The Counterparty Risk Exposure of ETF Investors

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Abstract

As most Exchange-Traded Funds (ETFs) engage in securities lending or are based on total return swaps, they expose their investors to counterparty risk. In this paper, we present a framework to study counterparty risk and provide empirical estimates for a sample of physical and synthetic funds. Our findings contradict the allegations made by international agencies about the poor quality of the collateral used by ETFs. We find that the counterparty risk exposure is higher for synthetic ETFs but that investors are compensated for bearing this risk. Finally, we theoretically show how to construct an optimal collateral portfolio for an ETF.

Keywords: Asset management, passive investment, derivatives, optimal collateral portfolio, systemic risk

JEL classification: G20, G23

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1 Introduction

With their low fees and ability to provide exposure to a variety of asset classes, exchange-traded funds (ETFs) have become popular investment vehicles among individual and institutional investors alike. The global ETF industry reaches a total of $2,600 billion in assets under management (AUM) in 2014-Q3 and has experienced an average growth of 30% per year for the past ten years (Blackrock, 2014).

ETFs come in two types. In a physical ETF, investors' money is directly invested by the fund issuer in the index constituents in order to replicate the index return. Differently in a synthetic ETF, the fund issuer enters into a total return swap with a financial institution which promises to deliver the performance of the index to the fund (Ramaswamy, 2011). An industry survey by Vanguard (2013) indicates that 17% of the ETFs in the US are synthetic compared to 69% in Europe.[1] Furthermore, most leveraged and inverse ETFs traded in the world are based on synthetic replications.[2]

One may wonder why synthetic replication exists to begin with. First, it is more convenient and cheaper for the ETF issuer to outsource the index replication rather then dealing itself with dividend flows, corporate events, changes in index composition, or storage for commodities.[3] Second, swap-based replications limit investors' exposure to tracking error risk (Vanguard, 2013). Third, these swaps constitute an important source of funding for banking groups and lead to synergies and cost saving with their investment banks which maintain large inventories of equities and bonds.

In this paper, we show that besides investment risk, tracking risk (Buetow and Henderson, 2012), and liquidity risk (Roncalli and Zheng, 2014), ETF investors can also be exposed to counterparty risk. Indeed, physical ETF issuers generate extra revenues by engaging in securities lending

[1]Synthetic ETFs are less common in the US because (1) some swaps, such as those between affiliated parties, are generally not permitted under the Investment Company Act of 1940 and (2) swap income faces a higher tax rate than the capital gains incurred by transacting in a physical ETF's underlying securities.

[2]Leveraged ETFs provide exposure that is a multiple (2×, 3×) of the performance of the index, whereas inverse ETFs generate the inverse performance of the index.

[3]An industry survey by Morningstar (2012a) indicates that swap fees are extremely low and can even be zero if the swap is entered with an investment bank from the same banking group.

(Amenc et al., 2012; Bloomberg, 2013). Hence, there is a possibility that the securities will not be returned in due time. Furthermore, for synthetic ETFs, there is a risk that the total return swap counterparty will fail to deliver the index return. In order to mitigate counterparty risk, both securities lending positions and swaps must be collateralized. Recently, the Financial Stability Board (2011), the International Monetary Fund (2011), and Ramaswamy (2011) warned about potential financial stability issues that may arise from synthetic ETFs. The latter were accused of being poorly collateraleralized and allowing banks to engage in regulatory arbitrage by using risky assets as collateral.[4]

< Insert Figure 1 >

We define counterparty risk of ETFs as the risk that the value of the collateral falls below the value of the Net Asset Value (NAV) of the fund when the fund counterparty is in default. To do so, we study the composition of the collateral portfolio of 55 physical ETFs managed by the largest ETF issuer, iShares, and 164 synthetic ETFs managed by the fifth largest ETF provider in the world, db X-Trackers. For each fund, we know the exact composition of the collateral portfolio every week between July 5, 2012 and November 29, 2012. This is to the best of our knowledge the first time that such a dataset is used in an academic study. The high granularity of our data allows us to study empirically the counterparty risk of ETFs for various asset exposures, regional exposures, and types of replication.

Our analysis of the collateral portfolios of the funds reveals several important features. First, collateral portoflios are well diversified and their value often exceeds the NAV of the fund (the average collateralization is 108.4%). Second, there is a good fit between the asset exposure of the fund (e.g. equity or fixed income) and the collateral used to secure the swap. This feature is extremely important given the fact that in the case of a default of the swap counterparty, the asset

[4]The debate on ETF reached its climax in November 2011 when Laurence D. Fink, chief executive officer of BlackRock (the leading physical ETF issuer), publically criticized the synthetic ETFs issued by Société Générale's asset management arm, Lyxor (the leading synthetic ETF issuer): "If you buy a Lyxor product, you're an unsecured creditor of SocGen" (Bloomberg, 2011).

manager would need to sell the collateral in order to meet redemptions from investors. Third, ETF collateral is of high quality. The equities used as collateral mainly come from large, non-financial firms that exhibit good level of liquidity and positive correlation with the index tracked by the fund. Reassuringly, collateralized equities correlate less with the stock return of the swap counterparty than with the ETF. Turning to fixed-income securities, we notice that bonds predominantly have a AAA rating (65.5%). Furthermore, we find that ETF issuers tend to match the duration of the collateral with the duration of the fixed-income index tracked by the fund, which is sound risk-management practice.

In order to quantify the counterparty risk exposure of investors, we propose two original risk metrics: (1) the probability for a fund of not having enough collateral on the following day and (2) the magnitude of the collateral expected shortfall conditional on not having enough collateral. Both measures are computed conditionally on the default of the fund counterparty. Overall we find that the estimated probability of being undercollateralized is substantial but that the collateral shortfall remains moderate. However, counterparty risk exposure turns out to be higher for inverse funds and for funds tracking commodities or exchange rates. When contrasting the level of counterparty risk of investors investing in synthetic and physical ETFs, we find that counterparty risk exposure is higher for synthetic funds but that investors are compensated for bearing this risk through lower tracking errors but similar fees. We also show that ETF investors do care about counterparty risk. Using a diff-in-diff approach, we show that there are more outflows for synthetic ETFs after an increase in counterparty risk.

In a final step, we show theoretically how to design an optimal collateral portfolio that aims to minimize the counterparty risk exposure of ETF investors. The composition of the optimal collateral portfolio is obtained by minimizing the variance of the collateral shortfall under the constraint that the fund will be sufficiently collateralized on average. We show that the optimal weights can be expressed as a function of the weights of the Markowitz's mean-variance portfolio but they also reflect the correlations between the collateral assets and the NAV.

Our paper contributes to the growing literature on the potential financial stability issues arising from ETFs.[5] Ben-David, Franzoni and Moussawi (2014) show that ETF ownership increases stock volatility through the arbitrage trade between the ETF and the underlying stocks and through inflows and outflows. In the same vein, Krause, Ehsani and Lien (2013) document that volatility-spillover from ETF to the index constituents depends on the ETF liquidity and on the proportion of the stock held in the ETF. Da and Shive (2013) find a strong relation between measures of ETF activity and return comovement at both the fund and the stock levels (see Sullivan and Xiong, 2012). Focusing on the real effects of ETF rebalancing activities, Bessembinder et al. (2014) report no evidence of predatory trading around the time of the Crude Oil ETFs rolls of crude oil futures (see Bessembinder, 2014). Focusing on leveraged and inverse funds, Bai, Bond and Hatch (2012) find that late-day leveraged ETF rebalancing activity significantly moves the price of constituent stocks (see Shum et al., 2014, and Tuzun, 2014). Finally, Cheng, Massa and Zhang (2013) show that ETFs strategically deviate from their indices and overweigh promising stocks for which the ETF affiliated bank has superior (lending-related) information.

Unlike previous academic studies, we do not focus on the interplay between the ETFs and the assets they track. Instead, our study considers a source of risk for ETF investors that attracted significant attention from regulators and the media but, so far, little academic research: the counterparty risk of ETFs. Looking at one MSCI Emerging Markets ETF in January 2011, Ramaswamy (2011) shows that the collateral composition of the fund has very little overlap with the composition of the MSCI Emerging Market index itself. Because it is based on one fund only and remains at the aggregated level, Ramaswamy' study does not tell us whether collateral mismatch is a widespread phenomenon. Furthermore, a rigorous study of the collateral of ETFs requires granular data about all collateral securities and, as acknowledged by Ramaswamy (2011) on page 9, "Extracting this information using the International Security Identifying Number (ISIN) provided for each of the

[5] For additional evidence on the link between asset management and financial stability, see Coval and Stafford (2007), Mitchell, Pedersen and Pulvino (2007), Boyson, Stahel and Stulz (2010), Chen, Goldstein and Jiang (2010), Jotikasthira, Lundblad and Ramadorai (2012), Manconia, Massa and Yasuda (2012), Kacperczyk and Schnabl (2013), and Schmidt, Timmermann and Wermers (2014).

collateral assets would be a cumbersome process". We take on this task in the present paper and reach different conclusions.

We make several contributions to the existing literature. To the best of our knowledge, our study is the first attempt to assess empirically the quality of the collateral for a large and representative sample of ETFs. Overall, our analysis of the quality of the securities pledged by the swap counterparties do not support the allegations made about the overall poor quality of ETF collateral. This, of course, does not mean that the agencies were wrong at the time but we find no support for this claim using more recent data. Our second contribution is methodological: we develop several risk management tools that can be used by asset managers, regulators, and investors to quantify the counterparty risk of an ETF. Our framework allows one to compare the counterparty risk of several ETFs or, alternatively, to build a collateral portfolio with the lowest possible level of counterparty risk. We believe that this is the first attempt to derive optimal allocation rules for collateral portfolios, which is a topic of growing importance given the emphasis put on collateral by the recent financial regulatory reform (Dodd Franck in the US and EMIR in Europe).

The rest of our study is structured as follows. Section 2 introduces the different types of ETF structures in a common framework and defines counterparty risk for ETFs. Section 3 presents our dataset and discusses our main empirical findings. In Section 4, we show how to build an optimal collateral portfolio that aims to minimize counterparty risk. We conclude our study in Section 5.

2 ETF Structures and Counterparty Risk

2.1 Physical Model

Physical ETFs attempt to track their target indexes by holding all, or a representative sample, of the underlying securities that make up the index (see Figure 1, Panel A). For example, if you invest in an S&P 500 ETF, you own each of the 500 securities represented in the S&P 500 Index, or some subset of them. Almost all ETF issuers have the provision in their prospectus for loaning out their stock temporarily for revenue. For instance, iShares recently changed its maximum on-loan limit from 95% of the AUM to 50% (ETF.com, 2014). In practice, however, little is known about the actual extend of securities lending, as well as the identity of the short-sellers.[6]

< Insert Figure 1 >

Securities lending exposes ETF investors to counterparty risk (Amenc et al., 2012). In order to mitigate this risk, short sellers have to post collateral with the ETF issuer. On a given point in time, if we denote by I_t, the AUM of the ETF, $\beta_t \in [0, 1]$ the fraction of the securities that are lent, and C_t the value of the collateral, the collateral shortfall at time $t + 1$ can be written as:

$$\Delta_{t+1} = \beta_t I_{t+1} - C_{t+1} \tag{1}$$

A situation where $\Delta_{t+1} > 0$ can be problematic if the short sellers cannot return the borrowed securities in due time, i.e., if some of them default. In that case, the fund will not be able to meet redemption requests from all ETF investors.

2.2 Unfunded Swap Model

First introduced in Europe in the early 2000's, synthetic ETFs are an interesting variant of physical ETFs. The most commonly used structure for synthetic replications is the unfunded swap model. In this model, the ETF issuer enters into a total return swap with a counterpartny, which can either be an affiliated bank from the same banking group or another bank (see Figure 1, Panel B). The swap counterparty commits to deliver the return of the reference index and sells a substitute basket of securities to the ETF issuer. The second leg of the swap consists of the performance of the basket of securities paid by the issuer to the swap counterparty. An important feature of this model is that the ETF issuer becomes the legal owner of the assets and enjoys direct access to them. This means that if the swap counterparty defaults, the ETF issuer can immediately liquidate the assets.[7]

[6] Lending securities is a natural thing to do for passive funds as they they rarely rebalance their portfolio.

[7] The unfunded swap model is particularly attractive for tax reasons. For instance, it allows ETFs to be eligible to the French tax-friendly share savings plans, *Plan d'Epargne en Actions* (PEA). In theory, eligible mutual funds

The counterparty exposure of the issuer, or swap value, is measured as the difference between the ETF's AUM, denoted I_t, and the value of the substitute basket used as collateral, denoted C_t. The swap is marked to market at the end of each day and reset whenever the counterparty exposure exceeds a given threshold, $\theta \in [0, 1[$, expressed as a percentage of the NAV:

$$I_t - C_t > \theta \ I_t. \tag{2}$$

In the event of a reset, the swap counterparty delivers additional securities for a value of $I_t - C_t$ to top up the substitute basket. The European directive UCITS sets θ to 10%. In other words, the daily NAV of the collateral must at least amount 90% of the NAV. However, a comprehensive industry survey conducted by Morgingstar (2012a) indicates that most ETF issuers affirm using a θ coefficient of less than 10% and some even claim to maintain full collateralization, *i.e.*, $\theta = 0$. Thus, in the rest of this analysis, without loss of generality, we will set θ to zero.

2.3 Funded Swap Model

In the funded swap model, the ETF issuer transfers investors' cash to a swap counterparty in exchange for the index performance plus the principal at a future date (see Figure 1, Panel C). The swap counterparty pledges collateral assets in a segregated account with a third party custodian. The posted collateral basket is made of securities which come from the counterparty's inventory and meet certain conditions in terms of asset type, liquidity, and diversification. In practice, appropriate haircuts apply to the assets posted as collateral to account for the risk of value fluctuations and for imperfect correlation between the index and the collateral value. As a consequence, funded-swap based ETFs are expected to be overcollateralized, $C_t > I_t$.

The swap counterparty exposure is measured by the difference between the NAV and the collateral value, once properly adjusted for haircut. If we denote by h the haircut, it means that if:

$$I_t - C_t\,(1 - h) > 0 \tag{3}$$

must be made up of at least 75% of shares of companies headquartered in a European member state. As a result, an ETF based on an unfunded swap and that tracks an index of Asian or US equities is eligible as long as the substitute basket is made of enough European equities.

additional collateral corresponding to $I_t - C_t(1-h)$ is requested in order to maintain appropriate collateralization.

2.4 Counterparty Risk Metrics

In any ETF structure, the counterparty risk faced by the ETF issuer and ultimately by the investors, corresponds to the risk that the ETF not being fully collateralized precisely when the counterparty defaults. Indeed, facing a collateral shortfall is only not an issue when the counterparty is unable to meet further margin calls. We propose a global framework for all types of ETF structures. For any fund, a collateral shortfall corresponds to:

$$\beta_t I_t - C_t(1-h) > 0 \tag{4}$$

where $\beta_t \in [0,1]$ and $h > 0$ for physical ETFs, $\beta_t = 1$ and $h = 0$ for unfunded-swap based ETFs, and $h > 0$ for funded-swap based ETFs.[8] If condition (4) is satisfied, additional collateral or reinvestment in the substitute basket is required to reach $C_t(1-h) = \beta_t I_t$ (Morningstar, 2012a). Denote by Δ_{t+1} the collateral shortfall of a given fund at time $t+1$:

$$\Delta_{t+1} = \beta_t I_{t+1} - C_{t+1}(1-h) = \beta_t I_t(1+r_{I,t+1}) - C_t(1-h)(1+r_{C,t+1}) \tag{5}$$

where $r_{I,t+1}$ and $r_{C,t+1}$ denote the return of the NAV and the return of the collateral portfolio, respectively. Given the information available at time t, the collateral shortfall Δ_{t+1} is stochastic since the returns $r_{I,t+1}$ and $r_{C,t+1}$ are unknown.

Within this framework, we propose two metrics for counterparty risk. The first metrics we define is the probability for a fund of being undercollateralized conditionally on a default of the counterparty.

Definition 1 (Probability of collateral shortfall) *The probability of a collateral shortfall at time $t+1$ is defined as:*

$$P_{t+1} = \Pr(\Delta_{t+1} > 0 \mid \mathcal{F}_t, D_{t+1}) \tag{6}$$

[8] We assume that the swap is not reset if the fund is overcollateralized ($\beta_t I_t - C_t(1-h) < 0$) since this does not induce any counterparty risk.

where $\mathcal{F}_t$ denotes the set of information available at time t and D_{t+1} denotes the counterparty default at time $t+1$.

This probability measures the likelihood of a collateral shortfall at the horizon of the next trading day, conditionally on a default of the counterparty, if no adjustment is made on the composition and the amount of collateral used at time t.

A second important dimension of collateral risk is the magnitude of the collateral shortfall.

Definition 2 (Expected collateral shortfall) *The expected collateral shortfall at time $t+1$ is:*

$$S_{t+1} = \mathbb{E}_t \left(\Delta_{t+1} \mid \Delta_{t+1} > 0, D_{t+1} \right) \tag{7}$$

where $\mathbb{E}_t$ denotes the expectation conditional on $\mathcal{F}_t$.

This metric denotes the expected collateral shortfall a fund is expected to experience conditional on the default of the counterparty.

2.5 Estimation and the Torino Scale

We estimate the counterparty risk metrics using a simple nonparametric method. We define $\omega_t = (\omega_{1,t}, ..., \omega_{K,t})$, the vector of the weights associated with the K assets that comprise the collateral portfolio of a fund at time t, with $\sum_{k=1}^{K} \omega_{k,t} = 1$. Given equation (5), the collateral shortfall at time $t+1$ can be written as:

$$\Delta_{t+1} = \beta_t I_t \left(1 + r_{I,t+1}\right) - C_t \left(1 - h\right) \left(1 + \sum_{k=1}^{K} \omega_{k,t} r_{k,t+1}\right) \tag{8}$$

where $r_{k,t+1}$ is the daily return of the k-th collateral security at time $t+1$. Given the information set $\mathcal{F}_t$, the potential collateral shortfall at time $t+1$ only depends on $r_{I,t+1}$ and $r_{k,t+1}$ for $k = 1, ..., K$.

One way to estimate the shortfall probability and the expected collateral shortfall is to assume a given distribution for these returns and to derive closed-form expressions for p and S. Alternatively, we follow Berkowitz and O'Brien (2002) and consider a series of hypothetical collateral shortfalls:

$$\Delta_\tau = \beta_t I_t \left(1 + r_{I,\tau}\right) - C_t \left(1 - h\right) \left(1 + \sum_{k=1}^{K} \omega_{k,t} r_{k,\tau}\right) \tag{9}$$

where $r_{I,\tau}$ is the historical daily return of the NAV, for $\tau = 1, ..., t$ and $r_{k,\tau}$ is the daily return of the k-th collateral security at time τ. The hypothetical collateral shortfall Δ_τ measures the shortfall that would have arisen in the past with the current values of I_t, C_t, and ω_t and past returns on the NAV and on the collateral securities. However, the counterparty in our sample never actually defaulted in the past. As a result, we estimate the counterparty risk metrics using past observations from a high counterparty-risk regime, i.e., a period during which the counterparty experienced a sharp increase in its default probability. We define two nonparametric estimators for the probability and the expected collateral shortfall:

$$\widehat{P}_{t+1} = \frac{1}{\dim(S)} \sum_{\tau \in \overline{\Upsilon}} \mathbb{I}(\Delta_\tau > 0) \times \mathbb{I}\left(\tau \in \overline{\Upsilon}\right) \tag{10}$$

$$\widehat{S}_{t+1} = \frac{\sum_{\tau=1}^{t} \Delta_\tau \times \mathbb{I}(\Delta_\tau > 0) \times \mathbb{I}\left(\tau \in \overline{\Upsilon}\right)}{\sum_{\tau=1}^{t} \mathbb{I}(\Delta_\tau > 0) \times \mathbb{I}\left(\tau \in \overline{\Upsilon}\right)} \tag{11}$$

where $\mathbb{I}(.)$ denotes the indicator function and $\overline{\Upsilon}$ denotes a high-counterparty risk regime.[9]

Finally, we propose an original approach to graphically summarize the counterparty risks of ETFs in a two-dimensional graph which is called a Torino scale. The latter is a tool developed in the mid 1990's by MIT Earth, Atmospheric, and Planetary Sciences Professor Richard P. Binzel for categorizing asteroids. The scale aims to measure asteroid-specific collision probabilities with the earth and the energy generated by such collisions. Torino scales are useful whenever one needs to jointly display the probability of an event, the "impact", and the effects of this event, the "energy". In our context, the "impact" corresponds to the occurrence of a collateral shortfall in a fund and the "energy" to the magnitude of the collateral shortfall.

The funds that are located in the North-East corner of the Torino scale are the ones that are the most exposed to counterparty risk, as shown in Figure 2. The key advantage of this graphic representation is that it permits to easily compare the risk for different ETFs or to monitor the risk of a given fund through time. Hence, it provides a global view of the counterparty risk in the ETF market, which may be of interest for both regulators and investors.

[9] If Δ_τ is a stationary process, these estimators are consistent and asymptotically normally distributed (Chen, 2008).

< Insert Figure 2 >

In Figure 2, the counterparty risk of a fund increases with the area of the rectangle delimited by the probability and expected collateral shortfall. We call this area the Aggregate Counterparty Risk (ACR):

$$ACR_{t+1} = P_{t+1} \times S_{t+1}. \tag{12}$$

These measures of counterparty risks, P, S, and ACR, will be estimated with actual data for all sample ETFs in the following section.

3 Empirical Analysis

Our empirical analysis is based on both synthetic and physical ETFs. The former are issued by db-X Trackers, the fifth largest ETF issuer in the world and the latter are issued by iShares, the largest ETF issuer in the world (by AUM, as of December 2014).

Our dataset of synthetic ETFs combines proprietary data provided to us by db X-trackers with public data retrieved from the db X-trackers website (www.etf.db.com). Data on physical ETFs have been collected from the iShares website (www.ishares.com). In our analysis, we only consider the funds for which we have been able to retrieve from Datastream and Bloomberg at least one year of data for the ETF price and its index.

We see in Table 1 that the 164 synthetic ETFs have a combined AUM of \$37.927 billion, which corresponds to more than 10% (respectively 30%) of the total AUM of all (synthetic) ETFs in Europe (Blackrock, 2012; Vanguard, 2013). Most funds are based on a funded swap (112 funds vs. 52 funds based on an unfunded swap). However, both types of funds account for comparable AUM (\$20.1 billion vs. \$17.8 billion). It is also important to notice that a significant fraction of the synthetic funds (30 funds and 5.1% of AUM) are inverse funds that deliver the inverse performance of an index.

< Insert Table 1 >

In terms of asset exposure, the majority of the synthetic funds track a stock index as equity funds account for 74.5% of AUM. Our sample is representative of the European ETF industry as most ETFs are synthetics and the share of equity ETFs is around 70% (Vanguard, 2013). Besides equity, the other funds allow investors to be exposed to a variety of asset classes including Government bonds (11% of AUM), treasuries and commercial papers (6.6%), commodities (3.8%), hedge funds (2.2%), credit (0.7%), corporate bonds (0.6%), and currencies (0.3%). Furthermore, half of the sample funds track European indices (79 funds out of 164 and 58.9% of AUM) whereas the remaining funds replicate the returns of some World indices (22.3%), Asia-Pacific indices (9.4%), North-American indices (7.2%), or indices from the rest of the World (2.2%).

We see in the last column of Table 1 that our sample of physical ETFs is larger than the synthetic one: the combined asset under management of the 55 funds is $77.874 billion (to be completed).

3.1 A First Look at the Collateral Portfolios

Allegations were recently made about the overall poor quality of ETF collateral. For instance, the Financial Stability Board (2011, page 4) states: "*As there is no requirement for the collateral composition to match the assets of the tracked index, the synthetic ETF creation process may be driven by the possibility for the bank to raise funding against an illiquid portfolio [...] the collateral basket for a S&P 500 synthetic ETF could be less liquid equities or low or unrated corporate bonds in an unrelated market.*"

To formally test the validity of these allegations, we collect for each sample fund the composition and the value of its collateral portfolio with a weekly frequency between July 5, 2012 and November 29, 2012. The collateral data have been retrieved from the db X-Trackers and I-Shares websites but because the websites keep no historical data, we had to download the collateral data for each fund, every week over our sample period. Then for each security used as collateral, we obtain its historical daily prices from Datastream.[10]

[10] For bonds, we use the time series of the bond index return that best matches the attributes of the bonds: its type (sovereign vs. corporate), country, rating, and maturity.

In Table 2, we see that the aggregate size of all collateral portfolios is equal to \$40.9 billion for synthetic ETFs, which indicates that, on average, the funds included in our analysis are overcollateralized (AUM = \$37.9 billion). For a given fund, the value-weighted average level of collateralization is 108.4%. However, the level of overcollateralization is higher for funded-swap based ETFs (114.6%) and for inverse funds (115.4%). The result for funded swaps comes from the fact that haircuts are applied to the value of the pledged collateral. Differently, no haircut applies for unfunded swaps as, in this case, the asset managers purchase some securities, i.e., the substitute basket, from its swap counterparty.

< Insert Table 2 >

In total, there are 3,299 different securities that are used as collateral in the synthetic ETF sample, which leads to 81 collateral securities per fund on average. We notice that the number of securities is much higher for equity or commodity funds (around 100 securities per fund) than for fixed-income funds tracking Government or corporate bond indices, money market funds, or CDS indices (10 to 20 securities per fund). In our sample, most of the collateral portfolio is made of equities. Out of the 3,299 securities, 2,591 are equities. When measured in value, equities account for around 75% of the collateral vs. 20% for Government bonds and 5% for corporate bonds.

We also report significant time-variation in the composition of the collateral portfolios of ETFs. For a given fund, we define the turnover of the collateral portfolio as the ratio between the number of different securities, as defined by their ISIN, that enter (n^+) or exit (n^-) the collateral portfolio between t and $t+1$ divided by the total number of collateral securities (N) on both dates:

$$turnover_{t,t+1} = \frac{n^+_{t,t+1} + n^-_{t,t+1}}{N_t + N_{t+1}}. \tag{13}$$

On average in our sample, more than a third of the pledged collateral in a given fund changes from one week to the next (see Table 2). Changes through time are particularly strong for unfunded-swap based ETFs, as almost half of their collateral is replaced from one week to the next. Furthermore, the managers of equity and commodity ETFs rebalance their collateral to a much greater extent (43.9% and 72.7%, respectively) than those of fixed income ETFs (around 2%).

3.2 The Match between Index and Collateral

One of the most persistent criticisms addressed to ETFs is the fact that the collateral may not be positively correlated with the index tracked by the ETF. Indeed, when the correlation is negative, the hedge provided by the collateral is less efficient: if the index return is large and positive and the swap counterparty defaults, the value of the collateral shrinks and a collateral shortfall mechanically arises. To look at this issue empirically, we compare the index tracked by the ETF and the securities included in the collateral portfolio. In Panel A of Table 3, we notice that there is a good match between the two as 92.5% of equity ETFs are backed with equity and 96.5 of Government bonds ETFs are collateralized with Government bonds. Differently, the collateral of Corporate bonds ETFs is not made of corporate bonds but, instead, exclusively of Government bonds. In Panel B, we see that the match between exposure and collateral turns out to be lower for geographic exposures. While 66% of the collateral are made of European securities, we find that this percentage rises to 71.8% for ETFs tracking European indices.[11] The matching score between exposure and issuers drops to 24.1% for ETFs tracking Asia-Pacific indices and to 19.5% for ETFs tracking North-American indices.

< Insert Table 3 >

We present in Figure 4 the distribution of the correlation between the return of the ETF and the return of its collateral portfolio.[12] We clearly see that most correlations are positive (the

[11] To understand the predominant role played by European collateral, which we dub "collateral home bias", one needs to understand the origin of the pledged collateral. Indeed, these securities come from the books of the swap counterparty, typically a large financial institution. In our sample, the swap counterparty is Deutsche Bank and as a result, its books predominantly include securities issued by local firms held for investment purposes, market making, or other intermediation activities.

[12] When the returns of one or several securities included in a collateral portfolio are missing, we apply the following rule: (1) when the cumulative weight of the missing securities exceeds 5%, we do not compute the portfolio return on this particular date and (2) when the cumulative weights of the missing securities is equal or less than 5%, we compute the portfolio return using all available returns (with weights properly rescaled to sum to 100%). When computing historical correlations or betas between a collateral security and another security, we impose a minimum of 50 days during which both returns are available.

average correlation $\bar{\rho}$ is 0.305), which confirms the close connection that exists between the pledged collateral and the index tracked by the ETF. However, for synthetic ETFs, the distribution is bimodal with significant mass associated with negative correlations. Looking more closely at our sample of funds reveals that most negative correlations are associated with inverse ETFs ($\bar{\rho}_{INV} = -0.595$).[13] Differently, the distribution obtained for physical ETFs is much more tilted toward positive values, which is consistent with the fact that this sample contains no inverse funds.

We regress the correlation between the return of the ETF and its collateral portfolio on a series of firm-specific variables using a panel linear specification with individual effects.[14] We show in Table 4 that the level of collateralization of the fund, the number of securities in the collateral portfolios, as well as the fraction of equities and European securitiese in the collateral portfolio are positively and significantly associated with the ETF-collateral correlation. Differently, inverse funds and funds that track commodities, currencies, corporate bonds, and money market funds tend to have a lower correlation with their collateral. This latter result is due to the fact that these asset classes are typically more difficult to include in the collateral portfolio.

< Insert Figure 3 and Table 4 >

3.3 Equities and Bonds Used as Collateral

To get a better sense of the type of securities used as collateral, we conduct an in-depth analysis of all equities, and then of all bonds. Because they attracted most critics from the media and the regulators, we focus in this sub-section only on synthetic ETFs. We start with equities in Table 5 and show that most equities used as collateral are issued by large, European, non-financial firms. Furthermore, collateralized equities exhibit good liquidity on average, with an average bid-ask spread of 0.21% and an average daily trading volume that corresponds to 4.67% of the market capitalization. We also find that 92.5% of the equities correlate positively with the index tracked

[13] When computing the ETF-collateral correlations for inverse ETFs we systematically use the short index and not the regular index. For instance, for the db X-tracker on the S&P 500 SHORT index, we use the latter and not the S&P 500 index.

[14] In order to guarantee that the estimated correlation remains within the $[-1, 1]$ range, the dependent variable is defined as $\ln\left((1 + corr_{i,t}) / (1 - corr_{i,t})\right)$. We also estimated a binary specification (panel logit model with random effects) for the sign of the correlation and obtained qualitatively similar results.

by the ETF. Another reassuring finding is the fact that, on average, the pledged equities have a higher beta, or conditional beta, with respect to the ETF than with respect to the stock return of Deutsche Bank, which is the swap counterparty for all ETFs. The average beta is 60% higher when computed with the index than with the swap counterparty and its distribution has more mass on any value greater than 0.4. Using collateral securities that correlate strongly with the counterparty or even worse, are issued by the counterparty, would be inefficient as their value would go to zero in the case of a default of the counterparty.

< Insert Table 5 >

When we look at the bond part of the collateral portfolio in Table 6, we notice that bonds predominantly have European issuers (88.3%), which is again consistent with the existence of a home bias (see Panel A). The fraction of European bonds is higher for corporate bonds (96.6%) than for Government bonds (86.0%). Note that the lower fraction of European Sovereign bonds may be due to the fact that our sample period corresponds to the end of the Eurozone crisis of 2012. Turning to bond ratings in Panel B, we see that 65.5% of the bonds have a AAA rating, which is significantly lower than the 74.7% mentioned by Deutsche Bank (2012) at the end of the year 2011. This difference could be due to the fact that many downgrades occurred in 2012, including many large banks and several European sovereign issuers. In our sample, the fraction of bonds with at least a AA rating is 84.5% for Government bonds and 64.9% for corporate bonds. Speculative-grade ratings account for 0.4% of the bonds used as collateral and undefined ratings for 1.3%. Our results drastically contrast with those of Ramaswamy (2011) that were based on a single equity ETF. He reports that 8.7% of the bonds are rated AAA, 13.1% at least AA, and 38% are unrated (vs. 65.5%, 80.3%, and 1.3%, respectively, in our sample), which seems to indicate that the fund selected by Ramaswamy is not representative of the entire ETF industry.

< Insert Table 6 >

The maturity spectrum of the bonds within the collateral portfolios covers a wide range of maturities from less than a year to more than 10 years (see Panel C). Overall, 40.8% of the bonds used as collateral have a maturity of less than 3 years and 23.1% have a maturity of more than 10 years. Government bonds and North-American bonds are more on the long side whereas the maturity of corporate bonds remains almost exclusively below 5 years. Interestingly, we find that the duration of the collateral matches well with the duration of the fixed-income index tracked by the fund. Indeed, ETFs that track an index with a maturity below 3 years mainly have collateralized bonds with a maturity less than 3 years (71.1%). The match is even stronger for funds that track medium maturity indices (3-10 years), with 89.3% of the collateralized bonds within the 3-10 year maturity bucket, and for funds that track long term indices (>10 years), for which 84.9% of the collateralized bonds also have a maturity of more than 10 years.

3.4 Counterparty Risk Analysis for Synthetic ETFs

We estimate for each of the 164 synthetic ETFs its counterparty risk measures, ACR. As explained in Section 2, the latter is obtained by multiplying the probability of the fund being undercollateralized and its expected collateral shortfall. Both risk measures are estimated non-parametrically with a one-day horizon and using haircuts that are specific to the type of securities: equities, Government bonds, corporate bonds, and others.[15] As explained in Section 2.5, we estimate the these risk metrics from past observations that belong to a high swap counterparty-risk regime, namely a 2-month period around the bankruptcy of Lehman Brothers (September 1, 2008 - October 31, 2008). During this period, the default probability of the swap counterparty, Deutsche Bank, implied from CDS spread got multiplied by three compared to its pre-crisis level.

In Panel A of Figure 4, we display for all synthetic funds both the average shortfall probability and expected collateral shortfall, expressed as a percentage of the NAV. For each fund, the risk metrics are averaged across time. The main result in this figure is that counterparty risk expose varies a lot across funds. Overall, in a high counterparty-risk regime, the probability of experiencing a

[15] The time-series of the security-specific haircuts have been provided to us by db X-trackers.

collateral shortfall is high but the expected shortfall always remains below 10% of the NAV. The high shortfall probility is due the high market volatility during our estimation period, which affects both the indices tracked by the ETF and the value of the collateral, and to the use of conservative haircuts by db X-Trackers. In order to assess the role of haircuts, we re-estimate the counterparty risk exposures assuming no haircut and we present the new risk map in the Panel B of Figure 4.

< Insert Figure 4 >

We complement our analysis by running some multivariate regressions in which we regress the counterparty risk meassures, ACR, on the characteristics of the funds. We see in Table 7 that ETFs based on funded swaps and funds with high levels of collateralization and fractions of European collateralized securities exhibit on average a lower level of counterparty risk. Furthermore, inverse funds tend to expose their investors to more counterparty risk. This latter result is consistent with our previous findings regarding the negative relationship between the values of the collateral and of the tracked index for inverse funds.

< Insert Table 7 >

3.5 Benchmarking with Physical ETFs

We conduct a similar risk assesment for physical ETFs as they also expose their investors to counterparty risk through securities lending. It is indeed possible that the short sellers who borrow some securities from the ETF issuer fail to return the securities in due time. We contrast counterparty risk estimates for physical and synthetic ETFs in Figure 5. We see that the difference in risk exposure is striking: counterparty risk exposures are several orders of magniture higher for synthetic ETF investors.

< Insert Figure 5 >

We extend the multivariate regressions presented in Table 7 by considering jointly the synthetic and physical ETFs, while using the ETF type as an extra control variable. Consistent with those in Figure 5, the results shown in Table 8 indicate that synthetic ETFs display a higher level of counterparty risk exposure. We also note that our conclusions about the other main drivers of counterparty risk remain robust in this extended sample: counterparty risk exposures is lower for funded swaps, and funds with high levels of collateralization and fractions of European collateralized securities. Furthermore, the negative coefficient associated with the number of securities is now significant at the 5% level.

< Insert Table 8 >

3.6 Are Investors Compensated for Being Exposed to Counterparty Risk?

We have seen that synthetic ETFs' investors tend to be more exposed to counterparty risk than physical ETFs' investors. A natural question is whether the former investors are compensated for bearing this additional risk. We answer this question by considering two important dimensions of an ETF: its costs and its performance. In this section, we formally show that synthetic ETFs are as cheap, if not cheaper (i.e., lower management fees), than physical ETFs and that syntheric ETFs have a better performance (i.e., lower tracking error) than physical ETFs.[16] As a result, synthetic investors are compensated for bearing this additional risks by getting a superior performance for the same price.

We conduct this tests in two ways. We first consider the entire sample of synthetic and physical funds. In Panel A of Table 9, we find no clear difference between the fees charged by the different types of funds. The average fee for physical ETFs are 44 bps vs. 43 bps for synthetic ETFs. However, we find major differences in the tracking error of these funds.: the average tracking error is 96 bps for physical ETFs and 13 bps for synthetic ETFs. Another interesting result is the much higher tracking error for funds that track a dividend-paying index, which indicates that a major source of tracking error for funds is the way dividends are handled and passed through to fund investors.

[16] We define the tracking error as the annualized volatility of the difference between the daily returns of the ETF and of the index.

We complement these unconditional results by running multivariate regressions for, in turn, fees and tracking errors. We find that the coefficient associated with the synthetic ETF's dummy variable is negative and significant for both the fees and the tracking errors. Interestingly, we uncover that inverse funds tend to charge higher fees and funds that distribute dividends exhibit larger tracking errors. Differently, larger funds have lower tracking errors.

Second, we only consider funds that track exactly the same index (i.e., same index provider, same asset class, same treatment of the dividends). We have been able to identify 13 pairs of synthetic ETF and perfectly equivalent synthetic ETF. We then compare their respective fees and tracking errors in the Panel B of Table 9. Consistent with the previous results based on the whole sample, we find that synthetic ETF outperform their physical counterparts in terms of tracking errors yet charging similar fees.

< Insert Table 9 >

4 Optimal Collateral Portfolio

4.1 Definition

We show in this section how to construct an optimal collateral portfolio that aims to protect the ETF issuer, as well as its investors, against counterparty risk. The collateral portfolio shall be mutually agreed upon by both the collateral provider (the counterparty) and the collateral receiver (the ETF issuer). The process that leads to the optimal collateral portfolio can be divided into three steps.

First, the counterparty and the ETF issuer have to determine a set of eligible securities. In practice, the securities pledged as collateral directly come from the inventory of the counterparty, which includes securities held for investment purposes, market making, underwriting, or other intermediation activities. As a result, there is no need for the collateral provider to purchase any new securities to meet collateral requirements. In general, when choosing the securities to be pledged, the collateral provider primarily transfers the ones that minimize the opportunity cost of holding collateral. Such securities include those with relatively low fees on the securities lending market; those with relatively low collateral value in the repo market (Bartolini et al., 2011); those that are not eligible as collateral for central-bank credit operations; and securities for which the demand is low on the secondary market (Brandt and Kavajecz, 2004). On the receiver side, only collateral with sufficient tradability will be admitted. For instance, securities that are not listed, issued by small firms, with wide bid-ask spreads, or low market depth may not be accepted as collateral. Similarly, the collateral receiver may prevent unrated bonds from being included in the collateral portfolio. This interaction between the provider and the receiver of collateral leads to the determination of a set Θ of K eligible securities that need to be allocated.

Second, both parties have to determine the level of collateralization. At the end of day t, the value of the collateral portfolio C_t is determined by the NAV of the fund:[17]

$$C_t = \frac{\beta_t I_t}{1-\theta}. \tag{14}$$

Third, given the eligible securities and the level of collateralization on day t, the composition of the collateral portfolio is set to minimize counterparty risk on day $t+1$, conditionally on the default of the counterparty, which is defined as the risk of being undercollateralized given the potential changes in the fund's NAV and in the value of the collateral securities. The collateral shortfall in the next day is defined as:

$$\Delta_{t+1} = \beta_t \left(1 + r_{i,t+1}\right) - (1-h)\left(1 + \sum_{k=1}^{K} \omega_{k,t} r_{k,t+1}\right) \tag{15}$$

where $\omega_{k,t}$ denotes the weight of the k^{th} security in the collateral portfolio, with $\omega_{k,t} \geq 0$ and $\sum_{k=1}^{K} \omega_{k,t} = 1$, $r_{k,t}$ the corresponding return, and $r_{i,t}$ the return of the fund's NAV. Note that if $\Delta_{t+1} > 0$, additional collateral is required at the end of day $t+1$. The collateral shortfall then depends on the return of the NAV and the returns of the collateral securities, which are unknown given the information available at the end of day t.

[17]Alternatively, we may consider the case in which the fund is overcollateralized, $C_t = \alpha I_t \left(1-\theta\right) / \left(1-h\right)$ with $\alpha > 1$.

We define the optimal collateral portfolio as the portfolio that minimizes both the probability of having a collateral shortfall and the expected collateral shortfall. Among all portfolios that can be generated by combining the securities included in Θ, the optimal portfolio is the closest one from the origin of the Torino scale (see Figure 3). In order to minimize the expected collateral shortfall (S_{t+1}), a solution consists in minimizing the volatility of the collateral shortfall (Δ_{t+1}). Indeed, the expected collateral shortfall corresponds to the truncated expectation of Δ_{t+1}, where the truncation parameter is set to 0. Then minimizing the variance of Δ_{t+1} necessarily reduces the average of the positive values of Δ_{t+1}, as can be seen in Figure 6.

However, minimizing the variance of the shortfall is not sufficient to minimize the probability of being undercollateralized (P_{t+1}). Indeed, when the variance of Δ_{t+1} tends to 0, the collateral shortfall converges to a constant $\mathbb{E}(\Delta_{t+1})$. If $\mathbb{E}(\Delta_{t+1}) > 0$, like in the bottom part of Figure 6, the fund will necessarily be undercollateralized on day $t+1$. As a result, in order to jointly minimize P_{t+1} and S_{t+1}, $\mathbb{E}(\Delta_{t+1})$ must be constrained to be negative. The economic interpretation of this constraint is that the expected return of the collateral portfolio must be larger than the expected return of the NAV.

< Insert Figure 6 >

This argument can be formalized under the normality assumption. In this case, P_{t+1} and S_{t+1} can be defined as functions of $\mathbb{E}\left(\Delta_{t+1}\right) = \mu_\Delta$ and $\mathbb{V}\left(\Delta_{t+1}\right) = \sigma_\Delta^2$:

$$P_{t+1} = \Phi\left(\frac{\mu_\Delta}{\sigma_\Delta}\right) \tag{16}$$

$$S_{t+1} = \mu_\Delta + \sigma_\Delta \lambda\left(\frac{\mu_\Delta}{\sigma_\Delta}\right) \tag{17}$$

where $\lambda(z) = \phi(z)/\Phi(z)$ denotes the inverse Mills ratio, $\Phi(z)$ denotes the cdf of the standard normal distribution, and $\phi(z)$ the pdf of the standard normal distribution. We show in Appendix A that, when $\mu_\Delta \leq 0$, the two collateral risk metrics are increasing functions of the variance of the collateral shortfall, σ_Δ^2.

4.2 The Optimization Program

We have shown that the composition of the optimal collateral portfolio can be obtained by minimizing the variance of the collateral shortfall under the constraint that the fund will be sufficiently collateralized on average. In turn, the weights of the optimal collateral portfolio, denoted $\boldsymbol{\omega} = (\omega_{1,t}, ..., \omega_{K,t})^\intercal$, with $k \in \Theta$, for $k = 1, ..., K$, are the solutions of the following program:

$$\begin{aligned} &\min_{\boldsymbol{\omega}} \mathbb{V}(\Delta_{t+1}) \\ &\text{subject to} \begin{cases} \mathbb{E}(\Delta_{t+1}) \leq 0 \\ \boldsymbol{\omega} \geq 0 \\ \boldsymbol{e}^\intercal \boldsymbol{\omega} = 1 \end{cases} \end{aligned} \tag{18}$$

where $\boldsymbol{e}$ is the $K \times 1$ unit vector. Note that we prevent the weights from being negative since short positions in collateral would be nonsensical.

Both $\mathbb{E}(\Delta_{t+1})$ and $\mathbb{V}(\Delta_{t+1})$ can be expressed as functions of the moments of the returns of the collateral securities and of the NAV.[18] If we denote by $\boldsymbol{r}_t = (r_{1,t}, ..., r_{K,t})^\intercal$ the $K \times 1$ vector of returns of the collateral securities, the moments are:

$$\mathbb{E}(\boldsymbol{r}_{t+1}) = \frac{1}{1-h}\boldsymbol{\mu} \tag{19}$$

$$\mathbb{E}\left((\boldsymbol{r}_{t+1} - \mathbb{E}(\boldsymbol{r}_{t+1}))(\boldsymbol{r}_{t+1} - \mathbb{E}(\boldsymbol{r}_{t+1}))^\intercal\right) = \frac{1}{(1-h)^2}\boldsymbol{\Sigma} \tag{20}$$

Define $\boldsymbol{z}_t = (r_{i,t}\beta_t \ , \ \boldsymbol{r}_t^\intercal (1-h))^\intercal$ the $(K+1) \times 1$ vector of transformed returns for the NAV and the collateral securities:

$$\mathbb{E}(z_{t+1}) = \left(\begin{array}{c} \underset{(1,1)}{\mu_i} \\ \hline \underset{(K,1)}{\boldsymbol{\mu}} \end{array} \right) \tag{21}$$

$$\mathbb{E}\left((\boldsymbol{z}_{t+1} - \mathbb{E}(\boldsymbol{s}_{t+1}))(\boldsymbol{z}_{t+1} - \mathbb{E}(\boldsymbol{s}_{t+1}))^\intercal\right) = \left(\begin{array}{c|c} \underset{(1,1)}{\sigma_i^2} & \underset{(1,K)}{\boldsymbol{\Sigma}_i^\intercal} \\ \hline \underset{(K,1)}{\boldsymbol{\Sigma}_i} & \underset{(K,K)}{\boldsymbol{\Sigma}} \end{array} \right). \tag{22}$$

where μ_i and σ_i^2 respectively denote the mean and variance of $\beta_t r_{i,t}$. The program in Equation

[18] These moments can be unconditional or conditional on the information available at time t. In the latter case, the moments are time-varying and must be indexed by time.

(18) becomes:

$$\begin{aligned} \min_{\boldsymbol{\omega}} \quad & \frac{1}{2}\boldsymbol{\omega}^\intercal\boldsymbol{\Sigma}\boldsymbol{\omega} + \frac{1}{2}\sigma_i^2 - \boldsymbol{\omega}^\intercal\boldsymbol{\Sigma}_i \\ \text{subject to} \quad & \begin{cases} \mu_i - \boldsymbol{\omega}^\intercal\boldsymbol{\mu} \leq 0 \\ \boldsymbol{\omega} \geq 0 \\ \boldsymbol{e}^\intercal\boldsymbol{\omega} = 1 \end{cases} \end{aligned} \tag{23}$$

If we neglect the positivity constraint, the Lagrange function $f(\boldsymbol{\omega},\lambda_1,\lambda_2)$ is:

$$f(\boldsymbol{\omega},\lambda_1,\lambda_2) = \frac{1}{2}\boldsymbol{\omega}^\intercal\boldsymbol{\Sigma}\boldsymbol{\omega} + \frac{1}{2}\sigma_i^2 - \boldsymbol{\omega}^\intercal\boldsymbol{\Sigma}_i - \lambda_1(\boldsymbol{e}^\intercal\boldsymbol{\omega} - 1) - \lambda_2(\boldsymbol{\omega}^\intercal\boldsymbol{\mu} - \mu_i) \tag{24}$$

with $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$. To solve this problem, we need to distinguish two cases: one in which the $\mu_i - \boldsymbol{\omega}^\intercal\boldsymbol{\mu} \leq 0$ constraint is not binding and another one when it is. First, if the expected return of the collateral portfolio is larger than the expected return of the NAV ($\mu_i - \boldsymbol{\omega}^\intercal\boldsymbol{\mu} < 0$ and $\lambda_2 = 0$), the optimal weights of the collateral portfolio can be expressed as a function of the weights of the Global Minimum Variance Portfolio (GMVP).

Proposition 1 *If $\mu_i - \boldsymbol{\omega}^\intercal\boldsymbol{\mu} < 0$, the optimal weights of the collateral portfolio are:*

$$\boldsymbol{\omega}^* = \boldsymbol{\Sigma}^{-1}\boldsymbol{\Sigma}_i + \left(1 - \boldsymbol{e}^\intercal\boldsymbol{\Sigma}^{-1}\boldsymbol{\Sigma}_i\right)^{-1}\widetilde{\boldsymbol{\omega}}_{GMVP} \tag{25}$$

where $\widetilde{\boldsymbol{\omega}}_{GMVP}$ corresponds to the weights of the GMVP:

$$\widetilde{\boldsymbol{\omega}}_{GMVP} = \frac{\boldsymbol{\Sigma}^{-1}\boldsymbol{e}}{\boldsymbol{e}^\intercal\boldsymbol{\Sigma}^{-1}\boldsymbol{e}}. \tag{26}$$

The proof of Proposition 1 is provided in Appendix B. The optimal weights depend (i) on the variance covariance matrix, $\boldsymbol{\Sigma}$, of the returns of the collateral securities and (ii) on the vector of covariances, $\boldsymbol{\Sigma}_i$, between the returns of these securities and the return of the NAV. Notice that if the returns of the collateral securities and the NAV are independent ($\boldsymbol{\Sigma}_i = \boldsymbol{0}_{K\times 1}$), the optimal weights simply correspond to those obtained by minimizing the variance of the collateral portfolio, that is to $\widetilde{\boldsymbol{\omega}}_{GMVP}$.

Second, when the constraint on the expected level of collateralization is binding ($\mu_i - \boldsymbol{\omega}^\intercal\boldsymbol{\mu} = 0$ and $\lambda_2 > 0$), the optimal weights can be expressed as a function of the weights of the Markowitz's mean variance portfolio. We define three scalar terms a, b, and c such that:

$$a = \boldsymbol{e}^\intercal \boldsymbol{\Sigma} \boldsymbol{e} \qquad b = \boldsymbol{e}^\intercal \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu} \qquad c = \boldsymbol{\mu}^\intercal \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu} \tag{27}$$

Proposition 2 *If $\boldsymbol{\omega}^\intercal \boldsymbol{\mu} - \mu_i = 0$, the optimal weights of the collateral portfolio are:*

$$\begin{aligned} \boldsymbol{\omega}^* &= \widetilde{\boldsymbol{\omega}}_{MV} + \boldsymbol{\Sigma}^{-1}\boldsymbol{\Sigma}_i + \left(\frac{c\boldsymbol{e}^\intercal - b\boldsymbol{\mu}^\intercal}{b^2 - ac}\right) \boldsymbol{\Sigma}^{-1}\boldsymbol{\Sigma}_i\boldsymbol{\Sigma}^{-1}\boldsymbol{e} \\ &+ \left(\frac{a\boldsymbol{\mu}^\intercal - b\boldsymbol{e}^\intercal}{b^2 - ac}\right) \boldsymbol{\Sigma}^{-1}\boldsymbol{\Sigma}_i\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} \end{aligned} \tag{28}$$

where $\widetilde{\boldsymbol{\omega}}_{MV}$ corresponds to the weights of the Markowitz's mean variance portfolio.

The proof of Proposition 2 is provided in Appendix C. In this case, the optimal weights depend on $\boldsymbol{\Sigma}$ and $\boldsymbol{\Sigma}_i$ as in the previous case, but they also depend on the vector of expected returns of the collateral securities $\boldsymbol{\mu}$ and on the expected return of the NAV μ_i. The latter can be viewed as a target for the expected return of the collateral portfolio, $\boldsymbol{\omega}^\intercal \boldsymbol{\mu}$. Notice that if the collateral securities and the NAV are independent, $\boldsymbol{\Sigma}_i = \boldsymbol{0}_{K \times 1}$, then the optimal weights simply correspond to the weights $\widetilde{\boldsymbol{\omega}}_{MV}$ of the mean variance portfolio with a target mean μ_i.

In practice, more constraints can be taken into account in the program. For instance, one can impose a positivity constraint on all weights. Alternatively, it is possible to prevent any issuer to account for more than a certain fraction of the collateral portfolio as it is the case under UCITS regulation. The constrained solution of the program may be obtained by solving the unconstrained problem as in Equation (18) but with a covariance matrix $\widetilde{\Sigma}$ that depends on the Lagrange coefficients (Jagannathan and Ma, 2003). Although it is not possible to obtain an analytic solution in the latter case, we can solve the optimization problem using a quadratic programming algorithm.

Finally, the results presented in this section can be extended by taking into account the correlations between the returns of the collateral assets and the constituents of the tracked index, rather than the index defined as a whole. Indeed, if we neglect the tracking error, the return of the NAV can be expressed as $r_{i,t} = \sum_{j=1}^{J} \delta_{j,t} r_{j,t}^i$ where $\delta_{j,t}$ denotes the weight of the j^{th} asset in the index, with

$\delta_{j,t} \geq 0$ and $r^i_{j,t}$ is the return of the j^{th} index constituent. Under these assumptions, it is possible to define the weights of the optimal portfolio as a function of the covariances between $r_{k,t+1}$ and $r^i_{j,t+1}$ for $k \neq j$ by using exactly the same methodology as the one presented in Section 4.2.

4.3 Illustration

As an illustration, we consider a simple example with two collateral securities ($K = 2$). Define the vector of transformed returns:

$$\boldsymbol{z}_t = \left(r_{i,t}\beta_t, r_{1,t}\left(1-h\right), r_{2,t}\left(1-h\right)\right)^{\intercal} \tag{29}$$

where $r_{i,t}$ is the return of the fund's NAV, $r_{1,t}$ and $r_{2,t}$ are the returns of the two securities. We assume that the vector $\boldsymbol{z}_t$ has a normal distribution with:

$$\mathbb{E}\left(\boldsymbol{z}_{t+1}\right) = \left(\begin{array}{c} 0.1 \\ \hline 0.5 \\ 0.5 \end{array}\right) \tag{30}$$

$$\mathbb{E}\left(\left(\boldsymbol{z}_{t+1} - \mathbb{E}\left(\boldsymbol{z}_{t+1}\right)\right)\left(\boldsymbol{z}_{t+1} - \mathbb{E}\left(\boldsymbol{z}_{t+1}\right)\right)^{\intercal}\right) = \left(\begin{array}{c|cc} 2 & 1 & 0.2 \\ \hline 1 & 2 & 0.2 \\ 0.2 & 0.2 & 1 \end{array}\right). \tag{31}$$

In this example, the two collateral securities are positively correlated, but the first security is more volatile than the second one. Furthermore, both securities are positively correlated with the NAV but the correlation with the first security is larger (0.5) than with the second one (0.1414). In this case, increasing the weight of the first security has two opposite effects on collateral risk: (1) since the first security is more risky, it increases the volatility of the shortfall but (2) since the first security is more strongly correlated with the NAV, it decreases the volatility of the shortfall.

Consider a collateral portfolio with a vector of weights $\boldsymbol{\omega} = (\omega_1, 1-\omega_1)^{\intercal}$ where $\omega_1 \geq 0$ corresponds to the weight of the first security. The Torino scale in Figure 7 displays the values of the probabilities of facing a collateral shortfall and the corresponding expected collateral shortfalls for all portfolios ranging from $\omega_1 = 0$ to $\omega_1 = 1$. We see that the optimal portfolio $\boldsymbol{\omega}^* = (0.6154, 0.3846)^{\intercal}$, given by Equation (25), corresponds to the lowest values for the two collateral risk metrics.

$<$ **Insert Figure 7** $>$

4.4 Hybrid ETF: Both Synthetic and Physical

One special case of the above optimization program is worthwhile mentioning. When the set of eligible securities for the collateral portfolio includes all the index constituents, the optimal collateral portfolio mirrors the index. In this case, the return of the NAV is $r_{i,t} = \sum_{j=1}^{J} \delta_{j,t} r_{j,t}$ where $\delta_{j,t}$ denotes the weight of the j^{th} asset in the index. In this particular case, an obvious way to set the collateral shortfall to zero is to maintaining a perfect match between the collateral portfolio and the index, $\omega^*_{k,t} = \delta_{k,t}$ for $k = 1, ..., J$, and $\omega^*_{k,t} = 0$ for $k > J$ (see Appendix D). This situation corresponds to a hybrid ETF combining some of the features of both synthetic and physical ETFs. While the fund is based on a swap, it benefits from a physical replication of the index.

5 Conclusion

How safe is the backup parachute of ETFs? To answer this question, we measure the collateral risk of ETFs using a $40.9 billion collateral portfolios. Overall, our results do not support the allegations made by the Financial Stability Board and other international agencies about the poor quality of the collateral used to produce ETFs and about the disconnect between the index tracked and the collateral. Funds in our sample tend to be overcollateralized and the collateral is mainly made of European securities and, for the most part, equities issued by large firms or highly-rated bonds. Furthermore, the collateral of equity (respectively bond) funds are mainly made of equities (respectively bonds) and the duration of the collateral matches well with the one of the bond index tracked by the ETF. We also provide evidence that collateral portfolio are actively managed with more than one third of the collateral of a given fund changing from one week to the next.

Our results concur with Louis Brandeis' saying that "*sunlight is said to be the best of disinfectants*". Indeed, scrutiny by the media, regulators, and investors following the criticisms of the ETF industry by international agencies lead to improved disclosure by ETF providers about their replication technology and collateral holdings (Morningstar, 2012). We show that increased transparency pressured industry participants to improve standards and practice on collateral management.

We find some heterogeneity in the collateral risk exposure of funds. We show that exposure to collateral risk is higher for funds that track commodities or currencies and for inverse ETFs which deliver the inverse performance of the underlying security. We find that the counterparty risk exposure is higher for synthetic ETFs but that investors are compensated for bearing this risk. Finally, we theoretically show how to construct an optimal collateral portfolio for an ETF. In order to facilitate collateral management, we provide closed-form solutions for minimum collateral-risk portfolios.

Future research could generalize our framework to study other sources of collateral risk. One may for instance relax the assumption that the pledged securities are not lent out and study this additional layer of collateral risk due to rehypothecation.

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Appendix

Appendix A: Proof of $\frac{\partial p}{\partial \sigma_\Delta} > 0$ and $\frac{\partial S}{\partial \sigma_\Delta} > 0$

Proof. The sensitivity of the probability of collateral shortfall to the variance of the collateral shortfall is:

$$\frac{\partial p}{\partial \sigma_\Delta} = \frac{\partial}{\partial \sigma_\Delta} \Phi\left(\frac{\mu_\Delta}{\sigma_\Delta}\right) = -\frac{\mu_\Delta}{\sigma_\Delta^2} \phi\left(\frac{\mu_\Delta}{\sigma_\Delta}\right) \tag{A1}$$

Since the pdf is always positive, the probability of collateral shortfall is an increasing function of σ_Δ as soon as the ETF is sufficiently collateralized on average, $\mu_\Delta < 0$. ■

Proof. A similar argument can be raised for the expected shortfall:

$$\begin{aligned} \frac{\partial S}{\partial \sigma_\Delta} &= \frac{\partial}{\partial \sigma_\Delta} \left(\mu_\Delta + \sigma_\Delta \lambda\left(\frac{\mu_\Delta}{\Delta_\Delta}\right)\right) \\ &= \lambda(u) - \sigma_\Delta \lambda(u)(u + \lambda(u)) \frac{\partial u}{\partial \sigma_\Delta} \\ &= \lambda(u) + \sigma_\Delta \frac{u}{\sigma_\Delta^2} \lambda(u)(u + \lambda(u)) \\ &= \lambda(u)\left(1 + u^2 + u\lambda(u)\right) \end{aligned} \tag{A2}$$

with $u = \mu_\Delta / \sigma_\Delta$. Since the inverse Mills ratio is always positive, the sign of $\partial S / \partial \sigma_\Delta$ corresponds to that of $1 + u^2 + u\lambda(u)$. So, the expected shortfall is an increasing function of σ_Δ as soon as:

$$1 + u^2 + u\frac{\phi(u)}{\Phi(u)} > 0 \tag{A3}$$

or equivalently when:

$$\Phi(u) + u^2 \Phi(u) + u\phi(u) > 0 \tag{A4}$$

Since that, for a standard normal distribution, we have $\phi(u) = -\Phi(u)\, u$, this condition becomes:

$$\Phi(u) > 0 \tag{A5}$$

Whatever the value of μ_Δ and σ_Δ, this condition is always satisfied. As a consequence, the expected shortfall is always an increasing function of the variance of the collateral portfolio σ_Δ^2, whatever the value of its expectation μ_Δ. ■

Appendix B: Proof of Proposition 1

Proof. The Lagrange function $f(\boldsymbol{\omega}, \lambda_1, \lambda_2)$ is defined as to be:

$$f(\boldsymbol{\omega}, \lambda_1, \lambda_2) = \frac{1}{2} \boldsymbol{\omega}^\intercal \boldsymbol{\Sigma} \boldsymbol{\omega} + \frac{1}{2} \sigma_i^2 - \boldsymbol{\omega}^\intercal \boldsymbol{\Sigma}_i - \lambda_1 \left(\boldsymbol{e}^\intercal \boldsymbol{\omega} - 1\right) - \lambda_2 \left(\boldsymbol{\omega}^\intercal \boldsymbol{\mu} - \mu_i\right) \tag{B1}$$

with $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$. The corresponding Kuhn-Tucker conditions are:

$$\boldsymbol{\Sigma}\boldsymbol{\omega} - \boldsymbol{\Sigma}_i - \lambda_1 \boldsymbol{e} - \lambda_2 \boldsymbol{\mu} = 0 \tag{B2}$$

$$\boldsymbol{e}^\intercal \boldsymbol{\omega} - 1 = 0 \tag{B3}$$

$$\min\left(\boldsymbol{\omega}^\intercal \boldsymbol{\mu} - \mu_i, \lambda_2\right) = 0 \tag{B4}$$

If $\boldsymbol{\omega}^\intercal \boldsymbol{\mu} - \mu_i > 0$ and $\lambda_2 = 0$, the Kuhn-Tucker conditions become:

$$\begin{aligned} \boldsymbol{\Sigma}\boldsymbol{\omega} - \boldsymbol{\Sigma}_i - \lambda_1 \boldsymbol{e} &= 0 \quad \text{(B5)} \\ \boldsymbol{e}^\intercal \boldsymbol{\omega} - 1 &= 0 \quad \text{(B6)} \end{aligned}$$

From Equation (32), we have:

$$\boldsymbol{\omega} = \boldsymbol{\Sigma}^{-1}\boldsymbol{\Sigma}_i + \lambda_1 \boldsymbol{\Sigma}^{-1}\boldsymbol{e} \quad \text{(B7)}$$

Next, multiply both sides by $\boldsymbol{e}^\intercal$ and use second equation to solve for λ_1 :

$$\begin{aligned} \boldsymbol{e}^\intercal \boldsymbol{\omega} &= \boldsymbol{e}^\intercal \boldsymbol{\Sigma}^{-1}\boldsymbol{\Sigma}_i + \lambda_1 \boldsymbol{e}^\intercal \boldsymbol{\Sigma}^{-1}\boldsymbol{e} = 1 \quad \text{(B8)} \\ \lambda_1 &= \frac{1 - \boldsymbol{e}^\intercal \boldsymbol{\Sigma}^{-1}\boldsymbol{\Sigma}_i}{\boldsymbol{e}^\intercal \boldsymbol{\Sigma}^{-1}\boldsymbol{e}} \quad \text{(B9)} \end{aligned}$$

and the optimal weights are equal to:

$$\boldsymbol{\omega}^* = \boldsymbol{\Sigma}^{-1}\boldsymbol{\Sigma}_i + \left(1 - \boldsymbol{e}^\intercal \boldsymbol{\Sigma}^{-1}\boldsymbol{\Sigma}_i\right) \frac{\boldsymbol{\Sigma}^{-1}\boldsymbol{e}}{\boldsymbol{e}^\intercal \boldsymbol{\Sigma}^{-1}\boldsymbol{e}} \quad \text{(B10)}$$

These weights can be expressed as a linear function of the weights of the Global Minimum Variance Portfolio (GMVP). The GMVP corresponds to the solution of the following optimization problem:

$$\begin{aligned} \min_{\widetilde{\boldsymbol{\omega}}} \quad & \frac{1}{2}\widetilde{\boldsymbol{\omega}}^\intercal \boldsymbol{\Sigma} \widetilde{\boldsymbol{\omega}} \quad \text{(B11)} \\ \text{subject to} \quad & \boldsymbol{e}^\intercal \widetilde{\boldsymbol{\omega}} = 1 \quad \text{(B12)} \end{aligned}$$

The corresponding optimal solution is:

$$\widetilde{\boldsymbol{\omega}}_{GMVP} = \frac{\boldsymbol{\Sigma}^{-1}\boldsymbol{e}}{\boldsymbol{e}^\intercal \boldsymbol{\Sigma}^{-1}\boldsymbol{e}} \quad \text{(B13)}$$

As a consequence, we have:

$$\boldsymbol{\omega}^* = \boldsymbol{\Sigma}^{-1}\boldsymbol{\Sigma}_i + \left(1 - \boldsymbol{e}^\intercal \boldsymbol{\Sigma}^{-1}\boldsymbol{\Sigma}_i\right)^{-1} \widetilde{\boldsymbol{\omega}}_{GMVP} \quad \text{(B14)}$$

■

Appendix C: Proof of Proposition 2

Proof. If $\boldsymbol{\omega}^\intercal \boldsymbol{\mu} - \mu_i = 0$ and $\lambda_2 > 0$, the Kuhn-Tucker conditions become:

$$\begin{aligned} \boldsymbol{\Sigma}\boldsymbol{\omega} - \boldsymbol{\Sigma}_i - \lambda_1 \boldsymbol{e} - \lambda_2 \boldsymbol{\mu} &= 0 \quad \text{(C1)} \\ \boldsymbol{e}^\intercal \boldsymbol{\omega} - 1 &= 0 \quad \text{(C2)} \\ \boldsymbol{\omega}^\intercal \boldsymbol{\mu} - \mu_i &= 0 \quad \text{(C3)} \end{aligned}$$

From the first equation, we have:

$$\boldsymbol{\omega} = \boldsymbol{\Sigma}^{-1}\boldsymbol{\Sigma}_i + \lambda_1 \boldsymbol{\Sigma}^{-1}\boldsymbol{e} + \lambda_2 \boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} \quad \text{(C4)}$$

Given this expression for $\boldsymbol{\omega}$, the two constraints can be rewritten as:

$$\begin{aligned} 1 - \boldsymbol{e}^\intercal \boldsymbol{\Sigma}^{-1}\boldsymbol{\Sigma}_i &= \lambda_1 \boldsymbol{e}^\intercal \boldsymbol{\Sigma}^{-1}\boldsymbol{e} + \lambda_2 \boldsymbol{e}^\intercal \boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} \quad \text{(C5)} \\ \mu_i - \boldsymbol{\mu}^\intercal \boldsymbol{\Sigma}^{-1}\boldsymbol{\Sigma}_i &= \lambda_1 \boldsymbol{\mu}^\intercal \boldsymbol{\Sigma}^{-1}\boldsymbol{e} + \lambda_2 \boldsymbol{\mu}^\intercal \boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} \quad \text{(C6)} \end{aligned}$$

Define three three scalar terms a, b, and c such that:

$$a = \boldsymbol{e}^\intercal \boldsymbol{\Sigma} \boldsymbol{e} \qquad b = \boldsymbol{e}^\intercal \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu} \qquad c = \boldsymbol{\mu}^\intercal \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu} \tag{C7}$$

The constraints can be expressed as:

$$1 - \boldsymbol{e}^\intercal \boldsymbol{\Sigma}^{-1} \boldsymbol{\Sigma}_i = \lambda_1 a + \lambda_2 b \tag{C8}$$

$$\mu_i - \boldsymbol{\mu}^\intercal \boldsymbol{\Sigma}^{-1} \boldsymbol{\Sigma}_i = \lambda_1 b + \lambda_2 c \tag{C9}$$

Solving for λ_1 and λ_2, we have:

$$\lambda_1^* = \frac{b\left(\mu_i - \boldsymbol{\mu}^\intercal \boldsymbol{\Sigma}^{-1} \boldsymbol{\Sigma}_i\right) - c\left(1 - \boldsymbol{e}^\intercal \boldsymbol{\Sigma}^{-1} \boldsymbol{\Sigma}_i\right)}{b^2 - ac} \tag{C10}$$

$$\lambda_2^* = \frac{b\left(1 - \boldsymbol{e}^\intercal \boldsymbol{\Sigma}^{-1} \boldsymbol{\Sigma}_i\right) - a\left(\mu_i - \boldsymbol{\mu}^\intercal \boldsymbol{\Sigma}^{-1} \boldsymbol{\Sigma}_i\right)}{b^2 - ac} \tag{C11}$$

By substituting λ_1^* and λ_2^* in the expression $\boldsymbol{\omega}$, we have:

$$\begin{aligned}\boldsymbol{\omega}^* = {} & \boldsymbol{\Sigma}^{-1} \boldsymbol{\Sigma}_i + \left(\frac{b\left(\mu_i - \boldsymbol{\mu}^\intercal \boldsymbol{\Sigma}^{-1} \boldsymbol{\Sigma}_i\right) - c\left(1 - \boldsymbol{e}^\intercal \boldsymbol{\Sigma}^{-1} \boldsymbol{\Sigma}_i\right)}{b^2 - ac}\right) \boldsymbol{\Sigma}^{-1} \boldsymbol{e} \\ & + \left(\frac{b\left(1 - \boldsymbol{e}^\intercal \boldsymbol{\Sigma}^{-1} \boldsymbol{\Sigma}_i\right) - a\left(\mu_i - \boldsymbol{\mu}^\intercal \boldsymbol{\Sigma}^{-1} \boldsymbol{\Sigma}_i\right)}{b^2 - ac}\right) \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu}\end{aligned} \tag{C12}$$

The standard mean variance portfolio with a target mean μ_i is the solution of the following program:

$$\min_{\boldsymbol{\omega}} \frac{1}{2} \boldsymbol{\omega}^\intercal \boldsymbol{\Sigma} \boldsymbol{\omega} \tag{C13}$$

$$\text{subject to} \quad \begin{cases} \boldsymbol{\omega}^\intercal \boldsymbol{\mu} = \mu_i \\ \boldsymbol{e}^\intercal \boldsymbol{\omega} = 1 \in \mathbb{N}^K \end{cases}$$

The corresponding optimal solution is:

$$\widetilde{\boldsymbol{\omega}}_{MV} = \left(\frac{b\mu_i - c}{b^2 - ac}\right) \boldsymbol{\Sigma}^{-1} \boldsymbol{e} + \left(\frac{b - a\mu_i}{b^2 - ac}\right) \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu} \tag{C14}$$

As a consequence, we have:

$$\begin{aligned}\boldsymbol{\omega}^* = {} & \widetilde{\boldsymbol{\omega}}_{MV} + \boldsymbol{\Sigma}^{-1} \boldsymbol{\Sigma}_i + \left(\frac{c\boldsymbol{e}^\intercal - b\boldsymbol{\mu}^\intercal}{b^2 - ac}\right) \boldsymbol{\Sigma}^{-1} \boldsymbol{\Sigma}_i \boldsymbol{\Sigma}^{-1} \boldsymbol{e} \\ & + \left(\frac{a\boldsymbol{\mu}^\intercal - b\boldsymbol{e}^\intercal}{b^2 - ac}\right) \boldsymbol{\Sigma}^{-1} \boldsymbol{\Sigma}_i \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu}\end{aligned} \tag{C15}$$

■

Appendix D: Hybrid ETF

Proof. Let us assume that $J \leq K$, and the indices j and k are ordered in the same way, *i.e.*, the first asset in the collateral portfolio corresponds to the first asset of the index, and so on. Under these assumptions, the collateral shortfall defined in Equation (15) can be rewritten as:

$$\Delta_{t+1} = \sum_{j=1}^{J} \delta_{j,t} r_{j,t+1}^i - (1 - h) \sum_{k=1}^{K} \omega_{k,t} r_{k,t+1} \tag{D1}$$

where $r^i_{j,t+1} = r_{j,t+1}$ for $j = 1, ..., J$. An obvious way to set this shortfall at zero consists in choosing the weights ω_k such that:

$$\delta_{k,t} = (1-h)\,\omega_{k,t} \quad \text{for } k = 1, ..., J$$
$$\omega_{k,t} = 0 \quad \text{for } k > J \tag{D2}$$

However, the weights ω_k do not sum to one. Indeed:

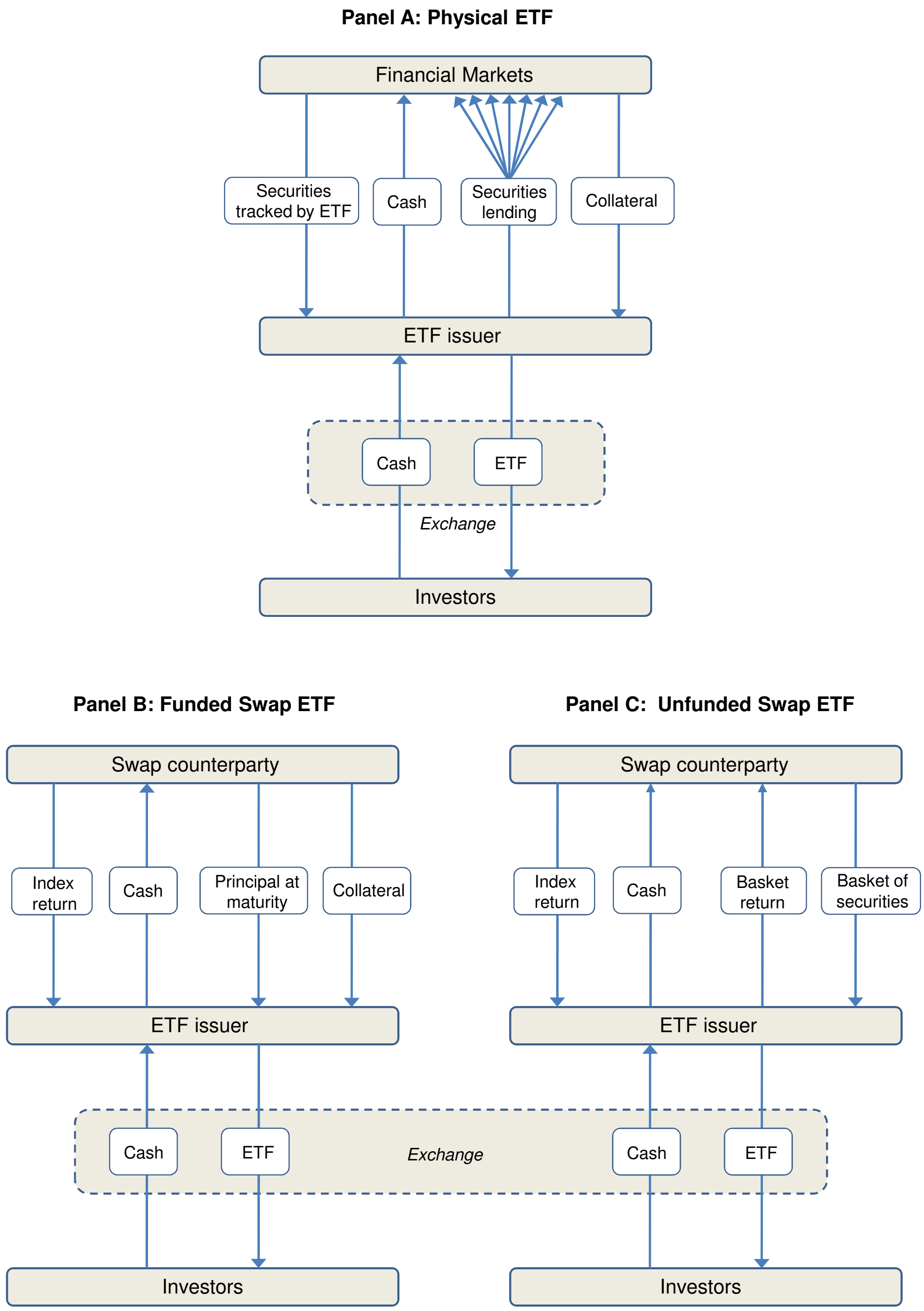
$$\sum_{k=1}^{K} \omega_{k,t} = \sum_{k=1}^{J} \frac{\delta_{k,t}}{1-h} = \frac{1}{1-h} \tag{D3}$$

since $\sum_{j=1}^{J} \delta_{j,t} = 1$. So, the normalized optimal weights are:

$$\omega^*_{k,t} = \frac{\omega_{k,t}}{\sum_{k=1}^{K} \omega_{k,t}} = \delta_{k,t} \quad \text{for } k = 1, ..., J \tag{D4}$$

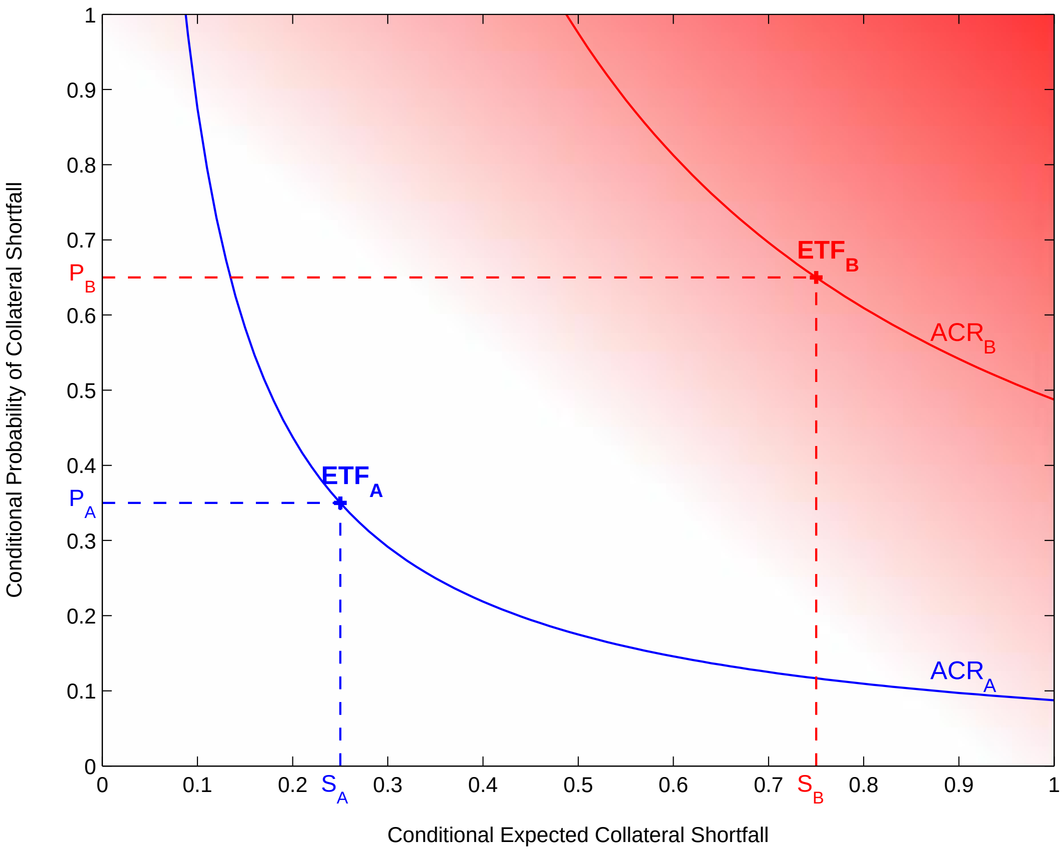
and zero otherwise. The composition of the index and the collateral are strictly identical whatever the value of h. ∎

Figure 1 – ETF Structures



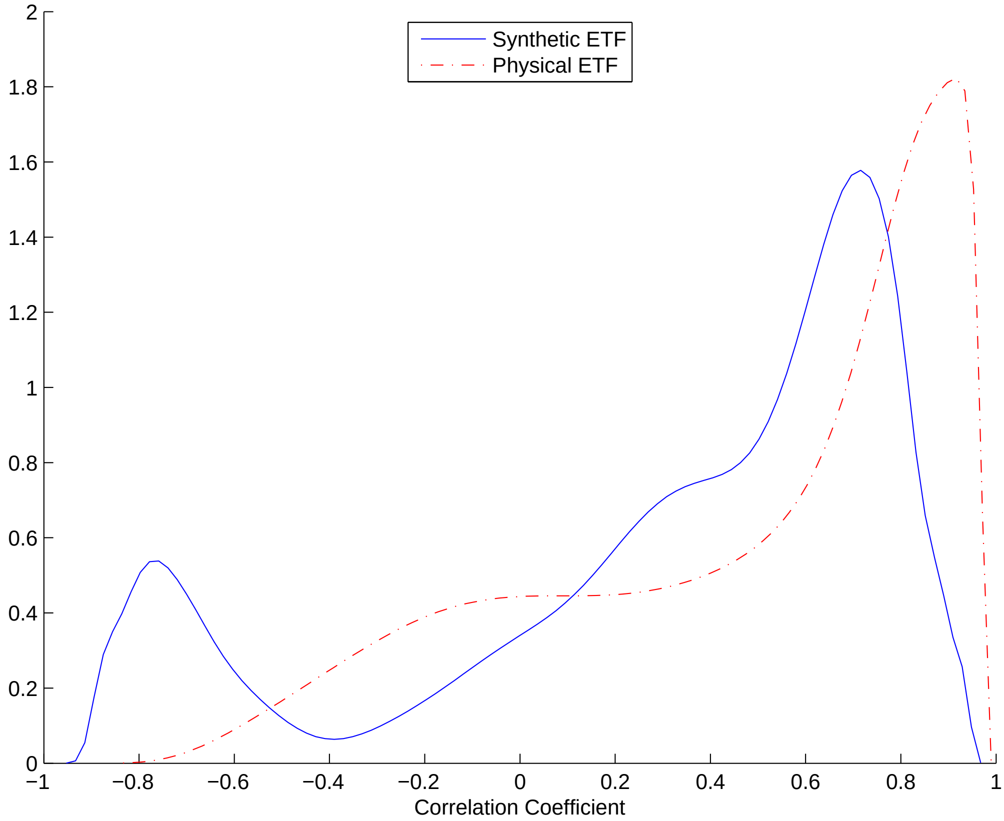
Notes: This figure describes the different cash-flows and asset transfers for three types of ETF structures: the physical ETF (Panel A), the funded-swap based ETFs (Panel B) and the unfunded-swap based ETFs (Panel C). The latter two are synthetic ETFs

Figure 2 – Displaying Counterparty Risk on a Torino Scale



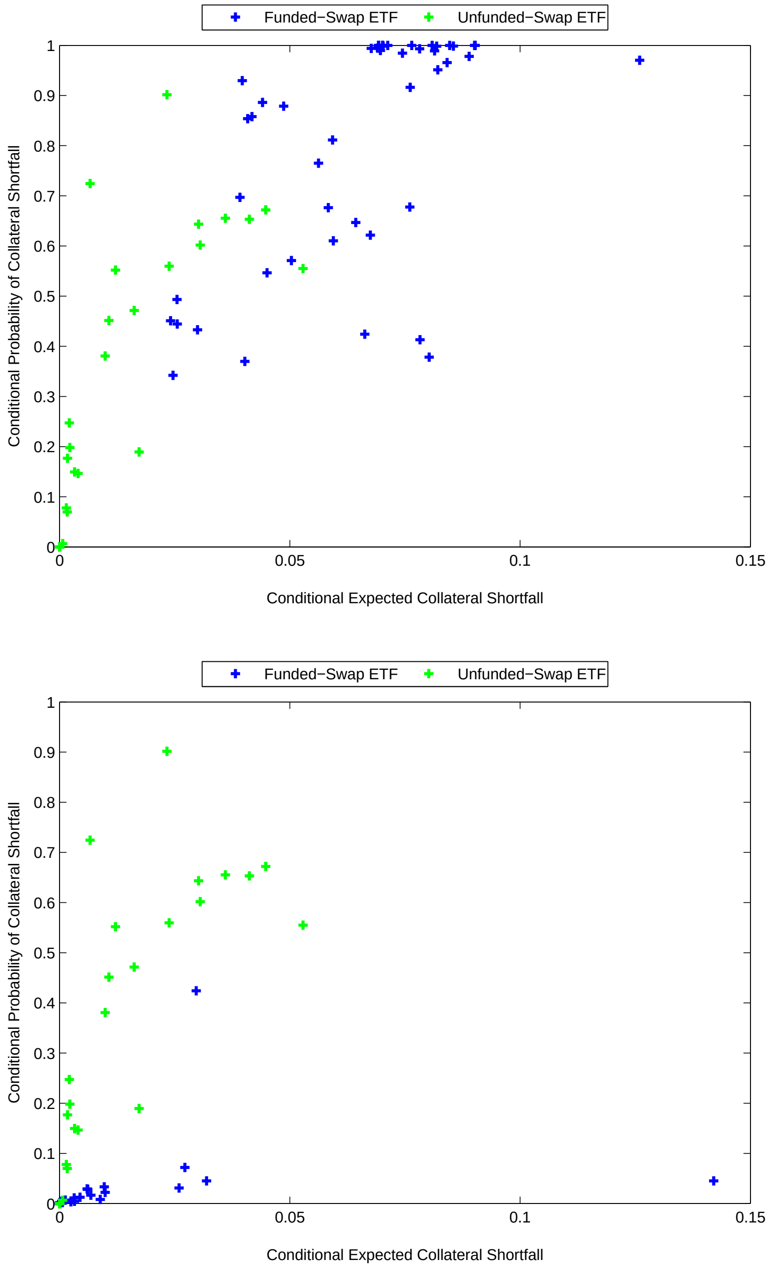
Notes: This figure presents a Torino scale that displays, for two hypothetical ETFs (A and B), their probability P of having a collateral shortfall (y-axis), their expected collateral shortfall S (x-axis), and their Aggregate Counterparty Risk $ACR = P \times S$. The funds located in the North-East corner of the Torino scale are the most exposed to counterparty risk. The two indifference curves show combinations of collateral probability and expected collateral shortfall that give the same level of Aggregate Counterparty Risk (ACR_A and ACR_B).

Figure 3 – Correlation between ETF Return and Collateral Return



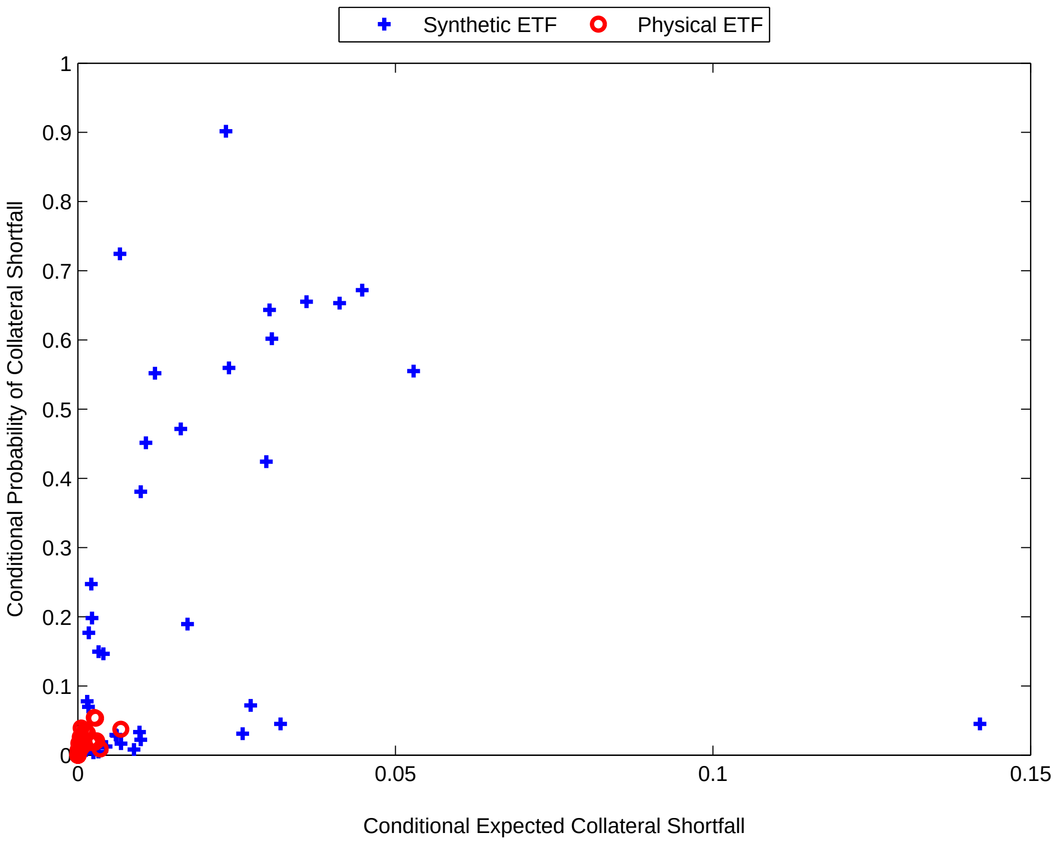
Notes: This figure presents the nonparametric kernel smoothing probability density function of the correlation coefficient between the ETF return and the return of the collateral portfolio. The estimation is based on all weekly combinations of ETF and collateral returns.

Figure 4 – Estimated Counterparty Risk for Synthetic ETFs



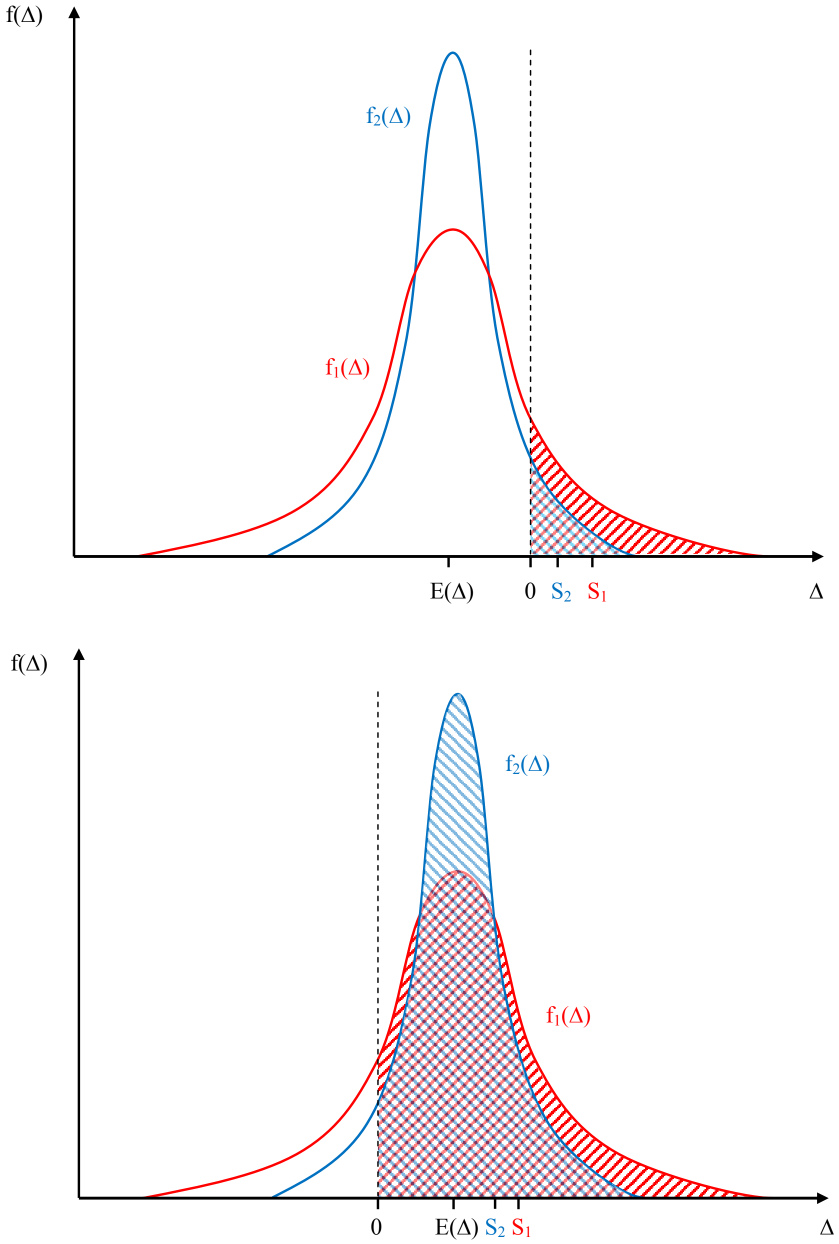
Notes: These figures present two Torino scales that display, for all synthetic ETFs, their probability P of having a collateral shortfall (y-axis, range = 0-100%), their expected collateral shortfall S (x-axis, range = 0-10%), and their Aggregate Counterparty Risk $ACR = P \times S$. We contrast ETFs that are based on funded swaps to those based on unfunded swaps. In the top figure we account for haircuts and we do not in the lower figure.

Figure 5 – Estimated Counterparty Risk for Physical and Synthetic ETFs



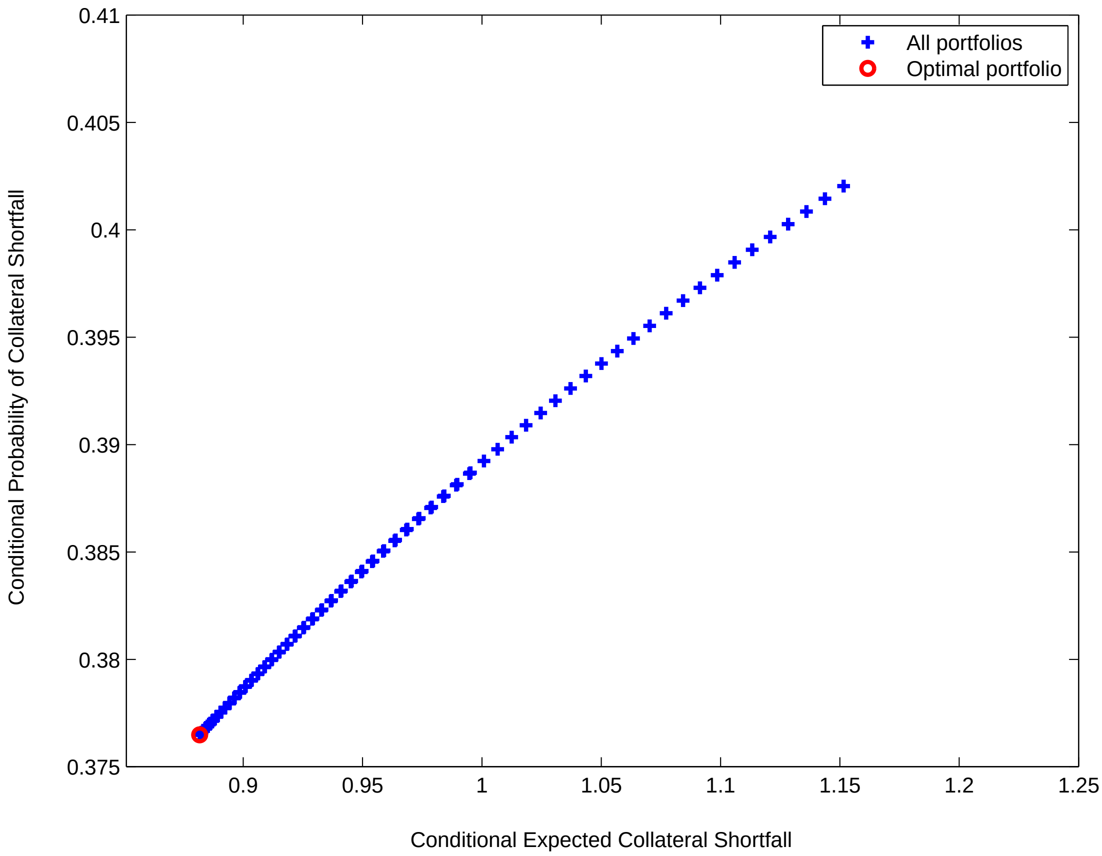
Notes: This figure presents a Torino scale that displays, for all sample ETFs, their probability P of having a collateral shortfall (y-axis, range = 0-100%), their expected collateral shortfall S (x-axis, range = 0-10%), and their Aggregate Counterparty Risk $ACR = P \times S$. We contrast physical ETFs and synthetic ETFs (both funded-swap and unfunded-swap ETFs).

Figure 6 – Some Visual Intuition about the Optimization Program



Notes: This figure presents two probability density functions f(Δ) that have the same expected values E(Δ) but different volatilities (high volatility for the red curve, $f_1(\Delta)$, and low volatility for the blue curve, $f_2(\Delta)$). The dashed areas correspond to the probabilities of having positive values. The S values correspond to the expected values of the truncated distribution.

Figure 7 – Illustration of the Optimal Collateral Portfolio



Notes: This Torino scale displays the probability of facing a collateral shortfall and the expected collateral shortfalls for all portfolios obtained by combining two collateral securities described in Section 4.2. The portfolios are obtained by varying the weight of the first security from $w_1 = 0$ to $w_1 = 1$ using a 0.01 increment.

Table 1 – Summary Statistics on the ETFs

		Synthetic	Funded	Unfunded	Long	Inverse
Number of ETF Funds		164	112	52	134	30
AUM ($ Mio)	Total	37,927	20,122	17,805	36,011	1,916
Asset Exposure	Equities	74.5% (111)	85.5% (100)	61.9% (11)	74.0% (90)	82.2% (21)
	Government Bonds	11.0% (24)	2.2% (2)	20.8% (22)	10.8% (21)	13.1% (3)
	Money Markets	6.6% (4)	-	14.0% (4)	7.0% (4)	-
	Commodities	3.8% (2)	7.2% (2)	-	4.0% (2)	-
	Hedge Funds Strategies	2.2% (6)	3.9% (3)	0.3% (3)	2.3% (5)	0.4% (1)
	Credits	0.7% (9)	-	1.6% (9)	0.5% (4)	4.3% (5)
	Corporate Bonds	0.6% (3)	-	1.4% (3)	0.7% (3)	-
	Currencies	0.3% (4)	0.6% (4)	-	0.4% (4)	-
	Multi Assets	0.3% (1)	0.6% (1)	-	0.3% (1)	-
Geographic Exposure	Europe	58.9% (79)	29.4% (41)	92.4% (38)	57.4% (54)	84.1% (25)
	World	22.3% (41)	37.0% (38)	5.5% (3)	23.6% (40)	1.1% (1)
	Asia-Pacific	9.4% (28)	17.6% (24)	0.2% (4)	9.9% (27)	1.3% (1)
	North America	7.2% (14)	11.9% (7)	1.9% (7)	6.8% (11)	13.5% (3)
	Rest of the World	2.2% (2)	4.1% (2)	-	2.3% (2)	-

Notes: This table presents some summary statistics for all synthetic ETFs, as well as separately for funded-swap based ETFs vs. unfunded-swap based ETFs, and long ETFs vs. inverse ETFs. The table displays the total number of funds, the combined assets under management (AUM) in USD million, the value-weighted averages of asset and geographic exposures, along with the number of funds in parentheses. The sample period is July 5, 2012 - November 29, 2012.

Table 2 – Size and Turnover of Collateral Portfolios

		Synthetic	Funded	Unfunded	Long	Inverse
Collateral Value ($ Mio)	All	40,939	23,083	17,856	38,706	2,233
Collateralization	All	108.4%	114.6%	101.3%	107.9%	115.4%
	Equities	109.6%	115.7%	99.9%	109.0%	117.8%
	Government Bonds	102.8%	100.7%	103.1%	102.8%	102.8%
Number of Collateral Securities	All	3,299	3,014	1,141	3,253	2,511
Average Number of Collateral Securities per ETF Fund	All	81	110	18	90	43
	Equities	109	117	35	120	58
	Government Bonds	14	13	16	15	9
	Money Markets	20	-	20	21	-
	Commodities	93	93	-	93	-
	Hedge Funds Strategies	37	66	9	43	7
	Credits	10	-	10	11	10
	Corporate Bonds	12	-	12	12	-
	Currencies	50	50	-	50	-
	Multi Assets	70	70	-	70	-
Turnover	All	34.0%	47.4%	5.0%	33.5%	36.3%
	Equities	43.9%	46.5%	18.8%	42.1%	51.3%
	Government Bonds	1.3%	0.5%	1.4%	1.3%	1.6%
	Money Markets	2.2%	-	2.2%	2.2%	-
	Commodities	72.7%	72.7%	-	72.7%	-
	Hedge Funds Strategies	29.7%	60.7%	0.3%	36.0%	0.7%
	Credits	1.4%	-	1.4%	1.6%	1.2%
	Corporate Bonds	2.4%	-	2.4%	2.4%	-
	Currencies	60.8%	60.8%	-	60.8%	-
	Multi Assets	76.7%	76.7%	-	76.7%	-

Notes: This table presents some summary statistics on the size and turnover of the ETFs' collateral portfolios. It displays for all synthetic ETFs, as well as separately for funded-swap based ETFs vs. unfunded-swap based ETFs, and long ETFs vs. inverse ETFs, the collateral value in USD million, the value-weighted average level of collateralization (collateral value/AUM), the total number of collateral securities, the average number of collateral securities, and the average turnover. Results are also broken down by asset exposure. Turnover is defined as the ratio between the number of different securities, as defined by their ISIN, that enter or exit the collateral portfolio between two dates divided by the total number of collateral securities on both dates. The sample period is July 5, 2012 - November 29, 2012.

Table 3 – Types of Collateral Securities

Panel A: Type of Collateral Securities		Equity	Government Bonds	Corporate Bonds
Number of Collateral Securities		2,591	490	218
ETF Asset Exposure	All	74.9%	19.7%	5.4%
	Equity	**92.5%**	2.7%	4.8%
	Government Bonds	-	**96.5%**	3.5%
	Corporate Bonds	-	100%	-
	Others	40.8%	48.8%	10.4%

Panel B: Geographic Origin of the Collateral Securities		Europe	Asia-Pacific	N. America	R. World
ETF Geographic Exposure	All	66.0%	17.5%	16.3%	0.2%
	Europe	**71.8%**	13.9%	13.9%	0.4%
	Asia-Pacific	56.0%	**24.1%**	19.8%	0.1%
	North America	58.3%	22.1%	**19.5%**	0.1%
	Rest of the World	58.1%	25.5%	16.3%	**0.1%**
	World	58.8%	21.7%	19.4%	0.1%

Notes: This table presents some summary statistics on the securities used as collateral. Panel A displays the number of collateral securities per type of collateral and the value-weighted average percentage of collateral that is held in equity, Governments bonds, and corporate bonds, respectively. Panel B presents, for each type of ETF geographic exposure, the value-weighted percentage of collateral that comes from Europe, Asia-Pacific, North America, and Rest of the World, respectively. The Government Bond category also includes Supranational Bonds (5), Government Guaranteed Bonds (3), Government Agency Bonds (2), and German Regional Government Bonds (1). The Corporate Bond category also includes Covered Bonds (16). The number in parentheses indicates the number of different securities in each category. The sample period is July 5, 2012 - November 29, 2012.

Table 4 – Regression Analysis of the ETF-Collateral Correlation

		Physical	Physical	Synthetic	Synthetic
	log(AUM)	0.092	0.009	-0.039	-0.040
		(0.94)	(0.16)	(-1.08)	(-1.10)
	Funded Swap	-	-	-0.269	-0.215
				(-1.53)	(-0.91)
	Inverse	-	-	-2.837***	-3.102***
				(-18.48)	(-17.15)
	Collateralization	-0.023	-0.049	0.003**	0.004**
		(-0.61)	(-1.32)	(2.40)	(2.41)
	log(# Securities)	0.083	0.077	0.058**	0.063**
		(1.61)	(1.44)	(2.42)	(2.55)
	Equity Fraction	0.017**	0.017*	0.003**	0.002*
		(1.97)	(1.95)	(2.34)	(1.94)
	Europe Fraction	0.006***	0.006***	0.003***	0.003***
		(2.82)	(2.70)	(4.46)	(4.44)
Asset Exposure	Government Bonds		-2.359***		-0.029
			(-10.39)		(-0.10)
	Money Markets		-		-1.379***
					(-4.47)
	Commodities		-		-0.866***
					(-3.81)
	Hedge Funds Strategies		-		-0.329
					(-0.76)
	Credits		-		0.015
					(0.03)
	Corporate Bonds		-2.019***		-1.161***
			(-8.21)		(-3.76)
	Currencies		-		-1.271***
					(-3.96)
	Multi Assets		-		-0.415***
					(-4.05)
Geographic Exposure	World		0.087		-0.072
			(0.43)		(-0.59)
	Asia Pacific		-1.417***		-0.827***
			(-8.42)		(-6.45)
	North America		0.518		-0.125
			(1.10)		(-1.05)
	Rest of the World		-		-0.409
					(-1.57)
	Fund Random Effects	Yes	Yes	Yes	Yes
	Observations	1,065	1,065	3,478	3,478
	R^2	0.101	0.738	0.671	0.756

Notes: This table reports the parameter estimates obtained by regressing the correlation coefficient between the ETF return and the return of the collateral portfolio on a series of ETF-specific variables. The estimation is based on all week-fund observations. The explanatory variables are the level of the ETF asset under management (in log), a dummy variable that takes the value of 1 if the ETF is based on a funded swap and 0 otherwise, a dummy variable that takes the value of 1 if the ETF is an inverse ETF and 0 otherwise, the level of collateralization of the fund, the number of securities in the collateral portfolio (in log), the fraction of equities in the collateral portfolio, the fraction of European securities in the collateral portfolio, as well as a dummy variable for each asset exposure and geographic exposure. In all columns, the model is estimated by GLS with random effects, a constant term, and robust standard errors. We transform the explained variable, $\ln(\frac{1+y}{1-y})$, to ensure it remains between -1 and 1. We display t-statistics in parentheses. ***, **, * represent statistical significance at the 1%, 5% or 10% levels, respectively.

Table 5 – Equities Used as Collateral

Panel A: Equity Issuer

Region	Europe 58.5% (752)	Asia-Pacific 21.6% (1,365)	N. America 19.6% (469)	R. World 0.3% (5)
Industry Classification	Industrial 76.6% (2,047)	Financial 11.4% (346)	Utility 10.4% (127)	Transportation 1.6% (71)
Market Capitalization	Micro-Cap 1.4% (30)	Small-Cap 9.3% (1,634)	Mid-Cap 9.9% (404)	Large-Cap 79.4% (523)

Panel B: Liquidity	mean	median	st.dev.	min	max
Average Daily Spread	0.21%	0.20%	0.48%	0.01%	12.73%
Average Daily Volume	4.67%	0.44%	21.83%	0.01%	115.15%

Panel C: Dependence		mean	median	st.dev.	min	max
Beta	ETF	0.48	0.45	0.70	-1.83	3.04
	Swap Counterparty	0.30	0.28	0.28	-0.06	1.11
Conditional Beta	ETF	0.50	0.47	0.74	-3.05	4.14
	Swap Counterparty	0.35	0.33	0.30	-0.46	1.06

		< 0	$[0; 0.2[$	$[0.2; 0.4[$	$[0.4; 0.6[$	$[0.6; 0.8[$	≥ 0.8
Beta	ETF	7.5%	11.3%	25.3%	21.4%	13.4%	21.1%
	Swap Counterparty	0.4%	36.6%	38.3%	19.3%	4.2%	1.2%
Conditional Beta	ETF	9.2%	9.1%	22.9%	20.9%	14.4%	23.5%
	Swap Counterparty	0.2%	23.9%	40.2%	25.3%	8.8%	1.6%

Notes: This table presents some summary statistics on the equities included in the collateral portfolios of synthetic ETFs. Panel A displays the value-weighted percentage of collateral equities by region, industry, and size of the issuer, along with the number of different equities in parentheses. We use the following definitions for size groups: Micro-Cap: below \$100 million; Small-Cap: \$100 million-\$4 billion; Mid-Cap: \$4 billion-\$10 billion; Large-Cap: Over \$10 billion. These ranges were selected to match the average market capitalization of the MSCI World Index of the respective categories. The size figures are as of November 29th, 2012. Panel B displays value-weighted statistics about the average daily percentage bid-ask spread and the average daily volume in percentage of the market capitalization. For each security, the percentage spread and volume are winsorized at the top 1%. Panel C displays value-weighted statistics on the beta coefficient ($\beta_{i,j}$) and conditional beta coefficient ($\beta_{i,j}|r_j < 0$) of the collateral equities with respect to the ETF return and to the swap counterparty return (Deutsche Bank stock return). We compute the conditional betas by using only days during which the index return or the swap counterparty return is negative. The lower part of Panel C presents a histogram of the betas and conditional betas, weighted by the equity value. In Panels B and C, the sample period is between January 1, 2007 and December 31, 2012.

Table 6 – Bonds Used as Collateral

Panel A: Bond Issuer		Europe	N. America	Asia-Pacific	R. World
Bond Type	All	88.3% (512)	7.1% (118)	4.5% (77)	0.1% (1)
	Gov. Bonds	86.0% (338)	8.6% (101)	5.4% (51)	-
	Corp. Bonds	96.6% (174)	2.0% (17)	1.3% (26)	0.1% (1)

Panel B: Rating		AAA	AA	A	BBB	BB	B	n/a
Bond Type	All	65.5%	14.8%	17.2%	0.8%	0.3%	0.1%	1.3%
	Gov. Bonds	75.2%	9.3%	15.3%	0.1%	0.1%	-	-
	Corp. Bonds	30.4%	34.5%	23.9%	3.5%	1.5%	0.1%	6.1%
Bond Issuer	Europe	66.0%	12.6%	19.2%	0.9%	0.2%	0.1%	1.0%
	North America	93.9%	0.4%	0.1%	0.1%	1.8%	0.2%	3.5%
	Asia-Pacific	19.8%	69.9%	3.5%	0.4%	1.1%	-	5.3%

Panel C: Maturity		<1Y	1-3Y	3-5Y	5-7Y	7-10Y	>10Y
Bond Type	All	16.4%	24.4%	14.7%	6.7%	14.7%	23.1%
	Gov. Bonds	11.7%	19.2%	13.3%	8.5%	18.1%	29.2%
	Corp. Bonds	33.4%	43.0%	19.6%	0.5%	2.3%	1.2%
Bond Issuer	Europe	17.4%	25.3%	14.7%	6.4%	13.2%	23.0%
	North America	4.5%	21.7%	18.8%	8.0%	16.7%	30.3%
	Asia-Pacific	17.9%	14.4%	12.4%	17.4%	28.1%	9.8%
Index Maturity	Short	21.6%	49.5%	23.7%	3.3%	1.0%	0.9%
	Medium	10.7%	8.9%	22.7%	34.0%	23.7%	-
	Long	-	-	-	7.5%	7.6%	84.9%

Notes: This table presents some summary statistics on the bonds included in the collateral portfolios of synthetic ETFs. Panel A displays the value-weighted percentages of collateral bonds by region, along with the number of different bonds in parentheses. Panel B presents the value-weighted percentages of collateral bonds by bond rating, for different bond types and issuers. The n/a category corresponds to unrated bonds. Panel C displays the value-weighted percentages of collateral bonds by bucket of maturity, for different bond types and issuers. Short, Medium, and Long refer to funds that track a bond index with, respectively, a short maturity (less than 3 years), a medium maturity (between 3 and 10 years), and a long maturity (more than 10 years). The sample period is July 5, 2012 - November 29, 2012.

Table 7 – Counterparty Risk of Synthetic ETFs

		ACR	ACR	ACR
	log(AUM)	0.093	0.112	0.126
		(0.72)	(1.01)	(1.10)
	Funded Swap	-1.514***	-2.464***	-2.525***
		(-3.07)	(-3.53)	(-3.95)
	Inverse	0.560	0.708	1.107**
		(1.24)	(1.55)	(2.44)
	Collateralization	-0.092***	-0.091***	-0.091***
		(-3.57)	(-3.55)	(-3.53)
	log(# Securities)	-0.167	-0.116	-0.189
		(-1.22)	(-0.85)	(-1.45)
	Equity Fraction	0.012**	-0.004	-0.004
		(2.41)	(-0.68)	(-0.63)
	Europe Fraction	-0.006**	-0.006***	-0.005**
		(-2.57)	(-2.61)	(-2.51)
Asset Exposure	Control Dummy Variables	no	yes	yes
Geographic Exposure	Control Dummy Variables	no	no	yes
	Fund Random Effects	Yes	Yes	Yes
	Observations	1,310	1,310	1,310
	R^2	0.373	0.417	0.528

Notes: This table reports the parameter estimates obtained by regressing the aggregate counterparty risk measure ACR on a series of ETF-specific variables. The estimation is based on all week-fund observations for all synthetic ETFs. The explanatory variables are the level of the ETF asset under management (in log), a dummy variable that takes the value of 1 if the ETF is based on a funded swap and 0 otherwise, a dummy variable that takes the value of 1 if the ETF is an inverse ETF and 0 otherwise, the level of collateralization of the fund, the number of securities in the collateral portfolio (in log), the fraction of equities in the collateral portfolio, the fraction of European securities in the collateral portfolio, as well as a dummy variable for each asset exposure and geographic exposure. In all columns, the model is estimated by GLS with random effects, a constant term, and robust standard errors. We transform the explained variable, $\ln(y+1)$, to ensure it remains non-negative. We display t-statistics in parentheses. ***, **, * represent statistical significance at the 1%, 5% or 10% levels, respectively.

Table 8 – Counterparty Risk of Physical and Synthetic ETFs

		ACR	ACR	ACR	ACR
	Synthetic ETF	1.312***	2.162***	2.807***	2.846***
		(5.81)	(5.53)	(7.04)	(7.02)
	log(AUM)		0.030	0.041	0.040
			(0.30)	(0.42)	(0.41)
	Funded Swap		-1.165***	-1.950***	-1.949***
			(-2.67)	(-3.91)	(-4.02)
	Inverse		0.454	0.650	0.831*
			(0.96)	(1.39)	(1.81)
	Collateralization		-0.091***	-0.091***	-0.091***
			(-3.63)	(-3.61)	(-3.61)
	log(# Securities)		-0.062**	-0.060**	-0.066**
			(-2.19)	(-2.17)	(-2.39)
	Equity Fraction		0.005*	0.002	0.003
			(1.88)	(1.13)	(1.22)
	Europe Fraction		-0.002**	-0.002**	-0.002**
			(-2.30)	(-2.34)	(-2.30)
Asset Exposure	Control Dummy Variables	No	No	Yes	Yes
Geographic Exposure	Control Dummy Variables	No	No	No	Yes
	Fund Random Effects	Yes	Yes	Yes	Yes
	Observations	2,056	2,056	2,056	2,056
	R^2	0.135	0.413	0.472	0.521

Notes: This table reports the parameter estimates obtained by regressing the aggregate counterparty risk measure ACR on a series of ETF-specific variables. The estimation is based on all week-fund observations. The explanatory variables are a dummy variable that takes the value of 1 if the ETF is synthetic and 0 otherwise, the level of the ETF asset under management (in log), a dummy variable that takes the value of 1 if the ETF is based on a funded swap and 0 otherwise, a dummy variable that takes the value of 1 if the ETF is an inverse ETF and 0 otherwise, the level of collateralization of the fund, the number of securities in the collateral portfolio (in log), the fraction of equities in the collateral portfolio, the fraction of European securities in the collateral portfolio, as well as a dummy variable for each asset exposure and geographic exposure. In all columns, the model is estimated by GLS with random effects, a constant term, and robust standard errors. We transform the explained variable, $\ln(y+1)$, to ensure it remains non-negative. We display t-statistics in parentheses. ***, **, * represent statistical significance at the 1%, 5% or 10% levels, respectively.

Table 9 – Fees and Tracking Errors

Panel A: Entire Sample		Total	Physical	Synthetic
Fees	All	0.43 (0.40)	0.44 (0.40)	0.43 (0.45)
	Capitalizing	0.43 (0.45)	0.42 (0.33)	0.43 (0.45)
	Distributing	0.44 (0.40)	0.44 (0.40)	0.44 (0.50)
Tracking Errors	All	0.36 (0.04)	0.96 (0.72)	0.13 (0.03)
	Capitalizing	0.06 (0.03)	0.93 (0.82)	0.04 (0.03)
	Distributing	0.84 (0.67)	0.96 (0.71)	0.60 (0.48)

Panel B: Matched Sample		Total	Physical	Synthetic
Fees	All	0.39 (0.35)	0.39 (0.40)	0.39 (0.35)
	Capitalizing	0.40 (0.34)	0.40 (0.33)	0.41 (0.35)
	Distributing	0.37 (0.35)	0.39 (0.40)	0.35 (0.35)
Tracking Errors	All	0.36 (0.08)	0.60 (0.21)	0.11 (0.04)
	Capitalizing	0.20 (0.07)	0.38 (0.11)	0.03 (0.03)
	Distributing	0.64 (0.42)	1.01 (0.47)	0.26 (0.08)

Notes: This table displays the average (and median) fees and tracking errors. In Panel A, we consider all sample ETFs, physical ETFs, and synthetic ETFs. In Panel B, we only consider pairs or physical and synthetic ETFs that track exactly the same index (same index provider, same asset class, and same treatment of the dividends). We distinguish funds that pay out dividends to their investors (Distributing) from those that do not (Capitalizing). Following industry practice, we remove some outliers likely due to misaligned data or other data errors (Morningstar, 2013). Fees correspond to total expense ratios and have been collected on November 29, 2014. Tracking errors are defined as the annualized standard-deviation of daily differences between the daily returns of the fund NAV and index. They are computed using two-year of daily returns covering the period November 29, 2010 - November 29, 2012.

Table 10 – Regressions Analyses of Fees and Tracking Errors

		Fees	Tracking Errors
	Synthetic ETF	-0.090***	-0.344***
		(-4.25)	(-4.49)
	Distributing	-0.025*	0.279***
		(-1.75)	(4.97)
	log(AUM)	-0.003	-0.030***
		(-1.25)	(-3.48)
	Funded Swap	0.026	-0.078
		(1.27)	(-1.54)
	Inverse	0.085***	0.027
		(5.63)	(1.09)
Asset Exposure	Control Dummy Variables	yes	yes
Geographic Exposure	Control Dummy Variables	yes	yes
	Observations	184	202
	R^2	0.809	0.674

Notes: This table reports the parameter estimates obtained by regressing the fees and tracking errors on a series of fund-specific variables. The estimation is based on the cross-section of all physical and synthetic ETFs. The explanatory variables are a dummy variable that takes a value of 1 if the ETF is synthetic and 0 otherwise, a dummy variable that takes a value of 1 if the fund pays out dividends to its investors (Distributing), the level of the ETF asset under management (in log), a dummy variable that takes a value of 1 if the ETF is based on a funded swap and 0 otherwise, a dummy variable that takes a value of 1 if the ETF is an inverse ETF and 0 otherwise, as well as a dummy variable for each asset exposure and geographic exposure. In both columns, the model is estimated by OLS with a constant term and robust standard errors. We transform the explained variable, $\ln(y+1)$, to ensure it remains non-negative. We display t-statistics in parentheses. ***, **, * represent statistical significance at the 1%, 5% or 10% levels, respectively.