

Monetary policy regimes and the forward bias for foreign exchange *

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Abstract

This paper provides a theoretical discussion of the forward premium anomaly. We reformulate the well-known Lucas (1982) model by allowing for the existence of monetary policy regimes. The monetary supply is viewed as having two stochastic components: a) a persistent component that reflects the preferences of the central bank regarding the long-run money supply or inflation target, and b) a transitory component that represents short-lived interventions. To generate agents' forecasts, we consider two scenarios: a) consumers can distinguish the permanent and the transitory components of the money supply, and b) consumers can observe only historical series of the aggregate monetary supply and face a signal-extraction problem. We simulate the model from a careful estimation of the parameters involved in the model. Numerical simulations reveal that, under complete information, forward unbiasedness cannot be rejected at conventionally significant levels. However, when learning about monetary policy is incorporated, the forward bias can be reproduced without artificially assuming an unreasonable degree of risk aversion.

Key words: forward bias, monetary policy, regime shifts, learning

JEL classification: C22;F31;F47.

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1 Introduction

According to the hypothesis of uncovered interest rate parity, expected changes in the spot exchange rates should be perfectly and positively correlated with the current forward premium. Therefore, the domestic currency is expected to depreciate whenever domestic nominal interest rates exceed foreign interest rates. However, this hypothesis has been empirically rejected in a large number of studies (see, Froot and Thaler (1995), Lewis (1995), Bansal and Dahlquist (2000), Tauchen (2001), Sercu and Vinaimont (2006), among many others), and sometimes a negative correlation has also been detected (see Backus et al. (1995) and More and Roche (2002, and 2007)). This empirical feature is known as the forward premium anomaly and has direct implications for expected returns from international currency deposits. When nominal exchange rates are negatively correlated with the lagged forward premium, a non-negative interest rate differential is associated with an expected depreciation of the currency for the high interest rate country.

This puzzle has been interpreted in several ways in the literature. From a theoretical point of view, although a substantial number of studies have addressed the ability of general equilibrium models that build up on Lucas (1982) to explain the forward premium puzzle (see, for example, Canova and Marriman (1993), Bekaert (1994), Engle (1996), Campbell and Cochrane (1999), and More and Roche (2002), they either require unreasonable risk aversion parameters or an incredibly volatile consumption processes. Relative to empirical work, recent studies focus on the non-stationarity and long-memory features of the exchanges rates and the forward premium (see, for example, Baillie and Bollerslev (2000), Tauchen, 2001, Maynard (2003, 2006), Maynard and Phillips (2001) and Maynard and Lu (2005), among many others). In sum, a good deal of empirical and theoretical work has been devoted to analyzing the forward premium anomaly, but a conclusive understanding of the issue continues to elude researchers. The anomaly is still regarded as a relevant puzzle in international finance.

In this paper, we propose a general equilibrium model to explain the forward bias for foreign exchange. The model is based on the Lucas (1982) model and is similar to that proposed in Dutton (1993). While the recent paper of Verdelhan (2011) provides an explanation based on the existence of external habit preferences and rational expectations, the work of Chakraborty and Evans (2008) suggests that the learning theory approach to expectation formation in the foreign exchange markets should be considered a serious contender in future empirical work on the forward-premium puzzle¹. These authors use a basic canonical monetary exchange-rate model, but they anticipate the impact of learning will remain prominent in more complex exchange-rate models that allow for risk aversion, incomplete information processing or heterogenous expectations. Following this intuition, our paper tries to provide additional insights on the forward premium bias from the perspective of general equilibrium theory.

In particular, we consider a modified version of the Lucas model in which incomplete information arises when agents must form expectations of future monetary policy actions by solving a signal extraction problem.

¹As Krugman said (The New York Review, 54(2), February 2007): "But Friedman never went there. His reality sense warned that this [rational expectations] was taking the idea of Homo economicus too far".

Also, our modelling is similar in spirit to that of Mark and Mohb (2007). These authors investigate the idea that the forward premium anomaly is caused by unanticipated central bank interventions in the foreign exchange market, and propose a theoretical model in which the violations to uncovered interest parity do not reflect unexploited profit opportunities or systematic risk. The numerical simulations of their model show that the forward premium anomaly intensifies during periods of active interventions by central banks.

In accordance with Mark and Moh (2007), our model mainly departs from the Lucas model in that individuals can observe the historical sequence of money supply, but they cannot distinguish whether a monetary shock implies a regime shift or a transitory intervention. Monetary policy is specified as having two components: one is determined at each time period by the prevailing monetary regime and reflects a particular target of the central bank regarding the money supply or inflation; the second represents short-lived interventions. Economic agents consequently face a signal-extraction problem through a learning mechanism that allows monetary shocks to be decomposed into estimated transitory and permanent components.

Consequently, a broad set of scenarios can be simulated in order to explore potential explanatory factors for the forward bias. Our main result is that when agents are capable of anticipating monetary policy shifts, that is, when they can exactly identify the transitory and persistent components of monetary disturbances, the forward unbiasedness hypothesis cannot be rejected at conventional significance levels. However, when agents need to solve a nontrivial signal extraction problem on the basis of the past history of the monetary policy, a significant downward forward bias systematically appears. These results do not require preferences with external habits and hold without any need for high risk aversion and for alternative degrees of substitution between domestic and foreign goods. Also, as expected, extremely high risk aversion intensifies the downward bias.

The remainder of the paper is structured as follows. Section 2 describes the theoretical model. Section 3 discusses the implications of the model using estimated parameters from US and the EMU. Finally, Section 4 summarizes and provides concluding remarks.

2 The Model

Our modelling strategy is to extend the well-known Lucas (1982) model by allowing for the existence of regime changes in monetary policy. We will consider two scenarios: in the first one, structural parameters concerning the evolution of money supply over time are assumed to be known. By contrast, we also consider the more realistic case in which agents need to learn from current history in order to forecast future monetary policy. We start by presenting our version of Lucas' model and describing the assumed structure for monetary policy.

Let X_t and X_t^* denote the exogenous endowments of consumption goods in the domestic and foreign countries. Endowments are stochastic, and their natural logarithm follows an autoregressive process with Normal innovations:

$$X_t = \mu_{X_t} X_{t-1}, \quad (1)$$

$$X_t^* = \mu_{X_t^*} X_{t-1}^*, \quad (2)$$

where:

$$\begin{pmatrix} \nabla \ln X_t \\ \nabla \ln X_t^* \end{pmatrix} = \begin{pmatrix} (1 - \rho_X) \ln \bar{\mu}_X \\ (1 - \rho_{X^*}) \ln \bar{\mu}_{X^*} \end{pmatrix} + \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix} \begin{pmatrix} \nabla \ln X_{t-1} \\ \nabla \ln X_{t-1}^* \end{pmatrix} + \begin{pmatrix} \xi_{X,t} \\ \xi_{X^*,t} \end{pmatrix}, \quad (3)$$

with, $\begin{pmatrix} \xi_{X,t} \\ \xi_{X^*,t} \end{pmatrix} \sim N \left(0_{2 \times 1}, \begin{pmatrix} \sigma_{\xi_{X,t}}^2 & \sigma_{\xi_{X,t}\xi_{X^*,t}} \\ \sigma_{\xi_{X,t}\xi_{X^*,t}} & \sigma_{\xi_{X^*,t}}^2 \end{pmatrix} \right)$.

This way, similarly to Lafuente and Ruiz (2006), the potential correlation between real shocks in the domestic and foreign countries is explicitly taken into account. Let us denote $\rho_{\xi_X, \xi_{X^*}}$ the linear correlation coefficient between these two shocks.

2.1 The consumer's problem

The optimization problem for the home consumer is:

$$\begin{aligned} \underset{\{C_{D,t}, C_{F,t}\}}{Max} \quad & E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} \left[[\phi (C_{D,t})^\epsilon + (1-\phi) (C_{F,t})^\epsilon]^{(1-\gamma)/\epsilon} - 1 \right] \\ \text{s.t.} \quad & P_{D,t} C_{D,t} + S_t P_{F,t} C_{F,t} \leq Y_t, \\ & Y_t = M_t + T_{t-1} \frac{S_t - F_{t-1}}{F_{t-1}}, \end{aligned} \quad (4)$$

where $C_{D,t}$ and $C_{F,t}$ are the consumption levels of domestic and foreign goods at time t , $\gamma > 0$ is the relative risk aversion coefficient, $\frac{1}{1-\epsilon}$ is the elasticity of substitution with $\epsilon < 1$, while parameter $\phi \in (0, 1)$ represents the weight of each consumption good in the utility function. As to the budget constraint, $P_{D,t}$ and $P_{F,t}$ denote the prices of domestic and foreign goods at time t , Y_t is the total income in period t , S_t is the spot exchange rate, F_{t-1} is the price of forward contract, T_{t-1} is the respective amount of its currency that the home country sold forward in the previous period. The money supply (M_t) plus the profits on each forward currency trade in period t equals total home income. A similar optimization problem is solved by the foreign consumer:

$$\begin{aligned} \underset{\{C_{D^*,t}, C_{F^*,t}\}}{Max} \quad & E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} \left[[\phi (C_{D^*,t})^\epsilon + (1-\phi) (C_{F^*,t})^\epsilon]^{(1-\gamma)/\epsilon} - 1 \right] \\ \text{s.t.} \quad & P_{D,t} C_{D^*,t} + S_t P_{F,t} C_{F^*,t} \leq Y_t^* S_t, \\ & Y_t^* = M_t^* + T_{t-1}^* \frac{S_t - F_{t-1}}{F_{t-1} S_t}. \end{aligned} \quad (5)$$

2.2 Optimum good choices.

In any period t the home consumer chooses levels of $C_{D,t}$ and $C_{F,t}$ that solves (4). First order conditions for choice of $C_{D,t}$ and $C_{F,t}$ are:

$$[\phi (C_{D,t})^\epsilon + (1 - \phi) (C_{F,t})^\epsilon]^{\frac{1-\gamma}{\epsilon}-1} (C_{D,t})^{\epsilon-1} - \lambda_t P_{D,t} = 0, \quad (6)$$

$$[\phi (C_{D,t})^\epsilon + (1 - \phi) (C_{F,t})^\epsilon]^{\frac{1-\gamma}{\epsilon}-1} (C_{F,t})^{\epsilon-1} - \lambda_t S_t P_{F,t} = 0, \quad (7)$$

$$Y_t - P_{D,t} C_{D,t} - P_{F,t} S_t C_{F,t} = 0, \quad (8)$$

where λ_t denotes the Lagrange multiplier. From equations (6) and (7), we obtain:

$$C_{F,t} = \left[\frac{(1 - \phi) P_{D,t}}{\phi P_{F,t} S_t} \right]^\sigma C_{D,t}, \quad (9)$$

where $\sigma = \frac{1}{1-\epsilon}$ denotes the elasticity of substitution. Using (8) and the budget constraint, the demand function for the domestic and foreign good is as follows:

$$C_{D,t} = \frac{Y_t P_{D,t}^{-\sigma}}{P_{D,t}^{1-\sigma} + \left(\frac{1-\phi}{\phi} \right)^\sigma (S_t P_{F,t})^{1-\sigma}}, \quad (10)$$

$$C_{F,t} = \left[\frac{(1 - \phi) P_{D,t}}{\phi P_{F,t} S_t} \right]^\sigma \frac{Y_t P_{D,t}^{-\sigma}}{P_{D,t}^{1-\sigma} + \left(\frac{1-\phi}{\phi} \right)^\sigma (S_t P_{F,t})^{1-\sigma}}. \quad (11)$$

Using a similar procedure, the demands for the foreign country can be found, that is:

$$C_{F,t}^* = \left[\frac{(1 - \phi) P_{D,t}}{\phi P_{F,t} S_t} \right]^\sigma C_{D,t}^*, \quad (12)$$

$$C_{D,t}^* = \frac{Y_t^* S_t P_{D,t}^{-\sigma}}{P_{D,t}^{1-\sigma} + \left(\frac{1-\phi}{\phi} \right)^\sigma (S_t P_{F,t})^{1-\sigma}}, \quad (13)$$

$$C_{F,t}^* = \left[\frac{(1 - \phi) P_{D,t}}{\phi P_{F,t} S_t} \right]^\sigma \frac{Y_t^* S_t P_{D,t}^{-\sigma}}{P_{D,t}^{1-\sigma} + \left(\frac{1-\phi}{\phi} \right)^\sigma (S_t P_{F,t})^{1-\sigma}}. \quad (14)$$

2.3 Forward Contracting

As well as the allocation of current resources between the two goods, the home consumer chooses the level of forward contracting in period t . The Euler condition is:

$$E_t \left[\lambda_{t+1} \beta^{t+1} \left(\frac{S_{t+1} - F_t}{F_t} \right) \right] = 0 \quad (15)$$

where E_t denotes the conditional expectation based on the information set available in period t . From (15):

$$E_t [\lambda_{t+1} S_{t+1}] = F_t E_t \lambda_{t+1}$$

and taking into account (6) the value of the forward exchange rate at time t consistent with consumer's optimal choice is:

$$F_t = \frac{E_t \left[\frac{\partial U_{t+1}}{\partial C_{F,t+1}} \frac{1}{P_{F,t+1}} \right]}{E_t \left[\frac{\partial U_{t+1}}{\partial C_{F,t+1}} \frac{1}{P_{F,t+1} S_{t+1}} \right]}. \quad (16)$$

A similar expression applies for the foreign country:

$$F_t = \frac{E_t \left[\frac{\partial U_{t+1}^*}{\partial C_{F,t+1}^*} \frac{1}{P_{F,t+1}} \right]}{E_t \left[\frac{\partial U_{t+1}^*}{\partial C_{F,t+1}^*} \frac{1}{P_{F,t+1} S_{t+1}} \right]}. \quad (17)$$

2.4 Equilibrium in the Goods Market

In equilibrium, the total endowment of the two goods must be equal to the consumption of each good in the two countries, that is:

$$C_{D,t} + C_{D,t}^* = X_{D,t}, \quad (18)$$

$$C_{F,t} + C_{F,t}^* = X_{F,t}. \quad (19)$$

Equilibrium prices of the two goods depend on the home and foreign money supplies as well as on their endowment. Taking into account that a) money is worthless after each period and b) each country's good can only be purchased with that country's currency, the following cash-in-advance spending constraints must hold:

$$P_{D,t} X_{D,t} = M_t, \quad (20)$$

$$P_{F,t} X_{F,t} = M_t^*. \quad (21)$$

Also, in equilibrium, the following relationship between home and foreign derivative positions holds:

$$T_t = -T_t^*.$$

2.5 Monetary policy

Following Andolfatto et al. (2004), we assume that the money supply at the beginning of period t comprises two stochastic components: one component which reflects the main regime which is determined by the long-run monetary policy target, and the second component being the short-run error made in control of monetary aggregates by the central bank. For the home country, we have:

$$M_t = \mu_{M_t} M_{t-1}, \text{ with } \ln \mu_{M_t} = \ln \bar{\mu}_M + z_t + u_t,$$

where M_t denotes the home money supply, μ is the average rate of growth of the natural logarithm of the money supply, and z_t and u_t denote the regime and transitory component, respectively. We also assume

that the regime component of the monetary policy to remain constant for a relatively long time period and a new regime appears only occasionally. Thus, the time evolution of z_t can be expressed as follows:

$$z_t = \begin{cases} z_{t-1}, & \text{with probability } p \\ g_t, & \text{with probability } 1 - p, \text{ where } g_t \sim N(0, \sigma_g^2). \end{cases}$$

Parameter p reflects the expected duration of any given regime, or alternatively, the persistence of the regime. Given that z_t represents the long-run monetary guidelines of the central bank, it is expected that such persistence would be fairly high. Parameter σ_g^2 reflects the potential size of the regime shift. The transitory money growth component of the u_t is assumed to follow a standard AR(1) specification:

$$u_t = \delta u_{t-1} + a_t,$$

with $0 < \delta \ll 1$ and $a_t \sim N(0, \sigma_a^2)$. The variable u_t can be interpreted as the outcome of a monetary intervention in financial markets as a reaction to shocks occurring in the world economy. In a similar way, the dynamics of the monetary policy of the foreign country is described as follows:

$$M_t^* = \mu_{M_t^*} M_{t-1}^*, \text{ with } \ln \mu_{M_t^*} = \ln \bar{\mu}_{M^*} + z_t^* + u_t^*,$$

$$z_t^* = \begin{cases} z_{t-1}^*, & \text{with probability } p^* \\ g_t^*, & \text{with probability } 1 - p^*, \text{ where } g_t^* \sim N(0, \sigma_{g^*}^2). \end{cases}$$

$$u_t^* = \delta^* u_{t-1}^* + a_t^*, \quad 0 < \delta \ll 1, a_t \sim N(0, \sigma_{a^*}^2)$$

Domestic and foreign monetary shocks on the transitory component (a_t and a_t^*) may be correlated, with correlation coefficient ρ_{aa^*} .

2.6 Spot exchange rates

Using the budget constraints and equations (18) to (21), the analytical expression of the equilibrium spot rate is:

$$S_t = \frac{1 - \phi}{\phi} \left(\frac{X_{F,t}}{X_{D,t}} \right)^\epsilon \frac{M_t}{M_t^*}. \quad (22)$$

2.7 Expectations and parameter estimation

To obtain simulated equilibrium time series for spot and forward exchange rates, there are two aspects of prior work that we need to deal with: a) the specification of parameter values and b) the computing of expectations.

Since we are trying to assess whether learning is an explaining factor for the forward bias in our model, we consider two frameworks for generating expectations concerning monetary policy. Firstly, let us assume that

consumers in the economy know the structural parameters. From now on we will refer this case as "complete information". In this case, consumers can perfectly distinguish the transitory and persistent components of money supply. For example, in the home country agents forecast the future money supply according to the following expression:

$$E_t \left(\ln \left(\frac{M_{t+1}}{M_t} \right) - \ln \bar{\mu}_M \right) = E_t (z_{t+1} + u_{t+1}) = pz_t + \delta u_t.$$

In the more realistic case of incomplete information, consumers are unable to exactly determine which policy regime applies at any given time, that is, they ignore how to decompose historical realizations of money supply into permanent and transitory drivers. This implies that agents need to solve a signal extraction problem which determines how agents learn using the new information that arrives at the market to estimate eventual regime changes in the monetary policy.

To compute expectations of z_{t+1} and u_{t+1} conditional to the information set available at time t under incomplete information researchers can use a state-space representation for the growth rate of the money supply above-mentioned in a similar way as in Andolfatto et al. (2004). For example, in the case of the domestic country:

$$y_{t+1} = H' \xi_{t+1}$$

$$\xi_{t+1} = F\xi_t + \tilde{v}_{t+1}$$

where:

$$y_{t+1} = \ln \left(\frac{M_{t+1}}{M_t} \right) - \ln \bar{\mu}_M,$$

$$\xi_t = (z_t, u_t)',$$

$$\tilde{v}_t = (N_t, a_t), \text{ where } N_t = \begin{cases} (1 - \varphi) z_t, & \text{with probability } p \\ g_t, & \text{with probability } 1 - p \end{cases}$$

$$F = \begin{pmatrix} \varphi & 0 \\ 0 & \delta \end{pmatrix}, |\varphi| \in (0, 1)$$

$$H' = (1, 1)$$

However, the use of the Kalman filter for the foregoing state-space representation is non-optimal because the noise vector \tilde{v}_t is not Gaussian. To overcome the absence of normality we use an alternative state-space representation that is equivalent in mean and variance to the representation considered in Andolfatto et al. (2004)². For example, for the domestic country we have:

$$y_{t+1} = H' \xi_{t+1} \tag{23}$$

²A detailed explanation on this alternative representation can be found in Lafuente et al. (2011).

$$\xi_{t+1} = F\xi_t + G_{S_{t+1}}E_t\xi_{t+1} + \Phi_{S_{t+1}}v_{t+1} \quad (24)$$

where:

$$y_{t+1} = \ln\left(\frac{M_{t+1}}{M_t}\right) - \ln\bar{\mu}_M,$$

$$\xi_t = (z_t, u_t)',$$

$$v_t = (g_{t+1}, a_{t+1}),$$

$$F = \begin{pmatrix} \varphi & 0 \\ 0 & \delta \end{pmatrix}, |\varphi| \in (0, 1)$$

$$H' = (1, 1)$$

$$G_{S_{t+1}} = \begin{pmatrix} \omega_{S_{t+1}} & 0 \\ 0 & 0 \end{pmatrix}, \omega_{S_{t+1}} = \begin{cases} \frac{1-\varphi}{p} \text{ if } S_{t+1} = 1, \text{ with probability } p \\ \frac{-\varphi}{p} \text{ if } S_{t+1} = 0, \text{ with probability } 1-p \end{cases},$$

$$\Phi_{S_{t+1}} = \begin{pmatrix} \delta_{S_{t+1}} & 0 \\ 0 & 0 \end{pmatrix}, \delta_{S_{t+1}} = \begin{cases} 0, \text{ if } S_{t+1} = 1, \text{ with probability } p \\ 1 \text{ if } S_{t+1} = 0, \text{ with probability } 1-p \end{cases},$$

where $S_{t+1} = 1$ reflects no regime shift, that is, no change for the current monetary policy target. On the contrary $S_{t+1} = 0$ corresponds to the case of changes in the policy target.

Now the state equation requires the use of state-contingent matrices, and explicitly incorporates the role of economic agents' expectations in learning about monetary policy-making. Interestingly enough, this representation allows us not only the optimal use of the Kalman filter as the signal extraction procedure to obtain $\hat{y}_{t+1|t}$, but also to perform the maximum likelihood estimation of the parameters involved in the monetary policy. Note that $\hat{y}_{t+1|t} = \hat{z}_{t+1|t} + \hat{u}_{t+1|t} = \hat{E}_t \left[\ln\left(\frac{M_{t+1}}{M_t}\right) - \ln\bar{\mu}_M \right]$, where \hat{E}_t denotes the Kalman filter estimation for the expectations of the growth rate of the money supply when agents have incomplete information.

3 Numerical results

The next table summarizes the point estimates (standard deviations are in brackets) for all the relevant parameters involved in the monetary policy based on the maximum likelihood estimation procedure using the state-space representation that we previously mentioned in subsection 2.7. From quarterly data of M1 for the US and the EMU covering the sample period from 1980:Q1 to 2011:Q1 and 1991:Q1 to 2011Q1, respectively we obtain:

| | | | | |
|-----|-----------------------|----------------------------|----------------------------------|----------------------------------|
| US | $p = .8695[.0674]$ | $\delta = .6103[0.0517]$ | $\sigma_g^2 = .0128[0.0058]$ | $\sigma_a^2 = .0048[0.0002]$ |
| EMU | $p^* = .8528[0.0056]$ | $\delta^* = .7902[0.0647]$ | $\sigma_{g^*}^2 = .0104[0.0044]$ | $\sigma_{a^*}^2 = .0038[0.0001]$ |

These probabilities imply a mean duration of shifts of about 8 years. Once the transitory components are estimated, we can compute the correlation coefficient coefficient $\rho_{aa^*} = 0.3188$.

Given that we are going to simulate quarterly data, we consider a discount factor of 0.99, a commonly used value in the real business cycle literature. As to the weight of each consumption good in the utility function we set $\phi = 0.5$. Concerning the risk aversion we follow Campbell and Cochrane (1999) by setting $\gamma = 1.5$. We also use time series data to estimate the parameters involved in the time evolution of real endowments. Using the common sample (1991:Q1 to 2011Q1), the estimation of the bivariate VAR for the growth rate of GDP leads to the following point estimates:

$$\begin{pmatrix} \nabla \ln X_t \\ \nabla \ln X_t^* \end{pmatrix} = \begin{pmatrix} 0.0032 \\ 0.0006 \end{pmatrix} + \begin{pmatrix} 0.5258 & -0.0521 \\ 0.2883 & 0.3883 \end{pmatrix} \begin{pmatrix} \nabla \ln X_{t-1} \\ \nabla \ln X_{t-1}^* \end{pmatrix} + \begin{pmatrix} \hat{\xi}_{X,t} \\ \hat{\xi}_{X^*,t} \end{pmatrix}$$

with $\sigma_{\hat{\xi}_{X,t}} = .0060$, $\sigma_{\hat{\xi}_{X^*,t}} = .0059$ and the correlation between the home and foreign real shocks $\rho_{\hat{\xi}_X, \hat{\xi}_{X^*}} = 0.1742$. Table 1 shows the baseline parameterization that we consider. We simulated the model 1,000 times generating observations for each time series with the same sample size of the largest dataset used in the estimation. Economic agents need to solve a signal-extraction problem for estimating the two individual components of the money supply. We assume that they face this problem by constructing an optimal forecast based on all the relevant information. Each simulation is performed under two scenarios: complete and incomplete information. The scenario of complete information corresponds to the case when agents can perfectly identify permanent and transitory components of monetary policy. In contrast, under incomplete information agents face a nontrivial signal extraction problem because they can observe only an aggregated noise.

The solution of the model requires the evaluation of highly non-linear expressions, that precludes the possibility of an analytical solution. Appendix 1 provides a detailed explanation of the solution method proposed to obtain simulated equilibrium time series for spot and forward exchange rates.

To test the unbiasedness of the forward exchange rate as a forecast of the future spot rate, following Fama (1984), the econometric specification most commonly used in the literature is the following:

$$s_{t+k} - s_t = \alpha + \beta (f_{t+k} - s_t) + \xi_{t+k}$$

where s_{t+k} is the log of the nominal exchange rate in time $t+k$, f_{t+k} is the log of the k -period forward rate traded at time t and ξ_{t+k} is a random error.

To clearly identify the ability of the model to reproduce the forward bias we not only present the median estimate and the confidence interval at the 10% significance level, but also the density functions of the estimated slopes with the 1,000 regressions for each scenario (complete and incomplete information). Figure 1 summarizes the results obtained from the baseline parameterization. It can be observed that the existence of learning clearly produces a downward bias in the estimated slope coefficient. On the contrary, under rational expectations the estimated slope is close to one. Indeed we cannot reject the null hypothesis of $\beta = 1$ against the alternative $\beta < 1$ at the 10% significance level. However, under incomplete information the null is rejected. Figure 2 shows an example of how the adjustment takes place under rational expectations

and bounded rationality using the baseline parameterization. The different paths might be interpreted as a consequence of higher dispersion beliefs. When agents need to disentangle monetary shocks into the permanent and transitory components. In summary, what the model shows is that, given that learning is time consuming, the price discovery role of forward market is not in the way expected with rational expectations.

To analyze the sensitivity of our results we also present numerical simulations by changing some parameters from the benchmark setting. In particular, we check the robustness of our results to changes in the curvature of the utility function (parameter γ) and the degree of substitutability (parameter ϵ).

Figure 3 presents the results with higher risk aversion, in particular for $\gamma = 4$. As is apparent, the nature of our results from the baseline parameterization remains qualitatively unchanged. Now we reject forward unbiasedness not only under incomplete information, but also under bounded rationality. The downward bias is now greater, regardless the scenario considered. When risk aversion is high a small consumption shock has a large impact on the change in marginal utility. Under such conditions agents prefer a more persistent consumption, which leads to higher persistence in the forward premium and therefore a regression imbalance, since the regressor is a long memory process while the regressand is clearly an $I(0)$ variable.

Figure 4 shows the results with $\epsilon = -1.0$, that is, considering a lower degree of substitution between domestic and foreign goods. Again the distribution of the slope coefficient under complete information departs from the one corresponding to the case of incomplete information. Similar to the baseline scenario, the null hypothesis of $\beta = 1$ against the alternative of $\beta < 1$ cannot be rejected at the 10% significance level under complete information, while it is rejected when learning takes place.

To reinforce the idea that learning should be taken into account in order to better understanding forward bias when monetary transmission is not similar, we finally simulate the model with a completely new scenario: specifically we consider a) very low persistence in the transitory components ($\delta = \delta^* = 0.1$), b) $\sigma_g^2 = \sigma_a^2 = \sigma_{a^*}^2$, and c) $\sigma_g^2 \gg \sigma_{g^*}^2$. The rest of the parameters in both countries taking benchmark values. Chart 4 shows the results. As can be observed, with a standard value for risk aversion the model clearly shows the relevance of learning. Given that regime shifts are quite different in the two countries, the updating process of expectations under learning is much slower. The robustness of our findings is reflected by the stability of the percentile that corresponds to the unitary slope in the density function under incomplete information, that, regardless the parameterization setting considered is at least the 90th percentile.

4 Concluding Remarks

This paper has explored an explanation for the forward bias in foreign exchange markets based on a stochastic and dynamic general equilibrium model that incorporates regime shifts in monetary policy. Based on the well-known Lucas' model, the main novelty lies in the existence of a representative agent that needs to estimate the current state of monetary policy from analyzing the past history. The money supply is viewed

as having two stochastic components: a) a persistent component that reflects the preferences of the central bank regarding the long-run money supply or inflation target, and b) a transitory component that represents short-lived interventions or errors in controlling the money supply. In addition the model is formulated to allow for different weights for each consumption good in the utility function and the possibility that home and foreign real shocks as well as transitory monetary shocks in the two countries may be correlated.

We present results from numerical simulations focusing on the role of the monetary policy. In particular, we consider two scenarios: a) complete information, with consumers that can perfectly distinguish the transitory and persistent components of money supply, and b) incomplete information: in this case consumers are unable to perfectly determine which policy regime applies at any given time, and they face a signal-extraction problem.

We simulated the model initially using a baseline parameterization which is based on a careful calibration from quarterly data for the US and the EU. Numerical simulations suggest that the need for learning can be a factor explaining the forward bias. The intuition is clear.

The hypothesis of rational expectations would require that market participants be homogeneous in their formation of expectations. But this assumption implicitly assumes that agents do not make systematic errors when forecasting the relevant variables and parameters (for example the probability of regime shift). However neither complete information nor perfect foresight are features of financial markets. And when learning about monetary policy does not take effect immediately, our results reveal that a significant downward bias arises. Given that learning is time-consuming, the price discovery role of the forward market is not in the way as expected under rational expectations.

5 References

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6 Appendix 1

This appendix contains the explanation of the solution method used. From the optimality conditions, we obtain the following set of equations:

$$S_t = \frac{1-\phi}{\phi} \left(\frac{X_{F,t}}{X_{D,t}} \right)^\epsilon \frac{M_t}{M_t^*} \quad (\text{A1})$$

$$P_{D,t} = \frac{M_t}{X_{D,t}} \quad (\text{A2})$$

$$P_{F,t} = \frac{M_t^*}{X_{F,t}} \quad (\text{A3})$$

$$C_{F,t} = \left[\frac{(1-\phi) P_{D,t}}{\phi P_{F,t} S_t} \right]^\sigma C_{D,t} \quad (\text{A4})$$

$$C_{F,t}^* = \left[\frac{(1-\phi) P_{D,t}}{\phi P_{F,t} S_t} \right]^\sigma C_{D,t}^* \quad (\text{A5})$$

$$C_{F,t} = \left[\frac{(1-\phi) P_{D,t}}{\phi P_{F,t} S_t} \right]^\sigma \frac{\left[M_t + T_{t-1} \left(\frac{S_t - F_{t-1}}{F_{t-q}} \right) P_{D,t}^{-\sigma} \right]}{P_{D,t}^{1-\sigma} + \left(\frac{1-\phi}{\phi} \right)^\sigma (S_t P_{F,t})^{1-\sigma}} \quad (\text{A6})$$

$$C_{F,t}^* = \left[\frac{(1-\phi) P_{D,t}}{\phi P_{F,t} S_t} \right]^\sigma \frac{\left[M_t^* + T_{t-1}^* \left(\frac{S_t - F_{t-1}}{F_{t-q}} \right) S_t P_{D,t}^{-\sigma} \right]}{P_{D,t}^{1-\sigma} + \left(\frac{1-\phi}{\phi} \right)^\sigma (S_t P_{F,t})^{1-\sigma}} \quad (\text{A7})$$

$$T_{t-1}^* = T_{t-1} \quad (\text{A8})$$

$$F_t = \frac{E_t \left[C_{D,t+1}^{\epsilon-1} \left(\phi C_{D,t+1}^\epsilon + (1-\phi) C_{F,t+1}^\epsilon \right)^{\frac{1-\gamma}{\epsilon}-1} \frac{S_{t+1}}{P_{D,t+1}} \right]}{E_t \left[C_{D,t+1}^{\epsilon-1} \left(\phi C_{D,t+1}^\epsilon + (1-\phi) C_{F,t+1}^\epsilon \right)^{\frac{1-\gamma}{\epsilon}-1} \frac{1}{P_{D,t+1}} \right]} \quad (\text{A9})$$

$$F_t = \frac{E_t \left[(C_{F,t+1}^*)^{\epsilon-1} \left(\phi (C_{D,t+1}^*)^\epsilon + (1-\phi) (C_{F,t+1}^*)^\epsilon \right)^{\frac{1-\gamma}{\epsilon}-1} \frac{1}{P_{F,t+1}} \right]}{E_t \left[(C_{F,t+1}^*)^{\epsilon-1} \left(\phi (C_{D,t+1}^*)^\epsilon + (1-\phi) (C_{F,t+1}^*)^\epsilon \right)^{\frac{1-\gamma}{\epsilon}-1} \frac{1}{S_{t+1} P_{F,t+1}} \right]} \quad (\text{A10})$$

Using equations (A1) to (A8), equations (A9) and (A10) can be expressed as follows:

$$F_t = \frac{E_t \left[g_1 (F_t, M_t; M_{t+1}, M_{t+1}^*, X_{D,t+1}, X_{F,t+1}) \right]}{E_t \left[g_2 (F_t, M_t; M_{t+1}, M_{t+1}^*, X_{D,t+1}, X_{F,t+1}) \right]} \quad (\text{A11})$$

$$F_t = \frac{E_t \left[g_3 (F_t, M_t; M_{t+1}, M_{t+1}^*, X_{D,t+1}, X_{F,t+1}) \right]}{E_t \left[g_4 (F_t, M_t; M_{t+1}, M_{t+1}^*, X_{D,t+1}, X_{F,t+1}) \right]} \quad (\text{A12})$$

where functions $g_i(\cdot)$, $i = 1, 2, 3, 4$ are:

$$g_1 = \Psi_D^{-\gamma} \left[\phi + (1 - \phi) \left(\frac{X_{F,t+1}}{X_{D,t+1}} \right)^{(1-\epsilon)\sigma\epsilon} \right]^{\frac{1-\gamma}{\epsilon} - 1} \left(\frac{X_{D,t+1}}{M_{t+1}} \right) \quad (\text{A13})$$

$$g_2 = \Psi_D^{-\gamma} \left[\phi + (1 - \phi) \left(\frac{X_{F,t+1}}{X_{D,t+1}} \right)^{(1-\epsilon)\sigma\epsilon} \right]^{\frac{1-\gamma}{\epsilon} - 1} \times \left(\frac{X_{D,t+1}}{M_{t+1}} \right) \frac{\phi}{1 - \phi} \left(\frac{X_{D,t+1}}{X_{F,t+1}} \right)^\epsilon \frac{M_{t+1}^*}{M_{t+1}} \quad (\text{A14})$$

$$g_3 = \left(\frac{X_{F,t+1}}{X_{D,t+1}} \right)^{(1-\epsilon)\sigma(\epsilon-1)} \left(\frac{X_{F,t+1}}{M_{t+1}^*} \right) \times \Psi_F^{-\gamma} \left[\phi + (1 - \phi) \left(\frac{X_{F,t+1}}{X_{D,t+1}} \right)^{(1-\epsilon)\sigma\epsilon} \right]^{\frac{1-\gamma}{\epsilon} - 1} \quad (\text{A15})$$

$$g_4 = \left(\frac{X_{F,t+1}}{X_{D,t+1}} \right)^{(1-\epsilon)\sigma(\epsilon-1)} \Psi_F^{-\gamma} \left[\phi + (1 - \phi) \left(\frac{X_{F,t+1}}{X_{D,t+1}} \right)^{(1-\epsilon)\sigma\epsilon} \right]^{\frac{1-\gamma}{\epsilon} - 1} \times \left(\frac{X_{F,t+1}}{M_{t+1}} \right) \frac{\phi}{1 - \phi} \left(\frac{X_{D,t+1}}{X_{F,t+1}} \right)^\epsilon \quad (\text{A16})$$

being:

$$\Psi_D = \frac{M_{t+1}^* + T_t \left[\frac{1-\phi}{\phi} \left(\frac{X_{F,t+1}}{X_{D,t+1}} \right)^\epsilon \frac{M_{t+1} - F_t}{M_{t+1}^*} \right]}{\left(\frac{M_{t+1}}{X_{D,t+1}} \right)^{-\sigma} + \left(\frac{1-\phi}{\phi} \right)^\sigma \left[\frac{1-\phi}{\phi} \left(\frac{X_{F,t+1}}{X_{D,t+1}} \right)^\epsilon \frac{M_{t+1}}{M_{t+1}^*} \frac{M_{t+1}^*}{X_{F,t+1}} \right]^{1-\sigma}} \left(\frac{M_{t+1}}{X_{D,t+1}} \right)^{-\sigma} \quad (\text{A17})$$

$$\Psi_F = \frac{M_{t+1} + T_t \left[\frac{1-\phi}{\phi} \left(\frac{X_{F,t+1}}{X_{D,t+1}} \right)^\epsilon \frac{M_{t+1} - F_t}{F_t} \right]}{\left(\frac{M_{t+1}}{X_{D,t+1}} \right)^{-\sigma} + \left(\frac{1-\phi}{\phi} \right)^\sigma \left[\frac{1-\phi}{\phi} \left(\frac{X_{F,t+1}}{X_{D,t+1}} \right)^\epsilon \frac{M_{t+1}}{M_{t+1}^*} \frac{M_{t+1}^*}{X_{F,t+1}} \right]^{1-\sigma}} \frac{1-\phi}{\phi} \left(\frac{X_{F,t+1}}{X_{D,t+1}} \right)^\epsilon \frac{M_{t+1}}{M_{t+1}^*} \left(\frac{M_{t+1}}{X_{D,t+1}} \right)^{-\sigma} \quad (\text{A18})$$

We use linear and log-linear approximation of functions $g_i(\cdot)$, $i = 1, 2, 3, 4$ around the estimated trend using the Hodrick-Prescott filter for $X_{D,t+1}$, $X_{F,t+1}$, M_{t+1} and M_{t+1}^* . Substituting such approximant functions into equations (A11) and (A12), we obtain a system of two equations with two variables (F_t, T_t) as a function of the following expectations: $E_t(\ln(\mu_{X_{t+1}}))$, $E_t(\ln(\mu_{X_{t+1}^*}))$, $E_t(\ln(\mu_{M_{t+1}}))$, $E_t(\ln(\mu_{M_{t+1}^*}))$. Expectations for endowments are always computed as $E_t(\ln(\mu_{X_{t+1}})) = (1 - \rho_X) \ln(\bar{\mu}_X) + \rho_{11} \ln(\mu_{X_t}) + \rho_{12} \ln(\mu_{X_t^*})$ and $E_t(\ln(\mu_{X_{t+1}^*})) = (1 - \rho_{X^*}) \ln(\bar{\mu}_{X^*}) + \rho_{21} \ln(\mu_{X_t}) + \rho_{22} \ln(\mu_{X_t^*})$ for the domestic and the foreign country, respectively. However, for the monetary policy we consider two alternative scenarios:

a) Complete information: in this case, expectations are computed as follows:

$$E_t(\ln(\mu_{M_{t+1}})) = \ln \bar{\mu}_M + p z_t + \delta u_t, \text{ and } E_t(\ln(\mu_{M_{t+1}^*})) = \ln \bar{\mu}_{M^*} + p^* z_t^* + \delta^* u_t^*$$

b) Incomplete information:

$E_t \left(\ln \left(\mu_{M_{t+1}} \right) - \ln \bar{\mu}_M \right) = \hat{y}_{t+1|t} = \hat{z}_{t+1|t} + \hat{u}_{t+1|t}$, and $E_t \left(\ln \left(\mu_{M_{t+1}^*} \right) - \ln \bar{\mu}_{M^*} \right) = \hat{y}_{t+1|t}^* = \hat{z}_{t+1|t}^* + \hat{u}_{t+1|t}^*$ by applying the Kalman filter using equations (23) and (24).

Once the relevant expectations have been computed either under complete or incomplete information, the above system for (F_t, T_t) can be solved.

Appendix 2

Table 1. Baseline parameterization

| Home country (US) | | Foreign country (EMU) | |
|-------------------|--------|-----------------------|--------|
| ρ_X | 0.5258 | ρ_{X^*} | 0.3883 |
| σ_X | 0.0060 | σ_{X^*} | 0.0059 |
| σ_g | 0.0128 | σ_{g^*} | 0.0104 |
| σ_a | 0.0048 | σ_{a^*} | 0.0038 |
| δ | 0.6103 | δ^* | 0.7902 |

Correlations: $\rho_{\xi_X, \xi_{X^*}} = 0.1742$; $\rho_{aa^*} = 0.4243$

Parameter to control the elasticity of substitution: $\epsilon = 0.50$

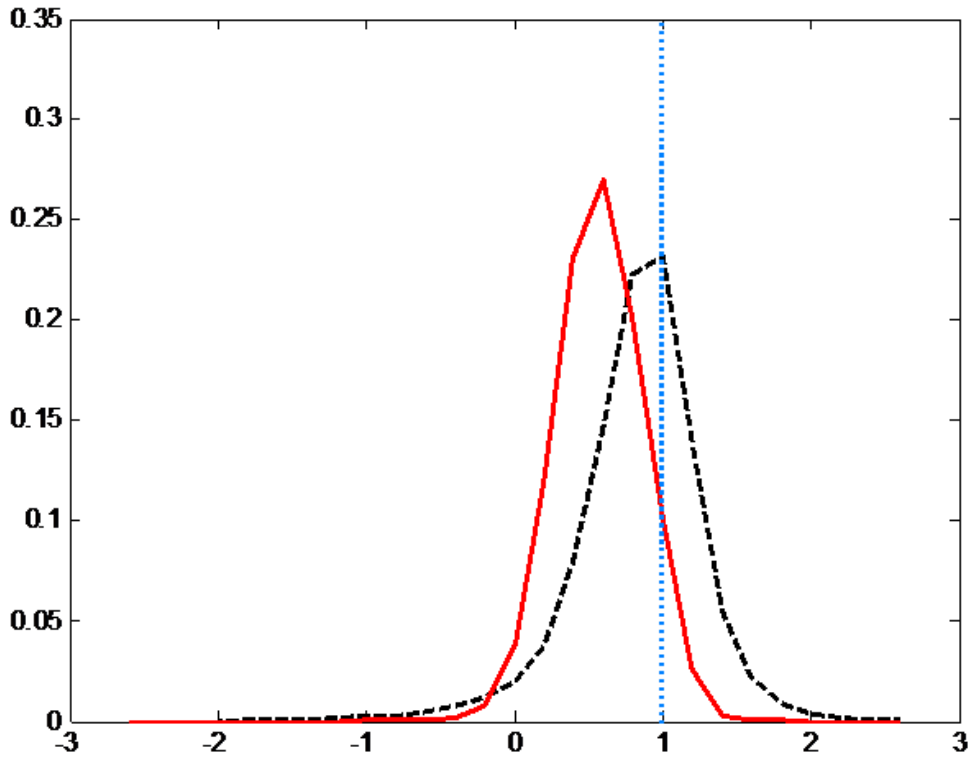
Discount factor: $\beta = 0.99$

Weight of each consumption good in the utility function: $\phi = 0.50$

Curvature of the utility function: $\gamma = 0.50$

Probability of regime shifts: $1 - p = 1 - 0.8695$; $1 - p^* = 1 - 0.8528$

Figure 1. Density functions of simulated slopes. Baseline parameterization



Note: the dotted line corresponds to complete information

Median slope

| | |
|----------------------|------------------------|
| Complete information | incomplete information |
| 0.8719 | 0.5742 |

90-th quantile

| | |
|----------------------|------------------------|
| Complete information | incomplete information |
| 1.2676 | 0.9412 |

th-quantile that corresponds to the unitary value

| | |
|----------------------|------------------------|
| Complete information | incomplete information |
| 64% | 91% |

Figure 2 Adjustment of forward rates under rational expectations and bounded rationality

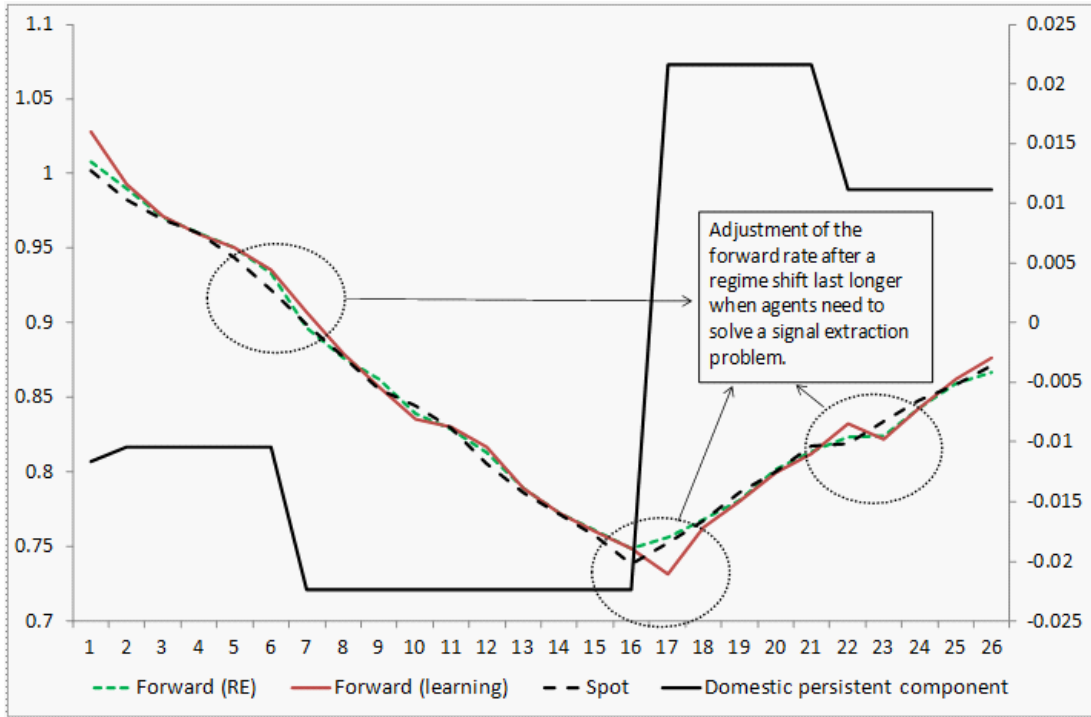
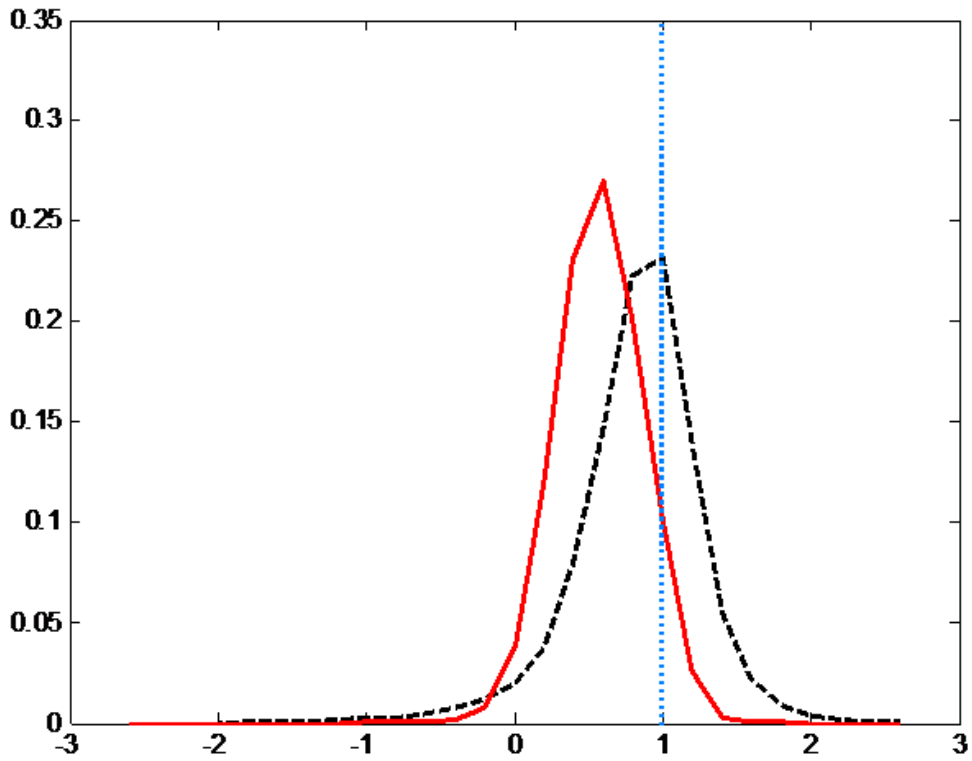


Figure 3. Density functions of simulated slopes. Sensitivity to parameter γ ($\gamma = 4.0$)



Note: the dotted line corresponds to complete information

Median slope

Complete information

0.5987

incomplete information

0.3477

90-th quantile

Complete information

1.0816

incomplete information

0.7997

th-quantile that corresponds to the unitary value

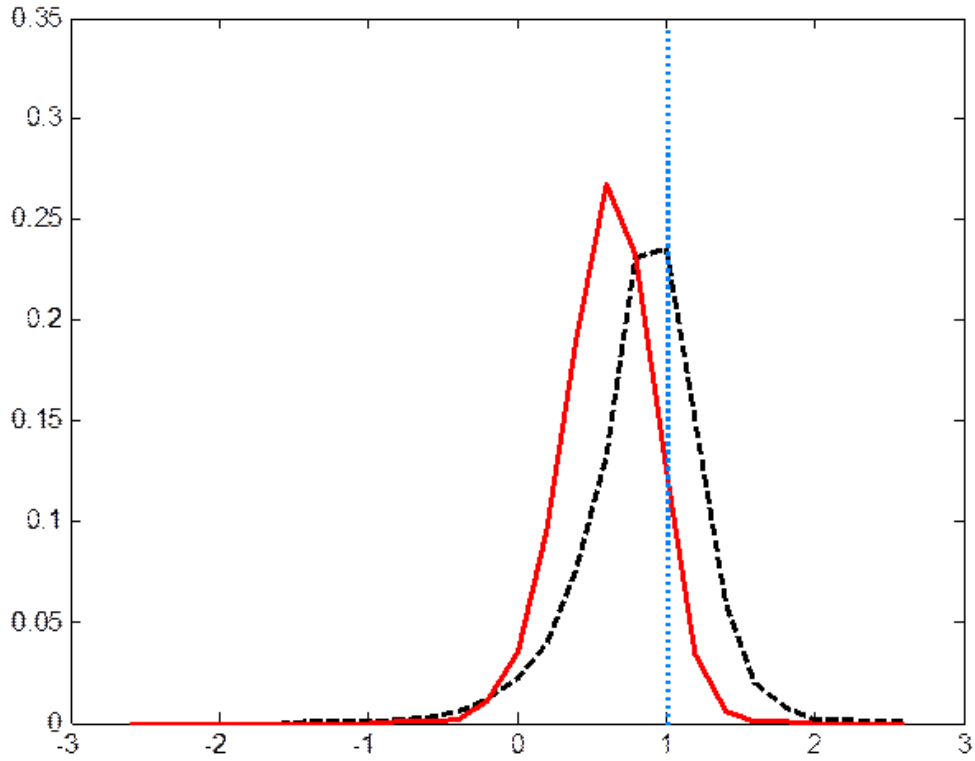
Complete information

89%

incomplete information

98%

Figure 4. Density functions of simulated slopes. Sensitivity to parameter ϵ ($\epsilon = -1.0$)



Note: the dotted line corresponds to complete information

Median slope

| | |
|----------------------|------------------------|
| Complete information | incomplete information |
| 0.8798 | 0.6261 |

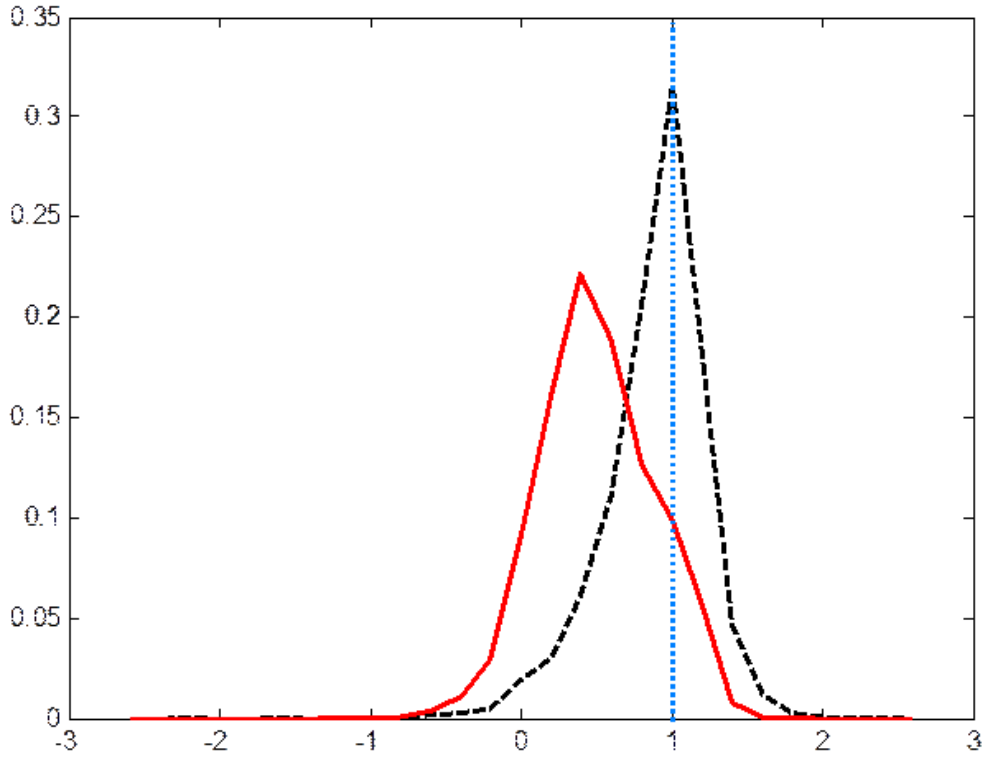
90-th quantile

| | |
|----------------------|------------------------|
| Complete information | incomplete information |
| 1.2813 | 0.9826 |

th-quantile that corresponds to the unitary value

| | |
|----------------------|------------------------|
| Complete information | incomplete information |
| 62% | 92% |

Figure 5. Density functions of simulated slopes. New baseline parameterization



Note: the dotted line corresponds to complete information

Median slope

| | |
|----------------------|------------------------|
| Complete information | incomplete information |
| 0.9486 | 0.4503 |

90-th quantile

| | |
|----------------------|------------------------|
| Complete information | incomplete information |
| 1.2398 | 0.9668 |

th-quantile that corresponds to the unitary value

| | |
|----------------------|------------------------|
| Complete information | incomplete information |
| 57% | 91% |