Hedge Fund
Asymmetric Dependences and Performance Evaluation

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Abstract

The study presents an application of multivariate regime switching copula models, in order to model the joint distributions of selected hedge fund classes / strategies and traditional capital markets. Our empirical interest focuses on testing for the presence of any asymmetric dependence structures between these sample hedge fund classes and benchmark capital market returns. A number of different specifications of canonical vine copulas are incorporated into a Markov switching two-regime framework to capture hedge fund reactions in bear and bull market phases. We apply this empirical framework on core and complementary hedge fund strategies as well as on traditional asset classes and produce substantial evidence of asymmetric dependence structures between hedge fund strategies and conventional capital markets. Furthermore, our empirical results reveal a high dependence regime with high estimated correlations and a low dependence regime with lower estimated correlations.

Keywords: hedge fund classes; traditional markets; dependence structure; alternative investment strategies

EFMA codes: 370, 380, 310, 570.

JEL: G11, G12, G13, G15.

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1. Introduction

During the last two decades hedge fund industry has attracted a significant portion of the global investment capitals as it is indicated by the exceptional growth rate of the number of funds and the capital inflow allocates in the different hedge fund strategies. The incorporation of alternative, non-traditional investment tools, the specific risk characteristics and the persistently superior 'absolute' returns have attracted investors' funds into hedge fund allocation. According to the Hedge Fund Research (HFR), an analytics and data-feed provide, in 1990 there were 530 hedge funds with approx. USD 50 mln. in assets under management. However, in the beginning of 2008, the exponential growth of the hedge fund industry shot these figures up to approximately 9,500 funds with more than USD 2 tln. in assets under management. The performance of alternative hedge fund investment strategies against traditional asset allocation choices (stocks, bonds, currencies, mutual funds etc.) indicates both high annual returns and low volatilities.

The environment of highly volatile capital markets, over the last years, has led private and institutional investors to seek alternative investment vehicles and diversify investment risks. At the same time, it has been argued that, since hedge funds implement market neutral strategies[3], they are capable of producing attractive risk adjusted returns, independent of market trends, through fund protection in volatile and uncertain market times. However, the attractive hedge fund risk-return characteristics against traditional investment alternatives are associated with a number of limitations. The complex and time-varying risk exposure of hedge fund strategies to different asset class factors induces substantial constraints in measuring their risk-return profile. Their flexibility in international investments in various asset classes facilitates the implementation of profitable but complicated strategies. Contrary to a typical buy-and-hold strategy of mutual funds, hedge fund managers design vehicles that take advantage of investment opportunities in illiquid securities, distressed firms, emerging markets, arbitrage, and mergers and acquisitions, inter alia. It has been argued that the abnormal returns of hedge funds are associated to a liquidity risk premium and a lockup period in their investments; hedge funds with a lockup provision attain annual returns that are about 4% higher relative to funds with no-lockup period (Aragon, 2007). Recent studies have indicated to the option-like payoff feature of hedge funds relative to underlying asset returns (Fung and Hsieh, 2001, 2002; Mitchell and Pulvino, 2001; Agarwal and Naik, 2004).

This paper intends to contribute to a growing body of literature on hedge fund investment strategies and performance evaluation. For that, we apply multivariate regime switching copula models to model the joint distributions of selected hedge fund classes and benchmark capital market returns. Our empirical interest focuses on testing for the presence of any asymmetric dependence structures between sample hedge fund classes and benchmark markets (equity, bond, interest rate and commodity instruments). A number of different canonical vine copulas are employed in a two-regimes framework to capture hedge fund reactions in bear and bull market conditions. Canonical vine copulas represent a flexible way of expressing multivariate joint distributions into bivariate conditional copulas. The rest of the paper is organized as follows: section 2 provides a summary of recent empirical literature; section 2 analyses the theoretical concepts of copula functions, dependence structures and marginal distributions; section 4 discusses an empirical application and findings on these theoretical concepts. Finally, section 5 concludes.

[3] Despite the widespread impression that hedge funds are in a position to attain 'market neutrality', recent empirical studies indicate the opposite outcome (Patton, 2007).

2. Literature Review

Past literature has studied different aspects of hedge funds, their investment strategies and their performance evaluation. The variety of investment strategies employed and the exposure to a wide range of financial markets ensure the absence of a typical representative investment style for hedge funds and support the diversifiable property of hedge funds. Empirical evidence indicates that hedge fund strategies exhibit low correlation with traditional asset returns as well as with themselves (Fung and Hsieh, 1999; Schneeweis and Spurgin, 1998; Amenc et al., 2003; Agarwal and Naik, 2004). The inclusion of hedge funds in portfolios of traditional assets improves substantially their risk-return profile (Schneeweis et al., 2002). On the other hand, it has been argued that hedge funds may not offer a superior performance as a stand-alone investment (Amin and Kat, 2003). However, investing a part of portfolio assets into hedge funds improves overall portfolio performance. Hedge funds appear to efficiently substitute bonds and cash in an equity portfolio because of their attractive high returns combined with low volatility and low correlation with equity markets. Cointegration analysis supports a long-run relationship between specific hedge fund strategies and traditional financial assets, while potential diversification benefits are available for some hedge fund strategies in the framework of tactical asset allocation (Füss and Kaiser, 2007).

Under the Markowitz mean-variance portfolio theory, the inclusion of hedge funds in a portfolio with traditional assets appears to enhance the risk-reward trade-off of a traditional portfolio (Agarwal and Naik, 2004). However, this framework assumes that either portfolio returns are normally distributed or investor's utility function is quadratic. However, the reliability of the mean-variance framework is questionable in case a portfolio is constructed by hedge funds (Amenc and Martellini, 2002; Brunel, 2004; Kat, 2005; Till, 2005; *inter alia*). The dynamic trading hedge fund strategies turn invalid the core assumption of normality and diversification benefits. A number of studies supports that the reported returns of hedge funds are not normally distributed (Brooks and Kat, 2001; Lo, 2001; Geman and Kharoubi, 2003; Alexiev, 2004)[4].

It is now argued that the low reported correlations between hedge funds and traditional assets do not represent the real relationship between asset classes. Evidence of skewness and 'asymmetric dependence' in the joint distributions and their importance in asset allocation has been indicated in the empirical finance literature. The term asymmetric dependence emphasizes the dependence between capital markets that appears to be stronger in period of low returns compared to normal or bullish market conditions. Patton (2004) demonstrates the impact of both skewness and asymmetric dependence on equity portfolio decisions for the constant relative risk aversion (CRRA) investor. Longin and Solnik (2001) use extreme value theory to provide evidence of asymmetric increased correlations in equity markets during turbulent periods. Ang and Bekaert (2002) and Ang and Chen (2002) reproduce the main implications of the study of Longin and Solnik (2001) based on Markov switching models. Investment strategies that account for upward and downward movements of financial markets clearly outperform static approaches (Campbell et al., 2002, Bae et al., 2003; Okimoto, 2008; *inter alia*).

Recent empirical studies in hedge fund portfolios reveal two important issues. First, the correlation between hedge funds and traditional financial markets has gradually increased. Second, the relationship between hedge funds and traditional asset classes should be examined in a framework beyond the conventional linear approaches, as the complex risk exposure (i.e. volatility risk, liquidity risk, credit risk etc.) of different hedge fund strategies indicates. Indeed, hedge fund risk exposure appears to be different at times of financial crises compared to normal market conditions (Billio et al., 2008). However, empirical literature on hedge funds has not investigated asymmetric dependences of hedge fund and traditional asset class returns in depth. Lo (2001) has studied the 'phase locking' behaviour of otherwise uncorrelated markets that suddenly become synchronized, especially during downward market periods. This phenomenon may partly explain the turbulent

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[4] Even if the distribution of hedge fund are not normal distributed, recent reports reveal that the majority of European hedge fund managers ignore the third and fourth moment (i.e. skewness and kurtosis) and rely on the traditional mean-variance analysis to evaluate hedge fund performance (EDHEC, 2003).

hedge fund response during and after major financial events (e.g. Asian crisis; Russian crisis; collapse of LTCM fund). More recently, Billio et al. (2006) have employed different switching regime models to demonstrate that, during turbulent market periods, hedge funds appear to exhibit higher exposure to traditional financial market movements. Furthermore, conventional, simple linear models do not account for the time-varying behaviour of hedge funds and the resulting asymmetric dependence. Schneeweis et al. (2002) conclude increased unconditional and conditional correlations between hedge funds and equity markets during turbulent market periods. On the other hand, Li and Kazemi (2007) reject the presence of asymmetry in conditional correlation between daily hedge fund returns and other investment instruments.

A number of past empirical studies investigate the presence of symmetric or asymmetric dependences across financial asset classes by incorporating a linear correlation coefficient (Pearson coefficient). However, this appears to be an inappropriate general dependence measure of a vector of random variables, as it is problematic in case the normality assumption is raised. Furthermore, this linear correlation coefficient is unstable under varying calculation methods and suffers from specific flaws when non-linear dependences are assumed across asset markets. When an elliptical distribution, such as the multivariate Gaussian or the Student's-*t* distribution, adequately describes asset returns, then the linear correlation coefficient is both capable and informative in depicting asset comovements. Otherwise, the linear correlation coefficient leads inevitably to misleading implications (Embrechts et al., 2002).

Clearly, the loss of information about the dependence structure of two markets by using the elliptical distribution assumption can be described by scatter plots as in figure 1. The collection of points describes the returns each time for the S&P index and a popular hedge fund strategy, the Event Driven strategy, are displayed. In the left part of figure 1, the scatter plot obviously support the positive dependence between the two assets, but, the linear correlation cannot capture the left tail dependence represents jointly high negative returns for the markets. In the right panel on the same figure, we plot with solid red line the contour at the 95% confidence intervals for the bivariate Gaussian distribution, mean values and covariance matrix corresponding to the empirical values. Both, the presence of outlier's and the numerous points outside the ellipsoid shape demonstrate the inefficiency of the bivariate Gaussian distribution.

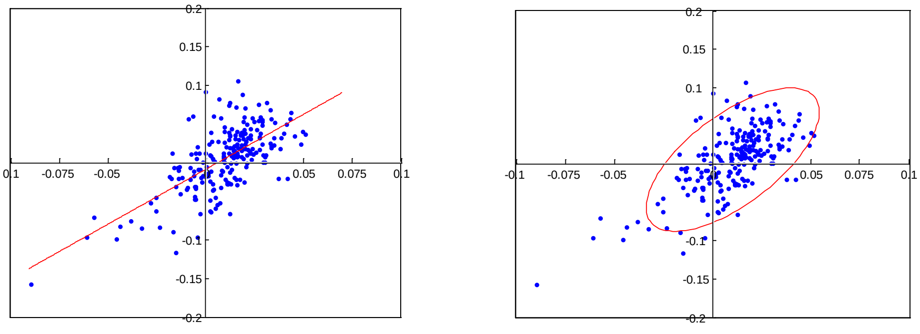


Figure 1: Left panel: Scatter plot of returns of S&P500 index vs. Event Driven hedge fund strategy. Right panel: Scatter plot of returns of S&P500 index vs. Event Driven hedge fund strategy and the Gaussian covariance ellipse for confidence level 95% with solid red line.

Copulas functions have been proposed recently in the financial economics literature as an alternative approach to overcome substantial drawbacks of the standard Pearson correlation coefficient. Generally, copula functions present a convenient way to describe the joint distribution between two or more random variables and are originally attributed to Sklar (1959). But, until very recently, the field of copula methodology has remained unreachable. Now it has been recognized as an important tool with significant applications in economics, insurance, finance and risk management. A concise survey on the use of the copula in finance and insurance is given by Embrechts et al. (2002). The application of copula methods to account for the dependences of insurance risk is adopted by professional bodies (see, International Actuarial Association) and the European Union "Solvency II" report (European Commission, 2004). Frees and Valdez (1999) provides an introduction of copulas in actuaries for the analysis of the relationships between multivariate outcomes. The explanation of the dependences developed between losses and claims in insurance industry is given by Isaacs (2003). In the field of risk management, Rosenberg and Schuermann (2006) use copulas to provide an integrated approach to deal jointly with market, credit and operational risks. Extensive review of dependence measures for risk management is found in the textbooks of McNeil *et al.* (2005), Denuit *et al.* (2005) and Alexander (2008). The advantages of dynamic evolution of copula models in risk management has suggested by Dias and Embrechts (2003), Patton et al., (2006c), Jondeau and Rochinger (2006), Xu (2009). Despite the flexibility of copula models to capture the dependence structure of time series Bee (2005) points out the theoretical and computational difficulties to set up and estimates copulas.

In the empirical studies, we have found applications of different copula models for modeling and estimating the dependence across international stock markets. Hu (2006) find left tail dependence using a mixed copula model for the S&P500 (US), FTSE100 (UK), Nikkei225(Japan) and Hang Seng (Hong Kong) indices, implying that these markets are more likely to crash together than to boom together. Jondeau and Rockinger (2006) support an increase of the dependency between European markets between 1980 and 1999, contrary to the dependency between the U.S. and European markets. They incorporate a dynamic version of the copula model with time-varying correlations and a Markov switching model. Also, the marginal distributions are modeled by the skewed student-t distribution with volatility, skewness, and kurtosis varying over time. Chen, et al (2004) recommend the Student's t-copula against the normal copula as an adequate measure for the dependence structure between equity returns as it can capture the asymmetric correlation and tail dependences. Likewise, the multivariate Student's t-copula seems to be a favorable choice than the multivariate normal copula when we test the dependence structure of a high number of assets (Mashal and Zeevi, 2002).

Until recently, the majority of empirical studies that use copula method have been limited in the bivariate case. Now a challenging task is the development of higher dimensional copula functions that will facilitate the investigation of dependence structure between more than two variables. Thus, the inclusion of many assets in the analysis of risk management, asset allocation etc. can be done in more realistic assumptions. In this direction recent works have addressed the implementation of higher dimension multivariate copula formulations either by extending the well known Archimedean copula family[5] (Joe, 1997) or proposing new methods such as the pair-copula constructions that is based on a graphical dependency models denoted as *regular vines* (Bedford and Cooke, 2001a, 2001b; Kurowicka and Cooke, 2002). In the first case, the multivariate Gaussian and Student-t copulas are more frequently used since they are easier to be built compared to the other Archimedean copulas. Mashal and Zeevi (2002) test the presence and statistically significance based on Student-t and normal copulas for three asset classes (equities, currencies and commodities). Fantazzini (2002) uses five future contracts traded in American market to highlight the importance of diversification benefits for the investors when they invest in low correlated assets. He controls for non-linear dependences by incorporating the copula theory in his analysis. The latter and recently proposed *regular vines* or *pair-decomposition* approach has been tested by Chollete et al. (2008) in a sample of nine equity indices. Also, a number of papers present the properties, compare and provide inferences the regular vines structure for modelling multivariate distributions (see among other, Kurowicka and Cooke, 2006; Aas et al, 2007, Berg and Aas).

[5] For a review of the most common multivariate extensions of the Archimedean's copula functions see Berg and Aas (2009).

3. Copula Functions, Dependence Structures and Marginal Distributions

Copula functions offer an attractive alternative empirical framework to capture linear, non-linear and tail dependences and can conveniently describe the joint distribution between two or more random variables (Sklar, 1959; Frees and Valdez, 1998; Embrechts et al., 2003). Indeed, copula functions allow for the link between different marginal distributions and dependence structures without imposing any restriction of normality on the joint distribution (such as linear dependence of financial assets). More specifically, copula functions describe the process in which uniform one-dimensional distributions can join to form multivariate distributions.

An important advantage of copulas against other stochastic dependence and multivariate distribution models is the simple two-step procedure for the estimation of the dependence structure across a sample of multivariate variables. In the first step, a parametric or non-parametric functional form is determined for each individual marginal distribution. In the second step, an appropriate copula function for reproducing the dependence structure between the marginal distributions is specified. However, the potential choice of models that can lead to misspecified margins inevitably affects the copulas function results. The scope of marginal modelling is to capture temporal dependencies of the financial univariate series, such as autocorrelation and time-varying volatility.

In the context of hedge funds, our empirical research investigates two joint directions: (a) the canonical vine copula that supports dependence structures in a multivariate context and is appropriate for multi-assets analysis and portfolio applications (Aas et al., 2007); and, (b) the Markov switching regime models that provide challenging characteristics for modelling asymmetric dependences in financial markets. In a similar framework, Okimoto (2008) investigates dependence structures in a bivariate setting of international equity markets incorporating three different Markov switching models: a pure Markov switching model; a Markov switching semi-asymmetric copula model (copula function introduced in only one regime); and a Markov switching asymmetric copula model (copula function introduced in both two regimes). The empirical findings support two types of asymmetric dependences: asymmetric dependence between bear and normal markets; and, asymmetric dependence with lower tail dependence in bear markets. Chollete et al. (2008) employ regime switching copula models incorporating the canonical vine approach to decompose the multivariate copula function into a product of bivariate conditional copulas. The empirical findings indicate an adequate performance of the canonical vine copula approach in replicating the ‘exceedance correlation’ in a sample of equity index returns.

3.1 Copula Function

Consider a n-dimensional random vector $X = (X_1, \dots, X_n)$ with a view to analyse the dependence of its components. The information on the distribution of a vector X is fully described by the following joint cumulative distribution function:

$$F_X(x) = F_X(x_1, \dots, x_n) = \mathbf{P}(X \leq x) = \mathbf{P}(X_1 \leq x_1, \dots, X_n \leq x_n) \quad (1)$$

where F_{X_i} describes the marginal distributions of X_i that is calculated through the joint distribution function. However, this structure of the joint distribution function mixes the information on the dependence between the different components of vector X with information of individual components themselves. Copulas are multivariate uniform distributions that decompose the above representation of the joint distribution function into two parts, the dependence structure and the marginal distributions. The fact that copulas functions apply to uniform distributions over the interval [0, 1] makes essential the extensive use of probability-integral and quantile transformations. Therefore, the initial random vector X with continuous marginal distribution functions $F_1, \ldots, F_n$, is transformed with the method of probability integral transformation to uniform marginal distributions $U_i \equiv F_i(X_i)$. A 'copula function' is the joint cumulative distribution function of the multivariate vector $U = (U_1, U_2, \ldots, U_n)$, that is denoted by:

$$C(u_1,\ldots,u_n) = P(U_1 \le u_1,\ldots,U_n \le u_n) \qquad (2)$$

A wide range of copula functions are available in the empirical literature, appropriate to capture different dependency relationships. The candidate copulas are Gaussian, Clayton, Gumbel, rotated Clayton and rotated Gumbel. This list includes some of the most popular copula in statistic and financial applications appropriate to capture important symmetric joint extreme events, asymmetric tail dependence etc. Gaussian copula is symmetric joint functional forms of the corresponding elliptical distributions. The Gaussian copula is quite restrictive as it provides only weak information about tail dependence. Different from elliptical distributions, Clayton and Gumbel copulas can capture asymmetric dependence as they assign more probability on single lower or upper tail dependence. In particular, Clayton copula is able to model only lower tail dependence, while Gumbel copula allows only upper tail dependence. The rotated Gumbel copula (survival copula) similar to Clayton has stronger lower tail dependence. In Appendix A, we provide the functional forms of the copulas.

3.2 Multivariate Dependence Structures

An interesting application of copula functions refers to developing and testing higher dimension copula functions to handle asymmetric and non-linear dependences of financial instruments in the construction and management of asset portfolios. The difficult task of handling higher dimension copulas is decomposed into a cascade of bivariate copulas. The following factorization for the joint density function shows clearly the pair-copula construction in capturing the portion of dependence structure for a random vector $X = (X_1, \ldots, X_n)$ in a joint density function:

$$f(x_1,\ldots,x_n) = f(x_n)\cdot f(x_{n-1} \mid x_n)\cdot f(x_{n-2} \mid x_{n-1}, x_n)\cdot \ldots \cdot f(x_1 \mid x_2,\ldots,x_n) \qquad (3)$$

In our study, we model the dependence structure of sample hedge funds and traditional equity markets following a special *regular vine* case called *canonical vine* (Kurowicka and Cooke, 2004). The density of the *n*-dimensional canonical vine is given by:

$$f(x_1,\ldots,x_n) = \prod_{k=1}^{n} f(x_k) \prod_{j=1}^{n-1}\prod_{i=1}^{n-j} c_{j,j+1|1,\ldots,j-1}\left(F(x_j \mid x_1,\ldots,x_{j-1}), F(x_{j+1} \mid x_1,\ldots,x_{j-1})\right) \qquad (4)$$

The advantage of canonical vine copula method is its ability to easily decompose any high dimension multivariate joint distributions in a sequence of bivariate copulas that capture the pair relationship between random variables[6]. The estimation process supposes that one variable plays the pivotal role. In our case we select each time a traditional indices (TR-ind) as the pivotal variable which is modelled with a group of hedge fund indices (HF-ind). In the first step we model the bivariate copulas of TR-ind with all other variables (HF-ind) in the system. Then we condition on TR-ind, we consider all bivariate conditional copulas of the first HF-ind with all other variables in the system etc. The dependence structure of a three dimensional canonical vine copula is graphically represented in figure 2.

[6] The joint probability function under the canonical structure in a three-dimensional case can be written as: $f(x_1, x_2, x_3) = c_{12}(F_1(x_1), F_2(x_2)) \cdot c_{13}(F_1(x_1), F_3(x_3)) \cdot c_{23|1}(F_{2|1}(x_2|x_1), F_{3|1}(x_3|x_1)) \cdot f_1(x_1) \cdot f_2(x_2) \cdot f_3(x_3)$, where c_{12} and c_{13} indicates the relationship between variables $x_1$and x_2; and x_1 and x_3, respectively, as they are captured by the selected copula and $c_{23|1}$ represents the relationship between the variables x_2 and x_3 conditioning on variable x_1. With features f_1, f_2 and f_3 we indicate the density functions for the variables x_1, x_2 and x_3, respectively.

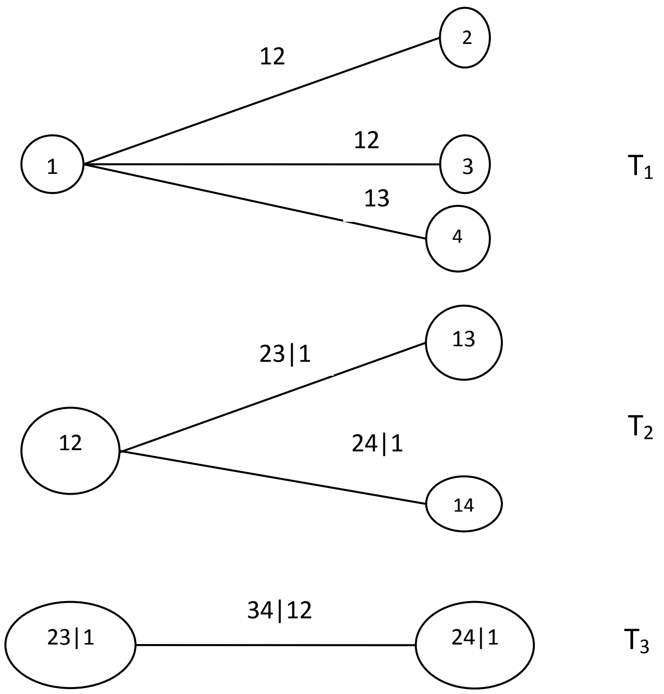


Figure 2. This figure presents the dependence structure of a canonical vine copula with four variables. In the first state, the dependence structure between variable 1 and all the other variables is modelled with bivariate copulas. In the second state, the dependence structure of variable 2 with variables 3 and 4, conditionally on variable 1. In the last state, we model the dependence structure between variables 3 and 4, conditionally on variables 1 to 2. A system with 4 variables require the estimation of 6 bivariate copulas.

3.3 Regime Switching Copula

A plausible way for modelling the asymmetric dependence structure between hedge funds and traditional assets is a regime-switching model. Thus, we expand the canonical vine copula model into a multivariate regime-switching copula model that supports the examination of dependence structure under two states, one high dependence regime and a low dependence regime. Our methodological approach is in line with the work of Chollete *et al.* (2008) that apply a similar approach to capture the asymmetric dependence in international equity market[7]. The seminal work of Hamilton (1989) that introduced the Markov-switching models in time series analysis could be useful not only to distinguish structure breaks in returns and variance of financial time series but also in the dependence structure between the variables. In our model the regimes only affect the dependence structure. Thus, the joint density of the data being in the regime j can be described as follows:

$$f(Y_t \mid Y_{t-1}, s_t = j) = c^{(j)}(F_1(y_{1,t},...,F_n(y_{n,t});\theta_c^{(j)})\prod_{i=1}^{n} f_i(y_{i,t};\theta_{m,i})$$

Where $Y_t = (y_{1,t}, ..., y_{n,t})$ is the vector of observations, s_t indicates the unobserved state variable, $c^{(j)}$is the copula in regime j with parameter $\theta^{(j)}$ and the figure f_i indicates the density function of y_i. The fact that marginal distributions are separately estimated from the dependence structure makes possible the introduction of regimes only in the dependence structure, simplifying significant the estimation process. According to the first-order Markov chain process, the probability of being in

[7] The empirical literature on Markov switching models in copula methodology remains limited with the exceptions of Rodriguez (2007) in a univariate setting; and Garcia and Tsafak (2008) and Chollete *et al.* (2008) in a multivariate extension.

a particular state in time *t* only dependent on the state prevailing *t-1*. For the two regimes $s_t = 1$ and $s_t = 2$ the probabilities p_{ij} are defined as:

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$$

where $p_{11} + p_{12} = p_{21} + p_{22} = 1$. The parameter p_{11} express the probability of remaining in the "high dependence" regime, while the p_{22} indicates the probability to maintaining in the "low dependence" regime. Also, the parameters p_{12} and p_{21} calculates the transition probability of moving from one regime to another. Specifically, the parameter p_{12} express the probability than a "low dependence" regime will be followed by a "high dependence" regime, while the parameter p_{21} measures the probability that "low dependence" regime will transit to a "high dependence" regime.

The explicit separation of individual series and their dependence structure supposes the specification of the marginal distributions from which the suitable distribution should be determined. Past empirical research indicates different distributional assumptions for the marginal distribution of the multivariate model. However, potential marginal distribution misspecification has forced researchers to employ non-parametric approaches for the margins, concentrating their interest to the modelling of the joint distribution (Chollete and Heinen, 2006; Ane and Kharoubi, 2003). The majority of past studies consider marginal distributions as 'nuisance parameters' and apply the empirical distribution function instead (Berg and Bakken, 2007; Kim et al., 2007). On the other hand, parametric approaches assume that the asset return generating process can be adequately described by parameters of known distributions. More recently, the full parameterization of copula and marginal distribution functions has been applied, supported by goodness-of-fit tests (Joe, 1997, 2005; Patton, 2004; Palmitesta and Provasi, 2005; Berg and Aas, 2007).

3.3 Marginal Distributions

We assume that the marginal distributions of both hedge funds and traditional capital markets follow an *ARMA(k,n)* process with a *GARCH(q,p)* volatility structure. Past empirical research supports GARCH models to conveniently model marginal distributions (Embrechts and Dias, 2005; Patton, 2006; Chollette, et al., 2008; Garcia and Tsafak, 2008; *inter alia*). Furthermore, few recent studies employ regime switching models as alternative parametric forms to model marginal distributions (Rodriguez, 2007; Okimoto, 2008). The univariare return $r_{i,t}$ for asset *i* is given by:

$$R_{i,t} = \Phi_{i,0} + \sum_{j=1}^{k} \Phi_j R_{i,t-j} + \varepsilon_{i,t} \qquad (5)$$

$$\sigma_{i,t} = \omega_i + \sum_{j=1}^{q} \alpha_{i,j} \varepsilon_{t-j} + \sum_{j=1}^{p} \beta_{i,j} \sigma_{t-j}$$

$$\varepsilon_{i,t} = \sqrt{\sigma_{i,t}} Z_{i,t}$$

where ω, α, β and φ are constant parameters. The *AR(k)* component in the mean equation can capture the presence of autocorrelation in the marginal series that may cause infrequent, non-synchronous and other market imperfections. For hedge funds in particular, monthly returns exhibit significant autocorrelation effects that are attributed to illiquid security holdings and the reported smoothed returns (Asness et al., 2001; Getmansky et al., 2004). The error term in conditional variance is assumed to follow a *GARCH* process that permits removing of volatility clustering. The innovations, z_t, are assumed to follow a skewed Student's-*t* distribution (e.g. Hansen, 1994; Poon and Granger, 2003).

Since the goodness-of-fit of a copula function is strongly dependent on the fit of the marginal distribution, we pay particular attention to test for the ability of the different *GARCH*-type specifications to fit our empirical sample. We apply goodness-of-fit tests on the probability integral transforms of the variables, in line with past empirical studies (Patton, 2006a, 2006b). We apply, more specifically, the Kolmogorov-Smirnov (K-S), the Kupier (KU) and the chi-squared (χ^2) tests to test for the hypothesis that the transformed series are uniformly distributed on [0,1]. In our case, the null hypothesis assumes that the transformed sample is drawn from a uniform distribution (the assumption of uniform distributed margins is the most important claim of copula framework). Furthermore, we employ the Ljung-Box statistic and the Lagrange multiplier (LM) test to evaluate whether the marginal uniform distributions $u_t = F(x)$ is serially correlated.

4. Empirical Application and Findings

4.1 Data

We employ hedge fund (index) data to proxy hedge fund returns produced by different hedge fund strategies. The dataset has been compiled from the Hedge Fund Research (HFR) that currently monitors about 7,500 funds and fund of funds (FoF). HFR provides index series data on different hedge fund strategies and constructs an equally-weighted aggregate hedge fund index, based on 2,000 funds of its database universe. HFR hedge funds are grouped according to their core investment strategies and complementary sub-strategies, including: (i) equity hedge; event-driven; macro and relative value arbitrage; emerging markets; (core strategies); and, (ii) short bias; fixed income-convertible arbitrage strategy; distressed/restructuring strategy; merger arbitrage; and, equity market neutral (sub-strategies).

The range of investment strategies in the selected hedge fund sample supports testing whether hedge funds undertake a market ‘directional’ or ‘non-directional’ exposure. The ‘directional’ (or ‘market timing’) hedge fund strategies exhibit high market correlations and bet on returns produced by major upward or downward market movements. The ‘non-directional’ (or ‘market neutral’) hedge fund strategies, on the other hand, attempt to systematically mediate risk exposure to financial market movements and gain higher returns by exploiting arbitrage opportunities and market inefficiencies. The selection of both directional and non-directional hedge fund strategies permits to empirically measure the dependence structure between hedge fund strategies and traditional asset classes. Based on monthly data, the study period expands from January 1990 to September 2008, a total of 225 observations.

In order to relatively compare and contrast hedge fund performance against traditional capital markets, we select a range of widely employed (benchmark) capital market indices as proxies for important investment asset classes. These benchmark markets under consideration include: stock markets - the S&P500 index for domestic equity portfolios to include largest public firms in the equity market; the MSCI (Morgan Stanley) world equity index to represent a globally diversified stock portfolio (including 23 developed countries); and, the Russell2000 index as a proxy for smallest firms that are of primary investment importance for non-directional hedge fund strategies; bond markets - the Barclays Global Aggregate index[8] to summarize different bond market returns of the most liquid international bond markets; the Citigroup/Salomon World Government Bond index; the Merrill Lynch US high yield index, that tracks the performance of the high yield US bond market; and, the 3-month Libor as the risk-free rate; commodity markets - we also consider the weighted S&PGS commodity index[9] as indicator for important commodity market returns and we include Brent crude oil and Gold price returns. Table 1

[8] The Barclays Global Aggregate index is the former Lehman Brothers Global Bond index, renamed as of November 1, 2008.
[9] The S&P GS commodity index is now the former Goldman Sachs commodity index.

summarizes the traditional market indices incorporated in this study to compare and contrast against established hedge fund strategies.

Table 1
Indices for Traditional Asset Classes

TRADITIONAL ASSETS	**Abbreviation**
Stock Market	
S&P500	TR-SP
MSCI World Index	TR-MS
Russell 2000	TR-RU
Bond Market	
Barclays Global Aggregate	TR-BG
CitiGroup Bond Index	TR-CB
Merrill Lynch US High Yield	TR-ML
Interest Rate Market	
3-months LIBOR	TR-LI
Commodities Market	
S&P GS Commodity Index	TR-CI
Brent Crude Oil Index	TR-OI
Gold	TR-GC

4.2 Hedge Fund Performance vs. Traditional Asset Classes

An initial overview of the historic performance of the markets under study is presented in Table 2; it summarizes descriptive statistics for hedge fund (index) returns and traditional asset classes. These figures reveal an important variation in the values of statistics across different strategies. The lower average return per month is recorded for short bias (0.34%) combined with highest standard deviation (5.6%). The emerging market funds appear to generate highest returns out of all hedge fund strategies (mean returns at 1.21 per month). Moreover, we do not observe any significant deviations of the directional vs. non-directional fund performance (average return 0.90% and 0.89% per month, respectively). On the contrary, the underlying risk, as measured by the standard deviation, is a multiple in the case of directional strategies (standard deviation at 4% and 1.5% for the directional and non-directional funds, respectively). Interestingly, all hedge fund strategies (except from short bias) attain higher, on average, returns compared to conventional asset classes. The most rewarding performance is recorded by crude oil that has exhibited a spectacular price increase over the last three years (mean return at 0.70% per month; standard deviation at 9.17%); and, the US corporate bonds (Merrill Lynch US High Yield index) (mean return at 0.70% per month; standard deviation at 2.13%).

Moreover, the estimated skewness for the sample hedge funds ranges from -4.1100 (convertible arbitrage) to 0.4059 (macro). The reported hedge funds with positive skewness are only macro, short bias and equity hedge; the remaining funds demonstrate negative asymmetry. Furthermore, the kurtosis coefficient is higher than 3 in all of the sample hedge fund strategies (indicating deviation from normality). Surprisingly, the most negatively skew is found in the arbitrage strategies (see convertible arbitrage, merger arbitrage and relative value arbitrage, *inter alia*) which are wrongly considered as low risky strategies defensive against major negative returns. On the contrarily, in the merger arbitrage strategy, hedge fund gains come at the maximum profit from the premium generated by the difference between the offered price and the last quoted market price of the target firm in an merger event and the corresponding risk is that the deal may be cancelled off causing major losses to the arbitragers. Thus, hedge fund distributions are characterized by fat tails and extreme return values. In general, both high kurtosis and negative asymmetry in asset return distributions are two adverse characteristics for risk-averse investors. However, it is widely argued that hedge fund investors can potentially attain an attractive risk-return trade-off at the cost of higher kurtosis and negative skewness. Our results are similar to previous empirical findings that confirm the presence of extreme values in hedge funds (Brooks and Kat, 2001).

We reach at similar conclusions for the equity return distributions that are leptokurtic and negative asymmetric. On the other hand, commodity indices present promising characteristics with lower kurtosis and positive asymmetry. Furthermore, the Jarque-Bera test indicates rejection of the normality assumption. Hence, hedge fund investors should consider alternative performance measures beyond standard mean and variance analysis to evaluate the hedge fund risk-return profile. The Ljung-Box *Q*-statistics for the presence of autocorrelation indicates serial correlation in eight out of ten hedge fund classes. A plausible explanation of high autocorrelation in hedge funds against traditional funds (e.g. mutual funds) is the extensive investment positions of the former on illiquid assets or over-the-counter markets that cannot be accounted for on a regular basis (Brooks and Kat, 2001; Getmansky et al. 2004). Thus, reported hedge fund performance reflects typically smoothed returns that suffer from both autocorrelation and noise in the data.

Table 2
Descriptive Statistics

	Mean	Std. Dev	Annual. Mean	Annual. Std. Dev.	Skew	Kurt	Min	Max	Jarque-Bera	Ljung-Box
HEDGE FUNDS										
Directional Strategies										
Macro	0.0116	0.0229	0.1390	0.0793	0.4059	3.7246	-0.0640	0.0788	11.101 [0.003]	14.928 [0.010]
Emerging Markets	0.0121	0.0413	0.1453	0.1431	-0.8307	6.7158	-0.2102	0.1480	155.319 [0.000]	24.431 [0.000]
Short Bias	0.0034	0.0566	0.0411	0.1961	0.1462	5.0639	-0.2121	0.2284	40.7366 [0.000]	8.5059 [0.130]
Non-Directional Strategies										
Convertible Arbitrage	0.0066	0.0139	0.0794	0.0483	-4.1100	35.4353	-0.1229	0.0333	10496.404 [0.000]	50.429 [0.000]
Distressed Securities	0.0106	0.0174	0.1275	0.0602	-0.6436	7.8684	-0.0850	0.0706	237.729 [0.000]	63.286 [0.000]
Equity Hedge	0.0118	0.0256	0.1421	0.0888	0.0019	4.5208	-0.0770	0.1088	21.683 [0.000]	10.990 [0.051]
Equity Market Neutral	0.0067	0.0092	0.0803	0.0320	-0.1329	4.1892	-0.0307	0.0359	13.920 [0.000]	12.987 [0.023]
Event Driven	0.0103	0.0191	0.1236	0.0660	-1.2340	6.9956	-0.0890	0.0513	206.776 [0.000]	21.964 [0.000]
Merger Arbitrage	0.0077	0.0124	0.0924	0.0429	-2.2549	11.9238	-0.0646	0.0312	937.230 [0.000]	12.489 [0.028]
Relative Value Arbitrage	0.0087	0.0111	0.1040	0.0383	-1.2358	12.4002	-0.0580	0.0572	885.672 [0.000]	29.904 [0.000]
Aggregate Hedge Fund Index										
Weighted Composite Index	0.0101	0.0198	0.1213	0.0685	-0.6227	5.4513	-0.0870	0.0765	70.871 [0.000]	16.084 [0.006]
TRADITIONAL ASSETS										
Stock Market										
S&P500	0.0053	0.0404	0.0636	0.1402	-0.6279	4.0500	-0.1575	0.1057	25.1247 [0.000]	2.6122 [0.759]
MSCI World Index	0.0033	0.0411	0.0392	0.1425	-0.6767	3.8915	-0.1445	0.0983	24.6235 [0.000]	2.7350 [0.740]
Russell 2000	0.0062	0.0528	0.0744	0.1829	-0.6717	4.2449	-0.2168	0.1520	31.4482 [0.000]	13.4138 [0.019]
Bond Market										
Barclays Global Aggregate	-0.0000	0.0090	-0.0004	0.0313	-0.3411	3.2887	-0.0262	0.0284	5.1436 [0.076]	10.9754 [0.051]
CitiGroup Bond Index	0.0058	0.0187	0.0696	0.0646	0.1480	2.9467	-0.0438	0.0577	0.8477 [0.654]	18.0351 [0.002]
Merrill Lynch US High Yield	0.0070	0.0213	0.0839	0.0738	-0.6717	5.9594	-0.0813	0.0711	99.0270	8.9190

									[0.000]	[0.112]
Interest Rate Market										
3-months LIBOR	-0.0032	0.0677	-0.0387	0.2344	-0.8104	13.6848	-0.4129	0.3659	1094.91 [0.000]	64.5758 [0.000]
Commodities Market										
S&P GS Commodity Index	0.0049	0.0571	0.0586	0.1976	0.0584	3.4679	-0.1817	0.1810	2.1809 [0.336]	7.9506 [0.158]
Brent Crude Oil Index	0.0070	0.0917	0.0845	0.3177	0.1017	4.4543	-0.3134	0.3610	20.2167 [0.000]	13.9723 [0.015]
Gold	0.0032	0.0387	0.0379	0.1339	0.4098	3.9983	-0.1011	0.1557	15.6410 [0.000]	5.4389 [0.364]

Notes: The table presents descriptive statistics for the returns of the sample hedge fund indices and the indices that proxy the traditional asset classes. Statistics include mean, standard deviation, skewness, kurtosis, maximum, minimum, Jarque-Bera normality test and the Ljung-Box Q-statistic on first 5 lags. Figures in [·] are p-values.

Table 3 summarizes additional statistical measures for the hedge fund sample under study: namely, the (risk-adjusted) Sharpe ratio; tail risk (value-at-risk estimates); modified Sharpe ratio and number of months with negative fund returns. Based on hedge fund Sharpe ratios, the non-directional strategies deliver in general a better risk-return trade-off against directional strategies (non-directional strategies avg. Sharpe ratio at 0.6236 vs. directional strategies at 0.2947). The most advantageous strategies for hedge fund investors appear to be two non-directional strategies, equity market neutral and relative value arbitrage (Sharpe ratios at 0.7568 and 0.8096, respectively). On the contrary, short bias strategy performs poorly compared to the other hedge fund strategies (Sharpe ratio at 0.0656). In total, eight out of nine hedge fund classes exhibit a Sharpe ratio higher than that of the S&P500 index. Across the conventional asset classes, the Merrill Lynch US High Yield index, a proxy for the high yield corporate bond market, performs better than any other market during the study period (Sharpe ratios at 0.3416). Moreover, the commodity asset class may not suffer from negative asymmetry and excess kurtosis as mentioned previously (see Table 2) but its performance is moderated according to the risk-adjusted Sharpe ratio which is the lowest among the conventional markets (Sharpe ratios at 0.0907, 0.0798 and 0.0892 for the aggregate commodity index, the crude oil index and gold, respectively). Neverthelesss, the commodity funds are recommended as a significant diversifier in portfolio composed by traditional assets and hedge funds (Edward and Caglayan , 2001).

The higher realized hedge fund Sharpe ratios relative to the stock market indicate a positive alpha. The estimated beta coefficients of hedge fund returns on equity market portfolio (S&P500) ranges from -0.9805 (short bias) to 0.5993 (emerging markets). Despite Sharpe ratio's and Jensen alpha's wide popularity as risk-adjusted-performance measures, the deviation of hedge funds from normality combined with their non-linear exposure to capital markets raises doubts about the validity of hedge fund empirical performance. Empirical evidence on heavy hedge fund losses, related, to major financial crises, indicates that hedge funds may be facing significant left-tail risk. The empirical hedge fund VaR estimates (realized maximum loss) range from 0.6% (relative value arbitrage) to 8.8% (short bias) within a time horizon of one month and probability 95%. The estimated S&P500 and aggregate hedge fund index VaRs are 7.01% and 2.21%, respectively. The modified Sharpe ratio (MSR)[10] is found to be substantially lower than conventional Sharpe ratio for the hedge fund classes under study. The presence of negative skewness on hedge fund returns moderates the earlier superior risk-return trade-off in hedge fund performance. Now, equity market neutral exhibits the highest estimated MSR (0.3257) but short bias remains the poorest performing strategy (0.0420).

[10] The modified Sharpe ratio (MSR) provides results that are more accurate as a risk-adjusted return measure compared to conventional Sharpe ratio, since it takes into account the non-normality characteristics of hedge fund returns.

Table 3
Additional Preliminary Results

	Sharpe ratio	VaR	Modified Sharpe ratio	Percentage of negative months	α	β	R^2
HEDGE FUNDS							
Directional Strategies							
Macro	0.5183	-0.0222	0.2649	31.11	0.0106*	0.2099*	0.131
Emerging Markets	0.3001	-0.0552	0.1481	31.11	0.0090*	0.5993*	0.342
Short Bias	0.0656	-0.0880	0.0420	50.66	0.0091*	-0.9805*	0.489
Non-Directional Strategies							
Convertible Arbitrage	0.4955	-0.0148	0.2197	19.55	0.0062*	0.1212*	0.121
Distressed Securities	0.6282	-0.0177	0.2764	20.88	0.0098*	0.1896*	0.194
Equity Hedge	0.4732	-0.0283	0.2349	29.33	0.0097*	0.4277*	0.456
Equity Market Neutral	0.7568	-0.0102	0.3257	19.55	0.0067*	0.0412	0.027
Event Driven	0.5559	-0.0219	0.2349	22.66	0.0088*	0.3141*	0.446
Merger Arbitrage	0.6461	-0.0143	0.2512	17.77	0.0071*	0.1582*	0.268
Relative Value Arbitrage	0.8096	-0.0064	0.3237	15.55	0.0083*	0.1146*	0.171
Aggregate Hedge Fund Index							
Weighted Composite Index	0.5255	-0.0221	0.2372	27.11	0.0084	0.3481	0.506
TRADITIONAL ASSETS							
Stock Market							
S&P500	0.1381	-0.0701	0.0740	37.77			
MSCI World Index	0.0863	-0.0703	0.0472	40.89			
Russell 2000	0.1229	-0.0837	0.0661	38.67			
Bond Market							
Barclays Global Aggregate	0.0282	-0.0156	0.0169	46.67			
CitiGroup Bond Index	0.3264	-0.0249	0.1761	40.00			
Merrill Lynch US High Yield	0.3416	-0.0308	0.1679	27.11			
Interest Rate Market							
3-months LIBOR	-0.0434	-0.1115	-0.0282	49.78			
Commodities Market							
S&P GS Commodity Index	0.0907	-0.0903	0.0552	45.78			
Brent Crude Oil Index	0.0798	-0.1393	0.0498	44.00			
Gold	0.0892	-0.0499	0.0585	51.56			

Notes: In this table some additional statistics are present for the hedge fund indices and traditional assets' indices. Column 2 presents a risk adjusted ratio, the monthly Sharpe ratio (SR). Column 3 presents the VaR at 95% confidence level of the empirical distributions. Columns 4-6 present the Jensen's alpha, beta coefficient and regression R-squared, respectively. For the calculation of the pricing model we consider the S&p500 index as the market portfolio and the 3 month Treasury bill proxies the risk free rate. T* denotes significance at 5% level.

We now turn into the discussion of a range of initial findings on the relationships across alternative hedge fund strategies and their correlation with conventional markets and hedge funds (Table 4). The hedge fund strategies appear to have a relatively higher correlation (negative or positive) with the aggregate hedge fund index that ranges from -0.7511 (short bias) to 0.9381 (equity hedge). Of core hedge fund strategies, equity hedge and event driven are strongly correlated (0.8013), while relative value arbitrage and macro strategies exhibit a low correlation (0.3985). High correlation coefficients indicate common systematic factors in different hedge fund strategies that drive these fund returns.

The correlation between hedge funds and the other asset classes vary considerably from negative to positive values allowing the international investor that is willing to consider hedge funds in his universal portfolio to enjoy substantial diversification benefits. The correlation coefficient between hedge fund indices and equity indices ranges from -0.82 (Short bias-Russell 2000) to 0.80 (Equity Hedge- Russell 2000). Also, we see limited variations in correlation between hedge funds and the three equity indices. Short bias is the only strategy that hedge funds have significant exposure and negative relationship. Interestingly, we found correlations very close to zero between sample hedge funds and the two of the three bond indices

(Barclays Global Aggregate index and CitiGroup Bond Index). Likewise, low values of correlation are also reported for the different fund strategies and commodity indices (do not exceed 0.19, 0.07 and -0.07 between Aggregate commodity index-Equity Hedge, Crude oil-Equity Hedge, Gold-Equity Market Neutral, respectively). Our results on the relationship of hedge funds and traditional asset classes may be in line with the dominant concept of low correlation but the outcomes of the Pearson correlation coefficients should be dealt with cautious. The main reason is the limitation of the correlation coefficient to be considered as a stable dependence measure. Correlation is very sensitive to the presence of outliers, non-stationarity and volatility clustering in the variables.

Table 4
Unconditional correlation coefficients across the Hedge Fund indices

	(HF-MC)	**(HF-EM)**	(HF-SB)	(HF-CA)	(HF-DS)	**(HF-EH)**	(HF-EN)	**(HF-ED)**	(HF-MA)	**(HF-RV)**	(HF)	(TR-SP)	(TR-MS)	(TR-RU)	(TR-BG)	(TR-CB)	(TR-ML)	(TR-LI)	(TR-CI)	(TR-OI)	(TR-GC)
Macro (HF-MC)	**1.00**																				
Emerging Markets(HF-EM)	0.59	**1.00**																			
Short Bias(HF-SB)	-0.37	-0.56	**1.00**																		
Convertible Arbitrage(HF-CA)	0.34	0.43	-0.26	**1.00**																	
Distressed Securities(HF-DS)	0.47	0.65	-0.47	0.58	**1.00**																
Equity Hedge(HF-EH)	0.59	0.68	-0.74	0.52	0.63	**1.00**															
Equity Market Neutral(HF-EN)	0.30	0.18	-0.11	0.36	0.29	0.44	**1.00**														
Event Driven(HF-ED)	0.55	0.71	-0.61	0.60	0.81	0.80	0.32	**1.00**													
Merger Arbitrage(HF-MA)	0.32	0.47	-0.39	0.51	0.56	0.55	0.31	0.75	**1.00**												
Relative Value Arbitrage(HF-RV)	0.39	0.53	-0.36	0.69	0.71	0.60	0.38	0.69	0.53	**1.00**											
Weighted Composite Index(HF)	0.67	0.83	-0.75	0.57	0.75	0.93	0.39	0.88	0.63	0.67	**1.00**										
S&P500(TR-SP)	0.36	0.58	-0.70	0.35	0.44	0.67	0.17	0.66	0.51	0.41	0.71	**1.00**									
MSCI World Index (TR-MS)	0.39	0.65	-0.65	0.37	0.44	0.66	0.18	0.64	0.49	0.43	0.70	0.87	**1.00**								
Russell 2000(TR-RU)	0.43	0.61	-0.82	0.34	0.59	0.80	0.23	0.79	0.57	0.49	0.83	0.75	0.69	**1.00**							
Barclays Global Aggregate(TR-BG)	0.36	0.04	-0.02	0.16	0.03	0.06	0.11	0.08	0.10	0.06	0.07	0.14	0.11	0.03	**1.00**						
CitiGroup Bond Index(TR-CB)	0.13	-0.03	0.02	0.02	-0.08	-0.00	0.09	-0.05	-0.01	-0.06	-0.02	0.01	0.15	-0.06	0.57	**1.00**					
Merrill Lynch US High Yield(TR-ML)	0.34	0.49	-0.43	0.49	0.62	0.50	0.13	0.67	0.49	0.53	0.60	0.55	0.51	0.57	0.21	0.03	**1.00**				
3-months LIBOR(TR-LI)	-0.18	-0.09	0.05	-0.32	-0.11	-0.10	-0.12	-0.11	-0.03	-0.15	-0.12	-0.08	-0.05	-0.04	-0.32	-0.21	-0.17	**1.00**			
S&P GS Commodity Index(TR-CI)	0.13	0.05	-0.02	0.11	0.04	0.19	0.16	0.07	0.04	0.12	0.14	-0.04	0.02	0.01	-0.04	0.10	-0.03	0.06	**1.00**		
Brent Crude Oil Index (TR-OI)	0.00	-0.01	0.05	0.06	-0.02	0.07	0.03	-0.01	-0.04	0.04	0.02	-0.13	-0.06	-0.09	-0.08	0.06	-0.09	0.06	0.81	**1.00**	
Gold(TR-GC)	-0.02	0.03	0.01	0.05	-0.00	-0.03	-0.07	-0.01	-0.04	-0.06	-0.03	-0.04	0.02	-0.09	0.01	0.17	-0.08	-0.15	0.01	0.01	**1.00**

Notes: This table reports the Pearson coefficient for the Hedge fund strategies and traditional asset classes. In first column the Main Hedge Fund strategies are underlined by bold.

Figures 3a and b depict a graphical representation of time-varying hedge fund and traditional market correlations. For that, we define a window of 48 monthly observations (1990 to 1993) and estimate all pair-correlations. A window of fixed size then rolls over one month ahead to estimate correlations again until the end-date (September 2008). Based on time-varying correlations of core hedge fund strategies with equity markets (S&P500), it becomes apparent that certain strategies (equity hedge; event driven; emerging markets) demonstrate a strong relationship with equity markets. During crisis periods, spiky correlations are seen at times. A similar market pattern of increased correlations is seen more recently during the profound sub-prime mortgage crisis. Of pair-correlations between hedge fund sub-strategies and equity markets, short bias - as anticipated - exhibits a strongly negative correlation.

The cumulative performance of hedge funds and equity markets over January 1990 to September 2008 is summarized in Figures 4a and b and Figures 5a and b. The performance of equity markets was impressive until mid-2000 (+119%); subsequently, an abrupt downward path was seen (until early 2003). Over the same time-period, hedge funds outperformed over traditional equity markets; the dominant hedge fund strategy was equity hedge, followed by event driven. Hedge funds are also seen to perform better than traditional markets over international financial crises.

Figure 3a. The graph presents a four year-based rolling window of pair correlations between Main hedge fund strategies and the equity market.

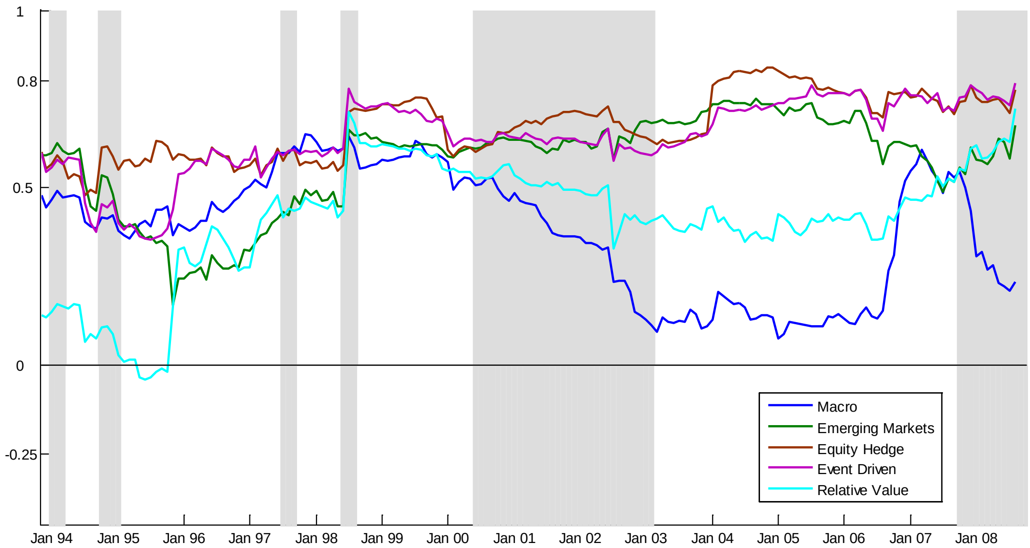


Figure 3b. The graph presents a four year-based rolling window of pair correlations between Sub-strategies of hedge funds and the equity market.

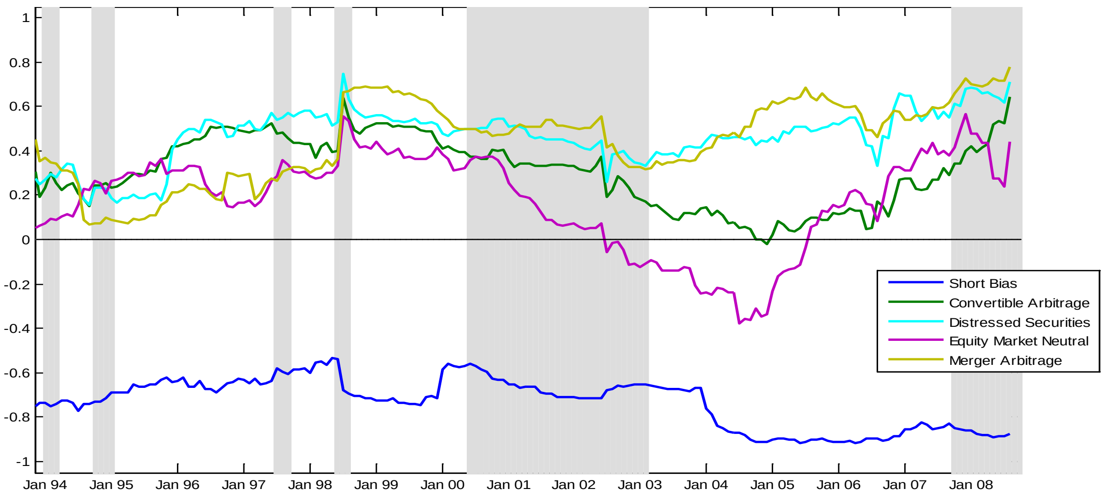


Figure 4a. Cumulative returns of core hedge fund strategies vs. S&P500 index: Jan 1994 to Sept 2008

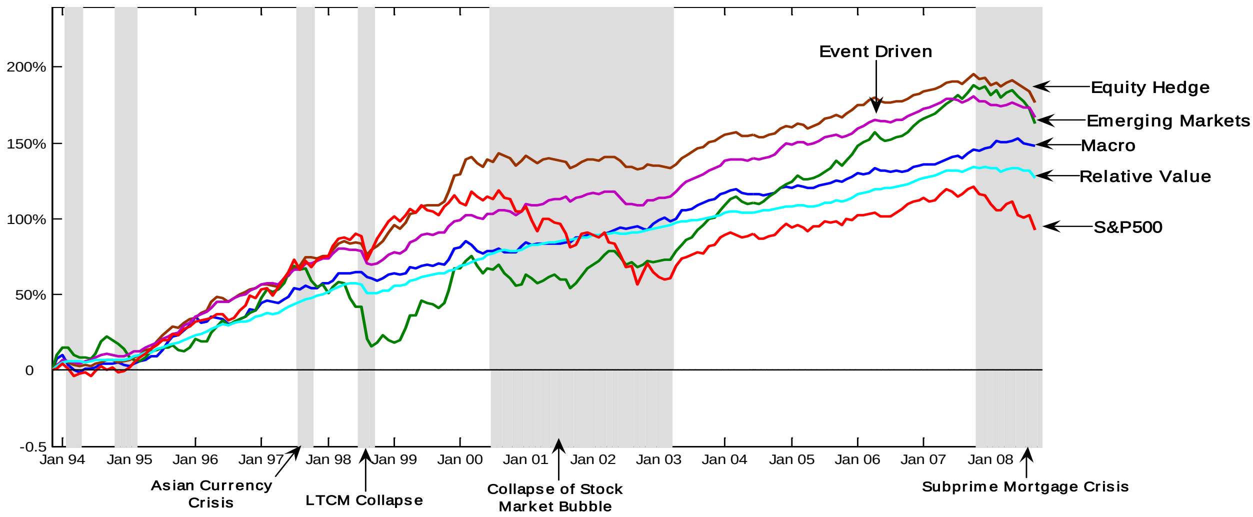
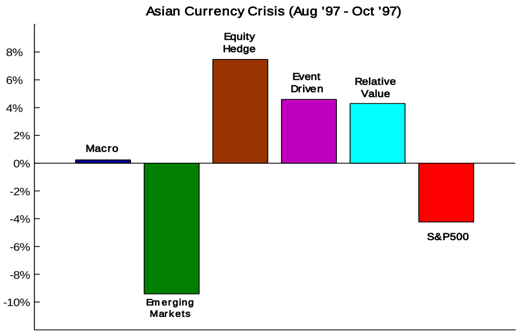
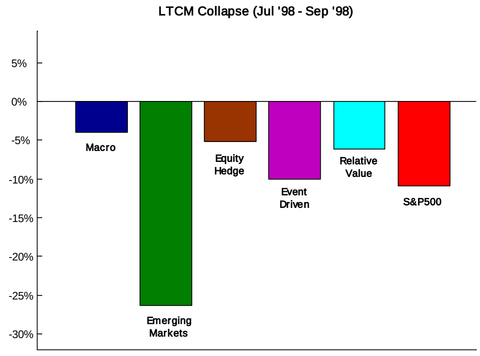


Figure 4b. Cumulative performance of core hedge fund strategies vs. S&P500 index: Major financial crises





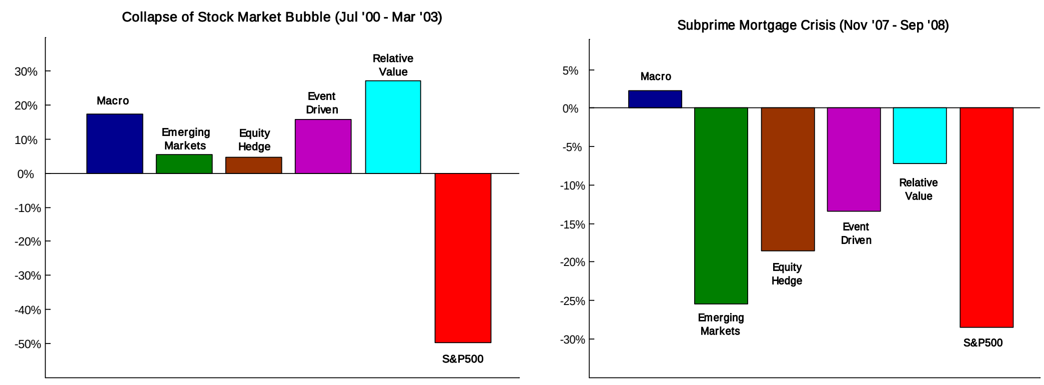
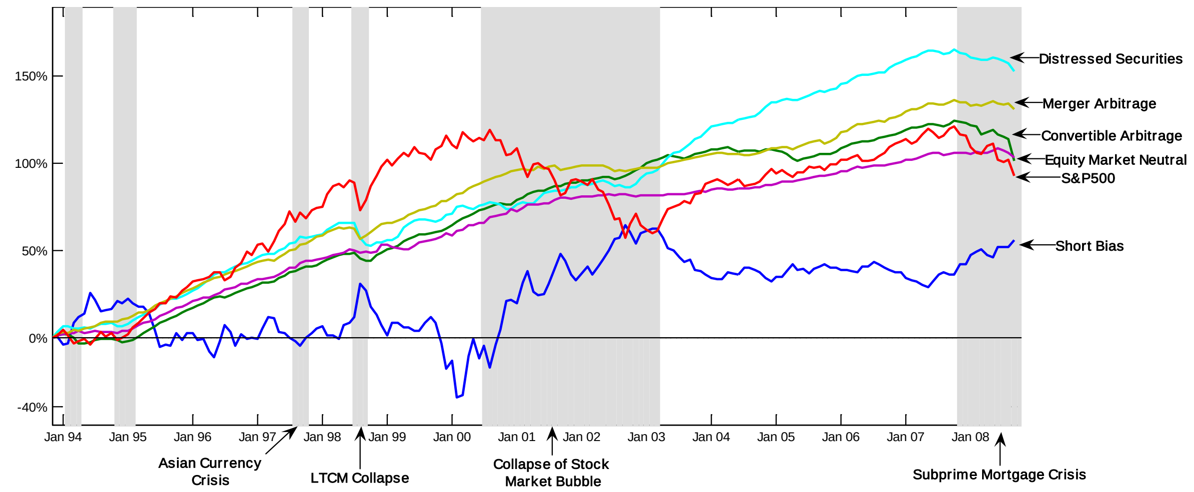
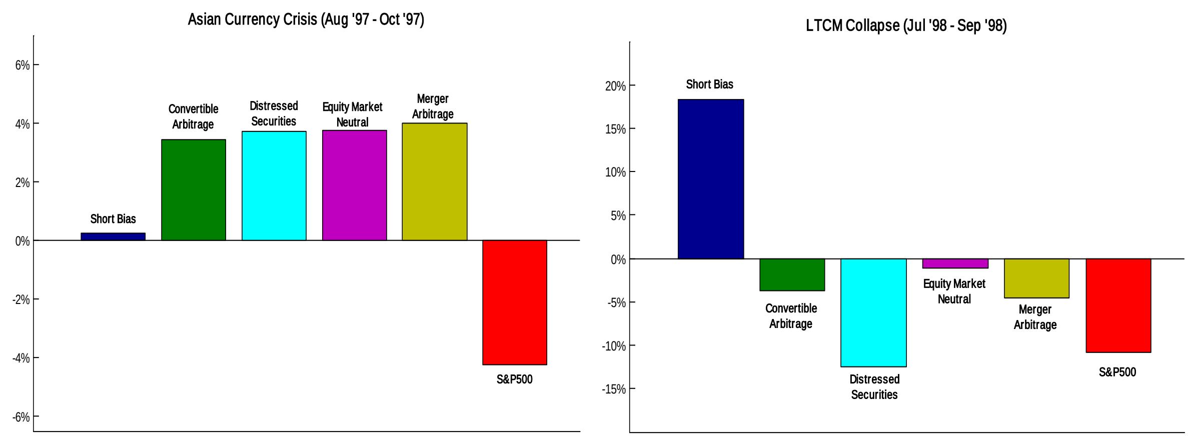
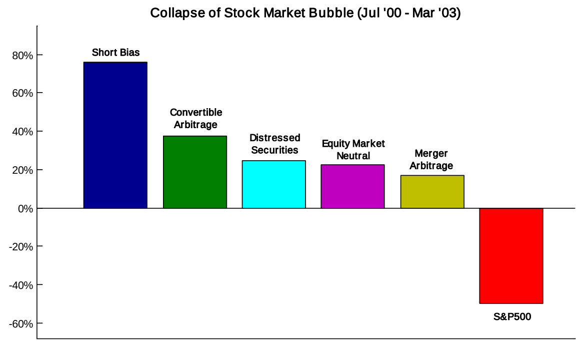


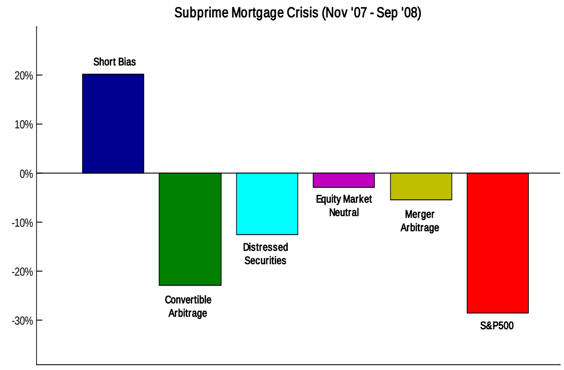
Figure 5a. Cumulative returns of hedge fund sub-strategies vs. S&P500 index: Jan 1994 to Sept 2008



Panel 5b. Cumulative performance of hedge fund sub-strategies vs. S&P500 index: Major financial crises







4.2 Estimation of Copula and Marginal Models

In our empirical approach, we specify the marginal model. Accurate specification and estimation of the marginal distribution is critical especially in the two-step estimation process of copula methodology. The marginal distributions of both hedge fund and traditional assets are assumed to follow an *ARMA(k,n)* process with *GARCH(q,p)* volatility, whereas innovations to follow a skewed Student's-*t* distribution (Table 5 & 6). The estimated coefficients ω, α and β are statistically significant in nearly all cases, supporting the existence of volatility clustering. The sum of α and β is close to one (ranging from 0.82283 to 0.99979), indicating a high level of persistence in volatility for both hedge fund and equity market returns. The skewed Student-*t* distribution appears to fit the data well most in the case of hedge funds indices, since the coefficients η and λ (control for distribution tail thickness and asymmetry respectively) are found to be statistically significant in many cases and suggest leptokurtic left-skewed conditional distributions. The estimates of the marginal models for variables of the traditional markets are slightly different since the coefficients η that controls the tail thickness of the distributions is statistically insignificant in most cases. These results are in line with the summary statistics presented above (Table 2) where the distributions of traditional indices do not appear significant kurtosis, compared to hedge fund indices.

Prior to the specification of the multivariate copula functions, we transform the standardized residuals into uniform series. Different goodness-of-fit test statistics are be employed to test for the null hypothesis that the margins are uniform (1,0) and *i.i.d.* processes (Kolmogorov-Smirnov (K-S); Kupier (KU); χ^2 tests); the LM-test and Ljung-Box *Q*-statistic to examine the independence of the first four moments. The goodness-of-fit findings ensure that the individually transformed series stem from the uniform distribution, do not suffer from serial correlation and can be employed into the estimation of the copula functions.

Table 5
Estimation result for the Skewed *t* GARCH model for the marginal distributions of HF indices

	ω	α	β	η	λ	$\alpha+\beta$
HEDGE FUNDS						
Directional Strategies						
Macro	0.00000 (0.000)	0.00000 (0.000)	0.99552 * (0.000)	12.69777 (107.093)	0.12967* (0.006)	0.99552
Emerging Markets	0.52389* (0.116)	0.05770* (0.000)	0.90761* (0.001)	5.36103 (3.864)	-0.26747* (0.006)	0.96531
Short Bias	0.29246* (0.092)	0.12925* (0.002)	0.86605* (0.001)	8.25721 (15.895)	0.11471* (0.007)	0.9953
Non-Directional Strategies						
Convertible Arbitrage	0.13992* (0.011)	0.17962* (0.010)	0.74955* (0.015)	3.31873* (0.653)	-0.15693* (0.005)	0.92917
Distressed Securities	0.05077* (0.001)	0.03134* (0.000)	0.93907* (0.000)	5.22639* (2.128)	-0.19568* (0.006)	0.97041
Equity Hedge	0.33682* (0.060)	0.13891* (0.004)	0.81354* (0.006)	14.42870 (138.054)	-0.08831* (0.009)	0.95245
Equity Market Neutral	0.06371* (0.002)	0.15120* (0.005)	0.78842* (0.010)	14.93667 (315.571)	0.00866 (0.007)	0.93962
Event Driven	0.22751* (0.041)	0.02390* (0.000)	0.89736* (0.004)	5.72722 (4.080)	-0.18625* (0.006)	0.92126
Merger Arbitrage	0.18929* (0.013)	0.06180* (0.001)	0.76103* (0.016)	5.38583* (2.812)	-0.46305* (0.007)	0.82283
Relative Value Arbitrage	0.05505* (0.002)	0.17265* (0.006)	0.80026* (0.007)	4.56091* (1.434)	-0.20731* (0.010)	0.97291
Aggregate Index						
Weighted Composite Index	0.12638* (0.008)	0.10145* (0.001)	0.87033* (0.002)	12.82886 (80.119)	-0.26426* (0.008)	0.97178

Notes: This table presents the results for the Skewed *t* GARCH model for the specification of the marginal distributions. The symbols of the table correspond to the volatility equation parameters. ω is the constant term while α, β measure the "reaction" and "persistence" of volatility, respectively. The parameters η and λ control the shape of the conditional distribution. The parameter, η, is the degree of freedom and controls tail thickness, while the parameter, λ, controls the asymmetry of the distribution. Figures in (·) are standard errors.

Table 6
Estimation result for the Skewed *t* GARCH model for the marginal distributions of traditional indices

	ω	α	β	η	λ	$\alpha+\beta$
TRADITIONAL ASSETS						
Stock Market						
S&P500	0.48332* (0.106)	0.15490* (0.002)	0.82976* (0.002)	12.87637 (89.582)	-0.37027* (0.009)	0.98466
MSCI World Index	0.83921* (0.203)	0.09490* (0.001)	0.85440* (0.002)	42.41437 (16899.4)	-0.40889* (0.011)	0.9493
Russell 2000	0.61718* (0.255)	0.07661* (0.000)	0.90130* (0.001)	12.21822 (69.308)	-0.33960* (0.007)	0.97791
Bond Market						
Barclays Global Aggregate	0.76480* (0.000)	0.00001 (0.004)	0.0002 (0.011)	14.87148 (165.721)	-0.22403* (0.010)	0.0021
CitiGroup Bond Index	0.20880 (0.140)	0.00932* (0.000)	0.92770* (0.014)	64.47083 (50872.79)	0.05270* (0.008)	0.93702
Merrill Lynch US High Yield	0.29028* (0.027)	0.30072* (0.011)	0.69907* (0.005)	3.86398* (0.751)	-0.30528* (0.006)	0.99979
Interest Rate Market						
3-months LIBOR	1.21683 (1.161)	0.20351* (0.005)	0.79628* (0.004)	2.97107* (0.156)	-0.13173* (0.004)	0.99979
Commodities Market						
S&P GS Commodity Index	1.47488 (1.978)	0.19383* (0.009)	0.78086* (0.011)	25.25811 (1763.62)	0.02555* (0.008)	0.97469
Brent Crude Oil Index	5.73064 (15.226)	0.11440* (0.004)	0.81403* (0.007)	13.46157 (130.867)	-0.02630* (0.008)	0.92843
Gold	1.22808 (1.065)	0.11123* (0.004)	0.81380* (0.008)	7.89796 (16.140)	0.20763* (0.007)	0.92503

Notes: This table presents the results for the Skewed *t* GARCH model for the specification of the marginal distributions. The symbols of the table correspond to the volatility equation parameters. ω is the constant term while α, β measure the "reaction" and "persistence" of volatility, respectively. The parameters η and λ control the shape of the conditional distribution. The parameter, η, is the degree of freedom and controls tail thickness, while the parameter, λ, controls the asymmetry of the distribution. Figures in (·) are standard errors.

Different specifications of copulas (with and without regime switching) model the dependence structure of the markets under study (hedge funds vs. traditional asset classes). In order to examine this dependence structure in depth, we form six groups of markets under study; each asset group includes a hedge fund class (strategy/index) and three traditional markets (indices) as proxies for traditional asset markets. We select the MSCI world index for equity markets, the Barclays index for bond markets and the S&P GS for commodity markets. In each group, the hedge fund class plays a pivotal role in the estimation process of the canonical vine copula, as our primary concern is to measure the dependence structure between hedge funds and conventional capital markets and not cross-dependences of capital markets[11].

Our approach is initially based on a straightforward canonical vine copula structure without regime switching to be considered. Some popular copula functions in empirical literature are employed in order to capture different expressions of the dependence structure between hedge funds and capital markets; the Gaussian copula, the Clayton copula and rotated Gumbel copula that is capable to capture joint dependence in lower tail and the Gumbel copula that is characterized by upper tail dependence. The canonical vine copula model appears to be a particularly flexible and realistic approach to model asymmetric dependences. Moreover, with the canonical vine copula approach, we overcome constraints in modelling multivariate copula functions, since the multivariate joint distributions are decomposed into bivariate conditional copulas.

Table 7 presents the dependence structure of core hedge fund classes (Macro, Emerging Markets, Equity Hedge, Event Driven and Relative Value) and benchmark capital markets (equity, bond, commodity markets) without regimes based on four different copulas. Our empirical results support the use of the Gaussian copula for the second and third stage of the canonical vine that includes the conditional copulas, since the estimates of the other copulas are close to their bounds. This result is in line with previous empirical findings (Chollete *et al.*; 2008). The table does not present the results for the conditional copulas that account for the cross dependence among capital markets (available upon request). The respective *t*-statistics for the estimated coefficients indicate that the majority of estimates are statistically significant. Our empirical results offer significant implications for the relationship of hedge funds and capital markets.

First, hedge funds are found to be more correlated with equity market compared to the bond and commodity markets. The estimated correlation based on Gaussian copula of each hedge fund strategy and the MSCI equity index ranges from 0.445 (Macro-MSCI) to 0.686 (Equity Hedge-MSCI). Except from the estimated coefficient of each copula function (table 7), we present the corresponding Kendall tau of each estimation in order to be compared our empirical findings with different copulas. The Equity Hedge is the strategy that demonstrate the strongest dependence with equity market according to the results of almost all copulas estimates (except from the Gaussian copula which indicates the emerging market strategy) with the Kendal tau coefficient to range from 0.4948 (rotated Gumbel copula) to 0.4497 (Clayton copula). Moreover, we find that the Macro strategy has the lowest correlation with equity market, although it is considered to be a directional strategy. Our empirical results are against previous findings that support higher exposure of directional strategies on equity market. The correlation between hedge fund strategies and the other capital markets (bond and commodity) are significantly lower, with our different copulas estimations to converge into the implication that Macro is the strategy that is most correlated with the bond market (highest Kendall tau value equal 1.874 and lowest equal to 1.542), while Macro and Equity Hedge are the strategies with stronger correlation with the commodity market compared to the other hedge fund strategies (although the

[11] For the specification of copulas, Aas at al. (2007) and Chollete *et al.* (2008) proceed to order the variables in decreasing correlations, in order to choose the variables with the larger correlation as the pivotal variables in the system. Hence, they achieve to capture the dependences that developed in the firsts stages of the canonical vines leaving only limited dependences to be modelled in the part of conditional copulas.

values of estimating Kendall are low). Evaluating our empirical estimates with different copulas, we see that our findings converge to the same implications about the dependence structure in the markets under study. Using the estimations of the maximum log likelihood as a selection criterion, we can decide which copula function perform better in modelling the dependence structure of different formulated groups of the markets presented in table 7. Hence, the Gaussian copula is preferred in the cases of Macro and Emerging Markets strategies. While, both Clayton copula and rotated Gumbel copula seem to provide more accurate dependence modelling for Equity Hedge and Event Driven strategies; Clayton copula is preferred in Relative Value and the rotated Gumbel copula in the Composite Hedge Fund index, respectively. We underline that Clayton copula and rotated Gumbel copula (that are found to be more appropriate for some hedge fund strategies) are asymmetric copulas and account for lower tail dependence.

We report (table 8) the estimates of the regime switching copula for the core hedge fund strategies and a proxy for each capital market. We have distinguished two regimes to investigate the dependence structure between hedge funds and conventional markets. In the regime switching framework, we initially use the Gaussian copula function in both high and low dependence states that is considered as a benchmark (panel A). The estimated correlations under these two regimes reveal a high dependence regime of high correlations indeed and a low dependence regime with lower correlations. All hedge fund strategies are found to be substantially correlated with equity with correlation coefficient that exceeds the value of 0.8. On the other hand, the correlations decline substantially in the low dependence regime. Indeed, three hedge fund strategies (Emerging Markets, Equity Hedge and Event Driven) are negatively correlated with the MSCI World equity index. The values of probabilities P_{11} that measures the probability of remaining in the 'high dependence' regime is low for Macro and Relative Value strategies as well as the Composite hedge fund index, indicating that they are not constantly highly correlated with conventional capital markets. On the contrary, the value of probabilities P_{22} is meaningfully large (except from Emerging Markets and Event Driven funds) indicating a high probability of remaining in the 'low dependence' regime. The fact that the hedge funds employ dynamic strategies and time-varying exposure to traditional markets can be a justification of our empirical findings.

In order to check for the significance of asymmetric dependences in the relationship between hedge funds and traditional capital markets, we extent our analysis in the context of regime switching models by incorporating asymmetric copulas in the canonical vine structure. We choose to conserve the Gaussian copula in the high dependence regime and the low dependence regime is now modeled by asymmetric copulas. For each hedge fund strategy, we select the copula that fits better according to our previous analysis and results (table 7). The empirical results of the regime switching specification are summarized in table 8 (panel B). Based on this specification, we observe increases in the likelihood values for 4 out of 6 joint distributions with a significant number of statistically insignificant estimated parameters, indicating that the asymmetric canonical vine model dominates only in some cases in terms of the likelihoods estimates. Further research and additional goodness of fit test are necessary for the selection of the best copulas that adequately describe the dependence structure of the hedge fund classes.

Table 7
Estimation results for canonical vine copulas

	Macro		Emerging		Equity Hedge		Event Driven		Relative Value		Composite index	
	Coeff	τ	Coeff	τ	Coeff	τ	Coeff	τ	Coeff	τ	Coeff	τ
Gaussian Copula												
MSCI Equity Index	0.4455 (0.288)	0.2939	0.6878* (0.028)	0.4828	0.6860* (0.029)	0.4812	0.6474* (0.031)	0.4483	0.4865* (0.046)	0.3234	0.7274* (0.025)	0.5186
Barclays Bond Index	0.2901* (0.100)	0.1874	0.0878 (0.066)	0.0560	0.0141 (0.066)	0.0090	0.0571 (0.066)	0.0364	0.0581 (0.067)	0.0370	0.0678 (0.066)	0.0432
S&P GS Commodity Index	0.1837* (0.035)	0.1176	0.0675 (0.037)	0.0430	0.1748* (0.037)	0.1119	0.1044* (0.037)	0.0666	0.1199* (0.037)	0.0765	0.1319* (0.037)	0.0842
Log-Likelihood	-50.409		-75.102		-83.022		-65.363		-35.984		-90.937	
AIC	-100.764		-150.151		-165.991		-130.674		-71.915		-181.821	
BIC	-100.673		-150.060		-165.899		-130.582		-71.824		-181.730	
Clayton Copula												
MSCI Equity Index	0.5329* (0.106)	0.2104	1.3282* (0.143)	0.3991	1.6344* (0.163)	0.4497	1.4974* (0.157)	0.4281	0.8222* (0.120)	0.2913	1.7345* (0.170)	0.4645
Barclays Bond Index	0.3646* (0.096)	0.1542	0.0915 (0.076)	0.0437	0.0139 (0.065	0.0069	0.0519 (0.071)	0.0253	0.1224 (0.081)	0.0577	0.0805 (0.077)	0.0387
S&P GS Commodity Index	0.2377* (0.048)	0.1062	0.0812 (0.043)	0.0390	0.2318* (0.049)	0.1039	0.1396* (0.047)	0.0652	0.1721* (0.047)	0.0792	0.1765* (0.050)	0.0811
Log-Likelihood	-44.787		-67.046		-93.507		-78.643		-43.582		-92.993	
AIC	-89.521		-134.039		-186.960		-157.234		-87.111		-185.934	
BIC	-89.430		-133.947		-186.869		-157.142		-87.019		-185.842	
Gumbel Copula												
MSCI Equity Index	1.3956* (0.074)	0.2835	1.7473* (0.093)	0.4277	1.8203* (0.099)	0.4506	1.6905* (0.089)	0.4085	1.3943* (0.111)	0.2828	1.9454* (0.111)	0.4860
Barclays Bond Index	1.1840* (0.054)	0.1554	1.0472* (0.042)	0.0451	1.0328* (0.038)	0.0317	1.0194* (0.043)	0.0190	1.0000 (1.016)	0	1.0378* (0.041)	0.0364
S&P GS Commodity Index	1.0885* (0.029)	0.0813	1.0450* (0.025)	0.0431	1.0491* (0.0290)	0.0468	1.0167* (0.027)	0.0164	1.0197* (0.030)	0.0193	1.0438* (0.027)	0.0420
Log-Likelihood	-40.026		-62.304		-64.355		-50.299		-24.312		-76.551	
AIC	-79.999		-124.554		-128.657		-100.545		-48.570		-153.050	
BIC	-79.908		-124.462		-128.565		-100.454		-48.478		-152.958	
Rotated Gumbel Copula												
MSCI Equity Index	1.3726* (0.071)	0.2715	1.8344* (0.097)	0.4549	1.9796* (0.111)	0.4948	1.8650* (0.100)	0.4638	1.4642* (0.078)	0.3170	2.0890* (0.114)	0.5213
Barclays Bond Index	1.2094* (0.058)	0.1731	1.0627* (0.042)	0.0590	1.0194* (0.032)	0.0190	1.0305* (0.036)	0.0296	1.0560* (0.039)	0.0530	1.0511* (0.039)	0.0486
S&P GS Commodity Index	1.1175* (0.029)	0.1052	1.0379* (0.024)	0.0365	1.1111* (0.029)	0.1000	1.0690* (0.024)	0.0646	1.0831* (0.027)	0.0767	1.0879* (0.028)	0.0808
Log-Likelihood	-49.055		-73.554		-93.716		-78.618		-42.297		-98.577	
AIC	-98.056		-147.054		-187.380		-157.182		-84.542		-197.102	
BIC	-97.965		-146.963		-187.288		-157.091		-84.450		-197.010	

Notes: This table presents the results of the canonical vine copulas model for the dependence structure between hedge fund strategies and the main conventional capital markets (equity market, bond market and commodity market). As proxy of the three markets the MSCI World index, the Barclays Bond index and the S&P500 GS commodity index are used. We presents only the results in the first layer of the canonical vine system that account for the dependences between hedge fund strategies and the three capital markets. Figures in (·) are standard errors. T* denotes significance at 5% level and τ is the Kendall coefficient. For the correlation coefficient p in the case of Gaussian copula the Kendall τ is obtained by the relation: τ=2arcsin(p)/π. For the Clayton copula coefficient θ the Kendall τ is obtained by the relation: $\tau = \theta/(2+\theta)$. And, for the rotated Gumbel copulas the following relation is used in the computation of Kendall coefficient: $\tau = 1 - 1/\theta$. For each estimated model the log likelihood, the Akaike information criterion (AIC) and the Bayesian information criterion (BIC) are reported.

Table 8
Estimation results for the regime switching canonical vine copulas

	Macro		**Emerging**		**Equity Hedge**		**Event Driven**		**Relative Value**		**Composite index**	
	Coeff	**τ**	**Coeff**	**τ**	**Coeff**	**τ**	**Coeff**	**τ**	**Coeff**	**τ**	**Coeff**	**τ**
PANEL A												
	High Dependence Regime											
Gaussian Copula												
MSCI Equity Index	0.8719* (0.083)	0.6742	0.7916 (1.134)	0.5815	0.8605* (0.096)	0.6597	0.8501* (0.203)	0.6469	0.8674* (0.300)	0.6684	0.9325* (0.180)	0.7648
Barclays Bond Index	0.7029* (0.132)	0.4962	0.0630 (7.396)	0.0401	0.0519* (0.075)	0.0331	0.0560 (0.082)	0.0357	0.5678* (0.000)	0.3844	0.6069* (0.230)	0.4152
S&P GS Commodity Index	0.7783* (0.098)	0.5678	0.4391 (3.384)	0.2894	0.1578* (0.077)	0.1009	0.1722* (0.036)	0.1102	0.6861* (0.000)	0.4814	0.6763* (0.190)	0.4728
	Low Dependence Regime											
Gaussian Copula												
MSCI Equity Index	0.2726* (0.025)	0.1758	-0.6194 (7.482)	-0.4252	-0.3442* (0.096)	-0.2237	-0.4367* (0.044)	-0.2877	0.4164* (0.101)	0.2734	0.7104* (0.019)	0.5030
Barclays Bond Index	0.3607* (0.033)	0.2349	-0.4239 (8.113)	-0.2787	-0.5643* (0.117)	-0.3817	-0.6332* (0.000)	-0.4365	-0.0880 (0.146)	-0.0561	-0.0918 (0.042)	-0.0585
S&P GS Commodity Index	0.0443* (0.016)	0.0282	-0.7956 (9.901)	-0.5857	0.0216 (0.075)	0.0138	-0.0871* (0.043)	-0.0555	0.0522 (0.131)	0.0332	0.0384 (0.044)	0.0245
P_{11}	0.4896* (0.002)		0.8460 (1.087)		0.9800* (0.026)		0.8826* (0.046)		0.6170* (0.003)		0.5886* (0.042)	
P_{22}	0.9207* (0.008)		0.6218 (0.462)		0.9764* (0.028)		0.6199* (0.069)		0.8682* (0.039)		0.9039* (0.020)	
Log-Likelihood	-73.992		-192.019		-110.727		-83.695		-68.503		-117.075	
AIC	-147.859		-383.912		-221.329		-167.266		-136.880		-234.026	
BIC	-147.646		-383.699		-221.116		-167.053		-136.667		-233.813	

PANEL B												
	High Dependence Regime											
Gaussian Copula												
MSCI Equity Index	0.5198 (0.326)	0.3480	0.2644 (0.530)	0.1703	0.6537 (6.066)	0.4536	0.8969* (0.241)	0.7084	0.9073* (0.340)	0.7237	0.4440 (0.900)	0.2929
Barclays Bond Index	0.2384 (0.243)	0.1532	0.5814* (0.120)	0.3950	0.6238 (62.114)	0.4288	0.5704 (0.704)	0.3864	0.7364* (0.199)	0.5270	0.0414 (0.107)	0.0264
S&P GS Commodity Index	0.1243 (0.238)	0.0793	0.5557 (0.890)	0.3751	0.5991 (0.206)	0.4089	0.3110 (1.003)	0.2013	0.7213* (0.230)	0.5129	0.6225 (0.870)	0.4278
	Low Dependence Regime											
Asymmetric Copula												
MSCI Equity Index	13.382* (0.772)	0.9253	2.2835* (0.194)	0.5621	1.3404 (20.810)	0.2540	1.6024* (0.304)	0.4448	0.7035* (0.119)	0.2602	2.3293* (0.093)	0.5707
Barclays Bond Index	32.2399* (3.491)	0.9690	1.0276* (0.441)	0.0269	1.2013 (5.285)	0.1676	0.1512 (0.076)	0.0703	0.0687 (0.187)	0.0332	1.1553* (0.032)	0.1344
S&P GS Commodity Index	4.7029 (0.450)*	0.7874	1.1619* (0.105)	0.1393	1.0386 (58.181)	0.0372	0.1639* (0.050)	0.0757	0.0741 (0.035)	0.0357	1.1320* (0.023)	0.1166
P_{11}	0.9481* (0.074)		0.9176* (0.067)		0.6192 (4.266)		0.6247* (0.021)		0.5828* (0.047)		0.5367 (0.434)	
P_{22}	0.4221 (0.991)		0.9674* (0.077)		0.8241 (1.862)		0.9420* (0.010)		0.8888* (0.018)		0.9099* (0.345)	
Log-Likelihood	-146.416		-120.586		-163.824		-93.797		-34.483		-164.0097	
AIC	-292.708		-241.047		-327.524				-68.842			
BIC	-292.4948		-240.834		-327.311				-68.629			

Notes: This table presents the results of the canonical vine copulas model for the dependence structure between hedge fund strategies and the main conventional capital markets (equity market, bond market and commodity market). As proxy of the three markets the MSCI World index, the Barclays Bond index and the S&P500 GS commodity index are used. We presents only the results in the first layer of the canonical vine system that account for the dependences between hedge fund strategies and the three capital markets. For the strategies of Macro, Emerging Markets, Equity Hedge and the Composite Hedge Fund index we select the rotated Gumbel copula for the low dependence regime. While, for the strategies Event Driven and Relative Value, we select the Clayton copula. Figures in (·) are standard errors. T* denotes significance at 5% level and τ is the Kendall coefficient. For the correlation coefficient p in the case of Gaussian copula the Kendall τ is obtained by the relation: τ=2arcsin(p)/π. For the Clayton copula coefficient θ the Kendall τ is obtained by the relation: τ= θ/ (2+θ). And, for the rotated Gumbel copulas the following relation is used in the computation of Kendall coefficient: τ= 1 – 1/θ. For each estimated model the log likelihood, the Akaike information criterion (AIC) and the Bayesian information criterion (BIC) are reported.

5. Conclusions

The empirical relationship between hedge funds and traditional asset classes is one of the most interesting issues in modern asset management practice. In this framework, this study has investigated the dependence structure of core hedge fund classes / strategies and traditional capital markets. Our empirical interest has focused on the assessment of potential asymmetric dependence structures between these fundamental instruments. For that, we have employed a copula methodology, a dynamic approach that has only recently been incorporated in quantitative asset management research. In this paper, the joint distributions of hedge fund and conventional market returns were modelled by multivariate regime switching copula models. Different possible specifications of canonical vine copulas were tested on two regimes that reflect high and low dependence structures. Canonical vine copulas represent a flexible way of expressing multivariate joint distributions into bivariate conditional copulas. In order to employ the copula-based models, the appropriate specification of the marginal distributions had to be considered first. For that purpose, we have considered a *GARCH*-type model with an assumed skewed Student-*t* distribution of innovations. Based on our empirical results, the *GARCH*-skewed Student-*t* model appears to adequately capture the temporal dependencies of the univariate series under study, such as autocorrelation and time-varying volatility. The estimated results produced by the multivariate regime switching copula models provide evidence of asymmetric dependence between hedge fund strategies and equity markets. Furthermore, our empirical results reveal a high dependence regime with high estimated correlations and a low dependence regime with lower estimated correlations. Since empirical literature on these issues remains thin, further research would be useful for investors, fund managers and other market practitioners.

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6. Appendix

6.1 Elliptical Copulas

The multivariate case of the Gaussian (normal) copula derives from the multivariate Gaussian distribution. The functional form of Gaussian copula that first defined by Lee (1983) is given by the following function:

$$C^G(u_1, u_2, ..., u_n; \Sigma) = \Phi_\Sigma\left(\Phi^{-1}(u_1), \Phi^{-1}(u_2), ..., \Phi^{-1}(u_n)\right)$$

Where Φ^{-1} represents the inverse of standard normal cumulative distribution and Φ denotes the standard multivariate normal cumulative distribution:

$$\Phi_\Sigma(x_1, ..., x_n) = \int_{-\infty}^{x_1} ... \int_{-\infty}^{x_n} \frac{1}{(2\pi)^{n/2} \, |\Sigma|^{1/2}} \exp\left(-\frac{1}{2} u' \Sigma^{-1} u\right) du$$

Where $u = (u_1, ... , u_n)$ and Σ is a symmetric, semi-definite positive matrix with ones on the diagonal and off diagonal elements the correlations that are bounded between -1 and 1. The density function of Gaussian copula is:

$$c^G(u_1, ..., u_n) = |\Sigma|^{-1/2} \exp\left[-\frac{1}{2}\left(x' \Sigma^{-1} x - x' x\right)\right]$$

Where $x = (\Phi^{-1}(u_1), \ldots , \Phi^{-1}(u_n))$.

Almost in the same way as the Gaussian copula, the multivariate distribution function of the student-t copula is defined as:

$$C_v^t(u_1, ..., u_n; \Sigma) = t_{\Sigma, v}\left(t_v^{-1}(u_1), ..., t_v^{-1}(u_n)\right)$$

Where t_v^{-1} is the inverse of the cumulative density function (*cdf*) of the student-t distribution function and v is the degree of freedom. If parameter v, that measures the heaviness of the tails, is smaller than 3 then the variance does not exist, while, if $v < 5$ the fourth moment does not exist. The multivariate t-distribution function $t_{\Sigma, v}$ is given as:

$$t_{\Sigma, v}(x_1, ...,) = \int_{-\infty}^{x_1} ... \int_{-\infty}^{x_n} \frac{\Gamma\left(\frac{v+n}{2}\right)}{\Gamma\left(\frac{v}{2}\right)\sqrt{(\pi v)^n} \, |\Sigma|} \left(1 + \frac{v' \Sigma^{-1} v}{v}\right)^{\frac{-v+n}{2}} dv$$

Where $u = (u_1, ... , u_n)$ and Γ is the gamma function and Σ is a symmetric, semi-definite positive matrix with ones on the diagonal and off diagonal elements the correlations that are bounded between -1 and 1. Comparing with the Gaussian copula, the student's t Copula introduced an additional parameter, namely the degree of freedom v. The density function of student-t copula is given as follows:

$$c_v^t(u_1, ..., u_n; \Sigma) = \frac{\Gamma\left(\frac{v+n}{2}\right)}{\Gamma\left(\frac{v}{2}\right)\sqrt{(\pi v)^n} \, |\Sigma|} \frac{1}{\prod_{i=1}^n f_v\left(t_v^{-1}(u_i)\right)} \left(1 + \frac{x' \Sigma^{-1} x}{v}\right)^{\frac{-v+n}{2}} dv$$

Where $x = (\Phi^{-1}(u_1), ... , \Phi^{-1}(u_n))$, v is the degree of freedom with $v > 2$, $f_v(\cdot)$ is the density function of the student-t distribution.

6.2 Archimedean Copulas

One of the most important class of copulas (except from the elliptical copulas) is the Archimedean family that initially named by the Ling (1965). The Archimedean copulas which accumulate a number of fruitful properties[12], firstly find a wide range of application in the field of actuarial science (Frees and Valdez, 1998). Contrary to the elliptical copulas, Archimedean copula functions are able to capture asymmetric tail dependence between time series. This is a basic advantage over the financial applications where the dependence of the losses across financial markets is stronger than the gains. The definition of an *n*-dimensional Archimedean copula is defined as follows:

Definition (Archimedean copula): *Let φ be a continuous strictly decrease, convex function mapping from [0, 1] onto [0, ∞], and that $\varphi(1) = 0$. Such a function is called generator, while the above conditions ensure that there is the inverse of function φ defined as φ^{-1}.*

If the pseudo-inverse of φ^{-1} that is written as $\varphi^{[-1]}$, defined as follows:

$$\phi^{[-1]}(t) = \begin{cases} \phi^{-1}(t) & 0 \le t \le \phi(0) \\ 0 & \phi(0) \le t \le +\infty \end{cases}$$

then the function

$$C(u_1, \dots, u_n) = \varphi^{-1}(\varphi(u_1) + \dots + \varphi(u_n))$$

is an n-dimensional Archimedean copula with generator φ.

The necessary condition for the construction of a multivariate Archimedean copula is that the generator φ is a strict copula generator ($\varphi : [0, 1] \rightarrow [0, \infty]$) as proposed by Kimberling (1974), although it is not sufficient. A key characteristic of Archimedean copulas is that all the information about the *n*-dimensional dependence structure is contained in a univariate generator φ. It follows a brief presentation in a multivariate setting of the most important Archimedean copula functions, the Clayton copula, Gumbel copula, Frank copula. These copula functions are capable to deal with different dependence structures between financial series. Specifically, Clayton copula can capture higher dependence at left tails, Gumbel copula models higher dependence at right tails and Frank copula gives symmetric probability mass in the tails.

Clayton Copula

The Clayton copula that was developed by Kimeldorf and Sampson (1975) and independently studied by Clayton (1978) is also referred to as Cook and Johnson (1981) copula. An expression of the *n*-dimensional Clayton copula

$$C_C(u_1, \dots, u_n; \theta) = \left(u_1^{-\theta} + \dots + u_n^{-\theta} - n + 1\right)^{-1/\theta}$$

or equivalent

$$C_C(u_1, \dots, u_n; \theta) = \left[\sum_{i=1}^{n} u_i^{-\theta} - n + 1\right]^{-\frac{1}{\theta}}$$

[12] As reported by the Nelsen (2006, Chapter 4) the popularity of Archimedean copulas is based on the ease way with which they are constructed, many parametric copula functions belong to this class and nice properties possessed by the members of the different Archimedean copulas.

Where $\theta \in (0, \infty)$ is the dependence measure and $(u_1, \ldots, u_n) \in [0, 1]^n$. As the dependence parameter tend to zero ($\theta \to 0$) the marginals approach the independence, while, as θ tend the infinity ($\theta \to \infty$) implies perfect dependence. The Claton copula cannot account for negative dependence. The Clayton copula can generate asymmetric dependence and lower tail dependence, but no upper tail dependence. The density function of Clayton copula is written as

$$c_C(u_1, u_2, \ldots, u_n; \theta) = \frac{\partial^n C}{\partial u_1 \partial u_2 \ldots \partial u_n} = \theta^n \frac{\Gamma\left(\frac{1}{\theta} + n\right)}{\Gamma\left(\frac{1}{\theta}\right)} \left(\prod_{i=1}^{n} u_i^{-\theta-1}\right) \left(\sum_{i=1}^{n} u_i^{-\theta} - n + 1\right)^{-\frac{1}{\theta} - n}$$

Where Γ indicates the Euler Γ function.

Gumbel Copula

Gumbel copula is also an asymmetric copula, but it mainly captures dependence in the upper (positive) tail rather than lower. The n-dimensional version of Gumbel copula can be written as:

$$C_{GU}(u_1, u_2, \ldots, u_n; \theta) = \exp\left(-\left[(\ln(u_1))^{\theta} + (-\ln(u_2))^{\theta} + \ldots + (-\ln(u_n))^{\theta}\right]^{1/\theta}\right)$$

or equivalent

$$C_{GU}(u_1, u_2, \ldots, u_n; \theta) = \exp\left(-\left[\sum_{i=1}^{n} (-\ln u_i)^{\theta}\right]^{1/\theta}\right)$$

Where the dependence measure θ is defined on the range $(1, \infty)$. When θ is equal to 1, it implies independence while for values of θ than tend to ∞ correspond to Frechet upper bound. The bivariate Gumbel copula can be expressed as:

$$c_{GU}(u, v; \theta) = \frac{C_{GU}(u, v; \theta) \cdot (\log u \log v)^{\theta-1}}{uv\left((-\log u)^{\theta} + (-\log v)^{\theta}\right)^{2-\frac{1}{\theta}}} \left(\left((-\log u)^{\theta} + (-\log v)^{\theta}\right)^{\frac{1}{\theta}}\right) + \theta - 1$$

Frank Copula

The n-dimensional Frank copula (Frank, 1979) takes the form

$$C_F(u_1, u_2, \ldots, u_n; \theta) = -\frac{1}{\theta} \log\left[1 + \frac{\left(e^{-\theta u_1} - 1\right)\left(e^{-\theta u_2} - 1\right) \ldots \left(e^{-\theta u_n} - 1\right)}{\left(e^{-\theta} - 1\right)^{n-1}}\right]$$

or equivalent

$$C_F(u_1, u_2, \ldots, u_n; \theta) = -\frac{1}{\theta}\left[1 + \frac{\prod_{i=1}^{n}\left(e^{-\theta u_i} - 1\right)}{\left(e^{-\theta} - 1\right)^{n-1}}\right]$$

where θ represents the dependence measure and is restricted on the region $(0,\infty)$ [13]. Values for the coefficient θ close to ∞ indicate perfect positive dependence while for θ = 0 implies independence. Compared to the Gaussian copula, Frank copula is suitable to capture strong positive dependence rather than tail dependences. Thus, Frank copula assigns a lower probability to joint extreme co-movements than a Gaussian copula. The density function of a bivariate Frank copula is written as

$$C_F(u,v;\theta) = \frac{-\theta\left(e^{-\theta}-1\right)e^{-\theta(u+v)}}{\left[\left(e^{-\theta}-1\right)+\left(e^{-\theta u}-1\right)\left(1-e^{-\theta v}-1\right)\right]^2}$$

[13] Restricted our analysis in a bivariate setting (n = 2), Frank copula is the only copula function that could take into account negative dependence between the marginals. Thus, for $\theta \to -\infty$ the Frank copula indicates perfect negative dependence between the marginals.