Automatic workout mortgage and housing consumption choice

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Abstract

Demand for housing increases if Automatic Workout Mortgage (AWM) is used and attains maxima for those intending to reverse-mortgage $\alpha \approx 40\%$ of equity (without workout) and about $\alpha \approx 80\%$ of equity (with AWM). We therefore assert that AWM is particularly suited for those who intend to sell or reverse-mortgage a significant fraction of their home equity before retirement. Insurance provided via AWM inhibis precautionary saving motives by making housing more attractive to risk averse borrowers. Consumption is reduced as more housing is purchased and borrowing increased. AWM improves expected utility by a dollar equivalent of about $40,000 to $55,000, which represents up to 55% of the initial value of the house ($100,000 in our simulations).

1 Introduction

One of the fundamental changes that should be done is to revise mortgage markets so that mortgages have workouts that are planned. During the recent crisis there were 50 millions households underwater, which means they were essentially insolvent. They werent getting any help.

Housing finance is still stuck in relatively a primitive stage. Insolvent households generate negative feedback loops in the economy. When home prices are falling people start realizing they are out of money i.e. they are in debt more than they own. For example, they cant get a home equity loan anymore. To keep on mortgage payments they may have no choice but to cut back their spending. This in turn can put the whole economy in a tailspin. Therefore, we should encourage that mortgages include a plan for the next crisis. Such plan can be provided by the private insurance sector. This would help reducing negative feedback loops in the economy.

There have been a growing number of proposals to change our mortgage institutions. In the US, the Dodd-Frank act recommnded a study of shared appreciation mortgages. In the UK a budget proposal [10] recommended the first half of negative equity losses to be covered by private insurance and the government acting as insurer of last resort for the second half.[1]

We focus on Automatic Workout Mortgages (AWMs), a two-in-one product including a home loan plus an automatic workout to prevent negative equity. First, we consider the standard insurance framework and recover the standard result that the homeowner prefers to be fully insured if the premium is fair. We then show that the homeowner underinsures if the premium is too high compared to the actual probability of the loss happening.

Second, we set up a basic two-period model where people will sell their house and consume the proceeds in retirement. Within this framework we show that there is a welfare gain to insure house values via AWMs. While assuming that people leave no bequests may seem unrealistic, there is evidence [CITE?] that the retirement trends are changing. Notably, the continuing care retirement community (CCRC) is a concept that is growing rapidly around the world. There is a CCRC crisis in the US today, after the drop in home

[1]In the end the government opted for a scheme called *Help to Buy Mortgage Guarantee*. It is aimed at increasing affordability by raising the maximum loan to value ratios to 95%. It does not provide automatic workouts and there is no help of the private sector. The government charges a premium. In return, the taxpayer covers up to 15% of the loan. The lender gets a refund if a default occurs within seven years.

prices, the CCRCs have a lot of vacancies, and this is no doubt because of the absence of AWMs.

Third,

This paper is organized as follows. In section 1 we cast our workout choice problem within the standard insurance framework. ETC.

2 House price workout as simple insurance

In this section we advocate a simple mortgage workout. To this end we work within the standard paradigm of insurance theory (see e.g. Picard [9], Eeckhoudt et al. [5]).

There are two important implications. First, in absence of imperfections the optimal workout is a full workout. Second, the homeowner will prefer a partial workout if the insurer raises the premium to e.g. pass through administrative costs.

The initial mortgage balance is H and equal to the value of the house.[2] At the end of the period there is risk that the value of the house will drop by l to $H - l$, where

$$0 \leq l \leq H \,. \tag{1}$$

The household has revenue y and the non-housing consumption is c. If the house price drops, the homeowner is underwater because the mortgage balance is higher than the house value. The question is whether or not the household will prefer insuring the house against real estate prices going down. When the house is sold at maturity, the terminal wealth is equal to

$$W = y - c + H_T - H \geqslant 0 \tag{2}$$

$$H_T = \begin{cases} H - l & \text{with probability} \quad p \\ H & \text{with probability} \quad 1 - p \end{cases} , \tag{3}$$

where $c \geq 0$ is the non-housing consumption. Both the consumption c and the terminal wealth W depend the random behaviour of the terminal house price H_T. However, if the individual who maximizes the expected utility of terminal consumption leaves no

[2] In our analysis the homeowner is repaying a 100% mortgage. It is straightforward to extend our case to situations with downpayment i.e. with loan to value ratio less than one.

bequest ($W = 0$), then:

$$c = y + H_T - H = \begin{cases} y - l & \text{with probability} \quad p \\ y & \text{with probability} \quad 1 - p \end{cases} . \tag{4}$$

Effectively, in this simple example, all the income y will be consumed if the house price remains at the level H. However, if the loss on the house value is $l > 0$, consumption also drops by l. In this case the house price drop affects consumption of the non-housing good and the expected utility of the terminal consumption is

$$E\left[u\left(c\right)\right] = pu\left(y - l\right) + \left(1 - p\right) u\left(y\right) \ . \tag{5}$$

However, if automatic workout is offered, after paying the insurance premium π the homeowner's consumption will not suffer any further loss, so that, in both cases the terminal consumption for the borrower is non-random

$$\bar{c} = y - \pi \ . \tag{6}$$

The insurance theory tells us (see for example Picard [9] or [5]) that the insurance premium for a risk-neutral insurer is actuarially fair when the premium π equals the expected loss

$$\pi = pl \ , \tag{7}$$

implying

$$\bar{c} = y - pl \ . \tag{8}$$

Proposition 1 *With automatic full workout against the risk of house prices falling, the homeowner achieves a higher utility*

$$u\left(\bar{c}\right) \geqslant E\left[u\left(c\right)\right] \tag{9}$$

Proof. By Jensen's inequality

$$u\left(p\left(y - l\right) + \left(1 - p\right) y\right) \geqslant pu\left(y - l\right) + \left(1 - p\right) u\left(y\right) \tag{10}$$

which implies (9). ■

Let's now assume that the homeowner is offered a choice between full and partial state-contingent workout and that the insurance premium is proportional to the refund (see Picard [9]). The state-contingent workout is

$$w = \begin{cases} l_p & \text{with probability} \quad p \\ 0 & \text{with probability} \quad 1-p \end{cases} \tag{11}$$

such that $l_p \leq l$ and the premium is kl_p where $k < 1$. Therefore, for any assumed coefficient $k > 0$ the amount of insurance l_p fully characterizes the partial workout contract $\{kl_p, l_p\}$.

Proposition 2 *Homeowner prefers a partial workout ($l_p < l$) if the insurance premium proportion k is higher than the probability p of the house becoming underwater.*

Proof. Homeowner maximizes the expected utility of terminal consumption by choosing the optimal workout amount l_p

$$\max_{l_p} E\left[u\left(c\right)\right] \tag{12}$$

$$\text{s.t. } c = y + H_T - H + w - kl_p \,. \tag{13}$$

We have

$$E\left[u\left(c\right)\right] = pu\left(y - l + l_p - kl_p\right) + \left(1-p\right)u\left(y - kl_p\right) \,. \tag{14}$$

The first order condition is

$$\left(1-k\right)pu'\left(y - l + \left(1-k\right)l_p\right) - k\left(1-p\right)u'\left(y - kl_p\right) = 0\,, \tag{15}$$

implying

$$\frac{u'\left(y - l + \left(1-k\right)l_p\right)}{u'\left(y - kl_p\right)} = \frac{k\left(1-p\right)}{p\left(1-k\right)}\,. \tag{16}$$

In particular, when $k = p$, we have

$$\frac{u'\left(y - l + l_p - kl_p\right)}{u'\left(y - kl_p\right)} = 1\,, \tag{17}$$

implying full insurance of the house price risk

$$l_p = l\,. \tag{18}$$

If, on the contrary, $k > p$ then

$$\left\{ \frac{k}{p} > 1, \frac{1-p}{1-k} > 1 \right\} \Longrightarrow \frac{k\,(1-p)}{p\,(1-k)} > 1\,, \tag{19}$$

implying

$$u'\left(y - l + l_p - kl_p\right) > u'\left(y - kl_p\right) \Longrightarrow l_p < l\,,$$

i.e. the homeowner under-insures if the premium charged is higher than the probability of the event. ■

In this section we motivated mortgage workouts using standard arguments of insurance theory. When insurance premia are based on actual probabilities, the insurer breaks even only on average, in the long run. Contracts are pooled together and there is reliance on the law of large numbers to reduce risk. In particular, the insurer cannot eliminate exposure to major risks such as bursting of a house price bubble, to which most of the contracts in the pool are exposed to.

There are, however, many alternative ways to approach the calculation of insurance premia. For a review of the different insurance paradigms see, for example Embrechts [6] or Laeven and Goovaerts [8]. In particular, when markets for underlying risks exist the risk-neutral construction can be used. The advantage of it is that the risk is eliminated at every time a delta hedge portfolio is constructed.[3] Therefore, in the next section we introduce the state-contingent approach to pricing mortgage workouts.

3 Automatic workout mortgage and housing consumption choice in a simple utility maximization model

In this section we introduce a simple intertemporal model where the housing consumption is financed by a workout mortgage. Our setup is derived from the classic model of housing choice first considered by Henderson and Ioannides [7]. Risk averse homeowner chooses housing consumption h_c financed by a mortgage loan L. When workout is offered, it is priced by a risk-neutral insurer who has access to markets for house price risk.

[3]For an excellent illustration of the risk-neutral approach v.s. the law of large numbers approach see the preamble in Baxter and Rennie [1], "The parable of the bookmaker," pp. 1–2, demonstrating that the bookmaker is *always* better off by *not* using the actual probabilities to compute odds.

3.1 No workout offered

Housing and consumption provide utility in period 1. In period 2 the house is sold at uncertain market price $P\left(1+\tilde{\theta}\right)$. Homeowner chooses $\{h_c, S\}$ to maximize

$$
\begin{aligned}
&\max_{\{h_c,S\}} U\left(c_1,h_c\right)+E\left[V\left(c_2,h_c\right)\right] \\
&\text{s.t.}\quad c_1 = y_1 - S - (P-L)h_c \\
&\qquad\;\; c_2 = y_2 + S(1+r) + (\alpha P(1+\tilde{\theta}) - L(1+r_m) - T)h_c\ ,
\end{aligned}
\tag{20}
$$

where:

- y_1, y_2 are incomes in period 1 and 2;
- c_1, c_2 are consumptions of non-housing good in period 1 and 2;
- h_c is the consumption of housing (chosen in period 1);
- S are savings at riskless rate r made in period 1;
- L is the mortgage per one unit of housing taken at the mortgage rate $r_m \geq r$;
- T is the cost of ownership (such as repairs and maintenance) of one unit of housing payable in period 2;
- P is the price of one unit of housing in period 1;
- $\tilde{\theta}$ is the random return on housing, to be observed in period 2;
- E is the expectation operator under the actual probability measure;
- α percentage of the house value reverse-mortgaged in period 2.

Note in particular that the period 1 and 2 utility functions both depend on h_c. Still, the person who buys a house is locking in an endowment of housing for period 2, but not of consumption c_2. Consistent with Cocco [4], Campbell and Cocco [2], [3] and Van Hemert [12] we specify the first utility function to

$$
U\left(c_1,h_c\right)=\frac{\left(c_1^{1-\eta}h_c^{\eta}\right)^{1-\gamma}}{1-\gamma} \tag{21}
$$

where γ is the relative risk aversion and η measures preference of housing v.s. non-housing consumption. Furthermore, we parametrise $V(c_2, h_c)$ as follows

$$V(c_2, h_c) = \beta U\left(c_2^{\frac{1-(1-\alpha)\eta}{1-\eta}}, h_c^{1-\alpha}\right) + \beta^{1.5}(1-\alpha)B(h_c). \tag{22}$$

where β is the time preference coefficient. The period 2 utility inherits its form from period 1 utility. The utility of unconsumed bequest left at the end of period 2 is $B(h_c)$

$$B(h_c) = \frac{\left(P(1+\tilde{\theta})h_c\right)^{1-\gamma}}{1-\gamma} \tag{23}$$

If the house is sold[4] ($\alpha = 1$) there is no bequest left, $(1-\alpha) = 0$, and the utility in period 2 is insensitive to the choice of h_c

$$V(c_2, h_c)\big|_{\alpha=1} = \beta U\left(c_2^{\frac{1}{1-\eta}}, h_c^0\right) = \beta\frac{c_2^{1-\gamma}}{1-\gamma}. \tag{24}$$

This assumes no housing demand in later years. This is realistic as a stylization of the idea that people no longer have children living with them and so sometimes do downsize or rent. This choice of utility for period 2 has them downsize housing to zero. So, people who expect to live in a retirement home can buy AWMs. Also, those who stay in their homes to the end let their houses depreciate as they age, do not modernize them, and so in a sense they are still downsizing. Also, those who plan to leave a bequest to their children might be concerned about the real value of their gift. The lump sum received at the beginning of period 2 can be used for purchasing e.g. retirement services such as CCRCs.[5]

However the world isn't a one-size-fits-all. While some people do plan to downsize dramatically and move to a retirement home when they retire, most people do not. While that may be changing, people don't usually downsize when they retire [CITE?], they just age in place. This alternative leads towards including bequests of the unconsumed housing. If the house is kept ($\alpha = 0$)

$$V(c_2, h_c)\big|_{\alpha=0} = \beta U(c_2, h_c) + \beta^{1.5}B(h_c)\ . \tag{25}$$

[4]Or, equivalently, fully reverse-mortgaged, see below.

[5]In our simplified setup these generate utility in period 2 via increased consumption c_2 rather than through housing capacity h_c.

Reverse-mortgaging amounts to selling a fraction $\alpha \in (0,1)$ of the house in period 2. The homeowner stays in place while taking advantage of the proceeds and the AWM guarantee. The mortgageor remains in the house and consumes periodic rent during a finite amount of time (until the end of period 2). The limiting case $\alpha \to 1$ i.e. full reverse mortgaging is akin to a sale of the property.

3.1.1 First order conditions

If the choice of savings S is not constrained, the first order conditions are

$$\begin{aligned} U^{(0,1)}(c_1,h_c) + (L-P)U^{(1,0)}(c_1,h_c) + \\ +E\left[V^{(0,1)}(c_2,h_c) + (\alpha P(1+\tilde{\theta}) - L(1+r_m) - T)V^{(1,0)}(c_2,h_c)\right] &= 0 \quad (26) \\ -U^{(1,0)}(c_1,h_c) + (1+r)\,E\left[V^{(1,0)}(c_2,h_c)\right] &= 0\,. \quad (27) \end{aligned}$$

After eliminating $U^{(1,0)}(c_1,h_c)$ the first condition becomes

$$\begin{aligned} &U^{(0,1)}(c_1,h_c) + E\left[V^{(0,1)}(c_2,h_c)\right] = \\ &\quad = E\left[\left((P-L)(1+r) - \alpha P(1+\tilde{\theta}) + L(1+r_m) + T\right) V^{(1,0)}(c_2,h_c)\right]\,. \end{aligned} \tag{28}$$

If the following conditions are satisfied:

1. Home is 100% financed by the mortgage ($P = L$)
2. The return on housing matches the mortgage rate $\theta = r_m$
3. The house is sold or 100% reverse-mortgaged in period 2 ($\alpha = 1$)

then the housing decision is based on how the expected marginal housing consumption and the expected marginal consumption of non-housing good in period 2 compare to marginal cost of ownership

$$\frac{U^{(0,1)}(c_1,h_c) + E\left[V^{(0,1)}(c_2,h_c)\right]}{E\left[V^{(1,0)}(c_2,h_c)\right]} = T\,, \tag{29}$$

which can also be written as a trade-off between housing and non-housing consumptions in period 1 v.s. the present value of the marginal cost of ownership

$$\frac{U^{(0,1)}\left(c_1, h_c\right) + E\left[V^{(0,1)}\left(c_2, h_c\right)\right]}{U^{(1,0)}\left(c_1, h_c\right)} = \frac{T}{1+r} \,. \tag{30}$$

In applications that follow we will assume that housing is 100% financed by the mortgage ($P = L$). The first FOC then is

$$\begin{aligned} U^{(0,1)}\left(c_1, h_c\right) + E\left[V^{(0,1)}\left(c_2, h_c\right)\right] = \\ = -E\left[\left(P\left(\alpha(1+\tilde{\theta}) - (1+r_m)\right) - T\right) V^{(1,0)}\left(c_2, h_c\right)\right] \,. \end{aligned} \tag{31}$$

The cost of ownership further increases in bad times when the realized house price growth doesn't make up for the mortgage rate ($\theta < r_m$) or the equity is not fully released via sale or reverse mortgaging ($\alpha < 1$).

3.2 Workout offered

A risk neutral insurer offers a fairly priced insurance to remove the risk of falling underwater. Such insurance can be standardized to depend on the loan to value ratio

$$\lambda = \frac{L}{P} \tag{32}$$

and the distributional characteristics of $\tilde{\theta}$. The mortgage provider adds an *automatic workout* W to the loan so that consumption in period 2 is shielded from the house price risk

$$c_2 = y_2 + S(1+r) + (\alpha P(1+\tilde{\theta}) - L(1+r_m) + \tilde{W} - T)h_c \tag{33}$$

where $\tilde{W} = W\left(\tilde{\theta}\right)$ is the insurance payoff contingent on realisation of $\tilde{\theta}$. As a first attempt to design the negative equity insurance we impose the following condition

$$\alpha P(1+\tilde{\theta}) - L(1+r_m) + \tilde{W} \geq 0 \qquad \text{for all} \qquad \theta \,, \tag{34}$$

so that the household never falls underwater. This is a strong requirement as most homeowners who get underwater do not default on mortgage payments. However, underwater households are likely to cut back on spending. This is bad for the economy and therefore the ultimate goal of workouts is to prevent that happening. To achieve this, the mortgage provider crafts the workout W by taking a long position in αP puts on θ, each having payoff $(K-\theta)^{+}$

$$\tilde{W} = W\left(\tilde{\theta}\right) = \alpha P \max\left(K-\tilde{\theta},0\right) = \alpha P\left(K-\tilde{\theta}\right)^{+} \ . \tag{35}$$

If the strike price K is equal to

$$K = \frac{L\left(1+r_m\right)-\alpha P}{\alpha P} = \frac{\lambda}{\alpha}\left(1+r_m\right)-1 \ . \tag{36}$$

then the following Proposition holds.[6]

Proposition 3 *A home loan with automatic workout W never never falls in negative equity.*

Proof. For $\alpha > 0$ we have

$$\begin{aligned}
\alpha P(1+\tilde{\theta}) - L(1+r_m) + \tilde{W} &= \alpha P(1+\tilde{\theta}) - L(1+r_m) + \alpha P\left(K-\tilde{\theta}\right)^{+} && (37)\\
&= \alpha P(1+\tilde{\theta}) - L(1+r_m) && (38)\\
&\quad +\alpha P \max\left(\frac{L\left(1+r_m\right)-\alpha P}{\alpha P} - \tilde{\theta},0\right) && (39)\\
&= \alpha P(1+\tilde{\theta}) - L(1+r_m) && (40)\\
&\quad + \max\left(L\left(1+r_m\right) - \alpha P\left(1+\tilde{\theta}\right),0\right) && (41)\\
&= \max\left(0, \alpha P(1+\tilde{\theta}) - L(1+r_m)\right) \geq 0\ , && (42)
\end{aligned}$$

which proves that the balance will always be non-negative. When $\alpha = 0$ there are no proceeds from sale of the house to hedge, the workout degenerates to $W = L\left(1+r_m\right)$ and the left hand side of (34) is zero. In this situation the loan has *de facto* to be repaid from earnings y_1, y_2. After this happens the protection expires and the bequest is then exposed to house price risk. ■

In practice, puts can be written on a risk variable ξ which is tradable and highly correlated with θ, such as a home price index. In this case workouts can be provided by a competitive risk-neutral insurer. Insurer will use the market where the θ risk is traded to delta hedge its exposure. This is in contrast to pooling house price insurance contracts

[6] When $\alpha = 0$ there are no proceeds from sale of the house and the workout degenerates to $W = L\left(1+r_m\right)$.

together. Typical risks, such as fire risk, are more amenable to traditional insurance practices, such as pooling policies together to diversify the risk. However, in case of house price risk such diversification is difficult to achieve. Here, the risk insured includes a burst of a price bubble. In practice all mortgaged contracts will be affected following a sudden drop in house prices. Insurer may default if traditional diversification is used. We assert that delta hedging via the financial market will do a better job to hedge the house price risk.

3.2.1 Workout premium

To illustrate how the house price risk can be priced and delta hedged we specify the risk variable to

$$\tilde{\theta} = \begin{cases} \theta_u & \text{with probability} \quad 1-p \\ \theta_d & \text{with probability} \quad p \end{cases}, \tag{43}$$

where p is the actual probability of house price going down. We also require $-1 \leq \theta_d \leq r < r_m \leq \theta_u < \infty$ to exclude arbitrage. Supplying one house price insurance contract provides the insurer with insurance premium π at time 1. However, it also exposes the insurer to the risk of being liable for paying $\left(K-\tilde{\theta}\right)^+$ at maturity if house prices fall.

Proposition 4 *Fairly priced continuous workout premium is*

$$\pi = \frac{1}{1+r}\mathbb{E}\left[\left(K-\tilde{\theta}\right)^+\right] = \frac{1}{1+r}\left[q\left(K-\theta_d\right)^+ + (1-q)\left(K-\theta_u\right)^+\right], \tag{44}$$

where $\mathbb{E}$ *denotes taking expectation under the pricing measure and*

$$q = \frac{\theta_u - r}{\theta_u - \theta_d} \tag{45}$$

is the corresponding (risk-neutral) probability.

Proof. Investing \$1 in the riskless asset gives $1+r$ in both states of nature. Investing \$1 in the asset perfectly correlated with house price index yields either $1+\theta_u$ or $1+\theta_d$. Therefore, the state price vector can be obtained by solving the system

$$\begin{cases} 1 = (1+r)\,\psi_d + (1+r)\,\psi_u \\ 1 = (1+\theta_d)\,\psi_d + (1+\theta_u)\,\psi_u \end{cases}, \tag{46}$$

which gives

$$\{\psi_d, \psi_u\} = \left\{ \frac{1}{1+r} \frac{\theta_u - r}{\theta_u - \theta_d}, \frac{1}{1+r} \frac{r - \theta_d}{\theta_u - \theta_d} \right\} \tag{47}$$

and the fair pricing (44) and (45) follows

$$\pi = (K - \theta_d)^+ \psi_d + (K - \theta_u)^+ \psi_u \,. \tag{48}$$

■

3.2.2 Hedging the house price risk

In addition to the cost π of the workout insurance[7] it is also possible to find the replicating strategy. The insurer uses the premium $\pi > 0$ collected from the household to form a delta hedging portfolio

$$\pi = \Delta + b \,, \tag{49}$$

where Δ is the position in the house price index and b is the position in the riskless asset. There is no cash left on hand in building the delta hedge portfolio, which is consistent with no arbitrage profit being made at time 1. Similarly, at time 2, the hedge portfolio $\{\Delta, b\}$ replicates the workout contract to generate exactly the amount of refund needed

$$\begin{cases} \Delta (1 + \theta_d) + b (1 + r) = (K - \theta_d)^+ \\ \Delta (1 + \theta_u) + b (1 + r) = (K - \theta_u)^+ \,. \end{cases} \tag{50}$$

Solving this system we obtain the hedge ratio Δ

$$\Delta = \frac{(K - \theta_u)^+ - (K - \theta_d)^+}{\theta_u - \theta_d} \leq 0 \,, \tag{51}$$

as well as the position in the riskless asset

$$b = \frac{(K - \theta_d)^+ (1 + \theta_u) - (K - \theta_u)^+ (1 + \theta_d)}{(\theta_u - \theta_d)(r + 1)} \geq 0 \,. \tag{52}$$

The sign of b (Δ) is positive (negative) because $\theta_u > \theta_d$ implies $(1 + \theta_u) > (1 + \theta_d)$ and $(K - \theta_d)^+ > (K - \theta_u)^+ \geq 0$. Therefore, the insurer takes a short position in the house

[7] Shiller, Wojakowski, Ebrahim and Shackleton [11] derive a closed form formula for the analogue of premium π in a continuous-time model.

price index and invests the proceeds and the premium π in the riskless asset at time 1. When the index decreases, this frees a positive amount of cash from the hedge portfolio to provide exactly the amount $\left(K-\tilde{\theta}\right)^{+} \geq 0$ required for paying the workout to the homeowner.

3.2.3 First order conditions with full workout

If full workout is offered the household chooses housing consumption and savings $\{h_c^*, S^*\}$ to maximize

$$\max_{\{h_c,S\}} U\left(c_1,h_c\right)+E\left[V\left(c_2,h_c\right)\right] \tag{53}$$

$$\text{s.t.} \quad c_1 = y_1 - S - (\alpha P\pi + P - L)h_c$$
$$c_2 = y_2 + S(1+r) + (\alpha P(1+\tilde{\theta}) - L(1+r_m) + \alpha P\left(K-\tilde{\theta}\right)^{+} - T)h_c\ .$$

For unconstrained choice of S the first order conditions are

$$\begin{aligned} U^{(0,1)}\left(c_1,h_c\right)+(L-P-\alpha P\pi)U^{(1,0)}\left(c_1,h_c\right)+ & \\ +E\left[V^{(0,1)}\left(c_2,h_c\right)+(\alpha P(1+\tilde{\theta})-L(1+r_m)+\alpha P\left(K-\tilde{\theta}\right)^{+}-T)V^{(1,0)}\left(c_2,h_c\right)\right] &= 0 \quad (54) \\ -U^{(1,0)}\left(c_1,h_c\right)+(1+r)\,E\left[V^{(1,0)}\left(c_2,h_c\right)\right] &= 0 \quad (55) \end{aligned}$$

By slightly abusing notation (introducing "curly" $\mathcal{U} = U + V$) denote

$$\mathcal{U}_{h_c} = U^{(0,1)}\left(c_1,h_c\right)+V^{(0,1)}\left(c_2,h_c\right) \tag{56}$$

$$\mathcal{U}_{c_1} = U^{(1,0)}\left(c_1,h_c\right) \tag{57}$$

$$\mathcal{U}_{c_2} = V^{(1,0)}\left(c_2,h_c\right) \tag{58}$$

Replacing $U^{(1,0)}\left(c_1,h_c\right)$ in the first equation (54) we obtain

$$\begin{aligned} \frac{E\left[\mathcal{U}_{h_c}\right]}{E\left[\mathcal{U}_{c_2}\right]} &= \left(\alpha P\pi+P-L\right)(1+r)+ \quad (59) \\ &+\frac{E\left[(L(1+r_m)-\alpha P(1+\tilde{\theta})-\alpha P\left(K-\tilde{\theta}\right)^{+}+T)\mathcal{U}_{c_2}\right]}{E\left[\mathcal{U}_{c_2}\right]} \end{aligned}$$

When housing is 100% financed by the mortgage ($P = L$, $\lambda = 1$) we have

$$K = \frac{1}{\alpha}(1 + r_m) - 1 \tag{60}$$

which gives

$$\begin{aligned} L(1+r_m) - & \quad \alpha P(1+\tilde{\theta}) - \alpha P \left(\frac{1}{\alpha}(1+r_m) - 1 - \tilde{\theta} \right)^+ \\ = & \quad P\left((1+r_m) - \alpha\left(1+\tilde{\theta}\right)\right) - P\left((1+r_m) - \alpha\left(1+\tilde{\theta}\right)\right)^+ \quad (61) \\ = & \quad P \min\left\{(1+r_m) - \alpha\left(1+\tilde{\theta}\right), 0\right\} \; . \quad (62) \end{aligned}$$

The above expression (59) further simplifies to

$$\begin{aligned} \frac{E\left[\mathcal{U}_{h_c}\right]}{E\left[\mathcal{U}_{c_2}\right]} = & \quad \alpha P \pi (1+r) + \quad (63) \\ & + \frac{E\left[(P \min\left\{(1+r_m) - \alpha\left(1+\tilde{\theta}\right), 0\right\} + T)\mathcal{U}_{c_2}\right]}{E\left[\mathcal{U}_{c_2}\right]} \; . \end{aligned}$$

Note that elements on the right hand side express costs or benefits of ownership. Costs are non-negative while benefits are non-positive quantities. The two costs are: the maintenance costs $T \geq 0$ and the future value of the workout insurance premium unit cost $\pi \geq 0$ (paid at time 1).

It is particularly simple to interpret the model when $\alpha = 1$. Benefits materialize only when the return on housing exceeds the cost of mortgage financing i.e. when $\theta > r_m$. Benefits appear as negative costs: $\min\left(0, r_m - \tilde{\theta}\right) \leq 0$. If, in addition, the return on housing never exceeds the mortgage rate $\theta \leq r_m$, the trade-off between housing h_c and future (present) consumption c_2 (c_1) is determined by the future (actual) value of the workout cost $P\pi$ and the (present value of) the maintenance cost T

$$\begin{aligned} \alpha = 1 \wedge \theta \leq r_m, \forall_\theta \quad \Longrightarrow & \quad \frac{E\left[\mathcal{U}_{h_c}\right]}{E\left[\mathcal{U}_{c_2}\right]} = P\pi(1+r) + T \quad (64) \\ \Longleftrightarrow & \quad \frac{E\left[\mathcal{U}_{h_c}\right]}{\mathcal{U}_{c_1}} = P\pi + \frac{T}{1+r} \quad (65) \end{aligned}$$

The trade-off between current consumption c_1 and future consumption c_2 is, as usual, determined by the riskless rate of borrowing r.

$$\frac{\mathcal{U}_{c_1}}{E\left[\mathcal{U}_{c_2}\right]} = 1 + r \tag{66}$$

3.3 Specification

We set $L = P$ and focus on comparing two cases:

1. **Without workout**, where we solve the system of FOCs (31) and (27)

$$\left\{ \begin{array}{rcl} E\left[\mathcal{U}_{h_c}\right] & = & -E\left[\left(P\left(\alpha(1+\tilde{\theta}) - (1+r_m)\right) - T\right)\mathcal{U}_{c_2}\right] \\ \mathcal{U}_{c_1} & = & (1+r)\,E\left[\mathcal{U}_{c_2}\right] \end{array} \right. \tag{67}$$

subject to budget constraints obtained from (20) for $L = P$

$$c_1 = y_1 - S \tag{68}$$

$$c_2 = y_2 + S(1+r) + \left[P\left(\alpha(1+\tilde{\theta}) - (1+r_m)\right) - T\right] h_c\,. \tag{69}$$

2. **With workout**, where we solve the system of FOCs (63) and (55)

$$\left\{ \begin{array}{rcl} E\left[\mathcal{U}_{h_c}\right] & = & \alpha P \pi\,(1+r)\,E\left[\mathcal{U}_{c_2}\right] - E\left[(P\left(\alpha\left(1+\tilde{\theta}\right) - (1+r_m)\right)^{+} - T)\mathcal{U}_{c_2}\right] \\ \mathcal{U}_{c_1} & = & (1+r)\,E\left[\mathcal{U}_{c_2}\right] \end{array} \right. \tag{70}$$

subject to budget constraints obtained from (53) for $L = P$ and for K given by (60)

$$c_1 = y_1 - S - \alpha P \pi h_c \tag{71}$$

$$c_2 = y_2 + S(1+r) + \left[P\left(\alpha\left(1+\tilde{\theta}\right) - (1+r_m)\right)^{+} - T\right] h_c\,. \tag{72}$$

We note in particular that with utility specifications (21), (22) and (23) we have

$$V^{(0,1)}\left(c_2, h_c\right)\Big|_{\alpha=1} = 0 \implies E\left[\mathcal{U}_{h_c}\right]\big|_{\alpha=1} = U^{(0,1)}\left(c_1, h_c\right). \tag{73}$$

Furthermore, it is clear that the major difference between systems (67)–(68) and (70)–(72) is that in the latter case the homeowner pays the premium $\alpha P \pi h_c$ upfront in order to remain "above water" later on

$$P\left(\alpha\left(1+\tilde{\theta}\right) - (1+r_m)\right)^{+} \geq 0 \tag{74}$$

3.4 Simulations

We numerically solve two systems, one with and one without workout, for h, S. When $\alpha = 1$ the system without workout is

$$\begin{cases} \frac{(1-\eta)\left(h^{\eta}(y_1-S)^{1-\eta}\right)^{1-\gamma}}{\beta(y_1-S)(r+1)} = p\left(B - hA_d\right)^{-\gamma} + (1-p)\left(B - hA_u\right)^{-\gamma} \\ \frac{\eta\left(h^{\eta}(y_1-S)^{1-\eta}\right)^{1-\gamma}}{\beta h} = pA_d\left(B - hA_d\right)^{-\gamma} + (1-p)A_u\left(B - hA_u\right)^{-\gamma} \end{cases} \tag{75}$$

where

$$A_d = P\left(r_m - \theta_d\right) + T \tag{76}$$

$$A_u = P\left(r_m - \theta_u\right) + T \tag{77}$$

$$B = S\left(r + 1\right) + y_2 \tag{78}$$

The system with workout is

$$\begin{cases} \frac{(1-\eta)(\theta_d-\theta_u)\left(h^{\eta}\left(\frac{C}{(r+1)(\theta_d-\theta_u)}\right)^{1-\eta}\right)^{1-\gamma}}{\beta C} = p\left(B - hA_{d,W}\right)^{-\gamma} + (1-p)\left(B - hA_{u,W}\right)^{-\gamma} \\ \frac{\eta(\theta_d-\theta_u)\left(h^{\eta}\left(\frac{C}{(r+1)(\theta_d-\theta_u)}\right)^{1-\eta}\right)^{1-\gamma}}{\beta h} = -p\left(PD + \left(\theta_u - \theta_d\right)A_d\right)\left(B - hA_{d,W}\right)^{-\gamma} \\ \qquad -(1-p)\left(PD - \left(\theta_u - \theta_d\right)A_u\right)\left(B - hA_{u,W}\right)^{-\gamma} \end{cases} \tag{79}$$

where

$$A_{d,W} = P\min\left(0, r_m - \theta_d\right) + T \tag{80}$$

$$A_{u,W} = P\min\left(0, r_m - \theta_u\right) + T \tag{81}$$

$$C = hPD - (r+1)\left(S - y_1\right)\left(\theta_d - \theta_u\right) \tag{82}$$

$$D = \left(r - \theta_d\right)\left(r_m - \theta_u\right)^{+} + \left(\theta_u - r\right)\left(r_m - \theta_d\right)^{+} \tag{83}$$

3.5 Parameters

First period is 20 years and household earns 50,000 a year, which gives $y_1 = 1,000,000$. We assume the same to occur during the second 20-years period, with $y_2 = 1,000,000$. In the alternative set of simulations we also assume $y_2 = 200,000$ (social security payments) instead of a million, so that the main source of living is housing and savings in period

2. The coefficient of time preference estimated by Cocco [4] is 0.98 per annum. Because periods 1 and 2 are 20 years "apart," therefore we use

$$\beta = (0.98)^{20} = 0.66761 \tag{84}$$

We adopt other numerical parameters estimated by Campbell and Cocco [2] and Cocco [4]: risk aversion ($\gamma = 3$) and the preference for housing ($\eta = 0.1$). We compound the riskless rate (2% per annum) and the mortgage rate (4% per annum) as follows

$$(1+0.02)^{20} = 1+r \implies r = 0.48595 \tag{85}$$

$$(1+0.04)^{20} = 1+r_m \implies r_m = 1.1911 \tag{86}$$

Note that the mortgage rate may seem high at first sight (119.11%), but this corresponds to interest cumulated at 4% p.a. over a period of 20 years. We also (arbitrarily) set $\theta_u = 0.5$, $\theta_d = -0.5$, the probability of the house terminating underwater $p = \frac{1}{4}$ and the house value to $100,000 (about 5% of the total income).

Also, in this first set of comparisons, the loan to value ratio λ has been set to 100% and the maintenance costs T to zero.

4 Results

	$y_2 = 1,000,000$		$y_2 = 200,000$	
	Workout			
	no	yes	no	yes
h^*	1.69216	2.78985	1.14761	1.89205
S^*	$-206,755$	$-323,960$	181,590	102,102
c_1	1,206,750	1,191,560	818,410	808,108
$E[c_2]$	533,520	518,612	361,828	351,718
U_1	$-6.34569 \cdot 10^{-12}$	$-5.94916 \cdot 10^{-12}$	$-1.37967 \cdot 10^{-11}$	$-1.29346 \cdot 10^{-11}$
Δc_1 [$]		$55,666.6		$37,752.6

Automatic workout mortgage improves the expected utility. In our simulations the utility increase amounts to increasing consumption in period 1 by about $40,000 to $55,000, which represents up to 55% of the initial value of the house ($100,000).

AWM feature *increases* housing consumption at optimum. It also increases borrowing (reduces savings). We assert that the extra insurance may inhibit the precautionary saving motive, inducing dis-saving. However consumption is reduced in both periods as more housing volume is purchased. The probable cause of this reduction is the presence of workout premium which represents extra cost to prospective homeowners. Also, there is no mechanism to borrow against home equity in our simple model.

Some interesting effects are illustrated on graphs. Dashed line denote standard mortgages. Solid lines are AWMs. Demand for housing h dominates if AWMs are used. This is the case for the entire range of housing preference parameter η as well as for the the risk aversion parameter γ. (Figures 1 & 2.) Converse observations apply to savings S. (Figures 3 & 4.) Interestingly, demand for housing attains maxima for those intending to reverse-mortgage $\alpha \approx 40\%$ of equity (without workout) and about $\alpha \approx 80\%$ of equity (AWMs). (Figure 5.) We therefore assert that AWMs are particularly suited for those who intend to sell or reverse-mortgage a significant fraction of their home equity. Consequently, savings under AWM and for high α's decrease accordingly. (Figure 6.)

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5 Figures

Solid line is AWM.

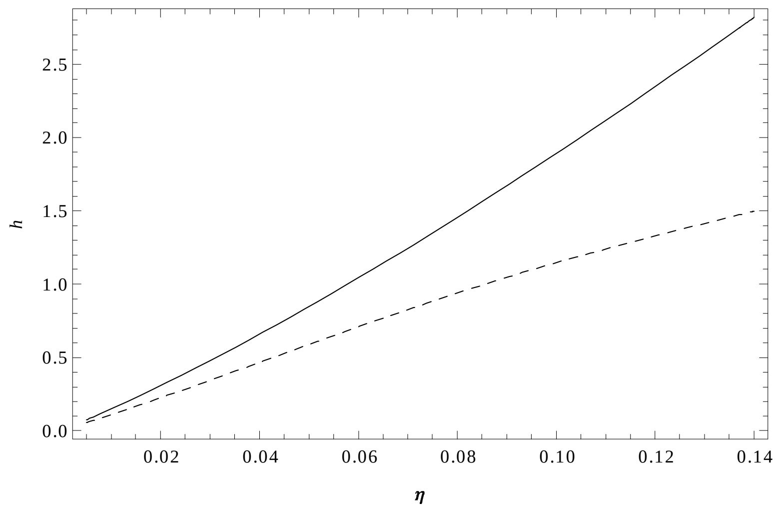

Figure 1: Demand for housing h as a function of housing preference parameter η.

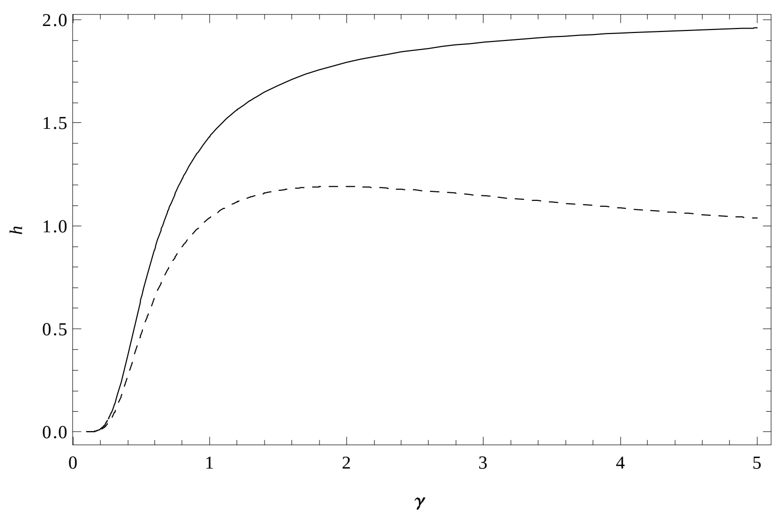

Figure 2: Demand for housing h as a function of risk aversion parameter γ.

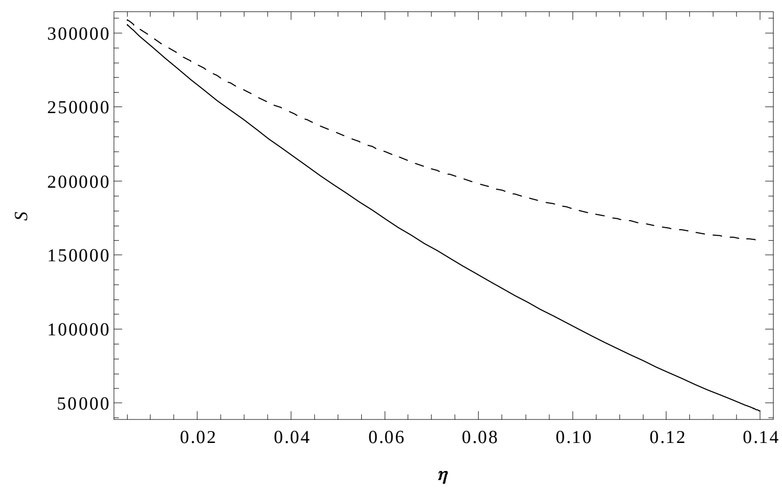

Figure 3: Savings S as a function of housing preference parameter η.

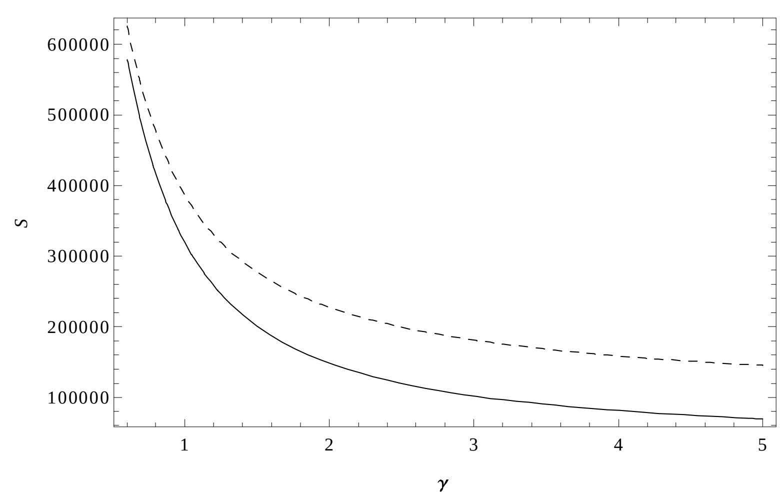

Figure 4: Savings S as a function of risk aversion parameter γ.

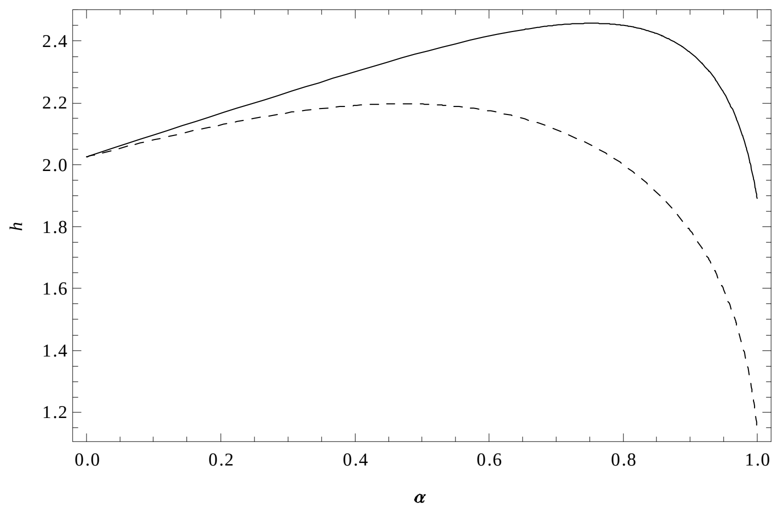

Figure 5: Demand for housing h as a function of reverse-mortgaging parameter α.

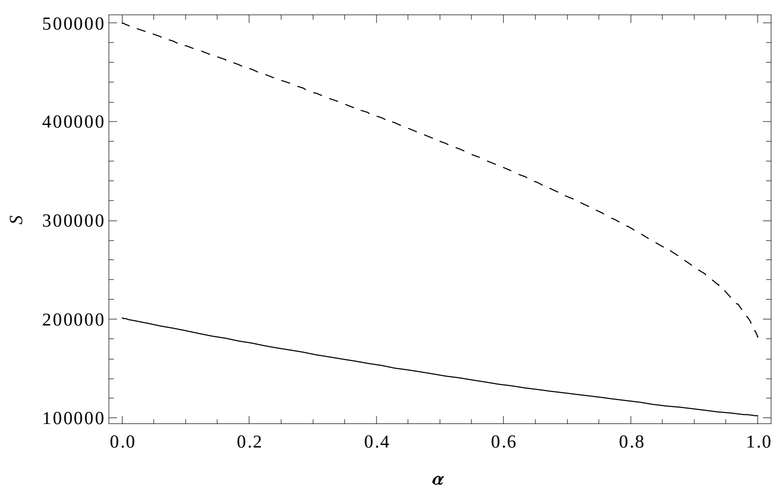

Figure 6: Savings S as a function of reverse-mortgaging parameter α.