Is Uninsurable Consumption Risk Priced ?

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Abstract

We argue that under incomplete consumption insurance consumption risk can be decomposed into two parts, insurable consumption risk (that can be hedged using financial instruments) and uninsurable consumption risk (caused by the incompleteness of consumption insurance) and examine whether exposure to uninsurable consumption risk helps explain expected returns. Exploiting household quarterly consumption data from the US CES, we find that uninsurable consumption risk, as captured by the rates of change in the normalized cross-sectional (stockholder) consumption moments of order two to four, is separately priced. There is no evidence that insurable risk in the consumption of stockholders is significantly rewarded in the stock market. The set of the Fama-French (1993) three risk factors and the set of the consumption moment (of order up to four) risk factors both have pricing significance when tested against each other in an integrated framework.

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1 Introduction

The Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1965) measures the risk of an asset by its beta with the market portfolio. Empirical evidence from testing the CAPM shows that the ability of the stock market indices to account for a large proportion of the intertemporal variability in other stock portfolios contrasts sharply with the insignificance of market beta in explaining the cross-section of expected stock returns. This suggests that some additional, beyond the market index, systematic risk factors may be required to explain cross-sectional differences in the expected returns on stocks.

In the Arbitrage Pricing Theory (APT) model, the single market beta is replaced by a vector of factor-specific betas that measure sensitivity of the return on a financial asset to changes in systematic risk factors deemed to affect the returns on all stocks. Using theoretical arguments, Chen, Roll, and Ross (1986) (hereafter CRR) argue that likely candidates for the systematic forces responsible for the comovements of asset prices are those that change discount factors and expected cash flows and model stock returns as functions of macroeconomic state variables and nonequity asset returns.[1] Fama and French (1993) (hereafter FF) suggest that market capitalization and book-value-to-price ratio explain better than the market as a whole different sensitivity of stock prices to the economy-wide systematic risks.[2]

In opposite to the APT, the consumption-based theories state that the single risk factor that influences the returns on all assets is the change in the aggregate marginal utility of consumption. Within this approach, it is common to measure the asset's systematic risk by the covariance (beta) of the asset return with the growth rate of average consumption in the economy. The beta coefficient in the implied single-beta pricing equation is known as the consumption beta. When testing the APT against the consumption beta theories in an integrated framework, CRR (1986) find no evidence that the growth rate of aggregate consumption in the economy is significantly priced as against the five economic state variables.

The assumption that the growth rate of aggregate consumption in the economy is an adequate proxy for the growth rate of the aggregate marginal utility of consumption implicitly relies on the hypothesis of complete consumption insurance. A growing literature in finance examines the impli-

[1] CRR (1986) show that the yield spread between long and short interest rates (the maturity premium), expected inflation and unexpected inflation, industrial production growth, and the yield spread between corporate high- and low-grade bonds (the default premium) should systematically influence stock returns.

[2] In the FF (1993) three-factor model, two additional (beyond the excess return on the market portfolio) risk factors are the excess returns on (i) small (market capitalization) over big (market capitalization) and (ii) value (high book-value-to-price ratio) over growth (low book-value-to-price ratio) stock portfolios.

cations of the incompleteness of consumption insurance for asset pricing.[3] In the existing literature, it is common to investigate whether allowing for incomplete consumption insurance improves the ability of the consumption CAPM to explain the excess return on the market portfolio and the return on the risk-free asset. Jacobs and Wang (2004) is the only paper that investigates the role of incomplete consumption insurance in explaining the cross-section of stock returns. They assume a stochastic discount factor to be a linear function of the FF risk factors and the consumption-based factors deemed to capture the influence of incomplete consumption insurance and examine whether the loadings of the stochastic discount factor on these consumption-based factors are statistically significant in the cross-section of stock returns. The approach adopted in this paper differs in many respects from the approach in Jacobs and Wang (2004). The main difference is that, in contrast with Jacobs and Wang (2004), we do not test the significance of the loadings of the stochastic discount factor on the risk factors, but rather whether the factors that proxy for uninsurable consumption risk (caused by incomplete consumption insurance) are priced, i.e., whether the risk premia for these factors are statistically significant in the cross-section of expected returns stocks. This article constitutes the first attempt to examine this issue.

Using a Taylor series approximation, we show that the average marginal utility of consumption is an affine function of the normalized cross-sectional moments of the individual consumption distribution with the coefficients defined in terms of the derivatives of the investor's utility function. Under complete consumption insurance, investors can use financial markets to fully insure their future consumption and hence are able to equalize, state-by-state, their optimal consumption growth rates. If so, the normalized cross-sectional consumption moments of order two and higher are all time-invariant. Only the change in average consumption (the first cross-sectional consumption distribution moment) influences the average marginal utility of consumption of stockholders and hence (through its covariance with security returns) the expected returns on stocks. If consumption insurance is incomplete, then the normalized cross-sectional consumption moments of order two and higher may change over time. Thus, the rates of change in these moments may also, along with the growth rate of average consumption of stockholders, influence (through their impact on the average marginal utility of consumption) expected stock returns and hence may also be regarded as sources of investment risk.

This suggests that risk in the consumption of stock market participants may be decomposed

[3] The potential of the incomplete consumption insurance hypothesis to help explain the equilibrium behavior of stock and bond returns, both in terms of the level of equilibrium rates and the discrepancy between equity and bond returns, was first suggested by Bewley (1982), Mehra and Prescott (1985), and Mankiw (1986). Constantinides and Duffie (1996), Brav, Constantinides, and Géczy (2002), Balduzzi and Yao (2007), and Kocherlakota and Pistaferri (2009) also argue that consumers' heterogeneity induced by the incompleteness of consumption insurance can be relevant for asset pricing.

into two parts, insurable consumption risk and uninsurable consumption risk. The rate of change in average consumption (the first cross-sectional consumption distribution moment) may be viewed as a proxy for the insurable part of consumption risk, while the rates of change in the higher-order normalized cross-sectional consumption moments may be regarded as a multivariate proxy for uninsurable risk in the consumption of stockholders. Hereafter, we refer to the rates of change in average consumption and normalized cross-sectional consumption moments of order two and higher as the consumption moment risk factors.

The signs of the risk premia associated with different consumption moment risk factors depend on whether a change in the corresponding cross-sectional consumption moment has a positive or negative effect on the average marginal utility of consumption. Non-satiation, risk aversion, prudence, temperance, and edginess in the investor's preferences enable us to sign the first five derivatives of the agent's utility function and hence the risk premia for the rates of change in average consumption and the second through fourth normalized cross-sectional moments of the consumption distribution. We demonstrate that the restrictions imposed by the preference assumptions imply the positive risk premia for the rates of change in average consumption and the third normalized cross-sectional consumption moment and the negative risk premia for rates of change in the second and fourth normalized cross-sectional moments of individual consumption distribution.

To examine whether incomplete consumption insurance has pricing significance in direct competition with other risk factors deemed to systematically affect stock returns, we consider a multifactor asset-pricing model, which extends the traditional FF (1993) three-factor model by including as additional risk factors the rates of change in average consumption and the second through fourth normalized cross-sectional consumption moments. The implied multibeta expected return relation nests as special cases a number of alternative specifications considered in the literature. The APT predicts that when the consumption moment risk factors are included along with the three portfolio risk factors, they should all be rejected as affecting stock market returns. Alternatively, the consumption-based theories predict that the FF (1993) three factors should not be significantly priced. The complete consumption insurance model is a special case of this model when the rates of change in the normalized cross-sectional consumption moments of order two and higher are not priced. If uninsurable risk in the consumption of stockholders is rewarded in the stock market, then the rates of change in the second through fourth normalized cross-sectional (stockholder) consumption moments (jointly or individually) have pricing significance as against the three portfolio risk factors and the aggregate (stockholder) consumption growth rate.

To estimate the factor risk premia, we employ a version of the Fama-MacBeth (1973) two-step estimation technique. Exploiting microlevel household quarterly consumption data from the US Consumer Expenditure Survey (CES) over 1982-2008, we find that uninsurable risk in the consumption of stockholders is separately rewarded in the stock market. This is the most notable finding of this paper. This result is based on the tests of the joint significance of the risk premia for the rates of change in the second through fourth normalized cross-sectional moments of individual consumption distribution.

Multicollinearity in the rates of change in the normalized cross-sectional consumption moments of order two to four implies high sampling variability of the estimates of the risk premia that makes it difficult to sort out the separate effects of these risk factors on expected stock market returns. Because of this, we also consider a model in which the rate of change in the normalized cross-sectional variance is the only factor that proxies for uninsurable consumption risk and conduct the test of whether the rate of change in the normalized cross-sectional variance is significantly priced. We find that, when the limited participation of households in the capital markets is taken into consideration, the risk premium of this risk factor is significantly different from zero and negative, as predicted by the preference theory. This result reinforces evidence that uninsurable risk in the consumption of stockholders is priced in the cross-section of expected stock returns. Another finding is that, in contrast with uninsurable risk, insurable risk in the consumption of stockholders fails to be rewarded in the stock market.

In testing the APT against the consumption beta theories in an integrated framework, we find that none of these competing theories is rejected empirically. The set of the FF (1993) three risk factors and the set of the consumption moment (of order up to four) risk factors are both found to be significantly priced in the cross-section of expected stock returns.

The rest of the paper proceeds as follows. In Section 2, we express the average marginal utility of consumption as an affine function of the normalized cross-sectional consumption distribution moments and show that changes in the (stockholder) consumption moments of order two and higher, along with the rate of change in average consumption of stockholders, are potentially important determinants of expected excess stock returns. Then, we show how preference assumptions enable to sign the risk premia for the consumption moment (of order up to four) risk factors and introduce a multifactor model that allows to test whether uninsurable consumption risk is priced in the cross-section of expected stock returns in direct competition with insurable consumption risk and the FF (1993) systematic risk factors. Section 3 tests empirically whether uninsurable consumption risk is significantly rewarded in the stock market as against the FF (1993) risk factors and insurable risk in consumption. Section 4 concludes.

2 Incomplete Consumption Insurance and Expected Stock Returns

2.1 Consumption Moment Risk Factors

Consumption-based asset pricing theories rely on the assumption that the asset's systematic risk is measured by the covariance of the asset return with the growth rate of aggregate marginal utility of consumption.

Consider an economy populated by a continuum of agents with homogenous preferences. Assume that the individual's utility function is $S+1$ times differentiable and take the Sth-order Taylor approximation of the time t average marginal utility of consumption, $I^{-1}\sum_{i=1}^{I} u'(C_{it})$, around average consumption $\overline{C}_t = I^{-1}\sum_{i=1}^{I} C_{it}$:

$$\frac{1}{I}\sum_{i=1}^{I} u'(C_{it}) \approx u'(\overline{C}_t) + \sum_{s=2}^{S} \frac{1}{s!} u^{(s+1)}(\overline{C}_t)\mu_{s,t} = u'(\overline{C}_t) + \sum_{s=2}^{S} \frac{1}{s!} u^{(s+1)}(\overline{C}_t)\overline{C}_t^s \widetilde{\mu}_{s,t}, \ t = 1, ..., T, \quad (1)$$

where C_{it} is the consumption of agent i in period t, u is the agent's continuously differentiable von Neumann-Morgenstern period utility-of-consumption function, u' is the first derivative of utility with respect to consumption,[4] $\mu_{s,t} \equiv I^{-1}\sum_{i=1}^{I}(C_{it} - \overline{C}_t)^s$ is the sth centered moment of the time t cross-sectional consumption distribution, and $\widetilde{\mu}_{s,t}$ (henceforth referred to as the sth normalized cross-sectional consumption moment) is $\mu_{s,t}$ normalized by $\overline{C}_t^s$, $\widetilde{\mu}_{s,t} \equiv I^{-1}\sum_{i=1}^{I}(C_{it}/\overline{C}_t - 1)^s$. Here and throughout the paper, $u^{(s)}$ denotes the sth derivative of u.

Suppose that investors can freely trade in the frictionless capital markets. If consumption insurance is complete, then the investors are able to fully insure their future consumption by equalizing, state-by-state, their intertemporal marginal rates of substitution in consumption and, therefore, their optimal consumption growth rates. Thus, with complete consumption insurance, the normalized cross-sectional consumption moments of order two and higher in (1) are all time invariant and, therefore, the rate of change in average consumption (the first cross-sectional consumption moment) may be regarded as the only factor that influences the average marginal utility of consumption and hence expected asset returns.

The incompleteness of consumption insurance makes it impossible for the agents to fully insure their consumption and hence realized consumption growth rates can differ across the investors. If so, the normalized cross-sectional consumption moments of order two and higher are not necessarily constant over time and, therefore, changes in these consumption distribution moments, along with a change in average consumption, may also contribute (through their impact on the average marginal utility of consumption) to expected asset returns.

[4] As is conventional in the literature, we assume that $u' > 0$ (non-satiation) and $u'' < 0$ (risk aversion).

The rates of change in the normalized cross-sectional consumption moments of order two and higher can hence be viewed as a multivariate proxy for uninsurable risk in consumption (caused by incomplete consumption insurance), while the rate of change in average consumption (the first cross-sectional consumption moment) proxies the influence of the part of consumption risk that can be fully insured by trading financial securities (i.e., insurable consumption risk).

As pointed out in Cochrane (2005), in the consumption-based framework the expected return-beta representation of a linear factor pricing model is

$$E\left[\mathbf{Z}_t\right] = \gamma_{0t}\boldsymbol{\iota}_J + \mathbf{B}_1\boldsymbol{\gamma}_{1t},\ t = 1, ..., T, \tag{2}$$

where $\mathbf{B}_1$ is defined as the $(J \times K_1)$ matrix of the assets' exposures to the risk factors or systematic risk measures in a multiple regression of asset excess returns on factors that proxy for marginal utility growth,

$$\mathbf{Z}_t = \boldsymbol{\alpha} + \mathbf{B}_1\mathbf{f}_{1t} + \boldsymbol{\epsilon}_t, \tag{3}$$

$\mathbf{Z}_t$ is a $(J \times 1)$ vector of returns for J risky assets in excess of the risk-free rate of return, $\mathbf{Z}_t = [R_{1,t} - R_{f,t}\ R_{2,t} - R_{f,t}\ ...\ R_{J,t} - R_{f,t}]'$, $E\left[\mathbf{Z}_t\right]$ is a $(J \times 1)$ vector of expected excess returns, $\boldsymbol{\alpha}$ is a $(J \times 1)$ vector of asset return intercepts, $\mathbf{f}_{1t}$ is a $(K_1 \times 1)$ vector of factors that are proxies for marginal utility growth, $\boldsymbol{\epsilon}_t$ is a $(J \times 1)$ vector of factor model disturbances, $E\left[\boldsymbol{\epsilon}_t\right] = \mathbf{0}$, $E\left[\boldsymbol{\epsilon}_t\boldsymbol{\epsilon}_t'\right] = \boldsymbol{\Sigma}_{\boldsymbol{\epsilon}}$, $\boldsymbol{\Sigma}_{\boldsymbol{\epsilon}}$ is the variance-covariance matrix of $\boldsymbol{\epsilon}_t$, $\mathbf{0}$ is a $(J \times 1)$ vector of zeroes, $\boldsymbol{\iota}_J$ is a $(J \times 1)$ vector of ones, γ_{0t} is the excess zero-beta rate, and $\boldsymbol{\gamma}_{1t}$ is a $(K_1 \times 1)$ vector of factor risk premia.

The Taylor series expansion (1) provides some insight into the determinants of the average marginal utility of consumption. It suggests that the rates of change in aggregate consumption and the normalized cross-sectional consumption moments of order two and higher may be viewed as proxies for marginal utility growth. An attractive feature of this approach to specifying the risk factors is that the set of the consumption moment risk factors in model (2)-(3) obtain endogenously from the Taylor series approximation (1), while the APT does not provide the identification of the factors that systematically affect asset returns. This allows to avoid some serious problems arising from an ad hoc specification of a factor structure.

To examine the signs of the expected return premia for the consumption moment risk factors in the beta model (2), assume first that there is an (normalized cross-sectional consumption moments of order two and higher preserving) increase in the mean of the cross-sectional consumption distribution. This is the (complete consumption insurance) case when the consumption of each agent in the cross-section increases by the same factor. It follows from non-satiation and risk aversion that an increase in the consumption of each investor lowers the marginal utility of his consumption and hence the average (over the investors) marginal utility of consumption. The asset with a greater covariance (beta) with the growth rate of average consumption has thus a higher return when the marginal utility of consumption is low and, inversely, a lower return when the marginal utility is high (i.e., when consumption is most valuable). The inability to insure against adverse movements in consumption makes such an asset riskier to the investor, driving up the risk premium the investor demands to hold the asset.

As implied by equation (1), if $u^{(s+1)} > 0$ $(s = 2, 3, ...)$, then an increase in the sth normalized cross-sectional consumption moment $\widetilde{\mu}_{s,t}$ (holding all other consumption moments constant) raises the average marginal utility of consumption. Thus, the asset with a greater covariance (beta) with the growth rate of the sth normalized cross-sectional consumption moment tends to have a higher return when the average marginal utility of consumption is high and, therefore, is, in this sense, less risky, driving down the risk premium required by the investor. If, by contrast, $u^{(s+1)} < 0$ $(s = 2, 3, ...)$, then an increase in the sth normalized cross-sectional consumption moment (holding all other consumption moments constant) lowers the average marginal utility of consumption, thereby implying that the asset with a greater covariance (beta) with the growth rate of the sth normalized cross-sectional consumption moment has a higher return when the average marginal utility is low and, oppositely, a lower return when the average marginal utility of consumption is high. This makes the asset riskier to the investor, raising the risk premium demanded by the investor to induce him to hold this asset.

This suggests that the risk premium for the growth rate of average consumption should be positive. The signs of the risk premia for the rates of change in the normalized cross-sectional consumption moments of order two and higher depend on the signs of the third and higher derivatives of the agent's utility function. The risk premium for the growth rate of the sth normalized cross-sectional consumption moment is positive if $u^{(s+1)} < 0$ and negative if $u^{(s+1)} > 0$.

2.2 Signs of the Risk Factor Premia

Having showed in the previous section that with incomplete consumption insurance the rates of change in the normalized cross-sectional consumption moments of order two and higher may also, along with the growth rate of average consumption, be viewed as risk factors that contribute to the expected asset returns, we must now decide how many consumption moment risk factors must be taken into account or, stated differently, at which point to truncate the Taylor expansion (1).

One way to determine the order at which the expansion should be truncated is to allow data to motivate the point of truncation.[5] This approach consists in repeating the estimation of a model for increasing values of S and truncating the expansion at the point when further increasing in S does not significantly affect the estimation results. As Dittmar (2002) points out, there are at least two difficulties with allowing data to determine the required order of a Taylor expansion. The first one is the possibility of overfitting the data. Another problem is that when a high-order expansion is taken, preference theory no longer guides in determining the signs of risk factor prices. To avoid the latter problem, Dittmar (2002) proposes to let preference arguments determine the point of truncation. He shows that non-satiation, risk aversion, decreasing absolute risk aversion, and decreasing absolute prudence imply $u^{(4)} < 0$. Since preference assumptions do not guide in determining the signs of the higher-order derivatives, Dittmar (2002) argues that the Taylor expansion terms of order higher than three do not matter for asset pricing and truncates a Taylor expansion after the cubic term.[6] He claims that the advantage coming from signing the Taylor expansion terms outweighs a loss of power due to omitting the terms of order four and higher.

Following Dittmar (2002), we also let preference theory determine the order of the Taylor series approximation. Below, we will show that, in contrast with the sign of u'' that characterizes the agent's attitudes towards market risk regardless of a specific choice problem, signing the higher-order derivatives of the utility function is based upon the context in which the risk associated with the investment decision arises.

The Taylor series approximation (1) implies that the sign of the contribution of the normalized cross-sectional variance to the average marginal utility of consumption is determined by the sign of u'''. Leland (1968), Sandmo (1970), and Drèze and Modigliani (1972), e.g., argue that if the agent's absolute risk aversion is decreasing (i.e., $u''' > 0$), then the agents save more in order to self-insure against the additional variability in their consumption streams caused by incomplete consumption insurance.[7] Kimball (1990) defines prudence ($u''' > 0$) as a measure of the sensitivity of the optimal choice of a decision variable to risk (of the intensity of the precautionary saving motive in the context of the consumption-saving decision under uncertainty). A precautionary saving motive is positive when $-u'$ is concave ($u''' > 0$) just as an individual is risk averse when u is concave.

The intuition is strong to suggest that the unavailability of insurance against consumption risk

[5] See Bansal, Hsieh, and Viswanathan (1993).

[6] Brav, Constantinides, and Géczy (2002) also limit their analysis to a third-order approximation when using a Taylor series expansion of the equal-weighted average of the household's intertemporal marginal rates of substitution. Cogley (2002) stops at a third-order polynomial when taking a Taylor series expansion of the individual's intertemporal marginal rates of substitution.

[7] Courbage and Rey (2007) stress that $u''' > 0$ is still a necessary and sufficient condition for a positive precautionary saving motive when a non-financial risk and the financial market risk are independent. They show that the set of sufficient conditions is more complex if the risks are dependent.

must increase the aversion of a decision maker to the risk associated with the investment in a risky asset. The preferences with such a property are said to exhibit risk vulnerability.[8] Gollier (2001) shows that preferences exhibit risk vulnerability if at least one of the following two conditions is satisfied: (i) absolute risk aversion is decreasing and convex and (ii) both absolute risk aversion and absolute prudence are positive and decreasing in wealth. This latter condition is referred to as standard risk aversion, the concept introduced by Kimball (1993).[9]

Non-satiation ($u' > 0$), risk aversion ($u'' < 0$), and prudence ($u''' > 0$) together imply that the absolute risk aversion coefficient, $ARA = -u''/u'$, and the coefficient of absolute prudence, $AP = -u'''/u''$, are both positive, as required by (ii). Prudence ($u''' > 0$) is also the necessary (but not sufficient) condition for decreasing absolute risk aversion. To complete the set of the conditions necessary for risk vulnerability, we, therefore, need to determine the properties of utility u required for absolute prudence to be decreasing in wealth (for the set of conditions (ii) to be met) or absolute risk aversion to be convex (for the set of conditions (i) to be met).

Intuitively, the willingness to save should be an increasing function of the expected marginal utility of future consumption. Since the marginal utility is decreasing in consumption, at a given level of uninsurable consumption risk the absolute level of precautionary savings must also be expected to decline as consumption rises.

Proposition 1 *Absolute prudence is decreasing if and only if $u^{(4)} < -APu'''$. The condition $u^{(4)} < 0$ is necessary for decreasing absolute prudence (see Appendix A for the proof).*

The condition $u^{(4)} < 0$ is referred to as temperance.[10] Kimball (1992) defines temperance as a type of behavior when the presence of an uninsurable risk leads the agent to reduce his exposure to another independent risk. Menezes and Wang (2005) provide an interpretation of $u^{(4)} < 0$ in the context of choice between pairs of risky prospects. They argue that $u^{(4)} < 0$ can also be interpreted as aversion to relocations of dispersion from the center of a distribution to its tails (aversion to outer risk).

It is easy to check that the condition $u^{(4)} < 0$ is also necessary (but not sufficient) for convex absolute risk aversion. Thus, the assumptions of non-satiation ($u' > 0$), risk aversion ($u'' < 0$), prudence ($u''' > 0$), and temperance ($u^{(4)} < 0$) form the set of necessary (but not sufficient) conditions for the agent's preferences to exhibit risk vulnerability.

It looks natural to assume that for each level of uninsurable consumption risk the absolute level

[8] See Gollier and Pratt (1996) and Gollier (2001), e.g.

[9] Kimball (1993) shows that standard risk aversion implies risk vulnerability.

[10] See Kimball (1992).

of precautionary savings should decline in wealth at a decreasing rate and hence that, like absolute risk aversion, absolute prudence is convex.

Proposition 2 *Absolute prudence is convex if and only if* $u^{(5)} > -2AP'u''' - APu^{(4)}$*. If preferences exhibit prudence and decreasing absolute prudence, then* $u^{(5)} > 0$ *is the necessary condition for convex absolute prudence (see Appendix B for the proof).*

Lajeri-Chaherli (2004) considers a two-period optimal consumption choice problem with income uncertainty in the second period. She shows that $u^{(5)} > 0$ is a necessary condition for decreasing absolute temperance and labels this condition as edginess.[11] Eeckhoudt, Schlesinger, and Tsetlin (2010) interpret edginess "as implying that a decrease in one risk (via second order stochastic dominance) helps to temper the effects of an increase in downside risk of another additive risk".

The restriction of convex absolute prudence enables us to sign $u^{(5)}$ and hence to expand the average of investors' marginal utilities of consumption up to the fourth cross-sectional consumption moment, further than it is done in Dittmar (2002).

Therefore, reference to the agent's behavior in the presence of uninsurable consumption risk (caused by incomplete consumption insurance) enables us to justify the signs of u''', $u^{(4)}$, and $u^{(5)}$. Combined with the conditions of non-satiation ($u' > 0$) and risk aversion ($u'' < 0$), this makes it possible to sign the risk premia for the rate of change in average consumption and the rates of change in the second through fourth normalized cross-sectional consumption moments. Denote the rate of change in average consumption as ACG and the rates of change in the normalized cross-sectional consumption moments of order two, three, and four as SMG, TMG, and FMG, respectively. As argued in Section 2.1, non-satiation ($u' > 0$) and risk aversion ($u'' < 0$) imply that the risk premium for ACG should be positive. From prudence ($u''' > 0$) and edginess ($u^{(5)} > 0$), it follows that the risk premia for SMG and FMG should be negative and, finally, temperance ($u^{(4)} < 0$) implies that the risk premium for TMG should be positive.

2.3 A Multifactor Model of Risk

To investigate the pricing significance of the consumption moment risk factors, we consider the multifactor pricing relation of the form

$$E\left[\mathbf{Z}_t\right] = \gamma_{0t}\boldsymbol{\iota}_J + \mathbf{B}_1\boldsymbol{\gamma}_{1t} + \mathbf{B}_2\boldsymbol{\gamma}_{2t},\ t = 1, ..., T, \tag{4}$$

where

$$\mathbf{Z}_t = \boldsymbol{\alpha} + \mathbf{B}_1\mathbf{f}_{1t} + \mathbf{B}_2\mathbf{f}_{2t} + \boldsymbol{\varepsilon}_t, \tag{5}$$

[11] Lajeri-Chaherli (2004) calls the term $-u^{(5)}/u^{(4)}$ absolute edginess.

$\mathbf{f}_{1t}$ is the $(K_1 \times 1)$ vector of the consumption moment risk factors, $\mathbf{B}_1$ is the $(J \times K_1)$ matrix of betas on the consumption moment risk factors, $\boldsymbol{\gamma}_{1t}$ is the $(K_1 \times 1)$ vector of consumption moment factor risk premia, $\mathbf{f}_{2t}$ is a $(K_2 \times 1)$ vector of the systematic risk factors motivated by the APT, $\mathbf{B}_2$ is a $(J \times K_2)$ matrix of the assets' exposures to the $\mathbf{f}_{2t}$ vector of risk factors, $\boldsymbol{\gamma}_{2t}$ is a $(K_2 \times 1)$ vector of associated risk premia, $\boldsymbol{\varepsilon}_t$ is a $(J \times 1)$ vector of factor model disturbances, $E[\boldsymbol{\varepsilon}_t] = \mathbf{0}$, $E[\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t'] = \boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}$, $\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}$ is the variance-covariance matrix of $\boldsymbol{\varepsilon}_t$. In this model, all the variables are expressed as decimals.

The consumption beta theories argue that the reward-to-risk ratio of an asset depends on its sensitivity to overall aggregate marginal utility of consumption. Implicitly assuming complete consumption insurance, one usually measures the asset's systematic risk by the covariance (beta) of the asset return with the growth rate of average consumption (the first moment of the cross-sectional consumption distribution) alone. As we argued in Section 2.1, the incompleteness of consumption insurance implies that the rates of change in the second and higher-order normalized cross-sectional consumption moments may also be regarded as pricing factors. Section 2.2 showed that reference to the agent's behavior in the presence of incomplete consumption insurance enables to sign the risk premia for the rate of change in average consumption, ACG, and the rates of change in the second through fourth normalized cross-sectional consumption moments, SMG, TMG, and FMG. This suggests that the $\mathbf{f}_{1t}$ vector of the consumption moment risk factors should include, in addition to ACG, variables SMG, TMG, and FMG. The inclusion of SMG, TMG, and FMG would allow to take into account the influence of incomplete consumption insurance on expected stock returns.

The approaches for identifying the APT factors in $\mathbf{f}_{2t}$ fall into two main categories. One approach consists in assuming that innovations in macroeconomic variables are risks that are rewarded in the stock market. An example of the this approach is CRR (1986), who argue that likely candidates for the systematic factors responsible for the comovements of asset prices are forces that influence changes in the discount factor and forces that affect expected cash flows. Based on this argument, CRR (1986) model stock returns as functions of macroeconomic state variables and nonequity asset returns. They show that the yield spread between long and short interest rates (the maturity premium), expected inflation and unexpected inflation, industrial production growth, and the yield spread between corporate high- and low-grade bonds (the default premium) can be regarded as factors that systematically influence all stocks. The evidence on the ability of the implied five-factor model to explain a cross-section of stock portfolio returns is mixed. While CRR (1986) find empirically that the identified sources of risk are significantly rewarded in the stock market, Shanken and Weinstein (2006), e.g., find the pricing of the CRR (1986) macroeconomic variables to be very sensitive to reasonable alternative techniques for calculating the returns on size portfolios and estimating the factor betas. Using the full-period post-ranking return approach, they find that only the quarterly growth rate of industrial production is significantly priced.

A second approach is to identify the firm characteristics deemed to be important in explaining the cross-sectional variation in average stock returns and then use the returns on portfolios of stocks formed based on these characteristics as risk factors. FF (1993) is a good example of this approach. FF (1993) construct a model with three risk factors, MKT, SMB, and HML, where MKT is the return on a proxy for the market portfolio in excess of the risk-free rate, SMB is the excess returns of small caps over big caps (small (market capitalization) minus big) portfolios, and HML is the excess returns of value over growth (high (book-to-market ratio) minus low) portfolios.

Model (4)-(5) with the risk factors being the APT factors and the consumption moment risk factors ACG, SMG, TMG, and FMG, nests as special cases a number of alternative specifications considered in the literature. If none of the consumption moment betas is significantly priced, then we obtain the multibeta expected return relation associated with the conventional APT model. Alternatively, under the null hypothesis of the consumption beta theories, no measure of risk, in addition to the consumption moment betas, is rewarded in the stock market and, therefore, the betas of the risk factors other than the consumption moment risk factors should all be rejected statistically as having an influence on excess stock returns. If uninsurable risk in consumption is priced, then the risk premia for SMG, TMG, and FMG are (individually or jointly) statistically significant. The influence of insurable consumption risk on excess stock returns is captured by the rate of change in aggregate consumption, ACG. Thus, one can test whether insurable risk in consumption is separately rewarded in the stock market by testing the null hypothesis that the risk premium for ACG is statistically positive.

To test the APT model against the model motivated by the consumption beta theory in an integrated framework, CRR (1986) augment the set of factors in their five-factor pricing equation by the inclusion of the percentage change in real aggregate (in the economy) per capita consumption. CRR (1986) find that average consumption have no explanatory power beyond the identified economic state variables in describing the cross-section of expected stock returns. The approach in CRR (1986) may be criticized at two levels. First, in their model CRR (1986) use as a consumption-based risk factor the growth rate of average consumption (the first cross-sectional consumption moment) alone and, therefore, implicitly assume the completeness of consumption insurance, while, as we argued in the previous section, under incomplete consumption insurance, higher-order consumption moments also affect marginal utility of consumption and hence changes in these moments of the consumption distribution may also be regarded as risk factors that systematically influence stock market returns. Second, when calculating the real per capita consumption growth rate, CRR (1986) suppose aggregate consumption in the economy to be an adequate proxy for the consumption of stockholders. However, it is observed that only a small fraction of individuals in the population hold stocks either directly or indirectly.[12] Because the consumption of non-stockholders is irrelevant to the determination of stock prices (but may be a large fraction of aggregate consumption in the economy) and the consumption of stock market investors is more highly correlated with stock returns than the consumption of the agents that do not participate in the stock markets,[13] it can then be supposed that the changes in per capita consumption of stockholders may have a more significant influence on expected stock returns compared with the changes in aggregate per capita consumption in the economy. Malloy, Moskowitz, and Vissing-Jørgensen (2009), e.g., use quarterly data and find empirical evidence that the use of per capita consumption growth of stockholders provides a better (than aggregate in the economy consumption growth) fit when capturing cross-sectional average stock returns. Similarly to CRR (1986), Malloy, Moskowitz, and Vissing-Jørgensen (2009), however, also ignore higher-order consumption moments and hence also rely on the assumption of complete consumption insurance.

3 Data, Estimation, and Testing

In a number of empirical studies, it has been shown that the FF (1993) three-factor model does a good job in explaining the cross-section of expected stock returns. Given this, to examine whether uninsurable risk is separately rewarded in the stock market in direct competition with insurable consumption risk and the systematic risk factors motivated by the APT, we consider the model in which the $\mathbf{f}_{1t}$ vector of factors includes, in addition to ACG, the variables SMG, TMG, and FMG and the $\mathbf{f}_{2t}$ vector contains the FF (1993) three risk factors MKT, SMB, and HML as the likely sources of investment risk. In the implied multibeta expected return relation, the risk premia for MKT, SMB, and HML are unrestricted, while the signs of the consumption moment risk premia are driven by preference assumptions. The findings in the previous section suggest that the risk-premium measures associated with ACG and TMG should be positive and the risk premia for the factors SMG and FMG should be negative for the estimation results to be consistent with the preference

[12] According to the US CES, e.g., only nearly 19% of households own stocks, bonds, mutual funds, and other such securities. Using data from the Family Expenditure Survey, Attanasio, Banks, and Tanner (2002) observe that, in keeping with the US, only about 20-25% of UK households own equities directly. Agell and Edin (1990) find that only 18.6% of Swedish households hold common stocks.

[13] See Vissing-Jørgensen (2002) and Malloy, Moskowitz, and Vissing-Jørgensen (2009), e.g. Campbell and Mankiw (1990), Mankiw and Zeldes (1991), Basak and Cuoco (1998), Alvarez and Jermann (2000), and Constantinides, Donaldson, and Mehra (2002), among others, argue that market frictions, such as transactions costs and limits on borrowing or short sales, can make aggregate consumption in the economy an inadequate proxy for the consumption of stock market investors.

theory. In the present section, we use this model to test empirically the pricing significance (in the cross-section of expected stock returns) of uninsurable consumption risk as against insurable consumption risk and the FF (1993) three risk factors. We start this section by describing the data set used in the study. Then, we explain the estimation and testing methodology and report the results.

3.1 The Data

The consumption moment risk factor variables are calculated using individual consumption data from the US CES. Since these data are at best available on a quarterly basis, we work with data at the quarterly frequency. Our sample period extends from 1982:Q1 to 2008:Q4, which makes 108 quarterly observations for each variable in the data set.

3.1.1 The Fama-French Factors

The data on the FF (1993) risk factors are downloaded from Kenneth French's web page. The factors are constructed using the six value-weighted portfolios formed on size and book-to-market. The variable MKT is the return on the value-weighted stock index (capital gain plus all dividends) on all NYSE, Amex, and NASDAQ stocks (from the Center for Research in Security Prices (CRSP) of the University of Chicago) in excess of the 3-month Treasury Bill return from Ibbotson and Associates. The variable SMB is calculated as the average return on the three small portfolios minus the average return on the three big portfolios and the variable HML is calculated as the average return on the two value portfolios minus the average return on the two growth portfolios.

3.1.2 The Consumption Moment Risk Factors

We use quarterly consumption data from the US CES produced by the US Bureau of Labor Statistics (BLS). The CES data available cover the period from 1980:Q1 to 2008:Q4. It is a collection of data on approximately 5000 households per quarter in the US. Each household in the sample is interviewed every three months over five consecutive quarters (the first interview is practice and is not included in the published data set). As households complete their participation, they are dropped and new households move into the sample. Thus, each quarter about 20% of the consumer units are new. The second through fifth interviews use uniform questionnaires to collect demographic and family characteristics as well as data on quarterly consumption expenditures for the previous three months made by households in the survey (demographic variables are based upon heads of households). Various income information and information on the employment of each household member is collected in the second and fifth interviews. As opposed to the Panel Study of Income Dynamics (PSID), which offers only food consumption data on an annual basis, the CES contains highly detailed data on quarterly consumption expenditures.[14] The CES attempts to account for an estimated 70% of total household consumption expenditures. Since the CES is designed with the purpose of collecting consumption data, measurement error in consumption is likely to be smaller for the CES consumption data compared with the PSID consumption data.

As suggested by Attanasio and Weber (1995), Brav, Constantinides, and Géczy (2002), and Vissing-Jørgensen (2002), we drop all consumption observations for the years 1980 and 1981 because the quality of the CES consumption data is questionable for this period. Thus, our sample covers the period extending from 1982:Q1 to 2008:Q4.[15] Following Brav, Constantinides, and Géczy (2002), in each quarter we drop from the CES data set households that do not report or report a zero value of consumption of food, consumption of nondurables and services, or total consumption. We also delete from the sample nonurban households, households residing in student housing, households with incomplete income responses, households that do not have a fifth interview, and households whose head is under 19 or over 75 years of age.

In the fifth (final) interview, the household is asked to report the end-of-period estimated market value of "all stocks, bonds, mutual funds, and other such securities" held by the consumer unit on the last day of the previous month as well as the difference in this estimated market value compared with the value of all securities held a year ago last month. Using these two values, we calculate each consumer unit's asset holdings at the beginning of a 12-month recall period in constant 2005 dollars. We consider four sets of households. First, we consider all households regardless of the reported holdings of "all stocks, bonds, mutual funds, and other such securities" at the beginning of a 12-month recall period (we denote this set of households as HS1). To recognize the limited participation of households in the stock markets, following Vissing-Jørgensen (2002) and Malloy, Moskowitz, and Vissing-Jørgensen (2009), we define as stockholders households that report to the CES holdings of "all stocks, bonds, mutual funds, and other such securities" above some threshold. We consider three sets of households classified as stockholders. The first set of stockholders (HS2) includes households that report an amount of total holdings of "all stocks, bonds, mutual funds, and other such securities" at the beginning of a 12-month recall period equal to or exceeding $1000 (in 2005 dollars). The second set of stockholders (HS3) consists of households with the reported amount of asset holdings equal to or exceeding $5000 (in 2005 dollars), and, finally, households that

[14] Food consumption is likely to be one of the most stable consumption components. Furthermore, as is pointed out by Carroll (1994), 95% of the measured food consumption in the PSID is noise due to the absence of interview training.

[15] We did not extend our dataset past 2008 because of the crash of the stock market in late 2008.

report an estimated market value of all securities held a year ago equal to or greater than \$10000 (in 2005 dollars) are grouped in the third set of households identified as stockholders (we refer to this set of households as HS4).[16]

As is conventional in the literature, the consumption measure used in this paper is consumption of nondurables and services. For each household, we calculate quarterly consumption expenditures for all the disaggregate consumption categories offered by the CES. Then, we deflate obtained values in 2005 dollars with the CPI's (not seasonally adjusted, urban consumers) for appropriate consumption categories.[17] Aggregating the household's quarterly consumption across these categories is made according to the National Income and Product Account (NIPA) definition of consumption of nondurables and services. To mitigate measurement error in individual consumption, we subject the households to a consumption growth filter and use the conventional z-score method to detect outliers. Following common practice for highly skewed data sets, in each quarter we consider the consumption growth rates with z-scores greater than two in absolute value to be due to reporting or coding errors and remove from the sample the households whose real per capita consumption has a z-score greater than two in absolute value for this period.[18]

Aggregate consumption per capita, C_t, within each set of households is calculated as the average per capita consumption expenditures of the households in the set. For each set of households, the aggregate per capita consumption growth rate between two quarters $t-1$ and t, C_t/C_{t-1}, is seasonally adjusted by using multiplicative adjustments obtained from the X-12 procedure. The quarter t rate of change in the normalized moment of order s $(s = 2, 3, 4)$ of the cross-sectional consumption distribution for each set of households is calculated as the ratio of the value of the normalized cross-sectional consumption moment in quarter t, $\widetilde{\mu}_{s,t}$, to the value of this moment in quarter $t-1$, $\widetilde{\mu}_{s,t-1}$, minus one, i.e., as $\widetilde{\mu}_{s,t}/\widetilde{\mu}_{s,t-1} - 1$. Individual consumption in the CES is the flow of consumption during a quarter and is not measured at the end of each quarter as the other variables. To deal with this problem, as suggested in CRR (1986), subsequent statistical work leads the time series of the quarterly rates of change in average consumption and the normalized moments of order two to four of the cross-sectional consumption distribution forward by one quarter.

[16] Since the CES reports only some limited information about asset holdings, we consider consumer units that report an amount of total holdings of "all stocks, bonds, mutual funds, and other such securities" equal to or exceeding \$1000, \$5000, and \$10000 (rather then a positive amount of total asset holdings) in order to reduce the likelihood of including households, who do not participate in the capital markets. See Cogley (2002) and Vissing-Jørgensen (2002), e.g., for more details.

[17] The CPI series are obtained from the BLS through CITIBASE.

[18] The quarterly consumption growth between dates $t-1$ and t is calculated if consumption is not equal to zero for both of the quarters (missing information is counted as zero consumption).

3.1.3 The Returns Data

Like Jacobs and Wang (2004), we use a set of test portfolios that includes the 25 size and book-to-market equity sorted portfolios of Fama and French (1993).[19] The nominal quarterly value-weighted portfolio returns (capital gain plus all dividends) are calculated by compounding monthly returns on these portfolios from Kenneth French's web page. The nominal quarterly risk-free rate is the 3-month Treasury Bill return from Ibbotson and Associates, Inc. The real quarterly returns are calculated as the nominal quarterly returns divided by the 3-month inflation rate based on the deflator defined for consumption of nondurables and services. Excess returns are calculated as the differences between the real portfolio returns and the real risk-free rate.

3.1.4 Descriptive Statistics

Tables I and II report statistical characteristics of the variables defined above computed over the entire sample period, 1982:Q1 - 2008:Q4. Table I displays the means, standard deviations, and correlation coefficients for the factors. The estimates reported in Table I show that ACG is significantly positively correlated with SMG (for the sets of households HS1, HS2, and HS3) and FMG (for the sets of households HS2 and HS3). For the sets of households identified as stockholders, the collinearity tends to weaken as the threshold value in the definition of stockholders is raised. Despite their statistical significance, these correlation coefficients are all small in absolute value. The SMG, TMG, and FMG series are all strongly positively correlated with each other suggesting that all the three convey essentially the same information.

The high collinearity of these risk factors as well as the multicollinearity of the FF (1993) risk factors MKT, SMB, and HML may lead to imprecise estimates of the loadings of the stock returns on these variables and hence may be viewed as a potential source of the errors-in-variables (EIV) complication. The MKT variable is significantly positively correlated with ACG for the set of all households regardless of the reported amount of assetholdings (HS1) and significantly negatively correlated with SMG, TMG, and FMG for the sets of households HS3 and HS4. This suggests that the growth rate of average consumption, ACG, is procyclical, while the growth rates of the second through fourth cross-sectional consumption moments, SMG, TMG, and FMG, respectively, are all countercyclical. The SMB series exhibits statistically significant negative correlation with SMG, TMG, and FMG. No significant correlation is estimated between HML and the consumption moment risk factors ACG, SMG, TMG, and FMG.

[19] The portfolios are constructed at the end of each June as the intersections of 5 portfolios formed on size and 5 portfolios formed on the ratio of book equity to market equity.

[Table I]

Table II displays the autocorrelations and Ljung-Box Q-statistics with five and ten autocorrelations. The FF (1993) factors generally display mild autocorrelations except for HML for which the first-order autocorrelation coefficient and the Ljung-Box Q-statistic with ten autocorrelations are both statistically significant at the 5% level.

Consumption moment risk factors ACG, TMG, and FMG exhibit statistically significant low-order serial correlation for the set of households regardless of the reported amount of assetholdings (HS1). As the definition of asset holders is tightened to recognize the limited participation of households in the capital markets, the ACG, TMG, and FMG series display less significant low-order autocorrelations and more statistically significant higher-order serial correlations. For the sets of households classified as stockholders, the number of statistically significant autocorrelation coefficients decreases as the thresholds value in the definition of stockholders is raised. Only for ACG the Ljung-Box Q-statistics are statistically significant at the 5% (for HS1 and HS2) and 10% (for all the sets of households) significance levels. This suggests that for the sets of households defined as stockholders the consumption moment risk factors SMG, TMG, and FMG are noisy enough to be treated as innovations, as required by the model.

[Table II]

3.2 Testing Methodology

To examine whether the identified risk factors explain pricing in the stock market, we employ the following version of the Fama-MacBeth (1973) two-step estimation technique. In a first-step time-series regression, we project each portfolio excess return on the underlying risk factors to estimate the portfolios' exposure (betas) to these factors over some estimation period (we used the previous 28 quarters). In the second step, for the following quarter we run a CSR of the portfolio excess returns (being the dependent variable) for the quarter on the first-step estimates of the factor betas (used as the independent variables). This provides an estimate of the risk premium associated with each risk factor for that quarter. Then, we move one quarter ahead and repeat these steps. As in CRR (1986), we use ordinary least squares (OLS) regressions.

Because with the 28-quarter prior betas the data for the first 28 quarters are used to estimate the factor loadings employed in the first CSR, this estimation technique yields for each risk factor a time series of quarter-by-quarter estimates of the associated risk premium for the period from 1989:Q1 to 2008:Q4 (i.e., 80 quarterly CSR estimates for each factor risk premium). A factor is assumed to be significantly priced in the stock market if the time-series mean of the estimate of its associated risk premium is statistically different from zero.

A potential problem with the two-step CSR methodology is that the true risk factor betas are unobservable and the second-step CSRs use betas estimated from the data. This introduces the EIV problem in the second step CSRs that potentially could lead to biased estimates of the risk premia. In order to reduce the EIV problem and reduce the noise in individual stock returns, it is common to aggregate stocks into portfolios and apply the two-step estimation methodology to portfolio returns rather than the returns on individual stocks. Using portfolios reduces estimation error in the asset betas and thus mitigates the EIV complication. CRR (1986), e.g., group stocks into 20 size portfolios according to the market values of stocks at the end of each five-year ranking (prior-beta-estimation) period. Ball and Kothari (1989) consider the case where systematic risk is measured by market beta and document that betas calculated for the portfolios formed by ranking stocks on their market values at the end of the ranking period (the most immediate past five years) give biased assessments of the portfolio betas beyond the ranking date. They observe that simple annual averages of the estimates of market beta for the ranking periods are biased downward (upward) estimates of the simple annual averages of the postranking-period (the five years following the ranking year) market betas for small (large) size portfolios. Even larger discrepancies between the ranking period and post-ranking period market betas are observed by Ball and Kothari (1989) for the portfolios constructed by ranking stocks according to their returns during the ranking period. It was found that the average portfolios' ranking-period beta estimates substantially understate (overstate) the average postranking betas for low-return (high-return) portfolios. Shanken and Weinstein (2006) argue that it is quite likely that the reduced spread in betas would lead to biased upward estimates of risk premia and adopt the portfolio formation approach used by Black, Jensen, and Scholes (1972), Black and Scholes (1974), Chan and Chen (1988), and FF (1993). They form size portfolios based on the market values of stocks at the end of December of each year and then compute the returns on these portfolios in each month of the following year. Shanken and Weinstein (2006) argue that the use of the returns on such annually updated portfolios enables to avoid the selection biases discussed above. Because Kenneth R. French uses a similar methodology of grouping stocks into portfolios, we may expect the use (in our empirical investigation) of the returns on the Kenneth R. French's size and book-to-market equity sorted portfolios to reduce a potential EIV complication in the second-step CSRs.

Recall from Section 3.1 that some of the considered in our empirical investigation risk factors (especially ACG) exhibit statistically significant autocorrelation. As argued in Shanken (1992), because the variance of an average depends not only on the population variance, but the covariances as well, if the risk factors are serially correlated, then the traditional approach to calculate the EIV-adjusted estimate for the standard error of the time-series average of the risk premium will either understate (if autocorrelation is positive) or overstate (in the case of negative autocorrelation) the variance of the CSR estimator and hence the neglect of serial dependence of the factors may lead to an additional misstatement of precision when the traditional time-series procedure to calculate the EIV-adjusted estimate for the standard error is implemented.

Thus, to control the second-step EIV problem, we ascertain whether the true factor risk premia are statistically significant by testing if the sample mean of the quarter-by-quarter CSR estimates of the risk premium for each factor is statistically significant at a given significance level using the conventional t-test based on the EIV-adjusted autocorrelation-consistent estimate for the standard error of the time-series average of the risk premium, as suggested in Shanken (1992) and Shanken and Weinstein (2006). This provides a more realistic assessment of the precision of the estimates of the factor risk premia in the presence of the autocorrelation in the factors.

3.3 Estimation and Testing

In this section, we report the estimation and testing results separately for two types of model (4)-(5) that differ by whether only one, $\mathbf{f}_{1t}$ or $\mathbf{f}_{2t}$, vector of factors or both, $\mathbf{f}_{1t}$ and $\mathbf{f}_{2t}$, vectors of the identified risk factors are included.

We start by considering the special cases of this model motivated by the APT and the consumption-based theories and evaluate the efficacy of each of these two competing approaches to stock pricing in isolation. Here, we first test a version of model (4)-(5) that includes the $\mathbf{f}_{2t}$ vector of factors only. The first model of this type that we test is the canonical CAPM. In this model, the single risk factor is the excess market return, MKT. Panel A of Table III reports the results of the tests of the pricing influence of the CAPM betas. The second model in this set is the FF (1993) three-factor model that includes, in addition to MKT, variables SMB and HML. Panel B of Table III reports the estimation and testing results for this model. Second, we test two models motivated by the consumption-based theories. These models include the $\mathbf{f}_{1t}$ vector of risk factors only. The first model in this set is the complete consumption insurance model. The single factor in this model is ACG. The results for this model are reported in Panel C of Table III. After that, we relax the assumption of complete consumption insurance and test the model with the $\mathbf{f}_{1t}$ vector including, in addition to ACG, the variables SMG, TMG, and FMG. Panel D of Table III displays the estimation and testing results for the incomplete consumption insurance model.

Next, we test the APT and consumption-based approaches against each other not separately, as before, but in an integrated framework. Here, we assume that the model includes both, $\mathbf{f}_{1t}$ and $\mathbf{f}_{2t}$, vectors of the identified risk factors and estimate three different models. All these models include the $\mathbf{f}_{2t}$ vector containing the FF (1993) variables MKT, SMB, and HML and differ by the variables in the $\mathbf{f}_{1t}$ vector of risk factors. The first model in this set is based on the assumption of complete consumption insurance and includes the $\mathbf{f}_{1t}$ vector containing ACG. The results for this model are shown in Panel E of Table III. The $\mathbf{f}_{1t}$ vector in the second model includes, in addition to ACG, the consumption moment risk factors SMG, TMG, and FMG. This model is based on the assumption that consumption insurance is incomplete. The estimation and testing results for this model are displayed in Panel F of Table III. High correlations between SMG, TMG, and FMG, documented in Section 3.1, can inflate the standard errors making it unlikely either of these variables is statistically significant when considered jointly, while they should be otherwise significant. To overcome this problem, we drop TMG and FMG from analysis and test the model that includes, in addition to the FF (1993) three risk factors, ACG and SMG. Panel G of Table III reports on this test.

Below, we discuss the estimation and testing results for the described models. For each model, the results are reported separately for the four sets of households defined in Section 3.1.

3.3.1 The CAPM

The results in Panel A of Table III show that the excess market return is a priced risk factor when considered alone. However, this model is able to explain only nearly 4% of the cross-sectional variation in stock returns. To test the significance of the model as a whole for the overall testing period (from 1989:Q1 to 2008:Q4), we use the shrinking factor approximation to the F distribution with the chi-square distribution, as proposed in Li and Martin (2002). We refer the resulting J statistic for this test to as J_1. Assuming that the quarterly statistics are independent, we form the overall aggregate test statistic as the sum of the quarterly J_1 statistics. Under the null hypothesis that none of the factor betas influence stock returns, the aggregate J_1 statistic is chi-square distributed with the number of degrees of freedom equal to the number of quarters times the number of degrees of freedom of the chi-square approximation for each quarter. According to the aggregate J_1 statistic, the CAPM is rejected statistically for the overall testing period at any conventional significance level. This result is in line with the results of earlier studies.

3.3.2 The Fama-French Model

Panel B of Table III reports on a test of the pricing significance of the FF (1993) three factors MKT, SMB, and HML in the absence of the consumption moment risk factors. This is the APT type model. The first-step OLS estimates of the loadings (betas) of the portfolio returns on the FF (1993) risk factors are obtained from multivariate time-series regressions with only these three variables included. When the FF (1993) three risk factors are assumed to be the only determinants of quarterly stock returns, the estimate of the risk premium for the excess market return, MKT, is negative, while the estimates of the risk premia for the SMB and HML variables are both positive. These results are similar to those in Liu and Zhang (2008).[20] Only the estimates of the risk premia for MKT and HML are significantly different from zero at the 5% significance level.[21] Using the aggregate J_1 statistic, the p-value for the overall testing period indicates that the null hypothesis that none of the FF (1993) three risk factor betas influence stock returns is rejected statistically at any conventional level of significance. The time-series average second-step coefficient of determination suggests that, on average, over the testing period, about 51% of the cross-sectional variation in the quarterly stock returns is predictable using the risk factors MKT, SMB, and HML. The associated average cross-sectional adjusted coefficient of determination is 0.44. These figures clearly indicate a significant improvement over the CAPM.

3.3.3 The Consumption-Based Models

The Complete Consumption Insurance Model. This and the other models below include one or more consumption moment risk factors. To make the estimation and testing results for these models comparable, we opt to estimate the betas in these models from time-series multiple regressions with the FF (1993) three risk factors and the consumption moment (of order two to four) factors included.

In the complete consumption insurance model, the single risk factor deemed to influence stock returns is the growth rate of aggregate consumption, ACG. The results in Panel C of Table III show that, whether the limited participation of households in the capital markets is taken into consideration or not, the risk premium for ACG is statistically insignificant at any conventional level of significance.[22] Our evidence contrasts with that of Malloy, Moskowitz, and Vissing-Jørgensen (2009), who argue that allowing for the limited capital market participation plays an important role

[20] Liu and Zhang (2008) estimate the FF (1993) three-factor model using monthly data from January 1960 to December 2004.

[21] Because the theory provides no strong guidance in signing the risk premia for the FF (1993) risk factors, here and henceforth we use a two-tailed test of statistical significance of these risk premia.

[22] As argued above, the preference theory allows us to sign the risk premia for the first four consumption moment risk factors. Thus, we conduct a one-tailed test when checking whether the risk premia for these factors are individually significant.

in explaining the cross-section of stock returns in the complete consumption insurance framework.

The time-series average second-step coefficient of determination suggests that, when the limited stock market participation is taken into account, this one-factor model can explain, on average, only 4-5% of the cross-sectional variation in expected quarterly stock returns. The associated average cross-sectional adjusted coefficient of determination is close to zero. The value of the aggregate J_1 statistic also indicates a low explanatory power of the model. According to this statistics, the model is rejected at the 5% significance level for the overall testing period as the threshold value in the definition of stockholders is raised to recognize the limited participation of households in the capital markets.

A plausible way to increase the explanatory power of a consumption-based model is to relax the assumption of complete consumption insurance. As noted above, under incomplete consumption insurance the rates of change in higher-order moments of the cross-sectional (stockholder) consumption distribution may also, along with the growth rate of average consumption of stockholders, influence stock returns and hence may also be regarded as the systematic risk factors.

The Incomplete Consumption Insurance Model. To test the pricing influence on uninsurable consumption risk, SMG, TMG, and FMG are added to the set of risk factors. The results in Panel D of Table III show that ACG has about the same significance as it did in Panel C of Table III. The risk premium for ACG has the wrong (negative) sign as the threshold value in the definition of stockholders is raised to recognize the limited capital market participation. Under the limited participation hypothesis, the estimates of the risk premium for SMG are negative, as predicted by the preference assumptions, and are statistically significant at the 10% significance level for the sets of households identified as stockholders HS3 and HS4. The estimates of the risk premia for TMG and FMG do not show up as significant in pricing and mostly have wrong signs. The value of the aggregate J_1 statistic suggests that the incomplete consumption insurance model is not rejected as a whole at any conventional significance level for the overall testing period.

The inability to sort out the separate effects of SMG, TMG, and FMG on stock market returns may be explained, at least partially, by their multicollinearity that implies high sampling variability of the estimates of the risk premia for these factors. Despite the inability to separate the individual effects of SMG, TMG, and FMG, we are still able to determine whether these risk factors are jointly significantly priced. If the joint effect of these variables is statistically significant, then uninsurable consumption risk is rewarded in the stock market. We use the aggregate J_2 statistic to test whether the uninsurable consumption risk variables SMG, TMG, and FMG jointly significantly influence stock returns for the overall testing period in direct competition with ACG. We find that the risk premia for these three factors are jointly significant, implying that for the overall testing period uninsurable risk in consumption has the independent explanatory influence on pricing.[23]

Despite the overall significance, the proportion of the cross-sectional variability of expected stock returns explained by the incomplete consumption insurance model under the assumption of limited capital market participation remains quite low (about 24%, on average, over the testing period) with the associated time-series average second-step adjusted coefficient of determination of only nearly 0.08-0.09.

3.3.4 Combined Fama-French and Consumption-Based Models

The Combined Fama-French and Complete Consumption Insurance Model. Thus far, our analysis has focused on whether the APT and the consumption-based theories are rejected statistically when considered in isolation from each other. We have shown that none of these two theories is rejected empirically. We now test these competing theories against each other within the framework of a single multifactor model. To do this, test first the model in which the risk factors are the FF (1993) three factors and ACG. This model is a mixture of the FF (1993) three-factor model and the complete consumption insurance model.

A comparison with Panel B of Table III reveals that neither the estimates nor the significance of the risk premia for the FF (1993) three risk factors is altered substantially by the presence of the ACG betas. The risk premium associated with the rate of change in average consumption of all households regardless of the reported amount of assetholdings has the wrong (negative) sign, but is statistically insignificant. This result is identical to the result in CRR (1986), who also obtain that the risk premium for the rate of change in aggregate consumption in the economy is negative and insignificant when it is tested how this risk factor fares in direct competition with the APT factors. As the definition of asset holders is tightened to recognize the limited participation of households in the capital markets, the inclusion of the FF (1993) risk factors makes the ACG betas positively related to average returns, as predicted by the preference assumptions, and statistically significant over the entire testing period for the two wealthiest sets of households, HS3 and HS4.

As suggested by the value of the aggregate J_1 statistic, the overall significance of the model over the entire testing period is not rejected statistically at any conventional level of significance.

[23] Like the J_1 statistic, the J_2 statistic for each quarter is calculated by scaling the F statistic for the test that the SMG, TMG, and FMG betas are jointly significant for that quarter. The aggregate J_2 statistic for the overall testing period is calculated as the sum of the quarterly J_2 statistics. The degrees of freedom of the null distribution of the aggregate statistic for J_2 is the degrees of freedom of the scaled chi-square approximation to the F distribution times the number of quarters in the testing period.

The analysis of the time-series average cross-sectional coefficient of determination suggests that, although the rate of change in average consumption of stockholders has a statistically significant influence on average stock returns, this risk factor has only a marginal contribution to explaining the cross-sectional variation in stock returns. As a comparison with Panel B of Table III shows, inclusion of ACG does not significantly affect the time-series average second-step coefficient of determination and the associated average cross-sectional adjusted coefficient of determination.

The Combined Fama-French and Incomplete Consumption Insurance Model. The results of Panel F of Table III show that the coefficients and the significance of the FF (1993) factor betas are unaltered when the factors that proxy uninsurable consumption risk, SMG, TMG, and FMG, are included along with ACG. Although the risk premia for ACG have the same signs as in the joint FF (1993) model and complete consumption insurance model, inclusion of SMG, TMG, and FMG lessen the ability of ACG to show up as significant. The variables SMG, TMG, and FMG are all rejected as having an influence on stock pricing.

For this model, we consider four test statistics. Denote as F_1 the F statistic for the test of the overall significance of the model. To further investigate the independent explanatory influence of incomplete consumption insurance on pricing, we test how SMG, TMG, and FMG fare in direct competition not only with ACG (that proxies insurable consumption risk), as it was done in the consumption-based model with incomplete consumption insurance, but also with the FF (1993) risk factors MKT, SMB, and HML. The F statistic for the test of the joint hypothesis that the prices of risk of SMG, TMG, and FMG all equal zero is referred to as F_3. The test of whether ACG, SMG, TMG, and FMG are jointly priced can be interpreted from the perspective of the APT as the test of whether the consumption moment risk factors have pricing significance as against the FF (1993) three risk factors. Let F_2 denote the test statistic for this test. We also test whether the FF (1993) risk factors in this model are jointly significantly priced. This test is best interpreted from the perspective of the consumption-based theories as the test of whether the set of consumption-based risk factors may be usefully augmented by the addition of risk factors motivated by the APT. The test statistic for this test is referred to as F_4. Figures 1 through 4 report the p-values for the F-test statistics F_1, F_2, F_3, and F_4 for each quarter from 1989:Q1 to 2008:Q4. The reported p-values suggest that the joint tests for factor pricing are all significant at the 5% and 10% levels in a large majority of quarters.

To test the underlying null hypotheses for the overall testing period, we use the shrinking factor approximation to the F distribution with the chi-square distribution, as suggested by Li and Martin

(2002). The resulting J statistics are referred to as J_1, J_2, J_3, and J_4, respectively. The aggregate statistics for J_1, J_2, J_3, and J_4 are the sum of the corresponding quarterly test statistics. Under the assumption that the quarterly statistics are independent, the distribution of the aggregate statistic under the null hypothesis is chi-square with degrees of freedom equal to the number of quarters times the degrees of freedom of the chi-square approximation for each quarter. The overall twenty-year period results present evidence that the model as a whole is not rejected statistically at any conventional level of significance. Neither the set of the FF (1993) risk factors MKT, SMB, and HML, nor the set of the consumption moment risk factors ACG, SMG, TMG, and FMG fail to have pricing significance as against each other. In contrast with insurable consumption risk, uninsurable risk in consumption (jointly captured by SMG, TMG, and FMG) is significantly priced over the entire testing period.

A comparison with Panel E of Table III shows that, after the consumption moment risk factors SMG, TMG, and FMG have been included, the model accounts on average for nearly 64% of the cross-sectional variability of stock returns. The associated average cross-sectional adjusted coefficient of determination for this model is about 0.49, i.e., slightly greater than in Panel E.

Recall from Section 3.1 that SMG, TMG, and FMG are highly correlated. This may explain why the risk premia for these factors are individually insignificant. A plausible solution to this multicollinearity problem is to delete some of the intercorrelated variables from the model. We decided to drop TMG and FMG. The resulting model contains as risk factors the FF (1993) three factors as well as the consumption-based variables ACG and SMG. This corresponds to the case of the second-order Taylor series approximation of the average marginal utility of consumption, when only the rates of change in the first two cross-sectional consumption distribution moments, ACG and SMG, are deemed to affect the average marginal utility of consumption of stockholders and hence (through their covariances with security returns) the expected returns on stocks.

As can be seen from Panel G of Table III, deleting TMG and FMG has no substantive effect on the FF (1993) risk factors and ACG. However, this significantly alters the results for SMG. When no provision is made for the limited participation of households in the capital markets, the SMG beta is statistically significant, but appear with a positive sign. As the definition of asset holders is tightened to recognize the limited capital market participation, the estimate of the risk premium for SMG is negative, as predicted by the preference assumptions, for any set of households classified as stockholders and statistically significant at the 5% significance level for the sets of households HS3 and HS4. This suggests that, under the limited capital market participation, uninsurable consumption risk (presented in the model under consideration by SMG) is significantly rewarded in the stock market. This result reinforces evidence (based on the test of the joint significance of SMG, TMG, and FMG) from the previous model that, in contrast with insurable consumption risk, uninsurable risk in consumption of stockholders is priced.

The model with the risk factors being the FF (1993) variables, ACG, and SMG as a whole is not rejected statistically for the overall testing period. Neither the set of the FF (1993) risk factors, nor the set of the consumption moment risk factors ACG and SMG is rejected statistically for the overall testing period as having a significant effect on pricing. The inclusion of SMG, in addition to the FF (1993) factors and ACG, in the list of the systematic factors only slightly increases the time-series average second-step coefficient of determination and the associated average cross-sectional adjusted coefficient of determination.

4 Concluding Remarks

In this paper, we relax the assumption of complete consumption insurance and investigate whether exposure to uninsurable consumption risk helps explain expected returns. We argue that under incomplete consumption insurance consumption risk can be decomposed into two parts, insurable consumption risk (that can be hedged using financial instruments) and uninsurable consumption risk (caused by the incompleteness of consumption insurance) and show that the rate of change in average consumption (the first cross-sectional consumption distribution moment) reflects the effect of insurable consumption risk on stock returns, while the rates of change in the normalized cross-sectional consumption moments of order two and higher jointly capture the pricing impact of uninsurable consumption risk and can thus be viewed as a multivariate proxy for uninsurable risk in consumption.

To test empirically whether uninsurable consumption risk is significantly priced in the cross-section of expected stock returns, we consider a multifactor asset pricing model that includes the FF (1993) risk factors along with the rates of change in average consumption and the normalized cross-sectional consumption moments of order two to four. We show that the implied multibeta expected return relation nests as special cases a number of alternative asset pricing models considered in the literature. To assess the pricing significance of uninsurable consumption risk, we test whether the rates of change in the second through fourth normalized moments of the cross-sectional consumption distribution are (individually or jointly) rewarded in the stock market.

Exploiting microlevel household consumption data from the US CES, we find strong evidence that neither the FF (1993) risk factors (motivated by the APT) nor the consumption moment (of order up to four) risk factors (motivated by consumption-based theories) fail to have pricing significance when considered jointly in an integrated framework. Another finding is that aggregate consumption of stockholders is unimportant for pricing when compared with the FF (1993) and consumption moment (of order two to four) risk factors. This suggests that insurable risk in consumption of stockholders is not significantly rewarded in the stock market. By contrast, the rates of change in the normalized cross-sectional (stockholder) consumption distribution moments of order two to four are jointly significantly priced when compared against the FF (1993) risk factors and the rate of change in aggregate consumption of stockholders. We interpret this result as suggesting that uninsurable risk in the consumption of stockholders captures important components of systematic risk not reflected in the FF (1993) risk factors. This is the most notable finding of this paper. Despite its statistical significance for pricing, uninsurable risk in consumption of stockholders accounts for only a small, relative to that of the FF (1993) risk factors, proportion of the cross-sectional variability in quarterly stock returns.

Appendix A: Proof of Proposition 1

Decreasing absolute prudence implies that

$$AP' = -(u^{(4)}u'' - (u''')^2)/\left(u''\right)^2 < 0. \tag{A.1}$$

In order to prove that the condition $u^{(4)} < 0$ is necessary for decreasing absolute prudence, suppose, in contrast, that $u^{(4)} \geqslant 0$. When $u^{(4)} \geqslant 0$, $u^{(4)}u'' \leqslant 0$ and, therefore, $AP' > 0$. This contradicts the assumption that absolute prudence is decreasing.

Inequality (A.1) implies that $u^{(4)}u'' - (u''')^2 > 0$ is the necessary and sufficient condition for decreasing absolute prudence. We can rewrite this condition as

$$u^{(4)} < \left(u'''\right)^2/u'' = -APu'''. \tag{A.2}$$

Since the agent is prudent, the term on the right-hand side of (A.2) is negative, proving that $u^{(4)} < 0$ is the necessary (but not sufficient) condition for decreasing absolute prudence. ■

Appendix B: Proof of Proposition 2

Absolute prudence is convex if the following condition is satisfied:

$$AP'' = -(A - B)/C > 0, \tag{B.1}$$

where $A = \left(u''\right)^2 \left(u^{(5)}u'' - u'''u^{(4)}\right)$, $B = 2u''u'''(u^{(4)}u'' - (u''')^2)$, and $C = (u'')^4$.

To prove that $u^{(5)} > 0$ is necessary for convex absolute prudence under prudence and decreasing absolute prudence, assume that $u^{(5)} \leqslant 0$. An agent is prudent ($AP > 0$) if and only if $u''' > 0$.

By Proposition 1, we know that the necessary condition for decreasing absolute prudence is that $u^{(4)} < 0$. Then, under prudence and decreasing absolute prudence, $A > 0$. Since $u^{(4)}u'' - (u''')^2 > 0$ is the necessary and sufficient condition for decreasing absolute prudence, prudence and decreasing absolute prudence also imply that $B < 0$. In consequence, $AP'' < 0$, which contradicts the initial assumption that absolute prudence is convex.

It follows from (B.1) that the necessary and sufficient condition for convex absolute prudence is $A - B < 0$. This condition can be written as

$$u^{(5)} > 2u'''(u^{(4)}u'' - (u''')^2)/\,(u'')^2 + u'''u^{(4)}/u'' \tag{B.2}$$

or, equivalently,

$$u^{(5)} > -2AP'u''' - APu^{(4)}. \tag{B.3}$$

If an agent exhibits prudence, then $AP > 0$ and $u''' > 0$. The condition $u^{(4)} < 0$ is necessary for decreasing absolute prudence. Therefore, under prudence and decreasing absolute prudence, the term $-2AP'u''' - APu^{(4)}$ is positive. Thus, $u^{(5)} > 0$ is necessary (but not sufficient) for convex absolute prudence. ■

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Table I. Summary Statistics for the Risk Factors

Set	Factor	Mean	Standard Deviation	Correlation Coefficient					
				MKT	SMB	HML	ACG	SMG	TMG
				Panel A: Fama-French Risk Factors					
	MKT	0.0178	0.0817	...	...	...	...	...	...
	SMB	0.0024	0.0494	0.401*	...	...	...	...	...
	HML	0.0134	0.0611	-0.535*	-0.202*	...	...	...	...
				Panel B: Consumption Moment Risk Factors					
HS1	ACG	0.0047	0.0038	0.288*	0.004	-0.112	...	...	...
	SMG	0.0116	0.0473	-0.041	-0.013	0.044	0.205*	...	...
	TMG	0.0629	0.1777	-0.084	-0.105	-0.033	0.117	0.820*	...
	FMG	0.0686	0.1783	-0.070	-0.097	-0.025	0.110	0.797*	0.977*
HS2	ACG	0.0032	0.0454	0.034	-0.012	0.017	...	...	...
	SMG	0.0179	0.1498	-0.222*	-0.210*	-0.043	0.175†	...	...
	TMG	0.2044	0.6338	-0.211*	-0.254*	-0.052	0.087	0.854*	...
	FMG	0.1737	0.5864	-0.194*	-0.254*	-0.040	0.195*	0.798*	0.899*
HS3	ACG	0.0029	0.0484	0.019	0.009	0.016	...	...	...
	SMG	0.0240	0.1872	-0.223*	-0.188†	-0.067	0.131	...	...
	TMG	0.3312	1.0546	-0.193*	-0.183†	-0.009	0.069	0.822*	...
	FMG	0.2495	0.7910	-0.202*	-0.227*	-0.050	0.158	0.859*	0.900*
HS4	ACG	0.0041	0.0535	0.027	-0.012	0.014	...	...	...
	SMG	0.0238	0.1864	-0.174†	-0.133	-0.067	0.123	...	...
	TMG	0.2766	0.8308	-0.136	-0.134	-0.034	0.025	0.838*	...
	FMG	0.2408	0.8549	-0.108	-0.122	-0.044	0.132	0.777*	0.863*

Note: MKT is the excess return on the market, SMB is the average return on the three small portfolios minus the average return on the three big portfolios, HML is the average return on the two value portfolios minus the average return on the two growth portfolios, ACG is the rate of change in average consumption, SMG, TMG, and FMG are the rates of change in, respectively, the second, third, and fourth normalized cross-sectional consumption moments. HS1 is the set of all households regardless of the reported amount of assetholdings, HS2, HS3, and HS4 are the sets of households with total assets $\geqslant$ \$1000, $\geqslant$ \$5000, and $\geqslant$ \$10000, respectively. Estimates marked with * and † are statistically different from zero at the 5% and 10% level, respectively. The sample extends from 1982:Q1 through 2008:Q4.

Table II. Autocorrelation in Risk Factors

Set	Factor	$\widehat{\rho}_1$	$\widehat{\rho}_2$	$\widehat{\rho}_3$	$\widehat{\rho}_4$	$\widehat{\rho}_5$	$\widehat{Q}_5$	$\widehat{Q}_{10}$
				Panel A: Fama-French Risk Factors				
	MKT	-0.0889	0.0136	0.0037	0.0061	-0.0361	0.95	4.17
	SMB	-0.0042	0.0019	-0.0143	0.0304	-0.0562	0.45	14.62
	HML	0.2523*	-0.0086	0.0110	-0.0765	-0.0486	7.24	23.61*
				Panel B: Consumption Moment Risk Factors				
HS1	ACG	0.4483*	0.3620*	0.4489*	0.1573	0.1277	58.19*	71.52*
	SMG	-0.0532	-0.1260	0.1219	0.1514	0.0840	6.51	14.23
	TMG	-0.2551*	0.0018	0.0788	0.0216	0.1029	8.30	11.03
	FMG	-0.1922*	-0.0587	0.0906	-0.0003	0.0903	5.73	9.12
HS2	ACG	0.0736	0.1313	-0.0025	-0.1079	0.1906*	7.28	17.11†
	SMG	-0.1046	0.1049	-0.0840	0.0436	-0.1410	5.20	6.54
	TMG	0.0416	-0.0114	-0.1063	0.1443	-0.1709†	6.54	7.61
	FMG	0.0788	-0.0282	-0.0308	0.0834	-0.1189	2.99	3.32
HS3	ACG	0.1296	0.0814	-0.0486	-0.1093	0.1745†	7.00	19.13*
	SMG	-0.0433	0.1042	-0.1387	0.0027	-0.1143	4.61	6.94
	TMG	-0.0586	0.0673	-0.0951	0.0469	-0.0885	2.77	4.07
	FMG	-0.0321	0.0081	-0.0466	0.0357	-0.1047	1.61	4.31
HS4	ACG	0.1350	0.0648	0.0591	-0.1571	0.1665†	8.04	16.81†
	SMG	0.0044	0.0540	-0.0491	0.0107	-0.0774	1.18	2.28
	TMG	0.1574	-0.0497	-0.0354	0.0047	-0.1480	5.14	8.48
	FMG	0.0900	-0.0745	-0.0004	-0.0406	-0.0733	2.10	2.81

Note: MKT is the excess return on the market, SMB is the average return on the three small portfolios minus the average return on the three big portfolios, HML is the average return on the two value portfolios minus the average return on the two growth portfolios, ACG is the rate of change in average consumption, SMG, TMG, and FMG are the rates of change in, respectively, the second, third, and fourth normalized cross-sectional consumption moments. HS1 is the set of all households regardless of the reported amount of assetholdings, HS2, HS3, and HS4 are the sets of households with total assets $\geqslant$ \$1000, $\geqslant$ \$5000, and $\geqslant$ \$10000, respectively. ρ_l is the autocorrelation coefficient of order l. Q_m is the Ljung-Box (1978) Q-statistic with m autocorrelations. Estimates marked with * and † are statistically different from zero at the 5% and 10% level, respectively. The sample extends from 1982:Q1 through 2008:Q4.

Table III. Estimates of the Factor Risk Premia

Set	Const	MKT	SMB	HML	ACG	SMG	TMG	FMG	R^2	R_a^2	J_1	J_2	J_3	J_4
Panel A: $\mathbf{Z}_t = \boldsymbol{\beta}_0 + \boldsymbol{\beta}_1 MKT_t + \varepsilon_t$														
	4.206*	-2.280†	...	...	...	...	...	...	0.041	0.000	78.7	...	...	...
	(1.871)	(1.195)	...	...	...	...	...	...			[0.519]	...	...	...
Panel B: $\mathbf{Z}_t = \boldsymbol{\beta}_0 + \boldsymbol{\beta}_1 MKT_t + \boldsymbol{\beta}_2 SMB_t + \boldsymbol{\beta}_3 HML_t + \varepsilon_t$														
	3.390*	-2.109*	0.374	0.846*	...	...	...	...	0.506	0.435	1514.7	...	...	...
	(1.188)	(0.830)	(0.373)	(0.275)	...	...	...	...			[0.000]	...	...	...
Panel C: $\mathbf{Z}_t = \boldsymbol{\beta}_0 + \boldsymbol{\beta}_4 ACG_t + \varepsilon_t$														
HS1	1.810†	...	...	...	0.042	...	...	...	0.060	0.019	120.0	...	...	...
	(1.025)	...	...	...	(0.101)	...	...	...			[0.003]	...	...	...
HS2	1.890†	...	...	...	0.249	...	...	...	0.051	0.010	99.9	...	...	...
	(1.033)	...	...	...	(1.006)	...	...	...			[0.065]	...	...	...
HS3	1.862†	...	...	...	-0.094	...	...	...	0.042	0.001	80.4	...	...	...
	(1.029)	...	...	...	(1.166)	...	...	...			[0.468]	...	...	...
HS4	1.864†	...	...	...	0.280	...	...	...	0.044	0.002	83.3	...	...	...
	(1.027)	...	...	...	(1.550)	...	...	...			[0.378]	...	...	...
Panel D: $\mathbf{Z}_t = \boldsymbol{\beta}_0 + \boldsymbol{\beta}_4 ACG_t + \boldsymbol{\beta}_5 SMG_t + \boldsymbol{\beta}_6 TMG_t + \boldsymbol{\beta}_7 FMG_t + \varepsilon_t$														
HS1	1.909	...	...	...	0.130	0.680	-0.893	-1.826	0.224	0.069	456.7	...	1243.5	...
	(1.149)	...	...	...	(0.105)	(1.012)	(3.755)	(4.469)			[0.000]	...	[0.000]	...
HS2	1.904	...	...	...	-0.302	-4.819	-14.287	0.230	0.240	0.088	491.7	...	1435.0	...
	(1.169)	...	...	...	(1.259)	(4.628)	(16.135)	(14.944)			[0.000]	...	[0.000]	...
HS3	1.940†	...	...	...	-0.797	-6.170†	-12.411	-12.937	0.231	0.077	473.8	...	1437.6	...
	(1.114)	...	...	...	(1.349)	(4.198)	(19.927)	(14.906)			[0.000]	...	[0.000]	...
HS4	1.969†	...	...	...	-1.591	-5.331†	-5.889	-5.786	0.241	0.089	491.9	...	1493.0	...
	(1.146)	...	...	...	(1.813)	(4.132)	(19.259)	(26.468)			[0.000]	...	[0.000]	...

Note: Estimates (in percentage per quarter) of the (excess) zero-beta rate and factor risk premia based on the two-step cross-sectional regression methodology with betas estimated from 28 quarters of prior data. The Shanken (1992) EIV-adjusted autocorrelation-consistent standard errors are in parentheses under the parameter estimates. Estimates marked with * and † are statistically different from zero at the 5% and 10% level, respectively. HS1 is the set of all households regardless of the reported amount of assetholdings, HS2, HS3, and HS4 are the sets of households with total assets $\geqslant$ \$1000, $\geqslant$ \$5000, and $\geqslant$ \$10000, respectively. R^2 and R_a^2 are the time-series average second-step coefficient of determination and adjusted coefficient of determination, respectively. J_1 is the aggregate statistic for the test of the overall significance of a model, J_2 is the aggregate statistic for the test of the joint hypothesis that the prices of risk of ACG, SMG, TMG, and FMG all equal zero (in Panel G, J_2 is the aggregate statistic for the test of the joint hypothesis that the prices of risk of ACG and SMG both equal zero). J_3 is the aggregate statistic for the test of whether the risk premia for SMG, TMG, and FMG are jointly significant, and J_4 is the aggregate statistic for the test of the joint hypothesis that the risk premia for the Fama-French (1993) three factors all equal zero. Their associated p-values are in brackets. The testing period extends from 1989:Q1 through - 2008:Q4.

Table III (continued)

Set	Const	MKT	SMB	HML	ACG	SMG	TMG	FMG	R^2	R_a^2	J_1	J_2	J_3	J_4
						Panel E: $\mathbf{Z}_t = \boldsymbol{\beta}_0 + \boldsymbol{\beta}_1 MKT_t + \boldsymbol{\beta}_2 SMB_t + \boldsymbol{\beta}_3 HML_t + \boldsymbol{\beta}_4 ACG_t + \varepsilon_t$								
HS1	3.608*	-2.338*	0.356	0.889*	-0.048	...	...	...	0.537	0.445	1594.3	...	...	...
	(1.308)	(0.918)	(0.360)	(0.274)	(0.086)	...	...	...			[0.000]	...	...	...
HS2	3.435*	-2.184*	0.367	0.901*	1.079	...	...	...	0.541	0.450	1632.6	...	...	...
	(1.164)	(0.835)	(0.368)	(0.274)	(0.876)	...	...	...			[0.000]	...	...	...
HS3	3.213*	-1.995*	0.403	0.904*	1.779*	...	...	...	0.530	0.436	1577.5	...	...	...
	(1.103)	(0.733)	(0.370)	(0.270)	(0.945)	...	...	...			[0.000]	...	...	...
HS4	3.118*	-1.913*	0.421	0.897*	2.002†	...	...	...	0.531	0.437	1570.7	...	...	...
	(1.119)	(0.813)	(0.374)	(0.273)	(1.236)	...	...	...			[0.000]	...	...	...
						Panel F: $\mathbf{Z}_t = \boldsymbol{\beta}_0 + \boldsymbol{\beta}_1 MKT_t + \boldsymbol{\beta}_2 SMB_t + \boldsymbol{\beta}_3 HML_t + \boldsymbol{\beta}_4 ACG_t + \boldsymbol{\beta}_5 SMG_t + \boldsymbol{\beta}_6 TMG_t + \boldsymbol{\beta}_7 FMG_t + \varepsilon_t$								
HS1	2.909*	-1.582	0.313	0.881*	-0.030	0.584	0.103	-0.557	0.643	0.496	1974.9	2070.6	1633.7	5729.1
	(1.394)	(1.182)	(0.369)	(0.269)	(0.095)	(0.823)	(3.132)	(3.457)			[0.000]	[0.000]	[0.000]	[0.000]
HS2	4.131*	-2.765*	0.338	0.746*	1.096	0.824	8.836	12.511	0.641	0.494	1931.9	2001.3	1441.5	5390.7
	(1.473)	(1.087)	(0.367)	(0.267)	(1.095)	(3.132)	(15.013)	(16.289)			[0.000]	[0.000]	[0.000]	[0.000]
HS3	4.234*	-2.878*	0.357	0.744*	0.713	1.453	24.099	10.753	0.642	0.495	1908.2	1985.2	1548.5	5382.5
	(1.374)	(0.999)	(0.371)	(0.263)	(1.231)	(3.598)	(19.118)	(17.262)			[0.000]	[0.000]	[0.000]	[0.000]
HS4	3.485*	-2.109†	0.402	0.679*	1.519	1.460	16.860	17.875	0.635	0.485	1883.6	1865.5	1484.7	5211.2
	(1.493)	(1.134)	(0.364)	(0.266)	(1.420)	(3.974)	(20.576)	(25.700)			[0.000]	[0.000]	[0.000]	[0.000]
						Panel G: $\mathbf{Z}_t = \boldsymbol{\beta}_0 + \boldsymbol{\beta}_1 MKT_t + \boldsymbol{\beta}_2 SMB_t + \boldsymbol{\beta}_3 HML_t + \boldsymbol{\beta}_4 ACG_t + \boldsymbol{\beta}_5 SMG_t + \varepsilon_t$								
HS1	4.487*	-3.139*	0.321	0.789*	-0.043	0.587†	...	...	0.570	0.457	1700.0	901.6	...	4460.8
	(1.423)	(1.062)	(0.366)	(0.275)	(0.092)	(0.400)	...	...			[0.000]	[0.000]	...	[0.000]
HS2	3.598*	-2.250*	0.320	0.828*	1.092	-0.466	...	...	0.588	0.479	1792.7	1190.7	...	4592.8
	(1.240)	(0.825)	(0.366)	(0.268)	(0.958)	(1.139)	...	...			[0.000]	[0.000]	...	[0.000]
HS3	3.432*	-2.141*	0.375	0.748*	1.086	-2.228*	...	...	0.574	0.462	1726.3	1025.0	...	4438.4
	(1.180)	(0.706)	(0.367)	(0.264)	(1.059)	(0.860)	...	...			[0.000]	[0.000]	...	[0.000]
HS4	3.626*	-2.353*	0.420	0.782*	1.592	-2.624*	...	...	0.571	0.458	1693.1	908.8	...	4241.0
	(1.274)	(0.909)	(0.371)	(0.270)	(1.297)	(0.702)	...	...			[0.000]	[0.000]	...	[0.000]

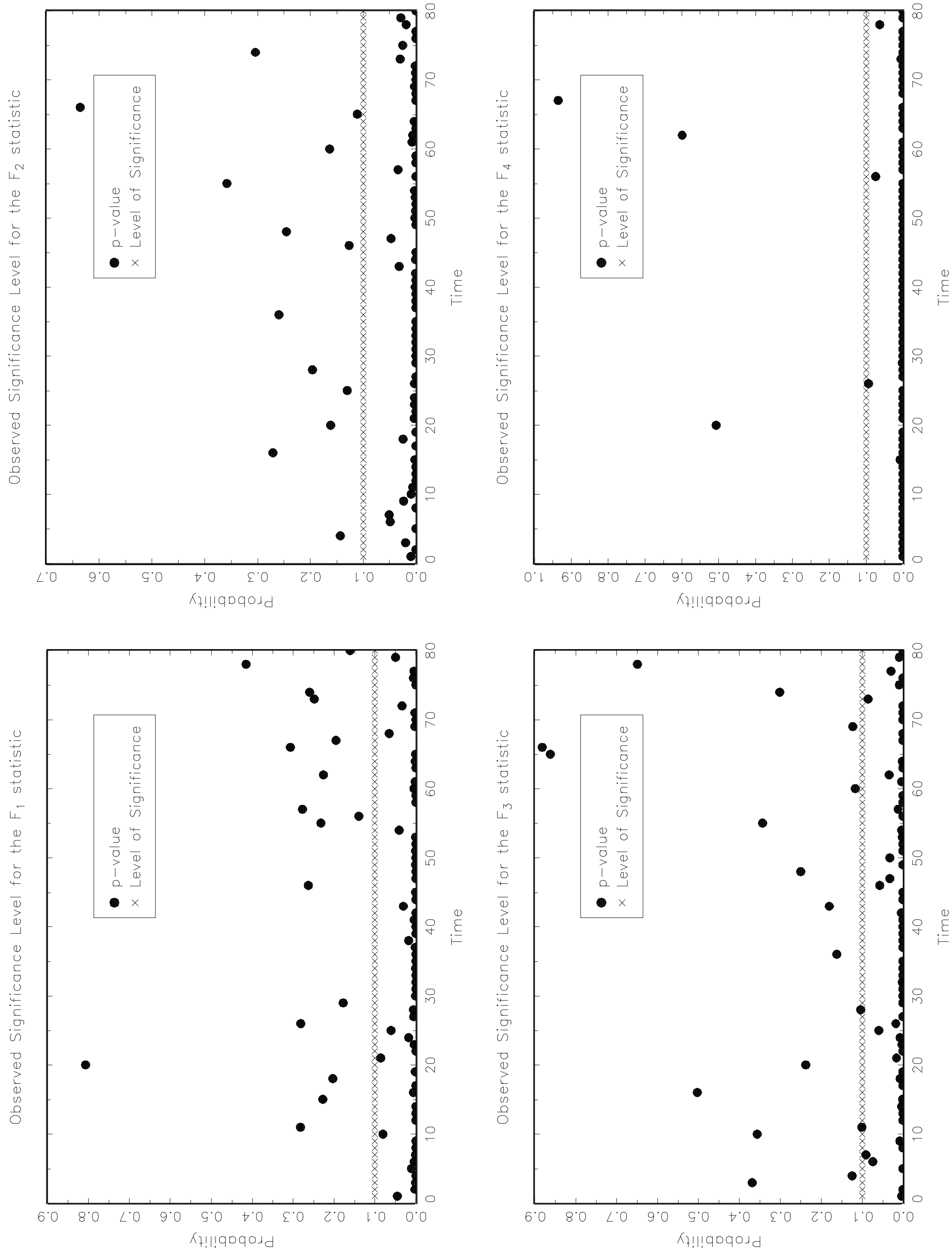

Figure 1: The p-values for the quarterly F statistics for the set of households HS1. F_1 is the F statistic for the test of the overall significance of the model. F_2 is the F statistic for the test of whether the variables ACG, SMG, TMG, and FMG are jointly priced. F_3 is the F statistic for the test of the joint significance of the variables SMG, TMG, and FMG. F_4 is the F statistic for the test of whether the Fama-French (1993) risk factors are jointly significantly rewarded in the stock market.

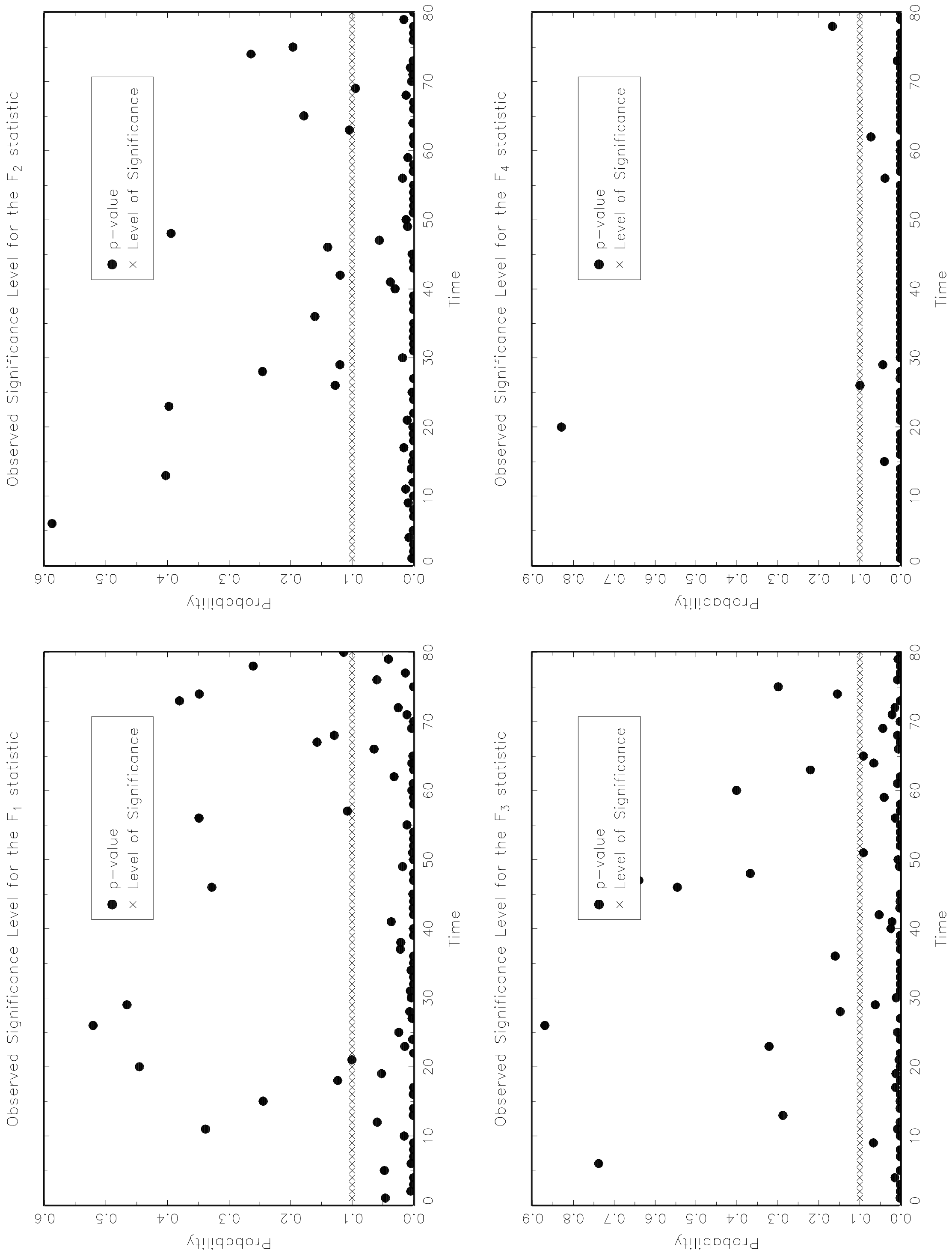

Figure 2: The p-values for the quarterly F statistics for the set of households HS2. F_1 is the F statistic for the test of the overall significance of the model. F_2 is the F statistic for the test of whether the variables ACG, SMG, TMG, and FMG are jointly priced. F_3 is the F statistic for the test of the joint significance of the variables SMG, TMG, and FMG. F_4 is the F statistic for the test of whether the Fama-French (1993) risk factors are jointly significantly rewarded in the stock market.

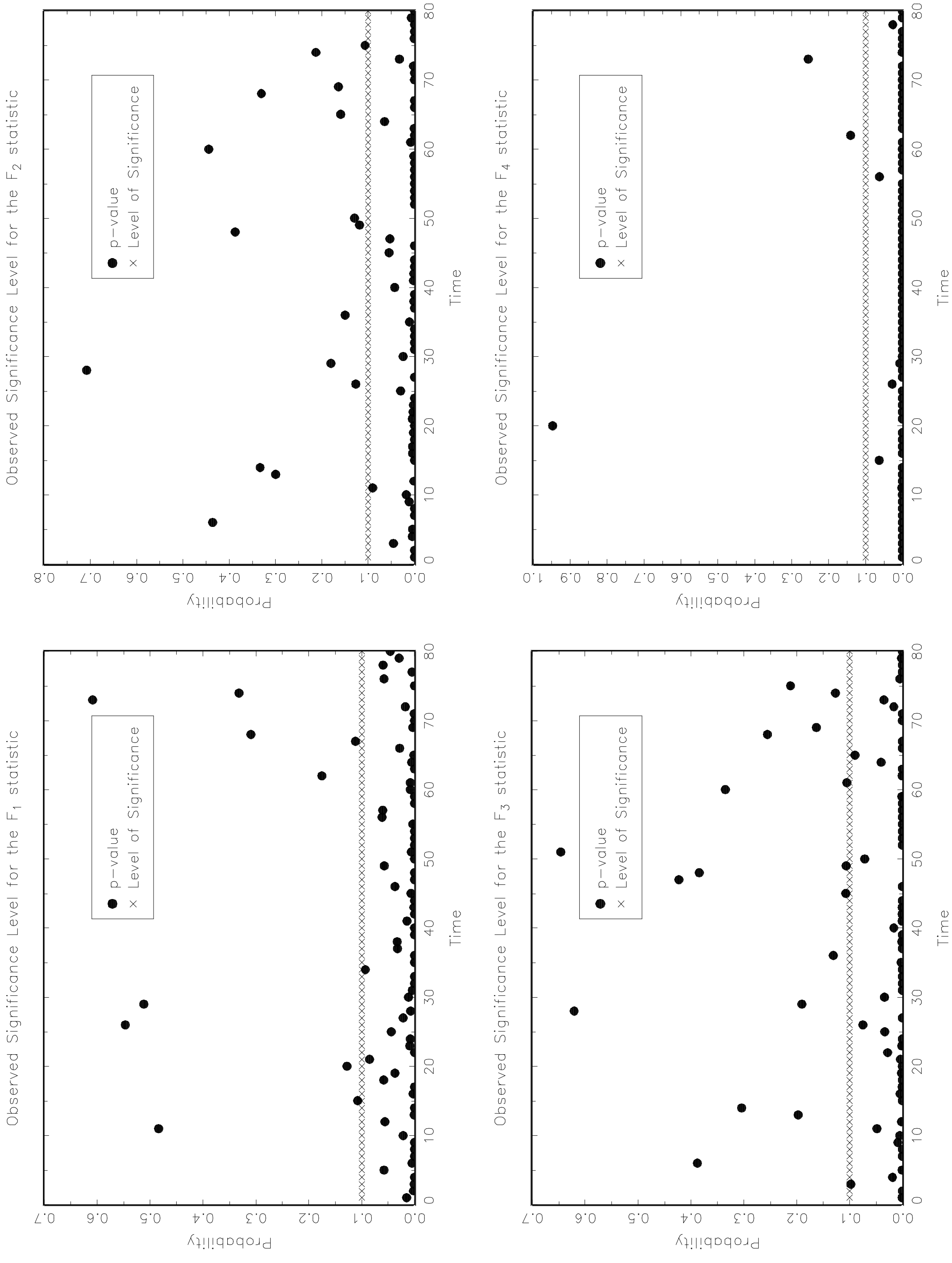

Figure 3: The p-values for the quarterly F statistics for the set of households HS3. F_1 is the F statistic for the test of the overall significance of the model. F_2 is the F statistic for the test of whether the variables ACG, SMG, TMG, and FMG are jointly priced. F_3 is the F statistic for the test of the joint significance of the variables SMG, TMG, and FMG. F_4 is the F statistic for the test of whether the Fama-French (1993) risk factors are jointly significantly rewarded in the stock market.

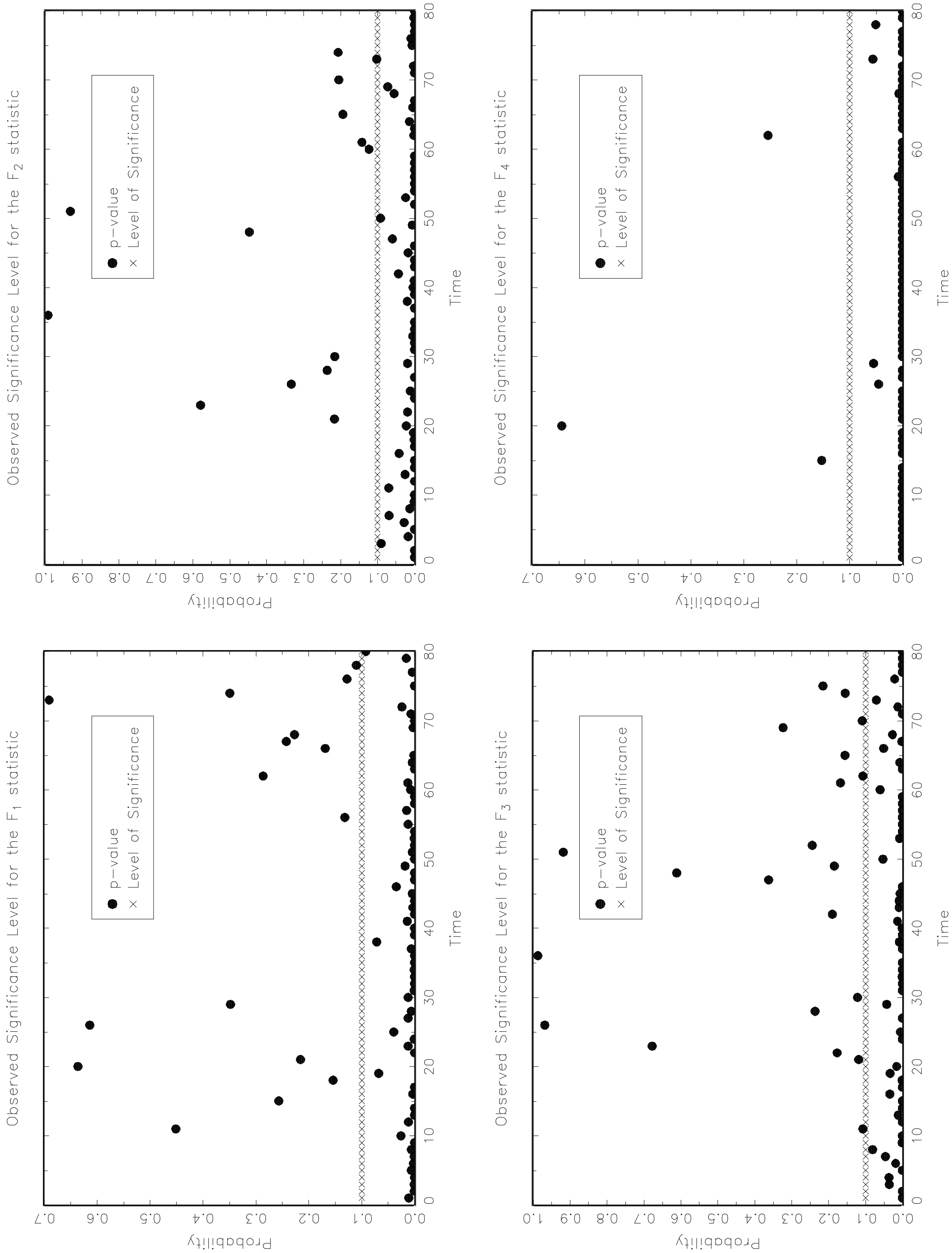

Figure 4: The p-values for the quarterly F statistics for the set of households HS4. F_1 is the F statistic for the test of the overall significance of the model. F_2 is the F statistic for the test of whether the variables ACG, SMG, TMG, and FMG are jointly priced. F_3 is the F statistic for the test of the joint significance of the variables SMG, TMG, and FMG. F_4 is the F statistic for the test of whether the Fama-French (1993) risk factors are jointly significantly rewarded in the stock market.