Contingent convertible bonds: a catastrophe insurance for banks?

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Abstract:

This paper justifies, in an agency context, the existence of hybrid securities that appeared very recently on the organised market: CoCo bonds (contingent convertible bonds). Like straight debt, CoCos make it possible to profit from the tax benefits of debt. And, like stocks, they provide protection against financial distress. Although CoCos cannot completely protect banks against bankruptcy, they significantly reduce their probability of failure independently of regulator actions. The structural model shows that CoCos allow increased valorisation of the banks without jeopardising their stability. However, special attention must be paid to their design on penalty of the inability to provide an efficient means of financing to investors. Particular attention should be paid to fixing the value of the "trigger". Its optimal value is highly dependent on the environment (structure and amount of bankruptcy costs, extent of dilution of shareholder claims, tax environment and so on).

Keywords: bank, capital structure, contingent capital, asset substitution, financial crisis, banking governance

JEL classification: G12, G13, G21, G28, G32

1. Introduction

The recent financial crisis has shown the difficulties that banks face in raising new equity other than from the government. So, contingent capital has been advanced as a solution. This instrument provides an automatic equity injection when a specified threshold or “trigger” is tripped. It is part of prudential measures defined by the Basel Committee to stabilise the financial system and reduce “Too Big To Fail” government assistance.

In fact, the first contingent capital known as “CoCo” bonds (contingent convertible bonds) was issued by Lloyds Bank in November 2009. This security is converted into equity if the Core Tier 1 ratio falls below 5%. Another CoCo issuance, also with a trigger based on accounting ratio, was by Rabobank in May 2010. Instead of converting into equity when the bank‘s capital ratio falls below 7%, the securities are written down by 75% of their face value and the remaining 25% is paid to investors. Indeed, Rabobank is a cooperative bank which cannot issue new equity. More recently (in February 2011), Crédit Suisse, the second-largest Swiss bank after UBS, successfully issued CoCos, again with a trigger level set at 7%. This issuance lies within the scope of the new equity requirements following the agreements of Basel III. In September 2010, the international regulators decided to raise (progressively until 2019) the minimum common equity requirement from 2% to 7% in order to enhance the soundness of banks. A second capital conservation buffer of 1% to 2% may also be imposed on systemically important banks with combinations of capital surcharges, contingent capital and bail-in debt. So, by including this systemic risk buffer, the minimum equity requirements would rise from 7% to 9%. Following the example of the Swiss regulator, the other regulators could call on CoCos for this second buffer.

The academic literature has already looked at this security and analysed its design, the interest of referring to a book value or a market value to set the threshold, the terms of conversion, the number of triggers to insert, the valuation of assets itself and, finally, the optimal structure of the capital.

In addition to the fact that they enable the issuing banks to recapitalise themselves and to reduce the probability of failure, CoCos also – according to Flannery (2005, 2009) – cause shareholders to internalise the costs associated with their risk-taking. Indeed, shareholders bear the downside outcomes from their investment decisions when the trigger is tripped. In return, they profit from lower expected bankruptcy costs and the tax benefits of debt until the conversion happens. Moreover, their bank can maintain less equity capital on its books. As for CoCos investors, they receive an equivalent value of shares in return for the face value of their debt so that they bear no loss of capital. The share price determines how many new stocks the investors receive for their bonds, knowing that only a proportion of outstanding CoCos may be converted to restore the capital ratio. Flannery recommends using a market-value trigger because the accounting-based triggers may be biased upwards. However, the market value of equity may also be subject to manipulation. Indeed, the debt holders receive more shares when the stock price drops, which is consistent with the some market manipulation.

Contrary to Flannery's view, two conditions are necessary to convert debt into equity for Squam Lake Working Group (2009): a declaration by regulators that the financial system is suffering a systemic crisis, and a violation by the bank of covenants specified in the hybrid-security contract. There are two reasons for this double trigger. On one hand, the debt disciplines the management: by limiting the conversion to only systemic crises, the benefit of the debt is maintained. On the other hand, the bank-specific component of the trigger enables the sound bank to avoid being forced to convert in a crisis period.

In the same way, McDonald (2010) proposes contingent capital with a dual price trigger. The conversion requires that both the bank's stock price and the financial institutions' index fall below a trigger value. The aim of this structure is to reduce the bank's debt in times of crisis and to permit a bank to fail during good times. He recommends, on one hand, using market-based triggers without reference to accounting-based measures of capital – contrary to Flannery, whose denominator is an accounting measure of assets, in order to be unaffected by changes in accounting rules. On the other hand, the bond must be converted into a fixed number of shares at a premium price, which means that the value of shares upon conversion is lower than the par value of the bonds. So, the manipulation is less profitable. A conversion ratio depending on the stock price can also lead to a "death spiral", as described by Hillion and Vermaelen (2004). So, if the stock price drops, the bondholders will receive more shares, diluting the existing stockholders' claims and so lowering the stock price even further. For Squam Lake Working Group (2009), this death-spiral problem disappears if the number of shares to be issued upon conversion is fixed.

Pennacchi (2010) analyses the influence of both the contractual terms of the bank's contingent capital and the level of its risk on the equilibrium pricing of CoCos and on the yields required by their holders. The return on bank assets follows a jump-diffusion process, introducing the possibility of sudden decline in the bank's asset value, which can occur during a financial crisis. The possibility of jumps has an impact on the pricing of contingent capital, which could convert at less than par even if the contract specifies the contrary. The bondholders will require a yield above default-free yields. Likewise, a higher yield is required for contingent capital that converts at discount, particularly when bank capital is low and apart from asset-value jumps. Pennacchi then examines the yields on contingent capital according to the maturity, and finds that they approach default-free yields when the bank has high capital. However, contingent capital is valued more as equity than as default-free bonds when the capital is low. He shows also that the incentives for risk-taking rise as the equity-conversion threshold for contingent capital decreases, confirming the importance of the conversion threshold in protecting bondholders. So, CoCos mitigate financial distress if their conversion threshold is set at a relatively high level of equity.

By extending the model of Leland (1994), Albul et al (2010) focus on the influence of the contingent capital on the optimal capital structure. They show that the banks must substitute CoCos for straight debt and not for equity in their capital structure. Replacing equity with CoCos increases the tax-shield benefit but provides no benefits for regulatory safety, and may raise the incentive for asset substitution. If the proceeds of CoCos are used to pay off existing straight debt, the reduction of bankruptcy costs may exceed the loss of tax-shield benefits at the expense of the shareholders, and to the benefit of the holders of the straight debt. Adding CoCos to a new optimal capital structure, instead of a small part of the optimal amount of straight debt, is worthwhile for the bank as long as the benefits from reduced expected bankruptcy costs compensate for the potential loss in the tax shield. Regulators exogenously fix the amount of CoCos so that the total amount of debt equals the optimal amount of straight debt without CoCos. So, the total amount of debt in the economy remains the same and there are no additional social costs in the form of extra tax subsidies. Moreover, the incentive for risk shifting is reduced, which is favourable for regulatory safety. Likewise, the higher the trigger, the greater bank safety is.

The key contribution of this current paper is to provide a formal financial model of CoCos as a component of the bank's capital structure. Specifically, the model makes it possible to achieve an optimal capital structure endogenously. This new instrument has the tax benefits of straight debt and can reduce the bankruptcy costs which arise when the cash flow of the bank is not sufficient to avoid failure. The model presented is an adaptation of that developed initially by Gavish and Kalay (1983), then generalised by Green and Talmor (1986). These authors analyse the effects of the leverage ratio on stockholders' incentives to increase risk. Green (1984) also examines the agency problems associated with debt financing and recommends the issuance of convertible debt and warrants to control risk incentives. From this literature, Maati-Sauvez (1996) proposes a model to explain the issuance of such financial instruments in an agency context. In the current paper, closed-form solutions are developed for the issue of CoCos, and conditions for internal optimal capital structure are discussed. Optimal leverage is linked to bank risk, taxes, bankruptcy costs and the properties of CoCos.

After specifying the theoretical model and the underlying assumptions, the results of model simulations will be considered.

2. The structural model

The framework of the analysis will first be specified before the value of the bank without and with asset substitution is analysed.

2.1. Framework of the analysis

We consider a one-period model with two dates, t_0 and t_1. Covenants restricting investment policies cannot be used because of the opacity characterising the assets of the bank. Once in possession of the funds, the shareholders have total freedom to choose the investment project. The model is founded on several assumptions:

- A.1: The shareholders of the bank can gain from an asset substitution;
- A.2: Debt holders know the characteristics of the projects but cannot observe the shareholders' decision about investment policy;
- A.3: All parties are risk-neutral;
- A.4: There is no personal taxation, and corporation tax is determined by the cash flow minus the total promised payment (principal and interest payments);
- A.5: There are no agency costs of equity.

The sequence of decisions in t_0 is as follows: the shareholders propose different financings (D_0; S_0), with D_0 being the market value of the debt and S_0 the market value of stocks.

The debt contract includes a provision of conversion which is automatically set off when the cash flow is below the trigger K. This affects the share of cash flow ex post because, according to its value, investors perceive the ex ante CoCos as a security with a design located between that of the debt and that of equity.

Automatic conversion will lead, if it happens, to a disappearance of the debt in favour of new equity. Cash flow will be shared into a greater number of stocks. Debt holders and shareholders respectively will receive the fraction $\alpha = m_0 \times h / (m_0 \times h + n_0)$ and (1-α) of the cash flow net of tax or indirect bankruptcy costs, knowing $\alpha + (1-\alpha) = 1$, with n_0 the number of stocks originally issued, m_0 the number of bonds issued and h the number of stocks obtained through the conversion of a bond. Consideration of a fixed number of stocks upon conversion reduces the risk of manipulation (McDonald, 2010) and the death spiral (Squam Lake Working Group, 2009).

The contractual terms of CoCos (α and K), fixed exogenously, are disclosed to debt holders at the same time as the potential financing plans and investment projects that may be undertaken. For each D_0, debt holders set i, the interest rate required, such as $D_1 = D_0$ (1 + i), by anticipating the choice of project j, the risk of which is transcribed through Ω with $\Omega \in$ [0; 1]. Moreover, debt holders are involved within a framework of perfect competition. Therefore, the interest rate i will be such that it prohibits any abnormal profit for the debt holders. For each pair (D_0; i), the shareholders determine the optimal investment policy Ω^* associated with this, and then choose D_0^* from the triplets (D_0; i(D_0); $\Omega^*(i(D_0); D_0)$).

The bank can choose from a multitude of projects with the same cost I, with $I = S_0 + D_0$, the cash flow of which at the end of the period is characterised by:

$$A_1 = \mu(\Omega) + \sigma(\Omega)\varepsilon \qquad (1)$$

with A_1 the random cash flow of the project in t_1 such as $A_1 \rightarrow N(\mu(\Omega); \sigma(\Omega)\sigma(\varepsilon))$ and $f(A_1)$ the density function of A_1.

The mean terminal value $\mu(\Omega)$ is a monotonically decreasing function of risk: $\delta\mu(\Omega)/\delta\Omega < 0$. This means there is an internal solution for the choice of investment policy by the shareholders: they choose a riskier project as long as the declining value of the bank is less than the transfer of wealth from which they benefit at the expense of debt holders. A random component $\sigma(\Omega)\varepsilon$ is added to the certain component. $\sigma(\Omega)$ represents the risk of the project, such as $\delta\sigma(\Omega)/\delta\Omega > 0$. ε is a random variable representing the specific risk with continuous density $g(\varepsilon)$, $E[\varepsilon] = 0$ and $\sigma(\varepsilon) > 0$. Given the limited liability of investors for the amount of their contribution, only the positive values of A_1 will be studied.

τ is the tax rate for corporations and $\overline{\beta}$ the rate representative of the total liquidation costs applied to cash flow when the bank cannot repay the debt. Given the one-period framework of the model, no reorganisation is possible. The provision to automatically convert debt into equity is intended to reduce the number of states of nature where bankruptcy occurs. But bankruptcy costs remain when $K \leq D_1$, and these have two components: direct and indirect costs. The conversion is a signal of the financial distress of the bank, generating indirect costs of bankruptcy in a proportion of $\underline{\beta}$, with $0 < \underline{\beta} < \overline{\beta}$. These costs are proportional to the difference between D_1 and A_1. In monetary units, indirect costs and total costs of bankruptcy associated with CoCos are, respectively, equal to $\underline{\beta}(D_1-A_1)$ and $\overline{\beta}(D_1-A_1)$. The limited liability for investors (shareholders and debt holders) restricts the field of A_1 in case of default. Financial distress generates indirect bankruptcy costs but the bondholders cannot be solicited to cover expenses in excess of cash flow: $A_1-\underline{\beta}(D_1-A_1)) \geq 0$. The field of A_1 is bounded such that $A_1 \geq \underline{\beta} D_1/(1+\underline{\beta})$, with $\underline{\beta} D_1/(1+\underline{\beta}) < \bar{\beta} D_1/(1+\bar{\beta})$.

There are four distinct pivot values in terms of future cash flows, corresponding to the amount of the:

- indirect bankruptcy costs $\underline{\beta} D_1/(1+\underline{\beta})$. If cash flow is below this marker, neither the shareholders nor bondholders can perceive income because the asset value is consumed by the indirect costs of bankruptcy even if the threshold is reached;
- total bankruptcy cost $\bar{\beta} D_1/(1+\bar{\beta})$ that support investors knowing that $\underline{\beta} D_1/(1+\underline{\beta}) < \bar{\beta} D_1/(1+\bar{\beta})$ since $\underline{\beta} < \bar{\beta}$. Otherwise, the direct costs of bankruptcy would be negative;
- debt to be repaid at maturity plus interest payments D_1 with $D_1 = D_0$ (1+i) and $\bar{\beta} D_1/(1+\bar{\beta}) < D_1$ if $\bar{\beta} > 0$;
- trigger K.

The first three pivot values, strictly different, are dependent on K, the latter being fixed exogenously by the shareholders at time 0. This decision leads to four scenarios (figure 1).

*** Insert Figure 1 ***

Scenario 4, corresponding to straight debt, will be studied first because it constitutes the benchmark. Then, scenarios 1 to 3 will be analysed.

2.1.1. Scenario 4: $K < \underline{\beta} D_1/(1+\underline{\beta})$

The trigger is strictly lower than the amount of indirect bankruptcy costs, while remaining positive. The CoCos are, from this perspective, straight debt since automatic conversion, if it occurs, will have no effect. Consequently, if:

- $0 \leq A_1 < \bar{\beta} D_1/(1+\bar{\beta})$: cash flow is consumed by the total bankruptcy costs;
- $\bar{\beta} D_1/(1+\bar{\beta}) \leq A_1 < D_1$: bondholders are the only ones who have non-zero earnings; $A_1 - \bar{\beta}(D_1-A_1)$, that is to say the cash flow net of bankruptcy costs, because legal liquidation occurs;
- $D_1 \leq A_1$: debt D_1 is repaid and shareholders receive the net cash flow after tax $(A_1-D_1)(1-\tau)$.

Substituting A_1 by (1), the NPV of the bank ($V_{t,4}$) in t_0 is:

$$V_{t,4} = \left\{ \int_{\bar{b}}^{d} (\mu(\Omega) + \sigma(\Omega)\varepsilon - \bar{\beta}(D_1 - (\mu(\Omega) + \sigma(\Omega)\varepsilon))\, g(\varepsilon)\, d\varepsilon + \int_{d}^{\infty} D_1\, g(\varepsilon)\, d\varepsilon \right.$$

$$\left. + \int_{d}^{\infty} (\mu(\Omega) + \sigma(\Omega)\varepsilon - D_1)(1-\tau)\, g(\varepsilon)\, d\varepsilon \right\} \theta - I \qquad (2)$$

with θ the discount factor, $\bar{b} = (\bar{\beta} D_1/(1+\bar{\beta}) - \mu(\Omega))/\sigma(\Omega)$ and $d = (D_1 - \mu(\Omega))/\sigma(\Omega)$.

2.1.2. Scenario 1: $D_1 < K$

With the trigger value exceeding the amount of debt, legal bankruptcy cannot occur. Only indirect bankruptcy costs can cut down the income of investors, and the design of CoCos is similar to that of stocks in terms of the distribution of cash flow. This scenario is associated with the highest probability of automatic conversion. Four configurations are possible, depending on the value of cash flow at maturity:

- $0 \leq A_1 < \underline{\beta} D_1/(1+\underline{\beta})$: cash flow being lower than $\underline{\beta} D_1/(1+\underline{\beta})$, the asset is worthless and investors receive no income. Although it is conceivable that automatic conversion occurs, there is nothing to share;
- $\underline{\beta} D_1/(1+\underline{\beta}) \leq A_1 < D_1$: the trigger is tripped and the bank suffers financial distress. So, investors lose (only) the indirect costs of bankruptcy. Shareholders and lenders respectively share a fraction of the cash flow net of the indirect costs of bankruptcy: $(1-\alpha)[A_1 - \underline{\beta}(D_1-A_1)](1-\tau)$ and $\alpha[A_1 - \underline{\beta}(D_1-A_1)](1-\tau)$;
- $D_1 \leq A_1 < K$: cash flow being larger than the debt to repay while remaining below the trigger, investors will again share this. Cash flow is reduced only by the amount of the tax, since the trigger is tripped. So, the debt disappears and the bank is not exposed to legal bankruptcy. Shareholders and lenders gain $(1-\alpha)A_1(1-\tau)$ and $\alpha A_1(1-\tau)$ respectively;
- $K \leq A_1$: the trigger is not tripped. The shareholders repay the debt and capture $(A_1-D_1)(1-\tau)$.

The NPV of the bank ($V_{t,1}$) in t_0 is:

$$V_{t,1} = \left\{ \int_{\underline{b}}^{d} [\mu(\Omega)+\sigma(\Omega)\varepsilon - \underline{\beta}(D_1-(\mu(\Omega)+\sigma(\Omega)\varepsilon))](1-\tau)\, g(\varepsilon)\, d\varepsilon \right.$$

$$\left. + \int_{d}^{k} (\mu(\Omega)+\sigma(\Omega)\varepsilon)(1-\tau)\, g(\varepsilon)\, d(\varepsilon) + \int_{k}^{\infty} ((\mu(\Omega)+\sigma(\Omega)\varepsilon)(1-\tau)+\tau D_1)\, g(\varepsilon)\, d\varepsilon \right\}\theta$$

$$-(S_0+D_0) \tag{3}$$

with $\underline{b} = (\underline{\beta} D_1/(1+\underline{\beta}) - \mu(\Omega))/\sigma(\Omega)$ and $k = (K-\mu(\Omega))/\sigma(\Omega)$.

2.1.3. Scenario 2: $\bar{\beta} D_1/(1+\bar{\beta}) < K \leq D_1$

With the trigger being lower than the value of the debt, legal bankruptcy can occur. Again, four configurations are possible:

- $0 \leq A_1 < \underline{\beta} D_1/(1+\underline{\beta})$: the asset value is consumed by the indirect costs of bankruptcy;
- $\underline{\beta} D_1/(1+\underline{\beta}) \leq A_1 < K$: shareholders and bondholders receive $(1-\alpha)[A_1 - \underline{\beta}(D_1-A_1)](1-\tau)$ and $\alpha[A_1 - \underline{\beta}(D_1- A_1)](1-\tau)$ respectively;
- $K \leq A_1 < D_1$: with cash flow being lower than the debt repayment, legal bankruptcy occurs. The asset value minus total costs of bankruptcy is received by bondholders, and shareholders do not achieve any gain;
- $D_1 \leq A_1$: the cash flow is sufficient to repay the debt. Since automatic conversion is not triggered, the bondholders gain D_1. Shareholders receive the residual claims after tax $(A_1-D_1)(1-\tau)$.

The net present value of the bank $V_{t,2}$ in t_0 is:

$$V_{t,2} = \left\{ \int_{\underline{b}}^{k} (\mu(\Omega)+\sigma(\Omega)\varepsilon - \underline{\beta}(D_1 - (\mu(\Omega)+\sigma(\Omega)\varepsilon)))(1-\tau)\, g(\varepsilon)\, d\varepsilon \right.$$

$$+ \int_{k}^{d} (\mu(\Omega)+\sigma(\Omega)\varepsilon - \bar{\beta}(D_1 - (\mu(\Omega)+\sigma(\Omega)\varepsilon)))\, g(\varepsilon)\, d\varepsilon$$

$$\left. + \int_{d}^{\infty} ((\mu(\Omega)+\sigma(\Omega)\varepsilon)(1-\tau) + \tau D_1)\, g(\varepsilon)\, d\varepsilon \right\} \theta - (S_0 + D_0) \tag{4}$$

2.1.4. Scenario 3: $\underline{\beta} D_1/(1+\underline{\beta}) < K \leq \bar{\beta} D_1/(1+\bar{\beta}) < D_1$

With the value of the trigger being intermediate between the indirect and total costs of bankruptcy, automatic conversion allows direct costs to be avoided. Five configurations can occur:

- $0 \leq A_1 < \underline{\beta} D_1/(1+\underline{\beta})$: the asset is consumed by the indirect costs of bankruptcy;

- $\underline{\beta}\, D_1/(1+\underline{\beta}) \leq A_1 < K$: automatic conversion occurs. This avoids the direct costs of bankruptcy. Shareholders and lenders obtain $(1-\alpha)[A_1 - \underline{\beta}(D_1-A_1)](1-\tau)$ and $\alpha[A_1 - \underline{\beta}(D_1-A_1)](1-\tau)$ respectively: their proportion of the cash flow minus indirect costs of bankruptcy. These gains would be lost for all parties with straight-debt financing;
- $K \leq A_1 < \bar{\beta}\, D_1/(1+\bar{\beta})$: the cash flow exceeds the trigger but is not sufficient to repay the debt. Since automatic conversion is not triggered, the costs of legal liquidation consume the entire cash flow. Investors thus do not receive any cash flow;
- $\bar{\beta}\, D_1/(1+\bar{\beta}) \leq A_1 < D_1$: the bondholders gain the cash flow net of the total bankruptcy costs $A_1 - \bar{\beta}(D_1-A_1)$, since the income does not make it possible to repay the debt totally;
- $D_1 \leq A_1$: the cash flow is sufficient to repay the debt D_1, shareholders capturing the residual claims after tax.

The net present value of the bank $V_{t,3}$ in t_0 is:

$$V_{t,3} = \left\{ \int_{\underline{\beta}}^{k} (\mu(\Omega) + \sigma(\Omega)\varepsilon - \underline{\beta}(D_1 - (\mu(\Omega) + \sigma(\Omega)\varepsilon)))(1-\tau)\, g(\varepsilon)\, d\varepsilon \right.$$

$$+ \int_{\bar{\beta}}^{d} (\mu(\Omega) + \sigma(\Omega)\varepsilon - \bar{\beta}(D_1 - (\mu(\Omega) + \sigma(\Omega)\varepsilon)))\, g(\varepsilon)\, d\varepsilon$$

$$\left. + \int_{d}^{\infty} ((\mu(\Omega) + \sigma(\Omega)\varepsilon)(1-\tau) + \tau D_1)\, g(\varepsilon)\, d\varepsilon \right\} \theta - (S_0 + D_0) \qquad (5)$$

Having set the framework of the study, this paper now analyses the value of the bank when there is no conflict in relations, and the value when there is a divergence of interests between shareholders and lenders.

2.2. Bank value without asset substitution

If the bank is a "black box", the shareholders, whatever the value of K, select the project characterised by the lesser risk and the greater expected return. Independently of the investment policy, the capital structure will be determined so that the NPV of the bank (V_t) is maximised:

$$D_0^* \in \arg \mathrm{Max}\ V_t(D_0;\ i(D_0);\ \Omega = 0)$$

$$\mathrm{sc}\ V_b(D_0;\ i(D_0);\ \Omega = 0) = 0$$

For each level of D_0, the bondholders will set the interest rate i which nullifies their NPV V_b, knowing that each pair (D_0; $i(D_0)$) is associated with one of the four scenarios. Indeed, the three pivot values – $\underline{b}$, $\bar{b}$ and d – are monotonically increasing functions of each pair, whereas K is a constant. Scenarios 1 and 4 will occur respectively ex post for higher and lower values of K. The intermediate levels of K lead to (exclusive) scenarios 2 or 3. The bondholders, anticipating the behaviour of the shareholders, know ex ante both the pair (D_0; $i(D_0)$) that the latter will choose and the associated scenario. As previously, scenario 4 (constituting the benchmark) will be studied first.

The first-order condition[1] depends on the value of K. When $K < \underline{\beta} D_1/(1+\underline{\beta})$ (scenario 4), the optimum is:

$$\frac{\delta V_{t,4}}{\delta D_0} = 0 \Leftrightarrow \tau \int_d^{\infty} g(\varepsilon)\, d\varepsilon = \bar{\beta} \int_{\bar{b}}^{d} g(\varepsilon)\, d\varepsilon \qquad (6)$$

Optimal leverage corresponds to that well known in trade-off theory, where the optimal amount of debt is such that the marginal interest tax shield equalises the marginal total cost of bankruptcy.

[1] Second-order conditions of this paper can be provided by the authors on request.

When $D_1 < K$ (scenario 1):

$$\frac{\delta V_{t,1}}{\delta D_0} = 0 \Leftrightarrow \tau\left[\underline{\beta}\int_{\underline{b}}^{d} g(\varepsilon)\, d\varepsilon + \int_{k}^{\infty} g(\varepsilon)\, d\varepsilon\right] = \underline{\beta}\int_{\underline{b}}^{d} g(\varepsilon)\, d\varepsilon \tag{7}$$

The optimal debt level is also reached when the marginal tax shield equalises the indirect marginal costs of liquidation. Nevertheless, this tax shield comprises two elements: the interest tax shield that occurs when the cash flow exceeds the trigger, and the tax shield on indirect costs of bankruptcy. The diminution of the cash flow related to investors being distrustful is an expense that has the same consequences as a tax cut (a non-product being like an expense). Indeed, the bank faces financial distress when $A_1 < D_1$. The debt disappears since the trigger is tripped, and investors share the cash flow net of both taxes and the indirect costs of bankruptcy.

Compared to straight debt (equation (6)), three fundamental differences appear. First, the interest tax shield relates to a restricted field: $[K; +\infty[$ against $[D_1; +\infty[$ (with $D_1 < K$). Second, a tax shield on indirect costs of bankruptcy appears. Third, the disappearance of the debt when $A_1 < K$ reduces the interest tax shield, but investors save the direct costs of liquidation because there is no bankruptcy.

Since $K > D_1$, the direct costs of bankruptcy are non-existent and the more K exceeds D_1, the more the design of CoCos tends towards that of equities. The expected value of the interest tax shield is thereby even more diminished. This product is, then, not very attractive relative to equities.

Optimal leverage will be higher than that prevailing for straight debt if the marginal gain composed of the tax shield on indirect costs (first integral of lhs of (8) = (6) – (7) plus the shield on direct costs of bankruptcy (last two integrals of lhs of (8)) exceeds the marginal cost corresponding to the loss of interest tax shield (rhs of (8)):

$$\tau\,\underline{\beta}\int_{\underline{b}}^{d} g(\varepsilon)\,d\varepsilon + \left[\bar{\beta}\int_{b}^{d} g(\varepsilon)\,d\varepsilon - \underline{\beta}\int_{\underline{b}}^{d} g(\varepsilon)\,d\varepsilon\right] > \tau\int_{d}^{k} g(\varepsilon)\,d\varepsilon \tag{8}$$

Since the debt becomes comparable to shares, the tax shield cuts down the bank value.

When $\bar{\beta}\,D_1/(1+\bar{\beta}) < K \leq D_1$ (scenario 2):

$$\frac{\delta V_{t,2}}{\delta D_0} = 0 \Leftrightarrow \tau\left[\underline{\beta}\int_{\underline{b}}^{k} g(\varepsilon)\,d\varepsilon + \int_{d}^{\infty} g(\varepsilon)\,d\varepsilon\right] = \underline{\beta}\int_{\underline{b}}^{k} g(\varepsilon)\,d\varepsilon + \bar{\beta}\int_{k}^{d} g(\varepsilon)\,d\varepsilon \tag{9}$$

Compared to scenario 1, the interest tax shield is greater because the probability of automatic conversion is lower. However, by construction, the tax shield on the indirect costs of bankruptcy will be less and the marginal cost is increased by the direct costs of bankruptcy (second integral of the rhs of (9)), since this event occurs in this scenario in the field [K; D_1[.

Optimal leverage is no longer restrained by the presence of a trigger, but a growing debt expands the field [K; D_1[on which the bank bears the total costs of bankruptcy. Since K $\leq D_1$, the optimal debt can be increased only with K. If not, the first scenario dominates. The differences in rates of variation of these two variables lead, in addition, to transition from one scenario to another.

Relative to straight debt, the marginal benefit is higher (lhs of (9)) because a tax shield on the indirect costs of bankruptcy is added to the interest tax shield. However, the latter remains lower than the rise in the marginal cost (first integral of rhs of (9)) compared to the straight debt, because $\tau < 1$. The marginal cost also evolves because the direct costs of bankruptcy appear in a more limited field: [K; D_1[(second integral of lhs of (9)) against [$\bar{\beta}\,D_1/(1+\bar{\beta})$; D_1[. Optimal leverage is thus higher than that prevailing for straight debt if the marginal benefit related to the tax shield on indirect costs of bankruptcy (first integral of the lhs of (10) = (6) – (9)), increased by the shield on direct costs of bankruptcy (second integral of lhs of (10)), dominates the marginal cost corresponding to the indirect cost of bankruptcy (rhs of (10)):

$$\tau \underline{\beta} \int_{\underline{b}}^{k} g(\varepsilon)\, d\varepsilon + \bar{\beta} \int_{\bar{b}}^{k} g(\varepsilon)\, d\varepsilon > \underline{\beta} \int_{\underline{b}}^{k} g(\varepsilon)\, d\varepsilon \tag{10}$$

The bank value is greater in scenario 2 because, compared to scenario 1, the interest tax shield is higher and, compared to straight debt, the failure costs are reduced.

When $\underline{\beta}\, D_1/(1+\underline{\beta}) < K \leq \bar{\beta}\, D_1/(1+\bar{\beta}) < D_1$ (scenario 3):

$$\frac{\delta V_{t,3}}{\delta D_0} = 0 \Leftrightarrow \tau \left[\underline{\beta} \int_{\underline{b}}^{k} g(\varepsilon)\, d\varepsilon + \int_{d}^{\infty} g(\varepsilon)\, d\varepsilon \right] = \underline{\beta} \int_{\underline{b}}^{k} g(\varepsilon)\, d\varepsilon + \bar{\beta} \int_{\bar{b}}^{d} g(\varepsilon)\, d\varepsilon \tag{11}$$

Despite being similar in symbolic form to (9), since only the total cost of bankruptcy is affected (second integral of rhs of (9) and (11)), the low value of the trigger has a significant impact. Automatic conversion makes it possible to save direct costs but in a very restricted field. The bank incurs bankruptcy costs in a larger field than in the previous scenario. The low value of the trigger leads the design of CoCos to tend towards that of straight debt.

The latitude of action regarding the optimal debt level is restricted because the two lower bounds ($\underline{b}$ and $\bar{b}$) are increasing monotonic functions of the debt, to be repaid at maturity. However, the value of the trigger is, by construction, between these two boundaries, a leverage too high leading the value of $\underline{b}$ to exceed the trigger, so switching to scenario 4.

Compared to straight-debt financing, optimal leverage is necessarily lower. Actually, the second integral of both the lhs and rhs of (11) is symbolically identical to that of (6). But since $\tau < 1$, the marginal benefit retranscribed by the first integral of the lhs of (11) is structurally lower than the marginal cost corresponding to the first integral of the rhs of (11). Thus, equation (12), obtained by subtracting (11) to (6), shows that it remains a marginal indirect cost of bankruptcy net of tax when CoCos are issued:

$$\underline{\beta}(1-\tau) \int_{\underline{b}}^{k} g(\varepsilon)\, d\varepsilon > 0 \tag{12}$$

The occurrence of a cash flow net of the indirect costs of bankruptcy and of tax in the field [$\underline{\beta} D_1/(1+\underline{\beta})$; K[increases the bank value compared to the issuance of straight debt. However, the lack of cash flow in the field [K; $\bar{\beta} D_1/(1+\bar{\beta})$[results in a lower value of the bank relative to scenario 2.

2.3. Bank value with asset substitution

In this section, we take into account the divergence of perspectives between shareholders and debt holders in terms of investment choices. Since debt holders anticipate the behaviour of shareholders, the agency cost is taken into account when they determine the interest rate i. For a given trigger and for each financing plan proposed by shareholders, the interest rate will be fixed, such as $V_b(D_0; i(D_0); \Omega^*(i(D_0); D_0)) = 0$. Once lenders reveal the loan conditions, shareholders will choose the investment policy and the amount of debt that maximise their NPV (V_s) and the associated scenario:

$$\Omega^* \in \arg\text{Max } V_s(D_0; i(D_0); \Omega(i(D_0); D_0))$$

$$\text{sc } V_b(D_0; i(D_0); \Omega^*(i(D_0); D_0)) = 0$$

and

$$D_0^* \in \arg\text{Max } V_s(D; i(D_0); \Omega^*(i(D_0); D_0))$$

$$\text{sc } V_b(D_0; i(D_0); \Omega^*(i(D_0); D_0)) = 0$$

2.3.1. Selection of $\Omega^*(i(D_0); D_0)$

For each financing plan (D_0; S_0), and according to the parameters setting (K and α), lenders will anticipate the behaviour of the shareholders in order to determine their optimal investment policy according to the interest rate they would be likely to demand: $\Omega^*(i(D_0)$;

D_0).

The first-order condition is, respectively, for the four scenarios:

$$\frac{\delta V_s}{\delta \Omega} = 0 \Leftrightarrow -\frac{\mu'(\Omega^*)}{\sigma'(\Omega^*)}$$

$$\text{Scenario 4:} \quad = \frac{\int_d^\infty \varepsilon\, g(\varepsilon)\, d\varepsilon}{\int_d^\infty g(\varepsilon)\, d\varepsilon} = E(\tilde{\varepsilon}|_{\tilde{\varepsilon} \geq d}) \tag{13}$$

$$\text{Scenario 1:} \quad = \frac{(1-\alpha)(1+\underline{\beta}) \int_{\underline{b}}^{d} \varepsilon\, g(\varepsilon)\, d\varepsilon + (1-\alpha) \int_d^k \varepsilon\, g(\varepsilon)\, d\varepsilon + \int_k^\infty \varepsilon\, g(\varepsilon)\, d\varepsilon}{(1-\alpha)(1+\underline{\beta}) \int_{\underline{b}}^{d} g(\varepsilon)\, d\varepsilon + (1-\alpha) \int_d^k g(\varepsilon)\, d\varepsilon + \int_k^\infty g(\varepsilon)\, d\varepsilon} \tag{14}$$

$$\text{Scenarios 2 and 3:} \quad = \frac{(1-\alpha)(1+\underline{\beta}) \int_{\underline{b}}^{k} \varepsilon\, g(\varepsilon)\, d\varepsilon + \int_d^\infty \varepsilon\, g(\varepsilon)\, d\varepsilon}{(1-\alpha)(1+\underline{\beta}) \int_{\underline{b}}^{k} g(\varepsilon)\, d\varepsilon + \int_d^\infty g(\varepsilon)\, d\varepsilon} \tag{15}$$

At the optimum, the implicit risk of the project is chosen by shareholders in such a way that the trade-off between the average gain, which decreases, and the standard deviation, which increases (lhs of equality) equalises the gain drawn from a riskier investment policy. This profit is related to the probability of achieving it. This is a conditional expectation which determines how the shareholders exchange the increased risk for a decline in average yield.

Concerning scenario 4, the equation (13) corresponds to that derived from straight-debt financing in the presence of asset substitution.

Concerning scenario 1 (equation (14)), the conditional expectation is composed of three elements. The first two correspond to the split of the cash flow because the trigger is tripped. From $\underline{b}$ to d, shareholders pocket part of the income from debt holders while saving the direct costs of bankruptcy. From d to k, their income is diluted because they must share the cash flow net of tax. However, shareholders do not have to repay the debt before collecting a single monetary unit. Beyond k (third item), they repay the debt and collect the residual cash flow net of tax.

Compared to straight debt with asset substitution (equation (13)), the shareholders will choose a riskier project if the lhs of (16) = (14) – (13) exceeds the rhs of (16):

$$\frac{(1-\alpha)(1+\underline{\beta})\int_{\underline{b}}^{d}\varepsilon\, g(\varepsilon)\, d\varepsilon+(1-\alpha)\int_{d}^{k}\varepsilon\, g(\varepsilon)\, d\varepsilon+\int_{k}^{\infty}\varepsilon\, g(\varepsilon)\, d\varepsilon}{(1-\alpha)(1+\underline{\beta})\int_{\underline{b}}^{d}g(\varepsilon)\, d\varepsilon+(1-\alpha)\int_{d}^{k}g(\varepsilon)\, d\varepsilon+\int_{k}^{\infty}g(\varepsilon)\, d\varepsilon} > \frac{\int_{d}^{\infty}\varepsilon\, g(\varepsilon)\, d\varepsilon}{\int_{d}^{\infty}g(\varepsilon)\, d\varepsilon} \tag{16}$$

The bank is likely to have a lower value when shareholders are incited to opt for riskier projects which are associated with a lower mean return.

Concerning scenarios 2 and 3 (equation (15)), the conditional expectation of the profit of shareholders has only two components. In the field [$\underline{b}$; k[, the trigger is tripped and they share the cash flow net of indirect costs with the debt holders. From d to infinity, they repay debt holders and keep the cash flow net of tax. Compared to scenario 1, the absence of the third component is derived from the presence of states of nature where legal bankruptcy occurs in [K; D_1[, a field in which shareholder wealth is worthless.

Compared to straight debt, the risk level chosen by the shareholders will be higher if the lhs of (17) = (15) – (13) exceeds the rhs of (17):

$$E(\tilde{\varepsilon}\big|_{\underline{b}\leq\tilde{\varepsilon}<k\cup\tilde{\varepsilon}\geq d}) > E(\tilde{\varepsilon}\big|_{\tilde{\varepsilon}\geq d}) \tag{17}$$

Since the permitted values of the trigger are lower than those in scenario 1, the bank may have a higher value than if it issues straight debt because investors benefit from two tax benefits (on the indirect costs of bankruptcy and on interest payments) in addition to reducing the direct costs of failure.

2.3.2. Selection of D_0^*

Knowing at this stage the interest rate required $i(D_0)$ by lenders for each level of debt D_0 and the optimal investment policy – $\Omega^*(i(D_0); D_0)$ – shareholders deduce from them the capital structure which maximises their NPV for the four scenarios:

$$\frac{\delta V_s}{\delta D_0} = 0 \Leftrightarrow$$

- Scenario 4:

$$(1-\tau)\theta \int_{d}^{\infty} (\mu'(D_0) + \sigma'(D_0)\varepsilon)\ g(\varepsilon)\ d\varepsilon = (1-\tau)\theta\ D_1' \int_{d}^{\infty}\ g(\varepsilon)\ d\varepsilon - 1 \qquad (18)$$

- Scenario 1:

$$(1-\tau)\theta \left\{ \int_{\underline{b}}^{d} (1-\alpha)(\mu'(D_0) + \sigma'(D_0)\varepsilon - \underline{\beta}(D_1' - (\mu'(D_0) + \sigma'(D_0)\varepsilon)))g(\varepsilon)\ d\varepsilon \right.$$

$$\left. + \int_{d}^{k} (1-\alpha)(\mu'(D_0) + \sigma'(D_0)\varepsilon)g(\varepsilon)\ d\varepsilon + \int_{k}^{\infty} (\mu'(D_0) + \sigma'(D_0)\varepsilon)g(\varepsilon)\ d\varepsilon \right\}$$

$$= (1-\tau)\theta\ D_1' \int_{k}^{\infty} g(\varepsilon)\ d\varepsilon - 1 \qquad (19)$$

- Scenarios 2 and 3:

$$(1-\tau)\theta \left\{ \int_{\underline{b}}^{k} (1-\alpha)(\mu'(D_0) + \sigma'(D_0)\varepsilon - \underline{\beta}(D_1' - (\mu'(D_0) + \sigma'(D_0)\varepsilon)))g(\varepsilon)\ d\varepsilon \right.$$

$$\left. + \int_{d}^{\infty} (\mu'(D_0) + \sigma'(D_0)\varepsilon)g(\varepsilon)\ d\varepsilon \right\} = (1-\tau)\theta\ D_1' \int_{d}^{\infty} g(\varepsilon)\ d\varepsilon - 1 \qquad (20)$$

At the optimum, the marginal cash flow allocated to the original shareholders (marginal gain) is equal to the additional remuneration required by debt holders (marginal cost). The marginal profit results from a split of the cash flow, which differs according to the scenario. The extent of sharing is the largest in scenario 1; it is non-existent in scenario 4, where shareholders receive a marginal benefit only if they are able to repay the debt.

The use of CoCos modifies the split because shareholders attain a marginal benefit even if the cash flow is lower than the debt. In scenario 1, they share the cash flow net of indirect costs of bankruptcy with the debt holders, if the bank's income is lower than D_1. For as long as the trigger remains tripped, the division is based on cash flow which is no longer decreased by the bankruptcy costs. It is necessary that the cash flow reaches the trigger, so that dilution is worthless. In scenarios 2 and 3, the marginal benefit has only two components because legal bankruptcy may arise. This then deprives the shareholders of any income.

The marginal cost also differs according to the scenario. In scenario 1, this corresponds to the additional income required by the debt holders when the trigger is not tripped. The same applies in scenarios 2, 3 and 4, where repayment will occur over a wider area since $K < D_1$.

What consequences for leverage and bank valuation can be deduced from the introduction of a provision to convert the security into equity? In scenario 1, compared to straight debt, the level of debt will be higher if the marginal cash flow – net of the indirect costs of bankruptcy which the shareholders collect on [$\underline{b}$; d[(first integral of lhs of (21) = (19) – (18)) plus the debt they will not have to repay on [d, k[because of the triggering (second integral of lhs of (21)) – exceeds the fraction of cash flow that they are forced to reassign to the debt holders in the field [d; k[(rhs of (21)):

$$(1-\alpha)\int_{\underline{b}}^{d}(\mu'(D_0)+\sigma'(D_0)\varepsilon-\underline{\beta}(D_1'-(\mu'(D_0)+\sigma'(D_0)\varepsilon)))g(\varepsilon)\,d\varepsilon+D_1'\int_{d}^{k}g(\varepsilon)\,d\varepsilon$$

$$> \alpha \int_{d}^{k} (\mu'(D_0) + \sigma'(D_0)\varepsilon) g(\varepsilon)\, d\varepsilon \tag{21}$$

However, the bank will not thereby be more highly valued. Even if the leverage increases, the probability of capturing the interest tax shield decreases because of the high value of the trigger.

In scenarios 2 and 3, the marginal benefit of shareholders is less than in scenario 1 because of a possible legal bankruptcy. Nevertheless, these two environments allow them to fully capture the interest tax shield. Compared to straight debt (scenario 4), shareholders choose a higher leverage if the additional cash flow they collect (the first two terms of (22) = (20) – (18)) exceeds the additional bankruptcy costs which they incur (last term of (22)):

$$\int_{\underline{b}}^{k} (1-\alpha)(\mu'(D_0) + \sigma'(D_0)\varepsilon - \underline{\beta}(D_1' - (\mu'(D_0) + \sigma'(D_0)\varepsilon))) g(\varepsilon)\, d\varepsilon \tag{22}$$

This phenomenon has repercussions for the bank value. With a low level of K, the shareholders profit from a high probability of collecting the interest tax shield, while the probability of saving the direct costs of bankruptcy is low. When K increases, the former probability decreases while the latter increases, thereby making it possible to achieve an internal solution regarding the bank value.

Compared to the analysis of the bank without asset substitution and for a given trigger, the conflicts between investors have two major consequences. First, the initial leverage is weaker because debt holders are more demanding in order to protect themselves against a wealth decline derived from asset substitution. Second, the asset substitution anticipated by debt holders leads shareholders to bear the associated agency costs, resulting in a lower valuation of the bank for the same level of trigger.

Next, we conduct simulations in order to verify the theoretical conclusions of the model.

3. Results

The one-period model with rational anticipation of the agents leads them to anticipate, when initially deciding on financing, the consequences on the cash flow sharing. This section first presents the results of the model by observing the interplay between the actors ex ante, which leads to an optimal endogenous capital structure without and with a divergence of interests between shareholders and debt holders. The impact of an evolution in economic risk on the optimal investment and financing policies will then be analysed.

3.1. Financing policy and the bank value

Simulations of table 1 confirm the conclusions of section 2.2. With straight-debt financing (scenario 4 with $K < 9$), there exists an optimal debt level which maximises the bank value. For an initial cost of the project equal to 90, the lenders will require an interest rate of 24.38%. The NPV of the bank is then equal to 9.02. Thus, for low values of the trigger, the advantages of CoCos cannot be activated since the level of optimal debt is bridled by the low value of K. So, scenario 4 dominates and there are corner solutions for scenarios 1 to 3.

*** Insert Table 1 ***

When $K = 9$, scenario 4 gives way to scenario 3, which allows the direct costs of bankruptcy to be reduced, and where the optimal debt amount remains lower, whatever the value of K. The conclusions drawn from the analysis of equation (12) are confirmed here: the indirect marginal cost of bankruptcy net of tax is strictly positive. The supervening of a cash flow both net of taxes and of indirect costs of bankruptcy in the field $[\underline{\beta} D_1/(1+\underline{\beta}); K[$ increases the bank value relative to straight debt financing.

Scenario 2 supplants scenario 3, starting with values of the trigger higher than 10. The optimal debt, in accordance with (10], is more important than in scenario 4 when K is higher than 15 (see table 2), where the marginal advantage (tax shield on indirect cost of bankruptcy and saving on direct costs of bankruptcy) dominates the marginal cost (indirect cost of bankruptcy). The bank value reaches a maximum of 9.43 for a value of K equal to 50. Indeed, sharing the cash flow is more interesting than bankruptcy for shareholders and debt holders when the values of K are low (in scenario 2) because the cash flow, net of the indirect costs of liquidation and net of the tax, is higher than the cash flow net of the total costs of bankruptcy. This relationship is reversed for high values of K, where bankruptcy is preferable because the income associated is higher. The inflection point of K is about 50, for which the bank value is the highest.

Scenario 1 replaces scenario 2 when K exceeds 90 (table 2). The optimal debt is higher than that prevailing for straight debt when K is equal to 95, because the marginal benefit associated with the tax shield on the indirect costs plus the saving on the direct costs of bankruptcy exceed the marginal cost corresponding to the loss of interest tax shield in accordance with (8). For higher values of K, the design of CoCos tends towards that of stocks, and the marginal cost dominates the marginal benefit. The probability of capturing the interest tax shield becomes weaker, while that of capturing the tax shield on the indirect costs of bankruptcy is at its peak; the bank value declines steadily while K increases.

CoCos allow both a better bank value and a higher leverage: 56.46 when K = 50 for scenario 2 against 52.49 for straight debt. However, they do not increase the financial risk of the bank, thanks to their faculty of automatic conversion. From the macroeconomic point of view, the systemic risk is not increased. A too-high trigger (K = 100) allows to reduce significantly the direct costs of failure but, on the other hand, cuts too sharply the tax benefits of debt.

Without conflicts of interest between shareholders and debt holders, CoCos lead to increased efficiency in the financial system. Banks have higher value and the financial risk is limited, thanks to automatic conversion.

The simulation results of table 2 confirm the theoretical conclusions. The ability of shareholders to undertake riskier ($\sigma(\Omega)$ = 70 against 80.18 for Ω^* = 0.5433) and less remunerative ($\mu(\Omega)$ = 105.5 against 100.98 for Ω^* = 0.5433) projects generates agency costs which they support since lenders anticipate their behaviour. These agency costs, corresponding to the difference in bank value without and with conflict, are equal to 1.61 = 9.02 - 7.41 (17.85% of the bank value). The model, then, reflects many results previously demonstrated in the financial literature.

The higher number of trigger values in table 2 relative to table 1 demonstrates the existence of a greater maximum leverage than the optimum (57.54 for K = 70 against 56.46 for K = 50). This maximum is explained by a strong increase in the requirements of lenders when the trigger reaches high values (K > 50). Indeed, increasing the trigger forces lenders to share the cash flow (widening the segment [$\underline{\beta} D_1/(1+\underline{\beta})$; K[for scenario 2), whereas previously they would have taken it all (reduction of the segment [K; D_1[). As compensation, they strongly increase the interest rate, thereby forcing shareholders to reduce leverage.

The introduction of agency conflicts leads to three major points. First, leverage declines because lenders increase the interest rate to fight against a transfer of wealth at their expense. For straight debt (K = 0.01), the initial debt drops from 52.49 to 26.16. This phenomenon prevails even when automatic conversion is introduced.

Then, the introduction of a trigger leads to a higher optimal leverage under scenarios 2 and 3. According to (22), shareholders opt for a higher leverage because the additional cash flow that they gain exceeds the additional bankruptcy costs they incur. Moreover, the trigger having a higher value reduces agency costs. For K = 120, the loss of value is 0.84 = 7.58 - 6.74 (11.08% of the bank value without conflicts) against 1.61 = 9.02 - 7.41 and 17.85% respectively for straight debt. However, the issuance of such CoCos is expensive for existing shareholders since the bank value is 6.74 against 7.47 when K = 20.

Finally, CoCos make it possible to improve the efficiency of the banking system because banks have a higher value relative to straight debt. Without conflict, highest bank value is equal to 9.43 for a trigger of 50, compared with 9.02 for straight debt. With agency conflicts, the maximum value of the bank is equal to 7.47 for a trigger of 20, against 7.41 for straight debt. Agency conflicts lower the optimal value of the bank: 9.43 against 7.47 for the same scenario 2. Thus, an optimal value for the trigger, which maximises the bank value, exists without or with conflict. For scenario 2, the shareholders prefer K to tend to D_1, but lenders do not necessarily have the same perspective; they may prefer bankruptcy to cash flow sharing when the trigger is too high. To curb the incentive of the shareholders, lenders will significantly increase the interest rate. Whereas the rate rises slightly until K reaches 20, it passes abruptly from 21.80% to 24.02% when K increases from 20 to 30. Beyond it, we fall in scenario 1.

*** Insert Table 2 ***

The pivot values of the trigger, which lead to a shift from one scenario to another, are higher in the absence than in the presence of conflict. In the first context, shareholders necessarily opt for the project associated with both the highest cash flow and the lowest risk.

Consequently, the probability of automatic conversion when K rises is lowered. Thus, scenario 4 gives way to scenario 3 when K increases from 7 to 9 (against 1 to 5 in the presence of conflict). At the other end of the scale, scenario 1 supplants scenario 2 when K goes from 90 to 100 (against 30 to 40 with conflict).

Although the model reveals that greater economic risk-taking (Ω) is associated with CoCos, total risk can be reduced. Indeed, the financial risk (reduction in the number of states of nature where bankruptcy occurs) decreases thanks to the automatic conversion of debt into equity, with a positive impact on the stability of both the bank and the banking system as a whole.

With CoCos, the state grants a subsidy to the bank in the form of an interest tax shield (and indirect costs of bankruptcy) as for straight debt. This is justified because the present cost avoids it injecting massive amounts of funds when a systemic banking crisis occurs.

CoCos are highly relevant to both bank valuation and risk management at the microeconomic and macroeconomic levels. However, special attention must be paid to their design, on penalty of the inability to provide an efficient means of financing to investors. Particular attention should be paid to fixing the value of the trigger. Its optimal value is highly dependent on the environment (structure and amount of bankruptcy costs, extent of the dilution of shareholder claims, tax environment and so on).

3.2. The influence of the economic environment

The certainty component of A_1 is defined symbolically as: $\mu(\Omega) = m_1 - m_2 \times e^{m_3 \times \Omega}$. Simulations of table 3 correspond to an increase in value from 1.5 to 1.7 for m_3, that is to say a decrease of the value of $\mu(\Omega) = 105.5 - 2e^{1.7\Omega}$ with m_1 = 105.5 and m_2 = 2. The expected return of the bank is reduced since $A_1 \rightarrow N(\mu(\Omega); \sigma(\Omega)\sigma(\varepsilon))$. This leads to the simulation of an economic crisis where the performance of projects available decreases.

*** Insert Table 3 ***

A more difficult economic environment leads to reduced risk-taking both for straight debt (Ω^* going from 0.54 to 0.39 for K = 0.01) and CoCos, whatever the value of the trigger. For example, bank value is maximised at 7.47 and 7.73 respectively when K = 20 for risk levels of 0.60 against 0.41. This is explained by the fact that shareholders are encouraged to engage in asset substitution as long as the decline in the value of the bank is less than that in the wealth of lenders. Shareholders' flexibility in this dimension is all the more restrained that risk reduces intensely the average value of the bank. It follows a general reduction in agency costs, with a maximum drop of 0.27 = 2.20 - 1.93 when K = 60 (scenario 1).

Despite the decline of $\mu(\Omega)$, the bank has a higher value because lenders are less fearful of suffering asset substitution. With straight debt, bank value rises to 7.66 against 7.41. The same phenomenon is observable for CoCos, whatever the value of K. The requirements of lenders are lower, allowing shareholders to increase leverage: 29.75 against 26.17 for straight debt, with this difference being observable for CoCos, whatever the value of K. It follows that there is a greater interest tax shield and a lower cost of failure, since the project undertaken is less risky.

Increased leverage leads to higher pivot values of K beyond which there is switching from one scenario to another. Thus, the bank operates, for K = 40, in scenario 2 when m_3 = 1.7 and in scenario 1 when m_3 = 1.5. When K = 40, D_1 = 39.37 (30.53× 1.2894) and 40.17 (31.76 × 1.2648) respectively for the values of m3 = 1.5 and 1.7. Scenario 1 thus prevails in the first case, and scenario 2 in the second.

4. Conclusion

The originality of this paper lies in the model proposed, which incorporates both the financing and investment policies of banks. It demonstrates for the first time the existence of an optimal capital structure using CoCos financing. The regulator refers to this new tool in its policy of prevention, treatment and resolution of banking crises. CoCos can be seen as a mechanism for absorbing losses and providing several advantages.

The paper demonstrates that CoCos allow banks to increase their leverage compared to the case in straight debt, thereby increasing their ability to raise funds to finance positive NPV projects. Although CoCos encourage increased economic risk-taking, they do not lead to a higher total risk, thanks to their ability to avoid bankruptcy in unfavourable situations. Although CoCos cannot completely protect a bank against bankruptcy, they significantly reduce its probability of failure independently of regulator actions. CoCos also improve the efficiency of the banking system, because bank value is higher than with straight debt. They can thus be understood as a means of mitigating some deficiencies of the financing system in force in the capitalist economies, but they are not a panacea. Special attention must be paid to their design on penalty of the inability to provide an efficient means of financing to investors. Particular attention should be paid to fixing the value of the trigger. Its optimal value is highly dependent on the environment (structure and amount of bankruptcy costs, extent of the dilution of shareholder claims, tax environment and so on).

The normative approach adopted here derives from the narrowness of the market at the present time, since some issues have been observed only in organised markets. It takes account of the agency cost related to asset substitution. Future research might enhance this framework by taking account of other agency costs and by making endogenous the threshold and the dilution level.

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Fig. 1. The four scenarios ex post according to the values of K and A_1

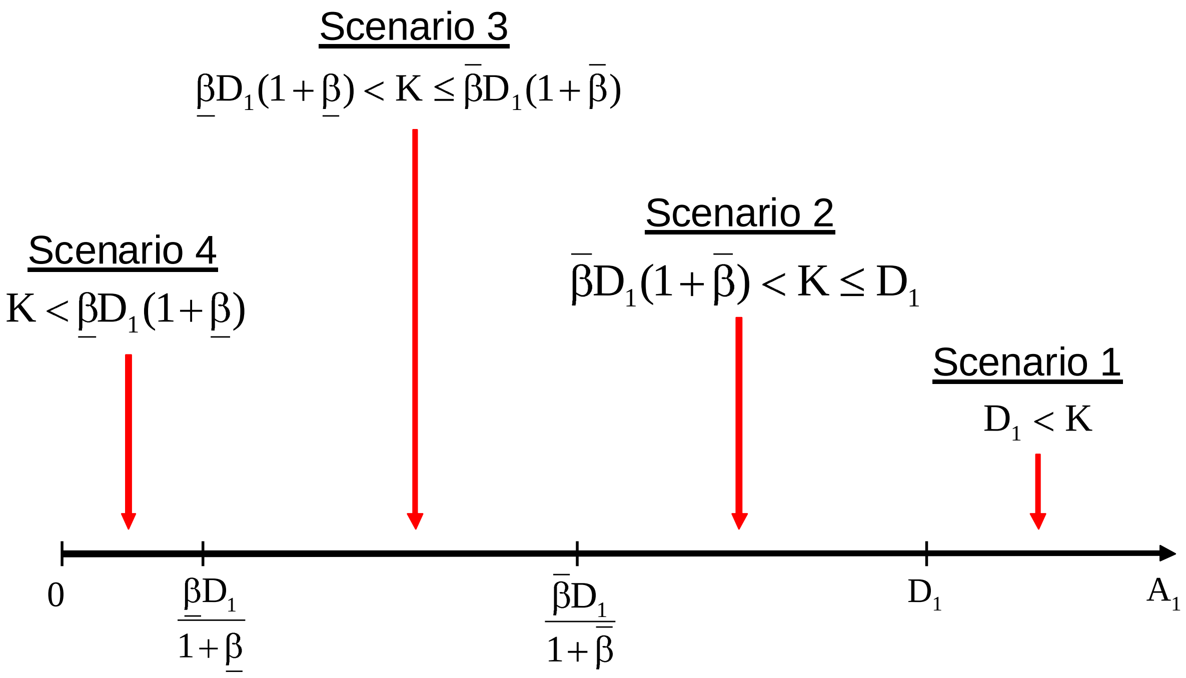

Table 1

Value of the bank without agency conflicts.

K	D_0	$i(D_0)$	V_t		Scenario
0.01					1
					2
					3
	52.49	**24.38%**	**9.02**	**IS**	**4**
5	4.52	9.59%	6.59	RS	1
	19.31	12.14%	7.67	RS	2
	33.01	15.89%	8.47	RS	3
	52.49	**24.38%**	**9.02**	**IS**	**4**
7	6.32	10.03%	6.73	RS	1
	26.52	13.95%	8.13	RS	2
	44.55	20.30%	8.92	RS	3
	52.49	**24.38%**	**9.02**	**IS**	**4**
9	8.13	10.50%	6.87	RS	1
	33.55	16.05%	8.51	RS	2
	52.31	**24.28%**	**9.03**	**IS**	**3**
	55.01	25.94%	9.01	LS	4
10	9.03	10.7422%	6.94	RS	1
	36.80	17.1562%	8.67	RS	2
	52.31	**24.27%**	**9.03**	**IS**	**3**
	59.52	29.13%	8.91	LS	4
20	17.50	13.45%	7.57	RS	1
	52.49	**24.33%**	**9.16**	**IS**	**2**
	64.93	33.81%	8.71	LS	3
	82.97	85.19%	1.83	LS	4
50	39.51	26.03%	8.89	RS	1
	56.46	**31.56%**	**9.43**	**IS**	**2**
					3
					4
60	45.28	32.20%	9.08	RS	1
	57.18	**36.16%**	**9.39**	**IS**	**2**
					3
					4
95	**53.57**	**63.90%**	**8.48**	**IS**	**1**
	57.54	65.67%	8.46	LS	2
					3
					4
100	**50.65**	**68.25%**	**8.28**	**IS**	**1**
	58.34	72.49%	8.19	LS	2
					3
					4

$\mu(\Omega) = 105.5 - 2e^{1.5\Omega}$ = 105.5; $\sigma(\Omega) = 70 \times e^{0.25\Omega}$; θ = 0.99; $\underline{\beta}$ = 0.1; $\overline{\beta}$ = 0.3; τ = 0.08; I = 90; α = 0333. RS: right corner solution; IS: inside solution; LS: left corner solution. There are three reasons for lack of value: credit rationing if the risk is too high: the lenders don't want to participate because their NPV is negative whatever the level of risk (meaning that no interest rate nullifies their NPV); negative NPV for shareholders whatever the leverage, because the requirements of lenders are too high; and refusal of the shareholders to participate in the initial financing plan because the NPV of the lenders is positive whatever the level of risk (meaning that no interest rate nullifies their NPV).

Table 2

Value of the bank according to the trigger level.

K	Without conflicts					With conflicts						
	D_0	$i(D_0)$	D_1	V_t	Sc. (1)	Ω	D_0	$i(D_0)$	D_1	V_s	Sc. (1)	V(ca) (2)
0.01	52.49	24.38%	65.29	9.02	4	0.54	26.17	19.53%	31.28	7.41	4	1.61
1	52.49	24.38%	65.29	9.02	4	0.54	26.17	19.53%	31.28	7.41	4	1.61
5	52.49	24.38%	65.29	9.02	4	0.55	26.29	19.60%	31.45	7.41	3	1.61
7	52.49	24.38%	65.29	9.02	4	0.56	26.93	19.99%	32.31	7.42	3	1.61
9	52.31	24.28%	65.01	9.03	3	0.55	26.25	19.20%	31.29	7.44	2	1.59
10	52.31	24.27%	65.01	9.03	3	0.56	26.96	20.02%	32.36	7.44	2	1.59
12	51.95	24.05%	64.44	9.04	3	0.56	26.84	20.02%	32.21	7.45	2	1.59
15	51.59	23.80%	63.87	9.08	2	0.56	28.80	20.20%	34.62	7.46	2	1.62
20	52.49	24.33%	65.26	9.16	2	0.60	28.46	21.80%	34.66	7.47	2	1.69
30	53.93	25.77%	67.83	9.30	2	0.61	28.21	24.02%	34.98	7.45	2	1.86
40	55.37	28.17%	70.97	9.40	2	0.68	30.53	28.94%	39.37	7.37	1	2.03
50	56.46	31.56%	74.27	9.43	2	0.68	30.54	33.55%	40.79	7.29	1	2.15
60	57.18	36.16%	77.85	9.39	2	0.72	32.33	39.50%	45.11	7.19	1	2.20
70	57.54	42.23%	81.84	9.25	2	0.66	29.54	42.38%	42.06	7.14	1	2.11
80	57.36	49.91%	85.99	9.02	2	0.64	29.02	46.60%	42.54	7.07	1	1.94
90	56.64	59.51%	90.34	8.67	2	0.63	29.83	52.47%	45.49	7.00	1	1.68
100	50.51	68.16%	84.93	8.28	1	0.61	29.78	57.34%	46.85	6.91	1	1.37
110	44.55	75.61%	78.24	7.91	1	0.61	31.11	65.27%	51.41	6.83	1	1.09
120	39.69	81.19%	71.90	7.58	1	0.58	31.71	72.54%	54.71	6.74	1	0.84

$\mu(\Omega) = 105.5 - 2e^{1.5\Omega} = 105.5$; $\sigma(\Omega) = 70 \times e^{0.25\Omega}$; $\theta = 0.99$; $\underline{\beta} = 0.1$; $\overline{\beta} = 0.3$; $\tau = 0.08$; $I = 90$; $\alpha = 0.333$.

Number of stocks originally issued: 1000; number of CoCos issued: 500; number of stocks obtained through conversion of a bond: 1. (1) Sc.: number of scenario according the trigger. (2) V(ac): agency costs = the bank value without conflicts - the bank value with conflicts for the same level of K.

Table 3

Economic shock and value of the bank.

K	Ω	D_0	$i(D_0)$	D_1	V_s	Sc. (1)	V(ca) (2)
0.01	0.39	29.75	19.10%	35.43	7.66	4	1.37
1	0.39	29.75	19.10%	35.43	7.66	4	1.37
5	0.39	29.75	19.10%	35.43	7.66	3	1.37
7	0.39	29.71	19.07%	35.38	7.66	3	1.36
9	0.39	29.57	18.98%	35.18	7.67	2	1.35
10	0.39	29.71	19.06%	35.37	7.68	2	1.34
12	0.40	29.86	19.19%	35.59	7.70	2	1.34
15	0.40	30.18	19.53%	36.07	7.71	2	1.37
20	0.41	30.13	20.02%	36.17	7.73	2	1.43
30	0.44	31.12	22.60%	38.15	7.71	2	1.59
40	0.47	31.76	26.48%	40.17	7.64	2	1.76
50	0.47	31.38	30.87%	41.06	7.55	1	1.89
60	0.48	31.54	35.73%	42.81	7.46	1	1.93
70	0.48	32.03	41.14%	45.21	7.38	1	1.88
80	0.46	32.46	46.88%	47.67	7.30	1	1.72
90	0.46	32.46	52.56%	49.52	7.21	1	1.46
100	0.44	32.92	59.16%	52.39	7.13	1	1.15
110	0.41	33.57	66.81%	55.99	7.05	1	0.87
120	0.37	34.46	76.00%	60.65	6.96	1	0.62

$\mu(\Omega) = 105.5 - 2e^{1.7\Omega}$; $\sigma(\Omega) = 70 \times e^{0.25\Omega}$; θ = 0.99; $\underline{\beta}$ = 0.1; $\overline{\beta}$ = 0.3; τ = 0.08; I = 90; α = 0.333.

Number of stocks originally issued: 1,000; number of CoCos issued: 500; number of stocks obtained through conversion of a bond: 1. (1) Sc.: number of scenario according the trigger. (2) V(ac): agency costs = the bank value without conflicts - the bank value with conflicts for the same level of K.