improved credit ratings in the future compared to firms that choose ordinary fixed-interest loans. They do not empirically test their pricing models for PSD contracts, nor do they study interest increasing or interest decreasing contracts separately. Asquith, Beatty, &

Weber (2005) do study these two categories separately, and find indications that interest decreasing PSD contracts are used when adverse selection costs are high.

We focus on the pricing of PSD contracts, and our model of a step-up/-down PSD contract differs from Manso et al. (2010) in the following ways. We analyze contracts with finite maturity, and derive a closed form solution in the special case of no jumps. Many PSD contracts include a covenant specifying when the borrower defaults on the contract. We randomly pick 50 loan contracts in our sample and manually review the terms of each contract. We find that all of these contracts have cross-default clauses, and this interpretation was confirmed by lawyers. Cross-default implies that a default of a single debt contract leads to default on the company's other debt contracts. It is, thus, reasonable to assume that default only occurs when all possible refinancing options are exhausted and the company may be expected to approach bankruptcy. This contractual default is exogenous as opposed to the optimal endogenous company liquidation-trigger analyzed in Manso et al. (2010). A possible interpretation is that the contractual default level is agreed between the company and its lender as *de facto* bankruptcy level. We only analyze PSD contracts where the performance measure is based on total firm cash flow and leverage. This assumption excludes, e.g., rating based contracts. Market evidence indicates that cash flow and debt based performance measures, which our model covers, are the most common performance measures in such PSD contracts (See Table 1, section 4.1). In addition, we separately consider both interest increasing PSD contracts, interest decreasing PSD contracts, and contracts with both provisions. Our empirical results improve the understanding of the signaling hypothesis in Manso et al. (2010). We find that interest increasing PSD contracts are priced consistent with a substantial amount of leverage risk, i.e., the risk that the borrower increases the total amount of debt before the maturity of the PSD contract.

Interest decreasing PSD contracts are priced as if no leverage risk is present. Comparing borrower characteristics we also find that borrowers using interest increasing PSD contracts are of an overall higher credit quality compared to borrowers using interest decreasing PSD contracts. These results indicate that borrowers may signal quality through the use of interest increasing PSD loans. Low quality borrowers, which are more credit constrained, do not mimic this behavior since they would have to incur the costs related to the lenders rational assessment of future leverage risk. Contrary to the findings of Asquith et al. (2005), we document that interest decreasing PSD contracts cannot be used for signaling purposes since these contracts do not induce a cost for low quality borrowers.

Several papers have empirically tested the ability of structural debt models to produce correct prices and/or spreads, e.g, Eom, Helwege, & Huang (2004), Huang & Huang (2003). Our paper contributes to this literature by empirically testing the pricing model using 3,052 loan contracts obtained from Thomson Reuter's Dealscan database. Our sample includes both interest increasing PSD contracts and interest decreasing PSD contracts, as well as PSD contracts including both provisions. Our empirical analysis confirms the initial overpricing in situations with large distance-to-default as found in earlier literature. We show that including future leverage risk in the performance measure reduces this problem.

The remainder of the paper is structured as follows. Section 2 discusses related literature. Section 3 presents the general economic set-up and some details of the theoretical pricing model. Section 4 provides a brief description of the market for PSD loans, as well as a description of the data we use. Section 5 includes an example of a PSD contract and calculates its theoretical price in the case of no jumps. In Section 6 we present and discuss empirical results for the whole sample.

Section 7 concludes. Technical calculations and supplementary descriptive statistics are collected in three appendices.

## 2 Related Literature

Our paper is related to the broad literature on credit risk, and especially to the part of the literature focusing on pricing performance sensitive debt. Credit risk is defined as the risk that a borrower will not honor his contractual obligations with the lender. There are two dominating approaches to model credit risk in the finance literature; *structural models* and *reduced form models*. Structural models view debt and equity as contingent claims on total firm value, and hence these claims could be valued using option pricing techniques. This approach was pioneered by Merton (1974) and further developed by, e.g., Black & Cox (1976), Geske (1977), Longstaff & Schwartz (1995) and Leland (1994). Reduced form models assume credit risk are modeled by introducing a default arrival intensity. This approach was pioneered by Jarrow & Turnbull (1992) and further developed by, e.g., Jarrow & Turnbull (1995) and Jarrow, Lando, & Turnbull (1997). In the recent years researchers have successfully merged the two modeling approaches by introducing alternative information filtrations and jumps in structural debt models. In short, reduced form models can be viewed as structural models with incomplete information or with jumps in the underlying asset. Duffie & Lando (2001) were the first to introduce incomplete information. Further advances have been made by, e.g., Collin-Dufresne, Goldstein, & Helwege (2003), Jarrow & Protter (2004), Coculesco, Geman, & Jeanblanc (2008), Guo, Jarrow, & Zeng (2009), and Lindset, Lund, & Persson (2011). The notion that jump risk might be important for pricing purposes was introduced by Merton (1976).

Both structural credit risk models and reduced form models have been successfully applied for the purpose of pricing performance sensitive debt contracts. Lando & Mortensen (2004) and Houweling, Mentink, & Vorst (2004) use the latter framework to develop pricing models for rating based PSD. Similarly, Bhanot & Mello (2006) and Koziol & Lawrenz (2010) develop structural pricing models for rating-based PSD contracts. Manso, Strulovici, & Tchistyi (2010) also use the structural model framework to derive theoretical models that could be used for pricing more general PSD contracts.

Our paper is both related to the general literature on performance sensitive debt and to the literature of empirical tests of structural credit risk models.

The existing literature on performance sensitive debt has mainly been focused on explaining the existence of these contracts. Assuming positive bankruptcy costs, performance sensitive debt contracts, at least at a first glance, seem inefficient. Whilst increased interest payments in bad states of the world may have an *ex ante* disciplining effect, the *ex post* conditional probability of bankruptcy increases, and a PSD contract destroys, rather than adds, firm value. In the light of this intuition, existing research on PSD has mainly focused on the efficiency and existence of these contracts. Regarding the existence, the problem of potential information asymmetry as pointed out in the seminal work of Myers & Majluf (1984), and the problem of agency costs, identified by Jensen & Meckling (1976), both may be important explanations.

Tchistyi (2006) studies optimal security design in a dynamic setting where the agency problem arises from the assumption that a manager in charge of a project could divert cash flows for his own consumption. Allowing cash flows to be correlated over time, he finds that the optimal contract could be implemented using a credit line with performance sensitive provisions.

Asquith, Beatty, & Weber (2005) make an important contribution to the understanding of performance sensitive bank debt. In addition to information asymmetry and agency costs, they claim that the existence of renegotiation costs provides another rationale for using PSD contracts. The authors find empirical evidence that performance sensitive debt is used when it has the largest net benefits, i.e., when moral hazard, adverse selection problems, or renegotiation costs are likely to be high. More specifically they find that interest decreasing PSD contracts are used when prepayment of the loan is more likely, i.e., when borrowers' relative bargaining power is assumed to be high and when costs related to adverse selection are large. They also find that interest increasing PSD contracts are used when moral hazard costs are high and that including interest increasing provisions in the debt contract has significant economic effects since, controlling for firm characteristics, borrowers are offered 26 basis points initial lower credit margins (over LIBOR) when these provisions are included in the contract.

Other important contributions to the PSD literature includes Roberts & Sufi (2009) on debt renegotiations and Tchistyi, Yermack, & Yun (2010) on CEO equity incentives.

Several papers have made empirical tests of different structural models of credit risk, focusing on the models' ability to replicate observed market prices and yield spreads. The evidence is mixed. Jones, Mason, & Rosenfeld (1984) find that predicted prices are, on average, 4.5% too high, and that the pricing error is largest for speculative-grade firms. More recently, Eom et al. (2004) compare five different models, and find that predicted spreads from some are too high, whereas some models generate too low spreads. Huang & Huang (2003) also test several different models. They use a calibration approach based on historical data, and find that credit risk accounts for only a small fraction of observed corporate yield spreads for investment grade bonds, but accounts for a larger share of high-yield bond spreads. They also find that different structural models predict fairly similar yield spreads. Our results are consistent with these findings.

# 3 The economic model

## 3.1 General set-up

This section reviews the general set-up and the main results needed for our pricing model. A filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, Q)$ is given. In particular, $Q$ represents a fixed equivalent martingale measure. We impose the standard frictionless, continuous time market assumptions, see, e.g., Duffie (2001).

Let $W_t$ be a standard Brownian motion under the equivalent martingale measure $Q$. We assume throughout that the time $t$ firm cash flow rate $\zeta_t$ under the equivalent martingale measure $Q$ is given by the stochastic differential equation

$$d\zeta_t = \mu\zeta_t dt + \sigma\zeta_t dW_t, \tag{1}$$

where the initial value $\zeta_0$ is a constant. Here the drift parameter $\mu$ and the diffusion parameter $\sigma$ are constants. Denoting the constant risk-free interest rate by $r$, where $r > \mu$, the time $t$ value of the firm's assets $A_t$ equals the risk-adjusted expected discounted value of all future cash flows

$$A_t = E_t^Q \left[ \int_t^\infty e^{-r(s-t)} \zeta_s ds \right] = \frac{\zeta_t}{r-\mu}, \tag{2}$$

where $E_t^Q[\cdot]$ denotes the expectation under the equivalent martingale measure $Q$ conditional on $\mathcal{F}_t$, the information available at time $t$. Hence, the market value of the firm's assets is given by expression (3) divided by the constant $(r-\mu)$. In particular, $A_0 = \frac{\zeta_0}{r-\mu}$.

Let $N_t$ denote a Poisson process with constant intensity $\lambda$ under the equivalent martingale measure $Q$. Let $\{Y_i\}, i \geq 1$, be a sequence of independent and identically distributed random variables under $Q$, independent of both $N_t$ and $W_t$. We assume that the firm is partly financed by debt and that the total book value of debt at time $t$ is

$$D_t = D_0 \prod_{i=1}^{N_t} Y_i,$$

where $D_t = D_0$ when $N_t = 0$. Here $Y_i$ can be interpreted as the change in debt due to jump $i$ relative to the amount of debt just before jump $i$. Realized values of $Y_i > 1$ implies an increase in the amount of debt, and realized values of $Y_i < 1$ implies a decrease in the amount of debt.

The state variable in our model is the ratio between the cash flow and debt. Define $\xi_t$ by

$$\xi_t = \frac{\zeta_t}{D_t} = \xi \exp\left( (\mu - \frac{1}{2}\sigma^2)t + \sigma W_t \right) \prod_{i=1}^{N_t} (Y_i)^{-1}, \tag{3}$$

where the initial value $\xi = \xi_0 = \frac{\zeta_0}{D_0}$ is a constant.

Let $T$ be the finite time horizon corresponding to the maturity of debt. Let the constant $C < \xi$ be an absorbing barrier, and define the stopping time $\tau$ (with respect to $\mathcal{F}_t$) as

$$\tau = \inf\{t \geq 0, \xi_t \leq C\}. \tag{4}$$

The constant $C$ can be interpreted as the contractual default barrier, and $\tau$ as the time of default.

### 3.2 A Valuation Model of a PSD Contract

This subsection explains the general structure of a PSD contract. In addition to the contractual default barrier $C$, a PSD contract includes $n+m$ constant levels or non-absorbing barriers $B_1, \ldots, B_{n+m}$ so that $B_1 > \cdots > B_{n+m} > C$. For notational simplicity only, we let $B_0 = \infty$ and $B_{n+m+1} = C$. We define $n$ by the initial value of the performance measure $\xi$ as $B_n > \xi > B_{n+1}$. That is, there are $n$ barriers above $\xi$. Similarly, $m$ represents the number of barriers below $\xi$. Observe that the contract is well defined in the cases where $n = 0$ and/or $m = 0$.

The contract specifies a sequence of interest rates, where $c_{i+1}$ is paid when $B_i > \xi_t > B_{i+1}$, $i = 0, ..., n+m$. All $c_i$s are assumed to be constants. An interest increasing contract is defined by $n = 0$ and $c_1 < c_2 < ... < c_m$, whereas an interest decreasing contract is defined by $m = 0$ and $c_1 < c_2 < ... < c_n$. See Figure 1 for an illustration.

The total time 0 market value of a PSD contract can be decomposed into the time 0 mar-

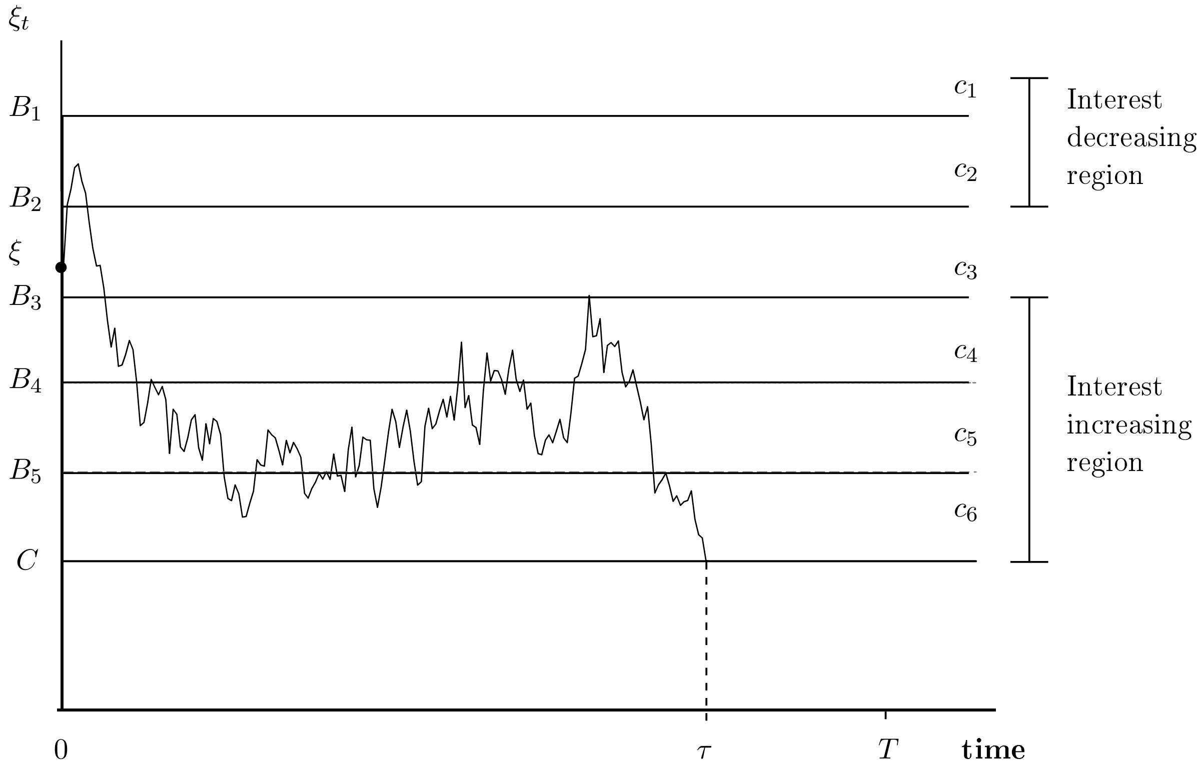


**Figure 1: PSD interest rate payments for an arbitrary performance measure development.** An illustration of the interest rate structure in a PSD contract. The graph contains an example of a path of the performance measure $\xi_t$ and indicates in which regions the interest rates are $c_1$,..., $c_6$ respectively. Also, $\xi$, $B_1$,...,$B_5$, $C$, $T$, and $\tau$ are depicted. $C$ denotes the contractual default barrier. The number of non-absorbing barriers above the starting level $\xi$ is $n = 2$, and the number of non-absorbing barriers below $\xi$ is $m = 3$. In order to make the two indicated regions interest decreasing and interest increasing, respectively, it is assumed that $c_1 < c_2 < ... < c_6$.

ket value of the interest payments and the time 0 market value of the repayment of the principal.

Let us first consider the market value of the interest payments. Consider the corridor $j$ defined by two adjacent barriers $B_j$ and $B_{j+1}$ for a fixed $j$. In this corridor the interest rate is $c_{j+1}$. The market value of interest payments in a time period $[0, T]$ from this corridor is

$$C_j(\xi) = E^Q \left[ \int_0^{\tau \wedge T} c_{j+1} e^{-rs} 1\{B_j > \xi_s > B_{j+1}\} ds \right],$$

where $E^Q[\cdot]$ denotes the expectation under the equivalent martingale measure $Q$. The total interest payments of the PSD contract can be seen as a portfolio of such corridors. To calculate the total time 0 market value of all interest payments from a PSD contract, we add the time 0 market values of the contract's corridors, i.e., $V(\xi) = \sum_{i=0}^{n+m} C_i(\xi)$ or

$$V(\xi) = E^Q \left[ \int_0^{\tau \wedge T} \sum_{i=0}^{n+m} c_{i+1} e^{-rs} 1\{B_i > \xi_s > B_{i+1}\} ds \right]. \tag{5}$$

The corridor decomposition in expression (5) is the basis for the subsequent simulation analysis of the interest payments in our sample of contracts.

We find it useful to further decompose a corridor by the use of above- and below annuities, see Mjøs & Persson (2010). A generic defaultable finite horizon *above* annuity pays the annuity rate of 1 when the performance measure is above some level $B$ until default or to the horizon, whatever comes first. Denote the time 0 market value of an above annuity by $\mathcal{A}(\xi, B)$, then

$$\mathcal{A}(\xi, B) = E^Q \left[ \int_0^{\tau \wedge T} e^{-rs} 1\{\xi_s > B\} ds \right].$$

A generic defaultable finite horizon *below* annuity pays the annuity rate of 1 when the performance measure is below some level $B$ until default or to the horizon, whatever comes first. Denote the time 0 market value of a below annuity by $\mathcal{B}(\xi, B)$, so

$$\mathcal{B}(\xi, B) = E^Q \left[ \int_0^{\tau \wedge T} e^{-rs} 1\{\xi_s < B\} ds \right].$$

Observe that for any $B$, $\mathcal{A}(\xi, B) + \mathcal{B}(\xi, B) = Z(\xi)$, where $Z(\xi)$ is the time 0 market price of finite horizon interest payments with rate identical to 1, i.e.,

$$Z(\xi) = E^Q \left[ \int_0^{\tau \wedge T} e^{-rs} ds \right].$$

A corridor $j$ can be decomposed in two equivalent ways. First, as a portfolio of a long defaultable above annuity with annuity payment $c_{j+1}$ and level $B_{j+1}$, and a short defaultable above annuity with annuity payment $c_{j+1}$ and level $B_j$. Second, as a portfolio of a long defaultable below annuity with annuity payment $c_{j+1}$ and level $B_j$, and a short defaultable below annuity with annuity payment $c_{j+1}$ and level $B_{j+1}$. The time 0 market value of corridor $j$, using above annuities, is

$$C_j(\xi) = (\mathcal{A}(\xi, B_{j+1}) - \mathcal{A}(\xi, B_j))c_{j+1}.$$

The time 0 market value of corridor $j$, using below annuities, is

$$C_j(\xi) = (\mathcal{B}(\xi, B_j) - \mathcal{B}(\xi, B_{j+1}))c_{j+1}.$$

Using above and below annuities, the total time 0 market value of all interest payments from a PSD contract can be expressed as

$$V(\xi) = c_{n+m+1} Z(\xi) - \sum_{i=1}^{m+n} \mathcal{A}(\xi, B_i)(c_{i+1} - c_i), \tag{6}$$

or

$$V(\xi) = c_1 Z(\xi) + \sum_{i=1}^{m+n} \mathcal{B}(\xi, B_i)(c_{i+1} - c_i), \tag{7}$$

*Proof.* Both these expressions follow from the relationship $V(\xi) = \sum_{i=0}^{n+m} C_i(\xi)$ and the above definitions of corridors, using above- and below annuities, respectively. Observe that $\mathcal{A}(\xi, B_0) = 0, \mathcal{A}(\xi, B_{m+n+1}) = Z(\xi), \mathcal{B}(\xi, B_0) = Z(\xi), \mathcal{B}(\xi, B_{n+m+1}) = 0$. □

These expressions may be interpreted in the following ways. Expression (6) suggests that, in principle, the borrower pays the highest interest rate $c_{n+m+1}$ throughout the term of the contract, but for each (additional) barrier away from the default level performance measure process is, the borrower is entitled to a lower interest rate. The interest rate discount is determined by the interest rate difference between each barrier and the time 0 market price of an above annuity. Alternatively, expression (7) suggests that, in principle, the borrower pays the lowest interest rate $c_1$ throughout the term of the contract, but for each barrier closer to the default level the performance measure process is, the borrower has to pay an increased interest rate. The additional interest rate is determined by the interest difference between each barrier and the time 0 market price of an below annuity. Although either of these interpretations may be used for any PSD contract of the type we consider, two special cases are worth emphasizing. An *interest decreasing* contract is characterized by the fact that the initial interest rate of the contract equals the highest possible interest rate. The first interpretation above, based on expression (6) seems more immediate for this contract. An *interest increasing* contract is characterized by the fact that the initial interest rate of the contract equals the lowest possible interest rate. The second interpretation above, based on expression (7) seems more immediate for this contract.

We now turn to the time 0 value of the repayment of the principal. First, let $Q(\xi) = Q(\tau > T)$ be the survival probability under the equivalent martingale measure $Q$. The time 0 market value of receiving the face value of debt ($D$) in the case of no default is, thus, $De^{-rT}Q(\xi)$. The time 0 market value of the recovery amount in the case of default is, similarly, $D(1-\kappa)H(\xi)$. The parameter $\kappa$ represents the debtholders' loss proportional to the face value of debt in case of contractual default. Here, $H(\xi) = E^Q[e^{-r\tau}1\{\tau \leq T\}]$ represents the time 0 value of one unit of currency paid upon default if default occurs before time $T$. The total time 0 value of a PSD contract is, thus,

$$L(\xi) = V(\xi) + De^{-rT}Q(\xi) + D(1-\kappa)H(\xi). \tag{8}$$

### 3.3 PSD Valuation Assuming No Jumps

In the special case of no jumps in the debt level $D_t$, i.e., $N_t \equiv 0$ and $\lambda = 0$, we can calculate a closed form formula for the present value of a PSD contract. In the following we use the superscript $c$ to denote closed form solutions. The closed form expression for $Q^c(\xi)$ and $H^c(\xi)$ are standard and can be found in Appendix A. In order to calculate $V(\xi)$ in the case with no jumps, our starting point is expression (6) or expression (7). Mjøs & Persson (2010) calculate closed form solutions for $\mathcal{A}(\xi, B)$ and $\mathcal{B}(\xi, B)$. These formulas depend on whether the initial value of the performance measure $\xi$ is above or below the barrier, $B$. We therefore write

$$V^c(\xi) = c_{n+m+1}Z^c(\xi) - \sum_{i=1}^{n} \mathcal{A}_b^c(\xi, B_i)(c_{i+1} - c_i) - \sum_{i=n+1}^{n+m} \mathcal{A}_a^c(\xi, B_i)(c_{i+1} - c_i), \tag{9}$$

where $\mathcal{A}_b^c(\xi, B_i)$ is the time 0 market value of an above annuity with barrier $B_i$ where $\xi < B_i$, and $\mathcal{A}_a^c(\xi, B_i)$ is the time 0 market value of an above annuity with barrier $B_i$ where $\xi > B_i$. The expressions for $\mathcal{A}_b^c(\xi, B_i)$ and $\mathcal{A}_a^c(\xi, B_i)$ are given in expressions (28) and (24) in Appendix A. The expression for $Z^c(\xi)$ is given in expression (21) in Appendix A. The similar expression based on below annuities is

$$V^c(\xi) = c_1 Z^c(\xi) + \sum_{i=1}^{n} \mathcal{B}_b^c(\xi, B_i)(c_{i+1} - c_i) + \sum_{i=n+1}^{n+m} \mathcal{B}_a^c(\xi, B_i)(c_{i+1} - c_i), \tag{10}$$

where $\mathcal{B}_b^c(\xi, B_i)$ is the time 0 market value of a below annuity with barrier $B_i$ where $\xi < B_i$, and $\mathcal{B}_a^c(\xi, B_i)$ is the time 0 market value of a below annuity with barrier $B_i$ where $\xi > B_i$. The expressions for $\mathcal{B}_b^c(\xi, B_i)$ and $\mathcal{B}_a^c(\xi, B_i)$ are given in expressions (31) and (29) in Appendix A. The closed form formula for the total value of the contract in the case of no jumps is, thus, given by

$$L^c(\xi) = V^c(\xi) + De^{-rT} Q^c(\xi) + D(1 - \kappa) H^c(\xi), \tag{11}$$

where $V^c(\xi)$ is given either by expression (9) or expression (10). Furthermore, $Q^c(\xi)$ and $H^c(\xi)$ are given by expressions (17), and (20), respectively.

# 4 Market and Data Description

This section gives an overview of the market for PSD contracts and describes the data we use to empirically test our pricing model.

## 4.1 Overview and Descriptive Statistics

The tables and statistics in this section describe the PSD contracts in the Thomson Reuter's Dealscan database. We have collected all available data for the years 1993 - 2010, resulting in a total of 218,204 loans. The database contains detailed information about the global commercial loan market, focusing primarily on corporate bank debt with longer maturities. The database provides information for both publicly traded and privately held debt[2]. The PSD part of the database includes 25,602 loans. The total outstanding principal of these PSD contracts is USD 9,900 bn. (25.6% of the total outstanding amount). One deal may consist of several loans, usually referred to as tranches. The data in Dealscan is reported by loans and, thus, all our data analysis is at loan level rather than deal level. Table 1 reports the use of different types of performance measures in PSD contracts. Total debt-to-cashflow and senior debt rating are the two most common performance measures in these contracts. In total, 51.3% of the PSD contracts are directly related to cash-flow (25.4% of loan amount), and could potentially be valued using our model.

| **Performance measure** | **Total number of deals** | **Total loan amount** |
|---|---|---|
| Total debt-to-cashflow | 47.6 % | 23.2 % |
| Senior debt rating | 25.8 % | 53.5 % |
| Leverage | 5.7 % | 3.9 % |
| Maturity | 4.2 % | 5.7 % |
| Senior debt-to-cash flow | 3.7 % | 2.2 % |
| Outstandings | 2.0 % | 3.4 % |
| Fixed charge coverage | 2.3 % | 0.7 % |
| Debt to tangible net worth | 2.0 % | 0.6 % |
| Interest Coverage | 2.5 % | 1.6 % |
| Debt Service Coverage Ratio | 0.8 % | 0.1 % |
| Other | 3.4 % | 5.1 % |

**Table 1:** This table shows the numbers of loans with different types of performance measures as a percentage of the total number of loans containing performance pricing provisions (N=25,602 loans), and as a percentage of the total amount issued (measured in USD). Datasource: Thomson Reuter's Dealscan database for the years 1993-2010.

Table 2 shows the distribution of such debt contracts according to some broadly defined financing purposes. PSD contracts specify a verifiable performance measure in order to trigger changes in interest rates. This fact may explain why such contracts are primarily used by firms for which audited financial reports are available. PSD contracts are seldom used for project financing. Table

[2]See Carey et al. (1998) for a more detailed description of the database.

| **Purpose** | **Total number of loans** | **Total loan amount** |
|---|---|---|
| Acquisition-related | 21.5 % | 26.7 % |
| Refinancing | 23.7 % | 16.9 % |
| Working Capital | 18.5 % | 11.5 % |
| Project Finance | 1.6 % | 1.3 % |
| All Other | 34.7 % | 52.0 % |

**Table 2:** This table shows the purpose of a issued performance sensitive loans as a percentage of the total number of issued loans containing performance sensitive provisions (N=25,602), as well as a percentage of the total amount issued (measured in USD). Datasource: Thomson Reuter's Dealscan database for the years 1993-2010.

3 shows the existence of a credit rating for 50.3% of PSD-borrowers compared to 13.7 % of other borrowers, probably explained by the use of credit rating as a performance measure. Of all PSD-borrowers, 29.8 % are rated below investment grade compared to 7.7 % of other borrowers. In addition, we find that with negligible exceptions, all performance sensitive loans are senior (99.9%). Also, 57.7% of the loans are secured, whereas 23.7% are unsecured (information regarding security is not available for the remaining 20%.). Table 4 shows that USA and Canada alone account for

| **Rating Category** | **PSD contracts** | **Non-PSD contracts** |
|---|---|---|
| AAA/Aaa | 0.3 % | 0.4% |
| AA/Aa | 0.9 % | 0.7% |
| A/A | 5.8 % | 2.2% |
| BBB/Baa | 13.5 % | 2.7% |
| BB/Ba | 13.6 % | 3.1% |
| B/B | 13.2 % | 4.1% |
| CCC/Caa | 1.0 % | 0.5% |
| CC/Ca | 0.0 % | 0.0% |
| C /C | 0.0 % | 0.0% |
| Sum Investment Grade | 20.5 % | 6.0 % |
| Sum Non-Investment Grade | 29.8 % | 7.7 % |
| Not Rated | 49.7 % | 86.3% |

**Table 3:** This table shows the distribution of borrower ratings (S&P/Moody's senior debt ratings respectively) at issue for both PSD (N=25,602) and non-PSD contracts (N=192,602). The numbers are calculated as the number of loans with a given borrower credit rating divided by the total number of loans for the given category. Datasource: Thomson Reuter's Dealscan database for the years 1993-2010.

almost 90% of these contracts which may be explained by the historically high level of sophistication of the financial markets in this region. Market participants indicate that more than half of recently issued syndicated bank loans in Europe include such provisions.

| **Borrower region** | **Total number of loans** |
|---|---|
| USA/Canada | 88.8 % |
| Western Europe | 6.5 % |
| Latin America/Caribbean | 1.7 % |
| Asia-Pacific | 1.9 % |
| Eastern Europe/Russia | 0.6 % |
| Middle East | 0.4 % |
| Africa | 0.2 % |

**Table 4:** This table shows the geographical distribution of issued performance sensitive loans as an equal-weighted percentage of the total number of such loans (N=25,602). Datasource: Thomson Reuter's Dealscan database for the years 1993-2010.

Figure 2 shows the use of performance sensitive loans, relative to all new loans during the last 17 years. We note that a substantial fraction of loans include performance sensitive provisions. The histogram indicates that the use of PSD features reached a peak in 1998, and has declined somewhat since.

Appendix B includes additional tables showing descriptive statistics for maturity and loan amounts comparing performance sensitive debt and non-PSD debt, as well as an overview of broad borrower industry classes. Loan amounts are larger for PSD contracts than for regular loans, but the maturities do not differ between the categories.

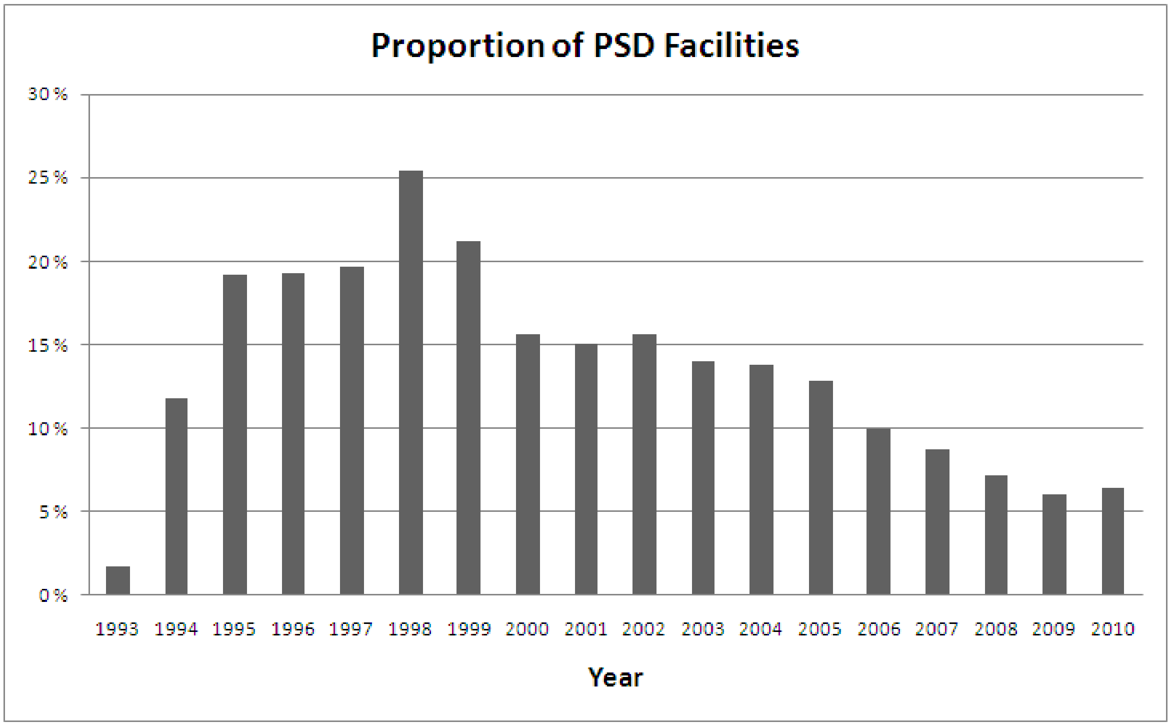


**Figure 2:** The histogram shows the annual proportion of performance sensitive loans relative to the total number of new loans. Numbers are based on data from Thomson Reuter's Dealscan database for the years 1993-2010.

## 4.2 Data Description

### 4.2.1 Sample Construction

Our sample is extracted from Thomson Reuter's Dealscan[3] in March 2011. We collect all loans with performance pricing provisions, a total of 27,994 loans issued in the period 1993 - 2010. We confine our analysis to contracts with interest rates linked to the company's debt-to-cash-flow ratio (Debt/CF), a condition for the model in Section 3. This restriction reduces the number of loans to 8,180. In addition to the performance sensitive feature, we require the existence of a debt-to-cash-flow default covenant in the contract. This requirement reduces our sample to 6,727 loans. We further restrict our sample to publicly listed borrowers with sufficient market and company

[3]See www.loanpricing.com for more information on the database and how to access it.

information from the databases CRSP and Compustat prior to the inception of the loan. Information from these databases is used to estimate the drift and volatility parameters of the borrowers' cash flow process in expression (1). Hence, we require the borrowing company to be listed in the two latter databases when the loan is established, and to have a minimum of 1 year of historical data for parameter estimation[4]. This last restriction reduces our sample to 5,143 loans. To ensure compatibility with our model, we also remove all contracts where the estimated starting value of the Debt/CF process is below the default covenant, the loan has no stated maturity, the number of barriers do not match the number of different loan spreads, or the loan spreads are not varying across different barriers. Some of the remaining loans in our sample are identical and come from the same loan deal. To avoid duplicating observations in our sample, we keep only one loan from each deal. Our final sample consists of 3,052 loans. The sample includes 342 interest decreasing contracts, 1,520 interest increasing contracts and 1,190 contracts containing both categories of performance sensitive interest rates. All loans are senior and secured, and are granted by banks in the time period 1993 - 2010. 93% of all loans are given to U.S. firms, whereas the remaining 7% are given to European or Canadian firms.

#### 4.2.2 Sample Presentation

Table 5 lists summary statistics of our sample. The loans have from 1 - 8 non-absorbing Debt/CF barriers with a mean of 3.46 and a median of 3. The size of the loans also varies from USD 0.6m to USD 10.7bn, whilst the average loan amount is USD 181m. No PSD loan has maturity above 21 years, whilst the average maturity is 4.54 years. The average credit spread at issue, measured by the all-in-spread (AIS), is 187 basis points, with a sample standard deviation of 86 basis points.

[4]To ensure correct matching of companies in Dealscan with Compustat/CRSP we use the Dealscan-Compustat links from Chava & Roberts (2008). We thank Michael Roberts for providing us with the matching file.

When we analyze firms using PSD loans we find no particular size distribution. The average borrower profitability, measured by the quarterly return on capital employed (ROCE), is 4.1%, with a median of 3.5% and a standard deviation of 4.9%. The average initial leverage, defined as the book value of debt divided by the by the sum of book value of debt and market value of equity, prior to entering into the PSD deal, is 0.27. The corresponding median and standard deviation are 0.23 and 0.20, respectively. To measure the relative significance of the PSD loan in the borrowers' total leverage, we estimate the ratio of the PSD loan divided by the total debt (the sum of existing debt and the new PSD loan). The average share of the new PSD loan relative to the borrower's total debt is 48%, with a median of 46% and a standard deviation of 26%. We do not know whether the borrowers' existing debt has PSD provisions. Here, 46% of the borrowers are rated, and 37% of the rated borrowers are rated investment grade[5]. The average and median of the sample estimated annual cash flow volatility are 11.9% and 4.4%, respectively. These volatility estimates, however, are widely distributed ranging from 2% to 80%. The average distance-to-default, defined as the starting value of the CF/debt measure less the contractual default barrier, and normalized by the cash flow volatility, is 49.2 with a standard deviation of 114.8. In Table 5 we include the distance-to-default characteristics for interest increasing and interest decreasing loans, respectively. Table 5 shows, as we would expect, that interest increasing contracts have a larger distance-to-default compared to interest decreasing contracts. Summary statistics for each of the three subsamples (interest increasing, interest decreasing, both provisions) can be found in Tables 15, 16, and 17 in Appendix C. See also Tables 11 and 12 in Section 6.3 for a statistical analysis by loan category. These tables show that borrowers using interest increasing PSD contracts are more profitable and less levered compared to borrowers using interest decreasing PSD contracts. Furthermore, they have

[5]This is the general senior S&P rating of the borrower's credit worthiness with respect to its long-term financial obligations such as senior debt. The bank loans we analyze are not rated separately.

a lower cash flow volatility and a much larger distance-to-default. Firms using interest increasing PSD contracts also pay a lower initial credit spread compared to borrowers using interest decreasing PSD contracts. These observations unanimously suggest that borrowers using interest increasing PSD loans are of an overall higher credit quality than those using interest decreasing PSD loans.

To assess how representative our sample is, we compare it to the population of PSD loans in the database. First of all, note that the sample borrower ratings correspond well with the observations in Table 3. For the entire database (our sample means in brackets) the average maturity is 4.5 (4.54) years, the average loan amount is USD 369m (USD 181m), the average borrower's quarterly sales volume is USD 2,658m (USD 986m) and the average AIS is 194.6 (187) bp. These statistics, see Table 6, suggest that our sample consists of somewhat smaller loans as well as smaller borrowers, and that the initial credit margins are close to the average of the population of PSD loans.

## 5 Pricing a PSD Contract in the Case of No Jumps

In order to show how the model input parameters are estimated and how these parameters influence pricing, we select the first loan contract in our sample (sorted alphabetically by borrower name), and illustrate the pricing of this contract using our closed form pricing formula (11). The results for the entire sample are included and analyzed in the next section.

| Variable | Mean | Median | Std. Dev. | Min. | Max. | N |
|---|---|---|---|---|---|---|
| **Borrower Characteristics** | | | | | | |
| Company Sales (MUSD) | 986 | 424 | 3,321 | 1.1 | 159,098 | 3,052 |
| ROCE (%,quarterly) | 4.09 | 3.53 | 4.94 | -17.09 | 112.17 | 3,052 |
| Leverage (Debt/Debt + Equity) | 0.27 | 0.23 | 0.20 | 0.00 | 1.00 | 3,052 |
| PSD Loan/Total Debt | 0.48 | 0.46 | 0.26 | 0.003 | 1.00 | 3,052 |
| Drift of cash flow $(r - \delta)$ | 0.023 | 0.025 | 0.014 | -0.09 | 0.06 | 3,052 |
| Volatility of cash flow $(\sigma)$ | 0.12 | 0.044 | 0.171 | 0.02 | 0.80 | 3,052 |
| **Loan Characteristics** | | | | | | |
| Loan Amount (MUSD) | 181 | 100 | 316 | 0.6 | 10,700 | 3,052 |
| Loan Maturity (Years) | 4.54 | 5.00 | 1.68 | 0.08 | 21 | 3,052 |
| All-In-Spread (Bp) | 187 | 175 | 86 | 23 | 750 | 3,052 |
| # of Barriers | 3.46 | 3 | 1.25 | 1 | 8 | 3,052 |
| Distance-to-default | | | | | | |
| - Full Sample | 49.2 | 6.6 | 114.8 | 0.00 | 1092.6 | 3,052 |
| - Interest increasing | 94.0 | 28.3 | 149.7 | 0.03 | 1092.6 | 1,520 |
| - Interest decreasing | 1.2 | 0.34 | 2.15 | 0.00 | 24.8 | 342 |

**Table 5:** This table shows summary statistics for various model input parameters and borrower characteristics for the final sample used in the paper. The loan contracts in the sample are issued in the period 1993 - 2010. Datasource: Thomson Reuter's Dealscan Database, Compustat and CRSP.

| | **Sample** | | | **Population** | | |
|---|---|---|---|---|---|---|
| **Variable** | **Mean** | **Median** | **Std. Dev.** | **Mean** | **Median** | **Std. Dev.** |
| Company Sales (MUSD) | 986 | 424 | 3,321 | 2,658 | 581 | 9,866 |
| Loan Maturity (Years) | 4.54 | 5.0 | 1.68 | 4.5 | 5.8 | 1.94 |
| Loan Amount (MUSD) | 181 | 100 | 316 | 369 | 149 | 879 |
| All-In-Spread (Basis points) | 187.5 | 175.0 | 86.5 | 194.6 | 175.0 | 116.5 |
| $N$ | 3,052 | 3,052 | 3,052 | 27,994 | 27,994 | 27,994 |

**Table 6:** Table shows sample averages as well as population averages for available variables (Borrowers' sales, loan maturity, loan amount and All-in-spread). Population defined as all PSD loans in Thomson Reuter's Dealscan database as of end 2010.

### 5.1 Pricing of an Example Contract - No Jumps

Actuant Corporation[6] borrowed USD 100m in the year 2000 using a PSD contract. The main terms of this contract are given in Table 7. The performance measure in this contract links the interest paid on the loan to the performance of the company via the company's Debt/CF ratio as

[6]See www.actuant.com for more information on the company.

| **Borrower** | **Actuant Corp.** |
|---|---|
| Deal Active Date | 31 Jul 2000 |
| Amount | USD 100 m |
| Loan type | Term Loan |
| Seniority | Senior |
| Maturity | 72 months |
| Distribution Method | Syndication |
| Lead Bank | Credit Suisse First Boston |
| Reference Rate | LIBOR 3 mth |
| Type of pricing grid | Interest decreasing |
| Initial CF/Debt ratio | 0.24 |
| Borrower Senior Debt Rating (S&P) | BB |

**Table 7:** This table provides an overview of the main terms in the chosen example PSD contract.

| | | **Performance Measure** | | **Interest Margins** | | |
|---|---|---|---|---|---|---|
| **Ranges** | **Barriers** | **Debt/CF** | **CF/Debt** | **LIBOR Spread** | **Commitment Fee** | **Total Spread** |
| 1 | $(B_1, B_0)$ | $[0, 1.75)$ | $(0.57, \infty)$ | 150 | 25 | 175 |
| 2 | $(B_2, B_1]$ | $[1.75, 2)$ | $(0.5, 0.57]$ | 162.5 | 37.5 | 200 |
| 3 | $(B_3, B_2]$ | $[2, 2.5)$ | $(0.4, 0.5]$ | 187.5 | 37.5 | 225 |
| 4 | $(B_4, B_3]$ | $[2.5, 3)$ | $(0.33, 0.4]$ | 212.5 | 37.5 | 250 |
| 5 | $(B_5, B_4]$ | $[3, 3.5)$ | $(0.29, 0.33]$ | 225 | 50 | 275 |
| 6 | $(B_6, B_5]$ | $[3.5, 4.25)$ | $(0.24, 0.29]$ | 250 | 50 | 300 |
| 7 | $(\mathbf{C}, B_6]$ | $[4.25, \mathbf{4.55})$ | $(\mathbf{0.22}, 0.24]$ | 275 | 50 | 325 |

**Table 8:** Table shows how the interest rate is linked to company cash flow through the Debt/CF ratio and the CF/Debt ratio respectively. A Debt/CF ratio equal to 4.55, or equivalently a CF/debt ratio of 0.22, represents the maximum (minimum) ratio that is accepted by the contract terms, and hence is the exogenously given contractual default barrier.

shown in Table 8[7]. Debt equals the book value of total debt, and is found by adding the company's long-term debt and debt in current liabilities (DLTTQ + DLCQ). We invert the Debt/CF ratio and assume that the borrower has a constant total debt level until the maturity of the loan. Also note that this is an *interest decreasing* contract since the starting level of the inverted performance measure (CF/debt) is 0.24, i.e, at the lowest non-absorbing barrier. In order to price this contract we estimate the drift and the volatility of the underlying cash flow process. To estimate the

[7]The LIBOR (London Interbank Offered Rate) spread indicates the contractual spread over LIBOR 3 mth USD rate measured in basispoints. The total spread equals the sum of the LIBOR spread and a commitment fee. CF equals cash flow and is proxied in the loan contract, and in our analysis, by reported EBITDA (Earnings Before Interest, Tax, Depreciations and Amortizations).

drift we use the insight of Goldstein et al. (2001) that the growth of the cash flow process, under the equivalent martingale measure $Q$, equals the risk-free rate $r$ if the company retains all of its earnings. However, a company with a payout rate $\delta$ proportional to earnings, has a lower drift under $Q$ equal to $r - \delta$. Payouts to investors and government typically consist of dividends to shareholders, interest payments to debtholders and tax payments to the government. Collecting quarterly data (Compustat codes in brackets) on EBITDA (OIBDQ), interest expenses (XINTQ), income taxes (TXTQ) and total dividends (DVTQ) from Compustat, we estimate $\delta$ by the ratio (XINTQ + TXTQ + DVTQ)/OIBDQ. The data are collected from the fiscal quarter prior to the loan issue to ensure that the data would be available to all parties in the deal[8]. For Actuant Corp. the estimates are $\delta = 0.0218$ and $\mu = r - \delta = r - 0.0218$.

The volatility of the cash flow process is the most important input parameter in our model. From equations (3) and (2) we know that the volatility of the cash flow is equal to the volatility of the firm's assets. Hence, the estimate of asset volatility is also the estimate of the cash flow volatility. In order to estimate the asset volatility we adopt the procedure used in Vassalou & Xing (2004) and Bharath & Shumway (2008). This procedure utilizes the insights from Merton (1974) that equity is as a call option on a firm's assets, where the strike of the call option is the face value of the firm's debt. The expiration date of the option corresponds to the maturity of the debt. Recall that $A_t$ and $D_t$ denote the market value of assets and the book value of debt, respectively. Define the value of equity at time $t$ by $E_t$. Using the Black-Scholes formula the time $t$ market value of

[8] One could alternatively use a longer time series to estimate $\delta$. As a robustness check we estimate $\delta$ for all borrowers in our sample, using information from the last 4 quarters prior to loan inception. This procedure tend to produce slightly higher payout rates, but the valuation effects are negligible.

equity is given by

$$E_t = A_t N\left(d_1\right) - D_t e^{-r(T-t)} N\left(d_2\right), \tag{12}$$

where

$$d_1 = \frac{\ln\left(\frac{A_t}{D_t}\right) + \left(r + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}, \qquad d_2 = d_1 - \sigma\sqrt{T-t}. \tag{13}$$

We estimate $\sigma$ using the following iterative procedure. For each firm we download daily stock price data from the past 12 months prior to the loan inception. Based on this time series we calculate the volatility of equity $\sigma_E$ and use this as our initial guess for the estimation of $\sigma$. Using expression (12) we compute $A_t$ for each trading day for the past 12 months, using the observed market value of equity of that particular day, and the last known observation of $D_t$. Thus, we create a new daily time series of $A_t$, and estimate $\sigma$ from this new time series. This estimate is then used as input for the next iteration. This procedure is repeated until the values of $\sigma$ from two consecutive iterations converge. The tolerance level for convergence is 0.0001. As in Bharath & Shumway (2008) we set $T = 1$ and define the face value of debt $D_t$ as debt in current liabilities (Compustat item DLCQ) plus one half of long-term debt (Compustat item DLTTQ). In the case of Actuant Corp our estimate of the asset volatility, equals to the cash flow volatility, is $\sigma = 0.1188$.

As an approximation for the risk-free rate we use the quote of the 3-month risk-free rate in the month prior to the the loan issuance date[9], i.e., in this case June 2000. The risk-free rate equals 5.83%, and implies that $\mu = r - \hat{\delta} = 0.0583 - 0.0218 = 0.0365$. The LIBOR 3 month rate is used as the reference interest rate in all loan contracts in our sample. To find the correct interest rates to

[9] These are collected from the Federal Reserve's official statistical releases (http://www.federalreserve.gov/releases/h15/data.htm). The website also contains descriptions of how these rates are measured.

use throughout the loan period as our model input, we add the contractual spreads to the forward LIBOR rates. Forward LIBOR rates are only available up to one year maturity, and hence we proxy longer-term forward rates by swap rates obtained from the quoted swap-curve at the time of issue[10]. In this example, the maturity of the loan is 6 years, and we use the 6 year forward swap rate quoted in June 2000 as our reference rate. Adding the contractually determined spreads yield the interest rates. As the contractual default barrier $C$, we use the financial covenant stating that the maximum Debt/EBITDA ratio should not be above 4.5. This ratio corresponds to a value of $C$ (i.e., CF/Debt) equal to $1/4.5 = 0.22$ in our model. The starting value of the asset process, i.e., the current value of CF/Debt is 0.24. As an approximation of the recovery rate (1 - $\kappa$) we use the estimated recovery rate for senior secured bank debt from Altman, Resti & Sironi (2004). This recovery rate[11] is 73%, implying that the liquidation cost parameter $\kappa$ equals 27%. The liquidation cost parameter determines the loss in the case of default. The size of the loss depends on whether the default leads to a full liquidation or not. Based on the explanation in the introduction regarding the presence of cross-defaults, it makes sense to apply the estimated liquidation cost parameter of 27%, even if we value a single debt contract and not necessarily a company's total debt. Practitioners confirm the magnitude of this parameter. The face (par) value of debt is normalized to 100. Table 9 summarizes the values of our input parameters.

Given the parameter values in Table 9 the market value of the PSD contract at issue is 95.44, calculated using expression (11). Thus, the theoretical market price is below the par value of 100, and this contract is underpriced.

[10] See also www.federalreserve.gov/releases/h15/data.htm for information on swap curves.
[11] The estimated recovery rate is based on recovery of principal 30 days after default.

| Parameters | Values | Explanations |
|---|---|---|
| $T$ | 6 | Maturity, in years |
| $D$ | 100 | Face value of debt, normalized |
| $B_1$ | 0.57 | Barrier 1 (CF/Debt) |
| $B_2$ | 0.50 | Barrier 2 (CF/Debt) |
| $B_3$ | 0.40 | Barrier 3 (CF/Debt) |
| $B_4$ | 0.33 | Barrier 4 (CF/Debt) |
| $B_5$ | 0.29 | Barrier 5 (CF/Debt) |
| $B_6$ | 0.24 | Barrier 6 (CF/Debt) |
| $C$ | 0.22 | Default Barrier (CF/Debt) |
| $c_1$ | 0.0892 | Interest rate paid when $A_t \geq B_1$ |
| $c_2$ | 0.0917 | Interest rate paid when $B_1 > A_t \geq B_2$ |
| $c_3$ | 0.0942 | Interest rate paid when $B_2 > A_t \geq B_3$ |
| $c_4$ | 0.0967 | Interest rate paid when $B_3 > A_t \geq B_4$ |
| $c_5$ | 0.0992 | Interest rate paid when $B_4 > A_t \geq B_5$ |
| $c_6$ | 0.1017 | Interest rate paid when $B_5 > A_t \geq B_6$ |
| $c_7$ | 0.1042 | Interest rate paid when $B_6 > A_t > C$ |
| $A$ | 0.24 | Starting value of the CF/Debt process |
| $\mu$ | 0.0365 | Risk-neutral drift of the CF/Debt process |
| $\sigma$ | 0.1188 | Volatility of the CF/Debt process |
| $r$ | 0.0583 | Risk-free interest rate |
| $\kappa$ | 0.27 | Liquidation cost parameter |

**Table 9:** This table states the value of all relevant input parameters needed to estimate the price of the example PSD contract, as described in Tables 7 and 8.

## 5.2 Decomposition of Interest Increasing and Interest Decreasing PSD Contracts

In order to interpret our empirical results we find it useful to decompose a PSD contract into a sum of a fixed rate loan and an option portfolio. From a lender's point of view an *interest increasing* PSD contract is equivalent to a fixed rate loan plus a portfolio of long put options. The put options give the lender rights to receive increased interest payments if the borrower's credit quality deteriorates. The lender has this right at every point in time interest payments are due within the contract period. Thus, we interpret this as a portfolio of options where the maturity of each option corresponds to an interest payment date. Denote the time 0 market value of the fixed rate loan and the put option portfolio by $F_0^I$ and $P_0$, respectively. Using expression (7), $P_0 = \sum_{i=1}^{m+n} \mathcal{B}(\xi, B_i)(c_{i+1} - c_i)$. Thus, the time 0 market value of the interest increasing PSD loan $L_0^I$ is given by $L_0^I = F_0^I + P_0$.

An interest decreasing PSD contract is, from a lender's point of view, equivalent to a fixed rate loan plus a portfolio of short call options. The call options give the borrower rights to pay reduced interest rates, at any interest payment date, if its credit quality improves. Since this right is held by the borrower, the lender is short in these options. Denote the time 0 market value of the fixed rate loan and the call option portfolio by $F_0^D$ and $C_0$, respectively. Using expression (6), $C_0 = \sum_{i=1}^{m+n} \mathcal{A}(\xi, B_i)(c_{i+1} - c_i)$. The market value of the interest decreasing PSD loan $L_0^D$ is $L_0^D = F_0^D - C_0$. The interest increasing/decreasing provisions are contractually determined and one could, thus, argue that the put and call options do not include the customary optionality at maturity included in regular options. However, any rational optionholder would exercise such options when they are in the money at maturity, so we can safely apply the option interpretation. Also, since the size of the increase or decrease in interest rate payments is independent of the underlying performance measure within a certain range of the credit performance measure, the put and call options are of digital type[12]. Normalizing the par value of the PSD loan to 100, the theoretically correct time 0 price is $F_0^I + P_0 = 100$ and $F_0^D - C_0 = 100$ for the interest increasing and interest decreasing cases, respectively.

A PSD contract which has both interest increasing and interest decreasing provisions can be

[12] A digital option is an option whose payout is fixed after the underlying asset exceeds a predetermined threshold or strike price. In the literature these options are also commonly referred to as 'cash-or-nothing' options, see, e.g., McDonald (2006).

decomposed into a portfolio of a fixed rate loan $F_0^B$, with fixed interest rate equal to the initial interest rate of the PSD contract, a portfolio of short digital call options, and a portfolio of long digital put options. This contract's time 0 market value can be written as $F_0^B - C_0 + P_0 = 100$.

To calculate the valuation effect that stems from the performance sensitive provisions of the example contract from the previous section, we compare the theoretical value of the PSD loan to the theoretical value of a fixed interest rate loan which pays an interest rate equal to the initial interest rate $c_7$ in Table 9. This value is calculated using a version of the pricing formula in Black & Cox (1976) modified to include finite maturity,

$$F(\xi) = c_i Z^c(\xi) + De^{-rT} Q^c(\xi) + D(1-\kappa) H^c(\xi), \tag{14}$$

where $c_i$ is the initial payment interest rate. $Q^c(\xi)$, $H^c(\xi)$ and $Z^c(\xi)$ are given by expressions (17), (20) and (21), respectively. In this example $c_i = c_7$.

Using expression (14) and contractual terms from Table 9, the time 0 market value of the fixed interest loan is 95.86, and, thus, there is a reduction in value from adding interest decreasing provisions to the loan contract, the size of which equals $95.86 - 95.44 = 0.42$. This number equals the value of the call option portfolio $C_0 = 0.42$.

The time 0 price $\hat{F}_0$ of a comparable risk-free contract with the same initial interest rate is 123.24, calculated using

$$\hat{F}_0 = \frac{c_i}{r}\left(1 - e^{-rT}\right) + De^{-rT}. \tag{15}$$

The difference of 27.80 between the market value of the risk-free contract and the PSD contract may be decomposed into 27.38 due to default risk and 0.42 due to the interest decreasing performance sensitive provision.

### 5.3 Sensitivity Analysis

To understand how sensitive the theoretical loan price is to model input parameters we perform a simple sensitivity analysis using the example PSD contract from above in the case of no jumps. In Figure 3 we plot the theoretical price of the example interest decreasing PSD contract and the corresponding fixed interest rate loan $F_0^D$ for varying levels of borrower's cash flow volatility. Recall that $F_0^D$ is defined as the time 0 market value of a fixed interest rate loan paying the initial interest rate of the corresponding PSD loan contract, with the remainder terms identical. The value of the corresponding call option portfolio is defined as the market value of the fixed-rate loan less the market value of the PSD loan, and is also plotted in Figure 3. Observe that the price of both debt contracts are monotonically decreasing in volatility, cf. standard results for debt from Merton (1974). The market value of the option portfolio is also decreasing in volatility. The latter result is consistent with the literature on vulnerable options, see, e.g., Johnson & Stulz (1987), and primarily due to the increased default risk. The theoretical price of the PSD contract is a monotonically decreasing function of borrower's cash flow volatility.

In Figure 4 we also plot the value of the call option portfolio for various combinations of $C$ and $\sigma$. The option value decreases when volatility increases, and the effect of increased volatility is larger for higher levels of $C$.

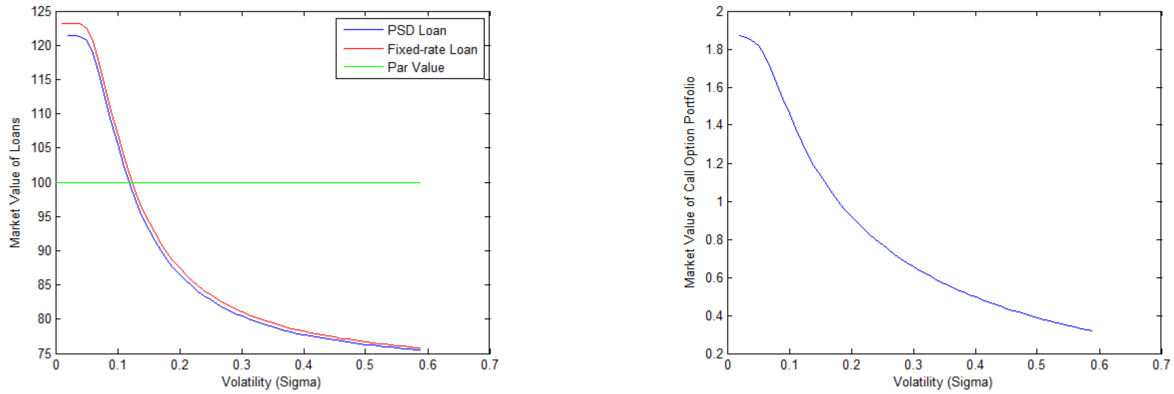


**Figure 3:** The market value of interest decreasing PSD contract consisting of the market value of a fixed-rate loan $F_0^D$ less the value of a call option portfolio $C_0$. Left plot shows the theoretical price of the example PSD contract and the fixed rate part $F_0^D$ plotted against **borrower's cash flow volatility** for the example PSD contract parameter values. Right plot shows the corresponding value of the call option portfolio as a function of borrower's cash flow volatility.

This sensitivity analysis is based on the example contract only and is partial in the usual *ceteris paribus* sense.

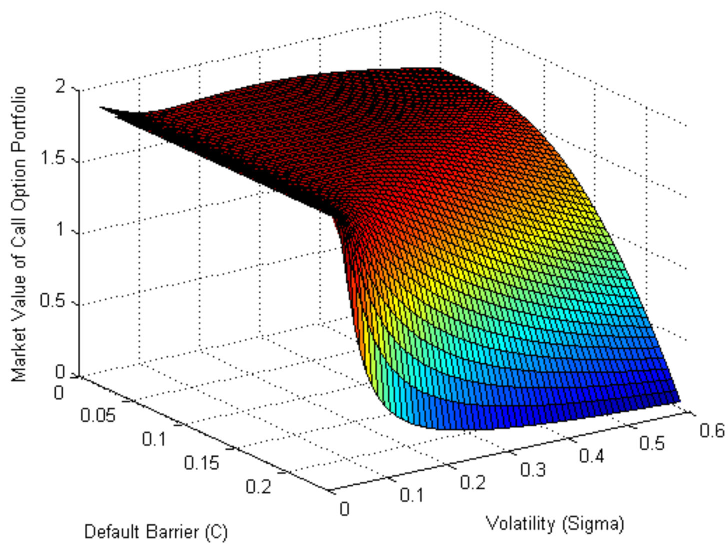


**Figure 4:** The market value of an interest decreasing PSD contract consists of the market value of a fixed-rate loan $F_0^D$ less the value of a call option portfolio $C_0$. This plot shows the theoretical market value of the call option portfolio as a function of both **the contractual default barrier** $C$ **and the borrower's cash flow volatility** $\sigma$, using the example contract parameter values.

# 6 Sample Analysis

We empirically test the pricing performance of our model for alternative assumptions regarding the jump frequency. We then analyze the pricing errors using t-tests and probit regressions and indicate some interpretations of our results.

## 6.1 Specification of Jumps

We assume that $Y_i$, the change in borrower's total debt caused by jump $i$, is lognormally distributed[13] under the equivalent martingale measure $Q$. We assume that the expectation and variance of $Y_i$ are $E^Q[Y] = 2$ and $\mathrm{Var}^Q[Y] = \frac{1}{12}$, respectively, where superscript $Q$ indicates the measure under which these quantities are given. The assumed value of $E^Q[Y]$ implies that the total amount

[13]The specific distributional assumptions of the jump size is not important. We get similar quantitative results using alternative distributions with the same numerical values of the first two moments.

of debt doubles, in expectation, in case of a jump. We define 10 scenarios with different jump risk, assuming the risk-adjusted jump intensity $\lambda$ from 0.05 to 0.5 in steps of 0.05. These intensities correspond to the following risk-adjusted frequencies, interpretable as, on average, one jump every 20, 10, $6\frac{2}{3}$, 5, 4, $3\frac{1}{3}$, $2\frac{6}{7}$, 2.5, $2\frac{2}{9}$, 2 'risk-adjusted years', respectively. The jump intensities could, in principle, be estimated from data if we assumed that the process for debt was stationary and we had sufficiently long time series, as well as the market risk premia for jump risk (size and frequency).

## 6.2 Test Statistics

The market price of a PSD loan at time 0 is given from our model as an expectation, see expression (8). Our first test statistic is the sample average of the time 0 market values, defined as

$$\bar{L} = \sum_{i=1}^{N} \hat{L}_i,$$

where $N$ is the number of observations. If our model is correct, the time 0 market value of each loan should be equal to the loan's normalized face value (100). This fact implies that also the average of the model's time 0 market values should be equal the normalized face value. Our second test statistic, $M$, is a measure of sample dispersion and is defined as the square root of the squared pricing errors (relative to the normalized face value). Denote the price of PSD contract $i$, based on estimated input parameters and assumptions regarding jumps as described in the previous subsection, by $\hat{L}_i$. Here, $M$ is given by

$$M = \sqrt{\frac{\sum_i^N \left(\hat{L}_i - 100\right)^2}{N}}. \tag{16}$$

If our pricing model correctly prices each individual loan contract, the numerical value of $M$ would be zero. This measure is related to the usual standard deviation, which uses the sample mean, instead of some theoretical value, as benchmark. Similar to standard deviation, our measure gives higher weight to observations further away from the benchmark.

### 6.3 Sample Results

Table 10 reports the average time 0 model price ($\bar{L}$), as well as $M$ (dispersion), separately for the full sample, interest increasing contracts, interest decreasing contracts, and contracts containing both features. Apart from the first column, these values have been estimated using standard Monte Carlo simulation techniques with 10,000 simulated[14] paths of the underlying performance measure for each loan in the sample. Row 1 reports the results assuming no jumps in debt value from our closed form solution in expression (11). The results from this assumption overprice PSD contracts, with a sample average price of 107.2. Considering the subsamples, interest increasing contracts are overpriced on average by 9.8% whereas contracts with both features are overpriced on average by 6.1%. For interest decreasing contracts the model produces a small underpricing on average by 0.7%, implying that estimated contract values are not significantly different from 100. The dispersion of prices ranges from 12.2 to 17.6 for the various subsamples.

Rows 2 - 10 in Table 10 report the average price and dispersion measure for values of the jump intensity $\lambda$ from 0.05 to 0.5. As expected, average prices are decreasing in jump intensity. For the full sample $\lambda = 0.2$ produces average loan price closest to par value, with an underpricing by only 0.7%. Price dispersion is smallest for $\lambda = 0.15$. For interest increasing contracts $\lambda = 0.4$

[14] Table 10 is based on 10,000 simulations of expression (8) for each contract using a C++ program. Each year is divided into 100 time steps. To reduce variance we use the standard antithetic variate technique, and, in addition, we apply the formula in expression (11) as control variate in the cases with jumps. All barriers are adjusted using the Broadie, Glasserman, & Kou (1997) adjustment. The average pricing difference between closed form values and simulated values without jumps is 0.025%. The total computing time for 3,052 contracts is about 6 hours and 20 minutes (2.8 GHz Intel Core i7 processor).

| | Full Sample | | Interest Increasing | | Interest Decreasing | | Both | |
|---|---|---|---|---|---|---|---|---|
| $\lambda$ | $\bar{L}$ | $M$ | $\bar{L}$ | $M$ | $\bar{L}$ | $M$ | $\bar{L}$ | $M$ |
| 0.00 | 107.199 | 13.272 | 109.782 | 12.184 | $99.269^{\$}$ | 17.605 | 106.089 | 13.149 |
| 0.05 | 104.993 | 11.634 | 108.992 | 11.498 | 95.311 | 15.264 | 102.580 | 10.544 |
| 0.10 | 102.934 | 10.738 | 107.999 | 10.781 | 92.137 | 14.519 | $99.482^{\$}$ | 9.305 |
| 0.15 | 101.022 | 10.413 | 106.879 | 10.104 | 89.543 | 14.744 | 96.755 | 9.226 |
| 0.20 | 99.249 | 10.494 | 105.664 | 9.524 | 87.438 | 15.453 | 94.366 | 9.893 |
| 0.25 | 97.618 | 10.842 | 104.418 | 9.097 | 85.708 | 16.323 | 92.273 | 10.912 |
| 0.30 | 96.105 | 11.346 | 103.150 | 8.840 | 84.266 | 17.215 | 90.429 | 12.045 |
| 0.35 | 94.712 | 11.935 | 101.883 | 8.770 | 83.074 | 18.041 | 88.817 | 13.168 |
| 0.40 | 93.430 | 12.559 | 100.646 | 8.859 | 82.055 | 18.809 | 87.404 | 14.228 |
| 0.45 | 92.245 | 13.189 | 99.438 | 9.098 | 81.215 | 19.470 | 86.151 | 15.209 |
| 0.50 | 91.159 | 13.806 | 98.274 | 9.445 | 80.492 | 20.064 | 85.060 | 16.094 |

**Table 10:** Table shows the time 0 average loan price ($\bar{L}$), as well as the price dispersion ($M$) for various jump intensities of debt, and using the estimated cash flow volatility in each contract. The results are reported for the full sample, interest increasing PSD contracts, interest decreasing PSD contracts and contracts with both features, respectively. The superscript $^{\$}$ indicate that the sample average is not significantly different from 100, on a 5% significance level.

produces the sample average price closest to par value. For this category the smallest $M$ is produced wih $\lambda = 0.35$. Interest decreasing contracts are priced closest to par value with $\lambda = 0$, although the model dispersion is smallest for $\lambda = 0.1$. Contracts with both interest increasing and interest decreasing features have the lowest $M$ at $\lambda = 0.15$, but the average price is closest to par value for $\lambda = 0.1$. The results for the full sample and the subsample of loans with both features are, necessarily subject to the underlying mix of contractual provisions in each sample. For all three subsamples the value of $\lambda$ which produces the value closest to par value is close to the value of the $\lambda$ which produces the lowest $M$. Figure 5 shows the distributions of loan prices for the full sample and the three subsamples.

From Table 10 we see that leverage risk in the debt process has large effects, both in terms of the level of prices, but also in terms of the dispersion of prices. This observation suggests that leverage risk might be an important source of risk for the value of PSD contracts. Observe that including a small leverage risk decreases dispersion compared to no leverage risk across all subsamples. We know from Subsection 4.2.2 that firms with interest increasing PSD contracts seem to be of an overall better credit quality than firms with interest decreasing PSD contracts. In particular, the average (median) leverage of the former group is 0.19 (0.14), versus the latter group at 0.42 (0.40), cf. Tables 15 and 16. Examining credit ratings for the two subsamples, we also find that 15% of borrowers using interest increasing PSD contracts are rated investment grade, compared to only 6% of the borrowers using interest decreasing PSD contracts. As an additional analysis, Figure 6 plots the normalized distributions of the simulated default probabilities $Q(\tau < T)$ for the three subsamples considered, using the subsample model specification that on average prices most correctly. The figure shows that borrowers using interest decreasing PSD contracts have more mass allocated in the right tail, supporting the hypothesis that these are of an overall lower credit quality compared to the two other groups. It is, therefore, likely that the former category of borrowers have larger debt capacity and, hence, have a higher probability of increasing debt in the future. The latter category of firms have lower debt capacity implying that they have less chance of obtaining additional debt financing in the future. Note that we do not specifically include investment opportunities or credit constraints in our analysis and can, thus, not rule out that the borrowers are at their optimal leverage. These findings suggest that banks price interest increasing PSD contracts by rationally taking into account the probability that borrowers increase leverage, and, hence, may increase risk for the lenders of existing loans throughout the maturity of the loan. Note that these borrowers due to their higher credit quality normally would be expected to be able to negotiate better terms on their debt. Our results indicate that leverage risk may be an explanation for this seemingly counterintuitive observation.

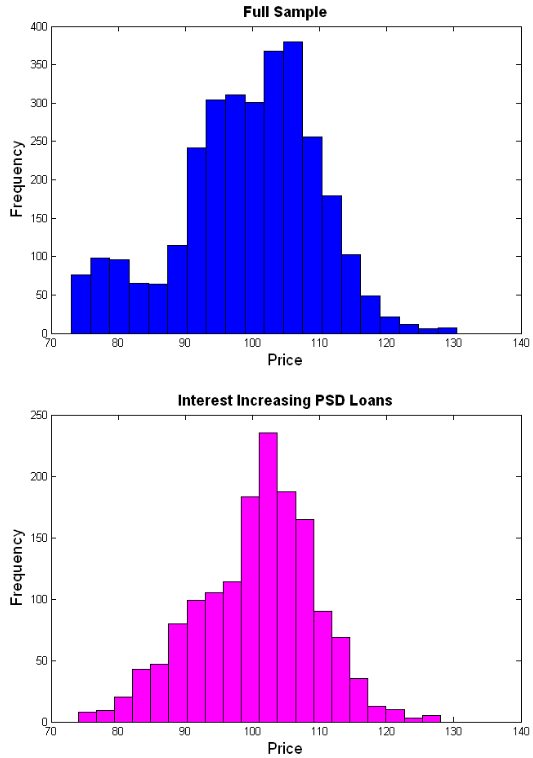


**Figure 5:** The histograms show the distribution of time 0 theoretical market values of the full sample of PSD contracts, interest increasing PSD contracts, interest decreasing PSD contracts, and contracts with both features, respectively.

## 6.4 Analyzing Pricing Errors

We analyze the pricing errors of each contract using the values of $\lambda$ which prices the class of contracts on average closest to par value. We include other potential factors, not necessarily included in our pricing model, that may affect prices of PSD loans. Following Eom et al. (2004), we compare loans that are overpriced to loans that are underpriced. We use the variables asset volatility, payout ratio, leverage and distance-to-default to capture the risk characteristics of the borrowers. We, furthermore include log of sales as a proxy for borrower size, and ROCE to measure borrower profitability. We also include loan specific characteristics like loan amount, maturity and initial credit spread paid on the loan. In addition we include PSD contract characteristics like number of

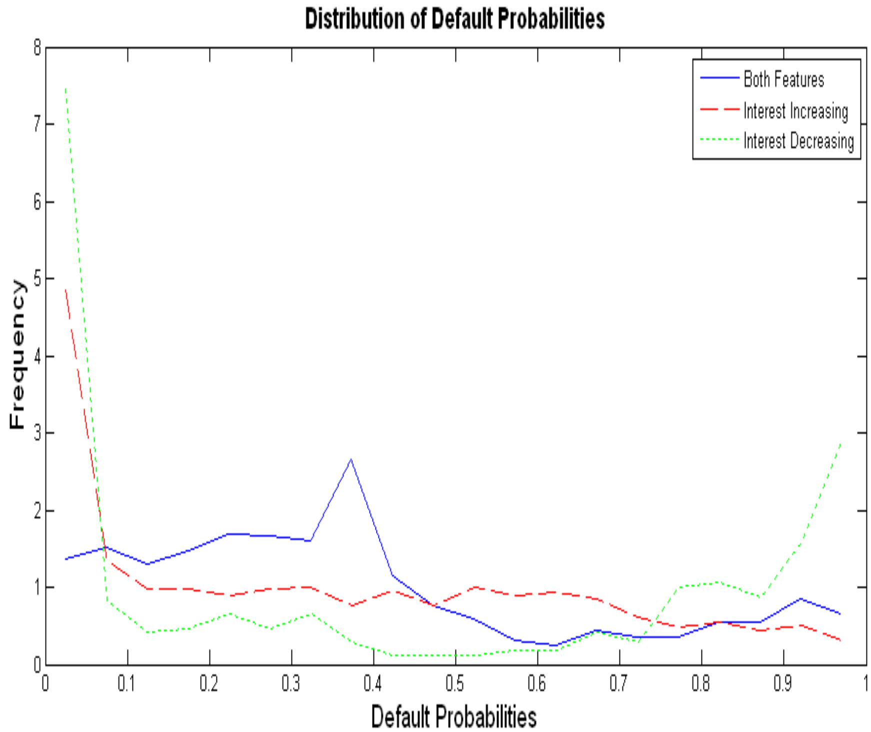


**Figure 6:** Figure shows the distribution of simulated default probabilities for the subsamples of interest increasing PSD loans ($\lambda = 0.4$), interest decreasing PSD loans ($\lambda = 0$), and loans with both features ($\lambda = 0.1$). The probabilities are calculated using the jump intensities $\lambda$ that produce the best model fit for each of the three subsamples.

non-absorbing barriers and performance sensitivity[15]. To measure the relative importance of PSD on the firm's capital structure we also use the ratio of the new PSD loan to other debt the company might have at the time of issue.

For each of the chosen variables we initially use a t-test to test whether or not there are any difference in means of the variables when comparing overpriced loans to underpriced loans. Table 11 reports the t-statistics from this test for the full sample and the subsamples of interest increasing, interest decreasing and loans with both features, respectively. As an example, the negative

[15]To measure performance sensitivity of a loan contract we define a variable called 'Diff', which is the maximum credit spread less the minimum credit spread as specified in the PSD contract.

coefficient on asset volatility indicates that overpriced loans are granted to firms with lower average asset volatility compared to underpriced loans. Note also that these firms have significantly lower leverage and payout ratio, and significantly higher distance-to-default and ROCE. Overall, our model tends to overprice loans given to firms which are less risky and more profitable. The two categories of loans also differ in amount, initial spread and maturity, with overpriced loans having smaller amounts, shorter maturities and larger initial spreads. The sensitivity parameter does not seem to influence pricing, but overpriced loans have, on average, fewer barriers specified in the contract. For overpriced loans the relative importance of the PSD loan on capital structure is much larger, probably reflecting the fact that these firms tend to have lower total leverage. The borrower size variable is not significant. All other explanatory variables are insignificant. By looking at the three subsamples, we observe that the signs and the significance of the test statistics are similar to the full sample. One exception is the sample of interest decreasing PSD loans, where neither asset volatility, leverage, ROCE, payout ratio, amount nor maturity are significant.

The results in Table 11 point to a number of systematic differences which may affect the pricing of PSD loans. A combination of factors may lead to higher or lower pricing errors, and therefore that an analysis in a multivariate regression setting is more appropriate. We define a new dummy variable 'overpricing', which is equal to 1 if the loan is overpriced, i.e., if $\hat{L}_i > 100$ and 0 if the loan is underpriced, i.e., $\hat{L}_i < 100$. We then use a probit model to regress this dummy variable on to our set of explanatory variables. In addition, to account for business cycle and market conditions, we include year dummies for the year the loan is initiated. Table 12 reports the coefficients from the probit regression and the corresponding standard errors. A regression of the whole sample shows that if a loan is of interest increasing type the probability that it will be overpriced is higher. We

| Variables | T-Statistics | | | |
|---|---|---|---|---|
| | **Full Sample** ($\lambda = 0.2$) | **Interest Increasing** ($\lambda = 0.4$) | **Interest Decreasing** ($\lambda = 0.0$) | **Both** ($\lambda = 0.1$) |
| **Borrower Characteristics** | | | | |
| Asset Volatility | -23.14*** | -10.60*** | -18.56*** | -18.27*** |
| Leverage (Debt/Debt + Equity) | -28.42*** | -19.34*** | -1.20 | -8.07*** |
| ROCE | 10.42*** | 4.71*** | 1.21 | 2.97** |
| DTF | 24.07*** | 23.98*** | 10.60*** | 16.76*** |
| Payout Ratio | -10.58*** | -3.19*** | -0.51 | -6.95*** |
| Size | 0.29 | -0.55 | 1.32 | -2.09* |
| **Loan Characteristics** | | | | |
| Loan Amount | -4.38*** | -3.10*** | 1.35 | -4.43*** |
| Loan Maturity | -5.25*** | -9.56*** | 1.15 | -2.43*** |
| Initial Spread | 1.12 | 1.82 | 0.26 | 3.09*** |
| **PSD Characteristics** | | | | |
| # Barriers | -5.58*** | -1.23 | -3.89*** | -3.11*** |
| Diff | -1.16 | 2.40** | -2.45** | 0.95 |
| PSD/Debt | 24.32*** | 18.12*** | 1.03 | 3.22*** |

**Table 11:** Table shows the value of a t-statistic when testing whether there is a difference in means comparing overpriced PSD loans to underpriced PSD loans. A negative value suggests that overpriced loans have a larger mean compared to underpriced loans. A superscript of *, ** and *** indicate a significance level of 5%, 1% and 0.1%, respectively.

also see that the probability of overpricing increases with the initial spread paid on the loan and the performance sensitivity of the PSD contract, whereas it decreases with maturity. Our model also overprice loans given to safer borrowers, as seen by the negative coefficients on payout-ratio and asset volatility, and on the positive coefficients on distance-to-default. No other explanatory variables are significant.

Motivated by the fundamental differences in borrower characteristics and contract features, we also do regressions for each of the three subsamples. The other regressions in Table 12 therefore report the results using the model specification that produce the sample average closest to par value for the sample of interest increasing PSD contracts ($\lambda = 0.4$), interest decreasing PSD contracts ($\lambda = 0$), and contracts with both features ($\lambda = 0.1$), respectively. From Column 2 we see that the probability that interest increasing PSD contracts will be overpriced is inversely related to leverage, loan amount and maturity, and positively related to initial spread paid on the loan, the relative size of the loan and the distance-to-default. The negative coefficient on current leverage supports our earlier interpretation that overpricing may be explained by leverage risk. A borrower with high leverage today is less likely to increase leverage in the future. However, we see that by introducing jumps the partial significance of leverage in explaining overpricing in Table 10 nearly disappears. All other explanatory variables are insignificant. The probability that an interest decreasing PSD contract is overpriced is inversely related to the asset volatility, whereas, it is positively related to initial spread and distance-to-default. The results are similar when analyzing the subsample of PSD contracts containing both interest increasing and interest decreasing features, except that the performance sensitivity of the contract and the number of barriers also become significant. Overall the only two variables that get significant coefficients in all 4 regressions are the initial spread paid on the loan and the distance-to-default. This observation indicates that the pricing of the PSD loans is crucially related to the initial spread, and that the performance sensitive features added to the loan contract are of lesser importance for any pricing deviations from par. We also see that loans with a high distance-to-default tend to be more overpriced, i.e., that the model tend to underestimate the probability of default for the safest firms. This is both consistent with high leverage risk and earlier results regarding the performance of structural credit models as discussed above.

### 6.5 Interpretation of Pricing Results

The results and the subsequent analysis in the previous subsections point to several interesting insights.

| | Full Sample ($\lambda = 0.2$) | Interest Increasing ($\lambda = 0.4$) | Interest Decreasing ($\lambda = 0.0$) | Both ($\lambda = 0.1$) |
|---|---|---|---|---|
| **Borrower Characteristics** | | | | |
| Asset Volatility | -1.473*** | -0.286 | -14.511*** | -6.328*** |
| | (0.275) | (0.423) | (3.688) | (0.676) |
| Leverage | -0.072 | -0.872* | -0.653 | 0.187 |
| | (0.258) | (0.443) | (0.607) | (0.361) |
| ROCE | 1.738 | 2.230 | 2.886 | 2.017 |
| | (1.346) | (1.938) | (5.144) | (2.193) |
| Distance-to-default | 0.226*** | 0.063*** | 0.539*** | 0.237*** |
| | (0.014) | (0.008) | (0.164) | (0.030) |
| Payout Ratio | -0.255*** | -0.102 | -0.022 | -0.111 |
| | (0.060) | (0.056) | (0.141) | (0.061) |
| Size | -0.063 | 0.099 | -0.106 | -0.082 |
| | (0.047) | (0.058) | (0.137) | (0.061) |
| **Loan Characteristics** | | | | |
| Amount | -0.006 | -0.139* | -0.217 | -0.087 |
| | (0.053) | (0.067) | (0.155) | (0.069) |
| Maturity | -0.054* | -0.309*** | 0.083 | -0.011 |
| | (0.027) | (0.037) | (0.090) | (0.038) |
| Initial Spread | 0.007*** | 0.009*** | 0.011*** | 0.007*** |
| | (0.0006) | (0.0009) | (0.002) | (0.0009) |
| **PSD Characteristics** | | | | |
| Number of Barriers | -0.046 | -0.097 | -0.079 | -0.119* |
| | (0.049) | (0.066) | (0.149) | (0.060) |
| Diff | 0.547*** | 0.263 | 0.485 | 0.816*** |
| | (0.147) | (0.184) | (0.595) | (0.199) |
| PSD/Debt | 0.301 | 1.116*** | 0.108 | 0.020 |
| | (0.230) | (0.279) | (0.706) | (0.323) |
| Interest Increasing | 1.010*** | | | |
| | (0.094) | | | |
| Interest Decreasing | -0.222 | | | |
| | (0.160) | | | |
| $N$ | 3,052 | 1,520 | 342 | 1,190 |
| Pseudo $R^2$ | 0.720 | 0.638 | 0.6489 | 0.538 |

Standard errors in parentheses
$^{*}$ $p < 0.05$, $^{**}$ $p < 0.01$, $^{***}$ $p < 0.001$

**Table 12:** Estimation results from a Probit regression of a dummy variabel taking the value 1 if the PSD loan is overpriced and 0 otherwise on borrower characteristics like asset volatility, leverage, ROCE, distance-to-default (DTF), payout ratio and size (proxied by log of sales). Furthermore we also use explanatory variables specific to the loan contracts like log of loan amount, maturity, initial spread paid on the loan, number of non-absorbing barriers, a variable (diff) measuring the performance sensitivity in the contract, as well as the relative size of the PSD loan to existing debt. We also include calendar year dummy variables and two dummy variables capturing whether the loan is of interest increasing or interest decreasing type. The regression is run using the best fitted pricing model for the full sample as well as the three subsamples. We report coefficients and robust standard errors in parenthesis.

Firstly, we price PSD contracts with, in expectation, downward jumps in the performance measure. We have chosen to interpret the jumps as changes in the borrower's total debt, allowing for the use of the standard continuous cash flow risk. Our performance measure is a ratio between these to quantities and to model each quantity separately seems like a natural approach. Practitioners tell us that the debt/CF performance measure is also used to discipline borrowers from additional borrowing. Alternatively, jumps could be included in the cash flow process although this approach would have required a more complicated procedure to estimate cash flow risk.

Secondly, the model does not correctly price both interest decreasing and interest increasing contracts using the same jump intensities. This fact suggests that there is a fundamental difference between the typical borrower using interest increasing PSD and interest decreasing PSD, respectively. According to our model, the market prices interest decreasing contracts as if no leverage risk is present. Interest increasing contracts are priced consistent with a risk-adjusted jump intensity of 0.4, suggesting that leverage risk is important in the pricing of these contracts. Based on these results we believe that the pricing of interest increasing PSD contracts reflects that the companies using these contracts have a larger debt capacity, and therefore have a higher probability of adding new debt to their capital structure in the future. The pricing of interest decreasing contracts reflects that companies using such contracts have a smaller debt capacity and have a lower probability of accessing additional debt funding in the future. This interpretation is supported by comparing borrower characteristics for the subsamples. Our findings supports the signaling hypothesis of Manso et al. (2010), and may also increase our understanding of their findings. High quality borrowers might be able to signal quality to the market by using interest increasing PSD loans. High quality borrowers acknowledge the presence of leverage risk and find the conditions of interest increasing contracts acceptable. Low quality borrowers which have less debt capacity find interest increasing contracts too expensive. Our results are in contrast to the findings of Asquith et al. (2005), who find that interest decreasing PSD loans tend to be used when costs related to adverse selection problems are larger.

Thirdly, our analysis of pricing errors shows that borrower's distance-to-default (DTF) is particularly important. A large DTF increases the probability that our model including jump risk overprices the loan. One way to compensate this problem is to increase the probability of jumps in the underlying performance measure. In our class of PSD-loans, a jump in the performance measure is caused by a jump in borrower leverage. Interest increasing loans, used by borrowers with a large DTF, require a higher jump intensity to be priced correctly, on average. Our finding is consistent with other empirical tests of structural debt pricing models, see, e.g., Eom et al. (2004) and Huang & Huang (2003).

# 7 Conclusions and Further Research

## 7.1 Future Research

An important implementation issue for PSD contracts is the observability and verifiability of the underlying performance measure. As in Manso et al. (2010), our model assumes that the performance measure is continuously observable to all contracting parties. For accounting based performance measures the observability is determined by the borrower's external financial reporting frequency, i.e., typically a maximum of 4 (quarterly) observations per year. These reports also present a delayed measure of the borrower's credit quality. Publicly listed companies typically file their financial reports at least one month after the end of the reporting period[16], subject to listing requirements. Any reported profit, cash flow, or other flow measures represent averages over a discrete time period and not the most recent continuous rates. Any valuation effects of these implementation issues are not included in our analysis, but might be an avenue for future research.

Our analysis suggests that borrowers using interest decreasing PSD contracts have higher leverage and, correspondingly, lower debt capacity. This result indicate that interest decreasing PSD loans might be used when problems related to debt overhang are severe. Intuitively, a debt contract that promises reduced interest rate payments when firm performance increase, might incentivize borrowers to invest earlier[17].

Our finding that interest increasing PSD loans are priced with a large risk of additional leverage is also a potential avenue for further studies. This includes a closer review of loan covenants potentially regulating the relationship between current and future creditors, which we have not included in this paper.

### 7.2 Conclusions

Performance Sensitive Debt (PSD) is a large class of debt contracts where the interest payment is contractually defined to change according to some predetermined performance measure reflecting credit risk. Our model prices PSD contracts with cash flow and leverage based performance measures. Based on financial valuation theory, we derive a valuation model and test its performance empirically. Our model incorporate finite maturity, jump risk in the borrower's total amount of

[16]Market participants inform us that borrowers tend to report observations of the performance measure early in case of interest decreasing contracts and late in case of interest increasing contracts, as would be expected.
[17]See Martin (2009) for an elaboration on this topic.

debt, as well as exogenous contract-specific default covenants. In the special case of no jump risk, we also derive a closed form solution for the market price of the contract.

In our empirical analysis, we test the model for interest increasing PSD contracts, interest decreasing PSD contracts, and loans including both provisions using alternative risk-adjusted jump specifications. We show that of the pure contractual categories, interest decreasing contracts are priced on average closest to par value using our closed form model disregarding leverage risk, whilst for interest increasing contracts a substantial leverage risk provides best model performance. This result fits well with characteristics of borrowers using such contracts which have a larger debt-capacity due to an overall higher credit quality, e.g., represented by a larger distance-to-default, and a lower current leverage. A comparison of borrower characteristics for the two subsamples confirm this interpretation. Our empirical results show that PSD contracts are priced recognizing the risk of future increases in debt with a potentially negative effect on current loan value. These results also contribute to a better understanding of the signaling hypothesis presented in Manso et al. (2010). High quality borrowers, which are less credit constrained, may signal quality through the use of interest increasing PSD loans. Low quality borrowers, which are more credit constrained, choose not to mimic this behavior since they have to incur a cost related to the lenders assessment of future leverage risk. Instead they would be granted interest decreasing PSD loans. The fact that these borrowers have significantly higher leverage lead us to believe that interest decreasing PSD loans may be used to reduce problems related to debt overhang.

In addition to an increased understanding of PSD loans, we also add to the more general field of empirically testing structural debt models. We show that, without jumps, our model tend to overprice loans given to the best borrowers. Including leverage risk in the form of jumps reduces this problem.

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# A Closed Form Solutions in the Case of No Jumps

Let $N(\cdot)$ denote the cumulative standard normal distribution function.

## A.1 Survival probability

The survival probability $Q^c(\xi)$ is given by

$$Q^c(\xi) = Q(\tau > T) = N(d_1) - \left(\frac{\xi}{C}\right)^{\alpha-\beta} N(-d_2), \tag{17}$$

where

$$d_1 = \frac{\ln\left(\frac{\xi}{C}\right) + (\mu - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}},$$

$$d_2 = \frac{\ln\left(\frac{\xi}{C}\right) - (\mu - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}},$$

$$\alpha = \frac{1}{\sigma^2}\left(\frac{1}{2}\sigma^2 - \mu + \sqrt{\left(\frac{1}{2}\sigma^2 - \mu\right)^2 + 2\sigma^2 r}\right), \tag{18}$$

and

$$\beta = \frac{1}{\sigma^2}\left(\mu - \frac{1}{2}\sigma^2 + \sqrt{\left(\frac{1}{2}\sigma^2 - \mu\right)^2 + 2\sigma^2 r}\right). \tag{19}$$

## A.2 Time 0 market price of 1 upon default

The time 0 market value $H^c(\xi)$ of a security paying 1 when the default barrier is hit is given by

$$H^c(\xi) = E^Q[e^{-r\tau}1\{\tau \leq T\}] = e^{b(u-w)}N\left(\frac{b-wT}{\sqrt{T}}\right) + e^{b(u+w)}N\left(\frac{b+wT}{\sqrt{T}}\right), \qquad (20)$$

where $u = (\mu - (1/2)\sigma^2)/\sigma$, $w = \sqrt{u^2 + 2r}$, and $b = \ln(C/\xi)/\sigma$, see Appendix B.2 in Lando (2004).

## A.3 Time 0 market price of finite unity interest payments

$$Z^c(\xi) = E^Q\left[\int_0^{\tau\wedge T} e^{-rs}ds\right] = \frac{1}{r}\left[1 - e^{rT}Q^c(\xi) - Q_l^\beta(\frac{\xi}{C})^{-\beta}\right], \qquad (21)$$

where $\beta$ and $Q^c(\xi)$ are given in expressions (19), and (17), respectively. Also,

$$Q_g^\beta = Q^\beta(\tau > T) = N\left(d_1^\beta\right) - \left(\frac{\xi}{C}\right)^{\alpha+\beta} N\left(-d_2^\beta\right), \qquad (22)$$

where

$$d_1^\beta = \frac{\ln\left(\frac{\xi}{C}\right) + \left(\mu - \sigma^2\beta - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}},$$
$$d_2^\beta = \frac{\ln\left(\frac{\xi}{C}\right) - \left(\mu - \sigma^2\beta - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}},$$

and $\alpha$ is given in expression (18). Furthermore,

$$Q_l^\beta = Q(\tau < T) = 1 - Q_g^\beta, \qquad (23)$$

where $Q_g^\beta$ is given in expression (22).

### A.4 Time 0 market values of above unity annuities

The following result is from expression (17) in Mjøs & Persson (2010). Define

$$x = \frac{\alpha}{\alpha+\beta},$$

$$y = \frac{\beta}{\alpha+\beta}.$$

Now,

$$\mathcal{A}_a^c(\xi, B) = \gamma_a(\xi, B)/r, \tag{24}$$

where

$$\gamma_a(\xi, B) = 1 - x\left(\frac{\xi}{B}\right)^{-\beta}(1 - Q_{gg}^\beta(B)) - y\left(\left(\frac{\xi}{B}\right)^{\alpha} Q_{lg}^\alpha(B) + \left(\frac{C}{B}\right)^{\alpha}\left(\frac{\xi}{C}\right)^{-\beta} Q_l^\beta\right) - e^{-rT}Q_{gg}(B).$$

The probabilitites

$$Q_{gg}^\beta(B) = Q^\beta(\xi_T > B, \tau > T) = N\left(d_3^\beta\right) - \left(\frac{\xi}{C}\right)^{\alpha+\beta} N\left(-d_4^\beta\right), \tag{25}$$

where

$$d_3^\beta = \frac{\ln\left(\frac{\xi}{B}\right) + \left(\mu - \sigma^2\beta - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}},$$
$$d_4^\beta = \frac{\ln\left(\frac{\xi}{C}\right) + \ln\left(\frac{B}{C}\right) - \left(\mu - \sigma^2\beta - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}},$$

and

$$Q^{\alpha}_{lg}(B) = Q^{\alpha}(\xi_T < B, \tau > T) = N\left(d_1^{\alpha}\right) - N\left(d_3^{\alpha}\right) + \left(\frac{\xi}{C}\right)^{-(\alpha+\beta)} \left(N\left(-d_4^{\alpha}\right) - N\left(-d_2^{\alpha}\right)\right), \tag{26}$$

where

$$\begin{aligned}
d_1^{\alpha} &= \frac{\ln\left(\frac{\xi}{C}\right) + \left(\mu + \sigma^2\alpha - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}, \\
d_2^{\alpha} &= \frac{\ln\left(\frac{\xi}{C}\right) - \left(\mu + \sigma^2\alpha - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}, \\
d_3^{\alpha} &= \frac{\ln\left(\frac{\xi}{B}\right) + \left(\mu + \sigma^2\alpha - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}, \\
d_4^{\alpha} &= \frac{\ln\left(\frac{\xi}{C}\right) + \ln\left(\frac{B}{C}\right) - \left(\mu + \sigma^2\alpha - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}.
\end{aligned}$$

Furthermore,

$$Q_{gg}(B) = Q(\xi_T > B, \tau > T) = N\left(d_3\right) - \left(\frac{\xi}{C}\right)^{\alpha-\beta} N\left(-d_4\right), \tag{27}$$

where

$$\begin{aligned}
d_3 &= \frac{\ln\left(\frac{\xi}{B}\right) + \left(\mu - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}, \\
d_4 &= \frac{\ln\left(\frac{\xi}{C}\right) + \ln\left(\frac{B}{C}\right) - \left(\mu - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}.
\end{aligned}$$

From the expressions (12) and (15) in Mjøs & Persson (2010) it follows that

$$\mathcal{A}_b^c(\xi, B) = \gamma_b(\xi, B)/r, \tag{28}$$

where

$$\gamma_b(\xi,B)=x\left(\frac{\xi}{B}\right)^{-\beta}Q_{gg}^{\beta}(B)+y\left(\left(\frac{\xi}{B}\right)^{\alpha}(1-Q_{lg}^{\alpha}(B))-\left(\frac{C}{B}\right)^{\alpha}\left(\frac{\xi}{C}\right)^{-\beta}Q_l^{\beta}\right)-e^{-rT}Q_{gg}(B),$$

where $Q_{gg}^{\beta}(B)$, $Q_{lg}^{\alpha}(B), Q_g^{\beta}, Q_{gg}(B)$ are given in expressions (25), (26), (22), (27), respectively.

### A.5 Time 0 market values of below unity annuities

The following result is from expression (18) in Mjøs & Persson (2010).

$$\mathcal{B}_a^c(\xi,B)=\eta_a(\xi,B)/r, \tag{29}$$

where

$$\eta_a(\xi,B)=$$

$$x\left(\frac{\xi}{B}\right)^{-\beta}(1-Q_{gg}^{\beta}(B))+y\left(\left(\frac{\xi}{B}\right)^{\alpha}Q_{lg}^{\alpha}(B)+\left(\frac{C}{B}\right)^{\alpha}\left(\frac{\xi}{C}\right)^{-\beta}Q_l^{\beta}\right)-e^{-rT}Q_{lg}(B)-\left(\frac{\xi}{C}\right)^{-\beta}Q_l^{\beta},$$

where $Q_{gg}^{\beta}(B)$, $Q_{lg}^{\alpha}(B)$, $Q_l^{\beta}$ are given in expressions (25), (26), (23), respectively. Here,

$$Q_{lg}(B)=Q^c(\xi)-Q_{gg}(B), \tag{30}$$

where $Q^c(\xi)$ and $Q_{gg}(B)$ are given in the expressions (17) and (27), respectively. From the expressions (13) and (16) in Mjøs & Persson (2010) it follows that

$$\mathcal{B}_b^c(\xi,B)=\eta_b(\xi,B)/r, \tag{31}$$

where

$$\eta_b(\xi, B) =$$

$$1 - x\left(\frac{\xi}{B}\right)^{-\beta} Q^{\beta}_{gg}(B) - y\left(\left(\frac{\xi}{B}\right)^{\alpha}(1 - Q^{\alpha}_{lg}(B)) - \left(\frac{C}{B}\right)^{\alpha}\left(\frac{\xi}{C}\right)^{-\beta} Q^{\beta}_{l}\right) - e^{-rT} Q_{lg}(B) - \left(\frac{\xi}{C}\right)^{-\beta} Q^{\beta}_{l},$$

where $Q^{\beta}_{gg}(B)$, $Q^{\alpha}_{lg}(B)$, $Q_{lg}(B)$, $Q^{\beta}_{l}$ are given in expressions (25), (26), (30), (23), respectively. Observe that $r(\gamma_a(\xi, B) + \eta_a(\xi, B)) = r(\gamma_b(\xi, B) + \eta_b(\xi, B)) = Z^c(\xi)$, the value of an above- and a below annuity should add to the value of a regular annuity, no matter if the initial value $\xi$ is above or below a given $B$.

# B Tables and Graphs

| **Industry** | **% of total** |
|---|---|
| Banks | 0.6 % |
| Corporates | 76.2 % |
| Government | 0.3 % |
| Media/Communications | 11.3 % |
| Non-bank Financial Inst. | 5.1 % |
| Utilities | 5.1 % |
| Others | 1.3 % |

**Table 13:** This table shows the industry distribution of borrowers of performance sensitive debt across broad industry classes. Datasource: Thomson Reuter's Dealscan database for the years 1993-2010 (N=25,602).

| | **Maturity (Years)** | | **Facility Amount (MUSD)** | |
|---|---|---|---|---|
| | **PSD Facilities** | **Non-PSD Facilities** | **PSD Facilities** | **Non-PSD Facilities** |
| **Mean** | 8.7 | 9.4 | 373 | 195 |
| **Median** | 10 | 8 | 150 | 60 |
| **St.Dev** | 4.6 | 8.3 | 887 | 602 |
| **Min** | 0.17 | 0.17 | 0 | 0 |
| **Max** | 64.2 | 146.8 | 30,000 | 61,607 |
| **N** | 25,602 | 192,602 | 25,602 | 192,602 |

**Table 14:** This table shows discriptive statistics for maturity and loan amounts for loans containing PSD features and for loans not containing PSD features. Datasource: Thomson Reuter's Dealscan database for the years 1993-2010.

| Variable | Mean | Median | Std. Dev. | Min. | Max. | N |
|---|---|---|---|---|---|---|
| **Borrower Characteristics** | | | | | | |
| Company Sales (MUSD) | 1,034 | 381 | 4,465 | 3.28 | 159,098 | 1,520 |
| ROCE (%,quarterly) | 5.01 | 4.27 | 6.24 | -12.86 | 112.17 | 1,520 |
| Leverage (Debt/Debt + Equity) | 0.19 | 0.14 | 0.18 | 0.00 | 0.99 | 1,520 |
| PSD Loan/Total Debt | 0.59 | 0.58 | 0.27 | 0.00 | 1.00 | 1,520 |
| Drift of cash flow | 0.024 | 0.027 | 0.014 | -0.09 | 0.058 | 1,520 |
| Volatility of cash flow | 0.084 | 0.038 | 0.13 | 0.02 | 0.80 | 1,520 |
| **Loan Characteristics** | | | | | | |
| Loan Amount (MUSD) | 164 | 100 | 350 | 0.06 | 10,700 | 1,520 |
| Maturity (Years) | 4.47 | 5.00 | 1.72 | 0.08 | 21 | 1,520 |
| All-In-Spread (Bp) | 186 | 175 | 89 | 23 | 750 | 1,520 |
| # of Barriers | 3.2 | 3 | 1.20 | 1 | 7 | 1,520 |
| Distance-to-default | 94.1 | 28.3 | 149.7 | 0.03 | 1092.6 | 1,520 |

**Table 15:** This table shows summary statistics for various model input parameters and firm characteristics for the **interest increasing PSD contracts** in the sample used in the paper. The loan contracts are granted in the period 1993 - 2010. Datasource: Thomson Reuter's Dealscan Database, Compustat and CRSP.

| Variable | Mean | Median | Std. Dev. | Min. | Max. | N |
|---|---|---|---|---|---|---|
| **Borrower Characteristics** | | | | | | |
| Company Sales (MUSD) | 880 | 475 | 1,217 | 9 | 10,877 | 342 |
| ROCE (%,quarterly) | 2.77 | 2.60 | 3.79 | -4.79 | 63.68 | 342 |
| Leverage (Debt/Debt + Equity) | 0.42 | 0.40 | 0.19 | 0.01 | 0.90 | 342 |
| PSD Loan/Total Debt | 0.30 | 0.27 | 0.18 | 0.02 | 1 | 342 |
| Drift of cash flow | 0.02 | 0.022 | 0.013 | -0.060 | 0.047 | 342 |
| Volatility of cash flow | 0.20 | 0.08 | 0.23 | 0.02 | 0.80 | 342 |
| **Loan Characteristics** | | | | | | |
| Loan Amount (MUSD) | 184 | 100 | 278 | 2 | 3,000 | 342 |
| Maturity (Years) | 4.91 | 5.00 | 1.72 | 0.92 | 10.08 | 342 |
| All-In-Spread (Bp) | 219 | 225 | 87 | 50 | 600 | 342 |
| # of Barriers | 2.95 | 3 | 1.35 | 1 | 7 | 342 |
| Distance-to-default | 1.21 | 0.34 | 2.15 | 0.00 | 24.88 | 342 |

**Table 16:** This table shows summary statistics for various model input parameters and firm characteristics for the **interest decreasing PSD contracts** in the sample used in the paper. The loan contracts are granted in the period 1993 - 2010. Datasource: Thomson Reuter's Dealscan Database, Compustat and CRSP.

| Variable | Mean | Median | Std. Dev. | Min. | Max. | N |
|---|---|---|---|---|---|---|
| **Borrower Characteristics** | | | | | | |
| Company Sales (MUSD) | 954 | 466 | 1,550 | 1.08 | 19,424 | 1,190 |
| ROCE (%,quarterly) | 3.29 | 3.06 | 2.55 | -17.09 | 24.57 | 1,190 |
| Leverage (Debt/Debt + Equity) | 0.33 | 0.29 | 0.19 | 0.01 | 0.99 | 1,190 |
| PSD Loan/Total Debt | 0.40 | 0.39 | 0.21 | 0.00 | 1 | 1,190 |
| Drift of cash flow | 0.022 | 0.024 | 0.013 | -0.07 | 0.053 | 1,190 |
| Volatility of cash flow | 0.14 | 0.06 | 0.18 | 0.02 | 0.80 | 1,190 |
| **Loan Characteristics** | | | | | | |
| Loan Amount (MUSD) | 201 | 125 | 277 | 1.00 | 4,500 | 1,190 |
| Maturity (Years) | 4.53 | 5.00 | 1.60 | 0.25 | 10.17 | 1,190 |
| All-In-Spread (Bp) | 180 | 175 | 81 | 25 | 450 | 1,190 |
| # of Barriers | 3.93 | 4 | 1.13 | 2 | 8 | 1,190 |
| Distance-to-default | 5.81 | 2.59 | 9.29 | 0.01 | 78.73 | 1,190 |

**Table 17:** This table shows summary statistics for various model input parameters and firm characteristics for sample **PSD contracts containing both interest increasing and interest decreasing provisions**. The loan contracts are granted in the period 1993 - 2010. Datasource: Thomson Reuter's Dealscan Database, Compustat and CRSP.

# C Variable Descriptions

| **Variable** | **Description** | **Means** | | | |
|---|---|---|---|---|---|
| | | Full Sample | Increasing | Decreasing | Both |
| Log of Borrower Sales | Logarithm of borrowers sales (in MUSD) | 19.87 | 19.79 | 19.91 | 19.97 |
| *Borrower Sales* | *Borrower sales (in MUSD)* | *986* | *1,034* | *880* | *954* |
| Asset Volatility | Estimated annual cash flow (asset) volatility | 0.12 | 0.084 | 0.203 | 0.139 |
| ROCE | Borrower's Quarterly Return on Capital Employed Based on the last quarterly report prior to the loan issuance | 4.09% | 5.02% | 2.77% | 3.30% |
| (Initial) Leverage | Borrower's leverage prior to loan issue | 0.27 | 0.19 | 0.42 | 0.33 |
| PSD Loan/Total Debt | Ratio of PSD loan amount to the borrower's total book value of debt (including the PSD loan) at the time of loan issuance | 0.48 | 0.59 | 0.30 | 0.40 |
| Log of Loan Amount | Logarithm of loan amount (in MUSD) | 18.37 | 18.25 | 18.41 | 18.53 |
| *Loan Amount* | *Loan amount (in MUSD)* | *181* | *164* | *184* | *201* |
| Maturity | Maturity of the PSD Loan (in years) | 4.54 | 4.47 | 4.91 | 4.53 |
| Barriers | Number of non-absorbing contractual barriers | 3.46 | 3.21 | 2.95 | 3.93 |
| Distance-to-Default | The distance between borrower's initial CF/Debt ratio and the contractually specified default barrier ($C$), normalized by borrower's cash flow volatility | 49.24 | 94.05 | 1.21 | 5.81 |
| Payout ratio | Variable measuring the total cash payouts made by a given firm | 0.013 | 0.012 | 0.017 | 0.013 |
| Initial Spread | The initial spread paid on a loan in bp. | 187 | 186 | 219 | 180 |
| Diff | Variable measuring the performance sensitivity of a loan using the difference of the maximum spread and the minimum spread in the contract | 91 | 87 | 80 | 101 |
| Interest Increasing | Dummy variable equal to one if the loan is of interest increasing type | | | | |
| Interest Decreasing | Dummy variable equal to one if the loan is of interest decreasing type | | | | |

**Table 18:** Table describes variables used in the probit regressions, and displays their respective means.