

# **Is Volatility Risk Priced in the Securities Market? Evidence from S&P 500 Index Options**

## **Abstract**

This article examines whether volatility risk is a priced risk factor in securities returns. Zero-beta at-the-money straddle returns of the S&P 500 index are used to measure volatility risk. It is demonstrated that volatility risk captures time variation in the stochastic discount factor, suggesting that straddle returns are important conditioning variables in asset pricing. The conditional model proposed here performs far better than its unconditional counterparts including the Fama-French three-factor model. Thus, we argue that investors use straddle returns when forming their expectations about securities returns. One interesting finding is that, different classes of firms react differently to volatility risk. For example, small firms and value firms have negative and significant volatility coefficients whereas big and growth firms have positive and significant volatility coefficients during high volatility periods, indicating that investors see these latter firms as hedges against volatile states of the economy. Overall, these findings have important implications for portfolio formation, risk management, and hedging strategies.

## INTRODUCTION

The notion that equity returns exhibit stochastic volatility is well documented in the asset pricing literature.<sup>1</sup> Furthermore, recent evidence indicates the existence of a negative volatility risk premium in the options market (Lamoureux & Lastrapes, 1993; Buraschi & Jackwerth, 2001; Coval & Shumway, 2001; Bakshi & Kapadia, 2003). However, the existence of volatility risk in the securities market and its impact on different classes of firms has not been extensively documented. Recently, Coval and Shumway (2001) examines the return characteristics of S&P 100 index straddles and gives preliminary evidence that volatility risk may be a common risk factor in securities markets - a finding that contradicts the classical CAPM.

CAPM suggests that the only common risk factor relevant to the pricing of any asset is its covariance with the market portfolio; thus an asset's beta is the appropriate quantity for measuring the risk of any asset. However, Vanden (2004) shows that when agents face non-negative wealth constraints, cross sectional variation in securities returns is not explained only by an asset's beta. Instead, excess returns on the traded index options and on the market portfolio explain this variation; implying that options are non-redundant securities. Furthermore, as Detemple and Selden (1991) suggests, if options in the economy are non-redundant securities, then there should be a general interaction between the

returns of risky assets and the returns of options. This implies that option returns should help explain security returns.

This article extends the preceding studies and presents evidence that straddle returns are important for asset pricing since they help capture time variation in the stochastic discount factor. The findings suggest that volatility risk is time-varying and that options are non-redundant securities at volatile states of the economy. This has important implications regarding the allocational role of options in the economy. The preliminary time-series regressions, Fama-MacBeth regressions, and GMM-SDF estimations in this article confirm the theory that options are effective tools in pricing securities and allocating wealth among agents as suggested by Vanden (2004). This article also examines the effect of volatility risk in pricing different classes of firms, i.e. small vs. big and value vs. growth, and finds distinct patterns in the returns of these firms, especially at volatile states of the economy.

Asset pricing theories thus far have been unable to provide a satisfactory economic explanation for the size and value vs. growth anomalies.<sup>2</sup> In a rational markets framework, we would expect these abnormal returns to be temporary. Once investors realize arbitrage opportunities, the abnormal profits of small and value stocks are expected to vanish. However, this has not been the case. The persistence of these two anomalies has led to extensive research and has yielded two alternative lines of explanations within the rational markets paradigm.

One line, led by Fama and French (1992, 1993, 1995), argues that a stock's beta is not the only risk factor. This approach suggests that fundamental additional variables such as book-to-market and market value explain equity returns much better, because they are proxies for some unidentified risk factors. However, the weakness of this explanation lies in its failure to address the economic variables underlying these factors. The other line of research within the risk-return framework argues that it is the time variation in betas and the market risk premium that cause the static CAPM to fail to explain these anomalies. There is now considerable evidence that conditional versions of CAPM perform much better than their unconditional counterparts.<sup>3</sup>

This article re-examines these two important asset pricing anomalies with an important but somewhat overlooked factor, the volatility risk. There is now a considerable amount of evidence that volatility risk is priced in the options market. First, Jackwerth and Rubinstein (1996) report that at-the-money implied volatilities of call and put options are consistently higher than their realized volatilities, suggesting that a negative volatility premium could be an explanation to this empirical irregularity. Furthermore, Coval and Shumway (2001) report that zero-beta at-the-money straddles on the S&P 100 index earn returns consistently lower than the risk free rate, suggesting the presence of a negative volatility risk premium in the prices of options. As an extension of this study, Driessen and Maenhout (2005) report that volatility risk is also priced in FTSE and Nikkei

index options. Finally, Bakshi and Kapadia (2003) show that delta-hedged option portfolios consistently earn negative returns and conclude that there exists a negative volatility risk premium in option prices.

While the above evidence indicates that volatility risk is priced in options markets, we are less confident that it is priced in securities markets. Recent studies find that volatility risk can explain the cross-section of expected returns. For example, Moise (2005) uses innovations in the realized stock market volatility, and demonstrate that volatility risk helps explain some of the size anomaly. Furthermore, by using changes in the VIX volatility index of Chicago Board Options Exchange (CBOE), Ang, Hodrick, Xing, and Zhang (2006) demonstrate that aggregate volatility is a cross-sectional risk factor. In this study, a measure from the options market, i.e. straddle returns on the S&P 500 index, is used as a proxy for volatility risk. The reasoning behind using straddle returns is intuitive. As Detemple and Selden (1991) argue, if options are non-redundant securities in the economy, then their returns should appear as factors in explaining the cross section of asset returns. Furthermore, Vanden (2004) reports that returns of call and put options indeed explain a significant amount of variation in securities return, but fail to explain the returns for small and value stocks. The failure of Vanden's model could be due to omitting an important risk factor, the volatility risk. Furthermore, straddles are volatility trades, and they provide

insurance against significant downward moves.<sup>4</sup> Thus, overall, straddle returns are ideal for studying the effects of volatility risk in security returns.

The remainder of this article is organized as follows. First, data and the methodology for calculating straddle returns are presented. Econometric issues in the estimation of the volatility risk premium are discussed in the next section. This is followed by empirical results. The final section offers concluding remarks.

## **DATA AND METHODOLOGY**

The data consist of two parts - S&P 500 options data and stock return data - covering the period January 1987 through October 1994.<sup>5</sup> Daily S&P 500 options data is obtained from the Chicago Board Options Exchange and consists of daily closing prices of call and put options, the daily closing level of the S&P 500 index, the maturities and strike prices for each option, the dividend yield on the S&P 500 index, and the one-month T-bill rate. For option volatilities, the closing level of CBOE's S&P 500 VIX index is used. For market portfolio, CRSP's value weighted index on all NYSE, AMEX and NASDAQ stocks are used. The return data on size and book-to-market portfolios are obtained from Kenneth French's data library.

The method for calculating daily option returns is as follows. First, options that significantly violate arbitrage-pricing bounds are eliminated. Then, options

that expire during the following calendar month are identified. This roughly coincides with options that have 14 to 50 days to expiry in our sample. The reason for choosing options that expire the next calendar month is that they are the most liquid data among various maturities.<sup>6</sup> Options that expire within 14 days are excluded from the sample, because they show large deviations in trading volumes, which casts doubt on the reliability of their pricing associated with increased volatility.<sup>7</sup> Next, each option is checked whether it is traded the next trading day or not. If no option is found in the nearest expiry contracts, then options in the second-nearest expiry contracts are used. To calculate the daily return of an option, raw net returns are used. The usage of raw net returns is justified by Coval and Shumway (2001) who argue that log-scaling of option returns can be quite problematic.

Once daily call and put returns are calculated, they are grouped according to their moneyness levels. Although there is no standard procedure for classifying at-the-money options, options with a moneyness level (S-K) between -5 and +5 are classified as at-the-money options. This classification also guarantees that there are at least two options around the spot price. One reason for focusing on zero-beta at-the-money straddles was to capture the effect of volatility risk, as mentioned previously. Another advantage of studying at-the-money options is that they are less prone to pricing errors compared to deep-out-of money options,

as cited in option pricing literature.<sup>8</sup> Using the above procedure results in 1937 days of return data out of 1980 trading days.

The straddle returns are calculated according to the methodology outlined by Coval and Shumway (2001). In order to capture the effect of volatility risk, zero-beta at-the-money straddle returns on the S&P 500 index are used. The advantage of using S&P 500 index options is that they are highly liquid, thus they are less prone to microstructure and illiquid trading effects. Zero-beta straddles are formed by solving for  $\theta$  from the following set of equations,

$$r_v = \theta r_c + (1 - \theta)r_p \quad (1)$$

$$\theta\beta_c + (1 - \theta)\beta_p = 0 \quad (2)$$

where  $r_v$  is the straddle return,  $r_c$  and  $r_p$  are the call and put returns,  $\theta$  is the fraction of the straddle's value in call options, and  $\beta_c$  and  $\beta_p$  are the market betas of the call and put options, respectively. It is straightforward to calculate returns on call and put options; however, in order to calculate the return of a straddle, the value of  $\theta$  is needed, which depends on  $\beta_c$  and  $\beta_p$ . By using the put-call parity theorem, Equation (2) can be reduced into a single unknown,  $\beta_c$ , and the value of  $\theta$  is derived as follows



$$\theta = \frac{-C\beta_c + s}{P\beta_c - C\beta_c + s} \quad (3)$$

where  $C$  is price of the call option,  $P$  is price of the put option, and  $s$  is the level of the S&P 500 index.

The only parameter that is not directly observable in the above equation is the call option's beta,  $\beta_c$ . We use Black-Scholes' beta, which is defined as

$$\beta_c = \frac{s}{C} N \left[ \frac{\ln(s/X) + (r - q + \sigma^2/2)t}{\sigma\sqrt{t}} \right] \beta_s \quad (4)$$

where  $N[\cdot]$  is the cumulative normal distribution,  $X$  is the exercise price of call option,  $r$  is the risk-free short term interest rate,  $q$  is the dividend yield for S&P 500 assets,  $\sigma$  is the standard deviation of S&P 500 returns, and  $t$  is the option's time to maturity.

The methodology to calculate zero-beta at-the-money straddle returns is as follows. First, an option's beta is calculated according to Equation (4). Then,  $\theta$  is derived by incorporating the previously calculated call and put option returns into Equation (3). Finally, straddle returns for each day are calculated according to Equation (1). The daily zero-beta straddle return is then simply the equally-weighted average of at-the money-straddle returns that are found in the final step.

Table I reports the summary statistics for the daily S&P 500 (SPX) straddle returns. The average daily S&P 500 straddle return is -1.06 % with a minimum return of -87.77% and maximum of 441.79%. The mean and median of the daily zero-beta straddle returns are negative as documented by the earlier literature. Note that call option betas are instantaneous betas, and therefore the straddles are zero-beta at the construction. However, we calculate the zero-beta straddle returns by using daily buy and hold returns. Thus, they are zero-beta instantaneously and their betas might change during the holding period. This might be the possible explanation of negative correlation of -0.54 between the straddle returns and market returns.<sup>9</sup> The straddle returns also exhibit positive skewness and relatively high kurtosis.

[Insert Table I here]

## **ECONOMETRIC SPECIFICATIONS**

In order to test the main hypothesis that volatility risk - proxied by zero-beta at-the-money straddle returns - is priced in securities returns, we first regress the excess returns of size and book-to-market portfolios on excess straddle returns and on the market factor<sup>10</sup>. The empirical model to be tested is

$$r_{it} - r_{ft} = \alpha_i + \sum_j \beta_{ij} (r_{jt} - r_{ft}) + \varepsilon_{it}, \quad (5)$$

where  $r_{it}$ 's are realized returns of size and book-to-market portfolios, and  $r_{jt}$ 's are the returns of factors that are included in the regressions.

The above analysis relies on monthly holding period returns, both because microstructure effects tend to distort daily returns, and to rule out non-synchronous trading effects that could be present in daily data. In order to calculate monthly at-the-money straddle returns, an equally weighted portfolio of at-the-money straddles is formed for each day and then each day's return is cumulated to find monthly holding period returns. This adds up to 94 monthly straddle returns, which are used as an independent variable in the preceding time-series regressions. Although these regressions are not formal tests of whether volatility risk is priced or not, they nevertheless give clues about the potential explanatory power of straddle returns in explaining the cross-section of expected returns.

Next the question of whether volatility risk is a priced risk factor is examined by performing Fama-MacBeth two-pass regressions by using the 25 size and book-to-market portfolios.<sup>11</sup> The model to be tested is

$$E[r_{it}] = \alpha_i + \beta' \lambda. \quad (6)$$

More specifically, in the first pass, portfolio betas are estimated from a single multiple time-series regression via Equation 5. Instead of using the 5-year rolling-window approach, a full sample period is used.<sup>12</sup> In the second pass, a cross-sectional regression is run at each time period, with full-sample betas obtained from the first pass regressions, i.e.

$$E[r_{it}] = \alpha_{it} + \beta'_{ij} \lambda_{jt}, \quad i = 1, 2, \dots, N \text{ for each } t. \quad (7)$$

Fama and MacBeth (1973) suggests that we estimate the intercept term and risk premiums,  $\alpha_i$  and  $\lambda_j$ 's, as the average of the cross-sectional regression estimates

$$\hat{\alpha}_i = \frac{1}{T} \sum_{t=1}^T \hat{\alpha}_{it}, \text{ and } \hat{\lambda}_j = \frac{1}{T} \sum_{t=1}^T \hat{\lambda}_{jt}.$$

One problem with the Fama-MacBeth procedure is that it ignores the errors-in-variables problem that results from the fact that in the second pass, beta estimates instead of the true betas are used. In order to avoid this problem, a Generalized Method of Moments (GMM) approach within the stochastic discount factor (SDF) representation is employed. The advantage of a GMM approach is that it allows the estimation of model parameters in a single pass, thereby avoiding the error-in-variables problem. The advantage of the SDF representation relative to the beta representation is that it is extremely general in its assumptions and can be applied to all asset classes, including stocks, bonds, and derivatives. Cochrane (2001) demonstrates that both representations express the same point,

but from slightly different viewpoints. However, the SDF view is more general, it encompasses virtually all other commonly known asset pricing models. Ross (1976) and Harrison and Kreps (1979) state that in the absence of arbitrage and when financial markets satisfy the law of one price, there exists a stochastic discount factor, or pricing kernel,  $m_{t+1}$ , such that the following equation holds

$$E[R_{it+1}m_{t+1}] = 1, \quad (8)$$

where  $R_{it+1}$  is the gross return (one plus the net return) on any traded asset  $i$ , from period  $t$  to period  $t+1$ . We denote this as the unconditional SDF model.

Since considerable evidence exists to suggest that expected excess returns are time-varying, the above unconditional specification may be too restrictive. Thus, in order to answer the question of whether or not there exists time-variation in the volatility risk premium, both unconditional and conditional models of asset pricing are tested. The conditional SDF model is denoted as

$$E_t[R_{it+1}m_{t+1}] = 1 \quad (9)$$

where  $E_t$  denotes the mathematical expectation operator conditional on the information available at time  $t$ .

Following Jagannathan and Wang (1996), we consider a linear factor pricing model with observable factors,  $f_t$ . Then,  $m_{t+1}$  can be represented as

$$m_{t+1} = a_t + b_t' f_{t+1} \quad (10)$$

where  $a_t$ , and  $b_t$  are time-varying parameters. Note that, when  $a_t$ , and  $b_t$  are constants, we obtain the unconditional version of linear factor models.

The question here is how one can incorporate the information that investors use when they determine expected returns in Equations (9) and (10). Because the investors' true information set is unobservable, one has to find observable variables to proxy for that information set. Cochrane (1996) shows that conditional asset pricing models can be tested via a conditioning time  $t$  information variable,  $z_t$ . One way of incorporating conditioning variable,  $z_t$ , into the model is to scale factor returns, as discussed in Cochrane (2001); and used in Cochrane (1996), Hodrick and Zhang (2001), and Lettau and Ludvigson (2001). This is done by scaling the factors with  $z_t$ , thus modeling the parameters  $a_t$ , and  $b_t$  as linear functions of  $z_t$  as follows

$$a_t = \gamma_0 + \gamma_1 z_t \quad (11)$$

$$b_t = \eta_0 + \eta_1 z_t \quad (12)$$

Plugging these equations into Equation (10), and assuming that we have a single factor, we have a scaled multifactor model with constant coefficients taking the form

$$\begin{aligned} m_{t+1} &= (\gamma_0 + \gamma_1 z_t) + (\eta_0 + \eta_1 z_t) f_{t+1} \\ &= \gamma_0 + \gamma_1 z_t + \eta_0 f_{t+1} + \eta_1 z_t f_{t+1} \end{aligned} \quad (13)$$

The scaled multifactor model can be tested by rewriting the conditional factor model in Equation (9), as an unconditional factor model with constant coefficients  $\gamma_0, \gamma_1, \eta_0$ , and  $\eta_1$  as follows,

$$E[R_{it+1}(\gamma_0 + \gamma_1 z_t + \eta_0 f_{t+1} + \eta_1 z_t f_{t+1})] = 1. \quad (14)$$

In the next section, empirical results of OLS time-series regressions (Equation 5), Fama-MacBeth regressions (Equation 6), and the GMM-SDF estimations (Equation 8) are presented.

## **EMPIRICAL FINDINGS**

### **Time Series Regressions**

Coval and Shumway (2001) (CS) argue that zero-beta at-the-money straddles can proxy for volatility risk, which can in turn explain the variation in the cross-section of equity returns. Usually, highly volatile periods are associated with significant downward market moves. Furthermore, index straddles earn positive (negative) returns in times of high (low) volatility, as can be seen by the negative correlation between the straddle and market returns in Table I. CS also argue that volatility risk is a possible explanation for the well-known size anomaly among securities returns. For a preliminary investigation of those two hypotheses, we use a two-factor model, and regress excess returns of CRSP's size deciles on

the excess returns of CRSP's value-weighted index on all NYSE, AMEX, and NASDAQ stocks and the excess returns of zero-beta at-the-money straddles. Table II presents the results of these regressions.

[Insert Table II here]

As can be seen from the table, there exists a statistically significant relationship between straddle returns and securities returns in 9 of the 10 size deciles. Thus, straddle returns and therefore volatility risk could be a significant variable in explaining securities returns. In their recent studies, Moise (2005) and Ang et al. (2006) also document statistically significant negative price of risk for aggregate volatility. In our case, the economic interpretation of this negative volatility risk premium could be that buyers of zero-beta at-the-money straddles are willing to pay a premium for downside market risk. If investors are assumed to be averse to downward market moves, the existence of a negative volatility risk premium would be justified, because downward moves are associated with high volatility periods. Following Vanden's theoretical framework, this would imply that straddles are effective tools in completing the market, because they help investors avoid insolvency and negative wealth levels, during high volatility periods.



A more interesting finding, which also confirms CS's predictions, is the significant pattern observed in the coefficients of straddle returns. The coefficients of straddle returns monotonically increase from the smallest size decile to the largest. This finding, if persistent, can be a potential explanation for the widely known size anomaly. Since stocks with small market capitalizations are the ones that are affected most by highly volatile states of the economy, the volatility coefficients of smaller decile firms are expected to be lower than larger decile firms; i.e., they are associated with more negative volatility risk premiums.<sup>13</sup> Moreover, the coefficients of the largest size decile turn out to be significantly positive, suggesting that investors see large firms as hedges against innovations in volatility. This finding suggests that, during volatile periods, large firms tend to protect their investors better than small firms.

The explanatory power of the regressions is relatively high with adjusted  $R^2$ 's ranging from 0.64 to 0.98. Furthermore, none of the intercept terms are significantly different from zero according to the  $t$ -statistics. However, the GRS F-test rejects the hypothesis that all the intercepts are jointly equal to zero at the 5% level. Overall, the above results favor the explanation that volatility risk might be a potential priced factor among securities returns.

Next, the relevance of the volatility risk factor on different classes of firms is examined. To do this, 25 portfolios formed on size and book-to-market are

used. One advantage of using this broader portfolio set is to see the robustness of the above results across book-to-market portfolios, as well.

Table III documents the time-series regression results for the 25 portfolios. As can be seen, straddle returns still explain the variation in the returns of 21 out of 25 portfolios formed according to size and book-to-market. Consistent with the previous results, small size portfolios (the lowest three size quintiles) have statistically significant negative coefficients for most of the book-to-market levels (14 out of 15 portfolios). Although, the intercept term  $\alpha_i$  is not statistically significant for 23 of the portfolios, the GRS- $F$  test rejects the hypothesis that intercepts are jointly equal to zero. This result is consistent with Vanden (2004) and Coval and Shumway (2001).

Looking across book-to-market portfolios, it is seen that high book-to-market (value) stocks consistently have significant and negative coefficients in the smallest four size quintiles and low book-to-market (growth) stocks have significant and positive coefficients in the biggest size quintile. The positive and significant coefficients for the big-growth portfolios are interesting. This result, if persistent, might indicate that among the big firms, investors see only growth firms as potential hedges against volatile states of the economy. This, in turn, can be a possible explanation for the value vs. growth anomaly.

[Insert Table III here]

To further check the robustness of this explanation, the sample is refined to 6 portfolios based on size and book-to-market. As can be seen from Table IV, small-sized firms still have negative and significant coefficients consistent with the previous documented results. Furthermore, among big firm portfolios it is only the growth portfolio, which exhibits a positive and significant volatility risk coefficient. These consistent results indicate that the volatility risk could not only explain the size anomaly but also the value vs. growth anomaly. When formed according to size, it is clearly seen that small firms are more prone to volatility risk, whereas big firms are seen as hedges against this kind of risk. However a detailed analysis reveals that it is actually the growth portfolios among big firms that provide a hedge against volatility risk.

[Insert Table IV here]

### **Is Volatility Risk Priced?**

Up to now, the documented evidence suggests that straddle returns are useful explanatory variables over the sample period studied, but we can not conclude whether volatility risk is priced in security returns or not. In an attempt to answer this question, Fama-MacBeth two-pass regressions are performed and Panel A of Table V reports the results of these tests for the conditional and

unconditional versions of various CAPM specifications. More specifically, risk premiums estimated according to Equation 6, their associated Shanken-corrected and uncorrected  $t$ -statistics, and adjusted  $R^2$  statistics for the cross-sectional regressions are shown.

The first row of Table V presents results for the traditional unconditional CAPM taking the form

$$E[r_{it}] = \alpha_i + \lambda_m \beta_i^m.$$

The statistically insignificant  $t$ -statistic for the market risk premium shows the inability of the value-weighted market beta to explain the cross-section of average returns. Moreover, the negative sign of the market risk premium contradicts the CAPM theory. These findings are also supported by the very low explanatory power for the model. The results are in line with the Fama and French (1992) findings.

Next, we test the significance of volatility risk as a priced factor with the following model

$$E[r_{it}] = \alpha_i + \lambda_m \beta_i^m + \lambda_{st} \beta_i^{st}.$$

Row 2 of Panel A shows that adding straddle betas significantly contributes to the explanatory power of the two-factor model. The adjusted  $R^2$  increases dramatically from 3 percent to 32 percent. Although the volatility risk premium is positive, the insignificant  $t$ -statistic shows that it is not a priced risk factor. This result needs further exploration, as it contradicts the previous findings

of significant volatility betas in time-series regressions. One explanation for this contradiction could be the time variation inherent in the volatility risk premium and the inadequacy of the unconditional models to capture this time variation. The literature on time-varying risk premiums argues that conditional versions of factor models better explain this time variation than their unconditional counterparts. Hence, a natural extension is to perform the preceding analysis with conditional factor models.

### **Conditional Factor Models**

Cochrane (1996, 2001) argues that conditional factor models can be represented in an unconditional form by using appropriate scaling variables. We posit that investors use time  $t$  straddle returns when forming their expectations about time  $t+1$  returns. For the conditional model with one factor (market return) and one scaling variable (straddle return), the scaled market factor would take the form,  $r_t^{st} \cdot r_{t+1}^m$ , and the cross-sectional regression for this scaled model would be

$$E[r_{it}] = \alpha_i + \lambda_m \beta_i^m + \lambda_{st} \beta_i^{st} + \lambda_{scaled} \beta_i^{scaled}$$

where  $\beta_i^{scaled}$  is the beta of the scaled market factor. Row 4 of Table 5 reports the estimated coefficients of the proposed conditional model. The estimated risk premia for straddle and market returns are still not statistically significant; however, the coefficient of the scaled market beta is negative and statistically

significant at the 5% level. The explanatory power of the model also improves from an  $R^2$  of 0.32 to 0.42.

Besides the statistical significance of the scaled factor in the conditional model, we examine the effect of a one standard deviation change in the estimated betas on average returns of various portfolios. This is done to see the sensitivity of average portfolio returns to changes in betas that are estimated in the first-pass. For example, taking the big-growth portfolio, a one standard deviation increase in the beta of the scaled factor causes a 0.19% decrease in the average return of the portfolio. The effect of a one standard deviation increase in the market beta results in a decrease of 0.03% in the average return, whereas a one standard deviation increase in straddle beta increases the average return of the big-growth portfolio by 1.25%. However, one need to be careful while interpreting the risk-premiums associated with the scaled returns. Lettau and Ludvigson (2001) argue that individual risk-premium estimates for the scaled multifactor model should not be interpreted as risk prices as in unconditional models. Cochrane (2001) note that scaled returns act as payoffs to managed portfolios, thus in incomplete market settings state contingencies can be provided through trading strategies using conditioning information. The significance of the scaled market factor in the conditional model indicate that investors use straddle returns in forming their expectations about the future prices of securities. This also supports the non-redundancy of options hypothesis by Vanden (2004). Overall, these results

suggest that there exist time variation in the volatility risk premium and that the scaled market return is an important factor for asset pricing.

Lettau and Ludvigson (2001) show that conditional versions of CAPM perform much better than the unconditional models, using the log consumption-wealth ratio as a conditioning variable. They document that these models perform about as well as the Fama-French three-factor model. In our case, Row 4 of Table V demonstrates that the conditional CAPM, using straddle returns as a conditioning variable, performs slightly worse than the Fama-French three factor model, where none of the risk premia is statistically significant. Furthermore, we test whether or not the addition of Fama-French factors can explain the cross-section of expected returns not explained by our model. The model to be tested is

$$E[r_{it}] = \alpha_i + \lambda_{st} \beta_i^{st} + \lambda_m \beta_i^m + \lambda_{SMB} \beta_i^{SMB} + \lambda_{HML} \beta_i^{HML} + \lambda_{scaled} \beta_i^{scaled},$$

where scaling is done in a similar manner as in the one factor model. Row 5 of Table 1 reports the results of this estimation. Although the explanatory power of the model increases to an  $R^2$  of 52%, the coefficients of the Fama-French factors are still insignificant. The only significant risk premium is that of the scaled market factor. This confirms that the conditional model using straddle returns as a scaling variable is successful in explaining the cross-section of average returns.

### GMM-SDF Tests

Because the Fama-MacBeth regressions is criticized for having errors-in-variables problem, we also examine whether the volatility risk is priced or not by using a GMM framework in various SDF representations. Panel B of Table V reports the estimates of SDF coefficients and their associated  $t$ -statistics, p-values, and Hansen-Jagannathan distances (HJ-dist.).<sup>14</sup> The first model to be tested is the unconditional CAPM, i.e.,

$$E[R_{it}(\delta_0 + \delta_m r_t^m)] = 1$$

where  $R_{it}$  is the gross return of 25 Fama-French portfolios and  $r_t^m$  is the return on the value-weighted index of all NYSE, AMEX, and NASDAQ stocks. Row 6 of Panel B presents the results of this estimation. Contrary to the previous findings, the unconditional CAPM yields a statistically significant coefficient for the market factor. However, the estimated HJ-dist. shows that the pricing error is very high, and significantly different from zero, suggesting that this model is a poor SDF representation.

Next we test whether straddle returns are a part of the stochastic discount factor or not. This gives the following SDF specification

$$E[R_{it}(\delta_0 + \delta_m r_t^m + \delta_{st} r_t^{st})] = 1.$$

Row 7 shows that, including straddle returns in the unconditional model results in slightly lower pricing errors. However, the insignificant coefficient for straddle



returns suggests that volatility risk does not play a significant role in constructing a stochastic discount factor in the unconditional form. This result is consistent with the previous Fama-MacBeth results. Next, we test whether the Fama-French factors are significant explanatory variables by the following SDF representation

$$E\left[R_{it}\left(\delta_0 + \delta_m r_t^m + \delta_{SMB} r_t^{SMB} + \delta_{HML} r_t^{HML}\right)\right] = 1.$$

As can be seen in Row 8, the coefficients are still insignificant and the pricing errors are slightly better than that of the traditional CAPM.

Row 9 of Panel B presents the results for the conditional CAPM using straddle returns as the conditioning variable. The model to be tested is

$$E\left[R_{it}\left(\delta_0 + \delta_m r_t^m + \delta_{st} r_t^{st} + \delta_{scaled} r_t^{scaled}\right)\right] = 1,$$

where  $r_t^{scaled}$  is calculated as before. The statistically significant coefficient for the conditioning variable suggests that this variable plays an important role in constructing a stochastic discount factor. This finding is consistent with our previous results and also confirms that there exists time variation in the volatility risk premium. However, although the pricing error is considerably lower, it is still significantly different from zero. Due to the small-sample problems with GMM estimation, it is not surprising to obtain large HJ-distances that are statistically different from zero. Altonji and Segal (1996), Cochrane (2001), and Lettau and Ludvigson (2001) suggest that using GMM estimates with the identity matrix is far more robust to small-sample problems. The last column of Panel B reports

estimates of Hansen-Jagannathan distances using the identity matrix. Note that, HJ-distances estimated with the identity matrix, and therefore pricing errors decrease drastically for all the models. However, only for the conditional models (Row 9 and 10) are the pricing errors not significantly different from zero. Furthermore, the addition of Fama-French factors to the conditional model does not considerably improve the explanatory power of the model, as reported in Row 10.

Consistent with the earlier findings from Fama-MacBeth regressions, conditional models using straddle returns as a scaling variable perform better than unconditional models examined in this study. Besides this statistical significance, in order to check the economic significance of the results, we examined the impact on the SDF of a one standard deviation change in factor returns. For example, for the conditional model in Row 9 in Table 5, a one standard deviation increase in scaled factor returns corresponds to a 0.15 standard deviation increase in the SDF. The effect of a one standard deviation increase in straddle returns is 0.47 standard deviation increase in the SDF, and a one standard deviation increase in market returns cause a 1.22 standard deviation increase in the SDF. As for the economic interpretation of the scaled returns, we can think of them as payoffs to managed portfolios as in Cochrane (2001). For example, an investor who observes high zero-beta straddle returns is expected to decrease her holdings in the market portfolio. Our findings confirm that investors use straddle returns as a

conditioning variable when forming their expectations of securities returns. Thus, they are important for asset pricing since they help capture the time variation in the SDF.

### **Effect of the 1987 Crash**

The effect of time variation in the volatility risk premium on asset returns can be tested by the threshold regression methodology. We applied the sup-LM test used in Hansen (1996) to explore the question of whether there are statistically significant discrete regime shifts in the risk factors due to certain instrumental variables. VIX Volatility of at-the-money options and the difference between volatilities of at-the-money and out-of-the-money options are used as instrumental variables, but no significant regime shifts are detected. However, the bootstrap p-values are likely to be poorly estimated in samples of the size encountered here.

Nevertheless, in an attempt to explore the possible effects of a high volatility periods on our results, the sample is divided into two sub-samples, one including the crash period and one excluding it.

[Insert Table VI here]

As can be seen from Table VI, when the crash period is excluded from the sample, the significance of the volatility risk factor vanishes for 9 of the 10 size portfolios. This result confirms that there exists time variation in the volatility risk

premium and it has several implications regarding the redundancy of options. According to Vanden (2004), options effectively complete the market when agents face non-negative wealth constraints. That is, options are non-redundant, because they help agents to avoid insolvency while still allowing them to obtain a degree of leverage that is not possible through direct borrowing. Thus, the high explanatory power of the proposed 2-factor model through the crash period makes sense in this manner. Straddles explain asset returns in periods of high volatility, because they allow their investors to hedge volatility risk and help them avoid insolvency in those periods. The failure of straddle returns to explain security returns in periods of low volatility arises because straddles are redundant securities at those times. As the highest volatility period in our sample is around October 1987 (see Figure 1), the exclusion of this time period results in less explanatory power for the volatility risk factor. Thus, although volatility risk is priced for all classes of assets at times of high volatility, we cannot assert the same for times of low volatility.

Asset return volatility literature documents that high volatility periods tend to coincide with business cycle downturns and recessions. (Turner, Startz, & Nelson (1989), Schwert (1989), Hamilton and Lin (1996), and Perez-Quiros and Timmerman (2001)) Also, Chauvet and Potter (2000) argue that bear markets have higher volatility than bull markets. Our finding of a significant volatility beta in a high volatility period like 1987 is in line with the literature. However, we also

report an insignificant volatility beta for the time period of 1991-1992, which is often cited as a period of poor business conditions and high volatility, is at odds with the above literature. We offer two possible explanations for this. First, as can be seen from Figure 1, VIX volatility index is much higher in the 1987 crash period compared to the volatility around 1991-1992 downturn. This large difference in the level of volatility, which is captured by straddle returns, might lead the volatility betas to be insignificant for the latter period. One can also argue that it might be the fear of a crash that drives these results. VIX measure is also considered to be a fear indicator among the professionals. High VIX levels are associated with a pessimistic market sentiment and conversely a low level of VIX is considered to be a sign of optimistic market sentiment. The relatively low levels of VIX measure for the second period studied might indicate that investors are optimistic about the market and hence lead the volatility betas to be insignificant for this period. Altogether, these results should be further investigated since the time period studied here covers only one peak and one trough, which makes it hard to reconcile our findings with that of the business cycle literature.

## CONCLUSION

The notion that volatility risk is priced in options markets is now widely documented. However, until recently, very few studies focused on the question of whether volatility risk is priced in the securities market. The answer to this question has important implications for asset pricing, portfolio and risk management, and hedging strategies.

The empirical findings in this article suggest that volatility risk explains a significant amount of variation in securities returns, especially during high volatility periods. In addition, the findings suggest that options are non-redundant securities during those periods. Investors use straddle returns when forming their expectations about securities returns. This implies that straddle returns can be used to price volatility risk.

The findings also indicate different patterns for different classes of firms. For example, during high volatility periods, small firms and value firms are more prone to downside market risk, hence they are associated with negative volatility coefficients. Thus, at times of high volatility, investors see value firms and small firms riskier than their growth and big counterparts and price this risk in their returns via an important factor, volatility risk. Furthermore, investors see big-growth firms as hedges against volatility, regardless of the level of volatility in the

market. This could be a potential explanation to why growth firms underperform value firms.

In conclusion, this article presents clear evidence that volatility risk, proxied by straddle returns, is an important factor in asset pricing since it helps capture time variation in the stochastic discount factor. Thus, options play an important role in pricing securities, and allocation of wealth among agents in the economy.

## ENDNOTES

<sup>1</sup> See Engle and Ng (1993), Canina and Figlewski (1993), Duffee (1995), Braun, Nelson, and Sunier (1995), Andersen (1996), Bollerslev and Mikkelsen (1999), and Bekaert and Wu (2000) for a theoretical discussion and distributional aspects of stochastic volatility of equity returns.

<sup>2</sup> Banz (1981) and Reinganum (1981) document that portfolios formed on small sized firms earn returns higher than the CAPM predicts. Rosenberg, Reid and Leinstein (1985) find that firms with high book-to-market ratios (value firms) earn higher returns than firms with low book-to-market ratios (growth firms). Davis, Fama, and French (2000) report that the value premium in United States' stocks is robust.

<sup>3</sup> See Ferson (1989), Ferson and Harvey (1991), Ferson and Korajczyk (1995), Jagannathan and Wang (1996), Lettau and Ludvigson (2001), and Altay-Salih, Akdeniz, and Caner (2003) for the theory behind time-varying beta and conditional CAPM literature.

<sup>4</sup> This is because increased market volatility coincides with downward market moves, a phenomenon which is reported by French, Schwert, and Stambaugh (1987), and Glosten, Jagannathan, and Runkle (1993). Engle and Ng (1993) show that volatility is more associated with downward market moves due to the leverage effect.

<sup>5</sup> We are grateful to Ramazan Gencay for providing the data.

<sup>6</sup> According to Buraschi and Jackwerth (2001), most of the trading activity in S&P500 options is concentrated in the nearest (0-30 days to expiry) and second nearest (30-60 days to expiry) contracts.

<sup>7</sup> Stoll and Whaley (1987) report abnormal trading volumes for options close to expiry.

<sup>8</sup> Macbeth and Merville (1979) report that the Black-Scholes prices of at-the-money call options are on average less than market prices for in-the-money call options. Also, Gencay and Salih (2001) document that pricing errors are larger in the deeper-out-of-money options compared to at-the-money options.

<sup>9</sup> In order to check the robustness of the results, we set the theoretical position beta in Equation 2 to a constant such that the in-sample straddle beta is exactly zero. Negative mean and median volatility risk premium still persists and furthermore conclusions from time series regressions do not change. Overall these results are in line with the literature on negative volatility risk premium, and the findings in Coval and Shumway (2001).



<sup>10</sup> Vanden (2004) uses a similar model, where he includes call and put option returns and a market factor as explanatory factors.

<sup>11</sup> The returns on 25 portfolios formed on size and book-to-market equity are obtained from Kenneth French's data library.

<sup>12</sup> Rolling regression approach is not appropriate in samples, which have fewer than 150 time series observations, as pointed out in Lettau and Ludvigson (2001).

<sup>13</sup> This finding is in line with Moise (2005)

<sup>14</sup> For a detailed discussion on the calculation of HJ-dist., see Jagannathan and Wang (1996).

## **BIBLIOGRAPHY**

- Altay-Salih, A., Akdeniz, L., & Caner, M. (2003). Time Varying Betas Help in Asset Pricing: The Threshold CAPM. *Studies in Nonlinear Dynamics and Econometrics*, 6(4), 1-16.
- Altonji, J.G. & Segal, L.M. (1996). Small Sample Bias in GMM Estimation of Covariance Structures. *Journal of Business and Economics*, 14, 353-366.
- Andersen, T.G. (1996). Return Volatility and Trading Volume: An Information Flow Interpretation of Stochastic Volatility. *Journal of Finance*, 51, 169-204.
- Ang, A. , R. Hodrick, Y. Xing, and X. Zhang (2006). The Cross Section of Volatility and Expected Returns. Forthcoming , *Journal of Finance*.
- Bakshi, G. & Kapadia, N. (2003). Delta-Hedged Gains and the Negative Market Volatility Risk Premium. *Review of Financial Studies*, 16, 527-566.
- Banz, R.W., (1981). The Relation Between Return and Market Value of Common Stocks. *Journal of Financial Economics* 9, 3-18.
- Bekaert, G. & Wu, G. (2000). Asymmetric Volatility and Risk in Equity Markets. *Review of Financial Studies*, 13(1), 1-42.
- Bollerslev, T. & Mikkelsen, H.O. (1999). Long-Term Equity Anticipation Securities and Stock Market Volatility Dynamics. *Journal of Econometrics*, 92, 75-99.
- Braun, P.A., Nelson, D.B., & Sunier, A.M. (1995). Good News, Bad News, Volatility and Betas. *Journal of Finance*, 50, 1575-1603.
- Buraschi, A., & Jackwerth, J. (2001). The Price of a Smile: Hedging and Spanning in Option Markets. *Review of Financial Studies*, 14, 495-527.
- Canina, L., & Figlewski, S. (1993). The Informational Content of Implied Volatility. *Review of Financial Studies*, 6, 659-681.
- Chauvet, M., & Potter, S. (2000). Coincident and Leading Indicators of the Stock Market. *Journal of Empirical Finance*, 7, 87-111.
- Cochrane, J.H. (1996). A Cross-Sectional Test of an Investment-Based Asset Pricing Model. *Journal of Political Economy*, 104, 572-621.
- Cochrane, J.H. (2001). *Asset Pricing*. Princeton, N.J.:Princeton University Press.

- Coval, J., & Shumway, T. (2001). Expected Option Returns. *Journal of Finance*, 56, 983-1009.
- Davis, J.L., Fama, E.F., & French, K. (2000) Characteristics, Covariances and Average Returns: 1929-1997. *Journal of Finance*, 55, 359-406.
- Detemple, J. & Selden, L. (1991). A General Equilibrium Analysis of Option and Stock Market Interactions. *International Economic Review*, 32, 279-303.
- Driessen, J., & Maenhout, P.J. (2005, January). The World Price of Jump and Volatility Risk. Paper presented at the annual meeting of American Finance Association, Philadelphia, U.S.
- Duffee, G.R. (1995). Stock Returns and Volatility: A Firm Level Analysis. *Journal of Financial Analysis*, 37, 399-420.
- Engle, R.F., & Ng, V.K. (1993). Measuring and Testing the Impact of News on Volatility. *Journal of Finance*, 48, 1749-1778.
- Fama, E.F., & French, K. (1992). The Cross-Section of Expected Returns. *Journal of Finance*, 47, 427-465.
- Fama, E.F., & French, K. (1993). Common Risk Factors in the Returns on Stocks and Bonds. *Journal of Financial Economics*, 33, 3-56.
- Fama, E.F., & French, K. (1995). Size and Book-to-Market Factors in Earnings and Returns. *Journal of Finance*, 50, 131-155.
- Fama, E.F., & MacBeth, J.D. (1973). Risk, Return, and Equilibrium: Empirical Tests. *Journal of Political Economy*, 81, 607-636.
- Ferson, W. (1989). Changes in Expected Security Returns, Risk and the Level of Interest Rates. *Journal of Finance*, 44, 1191-1217.
- Ferson, W., & Harvey, C.R. (1991). The Variation of Economic Risk Premiums. *Journal of Political Economy*, 99, 385-415.
- Ferson, W., & Korajczyk, R.A. (1998). Do Arbitrage Pricing Models Explain the Predictability of Stock Returns? *Journal of Business*, 68, 309-349.
- French, K., Schwert, W., & Stambaugh, R. (1987). Expected Stock Returns and Volatility. *Journal of Financial Economics*, 19, 3-29.
- Gencay R., & Altay-Salih, A. (2003). Degree of Mispricing with Black-Scholes and Nonparametric Cures. *Annals of Economics and Finance*, 4, 73-101.

Gibbons, M.R., Ross, S.A, & Shanken J. (1989). A Test of the Efficiency of a Given Portfolio. *Econometrica*, 57, 1121-1152.

Glosten, L.R., Jagannathan, R. & Runkle, D.E. (1993). On the Relation Between the Expected Value and the Volatility of the Nominal Excess Return on Stocks. *Journal of Finance*, 48, 1779-1801.

Hamilton, J.D., & Lin, G. (1996). Stock market volatility and the business cycle. *Journal of Applied Econometrics*, 11, 573–593.

Hodrick, R.J. & Zhang, X (2000). Evaluating the Specification Errors of Asset Pricing Models. Manuscript. New York, Columbia University.

Jackwerth, J., & Rubinstein, M. (1996). Recovering Probability Distributions from Option Prices. *Journal of Finance*, 51, 1611-1631.

Jagannathan, R., & Wang, Z. (1996). The Conditional CAPM and the Cross-Sections of Expected Returns. *Journal of Finance*, 51, 3-53.

Jegadeesh, N., & Titman, S. (1993). Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency. *Journal of Finance*, 48, 65-91.

Jones, C. (2001). A Nonlinear Factor Analysis of S&P 500 Index Option Returns. Working Paper, University of Rochester.

Lamoureux, C. G. & Lastrapes, W. D. (1993). Forecasting Stock-Return Variance: Toward an Understanding of Stochastic Implied Volatilities. *The Review of Financial Studies*, 6, 293-326.

Lettau, M. & Ludvigson, S. (2001). Resurrecting the (C)CAPM: A Cross-Sectional Test When Risk Premia Are Time-Varying. *Journal of Political Economy*, 109, 1238-1287.

Macbeth J., & Merville, L. (1979). An Empirical Investigation of the Black-Scholes Call Option Pricing Model. *Journal of Finance*, 34, 1173-1186.

Moise, C..E. (2005), Stochastic Volatility Risk and the Size Anomaly. Working Paper, University of Chicago.

Newey, W.K., & West, K.D. (1987). A Simple Positive Semi-Definite Heteroskedasticity and Autocorrelation Consistent Covariance Matrix. *Econometrica* 55, 703–708.

Perez-Quiros, G., & Timmermann, A. (2001). Business Cycle Asymmetries in Stock Returns: Evidence from Higher Order Moments and Conditional Densities. *Journal of Econometrics*, 103, 259-306.

- Reinganum, M.R. (1981). Misspecification of Capital Asset Pricing: Empirical Anomalies Based On Yields and Market Values. *Journal of Financial Economics* 9, 19-46.
- Rosenberg, B., Reid, K. & Lanstein, R. (1985). Persuasive Evidence of Market Inefficiency. *Journal of Portfolio Management*, 11, 9-17.
- Ross, S.A. (1976). The Arbitrage Theory of Capital Asset Pricing. *Journal of Economic Theory*, 13, 341–360.
- Schwert, G.W. (1989). Why does stock market volatility change over time?. *Journal of Finance*, 44, 1115–1153.
- Shanken, J. (1992). On the Estimation of Beta-Pricing Models. *Review of Financial Studies*, 5(1), 1-33.
- Stoll, H.R. & Whaley, R.E. (1987). Program Trading and Expiration Day Effects. *Financial Analysts Journal*, 43, 16–28.
- Turner, C.M., Startz, R., & Nelson, C.R. (1989). A Markov Model of Heteroskedasticity, Risk, and Learning in the Stock Market, *Journal of Financial Economics*, 25, 3-22.
- Vanden, J.M. (2004). Options Trading and the CAPM. *Review of Financial Studies*, 17, 207-238.

**TABLE I**

**Summary Statistics for Daily Zero-Beta Straddles**

Daily Straddle Returns (%)	
Mean	-1.06
Median	-1.58
Minimum	-87.77
Maximum	441.79
Skewness	17.03
Kurtosis	520.03
Correlation	-0.54

*Note.* This table reports the summary statistics for the returns of daily zero-beta at-the money straddles. The sample covers the period January 1987 to October 1994 (1980 days). After adjusting for moneyness and maturity criteria, we end up with 1937 days of data. Correlation is the correlation of straddle returns with market returns.

**TABLE II**  
**2-Factor Time Series Regressions**

$$r_{it} - r_{ft} = \alpha_i + \beta_{im} (r_{mt} - r_{ft}) + \beta_{iv} (r_{vt} - r_{ft}) + \varepsilon_{it}$$

$r_{it} - r_{ft}$	$\alpha_i$	t-statistic	$\beta_{im}$	t-statistic	$\beta_{iv}$	t-statistic	Adj. $R^2$
Small 10	-0.0024	-0.61	0.7555	6.91***	-0.0109	-4.55***	0.64
Decile 9	-0.0039	-1.23	0.9612	11.37***	-0.0080	-4.29***	0.78
Decile 8	-0.0004	-0.18	1.0106	13.69***	-0.0063	-3.98***	0.84
Decile 7	-0.0017	-0.70	1.0612	14.86***	-0.0052	-3.33***	0.86
Decile 6	0.0009	0.40	1.0553	14.83***	-0.0040	-2.74***	0.88
Decile 5	0.0009	0.51	1.0337	20.91***	-0.0031	-3.02***	0.92
Decile 4	0.0004	0.37	1.0343	27.10***	-0.0024	-2.31**	0.95
Decile 3	0.0007	0.60	1.0917	27.76***	0.0003	0.36	0.96
Decile 2	0.0004	0.55	1.0801	34.26***	0.0019	2.67***	0.98
Big 1	0.0006	0.56	0.9953	32.97***	0.0024	2.99***	0.96

GRS F-Test = 2.3314 (p=0. 0179)

*Note.* This table reports monthly time-series regression results of excess returns of CRSP's size deciles on market factor and excess straddle returns. The dependent variable is the excess return of CRSP's size-decile portfolio,  $r_{mt}$  is the return of CRSP's value-weighted index on all NYSE, AMEX, and NASDAQ stocks,  $r_{vt}$  is the monthly zero-beta straddle return, and  $r_{ft}$  is the 1-month T-bill rate. \*\*\*, \*\*, \* denote 0.01, 0.05, and 0.10 significance levels, respectively. All t-values are corrected for autocorrelation (with lag=3) and heteroskedasticity as suggested by Newey and West (1987). GRS F-Test reported at the bottom of the table is from Gibbons, Ross, and Shanken (1989).

**TABLE III**  
**25 (5x5) Portfolio Regressions**

$$r_{it} - r_{ft} = \alpha_i + \beta_{im} (r_{mt} - r_{ft}) + \beta_{iv} (r_{vt} - r_{ft}) + \varepsilon_{it}$$

Size	B/M	$\alpha_i$	t-statistic	$\beta_{im}$	t-statistic	$\beta_{iv}$	t-statistic	Adj. R <sup>2</sup>
S	L	-0.0115	-2.67***	1.0271	9.51***	-0.0100	-3.89***	0.70
S	2	-0.0018	-0.48	0.9158	9.06***	-0.0098	-4.33***	0.70
S	3	-0.0012	-0.37	0.8589	9.63***	-0.0085	-4.53***	0.76
S	4	0.0011	0.36	0.7602	8.20***	-0.0105	-4.91***	0.72
S	H	0.0018	0.41	0.7808	8.08***	-0.0105	-4.82***	0.65
2	L	-0.0052	-1.67*	1.2560	14.09***	-0.0041	-1.95**	0.81
2	2	-0.0014	-0.47	1.0796	14.50***	-0.0067	-4.16***	0.82
2	3	0.0026	1.05	0.8742	11.08***	-0.0080	-5.19***	0.84
2	4	0.0011	0.49	0.7999	12.43***	-0.0080	-5.70***	0.82
2	H	0.0012	0.35	0.9861	10.79***	-0.0062	-2.90***	0.77
3	L	-0.0013	-0.45	1.2517	18.22***	-0.0014	-0.83	0.83
3	2	0.0010	0.45	1.0854	16.14***	-0.0045	-2.96***	0.88
3	3	-0.0001	-0.07	0.8722	13.27***	-0.0047	-3.03***	0.86
3	4	0.0024	1.07	0.8723	13.77***	-0.0033	-2.27**	0.85
3	H	0.0027	0.98	0.9250	15.26***	-0.0062	-4.08***	0.82
4	L	0.0013	0.73	1.1890	26.99***	0.0013	1.13	0.89
4	2	-0.0006	-0.35	1.0294	25.29***	-0.0050	-4.69***	0.93
4	3	-0.0011	-0.53	1.0834	13.95***	-0.0005	-0.27	0.90
4	4	0.0023	1.35	0.9081	15.45***	0.0022	1.81*	0.89
4	H	0.0027	1.11	0.9264	12.16***	-0.0038	-2.19**	0.82
B	L	0.0012	0.52	1.1202	24.26***	0.0037	3.39***	0.88
B	2	0.0002	0.10	1.1129	24.46***	0.0027	2.55**	0.92
B	3	0.0005	0.25	0.8575	17.83***	-0.0025	-2.54**	0.87
B	4	0.0004	0.26	0.9113	24.29***	0.0043	2.91***	0.83
B	H	0.0027	0.79	0.9354	14.67***	0.0008	0.38	0.70

GRS F-Test = 2.7293 (p=0.0071)

*Note.* This table reports monthly time-series regression results of excess returns of CRSP's 25 size and book-to-market portfolios on market factor and excess straddle returns. The returns on 25 portfolios formed on size and book-to-market equity are obtained from Kenneth French's data library. The 25 portfolios constructed at the end of each June, are the intersections of 5 portfolios formed on size (market equity, ME) and 5 portfolios formed on the ratio of book equity to market equity (BE/ME). The size breakpoints for year t are the NYSE market equity quintiles at the end of June of t. BE/ME for June of year t is the book equity for the last fiscal year end in t-1 divided by ME for December of t-1. The BE/ME breakpoints are NYSE quintiles. S and B stands for the smallest and biggest size quintiles; L and H stands for the lowest and highest book-to-market quintiles.  $r_{it}$  is the dependent variable which denotes the return on each of the 25 portfolios from January 1987-October 1994.  $r_{mt}$  is the return of CRSP's value-weighted index on all NYSE, AMEX, and NASDAQ stocks,  $r_{vt}$  is the monthly zero beta straddle return, and  $r_{ft}$  is the 1-month T-bill rate obtained from Ibbotson and Associates. \*\*\*, \*\*, \* denote 0.01, 0.05, and 0.10 significance levels, respectively. All t-values are corrected for autocorrelation (with lag=3) and heteroskedasticity as suggested by Newey and West (1987). GRS F-Test reported at the bottom of the table is from Gibbons, Ross, and Shanken (1989).



**TABLE IV**  
**6 (2x3) Portfolio Regressions**

$$r_{it} - r_{ft} = \alpha_i + \beta_{im} (r_{mt} - r_{ft}) + \beta_{iv} (r_{vt} - r_{ft}) + \varepsilon_{it}$$

Size	B/M	$\alpha_i$	t-statistic	$\beta_{im}$	t-statistic	$\beta_{iv}$	t-statistic	Adj. R <sup>2</sup>
S	L	-0.0046	-1.57	1.1557	14.74***	-0.0058	-3.30***	0.83
S	2	0.0056	2.52**	0.8997	13.06***	-0.0066	-4.43***	0.86
S	H	0.0059	2.07**	0.8642	11.58***	-0.0076	-4.52***	0.80
B	L	0.0053	3.24***	1.1287	35.50***	0.0027	3.54***	0.94
B	2	0.0047	4.21***	0.9329	32.55***	0.0010	1.60	0.94
B	H	0.0056	2.97***	0.8659	25.07***	-0.0002	-0.15	0.86

GRS F-Test = 2.3260 (p=0. 0178)

*Note.* This table reports monthly time-series regression results of excess returns of CRSP's 6 size and book-to-market portfolios on market factor and excess straddle returns. Portfolios are constructed at the end of each June, which are the intersections of 2 portfolios formed on size (market equity, ME) and 3 portfolios formed on the ratio of book equity to market equity (BE/ME). The size breakpoint for year t is the median NYSE market equity at the end of June of year t. BE/ME for June of year t is the book equity for the last fiscal year end in t-1 divided by ME for December of t-1. The BE/ME breakpoints are the 30th and 70th NYSE percentiles. S and B stands for the smallest and biggest size quintiles; L and H stands for the lowest and highest book-to-market quintiles.  $r_{it}$  is the dependent variable which denotes the monthly return on each of the 6 portfolios from January 1987-October 1994.  $r_{mt}$  is the monthly return of CRSP's value-weighted index on all NYSE and AMEX stocks,  $r_{vt}$  is the monthly zero beta straddle return, and  $r_{ft}$  is the 1-month T-bill rate obtained from Ibbotson and Associates. \*\*\*, \*\*, \* denote 0.01, 0.05, and 0.10 significance levels, respectively. All t-values are corrected for autocorrelation (with lag=3) and heteroskedasticity as suggested by Newey and West (1987). GRS F-Test reported at the bottom of the table is from Gibbons, Ross, and Shanken (1989).

TABLE V

Evaluation of Various CAPM Specifications using 25 Fama-French Portfolios

Panel A: Risk premium estimates using two-pass Fama-MacBeth regressions								
ROW	$\alpha_i$	$\lambda_m$	$\lambda_{st}$	$\lambda_{SMB}$	$\lambda_{HML}$	$\lambda_{scaled}$	Adj. R <sup>2</sup>	
1	1.4486 (2.17 <sup>**</sup> ) (2.16 <sup>**</sup> )	-0.7850 (-0.96) (-0.95)					0.03	
2	1.4274 (2.16 <sup>**</sup> ) (2.15 <sup>**</sup> )	-0.7254 (-0.92) (-0.91)	23.4020 (0.79) (0.78)				0.32	
3	0.7525 (1.81 <sup>*</sup> ) (1.80 <sup>*</sup> )	-0.0643 (-0.10) (-0.10)		-0.1794 (-0.68) (-0.67)	0.2110 (0.83) (0.82)		0.44	
4	1.6442 (2.43 <sup>**</sup> ) (2.34 <sup>**</sup> )	-1.1322 (-1.42) (-1.32)	37.8143 (1.20) (1.11)			-5.6965 (-2.37 <sup>**</sup> ) (-2.21 <sup>**</sup> )	0.42	
5	1.2121 (3.05 <sup>***</sup> ) (2.94 <sup>***</sup> )	-0.6912 (-1.17) (-1.08)	15.4201 (0.71) (0.66)	-0.1077 (-0.41) (-0.38)	0.2964 (1.17) (1.08)	-6.0019 (-2.37 <sup>**</sup> ) (-2.20 <sup>**</sup> )	0.52	
Panel B: Stochastic Discount Factor (SDF) estimates using GMM								
	$\delta_0$	$\delta_m$	$\delta_{st}$	$\delta_{SMB}$	$\delta_{HML}$	$\delta_{scaled}$	HJ-dist.	HJ-dist. identity
6	0.9179 (8.55 <sup>***</sup> )	5.8378 (2.13 <sup>**</sup> )					1.0445 (0.00)	0.0121 (0.00)
7	0.9288 (8.59 <sup>***</sup> )	5.9155 (1.49)	0.0765 (0.44)				1.0440 (0.00)	0.0116 (0.01)
8	0.9108 (8.06 <sup>***</sup> )	6.2797 (1.92 <sup>*</sup> )		0.6760 (0.15)	-1.0204 (-0.20)		1.0438 (0.00)	0.0112 (0.00)
9	0.9390 (8.63 <sup>***</sup> )	6.2940 (1.38)	0.0845 (0.42)			0.3772 (1.82 <sup>*</sup> )	1.0155 (0.00)	0.0100 (0.11)
10	0.9435 (8.35 <sup>***</sup> )	6.0327 (1.23)	0.0857 (0.40)	0.2176 (0.04)	-0.7585 (-0.15)	0.3794 (2.15 <sup>**</sup> )	1.0153 (0.00)	0.0096 (0.13)

Note. This table gives the estimates for the cross-sectional Fama-MacBeth regression model

$$E[r_{it}] = \alpha_i + \lambda_{st} \beta_i^{st} + \lambda_m \beta_i^m + \lambda_{SMB} \beta_i^{SMB} + \lambda_{HML} \beta_i^{HML} + \lambda_{scaled} \beta_i^{scaled}$$

and the model for the moments

$$E\left[(1 + r_{it}) \left( \delta_0 + \delta_{st} r_t^{st} + \delta_m r_t^m + \delta_{SMB} r_t^{SMB} + \delta_{HML} r_t^{HML} + \delta_{scaled} r_t^{scaled} \right)\right] = 1$$

with either a subset or all of the variables. Panel A reports the individual risk-premium,  $\lambda_j$ , estimates from the second-pass cross-sectional regressions. In the first stage, the time-series betas are computed in one multiple regression of the portfolio of excess returns on the factors. The term  $r_{it}$  is the return on 25 Fama-French portfolios ( $i=1,2,\dots,25$ ) in month  $t$  (January 1987-October 1994). The numbers in parantheses are the two t-statistics for each coefficient estimate. The top statistic uses uncorrected Fama-MacBeth standard errors; the bottom statistic uses Shanken (1992) correction. The term adjusted R<sup>2</sup> denotes the cross-sectional R<sup>2</sup> statistic adjusted for the degrees of freedom. Panel B reports GMM estimates for various SDF representations and their associated t-

and p-values. The model for the moments are estimated using the GMM approach with the Hansen-Jagannathan weighting matrix.  $r_t^{st}$  is the straddle return,  $r_t^m$  is the return on the value-weighted index of all NYSE, AMEX, and NASDAQ stocks,  $r_t^{SMB}$ , and  $r_t^{HML}$  are the returns on Fama-French mimicking portfolios related to size and book-to-equity ratios, and  $r_t^{scaled}$  is the return of the scaled variable, i.e.  $r_t^{st} \cdot r_{t+1}^m$ . The numbers in parantheses are the t-statistics for each coefficient estimate. \*\*\*, \*\*, \* denote 0.01, 0.05, and 0.10 significance levels, respectively. The minimized value of the GMM criterion function is the first item under the "HJ-dist.", with the associated p-values immediately below it. The final column reports HJ-dist. using the identity matrix as suggested by Lettau and Ludvigson (2001).

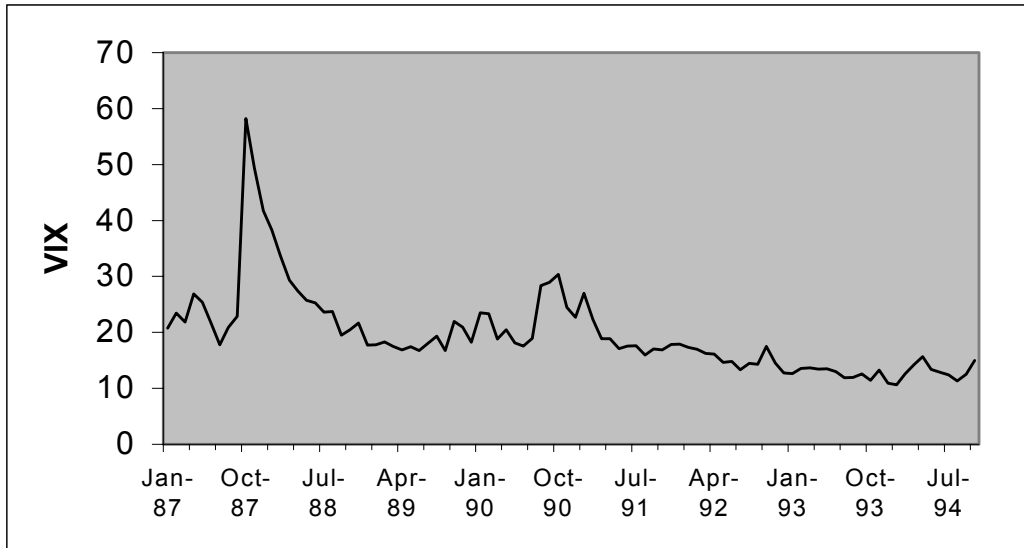
**TABLE VI**  
**10 Size Regressions With and Without 1987 Crash**

$$r_{it} - r_{ft} = \alpha_i + \beta_{im}(r_{mt} - r_{ft}) + \beta_{iv}(r_{vt} - r_{ft}) + \varepsilon_{it}$$

<b>January 1987 - November 1990</b>							
$r_{it} - r_{ft}$	$\alpha_i$	t-statistic	$\beta_{im}$	t-statistic	$\beta_{iv}$	t-statistic	Adj. R <sup>2</sup>
Small 10	-0.0111	-2.54**	0.7806	9.64***	-0.0097	-5.73***	0.85
Decile 9	-0.0099	-2.37**	0.9141	13.64***	-0.0085	-6.23***	0.90
Decile 8	-0.0039	-1.12	0.9848	13.55***	-0.0066	-4.44***	0.91
Decile 7	-0.0062	-1.63	1.0383	13.85***	-0.0055	-3.35***	0.90
Decile 6	-0.0038	-1.24	1.0139	12.72***	-0.0047	-2.93***	0.92
Decile 5	-0.0029	-1.16	1.0052	16.74***	-0.0035	-2.85***	0.94
Decile 4	-0.0003	-0.16	1.0172	24.49***	-0.0029	-2.60**	0.96
Decile 3	-0.0014	-0.79	1.0868	20.06***	0.0004	0.29	0.97
Decile 2	-0.0009	-0.69	1.0770	26.37***	0.0019	2.09**	0.98
Big 1	0.0024	1.51	1.0035	28.42***	0.0025	2.67**	0.97
GRS F-Test = 2.3249 (p=0. 0183)							
<b>December 1990- October 1994</b>							
$r_{it} - r_{ft}$	$\alpha_i$	t-statistic	$\beta_{im}$	t-statistic	$\beta_{iv}$	t-statistic	Adj. R <sup>2</sup>
Small 10	0.0080	1.58	0.7413	2.31**	-0.0043	-0.25	0.24
Decile 9	0.0030	0.71	1.0906	4.36***	-0.0009	-0.05	0.53
Decile 8	0.0047	1.37	1.1021	6.00***	0.0027	0.22	0.65
Decile 7	0.0058	1.97*	1.1727	8.24***	0.0099	1.06	0.74
Decile 6	0.0087	2.63**	1.2120	11.02***	0.0131	1.04	0.80
Decile 5	0.0062	2.45**	1.1301	15.73***	0.0057	0.54	0.85
Decile 4	0.0033	1.94*	1.1127	15.10***	0.0084	1.63	0.92
Decile 3	0.0034	2.50**	1.1162	34.07***	0.0034	0.90	0.96
Decile 2	0.0028	3.08***	1.1091	40.15***	0.0072	2.29**	0.97
Big 1	-0.0023	-1.78*	0.9564	17.42***	-0.0027	-0.62	0.93
GRS F-Test = 2.8324 (p=0. 0045)							

*Note.* This table reports monthly time-series regression results of excess returns of CRSP's size deciles on market factor and excess straddle returns. The effect of the crash is examined by dividing the sample period into two sub-samples, one from January 1987-November 1990 (47 months), and the other from December 1990-October 1994 (47 months). \*\*\*, \*\*, \* denote 0.01, 0.05, and 0.10 significance levels, respectively. All t-values are corrected for autocorrelation (with lag=3) and heteroskedasticity as suggested by Newey and West (1987). GRS F-Test reported at the bottom of the table is from Gibbons, Ross, and Shanken (1989).

**FIGURE 1**  
**Monthly Average Implied Volatility of the S&P 500 Index**



*Note.* This figure shows the monthly implied volatilities of the S&P 500 index (VIX) for the period January 1987 through October 1994.. Daily VIX data for the sample period is obtained from the Chicago Board of Options Exchange. Monthly implied volatility is the average of daily VIX levels for that month.